Title: Quasi-Topological Quantum Error Correction Codes

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Abstract: <p>Existing proposals for topological quantum computation have encountered
difficulties in recent years in the form of several ``obstructing" results.
These are not actually no-go theorems but they do present some serious
obstacles. A further aggravation is the fact that the known topological
error correction codes only really work well in spatial dimensions higher
than three. In this talk I will present a method for modifying a higher
dimensional topological error correction code into one that can be embedded
into two (or three) dimensions. These projected codes retain at least some
of their higher-dimensional topological properties. The resulting subsystem
codes are not discrete analogs of TQFTs and as such they evade the usual
obstruction results. Instead they obey a discrete analog of a conformal
symmetry. Nevertheless, there are real systems which have these features,
and if time permits I'll discuss some of these. Many of them exhibit
strange low temperature behaviours that might even be helpful for
establishing finite temperature fault tolerance thresholds.

This research is still very much a work in progress... As such it has
numerous loose ends and open questions for further investigation. These
constructions could also be of interest to quantum condensed matter
theorists and may even be of interest to people who like weird-and-wonderful
spin models in general.</p>

Quasi-topological quantum error correction codes

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Outline

Introduction and motivation

A geometrical trick

Features of the error correction code

What might this allow us to do?

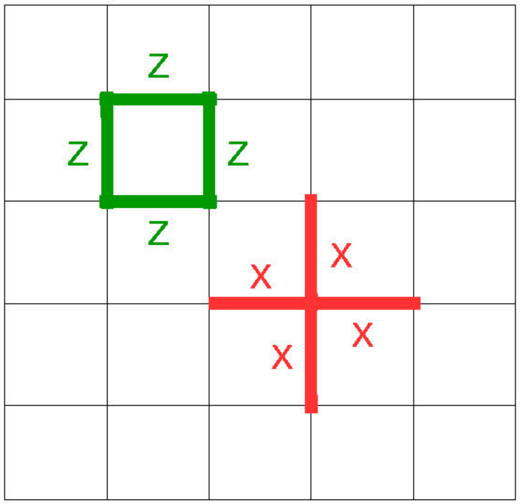
Mimicking higher dimensional topological behaviours

Finite temperature thresholds and naturally occurring systems

...which can have some unexpected properties: toric bosons?!

Summary and open questions

Introduction and motivation



Topological error correction codes

Known problems with these

Dimensionality

Thermally stable in 4D (Dennis et al, JMP 2002)

Entanglement percolation transition in 5-6D
(Hastings et al, PRL 2014)

