Title: Entanglement area law in superfluid 4He

Date: May 25, 2017 11:30 AM

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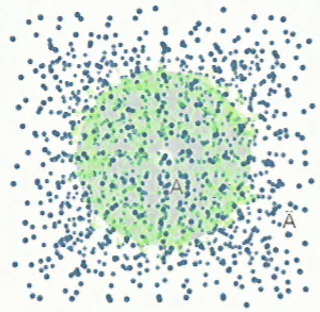
Abstract: Area laws were first discovered by Bekenstein and Hawking, who found that the entropy of a black hole grows proportional to its surface area, and not its volume. Entropy area laws have since become a fundamental part of modern physics, from the holographic principle in quantum gravity to ground state wavefunctions of quantum matter, where entanglement entropy is generically found to obey area law scaling. As no experiments are currently capable of directly probing the entanglement area law in naturally occurring many-body systems, evidence of its existence is based on studies of simplified theories. Using new exact microscopic numerical simulations of superfluid 4He, we demonstrate for the first time an area law scaling of entanglement entropy in a real quantum liquid in three dimensions. We validate the fundamental principles underlying its physical origin, and present an "entanglement equation of state" showing how it depends on the density of the superfluid.



Entanglement area law in superfluid ^{4}He

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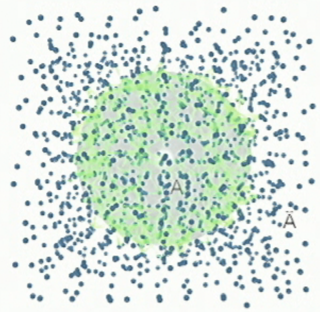

Entanglement area law in superfluid ^4He

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Reference:

Herdman, Roy, Melko & Del Maestro, ***Nature Physics*** (2017)
doi:10.1038/nphys4075

Outline

Part I

Background: Entanglement in many-body physics

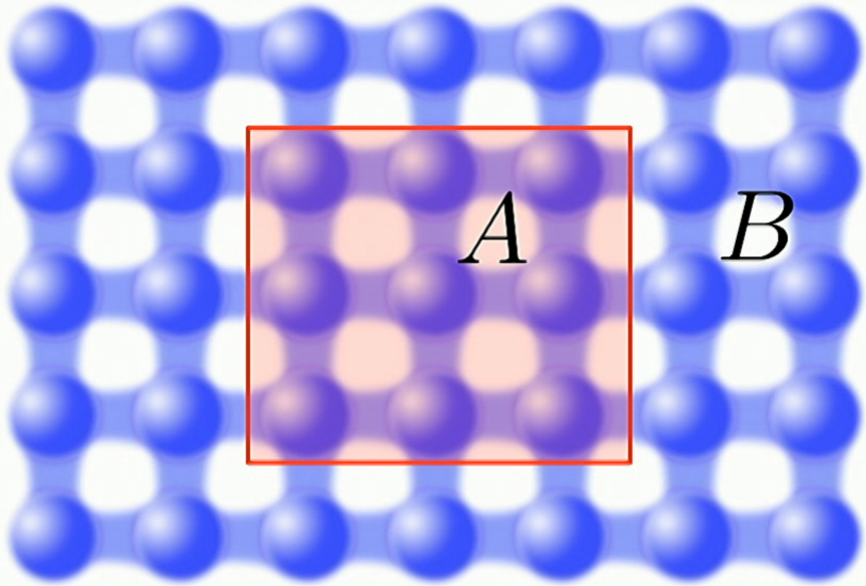
Part II

Method: Entanglement entropy in quantum fluids via quantum Monte Carlo

Part III

Application: Entanglement in superfluid ^4He

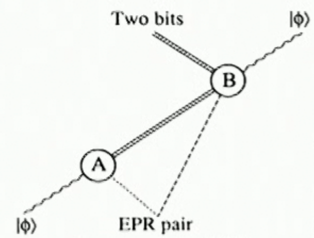
Bipartite entanglement in many-body systems



$$|\Psi_{AB}\rangle \stackrel{?}{=} |\psi_A\rangle \otimes |\psi_B\rangle$$

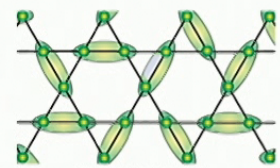
Motivation:
Entanglement in many-body systems

