Title: Self-gravitating fluid solutions of Shape Dynamics

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Abstract: Shape Dynamics possesses a large set of solutions in common with General Relativity. Upon close inspection, these solutions behave in surprising ways, so in order to probe the fitness of Shape Dynamics as a viable alternative to General Relativity one must understand increasingly complex solutions, on which to base perturbative studies and numerical analyses. We show that a class of time-dependent exact solutions of Shape Dynamics exists from first principles, representing a central inhomogeneity in an evolving cosmological environment. By assuming only a perfect fluid source in a spherically symmetric geometry, we show that this solution satisfies in all generality the Hamiltonian structure of Shape Dynamics. The solutions are characterized by shear-free flow of the fluid and admit an interpretation as cosmological black holes.

Self-gravitating fluid solutions of Shape Dynamics

Daniel C. Guariento

in collaboration with F. Mercati

based on [1606.01215]

Shape Dynamics Workshop – May 15, 2017

Motivation

- Shape Dynamics has potentially many solutions in common with GR
- Identical to GR in ADM formulation, provided:
  - Compact 3-space
  - CMC foliation
- Important difference with respect to GR
  - No spacetime: *No causal structure*
- Exact solutions
  - Black holes in the presence of self-gravitating matter
  - Time-dependent solutions
  - Global differences with respect to GR counterparts

The Shape Dynamics system

- Shape Dynamics action with matter source

$$S = S_{\mathsf{M}} + \int \mathrm{d}t \left[\pi^{ab}\dot{\gamma}_{ab} - N\mathcal{H} - \xi_a \mathcal{H}^a \right]$$

- Spherical symmetry

$$\mathrm{d}s^2 = \mu^2(r,t)\,\mathrm{d}r^2 + Y^2(r,t)\,\mathrm{d}\Omega^2$$
$$N = N(r,t) \qquad \xi^a = \xi(r,t)\delta^a_r$$

- Comoving perfect fluid source

$$-\frac{\delta S_{\mathsf{M}}}{\delta N} \propto \rho(r,t)\,; \qquad \frac{\delta S_{\mathsf{M}}}{\delta \xi^a} = 0\,; \quad -\frac{\delta S_{\mathsf{M}}}{\delta \gamma^{ab}} \propto p(r,t)\gamma_{ab}$$

- Asymptotically FLRW behavior
- A singularity at the center

Solution of the constraints

- Momentum constraint and CMC condition

$$s = \mu \left(\langle \pi \rangle - \frac{f}{Y^2} \right)$$
$$f = \frac{\langle \pi \rangle Y^2}{3} + \frac{A(t)}{Y}$$

- Hamiltonian constraint

$$\rho = \frac{1}{\mu^2} \left[\frac{2}{Y} \left(\frac{\mu'}{\mu} Y' - Y'' \right) - \left(\frac{Y'}{Y} \right)^2 \right] - \frac{1}{Y^2} - \frac{\langle \pi \rangle^2}{12} + \frac{3}{4} \frac{A^2}{Y^6}$$

(not very informative)

Solving the equations of motion

- Metric equations of motion

$$\frac{\dot{Y}}{Y} = -\frac{\langle\pi\rangle}{6}N + \frac{Y'}{Y}\xi$$
$$\dot{\mu} = -\frac{\langle\pi\rangle}{6}N\mu + (\xi\mu)'$$

- Choosing a gauge with no shift, $\xi = 0$

$$Y = \mu R(r)$$

$\Rightarrow$ The metric is conformally static

$$\mathrm{d}s^2 = \mu^2(r,t)\left[\mathrm{d}r^2 + R^2(r)\,\mathrm{d}\Omega^2\right]$$

Time evolution encoded in μ

Solving the equations of motion

- Solution for the lapse

$$N = -\frac{6}{\langle \pi \rangle}\frac{\dot{\mu}}{\mu}$$

- Equation of motion for the momentum

$$\frac{6}{\mu^3 \langle \pi \rangle}\left[\dot{\mu}'' + 2\frac{\mu'}{\mu}\left(2\dot{\mu}' - \frac{\mu'\dot{\mu}}{\mu}\right) - \frac{R'}{R}\dot{\mu}' + \frac{1}{R^2}\left(RR'' - R'^2 + 1\right)\right] = 0$$

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- Last available gauge choice: $R(r)$

$$R'^2 = 1 - kR^2 \quad \Rightarrow \quad R = \begin{cases} \sinh r & k = -1\,, \\ r & k = 0\,, \\ \sin r & k = 1\,. \end{cases}$$

Solving the equations of motion

- The momentum equation can be integrated in t

$$\mu'' - 2\frac{\mu'^2}{\mu} - \mu'\frac{R'}{R} = \chi(r)$$

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- Exact solutions of χ: *Kustaanheimo–Qvist* class *(Kustaanheimo & Qvist, 1947)*
  - CMC foliation, perfect fluid, spherical symmetry $\Rightarrow$ Shear-free
  - Physically interesting subcases
    - Wyman subclass: $p(\rho)$
    - Scalar field and k-essence sources
- Constantly rediscovered in literature, with rich causal structure

(Sussman, 1988)