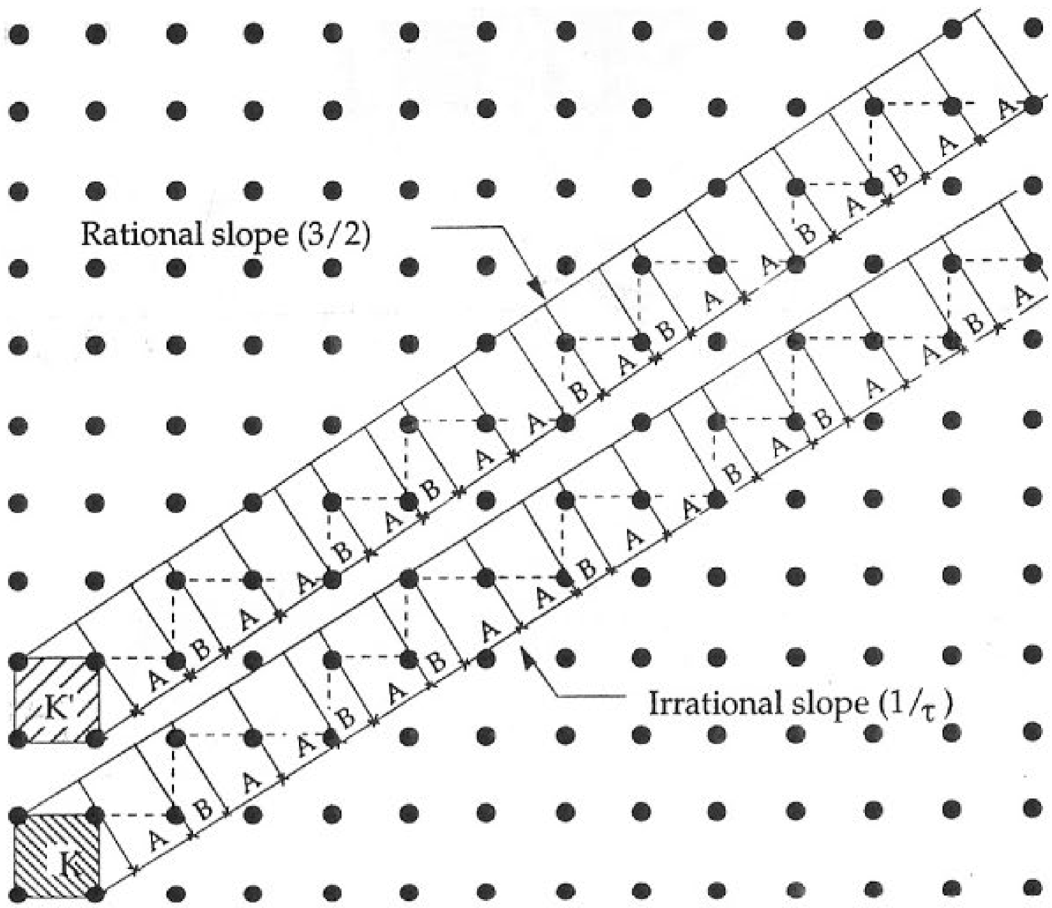
Gate errors

Beverland et al., JMP 2016

A geometrical trick, I

Note how the Quantum Monte Carlo simulations saw data collapse at about L=3, for D=5-6

Use a “cut and project” at an irrational angle.



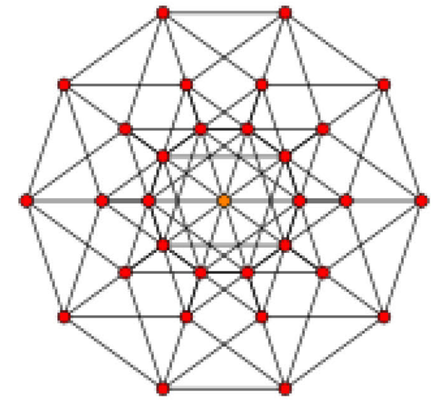
These are generally defined to be one lattice spacing wide...

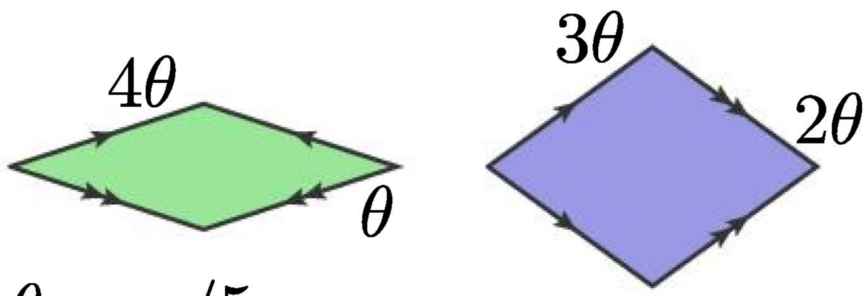
(but they don’t have to be!)

Maciá, Rep. Prog. Phys. 2005

A geometrical trick, II

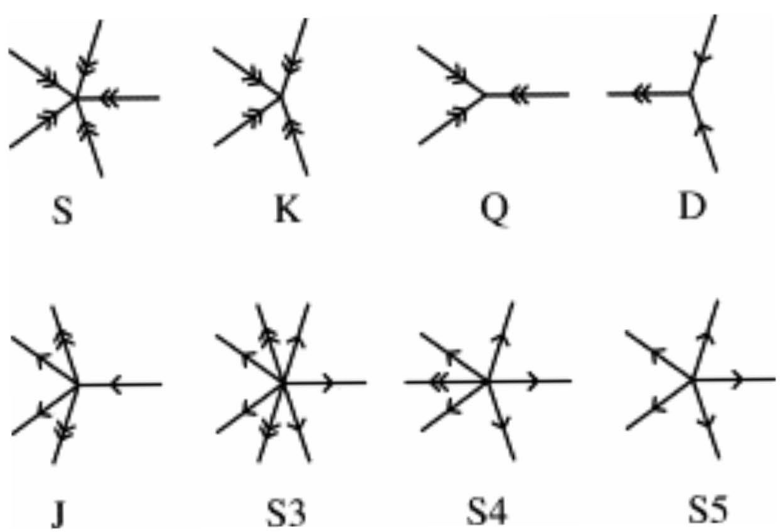
If we starting from a 5D hypercubic lattice and project to 2D, we get a Penrose tiling:

