Title: The i epsilon prescription in the SYK model

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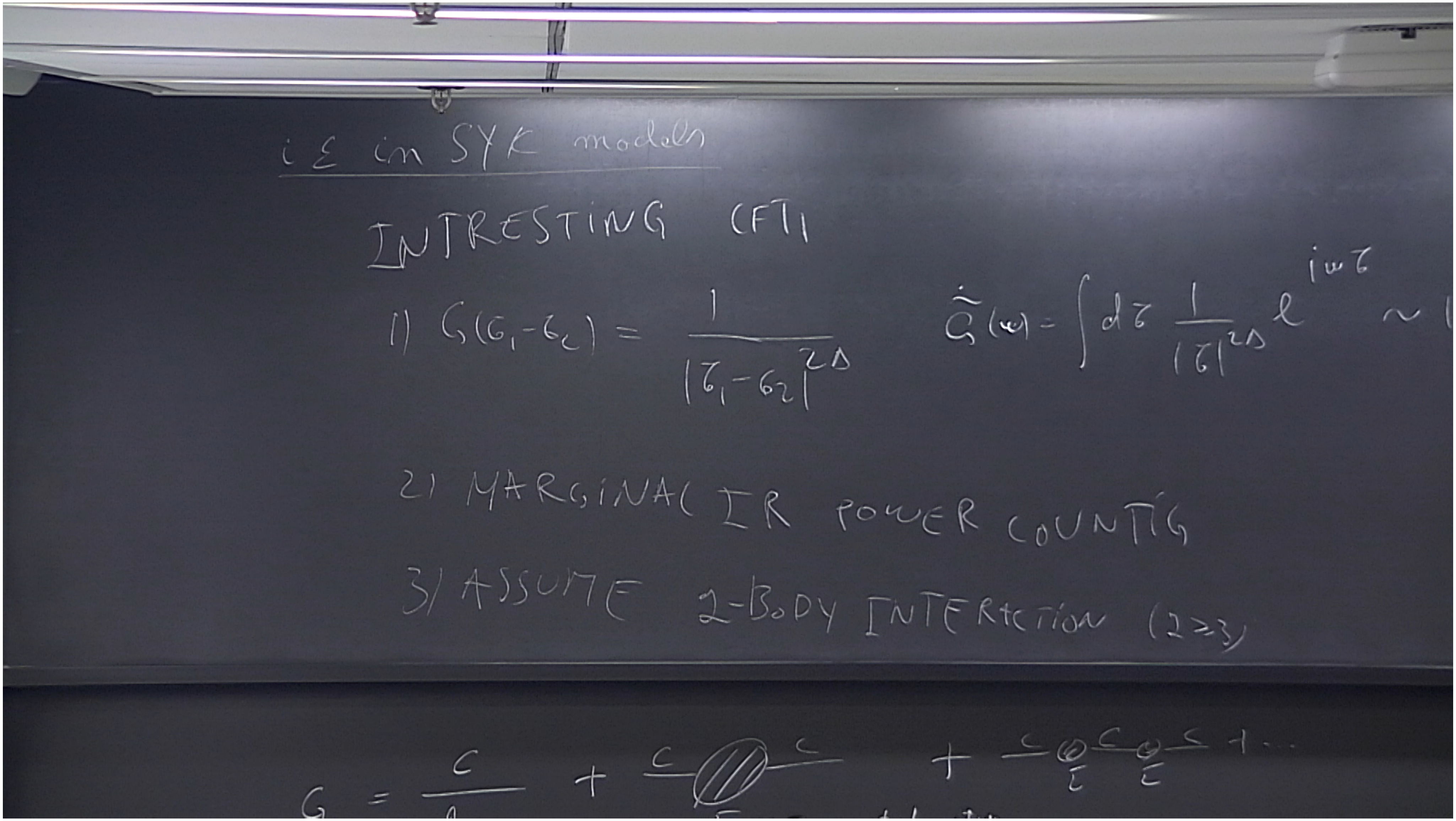
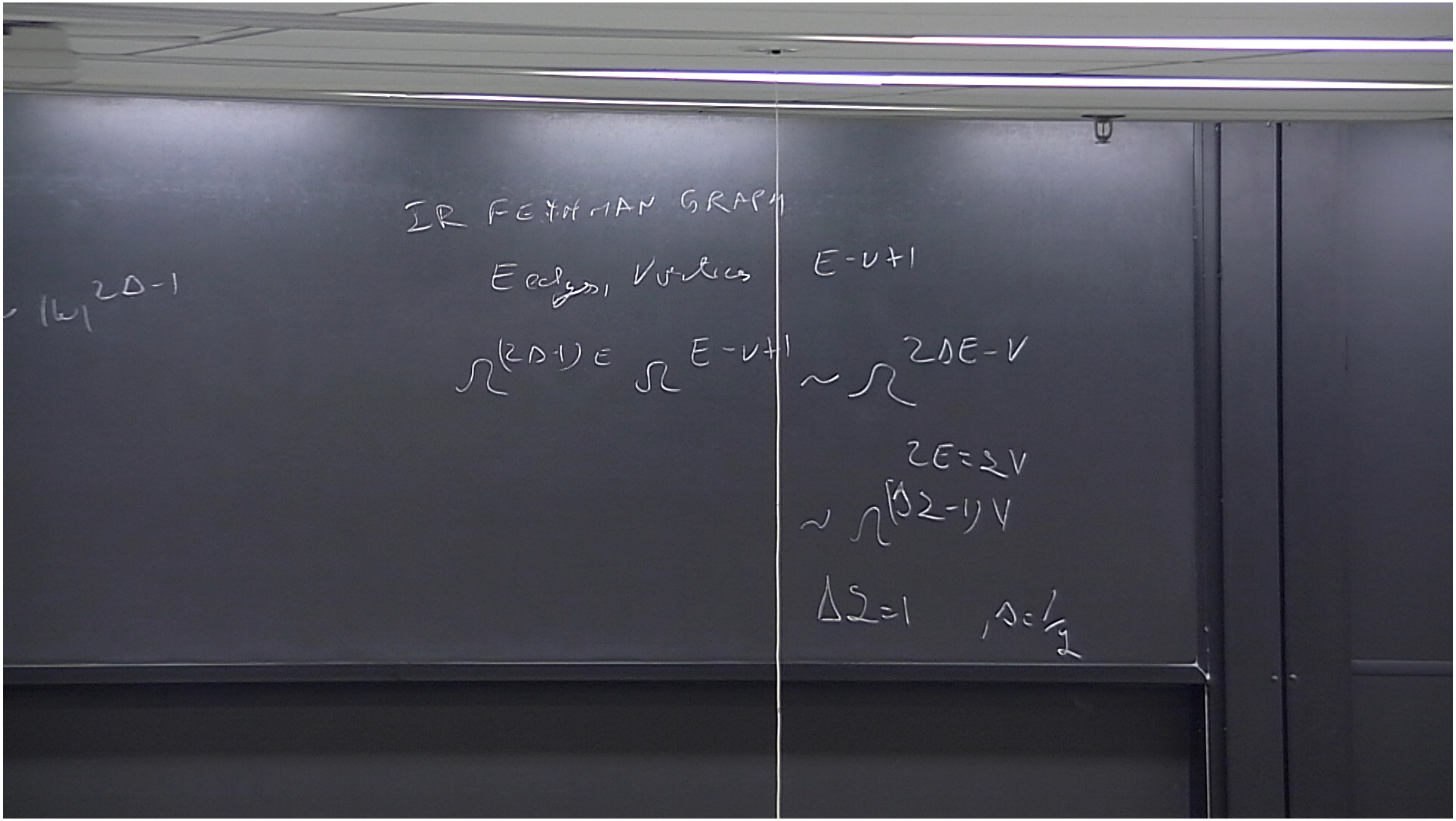
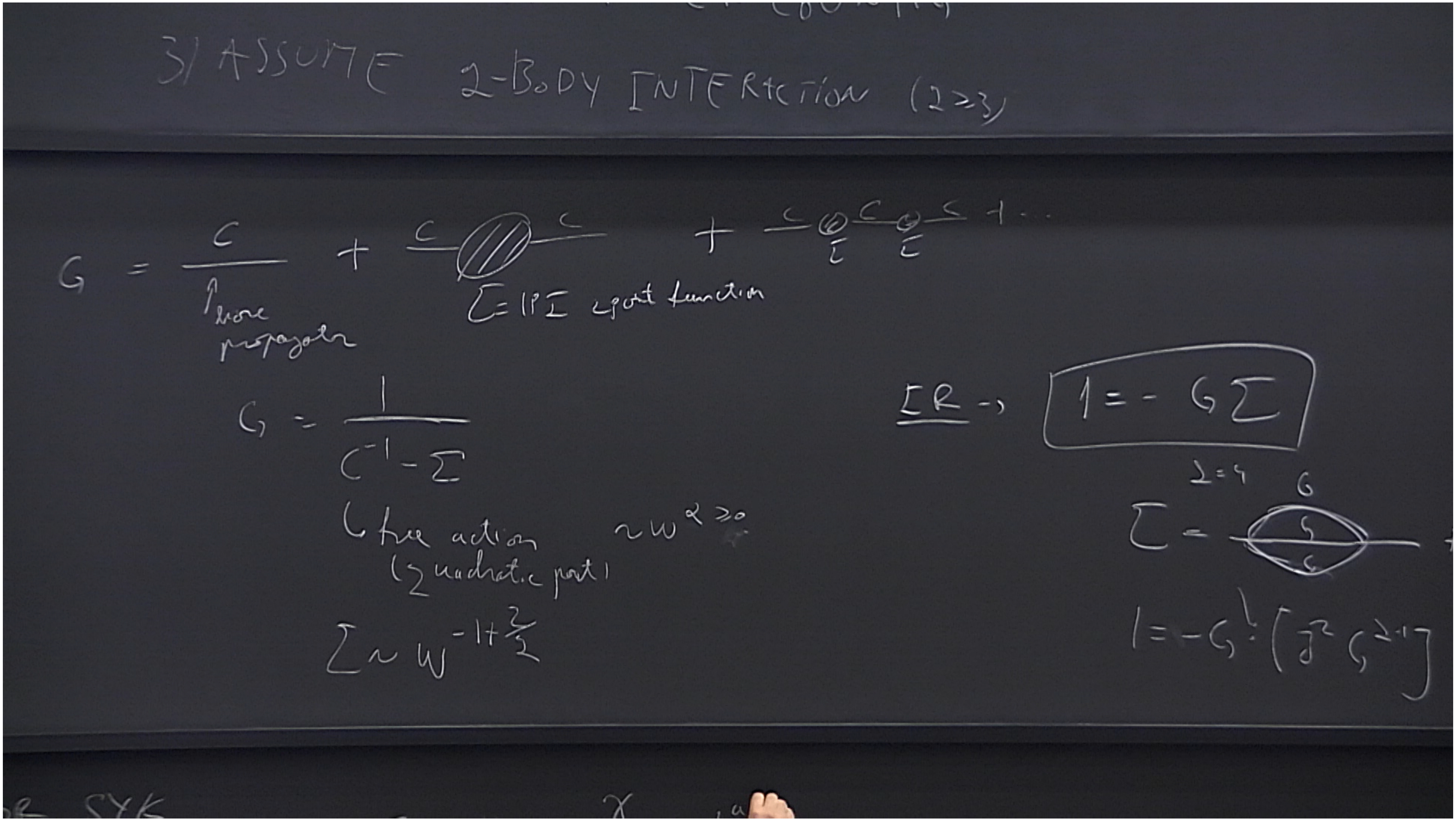
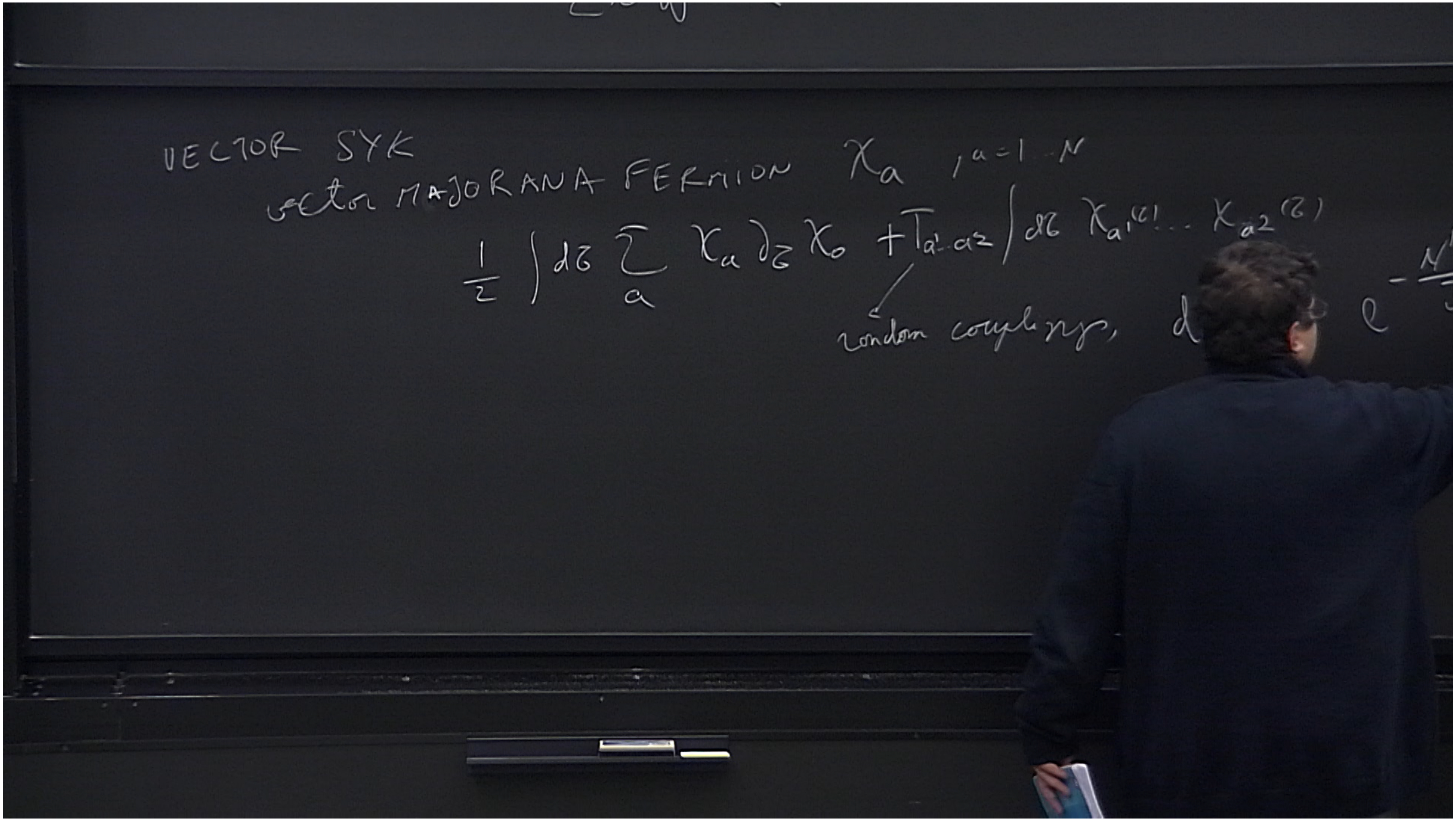
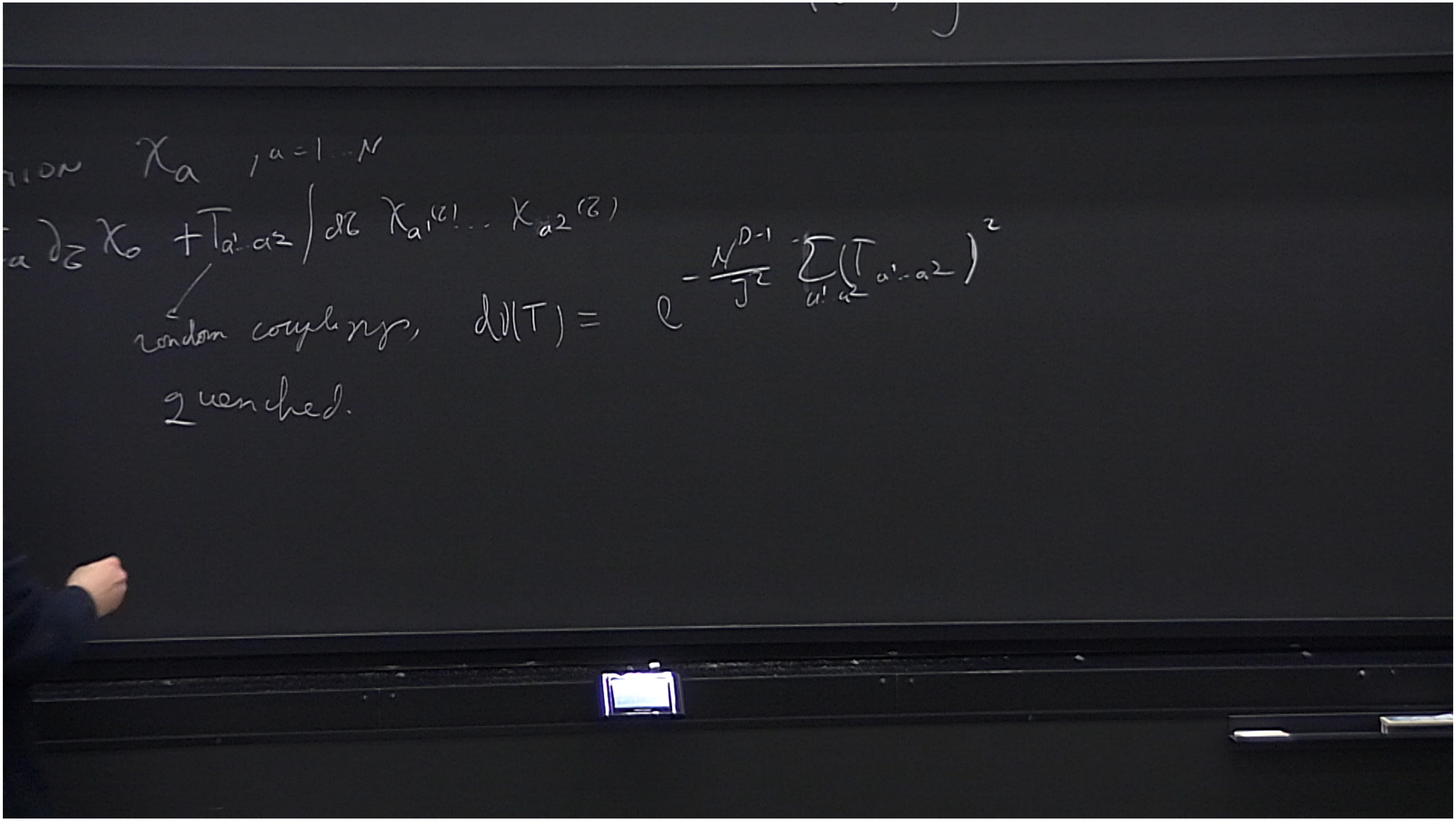
Abstract: <p>In this talk I will discuss the SYK model from a quantum field theoretical perspective and present an i epsilon prescription which regularizes the divergences of the model.</p>


iε in SYK models
INTRESTING CFT1
1) G(τ1−τ2) = 1/|τ1−τ2|^2Δ
G̃(ω) = ∫dτ 1/|τ|^2Δ e^iωτ ~
2) MARGINAL IR POWER COUNTING
3) ASSUME 2-BODY INTERACTION (2≥3)


