Title: Estimating entanglement from only a few moments

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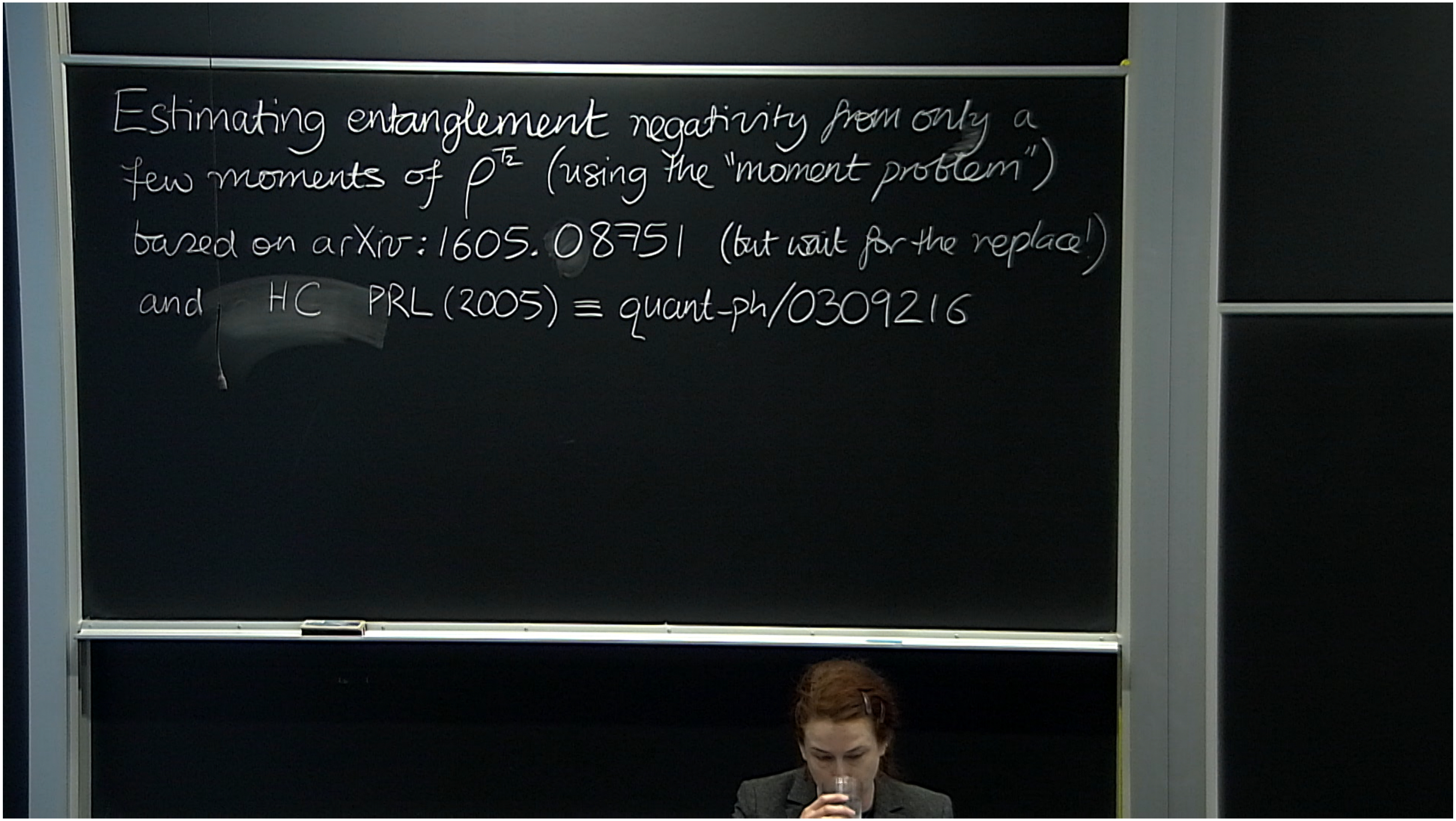
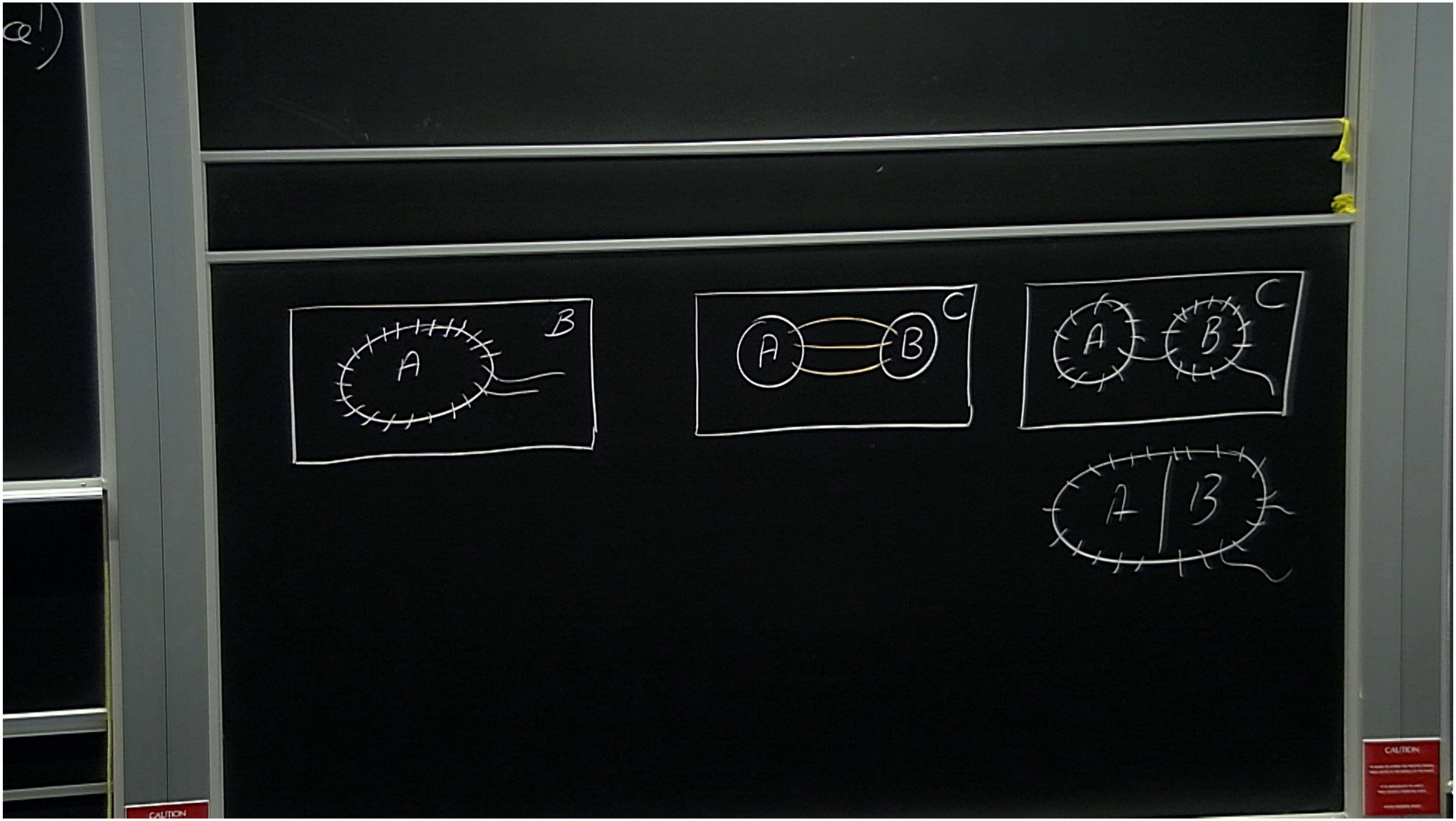
URL: http://pirsa.org/17050071

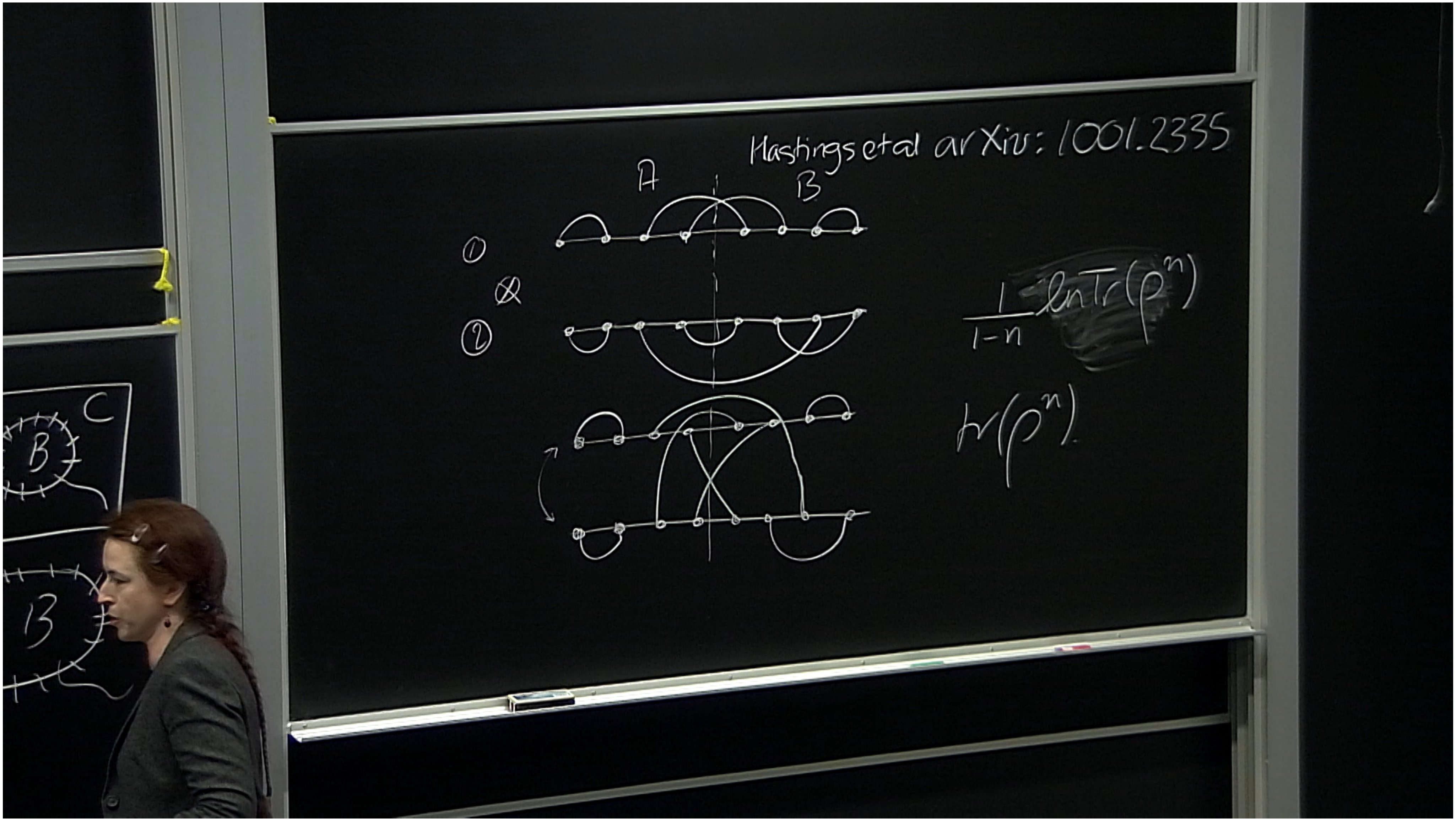
Abstract: <p>How can we quantify the entanglement between subsystems when we only have access to incomplete information about them and their environment?