Title: Z_2 topological order near the Neel state of the square lattice antiferromagnet

Date: May 18, 2017 10:00 AM

URL: http://pirsa.org/17050081

Abstract: <p>We classify quantum states proximate to the semiclassical Neel state of the spin S=1/2 square lattice antiferromagnet with two-spin near-neighbor and four-spin ring exchange interactions. Motivated by a number of recent experiments on the cuprates and the iridates, we examine states with Z_2 topological order, an order which is not present in the semiclassical limit. Some of the states break one or more of reflection, time-reversal, and lattice rotation symmetries, and can account for the observations. We discuss implications for the pseudogap phase.</p>

Shubhayu Chatterjee

Mathias Scheurer

Alex Thomson

1. Introduction to Z_2 topological order

   Z_2 and U(1) lattice gauge theories

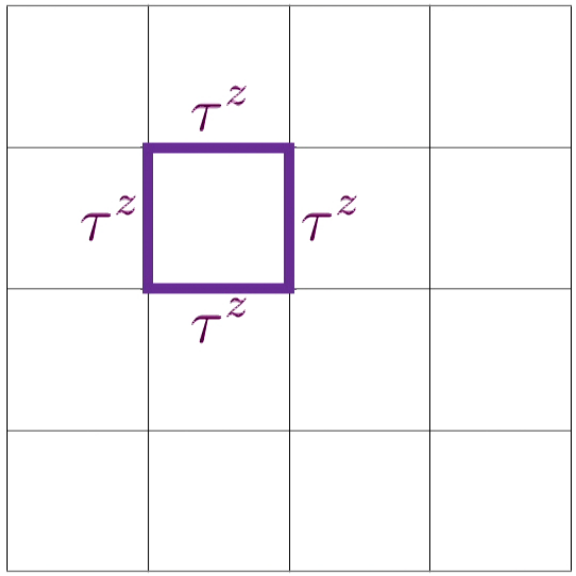
2. The uses of Z_2 topological order

   (A) Intertwining topological order and symmetry breaking: CP^1 theory of the square lattice antiferromagnet

   (B) The pseudogap: metals with non-Luttinger volume Fermi surfaces

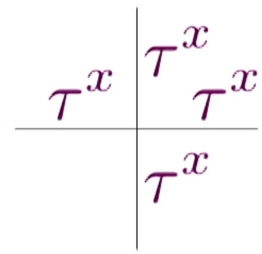
Z_2 lattice gauge theory

(Wegner, 1971)



$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x$$

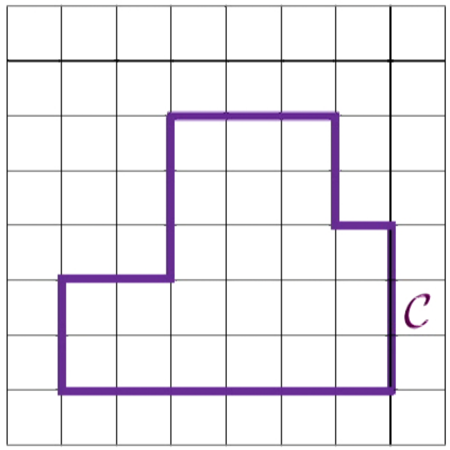
$G_i =$



Gauss's Law: $[H, G_i] = 0$, $\boxed{G_i = 1}$

Z_2 lattice gauge theory

(Wegner, 1971)



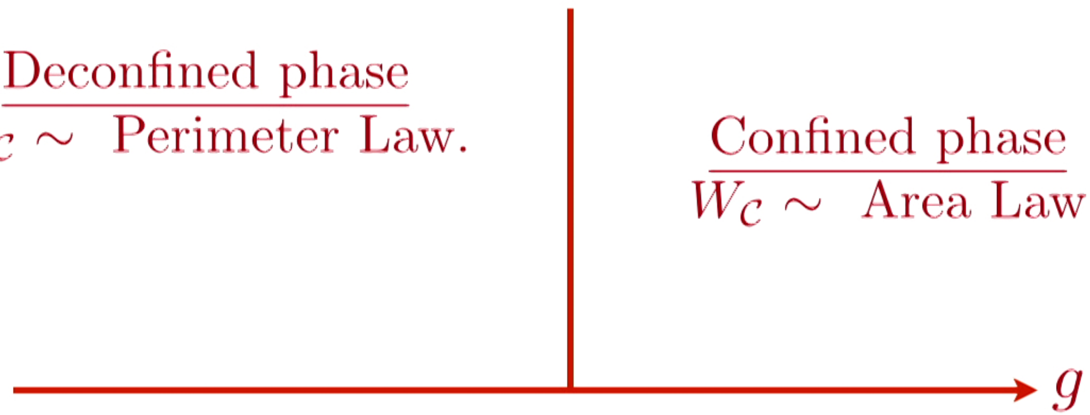
$$W_{\mathcal{C}} = \prod_{\mathcal{C}} \tau^z$$

Deconfined phase

$W_{\mathcal{C}} \sim$ Perimeter Law.

Confined phase

$W_{\mathcal{C}} \sim$ Area Law



Z_2 lattice gauge theory

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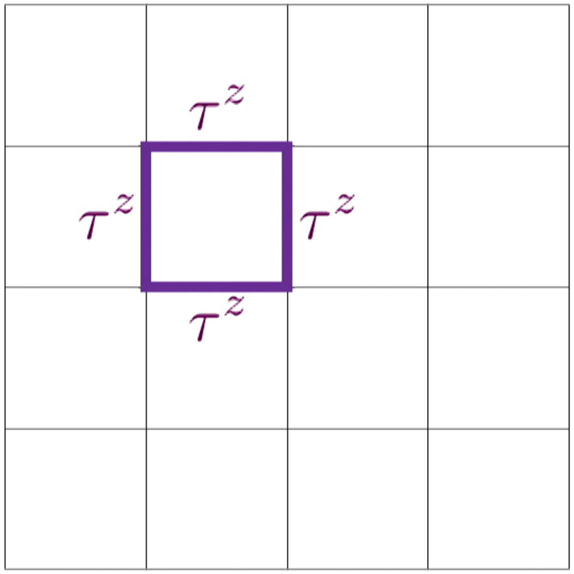

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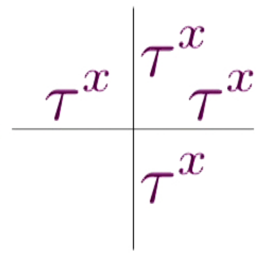
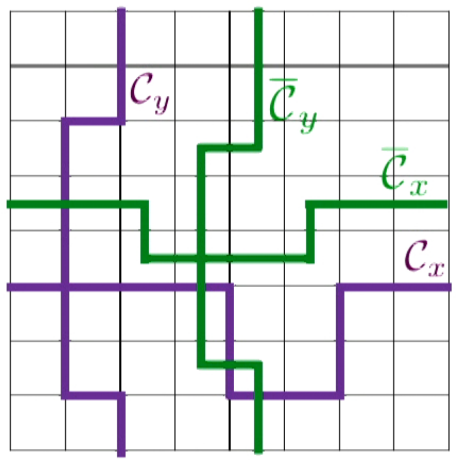


Gauss's Law: $[H, G_i] = 0$, $\boxed{G_i = 1}$

Topological order

$$V_x = \prod_{\overline{\mathcal{C}}_x} \tau^x \quad , \quad V_y = \prod_{\overline{\mathcal{C}}_y} \tau^x$$

$$W_x = \prod_{\mathcal{C}_x} \tau^z \quad , \quad W_y = \prod_{\mathcal{C}_y} \tau^z$$



Torus

$$V_x W_y = -W_y V_x \quad , \quad V_y W_x = -W_x V_y$$

and all other pairs commute.

On a torus, there are two additional independent operators, V_x and V_y which commute with the Hamiltonian:

$$[H, V_x] = [H, V_y] = 0$$

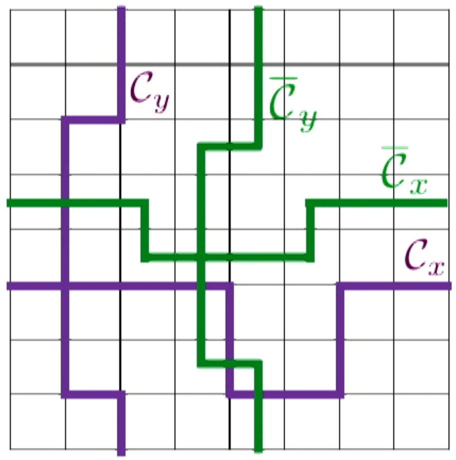
<u>Deconfined phase</u>
$W_{\mathcal{C}} \sim$ Perimeter Law.
Deconfined Z_2 flux
$W = -1$ particles

<u>Confined phase</u>
$W_{\mathcal{C}} \sim$ Area Law

g

Topological order

$$V_x = \prod_{\overline{\mathcal{C}}_x} \tau^x \quad , \quad V_y = \prod_{\overline{\mathcal{C}}_y} \tau^x$$



$$W_x = \prod_{\mathcal{C}_x} \tau^z \quad , \quad W_y = \prod_{\mathcal{C}_y} \tau^z$$

$$V_x W_y = -W_y V_x \quad , \quad V_y W_x = -W_x V_y$$

and all other pairs commute.