The Hawking–Hayward mass

- Hamiltonian 2-form integrated over a compact 2-surface

$$M_{\mathrm{HH}} \equiv \frac{1}{(4\pi)^{3/2}} \frac{\sqrt{A}}{4} \int_{\mathcal{S}} \mathrm{d}\mathcal{S} \left[{}^{(2)}\mathcal{R} + \theta_{(+)}\theta_{(-)} - \frac{1}{2}\sigma^{(+)}_{\mu\nu}\sigma^{\mu\nu}_{(-)} - 2\omega_\mu\omega^\mu \right]$$

- Spherical symmetry: *Misner–Sharp mass*

$$M_{\mathrm{HH}} = \frac{Y}{2}\left(1 - \frac{Y'^2}{\mu^2}\right) + \frac{f^2}{8Y}$$

- Asymptotically flat spacetime: *ADM mass*
- Can be decomposed in two parts *(Carrera & Giulini, 2010)*

$$M_{\mathrm{HH}} = M_{\mathcal{R}} + M_{\mathrm{W}}\,,$$

  - $M_{\mathcal{R}}$ *Ricci part*: Matter distribution
  - M_{W} *Weyl part*: Central compact object

Fixing the Misner–Sharp mass

- Weyl part of the Misner–Sharp mass

$$M_{\mathrm{W}} = \frac{R^3}{3}\chi$$

- Desired properties of the particular solution
  - ☐ Central singularity
    $\Rightarrow$ Divergence of the Weyl tensor at $r = 0$
  - ☐ Asymptotically FLRW
    $\Rightarrow$ Smooth Weyl at the antipode $r = \pi$
- Simple Ansatz: define $w(r) \equiv 2R\left({}^{r}/_{2}\right)$, so that

$$\chi = 3m\frac{w'^2}{w^3} = \begin{cases} 3\frac{m}{r^3} & k = 0 \\ 3m\frac{\cos^2(r/2)}{2\sin^3(r/2)} & k = 1 \end{cases}$$

Metric functions

- This ansatz solves the system completely

$$\mu = a(t)\left(1 + \frac{m}{2a(t)w}\right)^2$$

- The lapse-fixing equation finally gives

$$\begin{aligned} \langle \pi \rangle &= -6\frac{\dot{a}}{a} \\ N &= \frac{1 - \frac{m}{2a(t)\,w(r)}}{1 + \frac{m}{2a(t)\,w(r)}} \end{aligned}$$

- This is the *McVittie* metric in a compact foliation

What is the difference?

- Hamiltonian constraint may become singular

$$\mu = \frac{6YY'}{\sqrt{Y^4\left(\langle\pi\rangle + \frac{3A}{Y^3}\right)^2 + 9Y\left(4Y - 8M_{\mathrm{HH}}\right)}}$$

- McVittie is too simple, but less constrained solutions may have "forbidden regions" (*cf.* Flavio's talk)
- Lapse-fixing equation may be violated
- GR analogues may break down

Statement of the problem

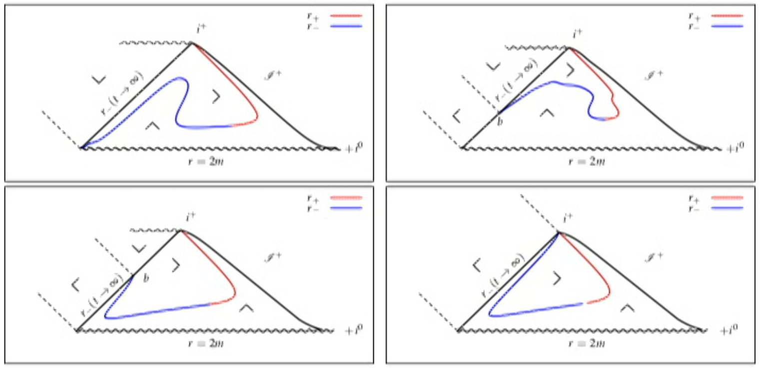
Solving the system

Particular solution

Conclusion

Next steps

- Causal structure of non-flat KQ metrics: black hole, horizon



(A. Maciel & DCG, 2015)

- Nature of the past singularity
  - ☐ Common feature of shear-free metrics
  - ☐ Not present in Shape Dynamics
- Consistency analysis of Shape Dynamics *(in progress)*

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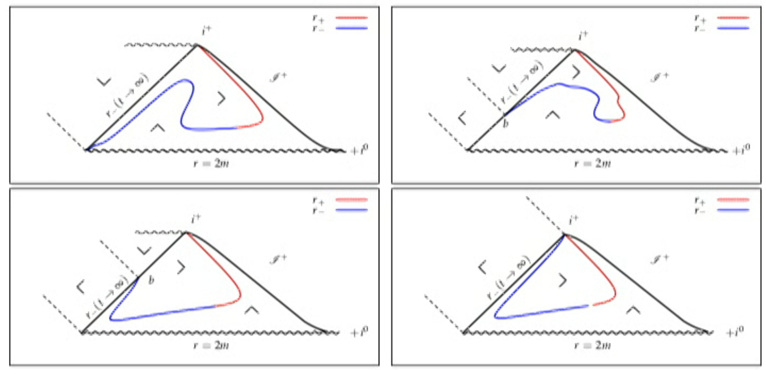

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Thank you!