- *physical* resource for quantum information processing
  - *e. g.* quantum speedup, teleportation



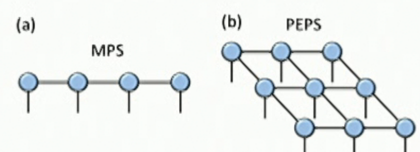
Bennett *et al.*, PRL (1993)

- characterize and classify quantum phases of matter
  - *e.g.* topological spin liquids



L. Clark, *Edinburgh*

- theoretical & computational methods to study of quantum many-body systems
  - *e.g.* DMRG, MPS, PEPS, MERA



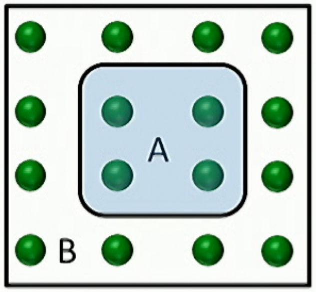
Orus *Ann. Phys.* (2014)

Entanglement entropy in many-body systems

Bipartition into subregion **A** and its complement **B**:

$$\rho_A = \mathrm{Tr}_B \, |\Psi_{AB}\rangle\langle\Psi_{AB}|$$

$$|\Psi_{AB}\rangle \overset{?}{=} |\psi_A\rangle \otimes |\psi_B\rangle$$



Rényi entanglement entropy:

Rényi index

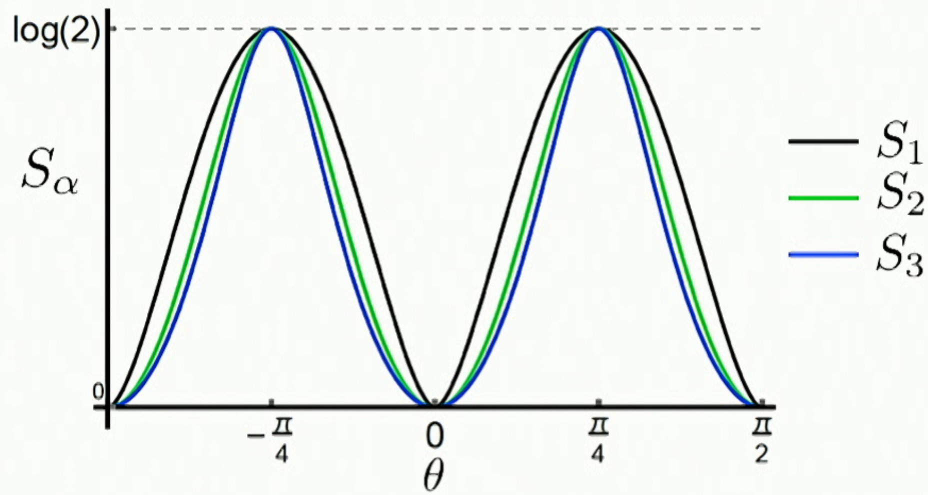
$$S_\alpha(A) \equiv -\frac{1}{\alpha - 1} \log\left[\mathrm{Tr}\rho_A^\alpha\right]$$

- von Neumann entanglement entropy:

$$\alpha \to 1 : S_\alpha \Rightarrow S_{\mathrm{vN}}(A) = -\mathrm{Tr}\rho_A \log \rho_A$$