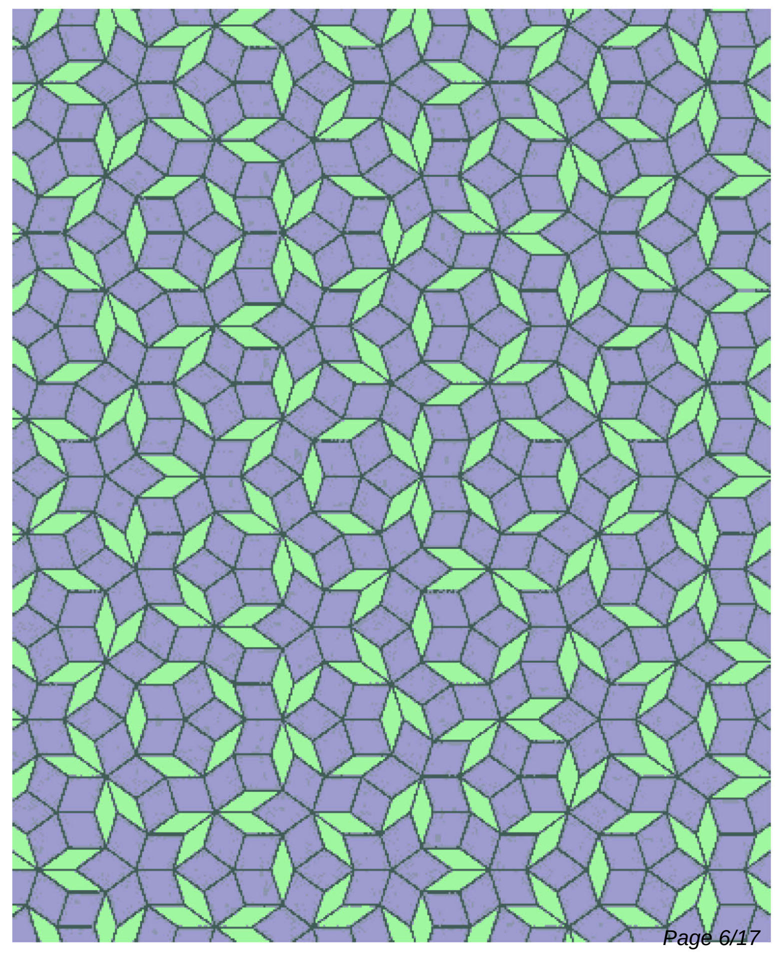




$$\theta = \pi/5$$

Allowed vertices:

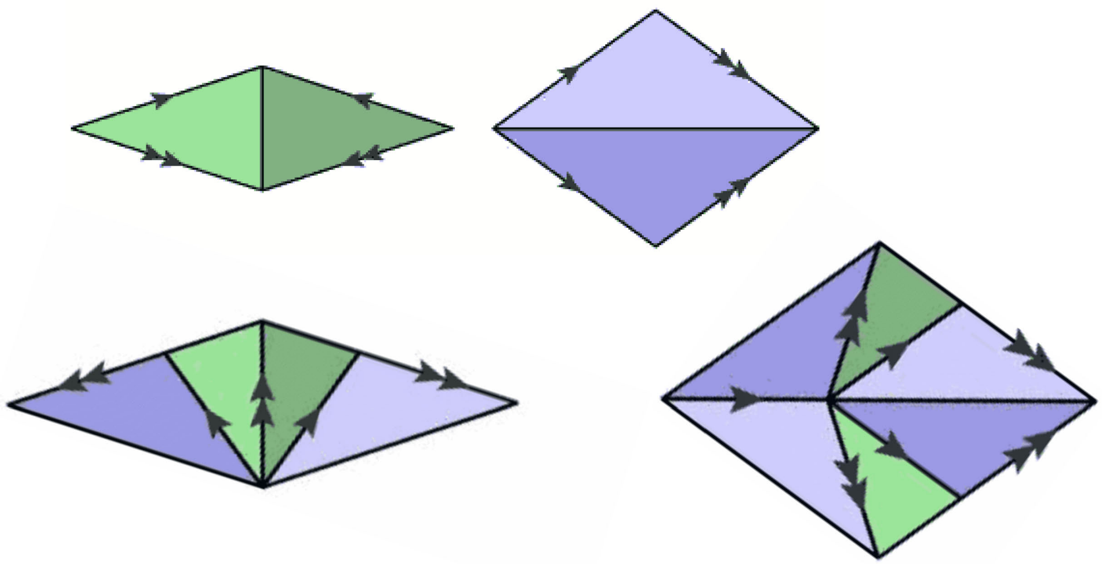


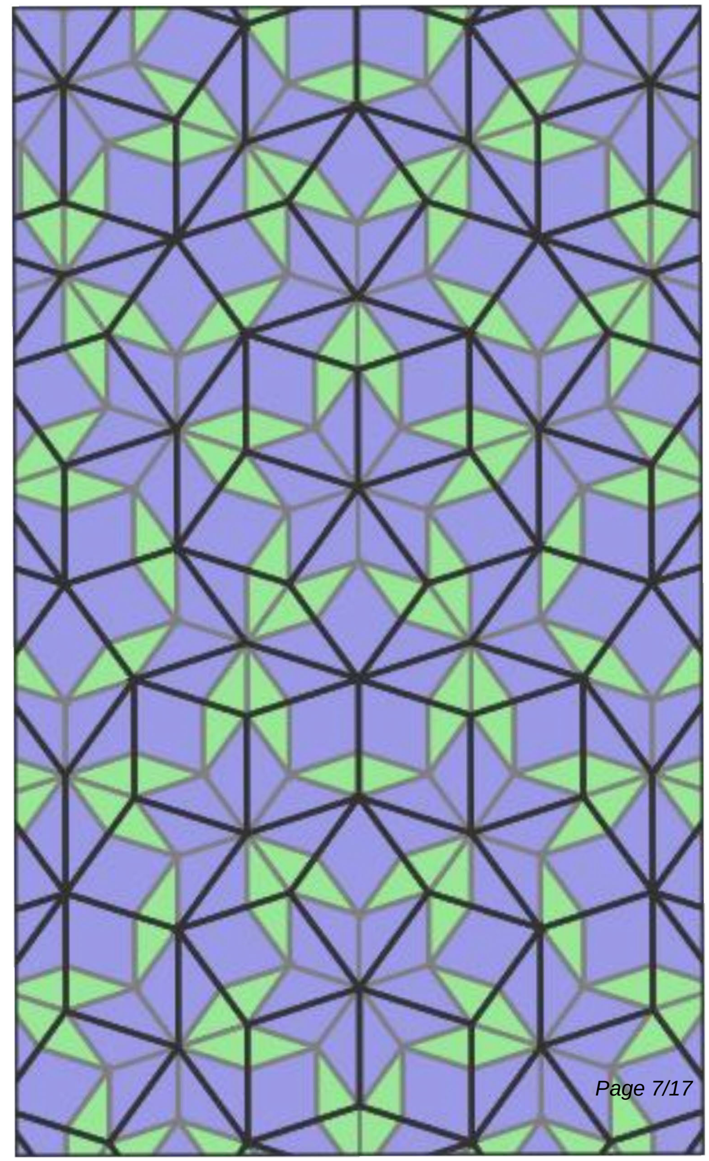


http://www.ams.org/samplings/feature-column/fcarc-penrose

A geometrical trick, III

Penrose tilings obey a discrete version of a conformal symmetry, known as deflation rules:

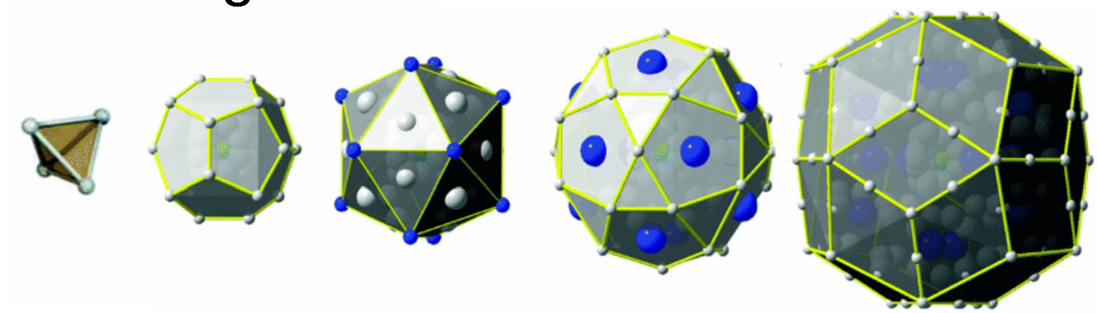




Tiling is neither translation nor rotation invariant, so does not respect Lorentz symmetry

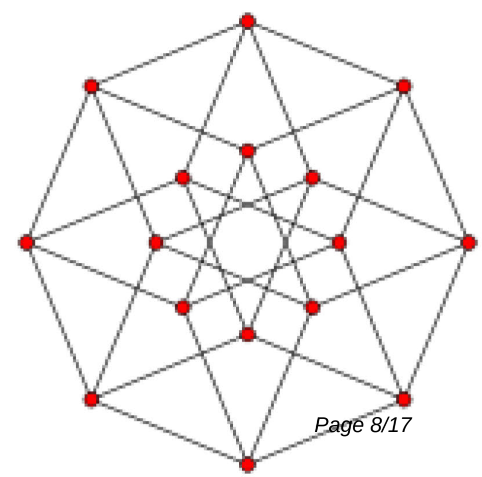
A geometrical trick, IV

If we had started from 6D and projected down into 3D, we would have obtained an icosahedral tiling:



In fact we can construct a Penrose tiling starting from a 4D hypercubic lattice, provided we switch to a face-centred lattice, also known as the A_4 "root lattice"

Put lattice sites in the middle of all the faces:



Then pick a 4D code if you can find one that fits...

