Title: Gravity and non-locality

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Abstract: <p>I will talk about the relation between non-local theories and gravity. The main thesis is that non-local field theories naturally induce gravity, even at the classical level. Supporting this idea, I will study bi-local scalar field theories, which involve minimal deviations from locality. We will treat them both, bi-local theories and gravity perturbatively. We will see that bi-local theories encode gravity together with higher spin fields.</p>

# Gravity and non-locality

PD, S. Das and M. Walton, arXiv:1609.08631
PD, S. Das, arXiv:1705.03893

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Perimeter Institute, May 18, 2017

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# Relation between gravity and non-locality

Non-locality is expected at the quantum level, generally at distances $\sim l_p$

- String theory.
- Loop quantum gravity.
- Discrete spacetimes.
- Non-commutative geometry.
- QFT on fuzzy spaces.

# Gravity and non-locality at the classical level

Impossibility of resolving two nearby points:

Attempt to resolve them → Probe spacetime with particles of wavelength of order distance → Spacetime gets warped (BH).
[**Gross, Mende, '88**] [**Maggiore, '93**]
[**Doplicher,Fredenhagen,Roberts, '94**]

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Semiclassical gravity implies non-locality

Reverse question: Does non-locality induce gravity?

# Bilocal (effective) theories

Non-local theory implies an action-at-a-distance. Different ways of organizing it. For instance, we could define our theory on fuzzy spaces.

# Bilocal (effective) theories

Non-local theory implies an action-at-a-distance. Different ways of organizing it. For instance, we could define our theory on fuzzy spaces.

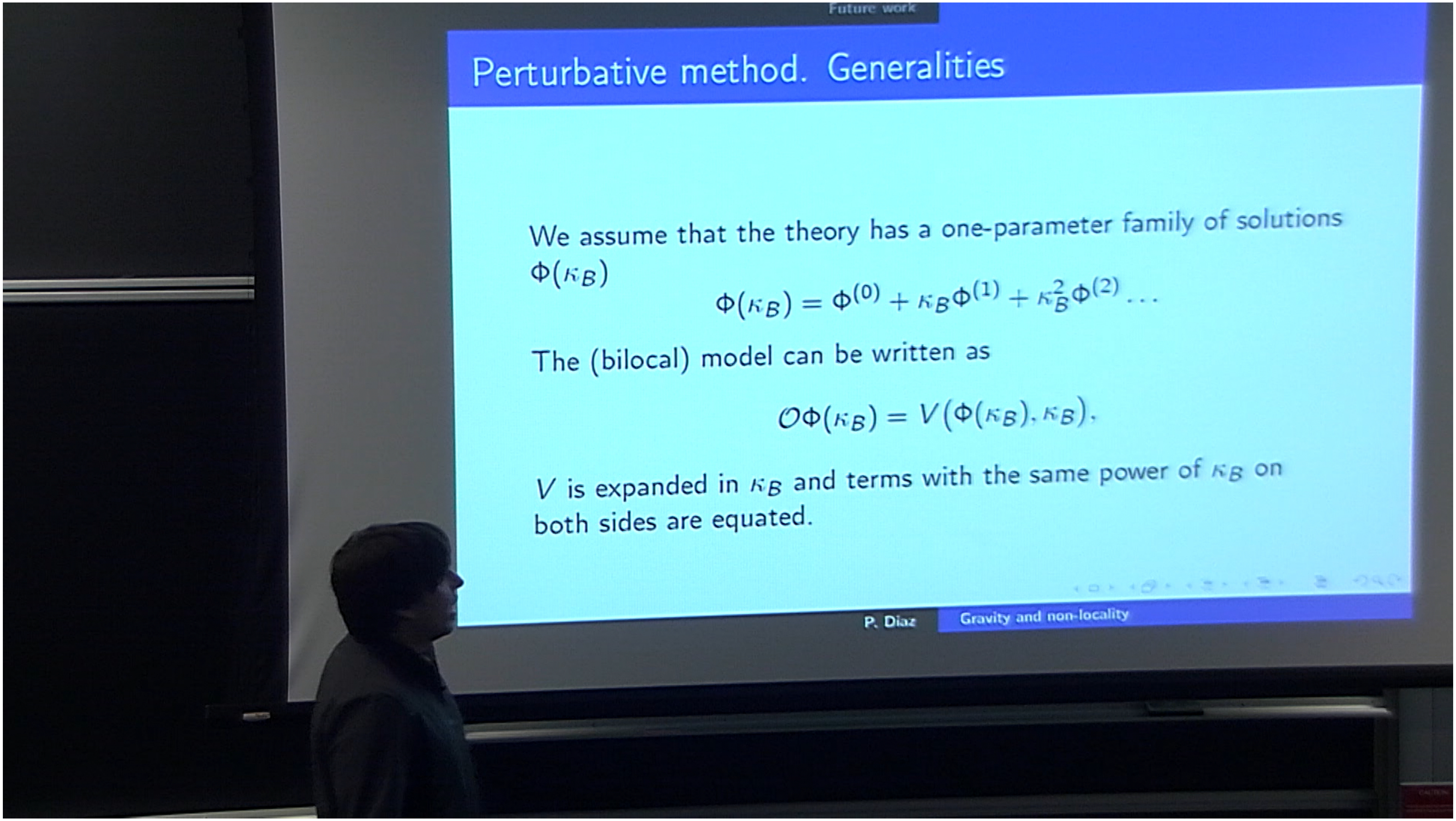
Another systematic way is to consider multilocal theories: $\phi(x), \phi(x,y), \phi(x,y,z) \ldots$.

