Title: Lorentzian quantum cosmology

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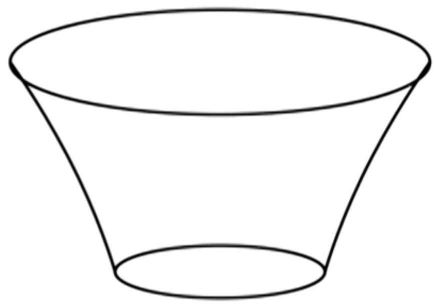
Abstract: Using Picard-Lefschetz theory we show that the Lorentzian path integral forms a good starting point for quantum cosmology which avoids the conformal factor problem present in Euclidean gravity. We study the Lorentzian path integral for a homogeneous and isotropic model with a positive cosmological constant. Applied to the â€œno-boundaryâ€• proposal, we show that this leads to the inverse of the result obtained by Hartle and Hawking. Including a inflation field, the Lorentzian path integral prefers to start at the 'top of the hill' leading to good initial conditions for slow roll inflation. However, when including gravitons the fluctuations seem to be unstable.

Lorentzian Quantum Cosmology



Job Feldbrugge

Neil Turok, Jean-Luc Lehners

June 1, 2017

JF, Jean-Luc Lehners, Neil Turok: arXiv:1703.02076, arXiv:1705.00192

Wick rotations in Quantum Mechanics

The Schrödinger equation

$$H\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

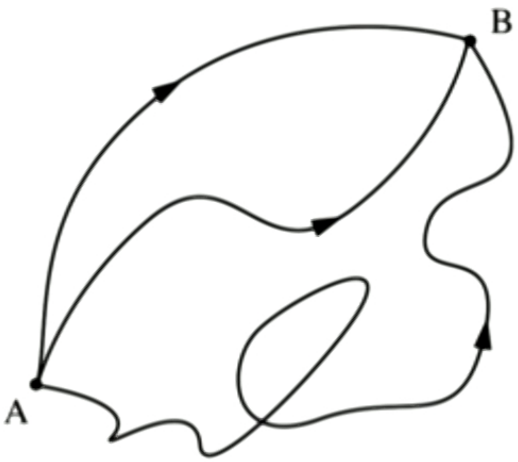
The Feynman path integral

$$G[x_1; x_0; T] = \int_{x_0}^{x_1} \mathcal{D}x \, e^{i \int_{t_0}^{t_1} \left[\frac{m}{2}\dot{x}^2 - V(x)\right] \mathrm{d}t}$$

$$\Psi[x_1, t_1] = \int G[x_1; x_0; T] \Psi[x_0, t_0] \mathrm{d}x_0$$

Wick rotating time, $t \mapsto \pm it$, ensures convergence

$$G[x_1; x_0; T] = \int_{x_0}^{x_1} \mathcal{D}x \, e^{- \int_{t_0}^{t_1} \left[\frac{m}{2}\dot{x}^2 + V(x)\right] \mathrm{d}t}$$



Lorentzian and Euclidean Quantum Gravity

Wheeler-DeWitt equation

$$\mathcal{H}_0 \Psi = 0$$

The Feynman propagator of minisuperspace

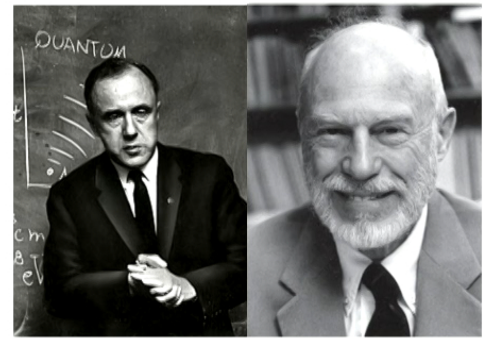
$$G[a_1; a_0] = \int_{0^+}^{\infty} \mathrm{d}N \int_{a=a_0}^{a=a_1} \mathcal{D}a \, e^{iS[N,a]}$$