IR FEYNMAN GRAPH
E edges, V vertices
E-V+1
|ω|^{2Δ-1}
Ω^{(2Δ-1)E} Ω^{E-V+1} ~ Ω^{2ΔE-V}
2E=qV
~ Ω^{(Δq-1)V}
Δq=1 , Δ=1/q


3) ASSUME q-BODY INTERACTION (q≥3)
G = C + C Σ C + C Σ C Σ C + ...
bare propagator
Σ = 1PI 2point function
G = 1 / (C⁻¹ − Σ)
free action (quadratic part)
~ω^α ≥ 0
Σ ~ ω^(−1+2/q)
IR → 1 = − G Σ
q = 4
Σ =
1 = −G [J² G^(q−1)]


VECTOR SYK
vector MAJORANA FERMION
random couplings,



random couplings, dν(T) =
quenched.

MAJORANA FERMION χ_a, $a=1\ldots N$

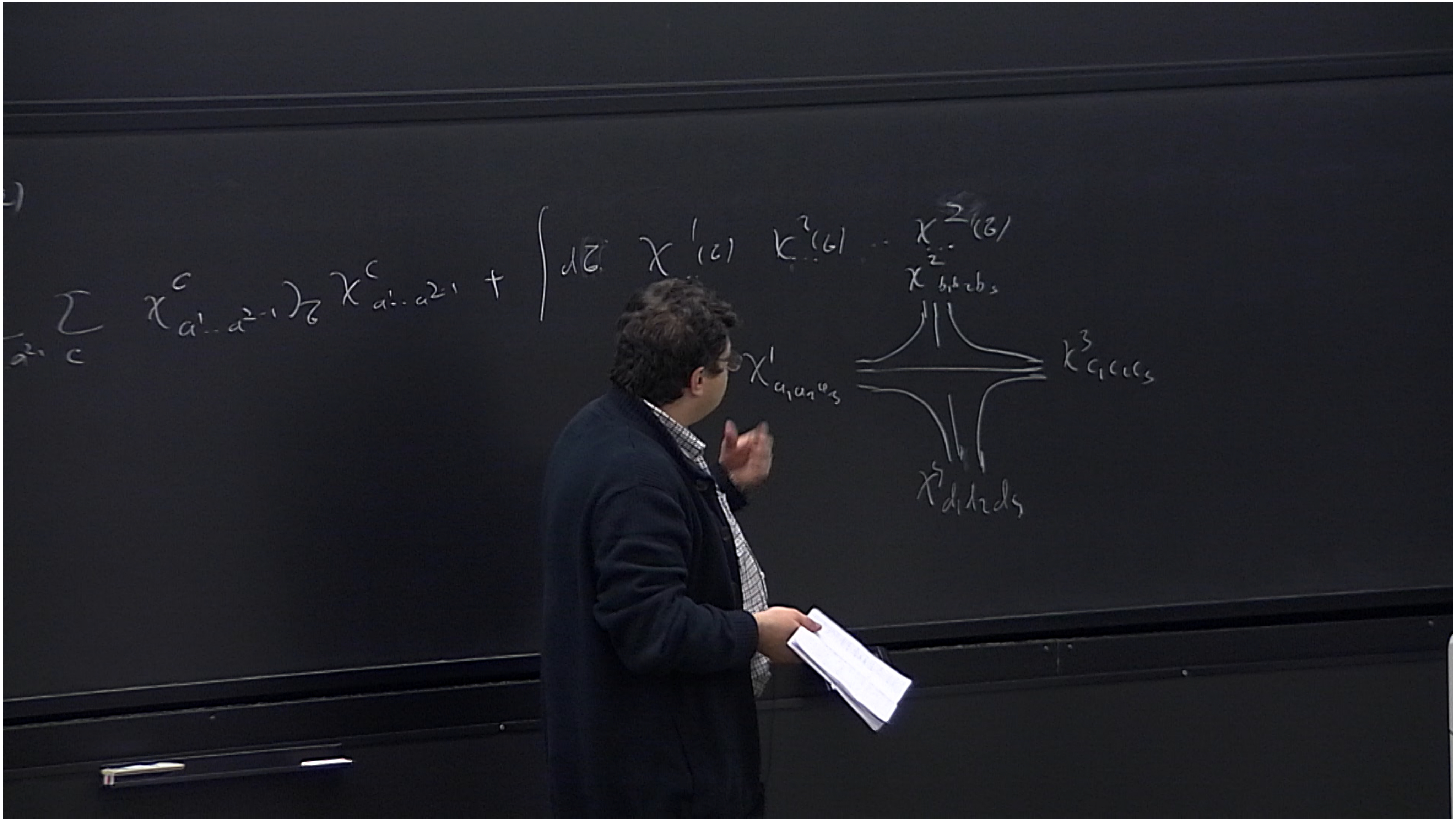
$$\frac{1}{2}\int d\tau \sum_a \chi_a \partial_\tau \chi_a + T_{a^1\ldots a^2}\int d\tau\, \chi_{a^1}(\tau)\ldots\chi_{a^2}(\tau)$$

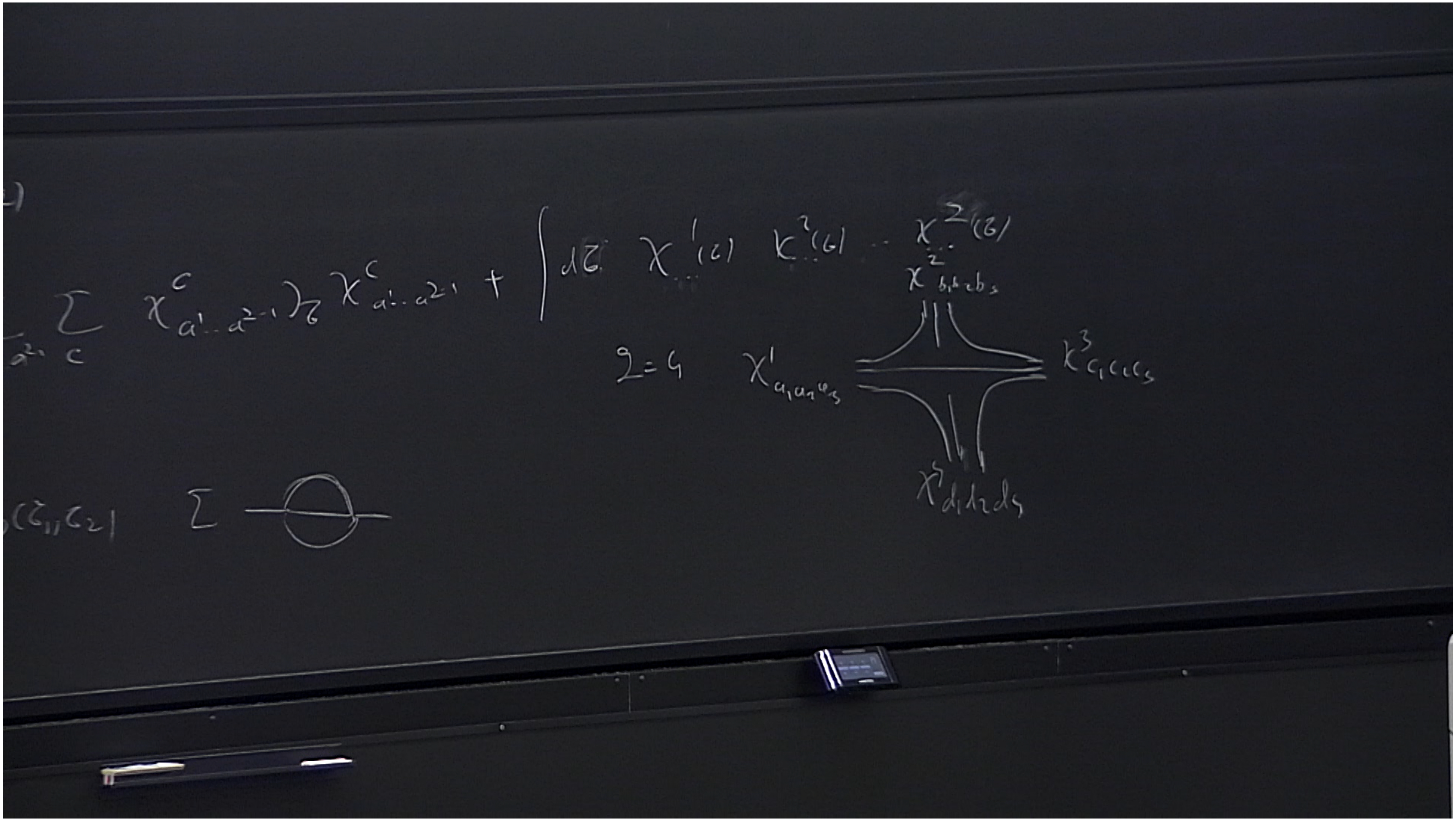
random couplings, $d\nu(T) = e^{-\frac{N^{D-1}}{J^2}\sum_{a^1\ldots a^2}(T_{a^1\ldots a^2})^2}$