â€Ž Existing approaches (such as RÃ©nyi entropies) can only detect the short-range entanglement across a boundary between a subsystem and its surroundings, and then only if the whole system is pure. These methods cannot detect the long-range entanglement between two subsystems embedded in a larger system. There is a natural choice of entanglement measure for this situation, called the entanglement negativity, which can do this and cope with mixed states as well. However it is defined in terms of the full density matrix, which we generally won't have access to.</p>

<p>I will begin this talk with a brief overview â€Žof some replica trick-based eigenspectrum reconstruction methods, and their various strengths and limitations. Then I will show how to modify these to find the moments of the partially transposed density matrix. Once those numbers have been obtained, it is possible to modify the earlier eigenspectrum reconstruction methods to obtain lower and upper bounds for the entanglement negativity.</p>

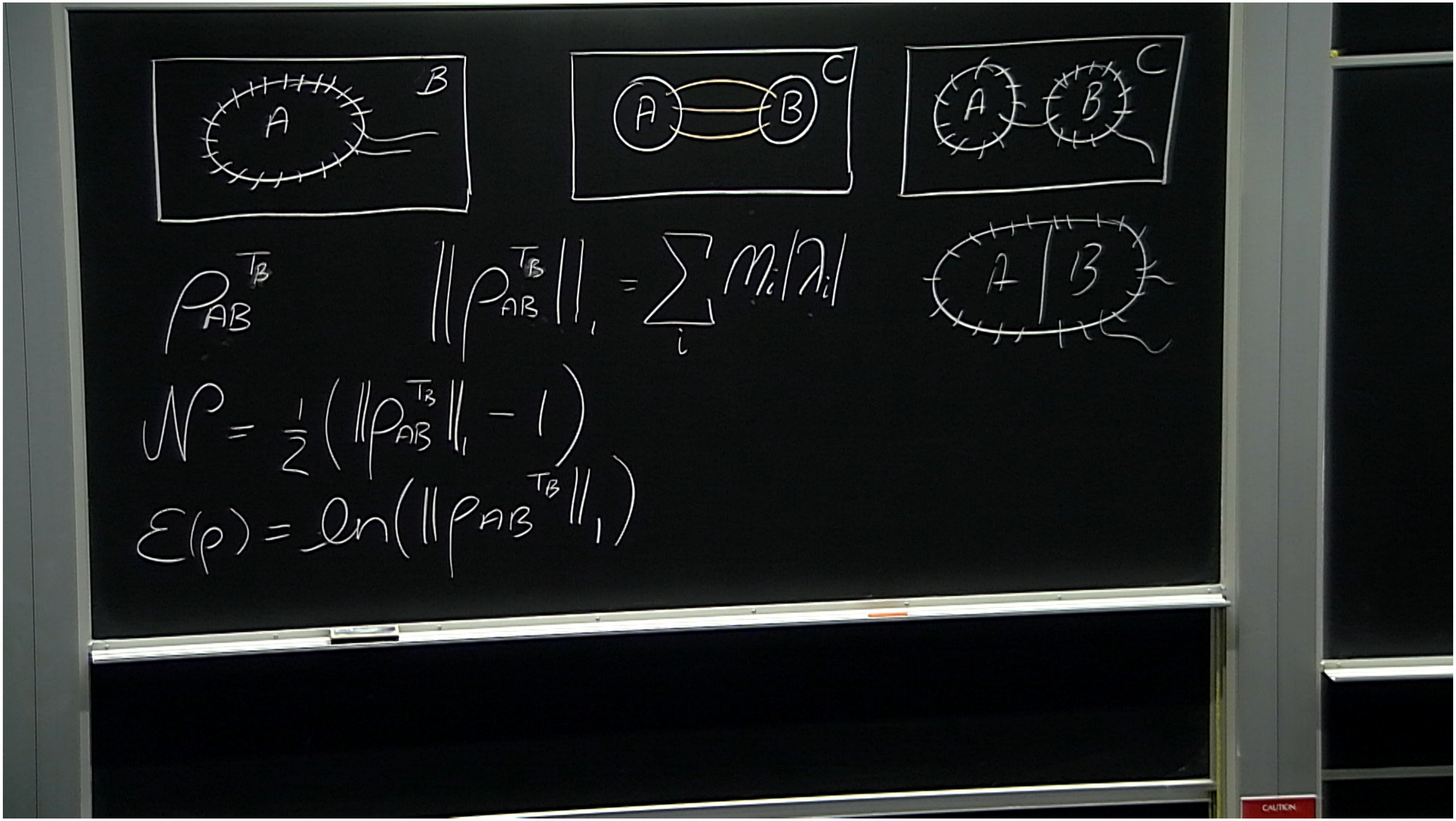

Estimating entanglement negativity from only a few moments of ρ^{T2} (using the "moment problem")
based on arXiv: 1605.08751 (but wait for the replace!)