| | |
|---|---|
| | Non-local theories |
| Integrating DOF | ↓ (?) |
| | Bilocal theory $\supset$ gravity |
| Integrating internal DOF | ↓ |
| | Local theories |

# Perturbative method. Generalities

We assume that the theory has a one-parameter family of solutions $\Phi(\kappa_B)$

$$\Phi(\kappa_B) = \Phi^{(0)} + \kappa_B \Phi^{(1)} + \kappa_B^2 \Phi^{(2)} \ldots$$


Future work
Perturbative method. Generalities
We assume that the theory has a one-parameter family of solutions $\Phi(\kappa_B)$
$$\Phi(\kappa_B) = \Phi^{(0)} + \kappa_B \Phi^{(1)} + \kappa_B^2 \Phi^{(2)} \dots$$
The (bilocal) model can be written as
$$\mathcal{O}\Phi(\kappa_B) = V\big(\Phi(\kappa_B), \kappa_B\big).$$
$V$ is expanded in $\kappa_B$ and terms with the same power of $\kappa_B$ on both sides are equated.
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# Perturbative gravity

One-parameter family of solutions $g_{\mu\nu}(\kappa)$, where $\kappa = \sqrt{16\pi G}$,

$$g_{\mu\nu}(\kappa) = \eta_{\mu\nu} + \kappa h^{(1)}_{\mu\nu} + \kappa^2 h^{(2)}_{\mu\nu} + \cdots ,$$

of the equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0, \qquad (1)$$

and equating equal powers of $\kappa$ in (1) one gets a tower of equations.
We will use the functions $G^{(k)}_{\mu\nu}[h_{ab}]$ given by

$$G_{\mu\nu}[\eta_{ab} + \kappa h_{ab}] = G^{(0)}_{\mu\nu}[\eta_{ab}] + \kappa G^{(1)}_{\mu\nu}[h_{ab}] + \kappa^2 G^{(2)}_{\mu\nu}[h_{ab}] + \cdots ,$$

# Linear order

First order equations $G^{(1)}_{\mu\nu}[h^{(1)}_{ab}] = 0$ lead to

$$-\Box h_{\mu\nu} + h_{\nu\ \ ,\mu\alpha}^{\ \alpha} + h_{\mu\ \ ,\nu\alpha}^{\ \alpha} - h_{,\mu\nu} - h^{\alpha\beta}_{\ \ ,\alpha\beta}\eta_{\mu\nu} + \eta_{\mu\nu}\Box h = 0.$$

$$\left.\begin{array}{rcl} \bar{h}_{\mu\nu} & = & h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu} \\ \partial^{\mu}\bar{h}_{\mu\nu} & = & 0 \end{array}\right\} \longrightarrow \Box\bar{h}_{\mu\nu} = 0$$

In the *Transverse-Traceless* coordinates we have gravitational waves

$$\bar{h}_{\mu\nu} = \Re\left(B_{\mu\nu}e^{iPx}\right), \quad P^2 = 0,$$

with

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & B_{+} & B_{\times} & 0 \\ 0 & B_{\times} & -B_{+} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