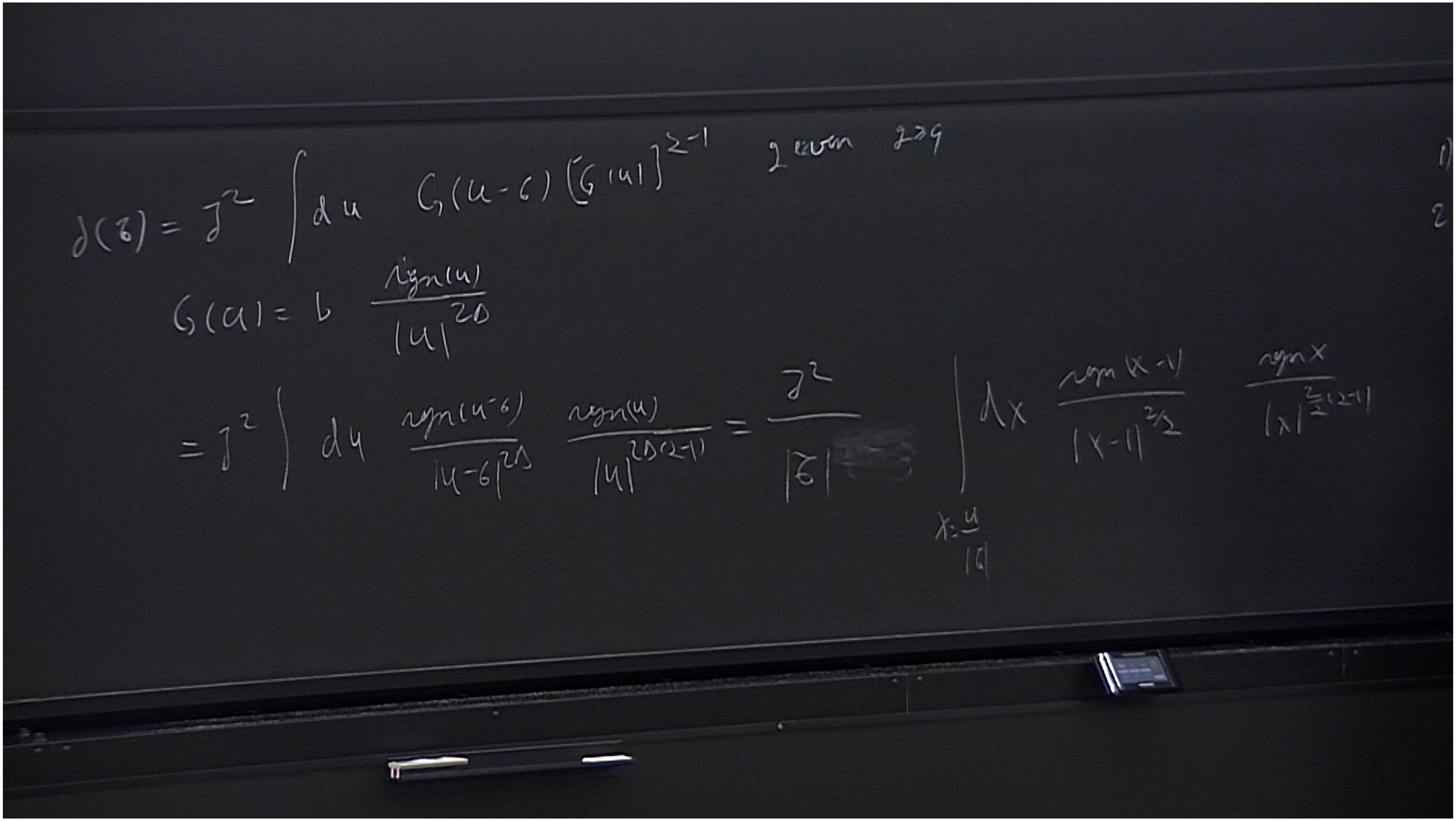
quenched.

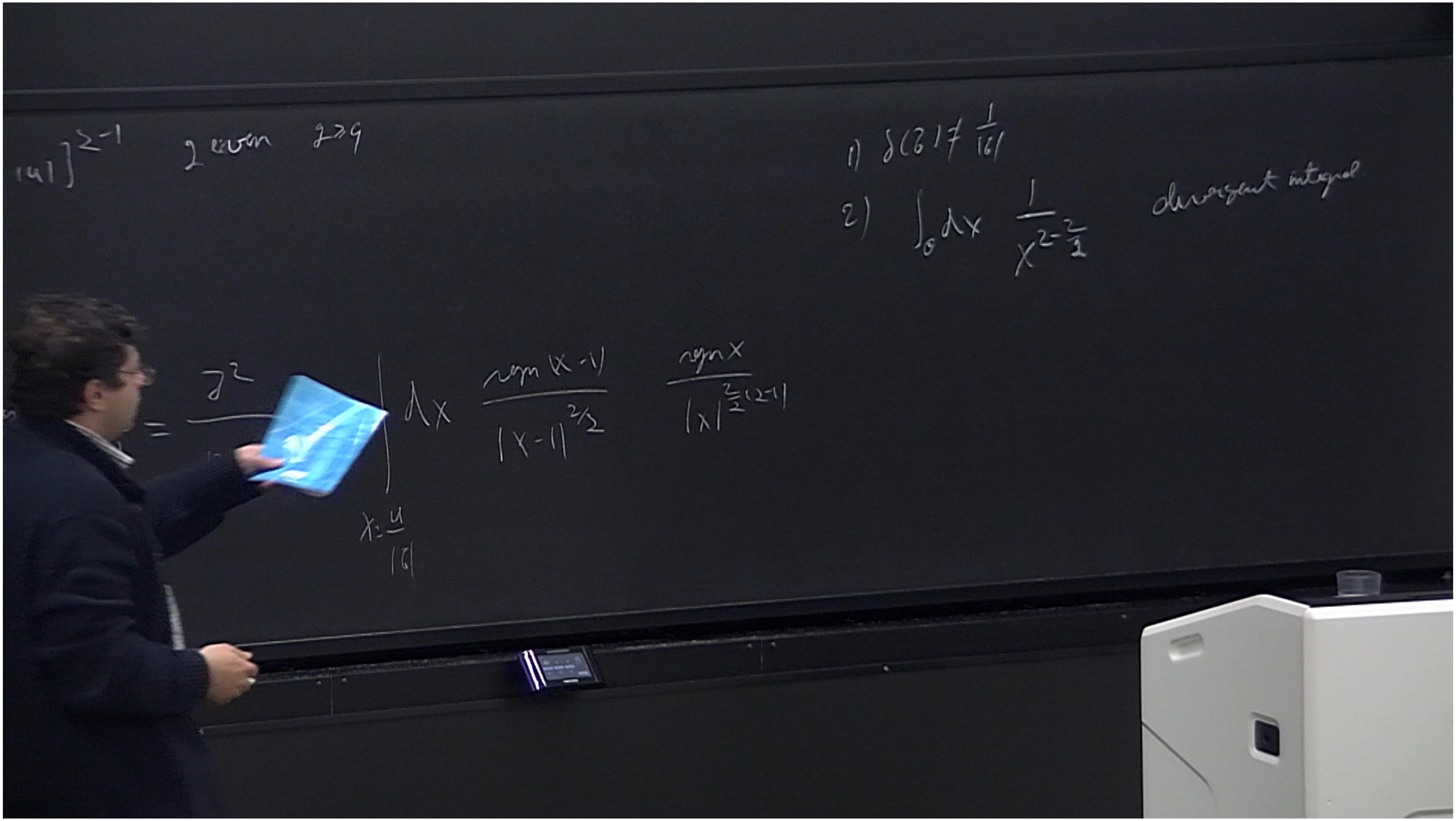
$$\delta_{ab}\, G(\tau_1,\tau_2) = \int d\nu(T)\, \frac{\int (d\chi)\, e^{-S}\, \chi_a(\tau_1)\chi_b(\tau_2)}{\int (d\chi)\, e^{-S}}$$

