

Title: The $i\epsilon$ prescription in the SYK model

Date: May 25, 2017 02:30 PM

URL: <http://pirsa.org/17050096>

Abstract:

In this talk I will discuss the SYK model from a quantum field theoretical perspective and present an $i\epsilon$ prescription which regularizes the divergences of the model.

IE in SYK models

INTERESTING CFT

$$1) G(\tau_1 - \tau_2) = \frac{1}{|\tau_1 - \tau_2|^{2\Delta}}$$

$$\tilde{G}(\omega) = \int d\tau \frac{1}{|\tau|^{2\Delta}} e^{i\omega\tau} \sim$$

2) MARGINAL IR POWER COUNTING

3) ASSUME 2-BODY INTERACTION ($2 \geq 3$)

$$G = \frac{c}{\omega} + \frac{c}{\omega} \text{ (diagram) } \frac{c}{\omega} + \frac{c}{\omega} \text{ (diagram) } \frac{c}{\omega} + \dots$$

IR FEYNMAN GRAPH

E external vertices

$$E - V + 1$$

$$\int \Omega^{(2D-1)} E \int \Omega E - V + 1$$

$$\sim \int \Omega^{2DE - V}$$

$$2E = 2V$$

$$\sim \int \Omega^{(2-1)V}$$

$$\Delta S = 1, \Delta = 1/2$$

$(2D-1)$

3) ASSUME 2-BODY INTERACTION ($2 \geq 3$)

$$G = \frac{C}{\uparrow \text{free propagator}} + \frac{C}{\text{circle with diagonal lines}} \frac{C}{\text{circle with diagonal lines}} + \frac{C}{\text{circle with diagonal lines}} \frac{C}{\text{circle with diagonal lines}} \frac{C}{\text{circle with diagonal lines}} + \dots$$

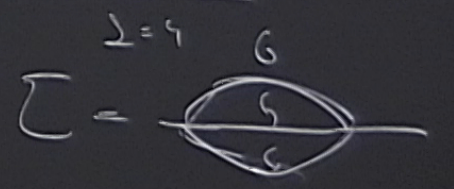
$\Sigma = \text{1PI self-energy function}$

$$G = \frac{1}{C^{-1} - \Sigma}$$

↳ free action $\sim W^2 \gg 0$
 (quadratic part)

$$\Sigma \sim W^{-1 + \frac{2}{d}}$$

IR \rightarrow $1 = -G \Sigma$



$$1 = -G \cdot \left[\int \frac{d^d k}{(2\pi)^d} G(k) \right]$$

VECTOR SYK

VECTOR MAJORANA FERMION χ_a $a=1 \dots M$

$$\frac{1}{2} \int d\tau \sum_a \bar{\chi}_a \partial_\tau \chi_a + T_{a_1 a_2} \int d\tau \chi_{a_1}(\tau) \dots \chi_{a_2}(\tau)$$

random couplings, d

$e^{-\frac{N}{d}}$

tion χ_a , $a=1 \dots N$

$$\int d\chi_0 \prod_a d\chi_a \exp\left(-\sum_{a_1, a_2} T_{a_1 a_2} \chi_{a_1} \chi_{a_2}\right)$$

random couplings,
quenched.