# Second order

To find $h_{ab}^{(2)}$ one must solve

$$0 = G_{\mu\nu}^{(1)}[h_{ab}^{(2)}] + G_{\mu\nu}^{(2)}[h_{ab}^{(1)}]$$

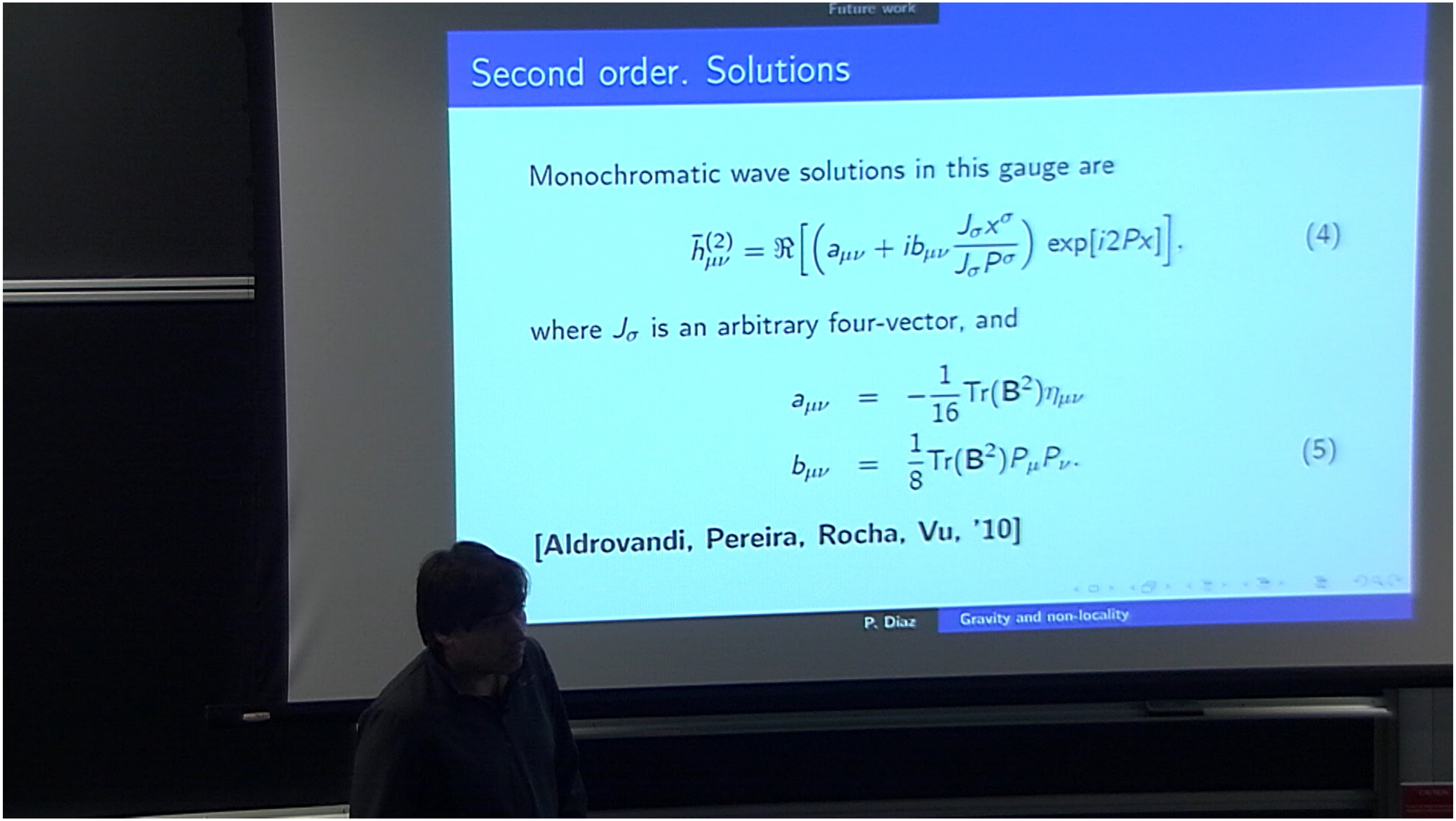
with the linear solutions $h_{ab}^{(1)}$ found by solving the first order.
If we take trace-reversed variables, and the Lorentz gauge

$$\partial^\mu \bar{h}_{\mu\nu}^{(2)} = 0, \tag{2}$$

we arrive at the equations

$$\Box \bar{h}_{\mu\nu}^{(2)} = -\frac{1}{2}\mathrm{Tr}(\mathbf{B}^2) P_\mu P_\nu \, \exp[i2Px], \tag{3}$$

where $\mathbf{B}$ is the polarization matrix coming from $\bar{h}_{\mu\nu}^{(1)}$.