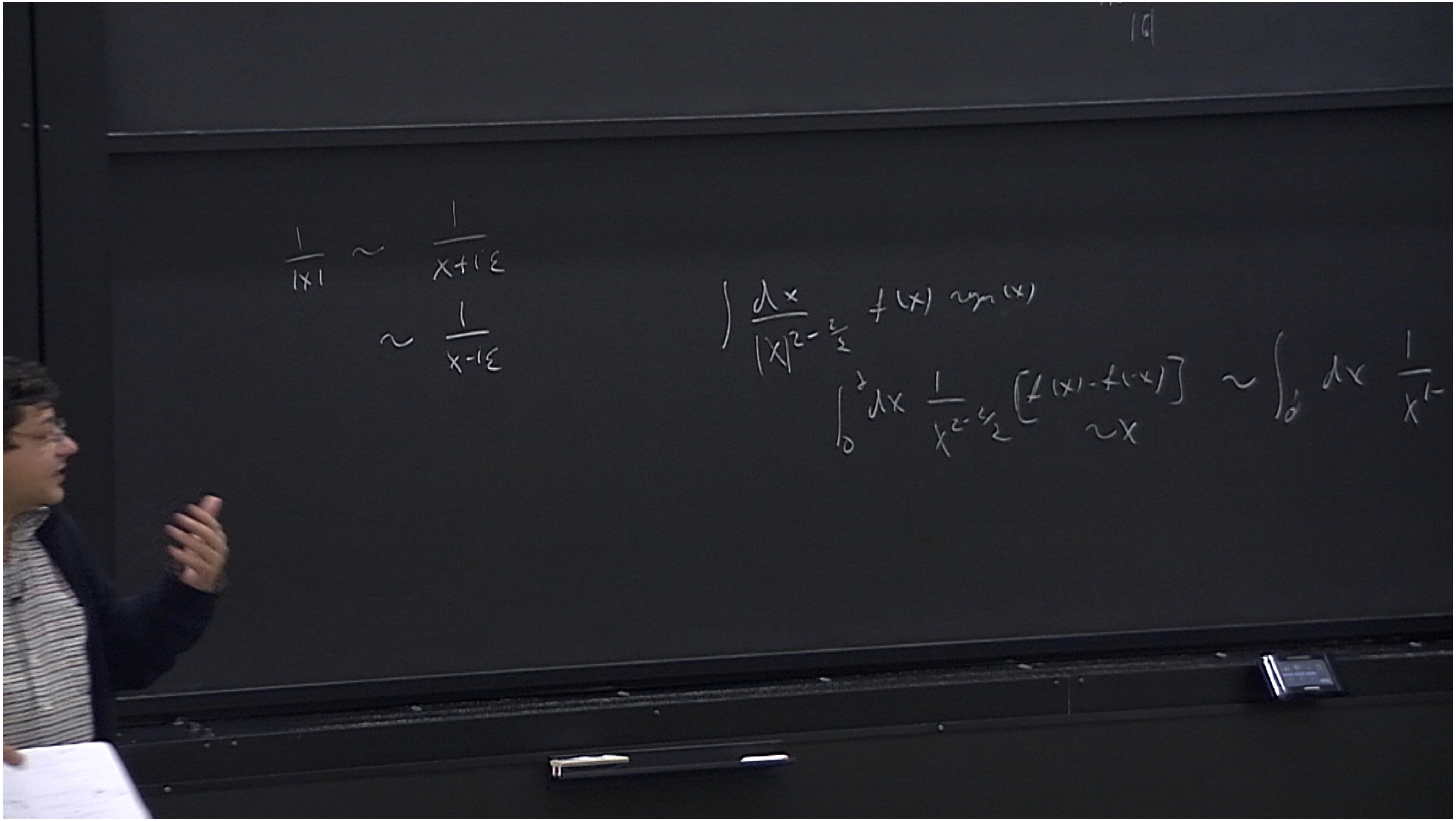
$\Sigma =$

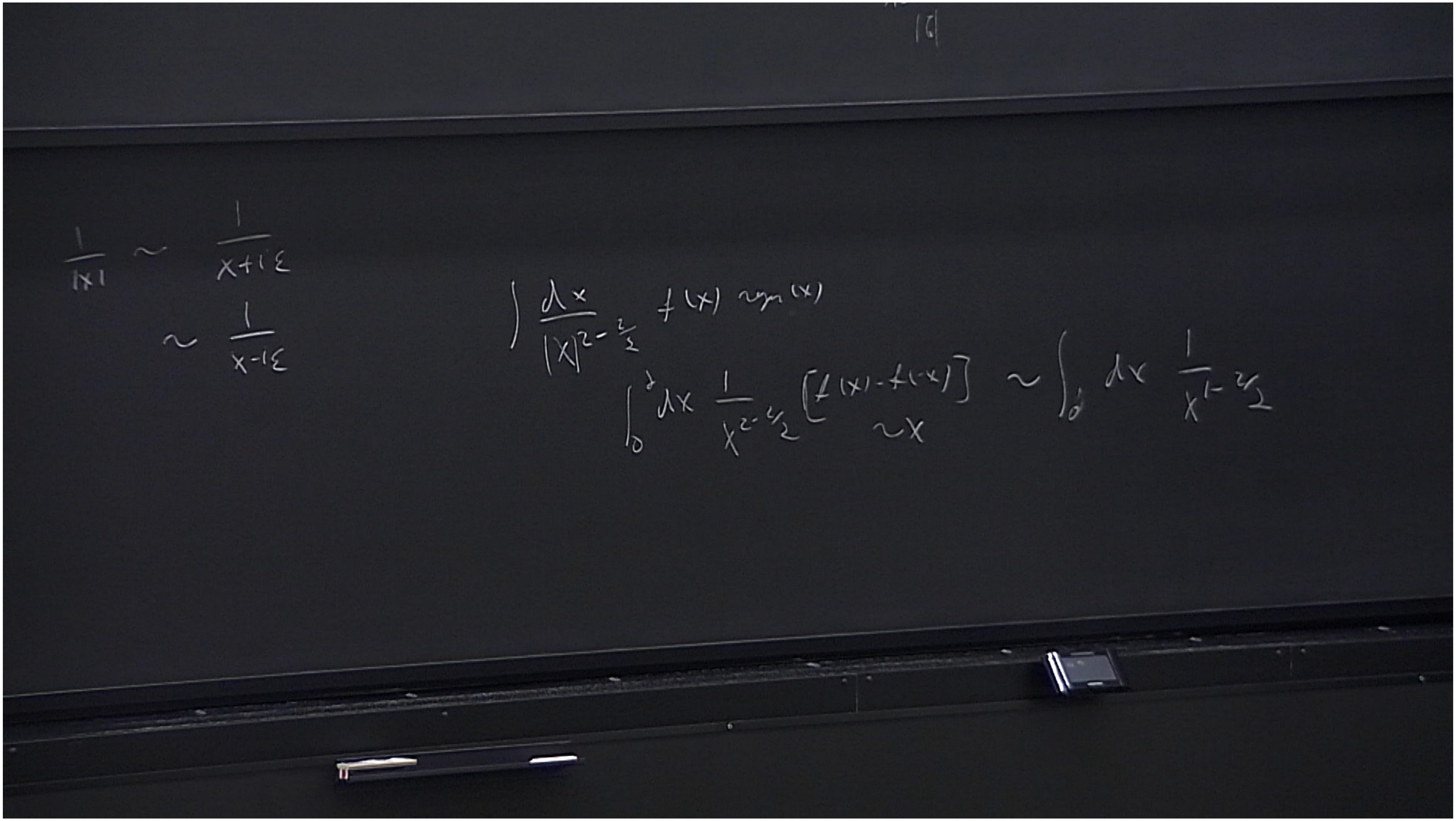


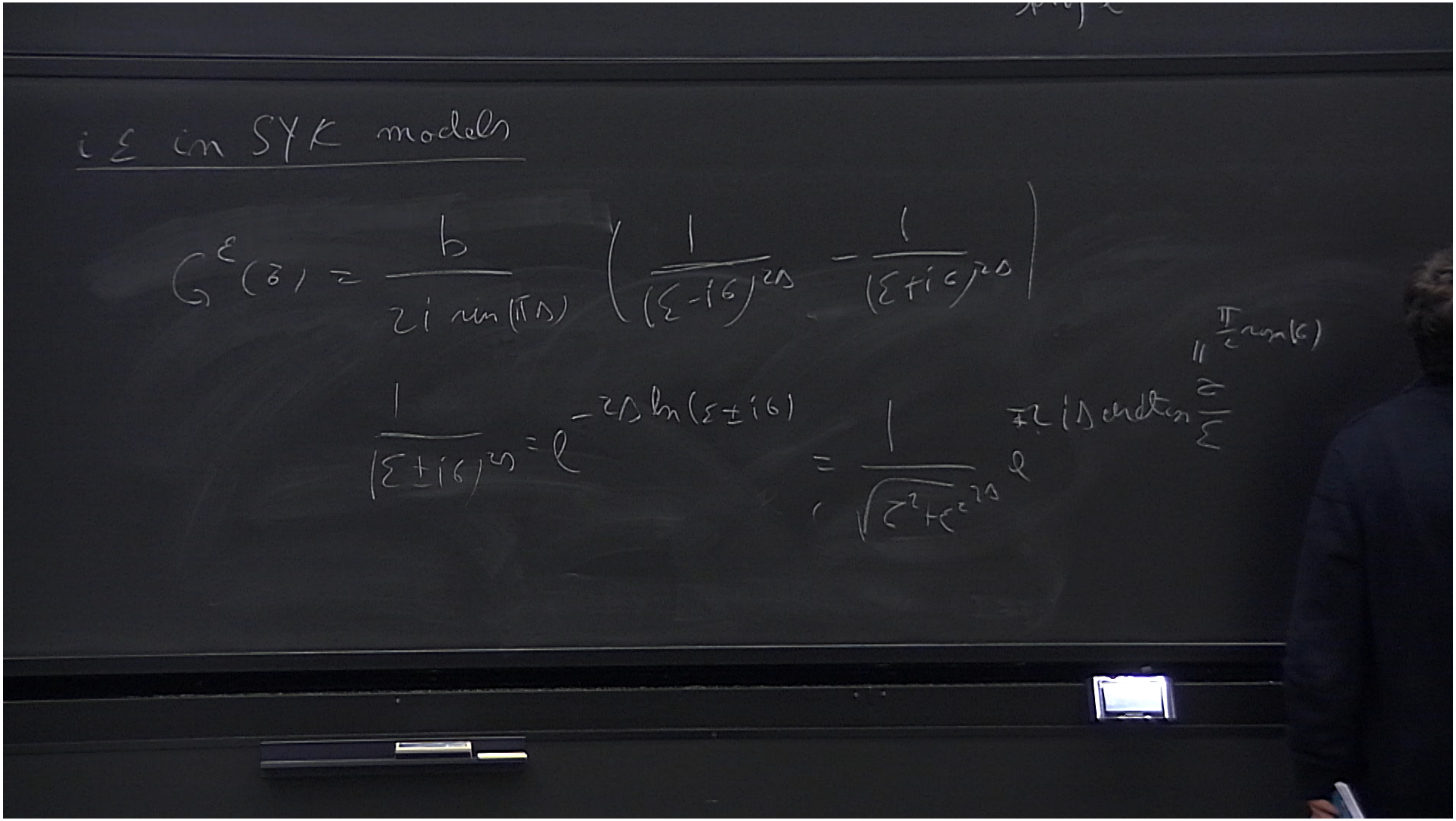

g = 4

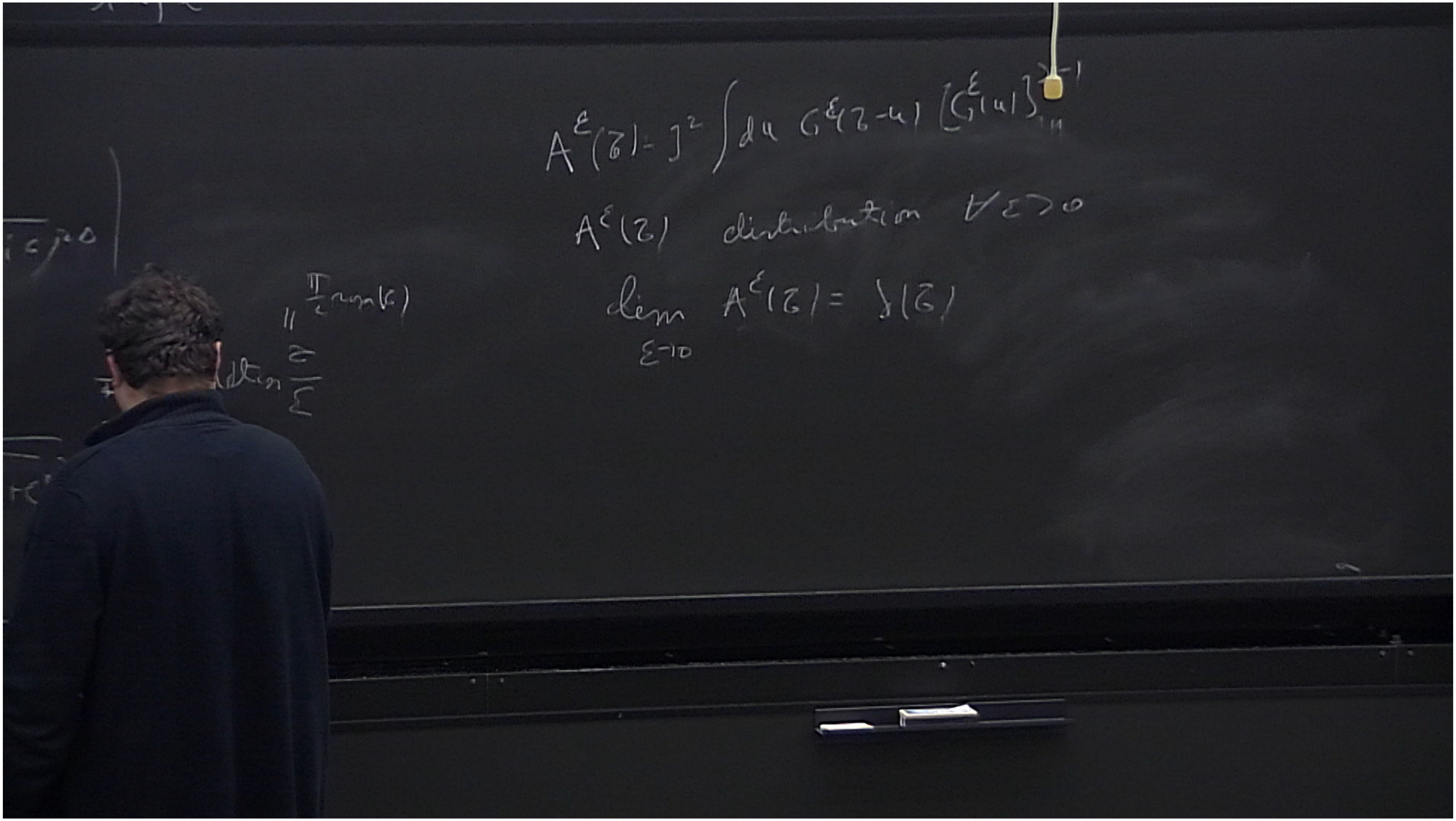

G(τ) = J^2 ∫ du G(u-τ) [G(u)]^{q-1}
q even q>4
G(u) = b sgn(u)/|u|^{2Δ}
= J^2 ∫ du sgn(u-τ)/|u-τ|^{2Δ} sgn(u)/|u|^{2Δ(q-1)} = J^2/|τ| ∫ dx sgn(x-1)/|x-1|^{2/q} sgn x/|x|^{2/q(q-1)}
x = u/|τ|