Discrete Conformal vs. Lorentz symmetry

A fact about fundamental groups

$$\pi_1[SO(p,q)] = \pi_1[SO(p)] \times \pi_1[SO(q)]$$

Conformal symmetry group is $SO(n+1,2)$

$\pi_1(SO^+(p, q))$	$p = 1$	$p = 2$	$p \geq 3$	**n**
$q = 1$	C_1	Z	C_2	Lorentz
$q = 2$	Z	Z × Z	Z × C_2	Conformal
$q \geq 3$	C_2	C_2 × Z	C_2 × C_2	

Choosing the code I

Will assume (for now) that we need to start with either a 5D simple hypercubic lattice, or a 4D face-centred hypercubic lattice.

(We are not guaranteed to get a Penrose tiling just because we started out with 5 dimensions and then projected down to 2 at the correct angle!)

There are several candidates. Look at more than one, as some Have potential problems...

1. The 4D toric code from (Dennis et al., JMP 2002)

The qubits are located at the centres of the faces of a hypercubic lattice. (However the corners are empty, which could complicate things later...)

Pros: low generator weight (6 and 6) and has a thermal stability proof in 4D (but that may not survive the projection step)

Cons: doesn't occupy the corners in the A_4 lattice

Choosing the code II

2. Some 5D toric code (e.g. 5D X-star, 4D Z-plaquette)

   Pros: occupies all the edges in the 5D hypercubic lattice

   Cons: ridiculously high generator weight:
   X-star: 10, Z-plaquette: 32
   (may lose some of that at the projection step, but see later)
   Don't know if it's thermally stable at finite T.

3. Some "A_4" code (that I just made up)

   Z-plaquettes are 3-cubes with qubits at the centre of all the faces *and* at all the corners (weight = 6+8 = 14)

   X-stars are 4 orthogonal intersecting 2x2 planes with qubits at the centre of all the faces *and* one at the centre, but none on the edges of the planes (weight = 4x4+1 = 17)

   Pros: occupies all the nodes in the A_4 lattice

   Cons: high generator weight; don't know about thresholds

Logic operators after the projection step

Both the natively 4D codes have logic operators that are surfaces in the original space. Think of the code as living on a higher dimensional torus: the logic operators have non-trivial winding. (If the code is unwrapped, need to follow the boundary rules.)

(*The 5D toric code will need its logical-$\overline{X}$ to be a volume.)

Will be left with the intersection of the operator and the remaining surface.

Can choose these to be stripes and patches for the 4D codes. (The 5D toric code might need a full surface for one of its $\overline{X}$s.)

Stripe-like logic operators are what you want to see (string-like ones cause problems for thermal stability).

Patches don't look so good: those qubits are more vulnerable to noise. They are now "gauge" qubits, and our code is a Subsystem code. (That's actually a good thing for fault tolerance!)

An unexpected subtlety...

Let’s play hunt the plaquette!