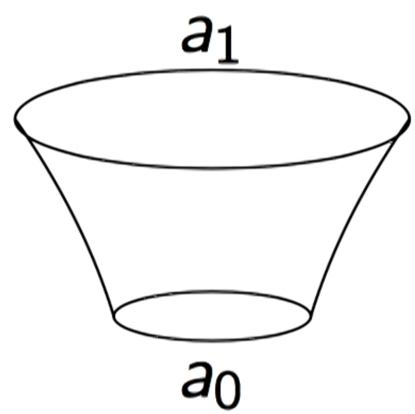
Wick rotating $N \mapsto \pm iN$

$$G[a_1; a_0] = \int_{0^+}^{\infty} \mathrm{d}N \int_{a=a_0}^{a=a_1} \mathcal{D}a \, e^{-\bar{S}[N,a]}$$

Conformal factor problem: $\bar{S}$ not bounded below

Gibbons, Hawking, Perry (1978),
Teitelboim (1982), Halliwell (1988)





De Sitter universe

Using the metric $\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + a(t)^2\mathrm{d}\Omega_3^2$, the Einstein-Hilbert action for a Λ-dominated universe

$$S^{(0)} = 2\pi^2 \int_0^1 \mathrm{d}t\, N \left[-3a\frac{\dot{a}^2}{N^2} + 3ka - a^3\Lambda \right]$$

Redefining the lapse $N \mapsto N/a$, the action is quadratic in $q = a^2$

$$S^{(0)} = 2\pi^2 \int_0^1 \mathrm{d}t \left[-\frac{3}{4N}\dot{q}^2 + N(3k - \Lambda q) \right]$$

The propagator $G[q_1; q_0] \propto \int_{0^+}^{\infty} \frac{\mathrm{d}N}{\sqrt{N}} e^{iS_0[q_1;q_0;N]}$ with the classical action

$$S_0[q_1; q_0; N] = N^3\frac{\Lambda^2}{36} + N\left[3k - \frac{\Lambda}{2}(q_0 + q_1)\right] - \frac{3}{4N}(q_1 - q_0)^2$$

Halliwell & Louko (1989), Brown & Martinez (1990)

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Picard-Lefschetz theory

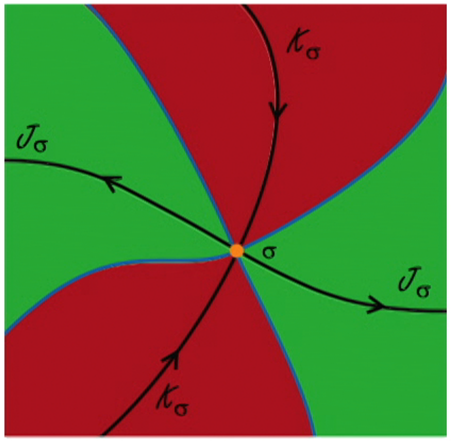
Deform the integration contour

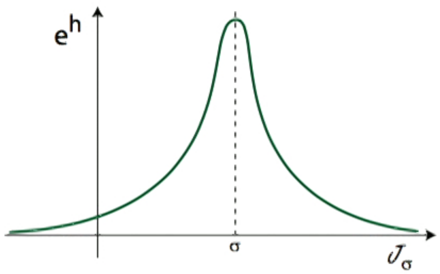
$$G[q_1; q_0] = \int_{0^+}^{\infty} \frac{\mathrm{d}N}{\sqrt{N}} e^{iS_0[q_1;q_0;N]}$$
$$= \int_{0^+}^{\infty} \frac{\mathrm{d}N}{\sqrt{N}} e^{h[q_1;q_0;N]} e^{iH[q_1;q_0;N]}$$