On a torus, there are two additional independent operators, V_x and V_y which commute with the Hamiltonian:

$$[H, V_x] = [H, V_y] = 0$$

Deconfined phase.
4-fold degenerate ground state: $V_x = \pm 1$, $V_y = \pm 1$.
Can take linear combinations to make eigenstates with $W_x = \pm 1$, $W_y = \pm 1$.
Topological order

Confined phase.
Unique ground state has $V_x = 1$, $V_y = 1$.
No topological order

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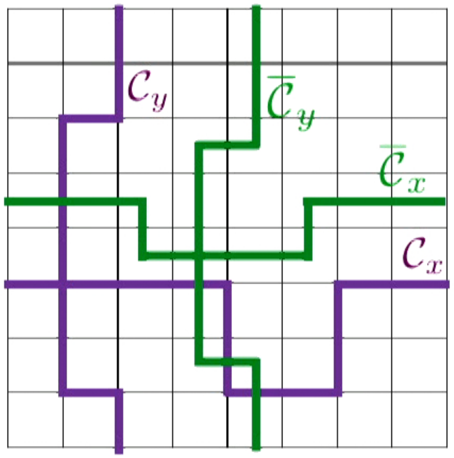
Topological order

$$V_x = \prod_{\overline{\mathcal{C}}_x} \tau^x \quad , \quad V_y = \prod_{\overline{\mathcal{C}}_y} \tau^x$$



$$W_x = \prod_{\mathcal{C}_x} \tau^z \quad , \quad W_y = \prod_{\mathcal{C}_y} \tau^z$$

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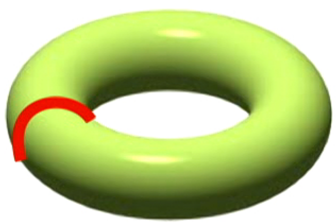

(N. Read and S.S., 1991 Freedman, Nayak, Shtengel, Walker, Wang, 2003)

Topological quantum field theory describes degenerate states with Z_2 flux $W = \pm 1$ through the holes of the torus

Confined phase. Unique ground state has $V_x = 1$, $V_y = 1$. No topological order

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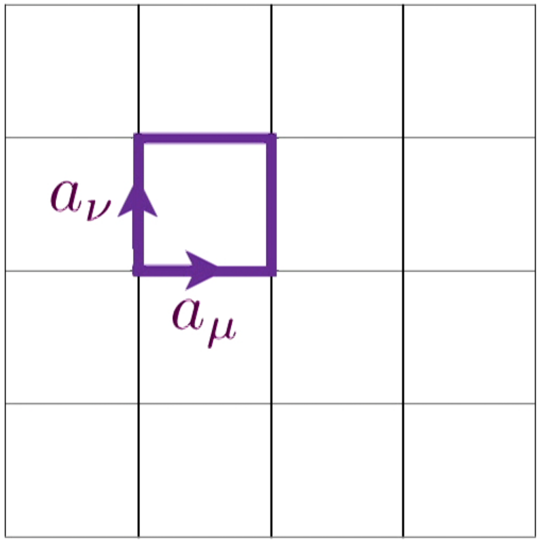
This criterion can distinguish the phases when dynamical (or even gapless) matter fields are present

Compact U(1) lattice gauge theory



(Fradkin and Shenker, 1979)

$$\mathcal{S} = -\frac{1}{g}\sum_{\square}\cos\left(\Delta_\mu a_\nu - \Delta_\nu a_\mu\right) - \sum_{i,\mu}\cos\left(\Delta_\mu \theta - 2a_\mu\right)$$

Same phases as Z_2 gauge theory

Deconfined phase
Monopoles suppressed.
Deconfined π (mod 2π) flux vortices

Confined phase
Monopoles proliferate

g

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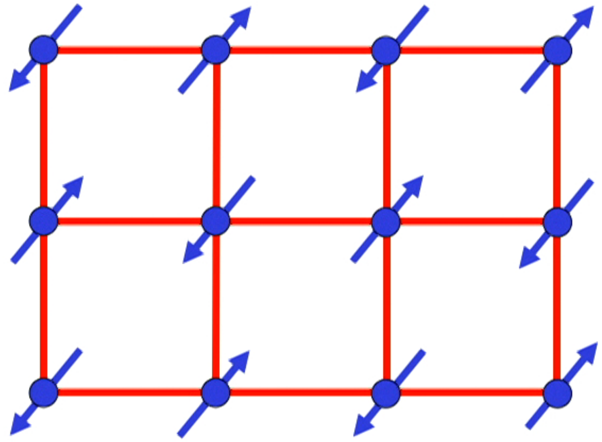
   (B) The pseudogap: metals with non-Luttinger volume Fermi surfaces



Insulating Antiferromagnet

Néel order parameter $\mathbf{n}(x_i, \tau) = \eta_i \mathbf{S}_i(\tau)$, where $\eta_i = \pm 1$ on two sublattices.
O(3) non-linear sigma model:

$$S = \frac{1}{2g} \int d^2x dt \, (\partial_\mu \mathbf{n})^2 \quad , \quad \mathbf{n}^2 = 1.$$

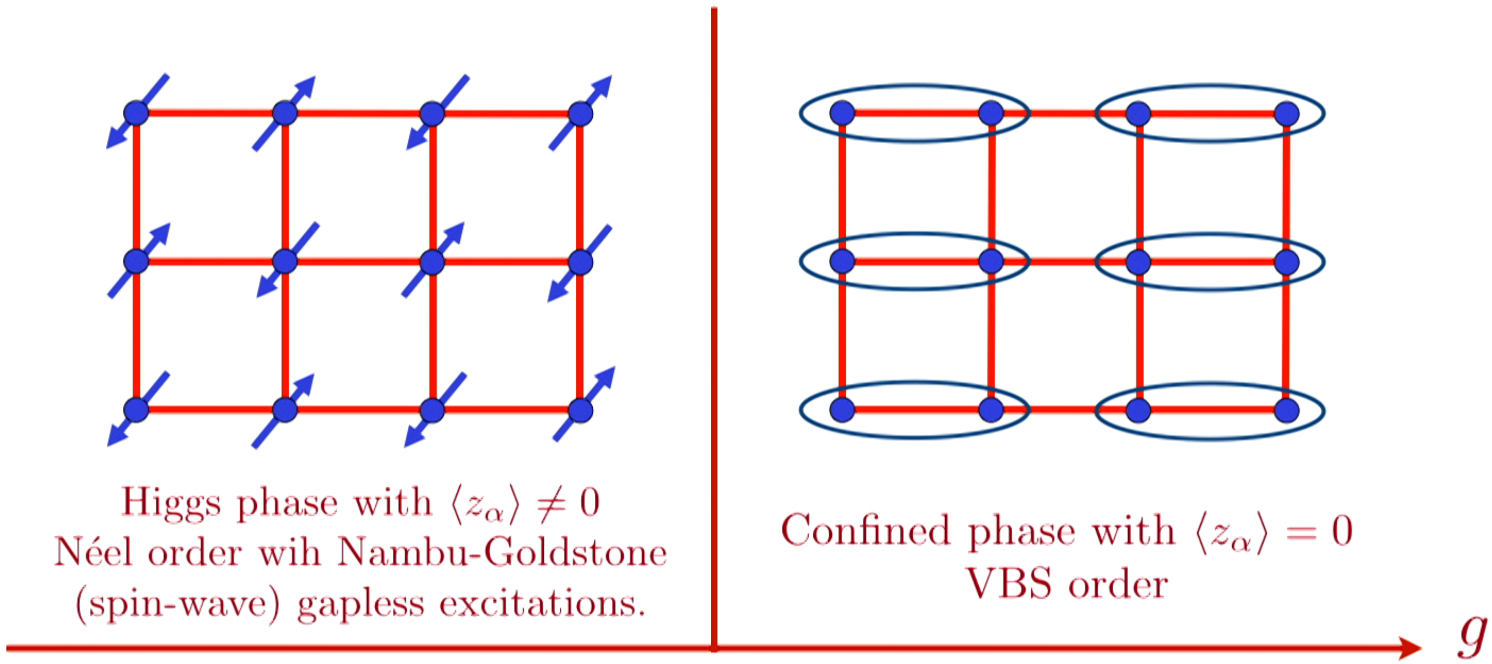
$\mathbb{CP}^1$ model: use $\mathbf{n} = z^*_\alpha \vec{\sigma}_{\alpha\beta} z_\beta$ with $\alpha, \beta = \uparrow, \downarrow$, and then

$$S = \frac{1}{g} \int d^2x dt \, |(\partial_\mu - i a_\mu) z_\alpha|^2 \quad , \quad |z_\alpha|^2 = 1 \, ,$$

where a_μ is an emergent U(1) gauge field.