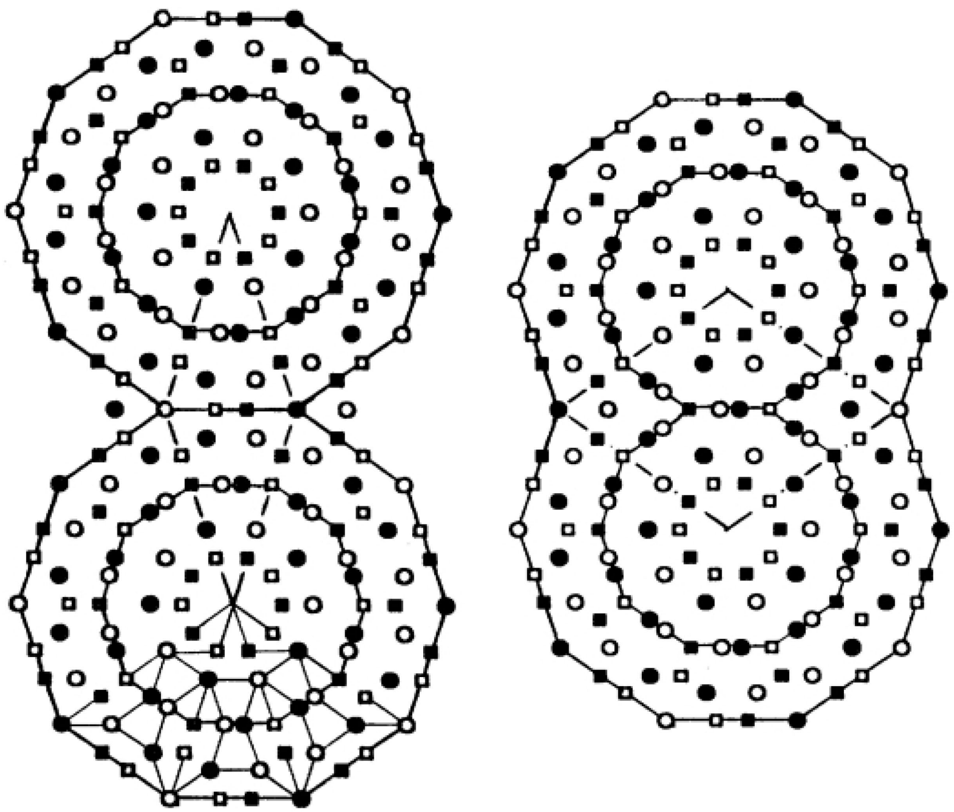
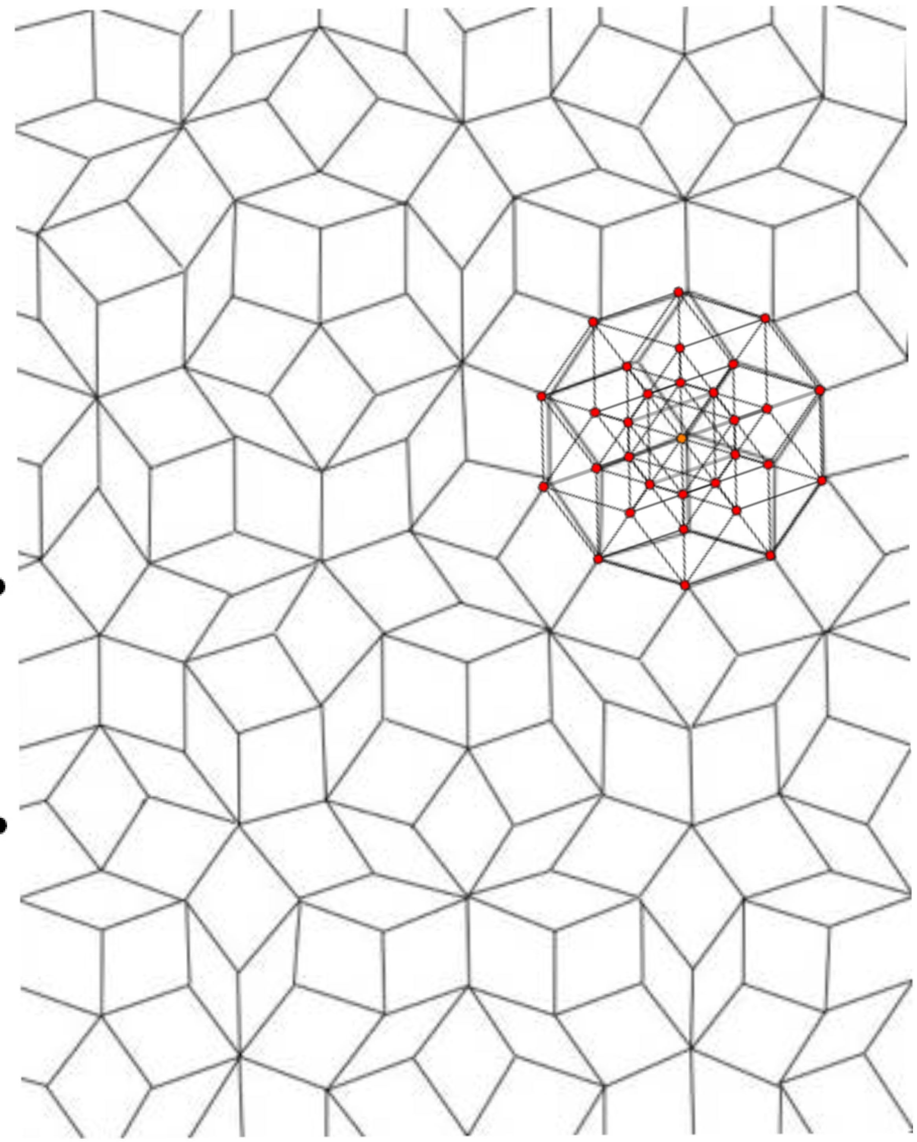


FIG. 1. Two allowed overlappings of decagonal clusters and corresponding tiles. A portion of the decagonal network hosting the atoms is shown. Circles, Al; squares, transition metal; open symbols, $z = 0$; solid symbols, $z = c/2$ layer.