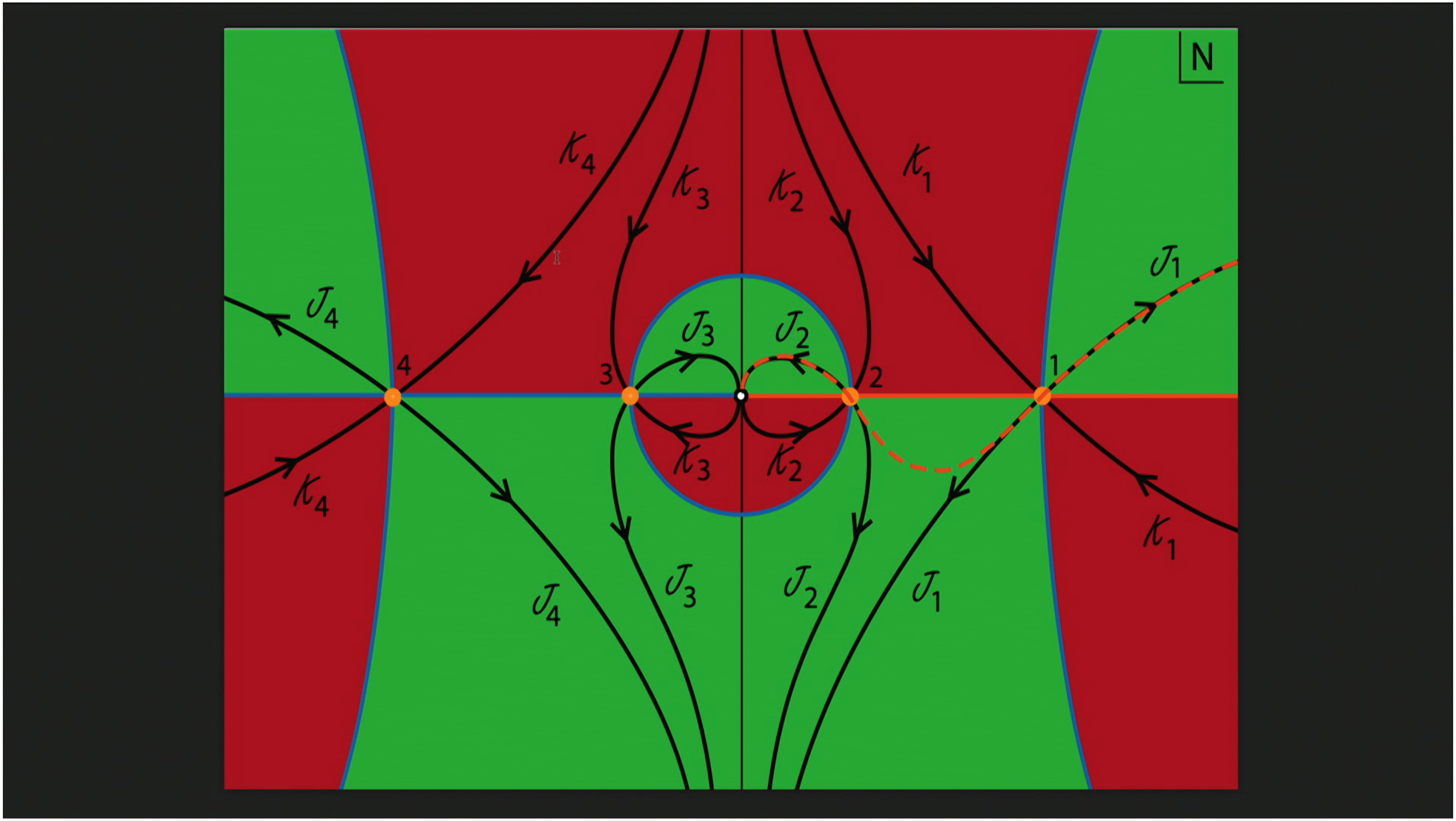
with $h = \mathrm{Re}[iS_0]$ and $H = \mathrm{Im}[iS_0]$

- Lefschetz thimbles $\mathcal{J}_\sigma$: Lines of steepest descent of h and constant H
- Include thimble when corresponding curves of steepest ascent $\mathcal{K}_\sigma$ intersect original contour

Arnol'd et al. (2012), Witten (2011)



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No-boundary proposal

Theory of initial conditions:
No-boundary proposal by Hartle & Hawking.