and HC PRL (2005) ≡ quant-ph/0309216

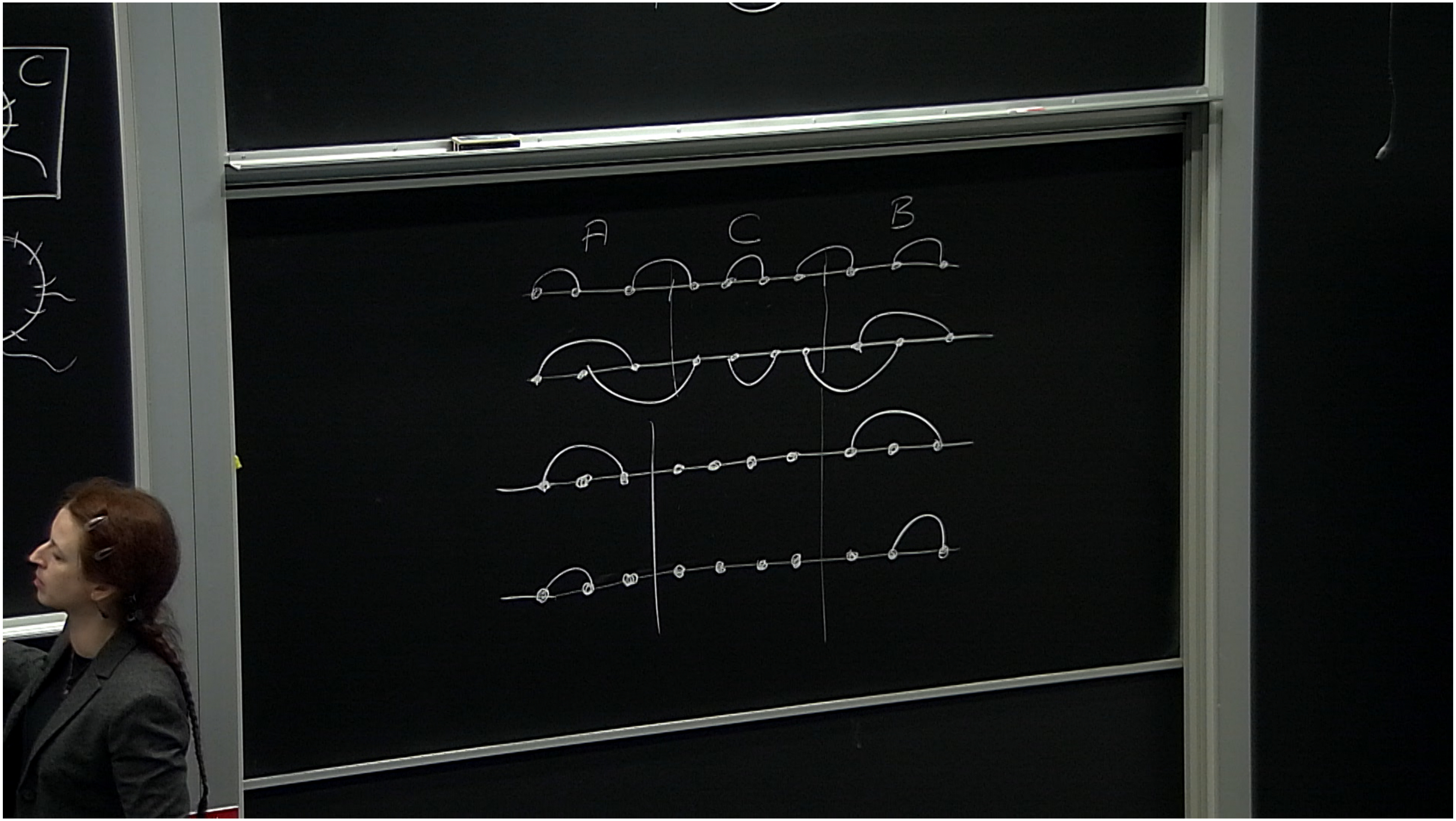