Future work
Second order. Solutions
Monochromatic wave solutions in this gauge are
$$\bar{h}^{(2)}_{\mu\nu} = \Re\Big[\Big(a_{\mu\nu} + ib_{\mu\nu}\frac{J_\sigma x^\sigma}{J_\sigma P^\sigma}\Big)\exp[i2Px]\Big]. \quad (4)$$
where $J_\sigma$ is an arbitrary four-vector, and
$$a_{\mu\nu} = -\frac{1}{16}\mathrm{Tr}(\mathbf{B}^2)\eta_{\mu\nu}$$
$$b_{\mu\nu} = \frac{1}{8}\mathrm{Tr}(\mathbf{B}^2)P_\mu P_\nu. \quad (5)$$
[Aldrovandi, Pereira, Rocha, Vu, '10]
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**[Aldrovandi, Pereira, Rocha, Vu, '10]**

# Bilocal scalar field

The main object of the theory is the bilocal scalar field $\bar{\Phi}(x, y)$.

We will use the coordinates

$$
\begin{aligned}
X^\mu &= \frac{1}{2}(x^\mu + y^\mu) \quad \text{Center of mass coordinates} \\
r^\mu &= (x^\mu - y^\mu)_E \quad \text{relative coordinates}
\end{aligned}
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so $\Phi(X, r) \equiv \bar{\Phi}(x, y)$.
And impose the symmetry

$$\bar{\Phi}(x, y) = \bar{\Phi}(y, x) \longrightarrow \Phi(X, r) = \Phi(X, -r)$$

# Expansion around locality

Taylor expansion on $r$

$$\Phi(X,r) = \phi(X) + H_{\mu\nu}(X) r^{\mu} r^{\nu} + D_{\mu\nu\sigma\rho}(X) r^{\mu} r^{\nu} r^{\sigma} r^{\rho} \cdots ,$$

with

$$\begin{aligned} \phi(X) &\equiv \Phi(X,0) \\ H_{\mu\nu}(X) &\equiv \frac{1}{2}\partial_{\mu}\partial_{\nu}\Phi(X,r)\Big|_{r=0} \\ D_{\mu\nu\sigma\rho}(X) &\equiv \frac{1}{4!}\partial_{\mu}\partial_{\nu}\partial_{\sigma}\partial_{\rho}\Phi(X,r)\Big|_{r=0} \\ &\vdots \end{aligned}$$

We see that $\Phi(X,r)$ is equivalent to an infinite tower of higher spin fields.

# Bilocal dynamics

The model we will use is minimal: just the necessary ingredients to assure

- Free with respect to CM coordinates $X^\mu$.
- Genuine bilocality. Action at a distance between points $x$ and $y$.
- Short-ranged non-locality.

Action-at-a-distance comes with violations of micro-causality $\longrightarrow$ essential to keep the non-local effects short-ranged.

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# Bilocal equations

The model consits of two equations

$$\begin{aligned}(\Box + \Box_r - \alpha^4 r^2 + 8\alpha^2)\Phi(X, r) &= V(\Phi) \quad \text{(dynamical)} \\ \frac{\partial}{\partial X_\mu}\left(\frac{\partial}{\partial r^\mu} + \alpha^2 r_\mu\right)\Phi(X, r) &= 0 \quad \text{(constraint)},\end{aligned}$$

- $\alpha$ (with dimensions of mass) measures non-locality: $\alpha \to \infty$ the theory is local.
- The term $\alpha^4 r^2$ says that the interaction (in relative coordinates) is harmonic $\longrightarrow$ short range.
- The constraint equation implies action-at-a-distance between points $x$ and $y$.

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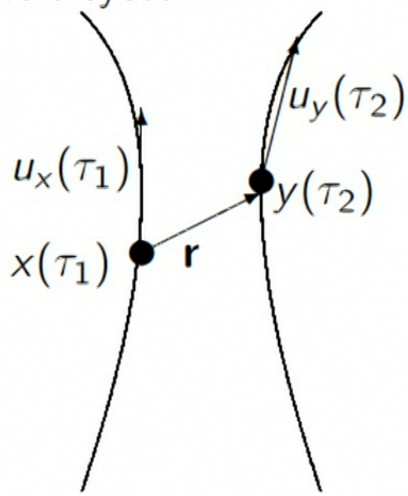
# Action at a distance