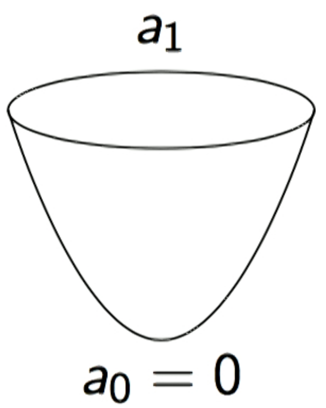
- Space-time is spherical $k = 1$ and started out Euclidean with initial condition

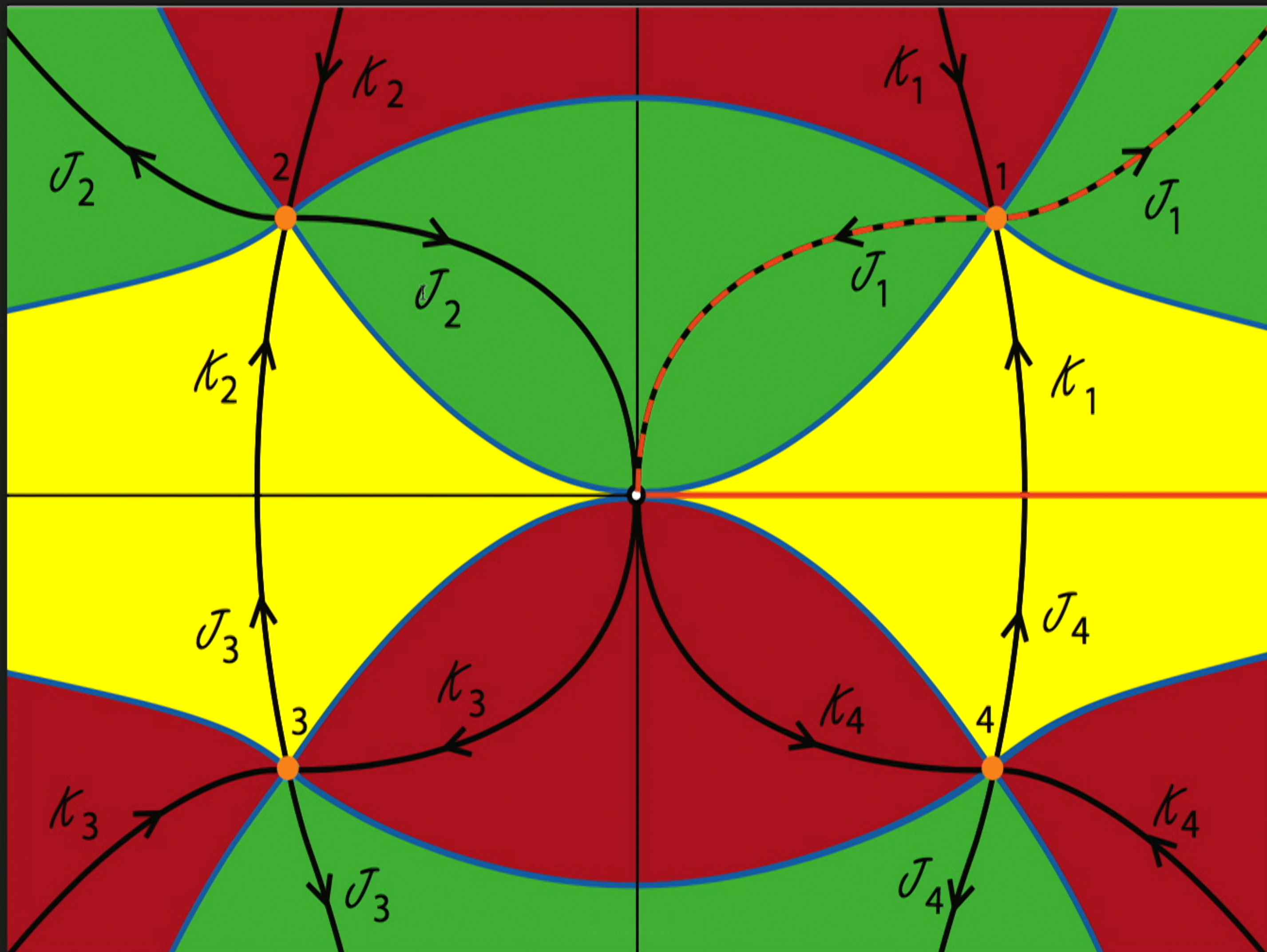
$$a_0 = 0$$

- Starting from Euclidean gravity deform to saddle point
- Weighting of the initial condition

$$|G[a_1; 0]|^2 \propto e^{12\pi^2/(\hbar\Lambda)}$$

Hartle & Hawking (1983), Vilenkin (1982)





