Title: How fundamental is the Immirzi parameter in Loop Quantum Gravity?

Date: May 11, 2017 02:30 PM

URL: http://pirsa.org/17050074

Abstract: <p>I will discuss the role(s) of the Immirzi parameter in Loop Quantum Gravity, insisting on the Poisson algebra formed by Thiemann's complexifier, the volume and the Hamiltonian constraint. In particular, we will see how loop quantum cosmology is a direct quantization of this CVH Poisson algebra and how cosmological evolution amounts to a flow in the Immirzi parameter.</p>

Immirzi parameter
& Thiemann Complexifier in LQG

Etera Livine

Laboratoire de Physique LP ENSL & CNRS

Perimeter Institute - May '17

Work with J. Ben Achour arXiv:1705.03772

What is the Immirzi Parameter ?

Loop Quantum Gravity introduces a new parameter γ

on top of the 3 fundamental constants $c, G_N, \hbar$

How should we consider the Immirzi parameter ?

New fundamental constant for Quantum Gravity

or just a regulator ?

- To be fixed and determined by experiments ?
- or allowed to flow under renormalization as a coupling constant ?
- or to be sent to $\gamma \to 0$ (or $\gamma \to i$) as a regulator ?

What is the Immirzi Parameter ?

Loop Quantum Gravity introduces a new parameter γ

on top of the 3 fundamental constants $c, G_N, \hbar$

How should we consider the Immirzi parameter ?

New fundamental constant for Quantum Gravity

or just a regulator ?

- To be fixed and determined by experiments ?
- or allowed to flow under renormalization as a coupling constant ?
- or to be sent to $\gamma \to 0$ (or $\gamma \to i$) as a regulator ?

Here: will become time along trajectories of the geometry

Immirzi Parameter and Thiemann Complexifier in LQG

Outline:

1. The Many Roles of the Immirzi Parameter in Loop Q Gravity
2. Thiemann Complexifier and CVH algebra for General Relativity

Immirzi Parameter and Thiemann Complexifier in LQG

Outline:

1. The Many Roles of the Immirzi Parameter in Loop Q Gravity
2. Thiemann Complexifier and CVH algebra for General Relativity
3. CVH algebra for Cosmology
4. CVH algebra for Loop Quantum Cosmology
5. CVH bubbles for (Loop) Quantum Gravity ?

What is the Immirzi Parameter ?

Many layers from the definition of the classical theory

to the features of the quantum theory

- A classical canonical transformation defining a choice of variables
- Cut-off in extrinsic curvature when using holonomy-flux algebra
- New coupling constant in GR action
- Topological parameter
- Controls coupling to fermions (CP-violation)
- Area gap & volume gap in standard loop quantization

The Canonical Transformation defining LQG

- **A classical canonical transformation defining a choice of variables**

Canonical Pair
triad-extrinsic curvature

$$\{K_a^i(x), E_j^b(y)\} = \delta_a^b \delta_j^i \delta^{(3)}(x-y)$$

Canonical Pair
triad-connection

$$\{A_a^i(x), E_j^b(y)\} = \gamma \delta_a^b \delta_j^i \delta^{(3)}(x-y)$$

$$K_a^i \longrightarrow A_a^i = \Gamma_a^i[E] + \gamma K_a^i$$

Thiemann Complexifier $C = \int d^3x \, K_a^i E_i^a$ generates shifts in Immirzi parameter

Key Property: $\{C, \Gamma_a^i[E]\} = 0$

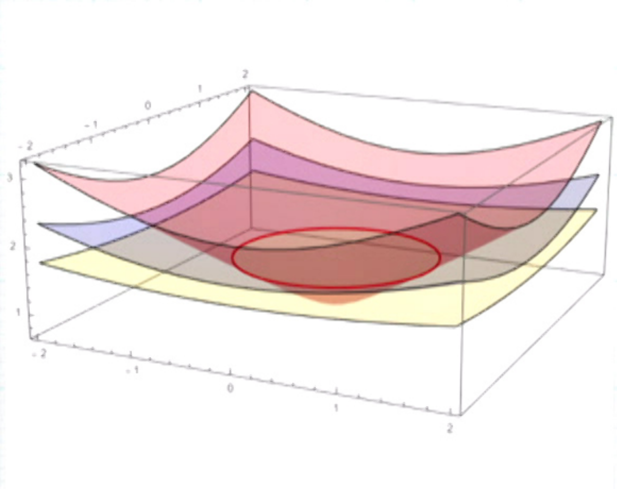
$$e^{\eta\{C,\cdot\}} A_\gamma = A_{e^{-\eta}\gamma}$$

$$e^{\eta\{C,\cdot\}} E = e^{+\eta} E$$

Cut-off in Curvature

- Cut-off in extrinsic curvature when using holonomy-flux algebra

Ashtekar-Barbero connection at pull-back of space-time connection: explicitly depends on space-time embedding



Can extract extrinsic curvature from holonomies ...