$$dV(T) = e^{-\frac{N^{D-1}}{J^2} \sum_{a_1, a_2} (T_{a_1 a_2})^2}$$

MAJORANA FERMION χ_a $a=1 \dots N$

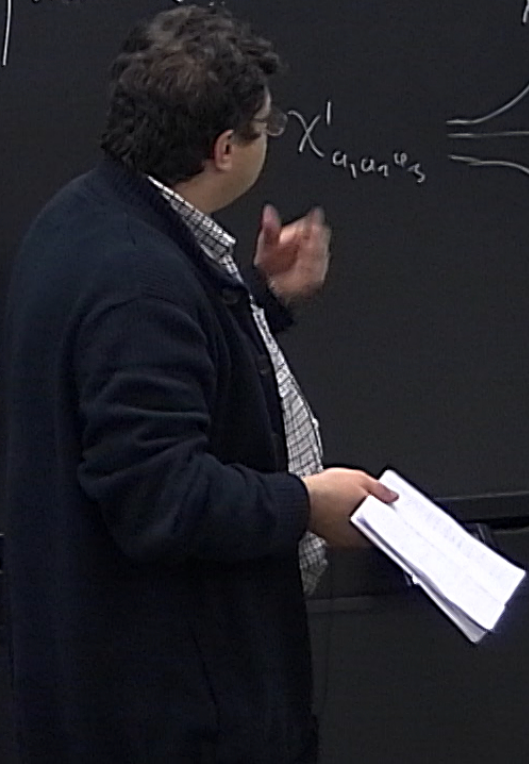
$$\frac{1}{Z} \int d\zeta \sum_a \chi_a \partial_{\bar{z}} \chi_a + T_{a_1 a_2} \int d\zeta \chi_{a_1}(\zeta) \dots \chi_{a_2}(\zeta)$$

random couplings,
quenched.

$$dV(T) = e^{-\frac{N^{D-1}}{J^2} \sum_{a_1, a_2} (T_{a_1 a_2})^2}$$

$$d_{a_1} G(\zeta_1, \zeta_2) = \int dV(T) \frac{\int (d\chi) e^{-S} \chi_{a_1}(\zeta_1) \chi_{a_2}(\zeta_2)}{\int (d\chi) e^{-S}}$$

$$\sum_{a^2=c} \chi_{a^1 a^2-1}^c \chi_{a^1 a^2-1}^c + \int d\bar{z} \chi^1(z) \chi^2(z) \dots \chi^2(z) \chi_{b^1 b^2}^2 \chi_{c^1 c^2}^3 \chi_{d^1 d^2}^3$$



$$\sum_{a^1, c} \chi_{a^1, a^2-1}^c \chi_{a^1, a^2-1}^c + \int d\mathcal{G} \chi^1(\mathcal{G}) \chi^2(\mathcal{G}) \chi^3(\mathcal{G})$$

$$g=4 \quad \chi_{a_1 a_2 a_3}^1 \quad \chi_{b_1 b_2 b_3}^2 \quad \chi_{c_1 c_2 c_3}^3 \quad \chi_{d_1 d_2 d_3}^4$$

$(c_1, c_2) \quad \Sigma \quad \bigcirc$

$$d(\zeta) = \int^2 du \quad G(u-c) (|G(u)|)^{2-1} \quad \text{2 even } 2 \rightarrow 9$$

$$G(u) = b \frac{\text{sign}(u)}{|u|^{2\alpha}}$$

$$= \int^2 dy \frac{\text{sign}(u-c)}{|u-c|^{2\alpha}} \frac{\text{sign}(u)}{|u|^{2\alpha(2-1)}} = \frac{\int^2}{|\zeta|} \left| dx \frac{\text{sign}(x-1)}{|x-1|^{2/\alpha}} \frac{\text{sign}(x)}{|x|^{2(2-1)}} \right.$$