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No-boundary proposal

Weighting *no-boundary proposal*:

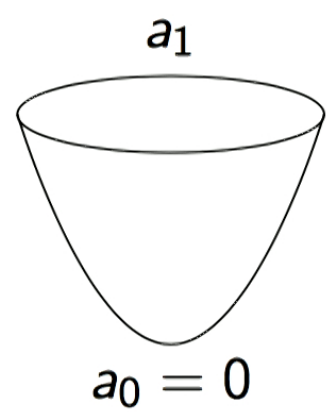
- Hartle-Hawking result

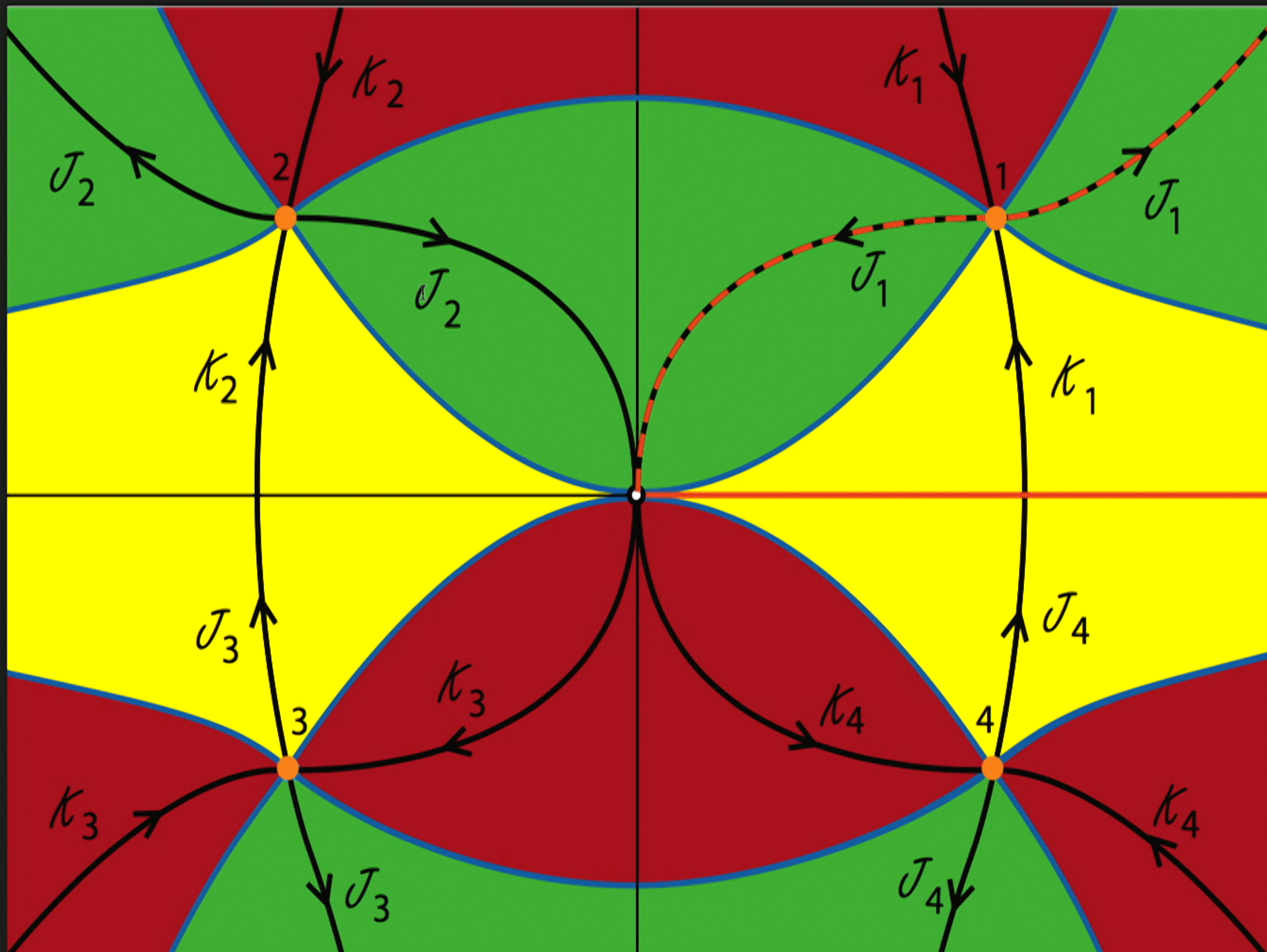
$$|G[a_1;0]|^2 \propto e^{12\pi^2/(\hbar\Lambda)}$$
$$\propto e^{12\pi^2/(\hbar V(\varphi))}$$