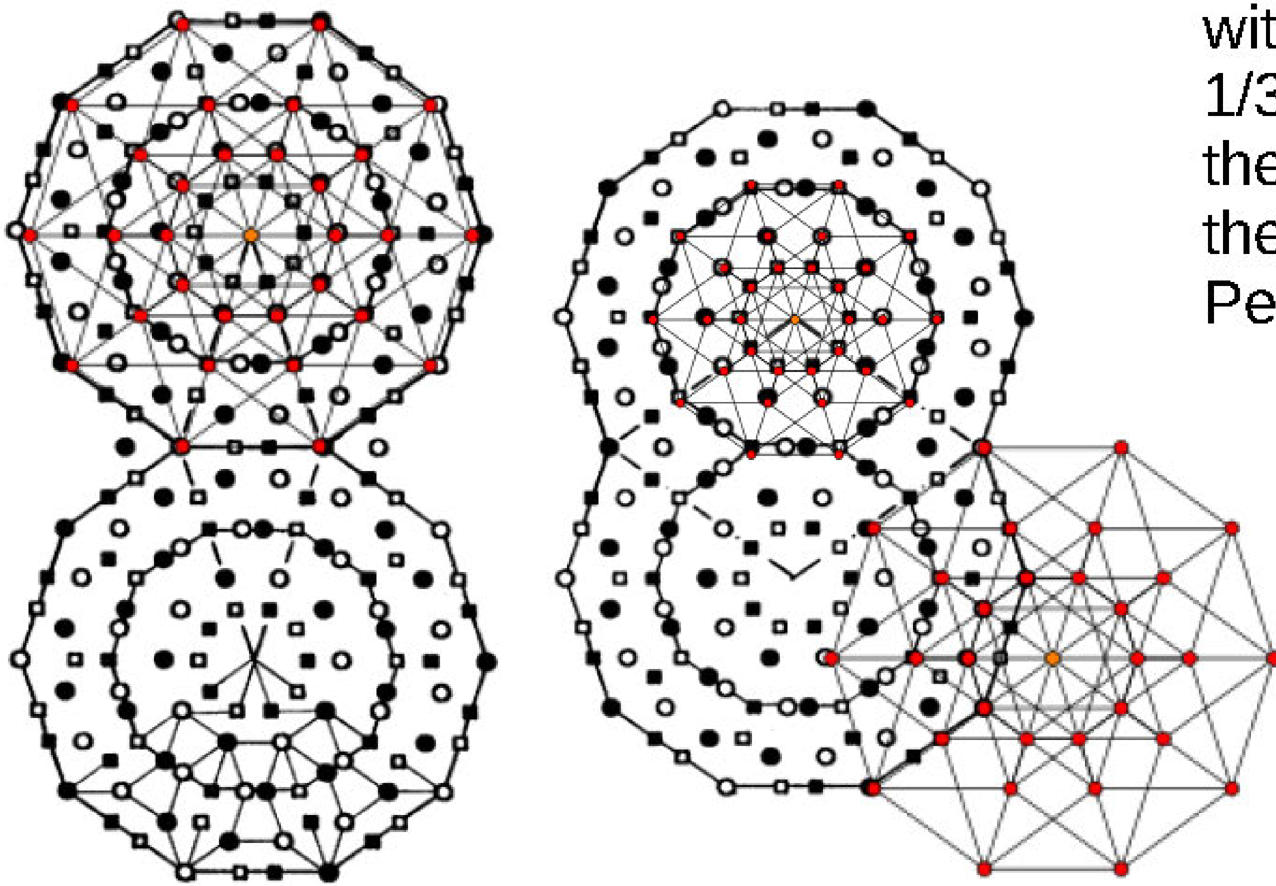
[S.E. Burkov, PRL, 1992]



$Al_{65}Cu_{20}Co_{15}$ and $Al_{70}Ni_{15}Co_{15}$

Too many hypercubes!

There are intact hypercubes with edge-lengths 1/3 and 2/3 the edge-length of the smallest Penrose tiling!



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[S.E. Burkov, PRL, 1992]

First put your lattice on a torus, then...

A lot of the standard results start with this step!

What happens if we try anyway?

5-fold topological phenomena!