Theory for $S = 1/2$ antiferromagnet also has spin Berry phase terms

$$S = \frac{1}{g}\int d^2x dt \, |(\partial_\mu - i a_\mu) z_\alpha|^2 + \sum_i \int dt \, \eta_i a_{it}$$



(N. Read and S.S., 1989; S.S. and R. Jalabert, 1990)

To obtain a Z_2 deconfined phase, we need to condense a Higgs field with U(1) charge 2. The simplest route is to condense spin-singlet pairs of long-wavelength spinons, z_α. There are two candidates for such Higgs fields, corresponding to the operators

$$P \sim \varepsilon_{\alpha\beta} z_\alpha \partial_t z_\beta \quad , \quad Q_a \sim \varepsilon_{\alpha\beta} z_\alpha \partial_a z_\beta \, ,$$

with $a = x, y$. By gauge-invariance and symmetry, we obtain the following effective action with additional tuning parameters s_1 and s_2

$$\begin{aligned} S =& \frac{1}{g} \int d^2x dt \, |(\partial_\mu - i a_\mu) z_\alpha|^2 + \sum_i \int dt \, \eta_i a_{it} \\ & \int d^2 x dt \Big[|(\partial_\mu - 2i a_\mu) P|^2 + |(\partial_\mu - 2i a_\mu) Q_a|^2 \\ & + \lambda_1 P^* \, \varepsilon_{\alpha\beta} z_\alpha \partial_t z_\beta + \lambda_2 Q_a^* \cdot \varepsilon_{\alpha\beta} z_\alpha \partial_a z_\beta + \text{H.c.} \\ & - s_1 |P|^2 - s_2 |Q_a|^2 - u_1 |P|^4 - u_2 (|Q_a|^2)^2 + \ldots \Big] \end{aligned}$$

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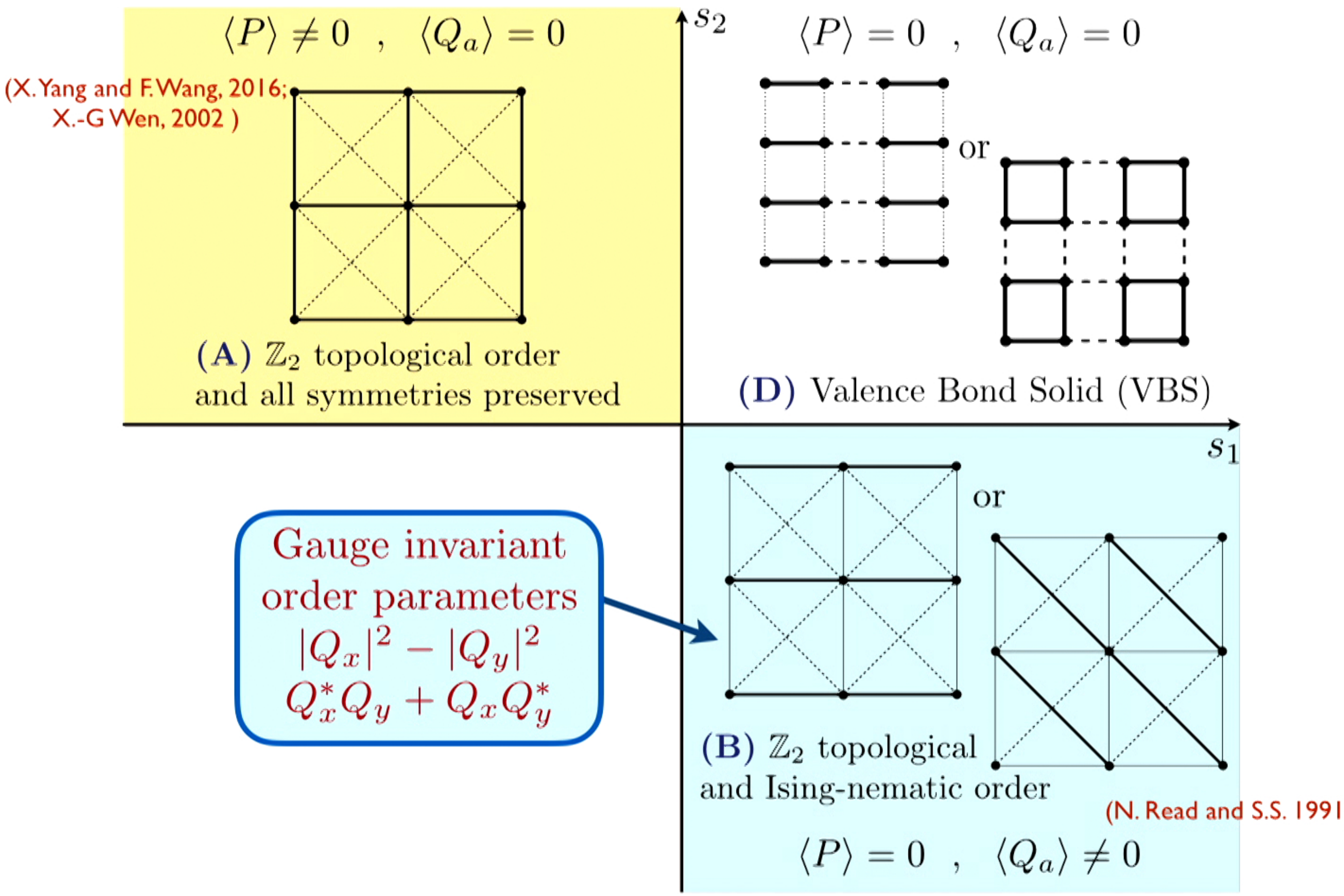
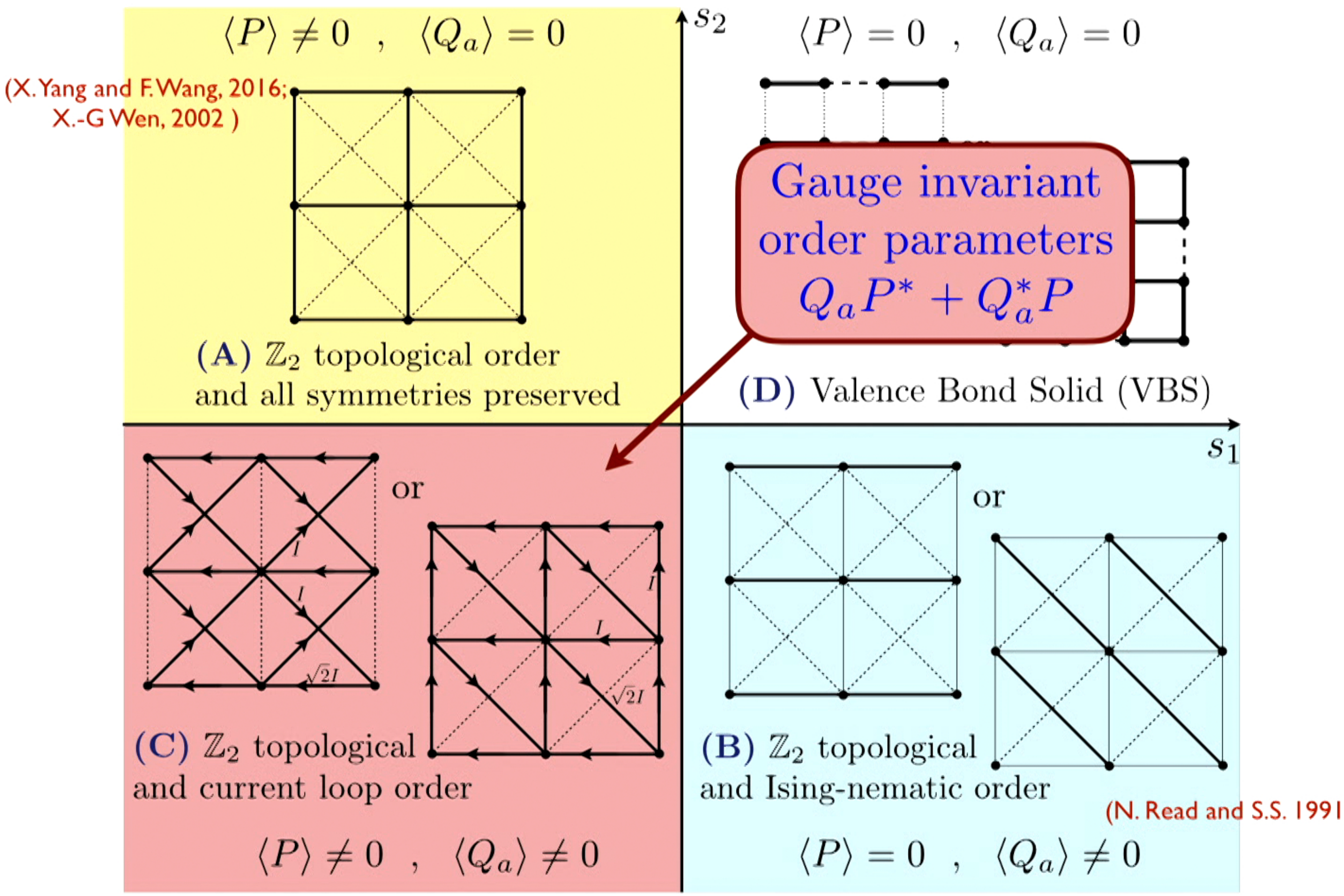
Phase diagram at large g with $\langle z_\alpha \rangle = 0$