$x = \frac{u}{|c|}$

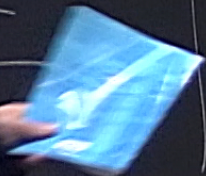
$|u|^{-1}$ $2 \leq u < 9$

1) $\int_{16}^{\infty} \frac{1}{|u|} du$

2) $\int_0^{\infty} dx \frac{1}{x^{2-\frac{2}{3}}}$

divergent integral

x^2

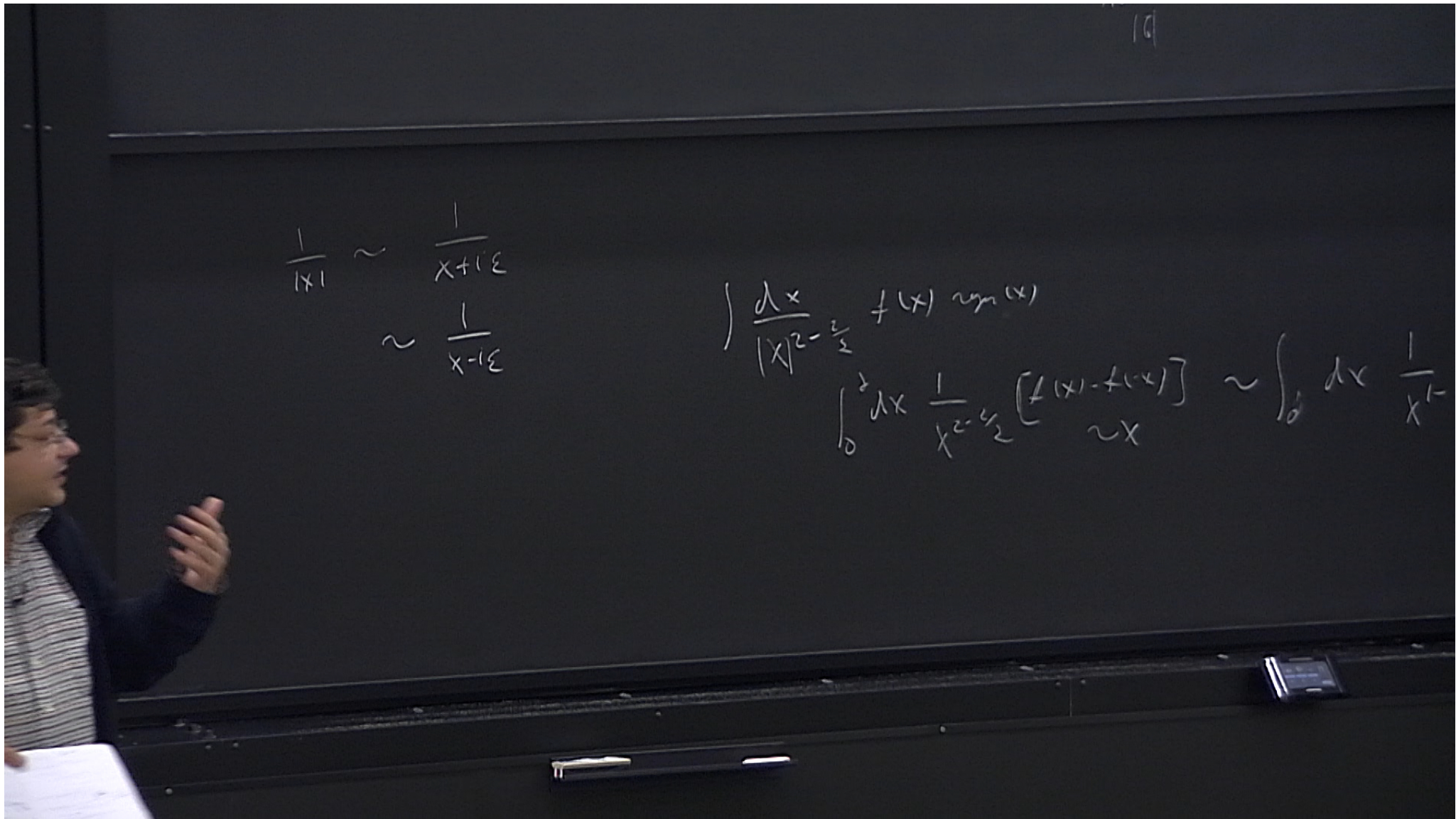


dx

$\frac{\text{negn } (x-1)}{|x-1|^{2/3}}$

$\frac{\text{negn } x}{|x|^{2(2-1)}}$

$x = \frac{u}{|u|}$



$$\frac{1}{|x|} \sim \frac{1}{x+i\epsilon}$$
$$\sim \frac{1}{x-i\epsilon}$$

$$\int \frac{dx}{|x|^{2-\frac{\epsilon}{2}}} f(x) \sim \int_0^\infty dx \frac{1}{x^{2-\frac{\epsilon}{2}}} [f(x) - f(-x)] \sim \int_0^\infty dx \frac{1}{x^{1-\frac{\epsilon}{2}}}$$