Rényi entanglement entropy

$$|\Psi_{AB}\rangle = \cos\theta\,|\uparrow_A\downarrow_B\rangle + \sin\theta\,|\downarrow_A\uparrow_B\rangle$$



$$S_1 \geq S_2 \geq S_3 \geq \ldots$$

Experimental measurements of 2nd Rényi entanglement entropy in many-body systems

nature International weekly journal of science

Measuring entanglement entropy in a quantum many-body system

Rajibul Islam[1], Ruichao Ma[1], Philipp M. Preiss[1], M. Eric Tai[1], Alexander Lukin[1], Matthew Rispoli[1] & Markus Greiner[1]

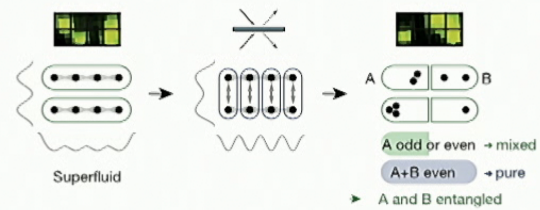
3 DECEMBER 2015 | VOL 528 | NATURE | 77

Science

Quantum thermalization through entanglement in an isolated many-body system

Adam M. Kaufman, M. Eric Tai, Alexander Lukin, Matthew Rispoli, Robert Schittko, Philipp M. Preiss, Markus Greiner*

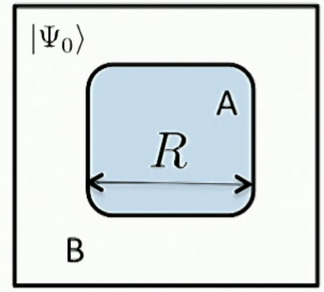
19 AUGUST 2016 • VOL 353 ISSUE 6301



Islam *et al.* (2015)

Ground state entanglement entropy

A "typical" state has *extensive* (i.e. volume-law) entanglement entropy
- *c.f.* thermal entropy



For ground states of local Hamiltonians for:
- gapped 1D systems
- free field-theories
- simplified lattice models (via numerics)

entanglement entropy is found to be *sub*-extensive

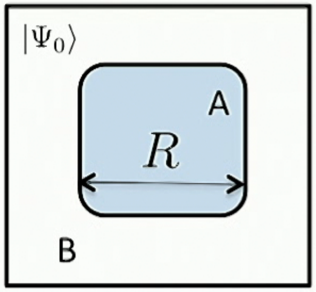
"Area Law": $S = c_0 R^d + c_1 R^{d-1} + \ldots$

Real phases of matter in 2D in 3D?

Page PRL (1993)
Hastings *J. Stat. Mech.* (2007)
Srednicki PRL (1993)

Beyond "Area law" scaling of entanglement entropy

"Area Law": $S = c_0 R^d + c_1 R^{d-1} + \ldots$

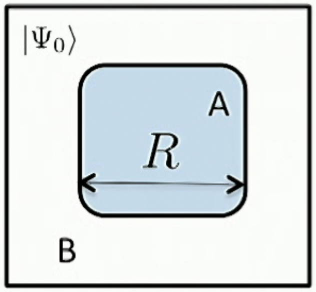


- Sub-leading corrections can encode universal properties of quantum phases of matter
  - topological order
  - Goldstone modes
  - criticality

Phenomenological entanglement entropy scaling in 3D

Fundamental physical principles:

- entanglement is generated by correlations local to the boundary
- contributions from relevant length scales



$$S_2(R) = 4\pi a \left(\frac{R}{r_0}\right)^2 + b \log\left(\frac{R}{r_0}\right) + c + \mathcal{O}\left(\left(\frac{R}{r_0}\right)^{-2}\right)$$

microscopic length scale

Liu & Mezei JEHP (2017)
Solodukhin *Phys. Lett. B* (2010)
Swingl*e* arXiv:1010.4023
Grover PRB (2011)

Part II

Method:

Rényi entanglement entropy via path integral quantum Monte Carlo

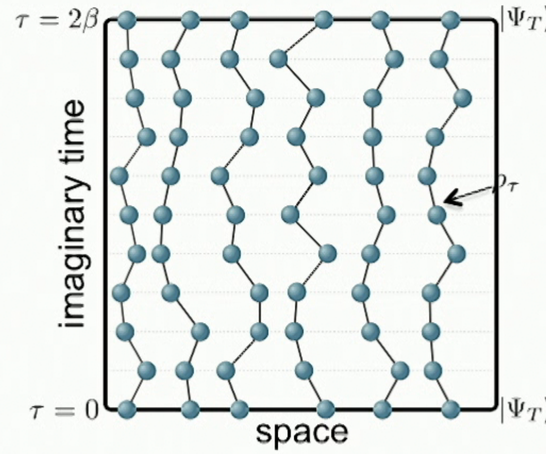
Path integral ground state quantum Monte Carlo studies of quantum fluids

System: itinerant non-relativistic bosons in continuous space

$$H = \sum_{i=1}^{N} \left(-\frac{\hbar^2}{2m_i} \nabla_i^2 + U_i \right) + \sum_{i<j} V_{ij}$$

- Project trial wave-function to ground state

$$|\Psi_0\rangle = \lim_{\beta \to \infty} e^{-\beta H} |\Psi_T\rangle$$



$$\rho_\tau(\boldsymbol{R}, \boldsymbol{R}') \approx \langle \boldsymbol{R} | e^{-\tau H} | \boldsymbol{R}' \rangle$$

- "Beads" linked by the short time propagator:
- Unbiased & scalable computational method

Ceperley, *RMP* (1995)
Sarsa *et al.*, *J. Chem. Phys.* (2000)

Rényi entropy via Monte Carlo

Consider a *replicated* system

$$\mathrm{SWAP_A}\Big(|\mathrm{C_{A_1}C_{B_1}}\rangle \otimes |\mathrm{C_{A_2}C_{B_2}}\rangle\Big) = |\mathrm{C_{A_1}C_{B_2}}\rangle \otimes |\mathrm{C_{A_2}C_{B_1}}\rangle$$

- The expectation value of the SWAP operator is a measure of the *2nd* Rényi entropy:

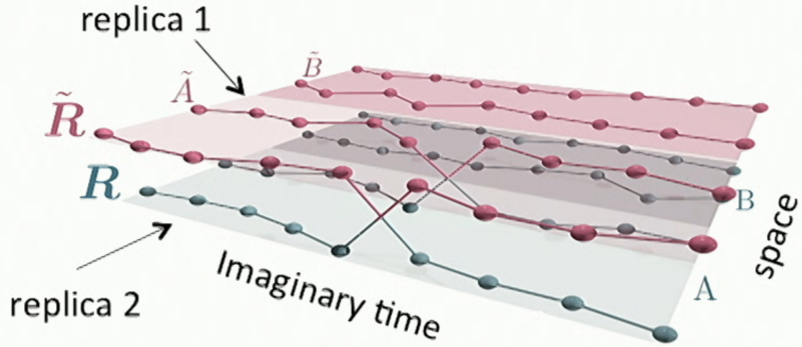
$$S_2 \equiv -\log\left[\mathrm{Tr}\rho_A^2\right] = -\log\langle \mathrm{SWAP_A}\rangle$$

Hastings, González, Kallin, & Melko, *PRL* (2010)

Rényi entropy via path integral QMC

- Monte Carlo sample two **replicas** of the same system
- entanglement entropy: allow worldlines to swap between replicas