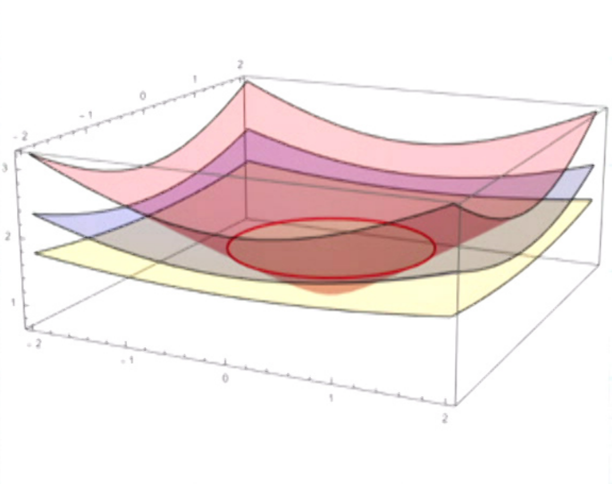
$$W_\kappa(R) = 1 + 2\cos\left[2\pi\sqrt{1 + (1+\gamma^2)\frac{R^2}{\kappa^2}}\right]$$

Periodic for real Imm parameter, so theory does not see high curvature excitations

Cut-off in Curvature

- **Cut-off in extrinsic curvature when using holonomy-flux algebra**

Ashtekar-Barbero connection at pull-back of space-time connection:
explicitly depends on space-time embedding



Can extract extrinsic curvature from holonomies ...

$$W_\kappa(R) = 1 + 2\cos\left[2\pi\sqrt{1+(1+\gamma^2)\frac{R^2}{\kappa^2}}\right]$$

Periodic for real Imm parameter, so theory does not see high curvature excitations

What is the Immirzi Parameter ?

Many layers from the definition of the classical theory

to the features of the quantum theory

- A classical canonical transformation defining a choice of variables
- Cut-off in extrinsic curvature when using holonomy-flux algebra
- New coupling constant in GR action
- Topological parameter
- Controls coupling to fermions (CP-violation)
- Area gap & volume gap in standard loop quantization

A New Coupling in the gravity action

- New coupling constant in GR action

$$S = \int \epsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}[\omega] - \frac{1}{\gamma} \int e^I \wedge e^J \wedge F_{IJ}[\omega]$$

Should compute flow under renormalization group of Immirzi parameter !

Remark: Spinfoam use Lorentz connection to define bulk amplitude, Ashtekar-B connection appears only on boundary as choice of boundary state/data

What is the Immirzi Parameter ?

Many layers from the definition of the classical theory

to the features of the quantum theory

- A classical canonical transformation defining a choice of variables
- Cut-off in extrinsic curvature when using holonomy-flux algebra
- New coupling constant in GR action
- Topological parameter
- Controls coupling to fermions (CP-violation)
- **Area gap & volume gap** in standard loop quantization

The Area gap in Loop Quantum Gravity

- **Area gap & volume gap** in standard loop quantization

Quantize canonical pair of fields $\{A_a^i, E_j^b\} = \gamma \delta_a^b \delta_j^i$

- Wave-functions of the connection $\Psi[A]$
- Triad as differential operator $\widehat{E} = -i\gamma \frac{\partial}{\partial A}$

Thus γ always factor in front of geometrical observables

as a **global scale factor** for spectrum of area and volume operators

$\mathcal{A}_S = \gamma \sqrt{j(j+1)}\, l_P^2$ with $j \in \mathbb{N}$ (or $\mathcal{A}_S = \gamma\, j\, l_P^2$)

What is the Immirzi Parameter ?

Many layers from the definition of the classical theory

to the features of the quantum theory

- A classical canonical transformation defining a choice of variables
- Cut-off in extrinsic curvature when using holonomy-flux algebra
- New coupling constant in GR action
- Topological parameter
- Controls coupling to fermions (CP-violation)
- Area gap & volume gap in standard loop quantization

Confusion about γ due to its many different roles

but, for sure, an essential parameter in LQG

CVH Algebra for Classical General Relativity

Hamiltonian constraint: $\mathcal{H} = \mathcal{H}^E + \mathcal{H}^K$

$$\mathcal{H}^E = -\frac{1}{2\gamma^2}\frac{E^a_i E^b_j}{\sqrt{\det(E)}}\,\epsilon^{ij}{}_k F^k_{ab} \qquad \mathcal{H}^K = \frac{(1+\gamma^2)}{\gamma^2}\frac{E^a_i E^b_j}{\sqrt{\det(E)}}\,K^{[i}_a K^{j]}_b$$