Phase diagram at large g with $\langle z_\alpha \rangle = 0$

Three phases with Z_2 topological order



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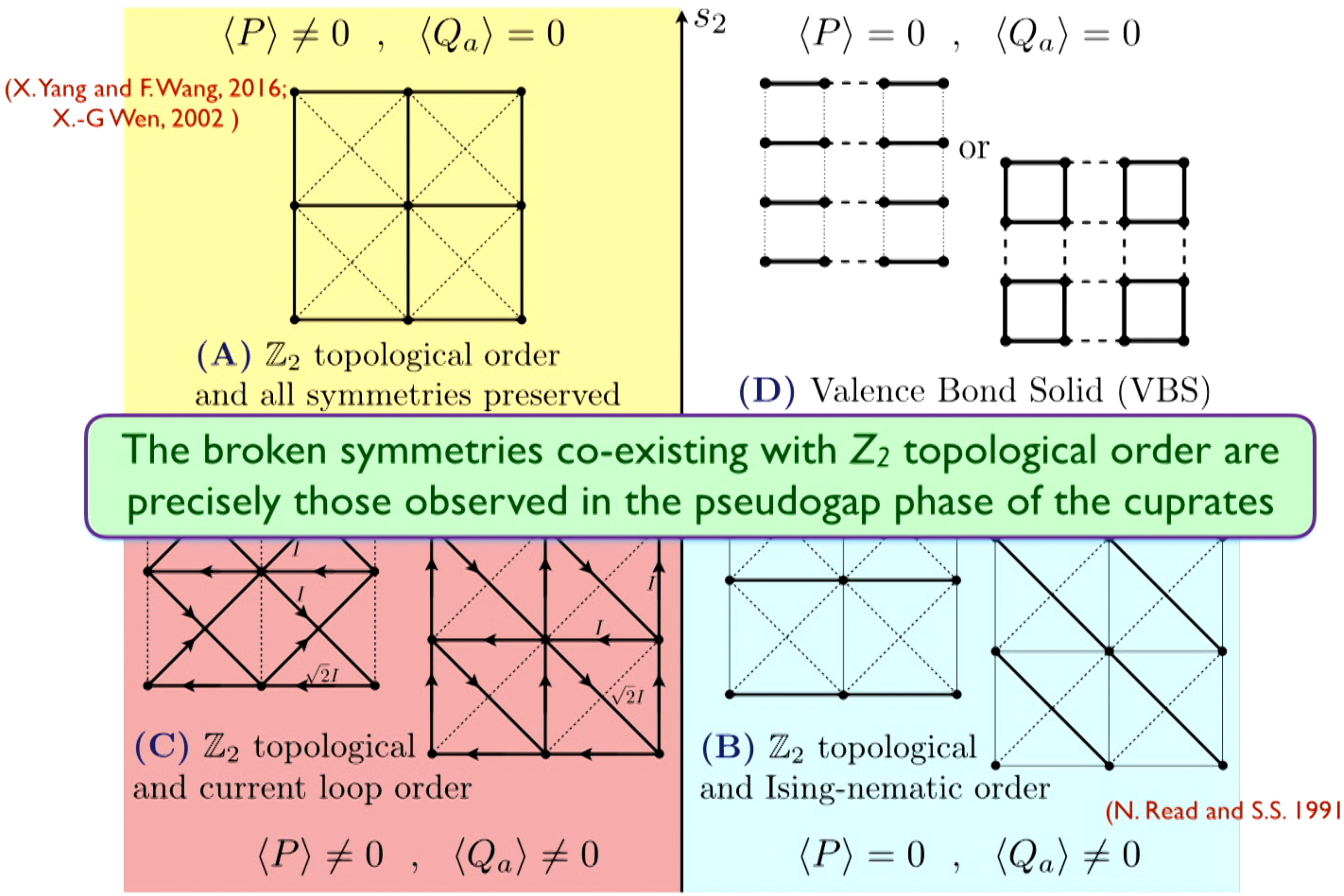
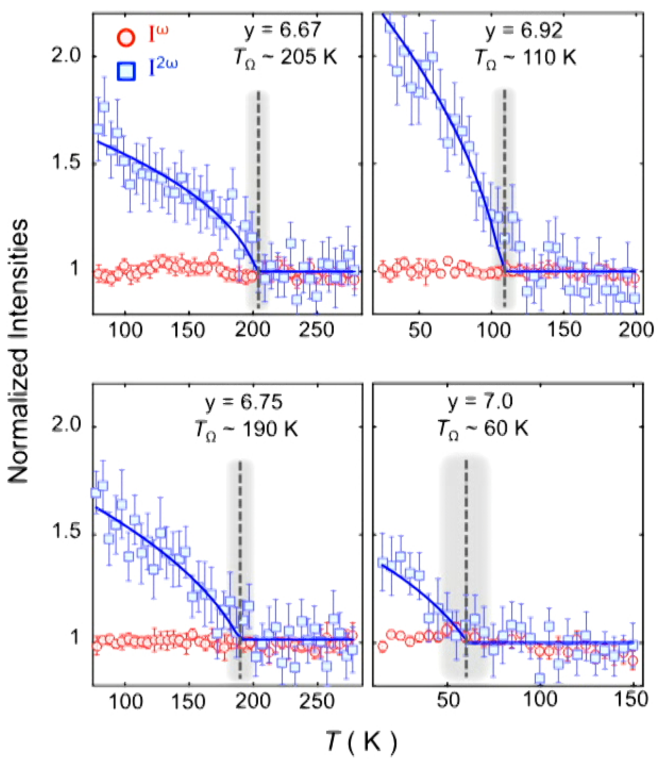
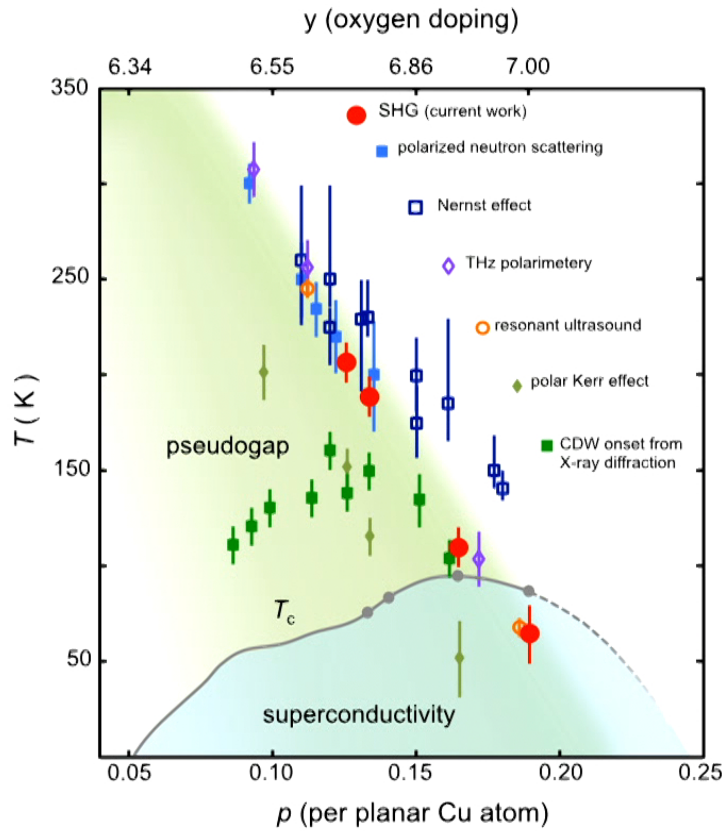