- Picard-Lefschetz theory result

$$|G[a_1;0]|^2 \propto e^{-12\pi^2/(\hbar\Lambda)}$$
$$\propto e^{-12\pi^2/(\hbar V(\varphi))}$$

- Same result as Vilenkin's tunneling proposal
- Lorentzian gravity $\neq$ Euclidean gravity





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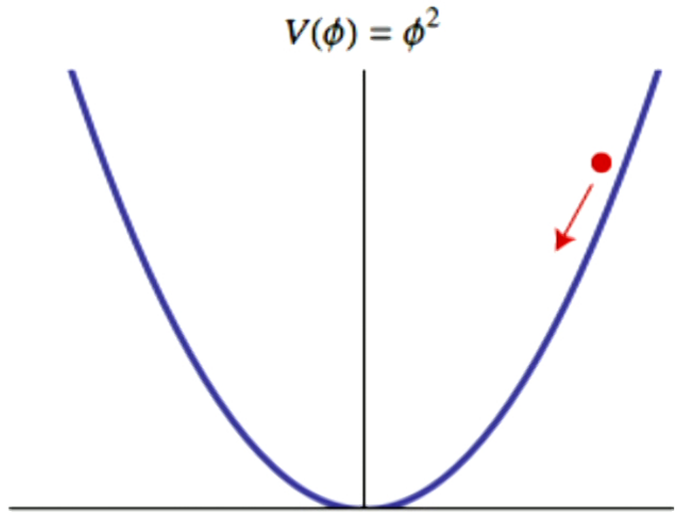
Inflaton potential

Top v.s. bottom of the hill



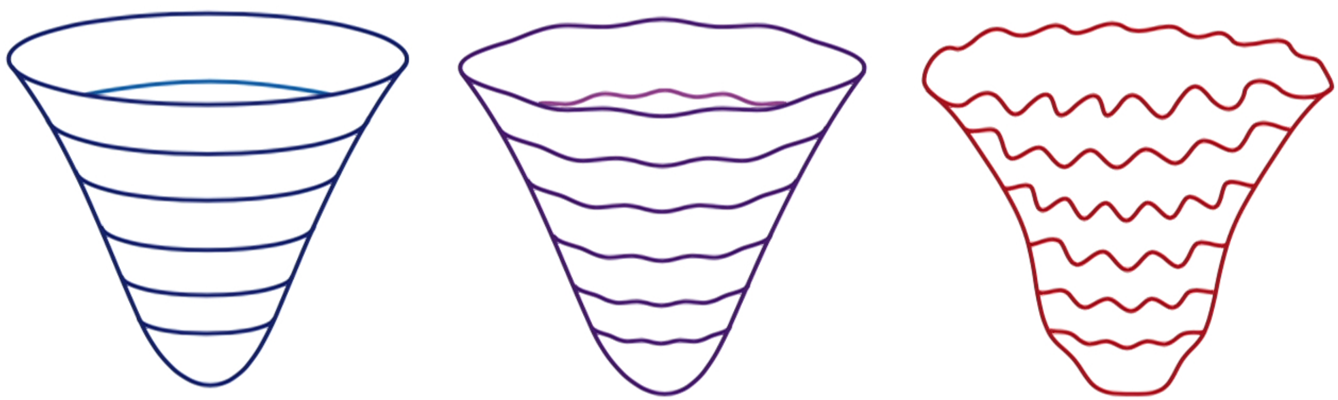
Gravitational waves in no-boundary proposal