Look at Poisson algebra with volume $V = \int_\Sigma d^3x \sqrt{\frac{1}{3!}\epsilon^{ijk}\epsilon_{abc}E^a_i E^b_j E^c_k}$

Forms a closed $\mathfrak{sl}_2$ algebra :

$$\begin{aligned}
\{\mathcal{C}, V\} &= V \\
\{\mathcal{C}, \mathcal{H}^E\} &= -\mathcal{H}^E \\
\{\mathcal{C}, \mathcal{H}^K\} &= -\mathcal{H}^K \\
\{V, \mathcal{H}^E\} &= \mathcal{C} \\
\{V, \mathcal{H}^K\} &= 0
\end{aligned}$$

What is the CVH algebra important?

Hamiltonian constraint: $\mathcal{H} = \mathcal{H}^E + \mathcal{H}^K$

$$\mathcal{H}^E = -\frac{1}{2\gamma^2}\frac{E_i^a E_j^b}{\sqrt{\det(E)}}\,\epsilon^{ij}{}_k F_{ab}^k \qquad \mathcal{H}^K = \frac{(1+\gamma^2)}{\gamma^2}\frac{E_i^a E_j^b}{\sqrt{\det(E)}}\,K_a^{[i}K_b^{j]}$$

Look at Poisson algebra with volume $V = \int_\Sigma d^3x \sqrt{\frac{1}{3!}\epsilon^{ijk}\epsilon_{abc}E_i^a E_j^b E_k^c}$

Forms a closed $\mathfrak{sl}_2$ algebra :

$$\begin{aligned} \{\mathcal{C}, V\} &= V \\ \{\mathcal{C}, \mathcal{H}\} &= -\mathcal{H} \\ \{V, \mathcal{H}\} &= \mathcal{C} \end{aligned}$$

- Complexifier is dilatation on (E,K) phase space
- Algebra contains Hamiltonian constraint, for once !
- Can be used as constraints on quantization of C, V and H

What is the CVH algebra important?

Hamiltonian constraint: $\mathcal{H} = \mathcal{H}^E + \mathcal{H}^K$

$$\mathcal{H}^E = -\frac{1}{2\gamma^2}\frac{E^a_i E^b_j}{\sqrt{\det(E)}}\,\epsilon^{ij}{}_k F^k_{ab} \qquad \mathcal{H}^K = \frac{(1+\gamma^2)}{\gamma^2}\frac{E^a_i E^b_j}{\sqrt{\det(E)}}\,K^{[i}_a K^{j]}_b$$

Look at Poisson algebra with volume $V = \int_\Sigma d^3x \sqrt{\frac{1}{3!}\epsilon^{ijk}\epsilon_{abc}E^a_i E^b_j E^c_k}$

Forms a closed $\mathfrak{sl}_2$ algebra :

$$\begin{aligned} \{\mathcal{C}, V\} &= V \\ \{\mathcal{C}, \mathcal{H}\} &= -\mathcal{H} \\ \{V, \mathcal{H}\} &= \mathcal{C} \end{aligned}$$

- Complexifier is dilatation on (E,K) phase space
- Algebra contains Hamiltonian constraint, for once !
- Can be used as constraints on quantization of C, V and H

CVH Algebra for FRW Cosmology

Look at FRW Cosmology with massless scalar field :

$$\{a,\pi_a\}=1\,,\quad \{\phi,\pi_\phi\}=1\,,\quad \mathcal{H}^0=\frac{\pi_\phi^2}{2a^3}-\frac{2\pi G}{3}\frac{\pi_a^2}{a}=0$$

Can switch to connection-triad variables : $A_a^i=c\delta_a^i \quad E_i^a=p\delta_i^a$

$$a=\sqrt{p}\,,\quad \pi_a=-\frac{3}{4\pi G\gamma}\sqrt{p}\quad ,\{c,p\}=\frac{8\pi G\gamma}{3}$$

depends explicitly on Imm param: $\mathcal{H}^0=\frac{1}{16\pi G}\left[8\pi G\frac{\pi_\phi^2}{p^{3/2}}-\frac{6}{\gamma^2}\sqrt{p}c^2\right]$

More convenient to work with volume for LQC and to look at CVH:

$$4\pi Gv=p^{3/2}=a^3\,,\quad b=\gamma^{-1}cp^{-1/2}=-\frac{4\pi G}{3}a^{-2}\pi_a\,,\quad \{b,v\}=1$$

Page 23 sur 47

CVH Algebra for FRW Cosmology

In v,b variables, no explicit dependence on Imm parameter :

$$\{b,v\}=1\,,\quad \{\phi,\pi_\phi\}=1\,,\quad \mathcal{H}^0=\mathcal{H}^0_g+\mathcal{H}^0_m=\frac{\pi_\phi^2}{8\pi G v}-\frac{3}{2}vb^2$$