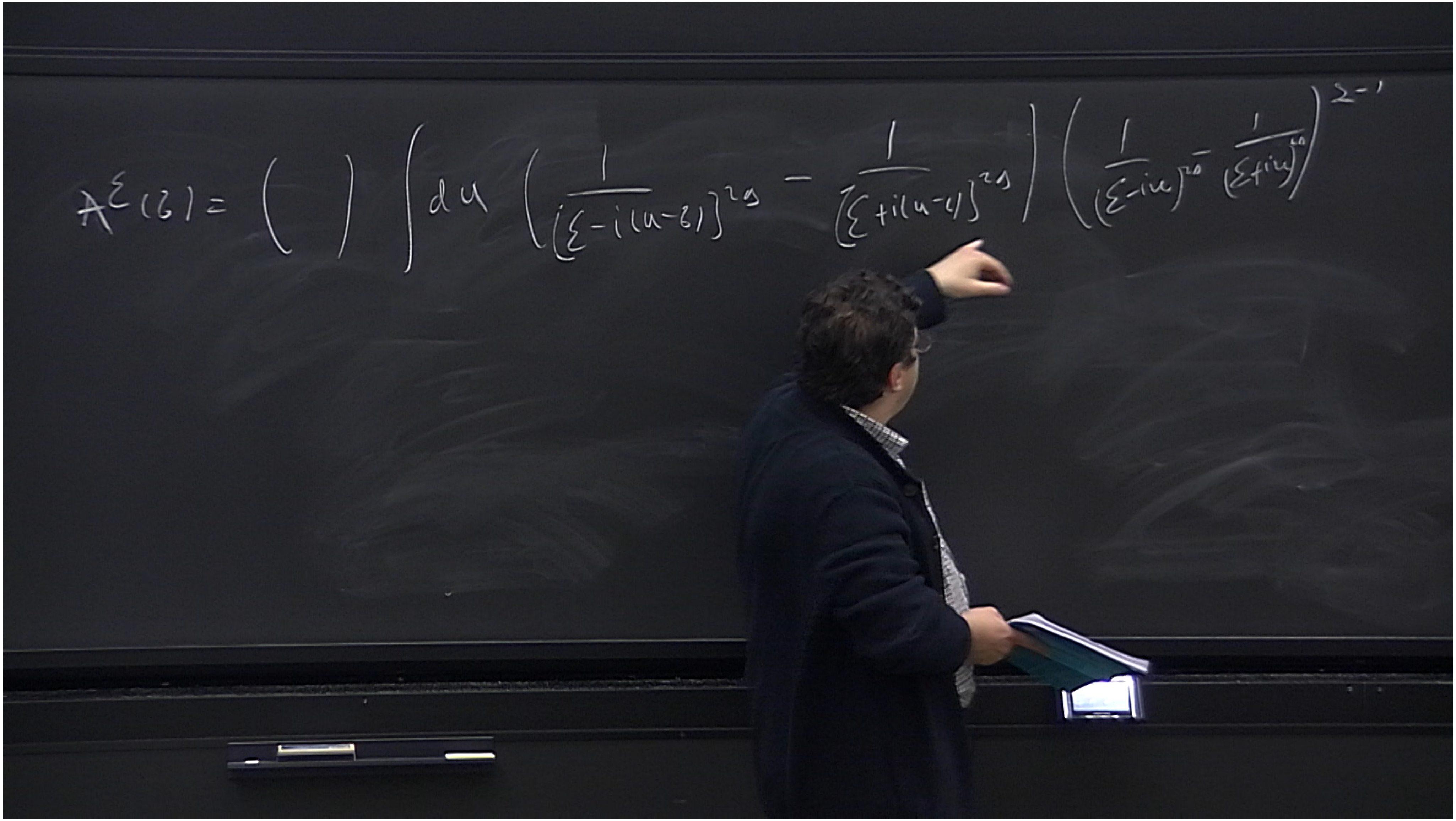

divergent integral

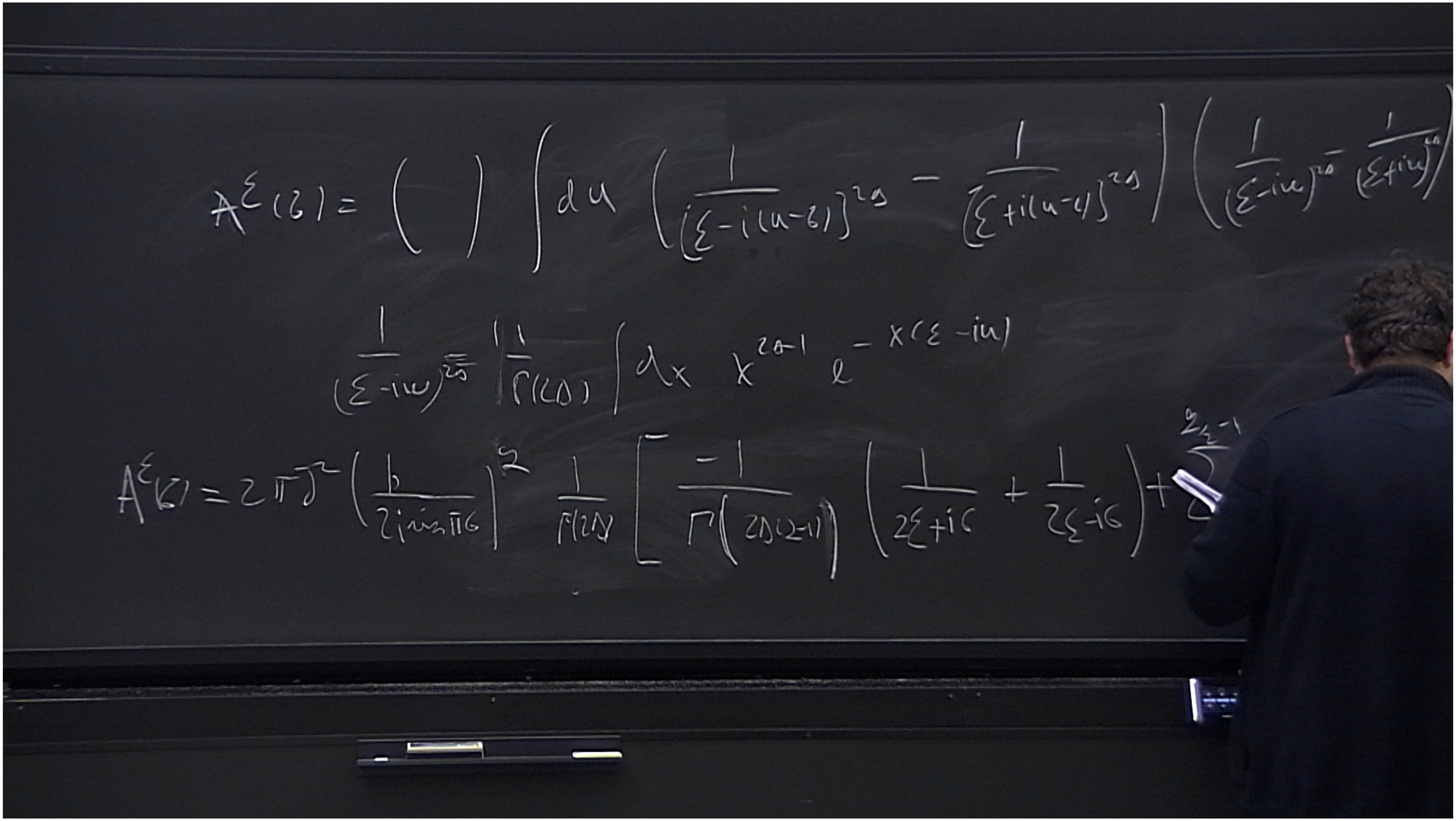

1/|x| ~ 1/(x+iε)
~ 1/(x−iε)
∫ dx/|x|^(2−2/2) f(x) sgn(x)
∫_0 dx 1/x^(2−ε/2) [f(x) − f(−x)] ~ ∫_0 dx 1/x^(1−
~x


1/|x| ~ 1/(x+iε)
~ 1/(x−iε)
∫ dx/|x|^(2−ε/2) f(x) sgn(x)
∫_0^δ dx 1/x^(2−ε/2) [f(x) − f(−x)] ~ ∫_0^δ dx 1/x^(1−ε/2)
~x


iε in SYK models
G^ε(τ) = b / (2i sin(πΔ)) (1/(ε−iτ)^{2Δ} − 1/(ε+iτ)^{2Δ})
1/(ε±iτ)^{2Δ} = e^{−2Δ ln(ε±iτ)}
= 1/(√(τ²+ε²))^{2Δ} e^{∓2iΔ arctan τ/ε}


$A^{\varepsilon}(\tau) = J^2 \int du\, G^{\varepsilon}(\tau-u)\, [G^{\varepsilon}(u)]^{-1}$
$A^{\varepsilon}(\tau)$ distribution $\forall \varepsilon > 0$
$\lim_{\varepsilon \to 0} A^{\varepsilon}(\tau) = \delta(\tau)$

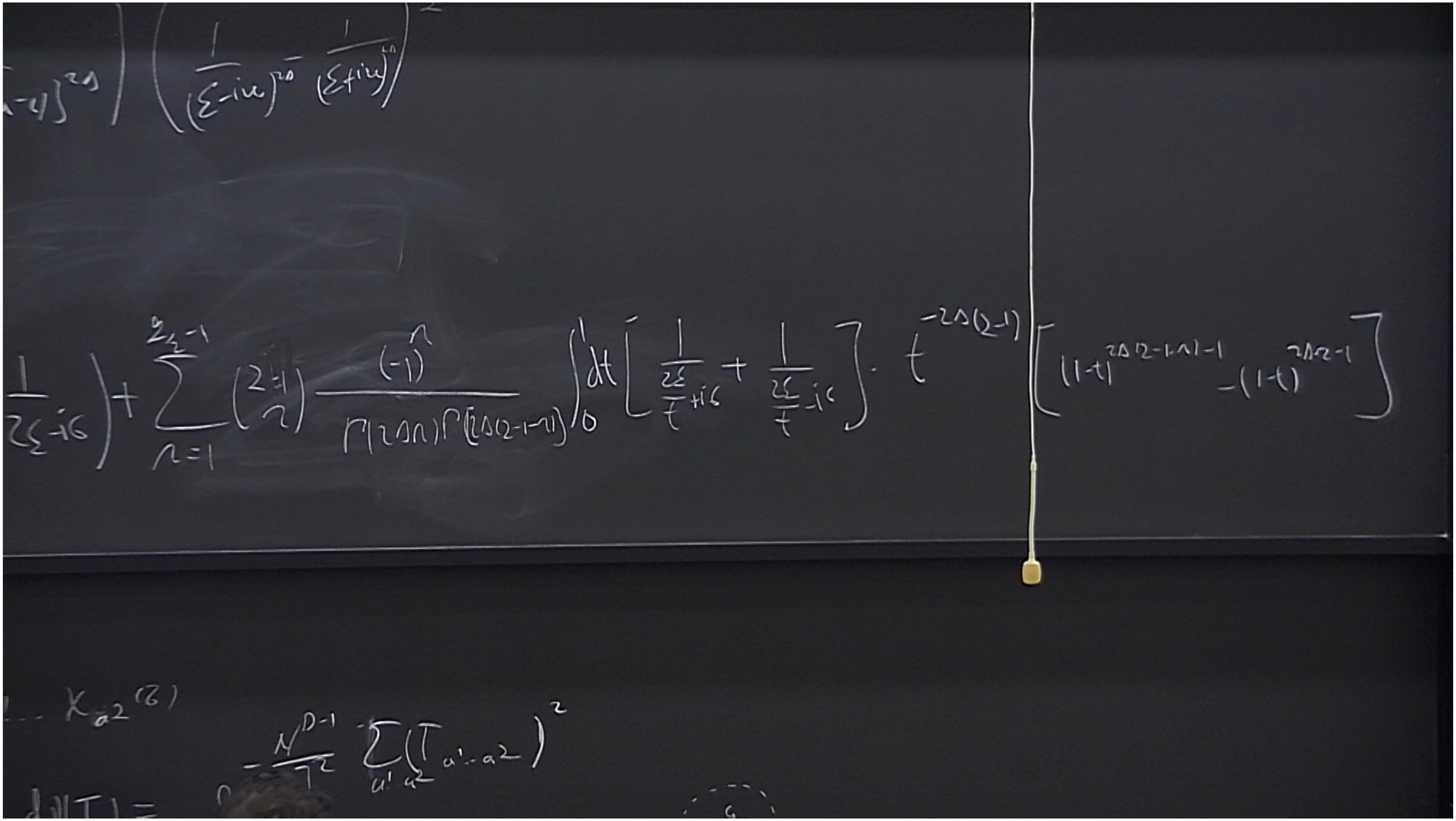

A^ε(ζ) = ( ) ∫du (1/[ε − i(u−ζ)]^{2Δ} − 1/[ε + i(u−ζ)]^{2Δ}) (1/(ε−iu)^{2Δ} − 1/(ε+iu)^{2Δ})^{Δ−1}