$$\langle \mathrm{SWAP_A} \rangle = \frac{Z_{\mathrm{SWAP}}}{Z}$$



2nd Rényi entropy

$$S_2 = -\log \langle \mathrm{SWAP_A} \rangle$$

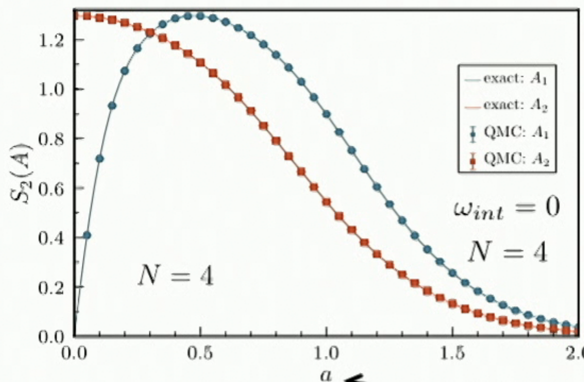
Hastings, González, Kallin, & Melko, PRL (2010)
CMH, Roy, Melko & Del Maestro PRB 2014
CMH, Inglis, Roy, Melko & Del Maestro PRE 2014

Proof of principle of QMC method

- *e.g.* bosons in a harmonic potential with a harmonic interaction in one spatial dimension:

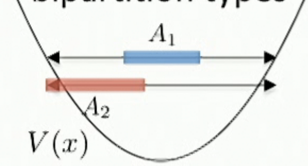
$$H = -\frac{\hbar^2}{2m}\sum_i \frac{d^2}{dx_i^2} + \frac{1}{2}m\omega_0^2\sum_i x_i^2 + \frac{1}{2}m\omega_{\rm int}^2\sum_{i<j}(x_i - x_j)^2$$

entanglement entropy



bipartition size

bipartition types



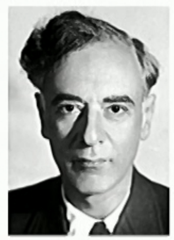
CMH, Inglis, Roy, Melko & Del Maestro, PRE (2014)

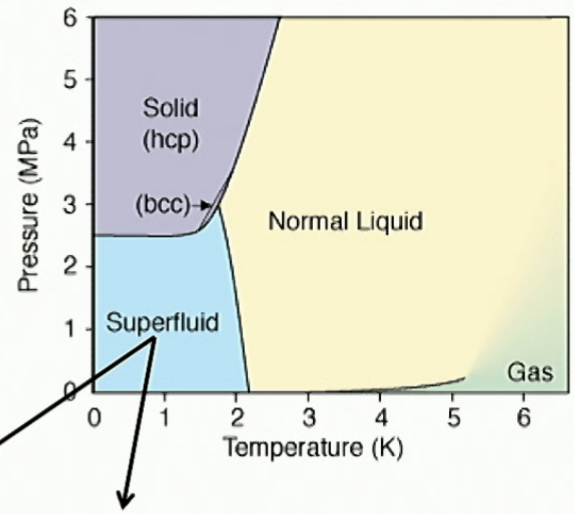
Part II

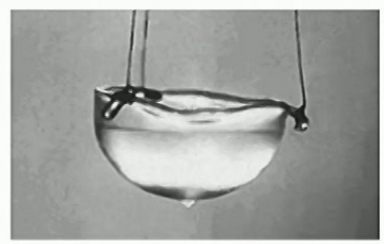
Application:

Entanglement in superfluid helium-4

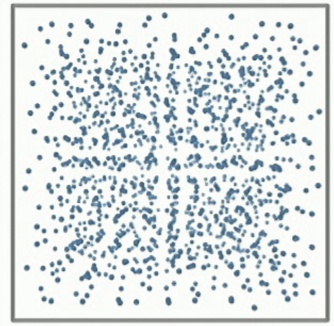
Superfluid helium-4







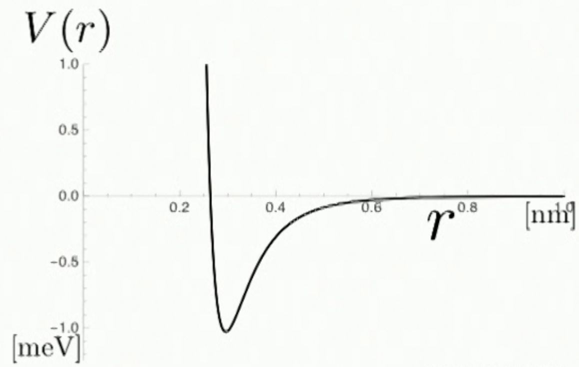
Quantum Monte Caro studies of ^{4}He



System: non-relativistic bosons interacting via a 2-body interatomic potential in 3D