Compute Complexifier from extrinsic curvature :

$$C=\frac{1}{4\pi G}\int_\Sigma d^3x\;E^a_iK^i_a=\frac{1}{4\pi G\gamma}\,pc=vb$$

Look at gravitational sector :

$$C=\{v,\mathcal{H}^0_g\}=vb$$

CVH algebra closes and forms $\mathfrak{su}(1,1)$ algebra

$$\{C,v\}=v\,,\quad \{C,\mathcal{H}^0_g\}=-\mathcal{H}^0_g$$

CVH Algebra for FRW Cosmology

Look at gravitational sector :

CVH algebra closes and forms $\mathfrak{su}(1,1)$ algebra

$$C = \{v, \mathcal{H}_g^0\} = vb$$

$$\{C, v\} = v\,, \quad \{C, \mathcal{H}_g^0\} = -\mathcal{H}_g^0$$

Write in terms of su(1,1) generators :

$$C = k_y\,, \quad v = \frac{1}{2}(j_z + k_x)\,, \quad \mathcal{H}_g^0 = k_x - j_z$$

Vanishing Casimir: $\mathfrak{C} = j_z^2 - k_x^2 - k_y^2 = -2v\mathcal{H}_g^0 - C^2 = 0$

- Complexifier as boost
- Gravitational Hamiltonian as null generator

Complexifier Flow for standard FRW

Complexifier is dilatation on (v,b) phase space:

$$v \longrightarrow \tilde{v} = e^{\eta\{C,\cdot\}}\, v = e^{\eta} v\,, \quad b \longrightarrow \tilde{b} = e^{\eta\{C,\cdot\}}\, b = e^{-\eta}$$

Simply rescales the Hamiltonian constraint:

$$\mathcal{H}^0 \longrightarrow \tilde{\mathcal{H}}^0 = e^{\eta\{C,\cdot\}}\, \mathcal{H}^0 = e^{-\eta} \mathcal{H}^0$$

Gives shift in Immirzi parameter: $\tilde{\gamma} = e^{-\frac{2}{3}\eta}\gamma$

$$c \rightarrow \tilde{c} = e^{\eta\{C,\cdot\}}\, c = e^{-\frac{2}{3}\eta} c \qquad \{c,p\} = \frac{8\pi G\gamma}{3} \longrightarrow \{\tilde{c},p\} = \frac{8\pi G\tilde{\gamma}}{3}$$

$$\mathcal{H}^0_\gamma[c,p] = \frac{\pi_\phi^2}{2p^{3/2}} - \frac{3}{8\pi G\gamma^2}\sqrt{p}c^2 \longrightarrow \quad \tilde{\mathcal{H}}^0[\tilde{c},p] = e^{-\eta}\mathcal{H}^0_{\tilde{\gamma}}[\tilde{c},p]$$

CVH Algebra for FRW Cosmology

Look at gravitational sector :

CVH algebra closes and forms $\mathfrak{su}(1,1)$ algebra

$$C = \{v, \mathcal{H}_g^0\} = vb$$

$$\{C, v\} = v\,, \quad \{C, \mathcal{H}_g^0\} = -\mathcal{H}_g^0$$

Write in terms of su(1,1) generators :

$$C = k_y\,, \quad v = \frac{1}{2}(j_z + k_x)\,, \quad \mathcal{H}_g^0 = k_x - j_z$$

Vanishing Casimir: $\mathfrak{C} = j_z^2 - k_x^2 - k_y^2 = -2v\mathcal{H}_g^0 - C^2 = 0$

- Complexifier as boost
- Gravitational Hamiltonian as null generator

CVH Algebra for Loop-deformed FRW Cosmology

LQC introduces regularization scale
and construct curvature
from finite holonomies of connection

$$b \to \frac{\sin(\lambda b)}{\lambda} = \frac{e^{i\lambda b} - e^{-i\lambda b}}{2i\lambda}$$

$$\mathcal{H}^{(\lambda)} = \mathcal{H}_m + \mathcal{H}_g = \frac{\pi_\phi^2}{24\pi G v} - \frac{1}{2} v \frac{\sin^2(\lambda b)}{\lambda^2}$$

CVH Algebra for Loop-deformed FRW Cosmology

LQC introduces regularization scale
and construct curvature
from finite holonomies of connection

$$b \to \frac{\sin(\lambda b)}{\lambda} = \frac{e^{i\lambda b} - e^{-i\lambda b}}{2i\lambda}$$

$$\mathcal{H}^{(\lambda)} = \mathcal{H}_m + \mathcal{H}_g = \frac{\pi_\phi^2}{24\pi G v} - \frac{1}{2} v \frac{\sin^2(\lambda b)}{\lambda^2}$$

Look at flow of standard complexifier $C = vb$

$$\mathcal{H}^{(\lambda)} \xrightarrow{e^{\eta\{C,\cdot\}}} \tilde{\mathcal{H}}^{(\lambda)}[\tilde{v},\tilde{b}] = \frac{\pi_\phi^2}{24\pi G\tilde{v}} - \frac{1}{2}\tilde{v}\frac{\sin^2(\lambda\tilde{b})}{\lambda^2} = \mathcal{H}^{(\lambda)}[\tilde{v},\tilde{b}] = \mathcal{H}^{(e^{-\eta}\lambda)}[v,b]$$

Shift in the regularization scale ...