The constraint equation can be understood if we think of a two-particle system



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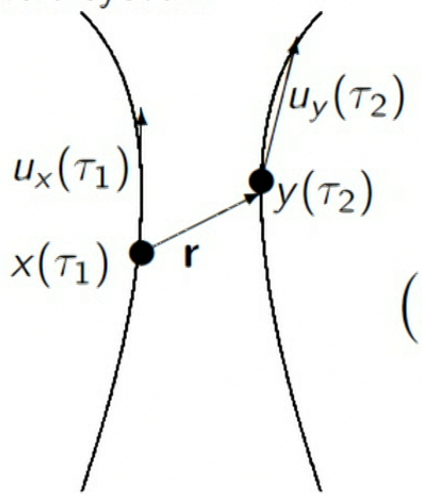


$$\left(u_x(\tau_1) + u_y(\tau_2)\right)_\mu r^\mu = 0$$

# Massless solutions. Linear order

$$
\begin{aligned}
(\Box + \Box_r - \alpha^4 r^2 + 8\alpha^2)\Phi^{(1)}(X,r) &= 0, \\
\frac{\partial}{\partial X^\mu}\left(\frac{\partial}{\partial r^\mu} + \alpha^2 r_\mu\right)\Phi^{(1)}(X,r) &= 0.
\end{aligned}
$$

General massless solutions of this system are

$$
\Phi^{(1)}(X,r) = r^\mu \bar{C}_{\mu\nu} r^\nu e^{-\frac{\alpha^2}{2}r^2} e^{iPX}, \quad P^2 = 0. \tag{6}
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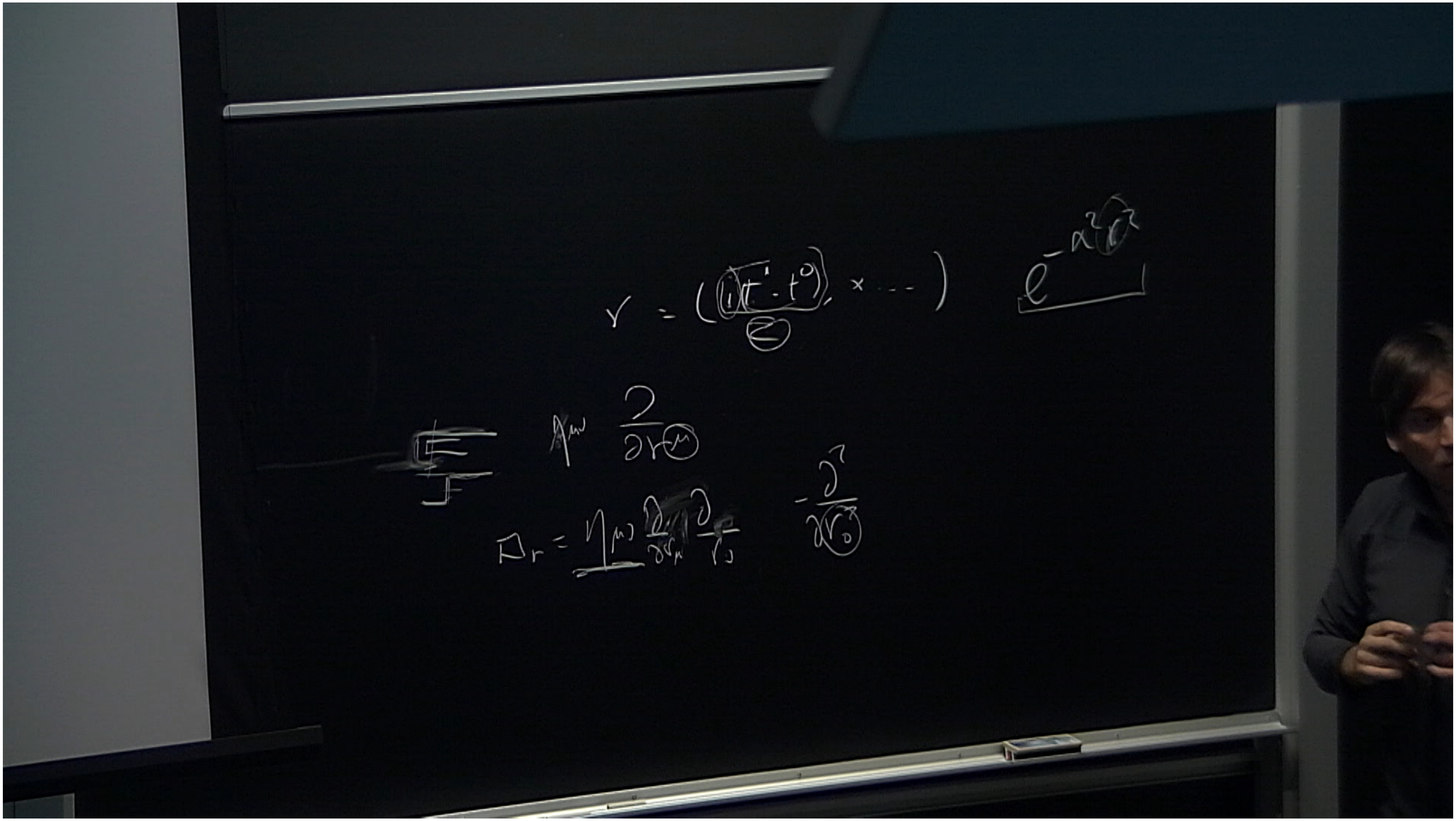
In coordinates for which the $z$-axis is paralell to $P_\mu$ we have