Broken inversion symmetry below T^* in $YBa_2Cu_3O_y$

No anomalies at T_c, T_{CDW} or T_{Kerr}

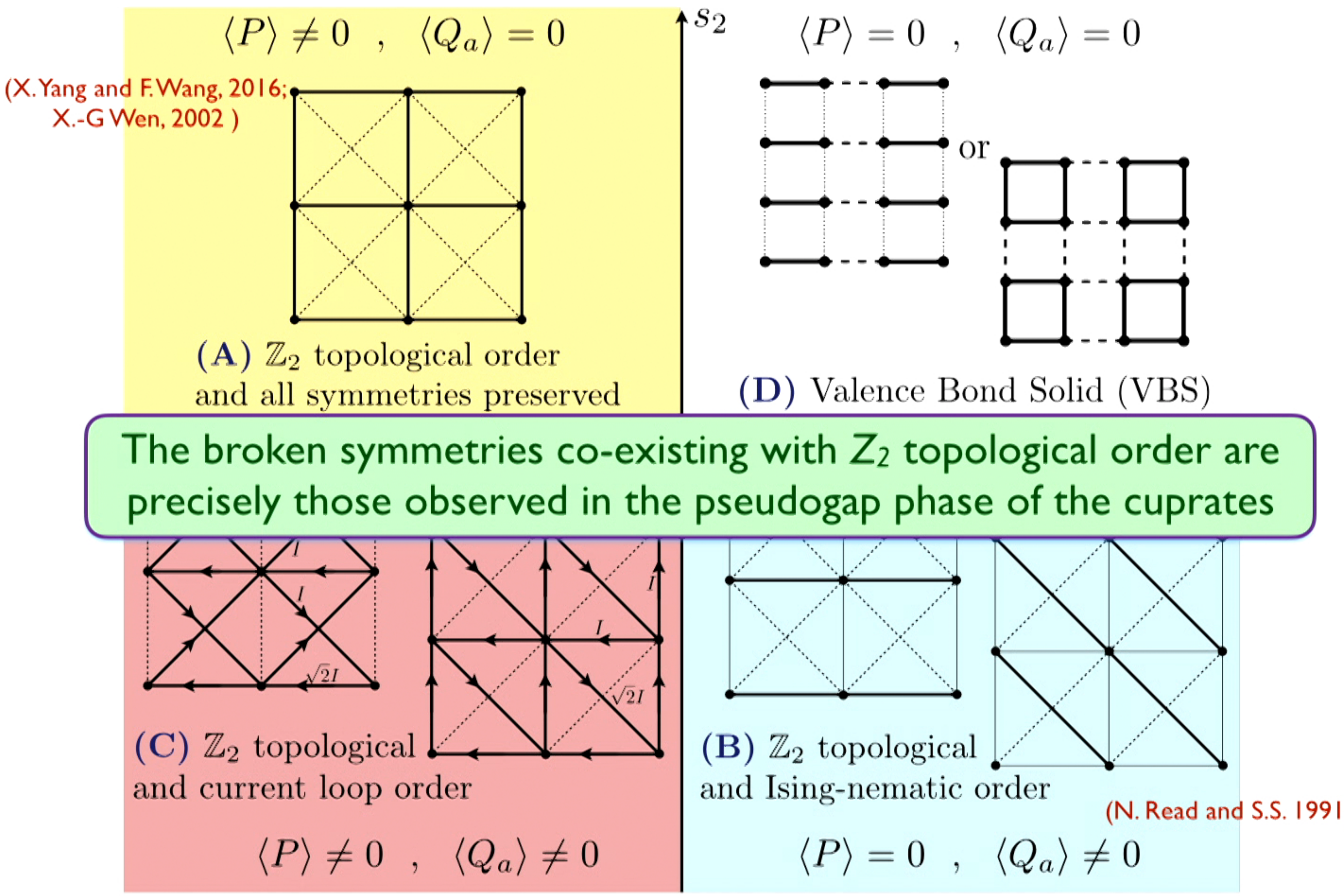




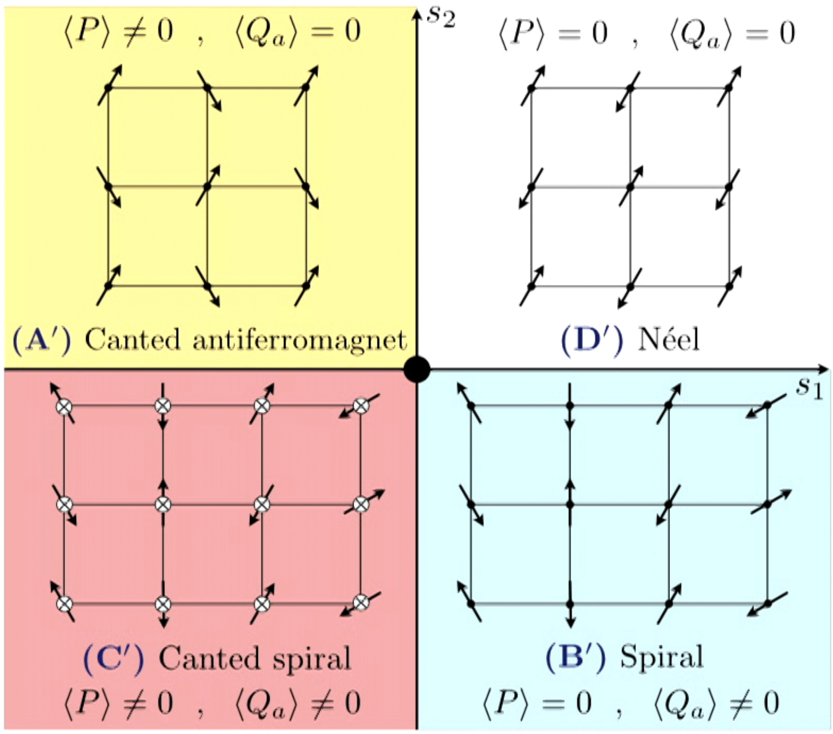
L. Zhao, C. A. Belvin, R. Liang, D. A. Bonn, W. N. Hardy, N. P. Armitage, and D. Hsieh, Nature Physics **13**, 250 (2017)

Phase diagram at large g with $\langle z_\alpha \rangle = 0$

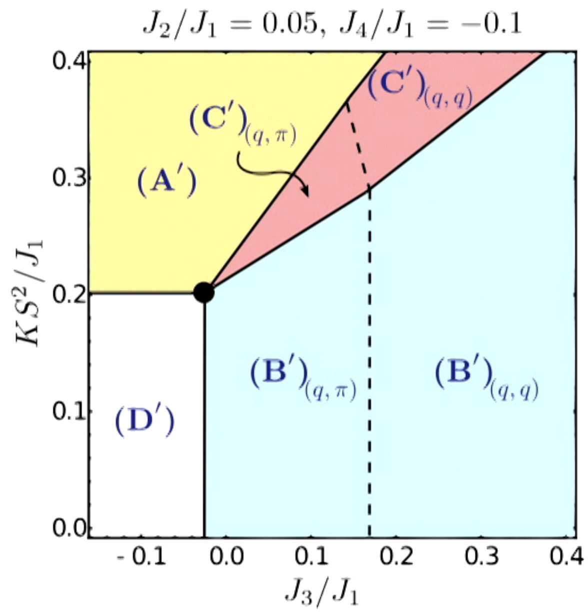
Three phases with Z_2 topological order



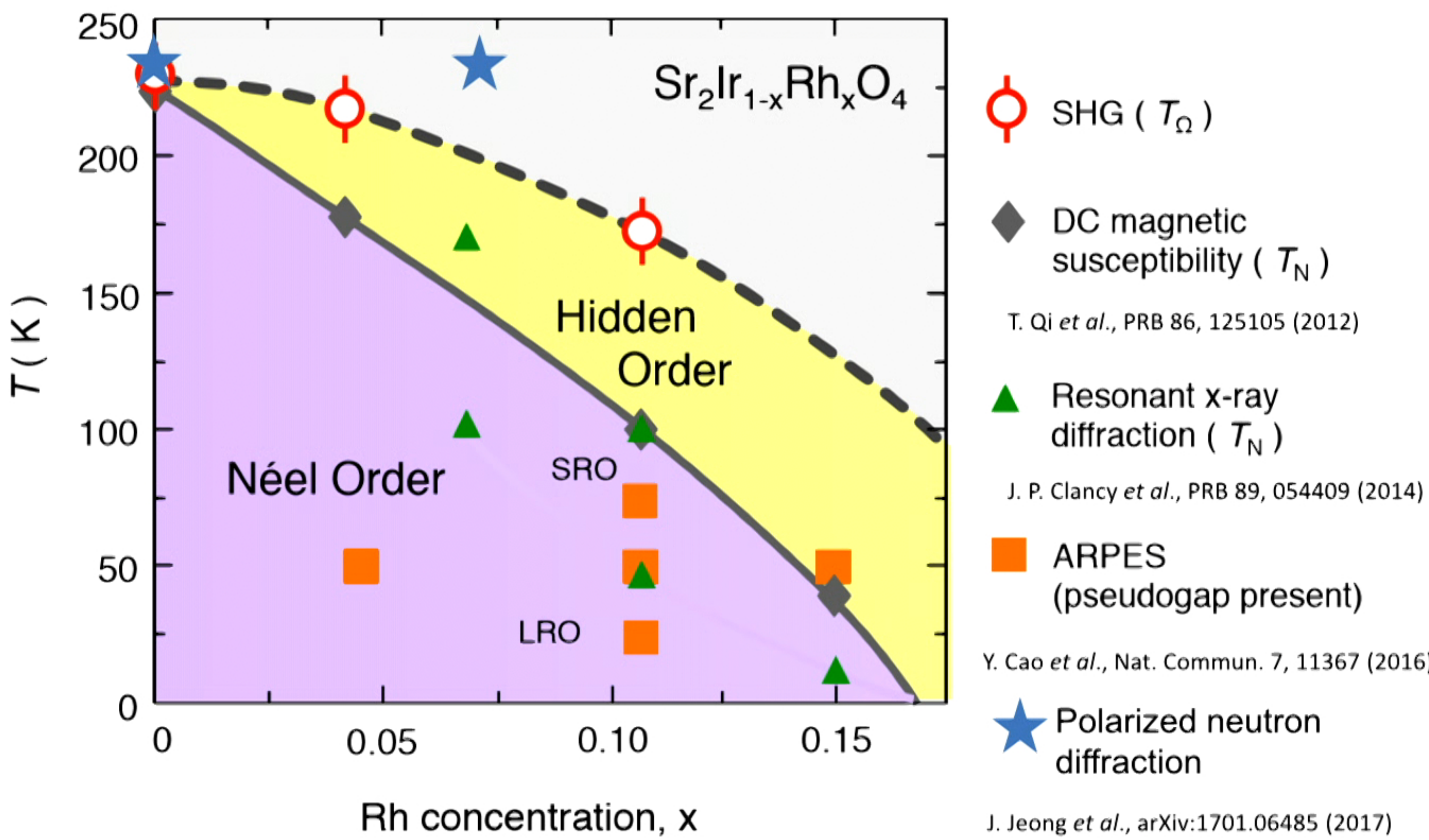
Phase diagram at small g with $\langle z_\alpha \rangle \neq 0$



Classical phase diagram of antiferromagnet with near-neighbor exchange J_p and ring-exchange K



Phase diagram of $Sr_2Ir_{1-x}Rh_xO_4$



L. Zhao, D. H. Torchinsky, H. Chu, V. Ivanov, R. Lifshitz, R. Flint, T. Qi, G. Cao, and D. Hsieh, Nature Physics **12**, 32 (2016)