... but actually involves b so should be also regularized

CVH Algebra for Loop-deformed FRW Cosmology

LQC introduces regularization scale
and construct curvature
from finite holonomies of connection

$$b \to \frac{\sin(\lambda b)}{\lambda} = \frac{e^{i\lambda b} - e^{-i\lambda b}}{2i\lambda}$$

$$\mathcal{H}^{(\lambda)} = \mathcal{H}_m + \mathcal{H}_g = \frac{\pi_\phi^2}{24\pi G v} - \frac{1}{2} v \frac{\sin^2(\lambda b)}{\lambda^2}$$

Construct regularized complexifier from Poisson bracket:

$$\mathcal{C} \equiv \{v, \mathcal{H}_g\} = v \frac{\sin 2\lambda b}{2\lambda}$$

$$\{\mathcal{C}, \mathcal{H}_g\} = -\mathcal{H}_g$$

$$\{\mathcal{C}, v\} = v + 4\lambda^2 \mathcal{H}_g$$

Closed regularized CVH algebra !

CVH Algebra for Loop-deformed FRW Cosmology

Construct regularized complexifier from Poisson bracket:

$$\mathcal{C} \equiv \{v, \mathcal{H}_g\} = v\frac{\sin 2\lambda b}{2\lambda}$$

$$\{\mathcal{C}, \mathcal{H}_g\} = -\mathcal{H}_g$$

$$\{\mathcal{C}, v\} = v + 4\lambda^2\mathcal{H}_g$$

Closed regularized CVH algebra !

Introduce su(1,1) generators:

$$j_z = (2\lambda)^{-1}v\,, \quad k_\pm = (2\lambda)^{-1}ve^{\pm 2\lambda i b}$$

$$\{j_z, k_\pm\} = \mp i k_\pm\,, \quad \{k_+, k_-\} = 2ij_z$$

$$v = 2\lambda j_z\,, \quad \mathcal{C} = k_y\,, \quad \mathcal{H}_g = (2\lambda)^{-1}(k_x - j_z)$$

Vanishing Casimir:

$$\mathfrak{C} = j_z^2 - k_+k_- = -2V\mathcal{H}_g - \mathcal{C}^2 = 0$$

$$V = \lambda(j_z + k_x) = v\cos^2\lambda b$$

Action of the regularized Complexifier

Write SU(1,1) group action on algebra su(1,1) as adjoint action:

$$M = \begin{pmatrix} j_z & k_- \\ k_+ & j_z \end{pmatrix} \in H_2(\mathbb{C}) \qquad M \longrightarrow \tilde{M} = e^{\{\vec{\eta}\cdot\vec{J},\,\cdot\,\}}\, M = G\, M\, G^\dagger$$

$$G = e^{i\,\vec{\eta}\cdot\vec{\tau}}$$

Get exponentiated action of complexifier as boost:

$$e^{\eta\{\mathcal{C},\cdot\}} M = G_\eta M G_\eta^\dagger\,, \quad G_\eta = \begin{pmatrix} \cosh\frac{\eta}{2} & \sinh\frac{\eta}{2} \\ \sinh\frac{\eta}{2} & \cosh\frac{\eta}{2} \end{pmatrix}$$

$$\begin{aligned} v &\longrightarrow \tilde{v} = e^\eta v + 4\lambda^2 \mathcal{H}_g \sinh\eta \\ \mathcal{H}_g &\longrightarrow \widetilde{\mathcal{H}}_g = e^{-\eta}\,\mathcal{H}_g\,. \end{aligned}$$

Transformed volume does not commute with original volume:

$$\{v, \tilde{v}\} = 4\lambda^2 \sinh\eta\, \mathcal{C} \neq 0$$

CVH algebra for whole Hamiltonian cosntraint

Complexifier simply rescales gravitation Hamiltonian ...

... but not matter Hamiltonian ...