$$\frac{1}{|x|} \sim \frac{1}{x+\epsilon}$$

$$\sim \frac{1}{x-\epsilon}$$

$$\int \frac{dx}{|x|^{2-\frac{2}{d}}} f(x) \sim \int dx$$

$$\int_0^2 dx \frac{1}{x^{2-\frac{2}{d}}} [f(x) + f(-x)] \sim \int_0^2 dx \frac{1}{x^{1-\frac{2}{d}}}$$

page 2

ζ in SYK models

$$G^E(\bar{z}) = \frac{b}{2i \sin(\pi \Delta)} \begin{pmatrix} \frac{1}{(\zeta - i\epsilon)^{2\Delta}} & -\frac{1}{(\zeta + i\epsilon)^{2\Delta}} \end{pmatrix}$$

$$\frac{1}{|\zeta + i\epsilon|^{2\Delta}} = e^{-2\Delta \ln(\zeta + i\epsilon)}$$

$$= \frac{1}{\sqrt{\zeta^2 + \epsilon^2}^{2\Delta}} e^{-2\Delta \ln(\zeta + i\epsilon)}$$

$\frac{\pi}{\epsilon} \text{Im}(G)$
 $\frac{2}{\epsilon}$
 $\frac{1}{\zeta}$

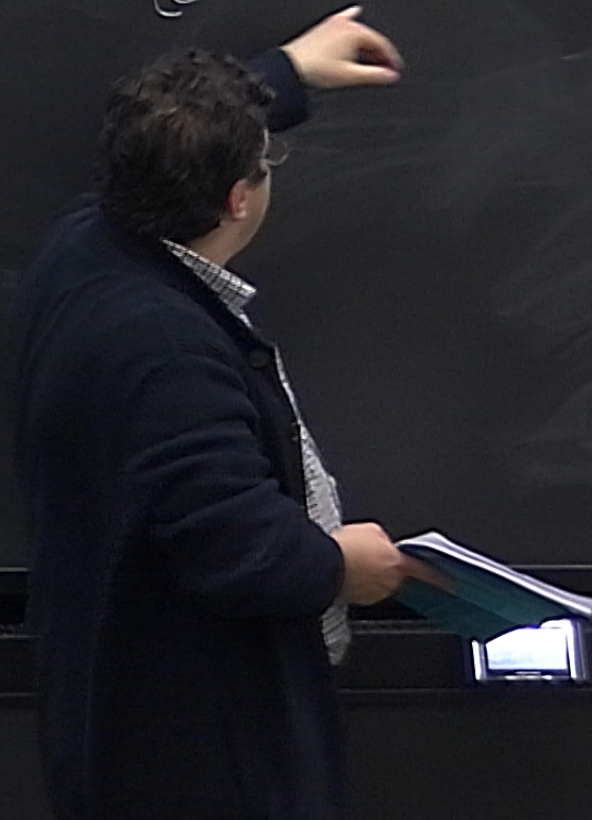
$$A^\varepsilon(z) = J^2 \int du G^\varepsilon(z-u) \left[G(u) \right]_{11}^{\varepsilon-1}$$