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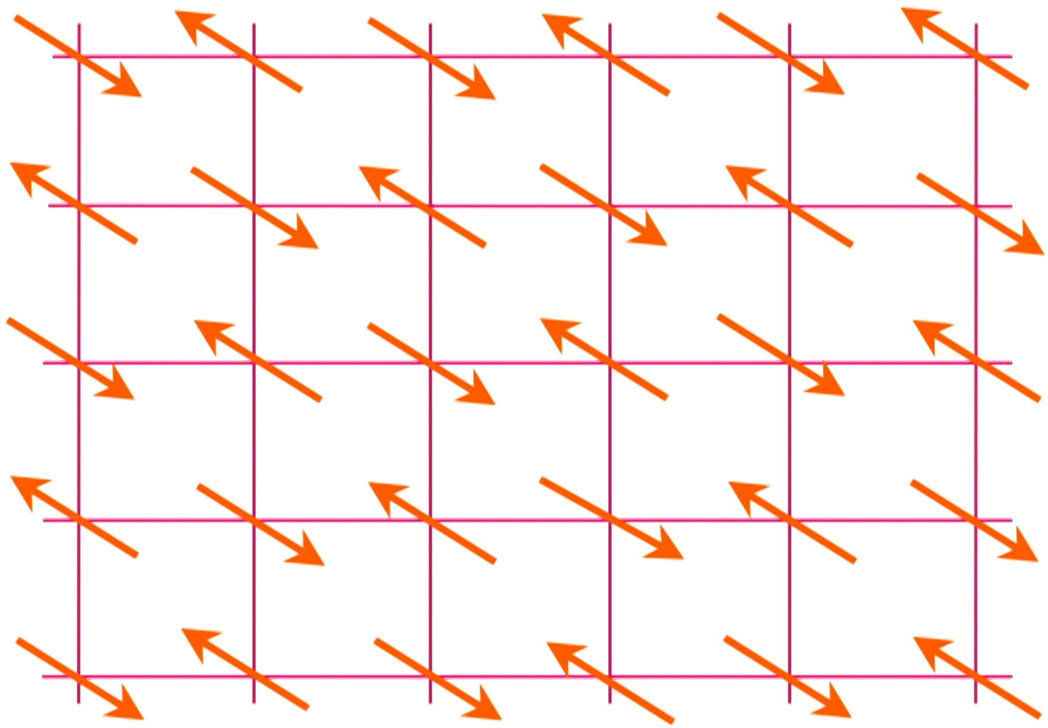
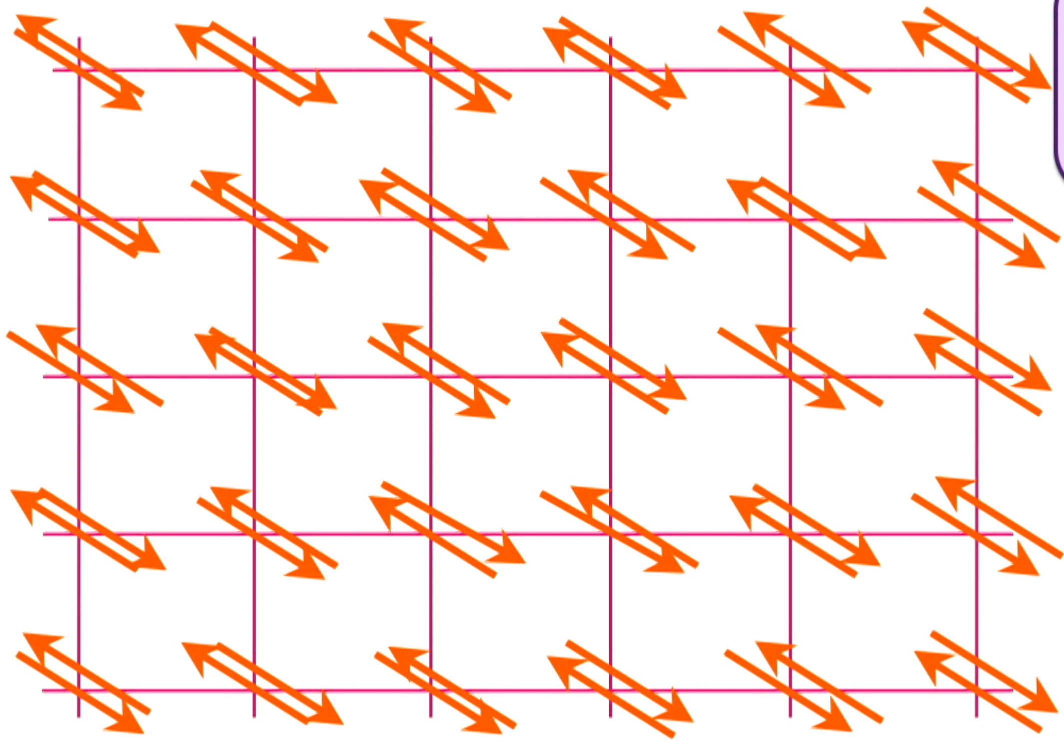
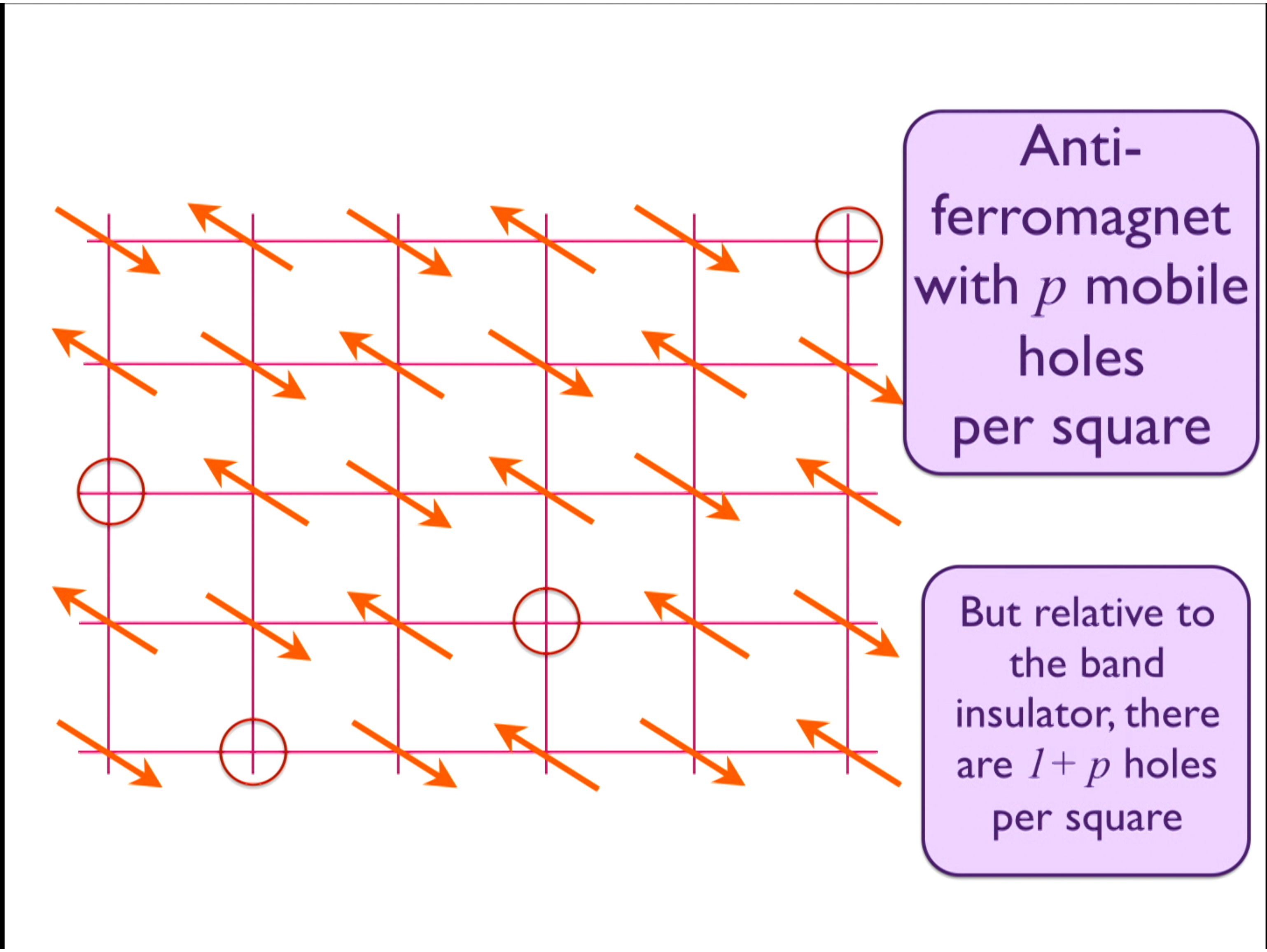


“Undoped” insulating anti-ferromagnet



Filled Band


Anti-
ferromagnet
with p mobile
holes
per square
But relative to
the band
insulator, there
are $1+p$ holes
per square

Begin with the "spin-fermion" model. **Electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = -\sum_{i,\rho} t_\rho \left(c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) - \mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\rm int}$$

are coupled to an **antiferromagnetic order parameter $\Phi^\ell(i)$,** $\ell = x, y, z$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i \Phi^\ell(i) c_{i,\alpha}^\dagger \sigma^\ell_{\alpha\beta} c_{i,\beta} + V_\Phi$$

where $\eta_i = \pm 1$ on the two sublattices.

When $\Phi^\ell(i)$ =constant independent of i, we have long-range AFM, and a gap in the fermion spectrum at the anti-nodes.

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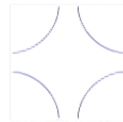

Fermi surface size $1+p$

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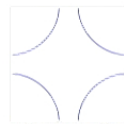

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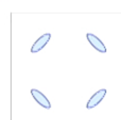
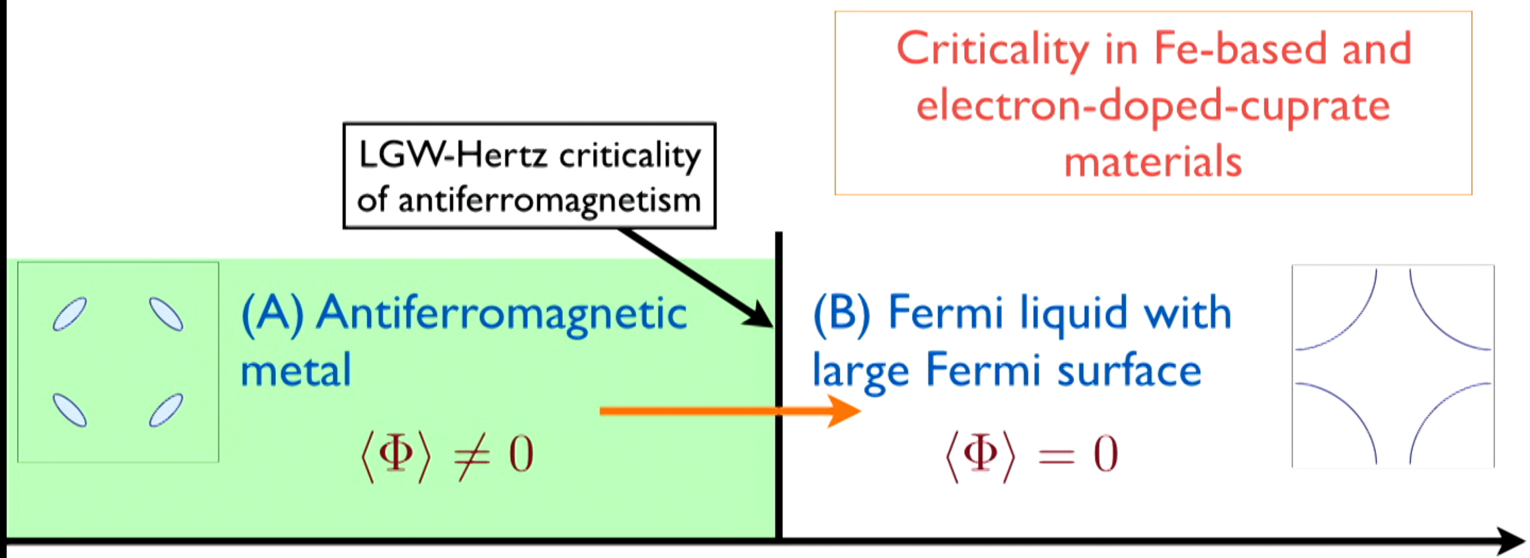