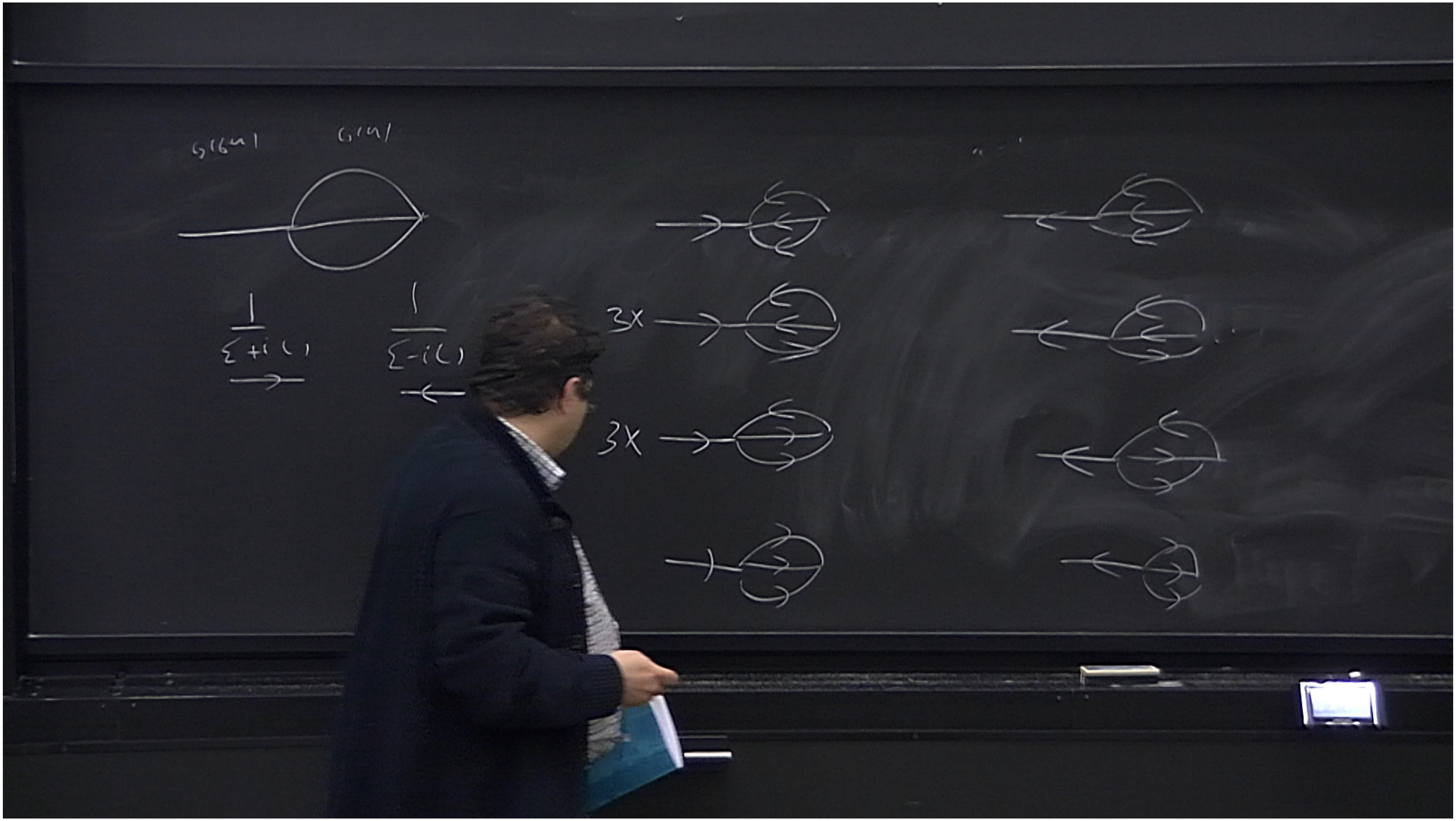

$A^{\varepsilon}(z) = (\ ) \int du \left(\frac{1}{[\varepsilon - i(u-z)]^{2\Delta}} - \frac{1}{[\varepsilon + i(u-1)]^{2\Delta}} \right) \left(\frac{1}{(\varepsilon - iu)^{2\Delta}} - \frac{1}{(\varepsilon + iu)^{2\Delta}} \right)$
$\frac{1}{(\varepsilon - iu)^{2\Delta}} = \frac{1}{\Gamma(2\Delta)} \int dx\, x^{2\Delta - 1} e^{-x(\varepsilon - iu)}$
$A^{\varepsilon}(z) = 2\pi J^2 \left(\frac{b}{2i \ldots \pi G} \right)^2 \frac{1}{\Gamma(2\Delta)} \left[\frac{-1}{\Gamma(2\Delta(2-1))} \left(\frac{1}{2\varepsilon + iG} + \frac{1}{2\varepsilon - iG} \right) + \ldots \right.$

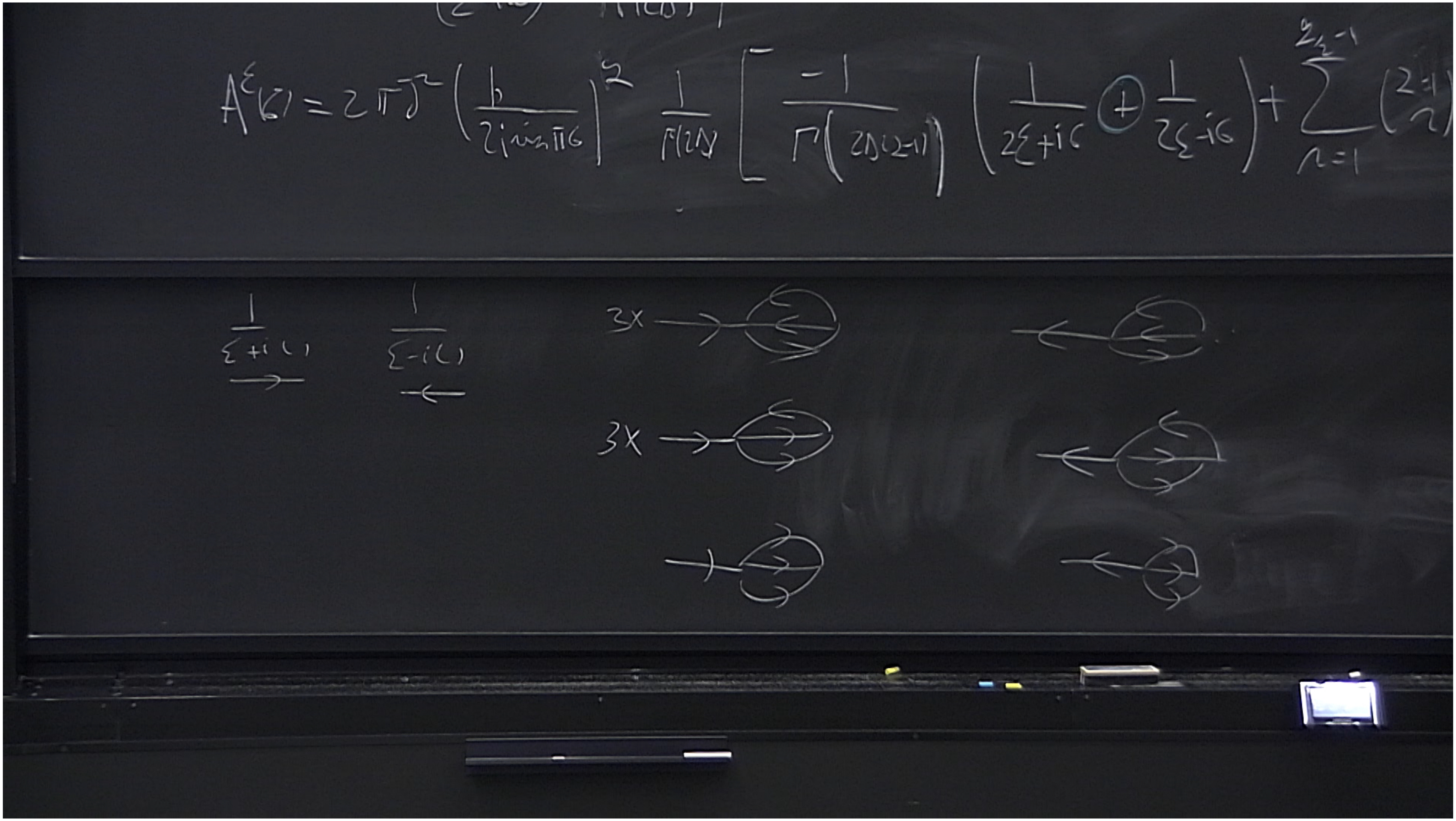
$$\ldots \Big) \int du \left(\frac{1}{[\xi - i(u-t)]^{2\Delta}} - \frac{1}{[\xi + i(u-t)]^{2\Delta}} \right) \left(\frac{1}{(\xi - iu)^{2\Delta}} - \frac{1}{(\xi + iu)^{2\Delta}} \right)^{q-1}$$

$$\ldots u)^{2\Delta} = \frac{1}{\Gamma(2\Delta)} \int dx \; x^{2\Delta - 1} e^{-x(\xi - iu)}$$

$$\ldots \Big|^2 \frac{1}{\Gamma(2\Delta)} \left[\frac{-1}{\Gamma(2\Delta(q-1))} \left(\frac{1}{2\xi + i\epsilon} + \frac{1}{2\xi - i\epsilon} \right) + \sum_{n=1}^{q-1} \binom{q-1}{n} \frac{(-1)^n}{\Gamma(2\Delta n)\Gamma(2\Delta(q-1-n))} \int_0^1 dt \left[\frac{1}{\frac{2\xi}{t} + i\epsilon} + \ldots \right. \right.$$




3x
3x


3x
3x