$A^\varepsilon(z)$ distribution $\forall \varepsilon > 0$

$$\lim_{\varepsilon \rightarrow 0} A^\varepsilon(z) = \delta(z)$$

$\frac{1}{\varepsilon} \int_{-\varepsilon}^{\varepsilon} f(x) dx$
 $\approx f(0)$
 $\frac{1}{\varepsilon} \int_{-\varepsilon}^{\varepsilon} f(x) dx \approx f(0)$

$$A^{\Sigma}(z) = \left(\int du \left(\frac{1}{(\Sigma - i(u-z))^{2\sigma}} - \frac{1}{(\Sigma + i(u-z))^{2\sigma}} \right) \left(\frac{1}{(\Sigma - iu)^{2\sigma}} \frac{1}{(\Sigma + iu)^{2\sigma}} \right)^{\Sigma-1} \right)$$



$$A^\xi(z) = \left(\right) \int du \left(\frac{1}{(\xi - i(u-2))^{2\alpha}} - \frac{1}{(\xi + i(u-4))^{2\alpha}} \right) \left(\frac{1}{(\xi - iu)^{2\alpha}} - \frac{1}{(\xi + iu)^{2\alpha}} \right)$$

$$\frac{1}{(\xi - iu)^{2\alpha}} \left| \frac{1}{\Gamma(2\alpha)} \int dx x^{2\alpha-1} e^{-x(\xi - iu)} \right.$$

$$A^\xi(z) = 2\pi i \left(\frac{b}{2i\sin\pi\alpha} \right)^2 \frac{1}{\Gamma(2\alpha)} \left[\frac{-1}{\Gamma(2\alpha(2-1))} \left(\frac{1}{2\xi + i\epsilon} + \frac{1}{2\xi - i\epsilon} \right) + \dots \right]$$

$$) \int du \left(\frac{1}{(\xi - i(u-3))^{2\sigma}} - \frac{1}{(\xi + i(u-4))^{2\sigma}} \right) \left(\frac{1}{(\xi - iu)^{2\sigma}} - \frac{1}{(\xi + iu)^{2\sigma}} \right)^{2-1}$$

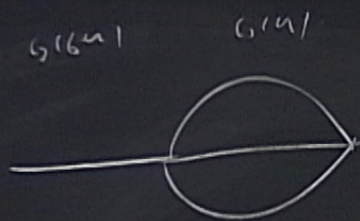
$$) \frac{1}{\Gamma(2\sigma)} \int dx x^{2\sigma-1} e^{-x(\xi - iu)}$$

$$) \frac{1}{\Gamma(2\sigma)} \left[\frac{-1}{\Gamma(2\sigma(2-1))} \left(\frac{1}{2\xi + i\xi} + \frac{1}{2\xi - i\xi} \right) + \sum_{n=1}^{\infty} \binom{2-1}{n} \frac{(-1)^n}{\Gamma(2\sigma n) \Gamma(2\sigma(2-1-n))} \int_0^1 dt \left[\frac{1}{t^{2\xi + i\xi}} + \frac{1}{t^{2\xi - i\xi}} \right] \right]$$

$$\left(\frac{1}{z-i\omega} \right)^{2\Delta} \left(\frac{1}{z+i\omega} \right)^{2\Delta}$$