Fermi surface size p


Criticality in Fe-based and electron-doped-cuprate materials
LGW-Hertz criticality of antiferromagnetism
(A) Antiferromagnetic metal
$\langle \Phi \rangle \neq 0$
(B) Fermi liquid with large Fermi surface
$\langle \Phi \rangle = 0$

Can we get a stable zero temperature state with "fluctuating antiferromagnetism" and a Fermi surface of size p *i.e.* a pseudogap metal ?

Yes, provided the metal has topological order

T. Senthil, M. Vojta and S. Sachdev, PRB **69**, 035111 (2004)

For fluctuating antiferromagnetism, we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^\ell \Phi^\ell(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the AFM order in the rotating reference frame. Note that this representation is ambiguous up to a SU(2) gauge transformation, V_i

$$\left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right) \rightarrow V_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right)$$

$$R_i \rightarrow R_i V_i^\dagger$$

$$\sigma^a H^a(i) \rightarrow V_i \, \sigma^b H^b(i) \, V_i^\dagger.$$

Fluctuating antiferromagnetism

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the AFM order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi^\dagger_{i,s} \psi_{i+\boldsymbol{v}_\rho,s} + \psi^\dagger_{i+\boldsymbol{v}_\rho,s} \psi_{i,s} \right) - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'} + V_H$$

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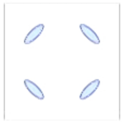
$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'} + V_H$$

IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of fluctuating AFM will inherit the anti-nodal gap of the electrons in the presence of static AFM.

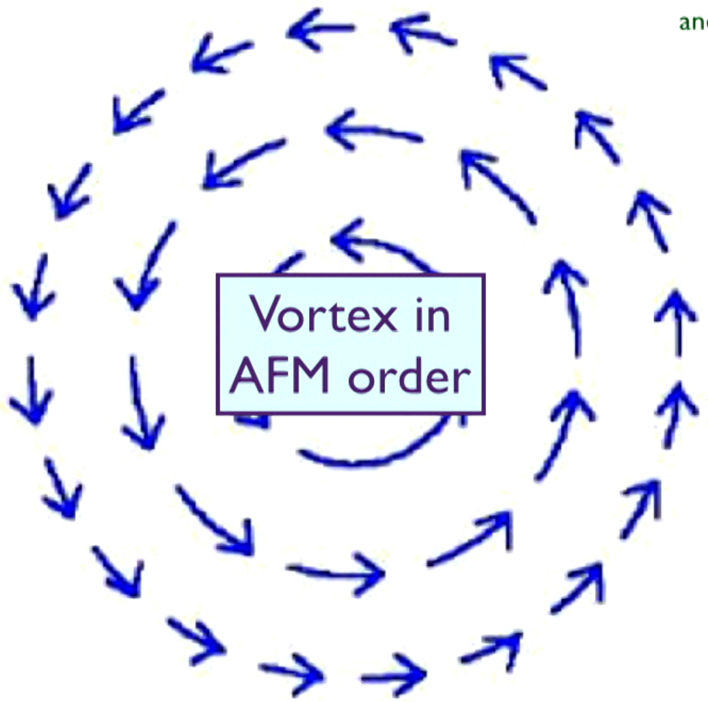


Fermi surface size p

Fluctuating antiferromagnetism

We cannot always find a single-valued SU(2) rotation R_i to make the Higgs field $H^a(i)$ a constant !

A.V. Chubukov, T. Senthil and S. Sachdev, PRL **72**, 2089 (1994);
S. Sachdev, E. Berg, S. Chatterjee, and Y. Schattner, PRB **94**, 115147 (2016)



Topological order

We cannot always find a single-valued SU(2) rotation R_i to make the Higgs field $H^a(i)$ a constant !