$$S = 2\pi^2 \int_0^1 \left[-\frac{3}{4N}\dot{q}^2 + N(3 - \Lambda q) \right] \mathrm{d}t + \frac{1}{2}\int_0^1 \left[q^2 \frac{\dot{\phi}^2}{N} - l(l+2)\phi^2 \right] \mathrm{d}t \quad (1)$$

Result:

$$\text{P-L:} \quad |G[q_1, \phi_1; 0, 0]|^2 \propto e^{-\frac{12\pi^2}{\hbar\Lambda} + \frac{l(l+1)(l+2)}{2\hbar H^2}\phi_1^2}$$

$$\text{H-H:} \quad |G[q_1, \phi_1; 0, 0]|^2 \propto e^{+\frac{12\pi^2}{\hbar\Lambda} - \frac{l(l+1)(l+2)}{2\hbar H^2}\phi_1^2}$$



Gravitational waves in adiabatic equation of state

Adiabatic equation of state, $q = a^2$ as time variable

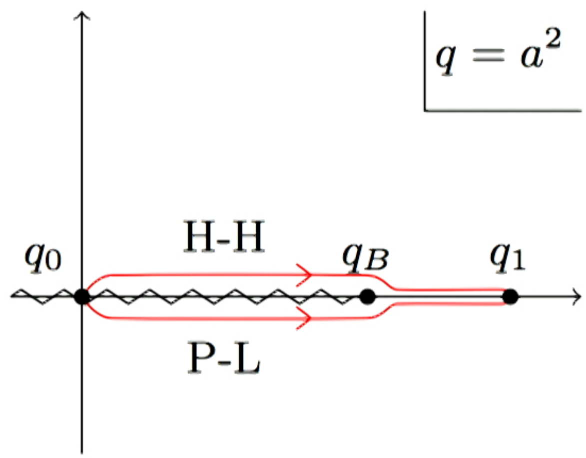
$$ds^2 = -\frac{dq^2}{4q\left(\frac{1}{3}\rho(q)q - 1\right)} + q\,d\Omega_3^2, \quad iS^{(0)} = -6\pi^2 i \int_0^{q_1} dq\sqrt{\rho\, q/3 - 1}$$

The quantum phase gives the real part

$$\mathrm{Re}[iS^{(0)}] = -6\pi^2 i \int_0^{q_B} dq\sqrt{\rho\, q/3 - 1}$$

The classical phase the imaginary part

$$\mathrm{Im}[iS^{(0)}] = -6\pi^2 i \int_{q_B}^{q_1} dq\sqrt{\rho\, q/3 - 1}$$



Gravitational waves in adiabatic equation of state

Adiabatic equation of state, $q = a^2$ as time variable

$$ds^2 = q(\eta)\left[-d\eta^2 + d\Omega_3^2\right], \quad d\eta = dq/(2q\sqrt{\rho q/3 - 1}), \quad q(0) = q_B$$