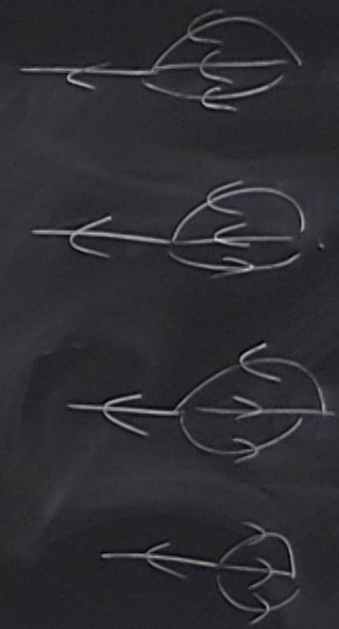
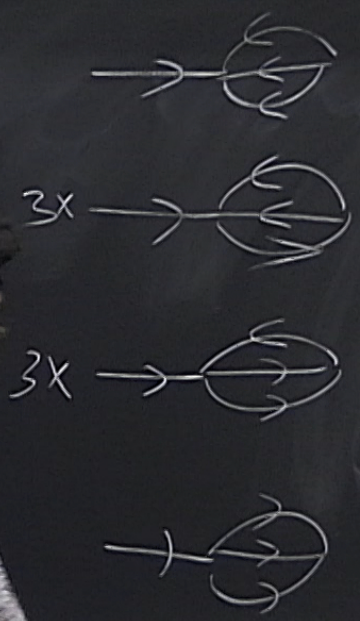
$$\frac{1}{z-i\omega} + \sum_{n=1}^{2\Delta-1} \binom{2\Delta-1}{n} \frac{(-1)^n}{\Gamma(2\Delta-n)\Gamma(2\Delta-1-n)} \int_0^1 dt \left[\frac{1}{t+i\omega} + \frac{1}{t-i\omega} \right] \cdot t^{-2\Delta(2-1)} \left[\begin{matrix} (1-t)^{2\Delta(2-1)-1} & (1-t)^{2\Delta(2-1)} \\ & -(1-t) \end{matrix} \right]$$

$$K_{a_2}(z) = \frac{N^{D-1}}{\Gamma^2} \sum_{a'_1, a_2} (T_{a'_1, a_2})^2$$

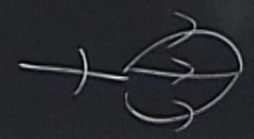
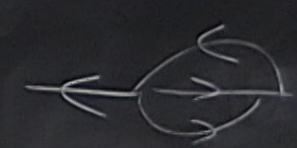
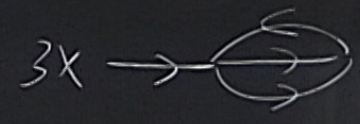
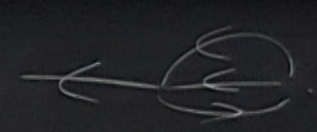
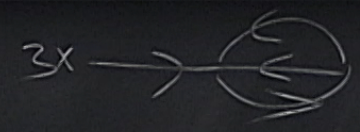
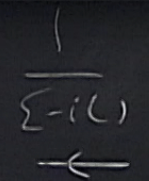
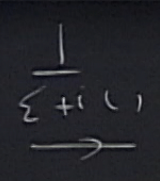