... except if we also regularize the inverse volume factor !

$$\mathcal{H} \equiv \frac{\pi_\phi^2}{24\pi G v} - \frac{1}{2} v \frac{\sin^2(\lambda b)}{\lambda^2} \quad \rightsquigarrow \quad \mathcal{H}^{reg} \equiv \frac{\pi_\phi^2}{24\pi G V} - \frac{1}{2} v \frac{\sin^2(\lambda b)}{\lambda^2}$$

Because boost simply rescales null directions:

$$\{k_y, (k_x - j_z)\} = -(k_x - j_z)$$

$$\{k_y, (k_x + j_z)\} = (k_x + j_z)$$

CVH algebra for whole Hamiltonian cosntraint

$$\mathcal{H}^{reg} = \frac{\pi_\phi^2}{24\pi G V} - \frac{1}{2} v \frac{\sin^2(\lambda b)}{\lambda^2}$$

Closed regularized CVH algebra, forms new su(1,1) algebra :

$$\mathcal{C} = K_y\,, \quad V = \lambda(K_x + J_z)\,, \quad \mathcal{H}^{reg} = (2\lambda)^{-1}(K_x - J_z)$$

$$K_y = \mathcal{C} = v\frac{\sin 2\lambda b}{2\lambda}\,, \quad K_x = v\frac{\cos 2\lambda b}{2\lambda} + \frac{\lambda \pi_\phi^2}{24\pi G v \cos^2 \lambda b}\,, \quad J_z = \frac{v}{2\lambda} - \frac{\lambda \pi_\phi^2}{24\pi G v \cos^2 \lambda b}$$

Non-vanishing Casimir:

$$\mathfrak{C} = -2V\mathcal{H}^{reg} - \mathcal{C}^2 = J_z^2 - K_x^2 - K_y^2 = -\frac{\pi_\phi^2}{12\pi G}$$

Hamiltonian flow for loop regularized FRW model

We integrate flow of Hamiltonian constraint (lapse N=1) as null SU(1,1) transformation :

- Evolution at constant Complexifier
- Volume v gets modified Friedman equation

$$\left(\frac{\partial_t v}{v}\right)^2 = \frac{8\pi G}{3}\rho\left[1 - \frac{1}{\left(1+\sqrt{1-\frac{\rho}{\rho_c}}\right)^2}\frac{\rho}{\rho_c}\right]^2$$

Critical density, Big bounce, ...

$$\frac{8\pi G}{3}\rho_c = \frac{1}{4\lambda^2}$$

$$v_{bounce} = 2\lambda\frac{\pi_\phi}{\sqrt{12\pi G}}$$

Deparametrizing Cosmology

Evolution as Flow Generated by the Complexifier

Use scalar field ϕ as clock and

matter momentum π_ϕ as deparametrized Hamiltonian

giving the evolution $v(\phi), b(\phi)$ with no reference to time t

For both standard FRW and loop-regularized FRW,

deparametrized Hamiltonian is Complexifier !

$$\mathcal{H}^0 = 0 \Rightarrow \pi_\phi = \pm\sqrt{12\pi G}\, C \qquad \mathcal{H}^{reg} = 0 \Rightarrow \pi_\phi = \pm\sqrt{12\pi G}\, C$$

Deparametrizing Cosmology

Evolution as Flow Generated by the Complexifier

For both standard FRW and loop-regularized FRW,

deparametrized Hamiltonian is Complexifier !

$$\mathcal{H}^0 = 0 \Rightarrow \pi_\phi = \pm\sqrt{12\pi G}\, C \qquad \mathcal{H}^{reg} = 0 \Rightarrow \pi_\phi = \pm\sqrt{12\pi G}\, C$$

Thus evolution given SU(1,1) boost action generated by Complexifier

i.e. as scale transformations shifting the Immirzi parameter !

with trajectories at constant Complexifier

Loop Quantum Cosmology as SU(1,1) Coherent States

Let's go to quantum level

Loop Quantum Cosmology as SU(1,1) Coherent States

Focus on deparametrized theory

and quantize gravitational CVH alg as SU(1,1) representation

$$
\begin{aligned}
\hat{\mathfrak{C}}\,|\mathfrak{C},m\rangle &= \left[\hat{\mathcal{J}}_z^2 - \tfrac{1}{2}\hat{\mathcal{K}}_+\hat{\mathcal{K}}_- - \tfrac{1}{2}\hat{\mathcal{K}}_-\hat{\mathcal{K}}_+\right]|\mathfrak{C},m\rangle = \mathfrak{C}\,|\mathfrak{C},m\rangle \\
\hat{\mathcal{J}}_z\,|\mathfrak{C},m\rangle &= m\,|\mathfrak{C},m\rangle \\
\hat{\mathcal{K}}_+\,|\mathfrak{C},m\rangle &= \sqrt{m(m+1)-\mathfrak{C}}\,|\mathfrak{C},m+1\rangle \\
\hat{\mathcal{K}}_-\,|\mathfrak{C},m\rangle &= \sqrt{m(m-1)-\mathfrak{C}}\,|\mathfrak{C},m-1\rangle
\end{aligned}
$$