Hamiltonian:

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\nabla_i^2 + \sum_{i<j} V\left(r_{ij}\right)$$

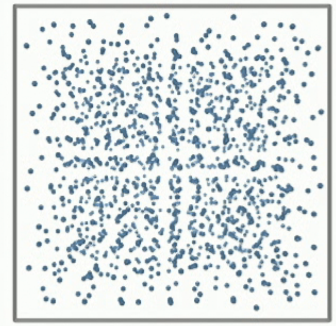


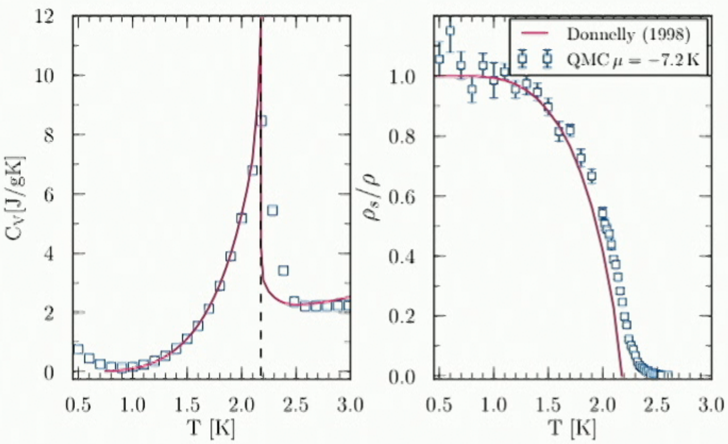
- *ab initio* based interatomic potential
- 4^{th} order propagator
- constant trial wave function
- periodic boundary conditions

Aziz (1979)
Chin (1997)
Jang, Jang & Voth (2001)

Quantum Monte Caro studies of ^{4}He

Microscopic description + QMC:
accurate calculations that *quantitatively* describe experimental ^{4}He systems



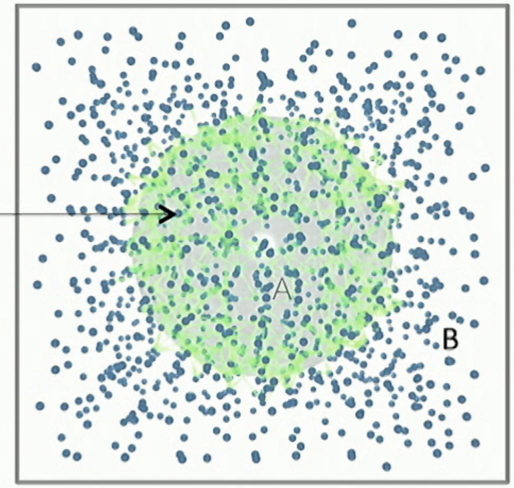


Kulchytskyy & Del Maestro (2013)

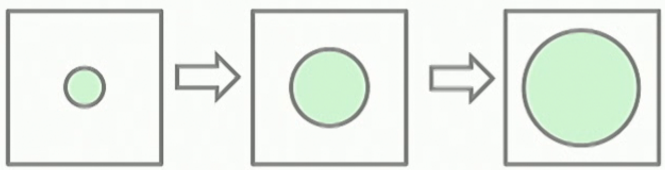
Entanglement in the ground state of ^{4}He

We consider entanglement between two spatial regions of the ground state of ^{4}He

- A: a spherical region, radius R
- S_2(R): entanglement between A and the rest of the box