If we drag these things around each other, we're going to Pick up phases that are multiples of π/5

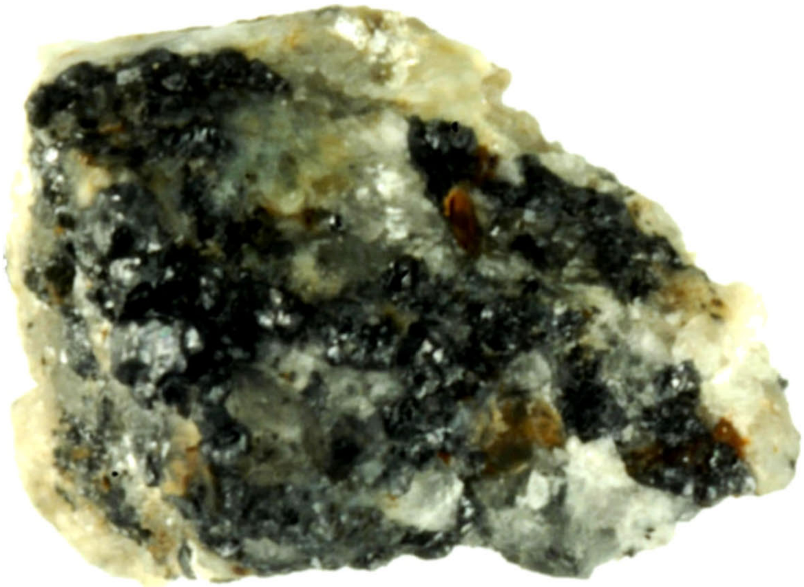
Breaking the bi-coloring may make those phases non-Abelian....

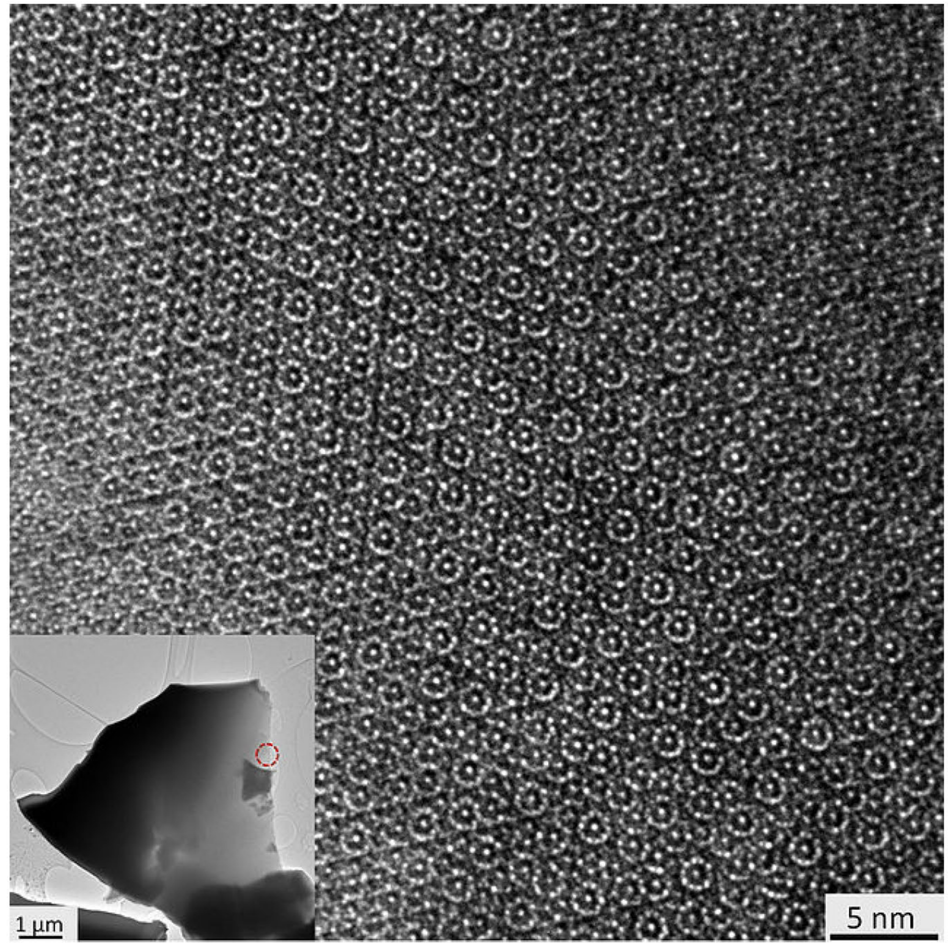
...we might start seeing braiding matrices like this:

$$\sigma_1 = \left(\begin{array}{cc|c} e^{-i4\pi/5} & 0 & \\ 0 & e^{i3\pi/5} & \\ \hline & & e^{i3\pi/5} \end{array}\right)$$

$$\sigma_2 = F^{-1}\sigma_1 F = \left(\begin{array}{cc|c} -\tau e^{-i\pi/5} & \sqrt{\tau}e^{-i3\pi/5} & \\ \sqrt{\tau}e^{-i3\pi/5} & -\tau & \\ \hline & & e^{i3\pi/5} \end{array}\right)$$

The Khatyrka meteorite





Contained (metastable) decagonal quasicrystal $Al_{71}Ni_{24}Fe_5$