$$
\bar{\mathbf{C}} = \begin{pmatrix} c & 0 & 0 & c \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ c & 0 & 0 & c \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{7}
$$



# Gravity from bilocal theory

We match first order solutions in gravity if $c = 0$.
For first order

$$\bar{h}^{(1)}_{\mu\nu}(X) = \partial_\mu \partial_\nu \Phi^{(1)}(X, r, c = 0)\big|_{r=0} + \alpha^2 \eta_{\mu\nu} \Phi^{(1)}(X, 0, c = 0).$$

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For second order

$$\begin{aligned} a_{\mu\nu} &= -\frac{1}{2}\frac{\alpha^2}{K} A(0, c = 0)\eta_{\mu\nu}, \\ b_{\mu\nu} &= \frac{\alpha^2}{K}\Big(\partial_\mu \partial_\nu A^2(r, c = 0)\big|_{r=0} + \alpha^2 A^2(0, c = 0)\eta_{\mu\nu}\Big). \end{aligned}$$

## Higher orders

It is surprising that there is any match between both theories:

- Interaction terms in gravity are of the form $\partial\partial h^n$.
- The bilocal counterparts are $\int \Phi^n$.

Solutions are expected to be related in higher orders.

$$\Phi^{(n)} \sim P^{(2n)}(r) e^{-\frac{1}{2}\alpha^2 r^2}.$$

In general, for any invertible $d \times d$-matrix $M$ and a polynomial $P(r)$ on the variables $r_1, \dots, r_d$, we have

$$\int P(r) e^{-\frac{1}{2} M_{\mu\nu} r^\mu r^\nu} \mathrm{d}^d r = \sqrt{\frac{(2\pi)^d}{\det M}} e^{\frac{1}{2}(M^{-1})_{\mu\nu}\partial^\mu \partial^\nu} P(r)\Big|_{r=0}, \quad (9)$$

# Relation with holography?

We are using a non-gravitational theory to describe gravity. That ressembles holography but

- Bilocal theory is non-local.
- Bilocal theory is not conformal since $\alpha$ has dimensions of mass.
- Bilocal theory in 4d describing gravity in 4d.

Consider the limit

$$\lim_{\substack{\alpha\to\infty \\ z\to 0}} \alpha z = \lambda, \tag{10}$$

where $z$ is a coordinate (it would be the radial coordinate in AdS).

## Summary

What we have learnt so far:

- All linearized and second order gravity solutions in a particular gauge are contained in $\Phi(X, r)$.
- Action at a distance in $\Phi \longleftrightarrow$ Lorentz gauge in $\bar{h}_{\mu\nu}$.
- There is more than gravity in $\Phi$: all the higher spin fields. (Good news)
- Likely that higher orders match. Can we find a closed form for $V(\Phi)$?

# Future work

- Find final shape (sum to all orders) of $V(\Phi)$. Nonperturbative statements.
- Work at the level of actions. How would we recover diffeomorphism invariance in the bilocal picture?
- Coupling to matter.
- Cosmological constant?

# Future work

- Find final shape (sum to all orders) of $V(\Phi)$. Nonperturbative statements.
- Work at the level of actions. How would we recover diffeomorphism invariance in the bilocal picture?
- Coupling to matter.
- Cosmological constant?
- Quantization bilocal field. Renormalizability? Finite theory?
- General conditions for non local theory to effective bilocal.
- Relation with holography.
- Modifications of GR.