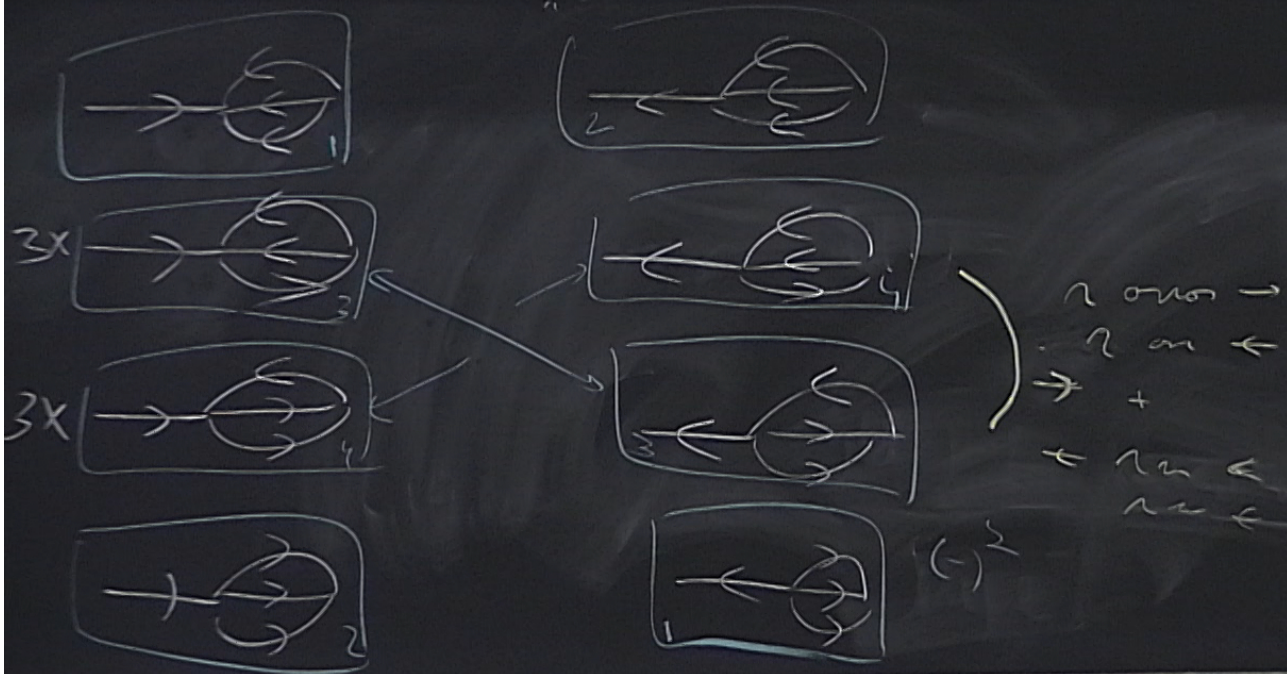
$$\frac{1}{\zeta + i(\epsilon)}$$

$$\frac{1}{\zeta - i(\epsilon)}$$



$$A^E(z) = 2\pi j^2 \left(\frac{b}{z \sin \pi G} \right)^2 \frac{1}{\Gamma(2N)} \left[\frac{-1}{\Gamma(2N-1)} \left(\frac{1}{2z+iG} \oplus \frac{1}{2z-iG} \right) + \sum_{n=1}^{2N-1} \binom{2N-1}{n} \right]$$





$$\left(\frac{1}{(\zeta - i(u-1))^{2\alpha}} - \frac{1}{(\zeta + i(u-1))^{2\alpha}} \right) \left(\frac{1}{(\zeta - iu)^{2\alpha}} - \frac{1}{(\zeta + iu)^{2\alpha}} \right)^{2-1}$$

$$x^{\alpha-1} e^{-x(\zeta - iu)} \quad 2\pi\delta(\zeta)$$

$$\frac{-1}{\Gamma(2\alpha-1)} \left(\frac{1}{2\zeta + i\epsilon} + \frac{1}{2\zeta - i\epsilon} \right) + \sum_{n=1}^{\infty} \binom{2n-1}{n} \frac{(-1)^n}{\Gamma(2n)\Gamma(2\alpha-1-n)} \int_0^1 dt \left[\frac{1}{2\zeta + i\epsilon} + \frac{1}{2\zeta - i\epsilon} \right] 2\pi\delta(\zeta)$$

$$\frac{1}{\epsilon - i0} - \frac{1}{(\epsilon + i0)^{2-1}}$$

$$\lim_{\epsilon \rightarrow 0} A^{\epsilon}(b) = \left(\begin{array}{c} \\ b-1 \end{array} \right) \delta(b)$$

$$\begin{aligned} & \frac{\binom{2-1}{2}}{\Gamma(2) \Gamma(1) \Gamma(2-1-1)} \int_0^1 dt \left[\frac{1}{t+i0} + \frac{1}{t-i0} \right] \cdot \overbrace{2\pi \delta(b)} \\ & \frac{(-1)^1}{\Gamma(2) \Gamma(1) \Gamma(2-1-1)} \int_0^1 dt \left[\frac{1}{t+i0} + \frac{1}{t-i0} \right] \cdot t^{-2\Delta(2-1)} \left[\frac{(1-t)^{2\Delta(2-1)-1}}{t^{-2+2\Delta}} - \frac{(1-t)^{2\Delta-1}}{t^{-1+2\Delta}} \right] \end{aligned}$$