Choose time-like irrep with vanishing or positive Casimir labeled by half-integer spin j $\quad \mathfrak{C} = j(j-1) \geq 0 \qquad j \geq 1$

v=Jz has discrete positive spectrum, volume gap given by j

Loop Quantum Cosmology as SU(1,1) Coherent States

Complexifier generates unitary transformations:

- unitary shifts in Immirzi parameter
- unitary deparametrized evolution

Can describe flow on coherent states:

$$|j,z\rangle = \sum_{m\in j+\mathbb{N}}^{+\infty} \sqrt{\frac{(m+j+1)!}{(m-j)!(2j+1)!}} \frac{(z^1)^{m-j}}{(\bar{z}^0)^{m+j+2}} \, |j,\ m\rangle$$

$$\langle \hat{V} \rangle = \lambda \, \langle \hat{k}_x + \hat{j}_z \rangle = \lambda \, j \, \frac{z^0 z^1 + \bar{z}^0 \bar{z}^1 + |z^0|^2 + |z^1|^2}{|z^0|^2 - |z^1|^2}$$

$$\langle \hat{C} \rangle = \langle \hat{k}_y \rangle = -ij \, \frac{\bar{z}^0 \bar{z}^1 - z^0 z^1}{|z^0|^2 - |z^1|^2}$$

$$\langle \hat{\mathcal{H}}_g \rangle = (2\lambda)^{-1} \, \langle \hat{k}_x - \hat{j}_z \rangle = \frac{j}{2\lambda} \, \frac{z^0 z^1 + \bar{z}^0 \bar{z}^1 - |z^0|^2 - |z^1|^2}{|z^0|^2 - |z^1|^2}$$

Loop Quantum Cosmology as SU(1,1) Coherent States

Complexifier generates unitary transformations:

- unitary shifts in Immirzi parameter
- unitary deparametrized evolution

Can describe flow on coherent states:

$$|j,z\rangle = \sum_{m\in j+\mathbb{N}}^{+\infty} \sqrt{\frac{(m+j+1)!}{(m-j)!(2j+1)!}} \frac{(z^1)^{m-j}}{(\bar{z}^0)^{m+j+2}} \, |j,\ m\rangle$$

- Transform simply under SU(1,1) action $U|j,z\rangle = |j, U \triangleright z\rangle$
- Satisfies Semi-classical Casimir eqn $-\left[2\langle V\rangle\langle\hat{\mathcal{H}}_g\rangle + \langle\hat{C}\rangle^2\right] = j^2 > 0$

Loop Quantum Cosmology as SU(1,1) Coherent States

Complexifier generates unitary transformations:

- unitary shifts in Immirzi parameter
- unitary deparametrized evolution

Flow as classical boost action: $\hat{W}_\eta|j,z\rangle = e^{i\eta\hat{\mathcal{C}}}|j,z\rangle = |j, e^{i\eta\tau_y} \triangleright z\rangle$

$$\begin{pmatrix} z^0_\eta \\ \bar{z}^1_\eta \end{pmatrix} = e^{i\eta\tau_y} \triangleright z = \begin{pmatrix} \cosh\frac{\eta}{2} & \sinh\frac{\eta}{2} \\ \sinh\frac{\eta}{2} & \cosh\frac{\eta}{2} \end{pmatrix} \begin{pmatrix} z^0 \\ \bar{z}^1 \end{pmatrix}$$

Trajectories at constant Complexifier and Matter Momentum:

$$\frac{\pi_\phi^2}{12\pi G} \equiv -2\,\langle V\rangle\langle\hat{\mathcal{H}}_g\rangle$$

Loop Quantum Cosmology as SU(1,1) Coherent States

Complexifier generates unitary transformations:

- unitary shifts in Immirzi parameter
- unitary deparametrized evolution

Can describe flow on coherent states:

$$|j, z\rangle = \sum_{m \in j+\mathbb{N}}^{+\infty} \sqrt{\frac{(m+j+1)!}{(m-j)!(2j+1)!}} \frac{(z^1)^{m-j}}{(\bar{z}^0)^{m+j+2}} |j,\, m\rangle$$

$$\langle \hat{V} \rangle = \lambda \langle \hat{k}_x + \hat{j}_z \rangle = \lambda\, j \, \frac{z^0 z^1 + \bar{z}^0 \bar{z}^1 + |z^0|^2 + |z^1|^2}{|z^0|^2 - |z^1|^2}$$

$$\langle \hat{C} \rangle = \langle \hat{k}_y \rangle = -ij \, \frac{\bar{z}^0 \bar{z}^1 - z^0 z^1}{|z^0|^2 - |z^1|^2}$$

$$\langle \hat{\mathcal{H}}_g \rangle = (2\lambda)^{-1} \langle \hat{k}_x - \hat{j}_z \rangle = \frac{j}{2\lambda} \, \frac{z^0 z^1 + \bar{z}^0 \bar{z}^1 - |z^0|^2 - |z^1|^2}{|z^0|^2 - |z^1|^2}$$