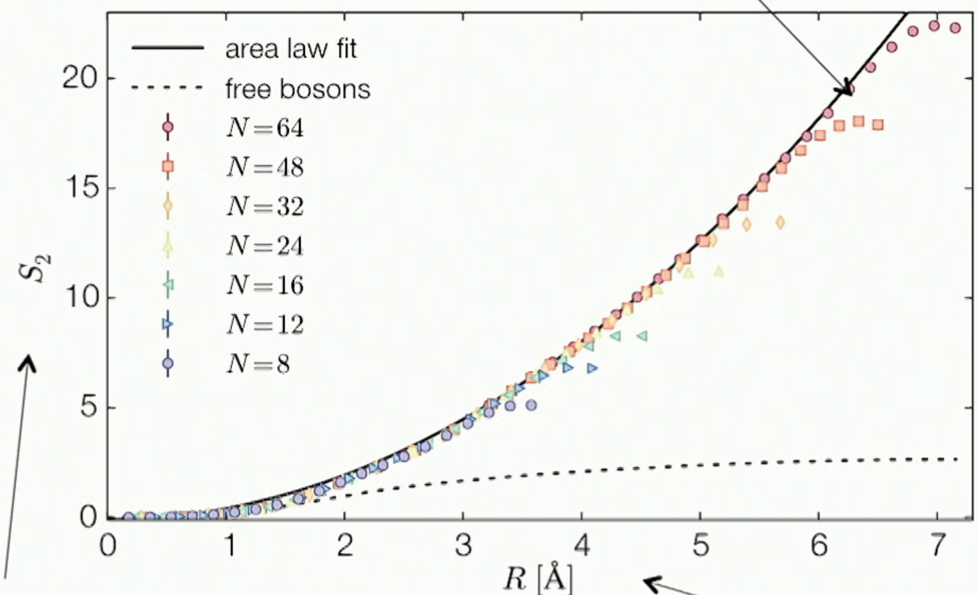
- Scaling: vary the radius:

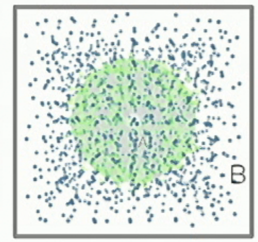
$S_2(R)$ 

CMH, Roy, Melko, Del Maestro, *Nat. Phys.* (2017)

Scaling of entanglement entropy in ^{4}He

Area-law scaling: $S_2(R) \propto 4\pi R^2$





entanglement
entropy

radius

CMH, Roy, Melko, Del Maestro, *Nat. Phys.* (2017)