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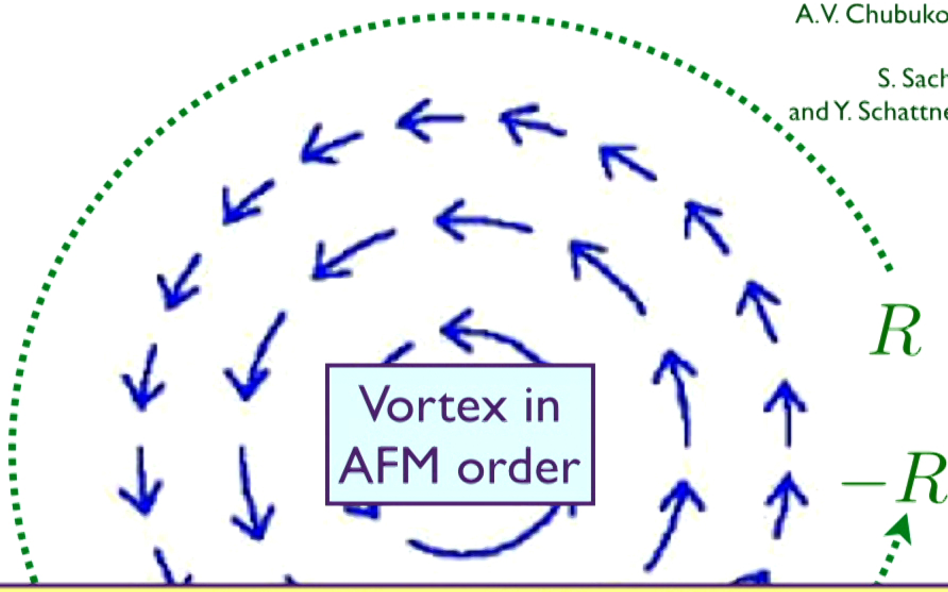
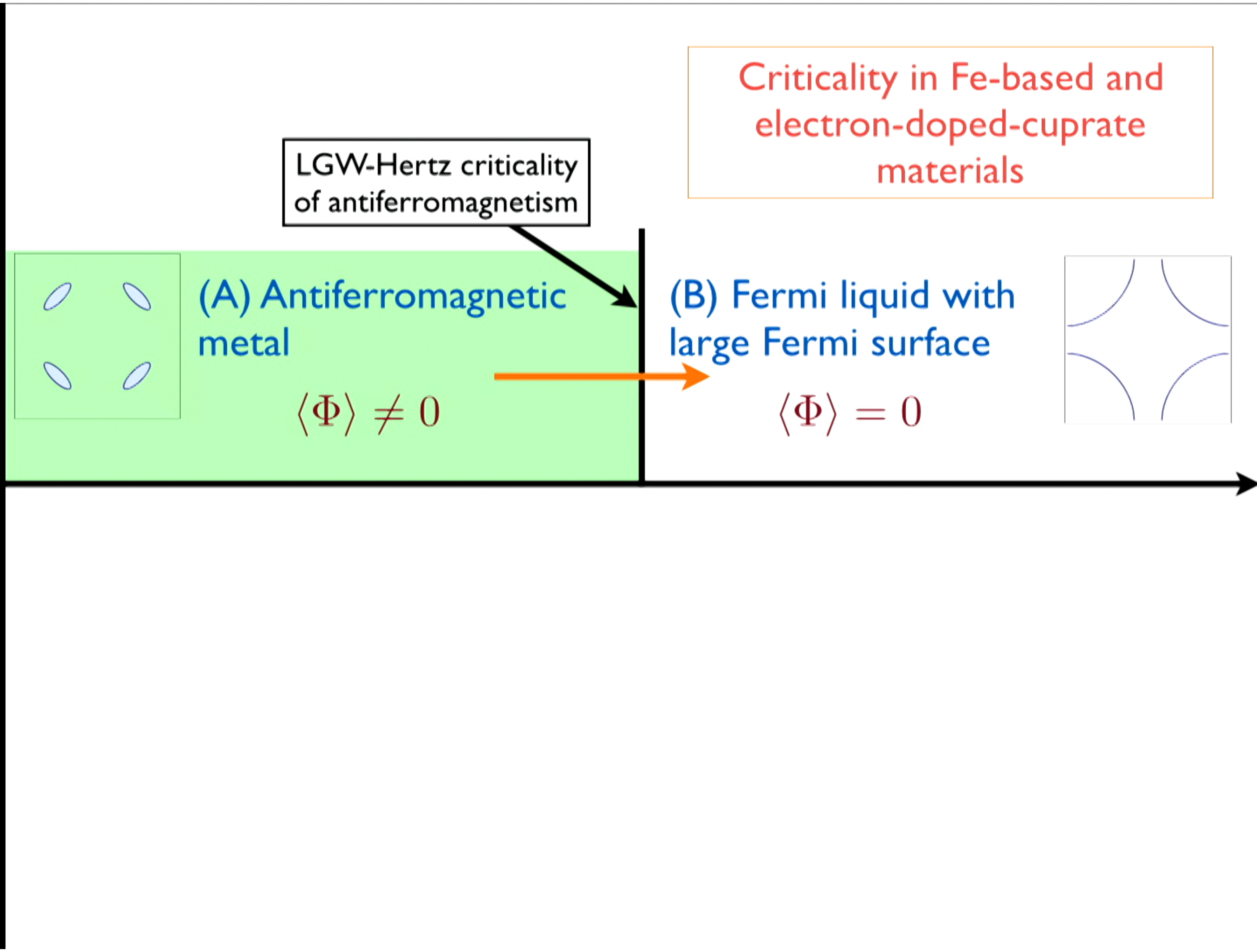
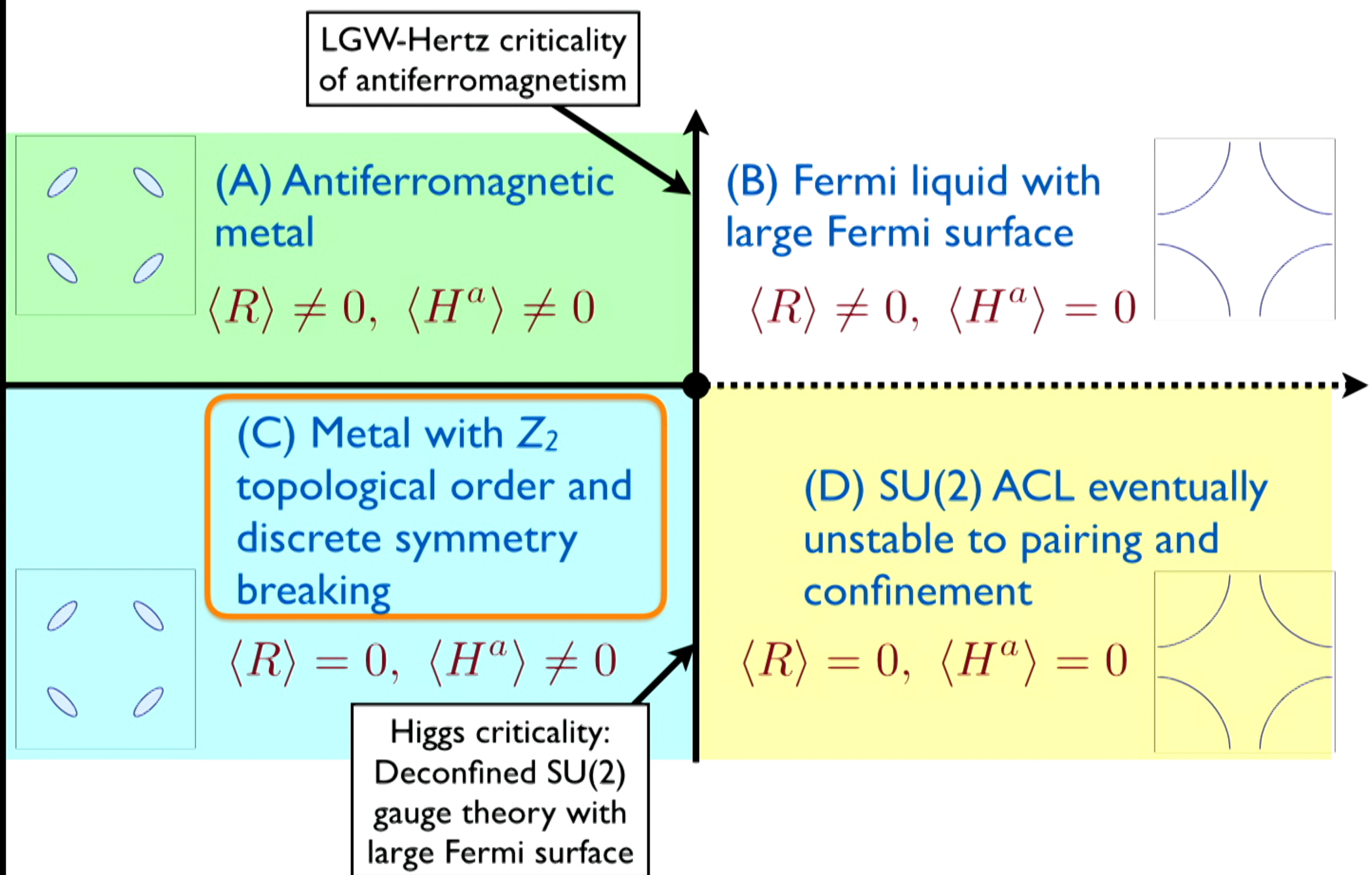


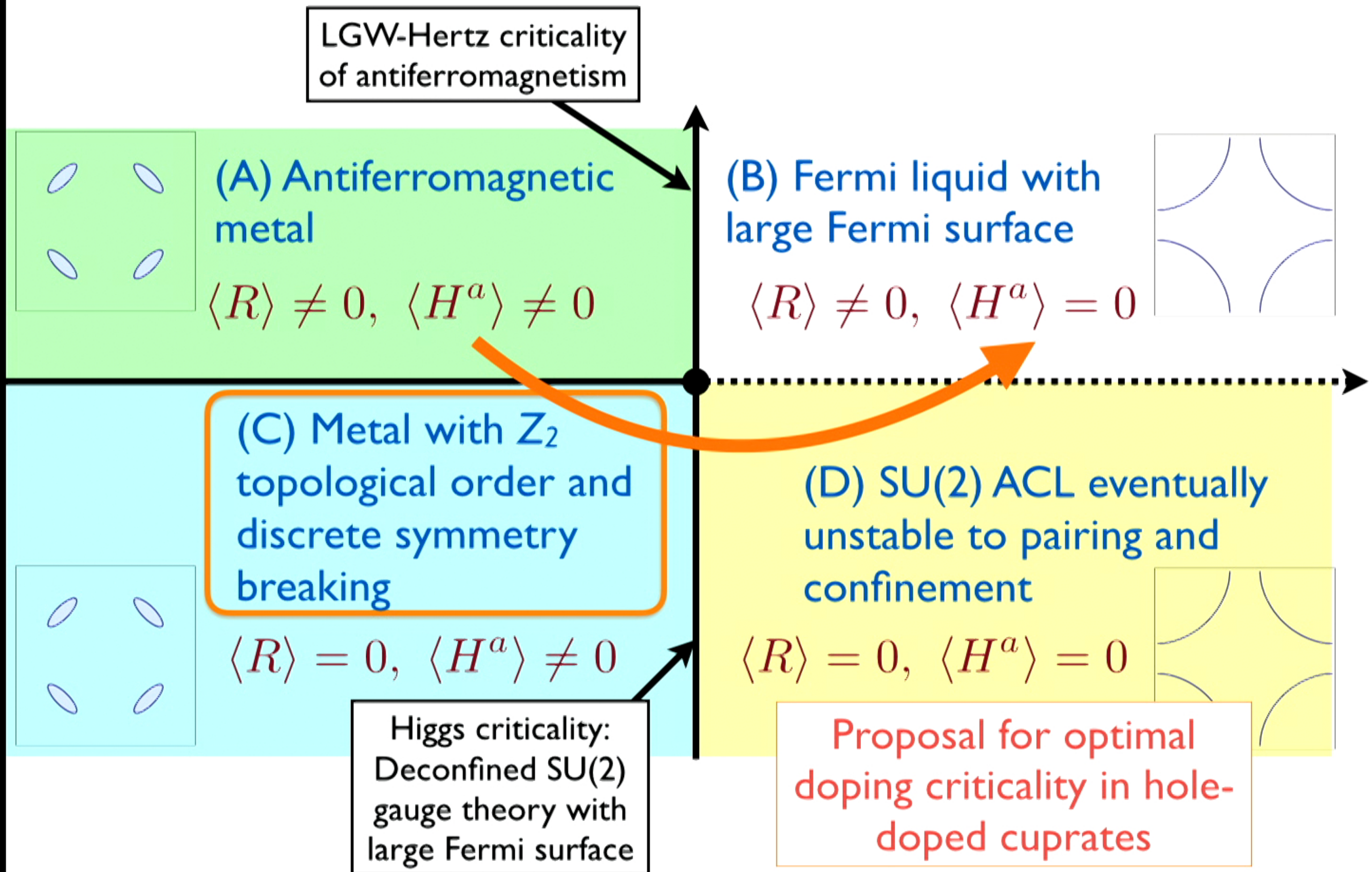
The Higgs field $\in$ SO(3).
Vortices associated wi th $\pi_1(\mathrm{SO}(3)) = \mathbb{Z}_2$ must be suppressed: such a metal with "fluctuating antiferromagnetism" has $\mathbb{Z}_2$ TOPOLOGICAL ORDER and a pseudogap.


Criticality in Fe-based and electron-doped-cuprate materials
LGW-Hertz criticality of antiferromagnetism
(A) Antiferromagnetic metal
$\langle \Phi \rangle \neq 0$
(B) Fermi liquid with large Fermi surface
$\langle \Phi \rangle = 0$

Global phase diagram



Global phase diagram



1. Introduction to Z_2 topological order

   Z_2 and U(1) lattice gauge theories

2. The uses of Z_2 topological order

   (A) Intertwining topological order and symmetry breaking: CP^1 theory of the square lattice antiferromagnet

   (B) The pseudogap: metals with non-Luttinger volume Fermi surfaces

Global phase diagram