Loop Quantum Cosmology as SU(1,1) Coherent States

Complexifier generates unitary transformations:

- unitary shifts in Immirzi parameter
- unitary deparametrized evolution

Flow as classical boost action: $\hat{W}_\eta|j,z\rangle = e^{i\eta\hat{\mathcal{C}}}|j,z\rangle = |j, e^{i\eta\tau_y} \triangleright z\rangle$

$$\begin{pmatrix} z^0_\eta \\ \bar{z}^1_\eta \end{pmatrix} = e^{i\eta\tau_y} \triangleright z = \begin{pmatrix} \cosh\frac{\eta}{2} & \sinh\frac{\eta}{2} \\ \sinh\frac{\eta}{2} & \cosh\frac{\eta}{2} \end{pmatrix} \begin{pmatrix} z^0 \\ \bar{z}^1 \end{pmatrix}$$

Trajectories at constant Complexifier and Matter Momentum:

$$\frac{\pi_\phi^2}{12\pi G} \equiv -2\,\langle V\rangle\langle\hat{\mathcal{H}}_g\rangle$$

Physical States for LQC from SU(1,1) Irreps

Quantize CVH alg for gravity+matter

using space-like SU(1,1) irreps $s^2 = \frac{\pi_\phi^2}{12\pi G}$

and solve Hamiltonian constraint $\hat{J}_z|\psi\rangle = \hat{K}_x|\psi\rangle$

Physical States as eigenspaces of Complexifier

CVH Algebra for Loop-deformed FRW Cosmology

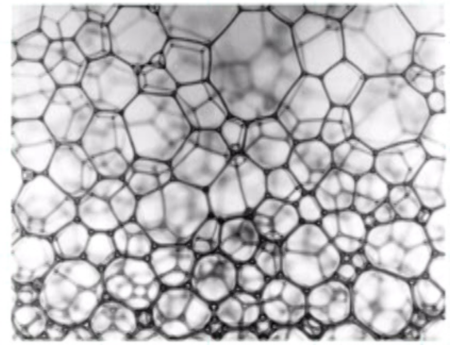
Remarks :

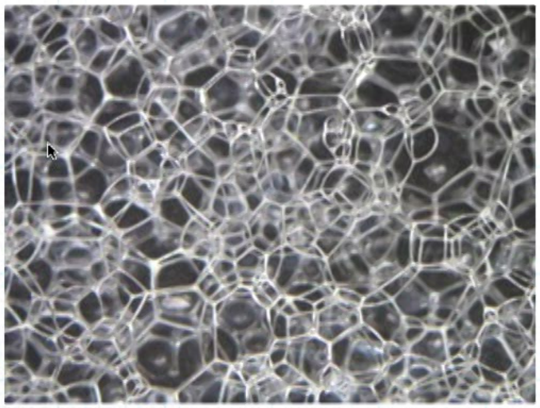
- Adding Cosmological constant term
- CVH selects $\bar{\mu}$ regularization scheme over μ_0 scheme
- Wick rotation to imaginary γ as thermal states for complexifier, i.e. deparametrized Hamiltonian

(Loop) Quantum Gravity as CVH Bubble Networks

Bubbles of
Quantum Geometry
with local CVH alg

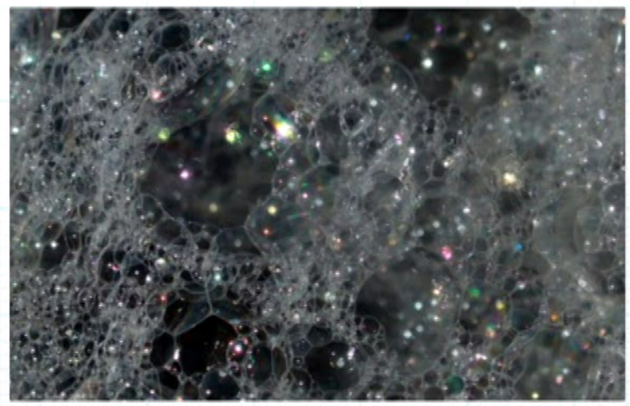
Boundary complexifier
as Gibbons-Hawking term





Extrinsic curvature
as Torsion for
Ashtekar-Barbero connection

... or use boundary
Virasoro alg ?



Surface measures
of torsion
for twisted geometries

so(3,2) boundary alg
for LQG bubbles ...

Immirzi Parameter & Thiemann Complexifier in LQG - Livine - PI '17