Area law vs. volume law scaling

Area law

$$S_2(R) = 4\pi a \left(\frac{R}{r_0}\right)^2 + c$$

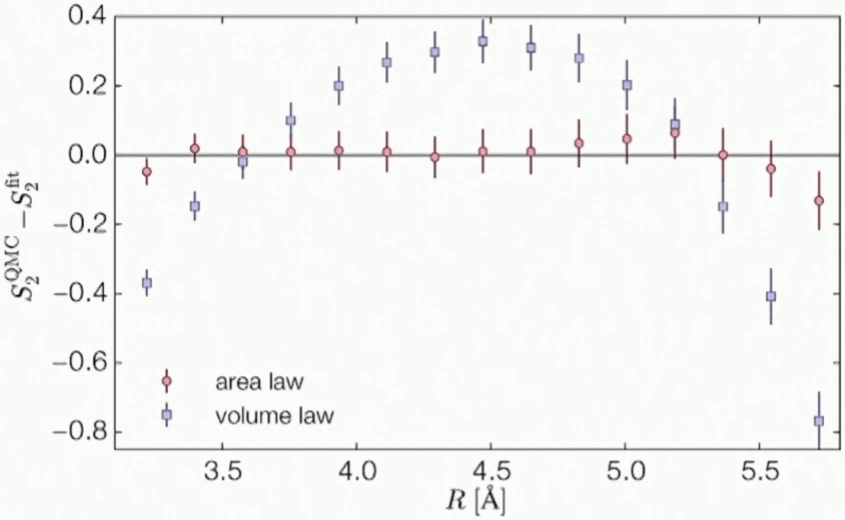
Volume law

$$S_2(R) = \frac{4}{3}\pi a' \left(\frac{R}{r_0}\right)^3 + c'$$

two fit parameters

r_0 : inter-particle separation

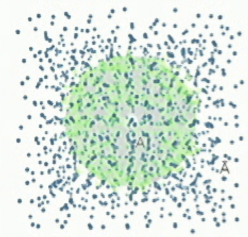
fit residuals



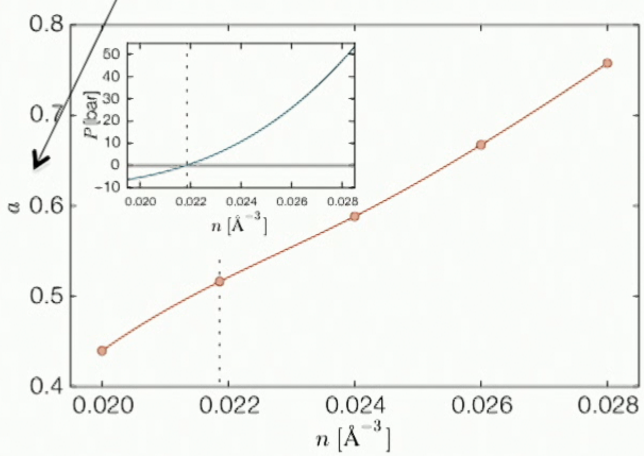
CMH, Roy, Melko, Del Maestro, *Nat. Phys.* (2017)

“Entanglement equation of state”

Area-law scaling: $S_2\left(R\right) = 4\pi a \left(\frac{R}{r_0}\right)^2 + c$



area law coefficient



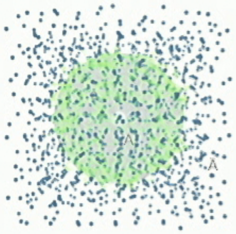
density

entanglement length $\ell_e \equiv \frac{r_0}{\sqrt{a}} \sim 5\text{Å}$

- Entanglement increases with density & pressure
- Pressure could be used to engineer entanglement

CMH, Roy, Melko, Del Maestro, *Nat. Phys.* (2017)

Future work: Beyond area law scaling in superfluids



Corrections to the area law are predicted to display fingerprints of superfluidity:

$$S\left(R\right) = 4\pi a \left(\frac{R}{r_0}\right)^2 + b \log\left[\frac{R}{r_0}\right] + c + \mathcal{O}\left(\frac{r_0^2}{R^2}\right)$$

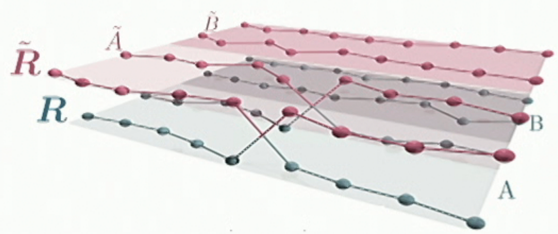
Universal corrections arise due to spontaneous breaking of a continuous symmetry

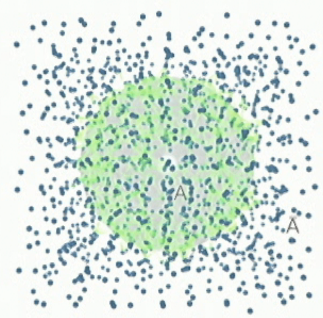
- counts the number of Goldstone Modes
- encodes a central charge of conformal field theory

Metlitski & Grover arXiv:1112.5166

Summary

- We have developed a numerical method to study entanglement in real quantum fluids
- We have demonstrated **area law** scaling of entanglement entropy in superfluid helium-4
- Future studies may help us understand how entanglement can be manipulated in experimental systems





Thank You!

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Reference:

Herdman, Roy, Melko & Del Maestro, ***Nature Physics*** (2017)
doi:10.1038/nphys4075

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