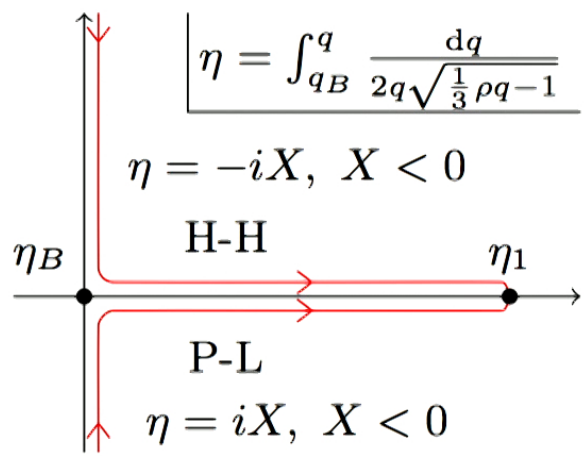
The perturbations $\chi = a\phi$ in terms of η,

$$iS^{(2)} = i\pi^2 \int_0^{\eta_1} d\eta \left[(\chi_{,\eta})^2 + U(\eta)\chi^2\right]$$

The perturbations χ in terms of X,

$$-S_E^{(2)} = \pi^2 \int_{-\infty}^{0} dX \left[\chi'^2 + U(X)\chi^2\right]$$

with positive U. Anti-Gaussian weighting



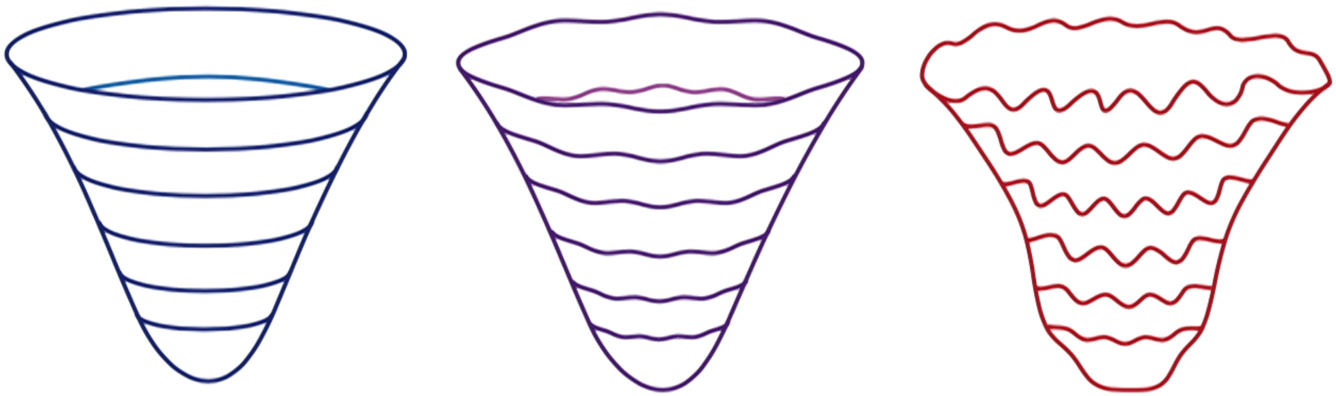
Gravitational waves in no-boundary proposal

$$S = 2\pi^2 \int_0^1 \left[-\frac{3}{4N}\dot{q}^2 + N(3 - \Lambda q) \right] \mathrm{d}t + \frac{1}{2}\int_0^1 \left[q^2 \frac{\dot{\phi}^2}{N} - l(l+2)\phi^2 \right] \mathrm{d}t \tag{1}$$

Result:

$$\text{P-L:} \quad |G[q_1, \phi_1; 0, 0]|^2 \propto e^{-\frac{12\pi^2}{\hbar\Lambda} + \frac{l(l+1)(l+2)}{2\hbar H^2}\phi_1^2}$$

$$\text{H-H:} \quad |G[q_1, \phi_1; 0, 0]|^2 \propto e^{+\frac{12\pi^2}{\hbar\Lambda} - \frac{l(l+1)(l+2)}{2\hbar H^2}\phi_1^2}$$



Conclusion

Summary:

- Euclidean Quantum Cosmology: *conformal factor problem*
- Picard-Lefschetz theory: unambiguous evaluation Lorentzian integral
- Lorentzian quantum gravity $\neq$ Euclidean quantum gravity

No-boundary proposal:

- Inverse of Hartle-Hawking result
- On the top of the hill
- Gravitational waves have anti-Gaussian distribution
- De Sitter space nucleated from a no boundary proposal is unstable