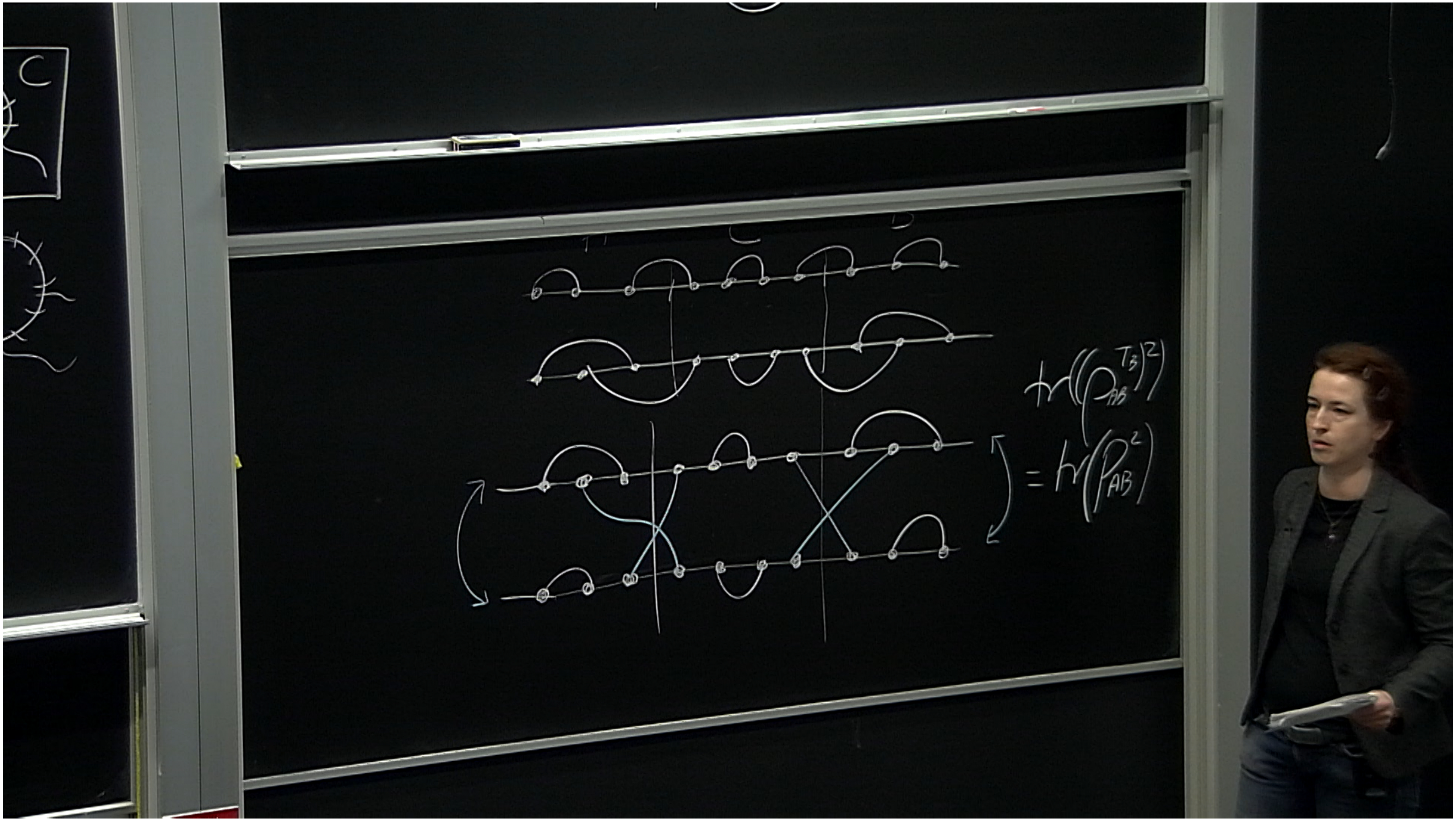
A
B
A
B
C
A
B
C
A
B



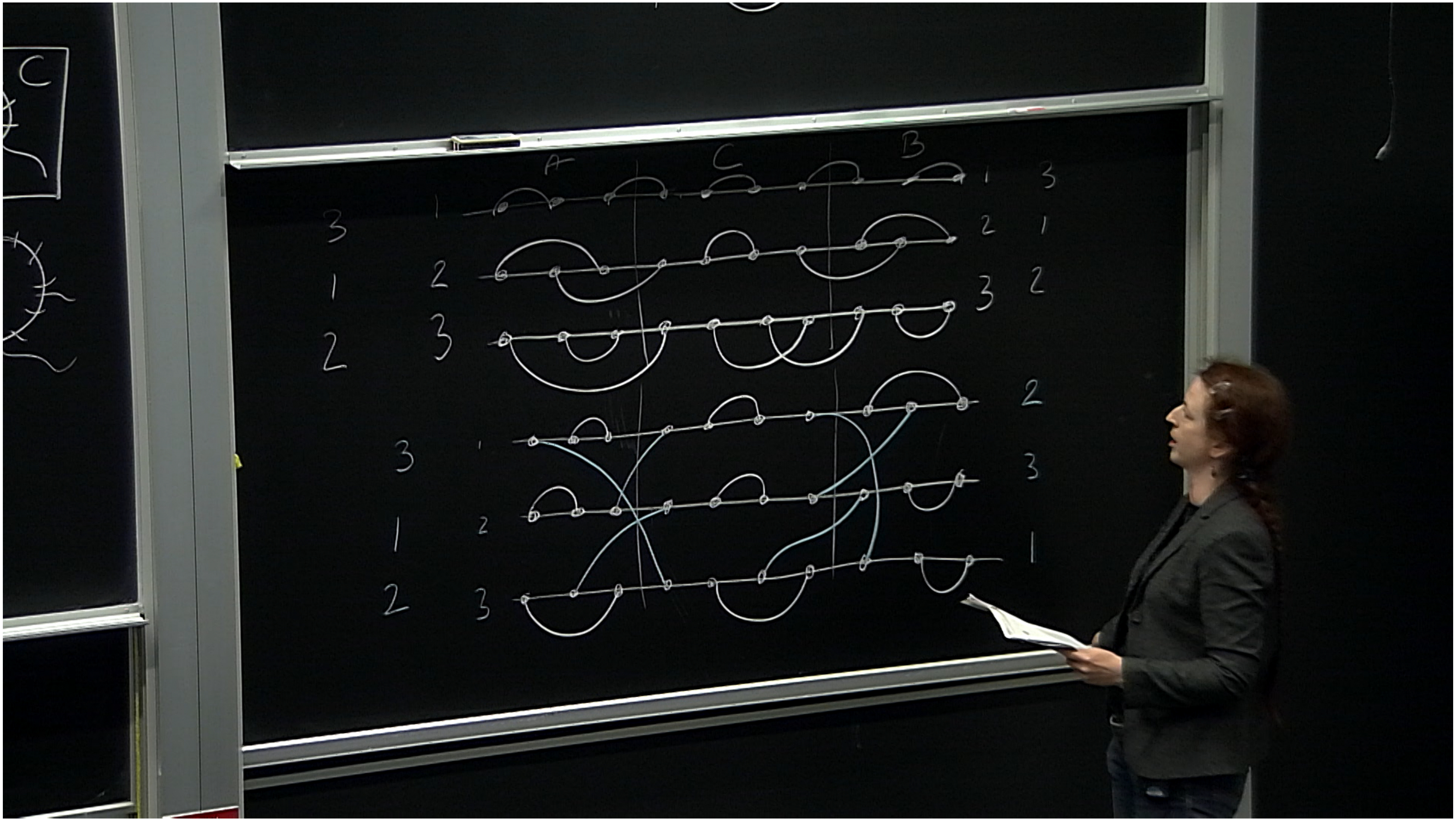
Hastings et al arXiv: 1001.2335
A
B
①
⊗
②
$\frac{1}{1-n}$ ln Tr(ρ^n)
tr(ρ^n)
C
B
B

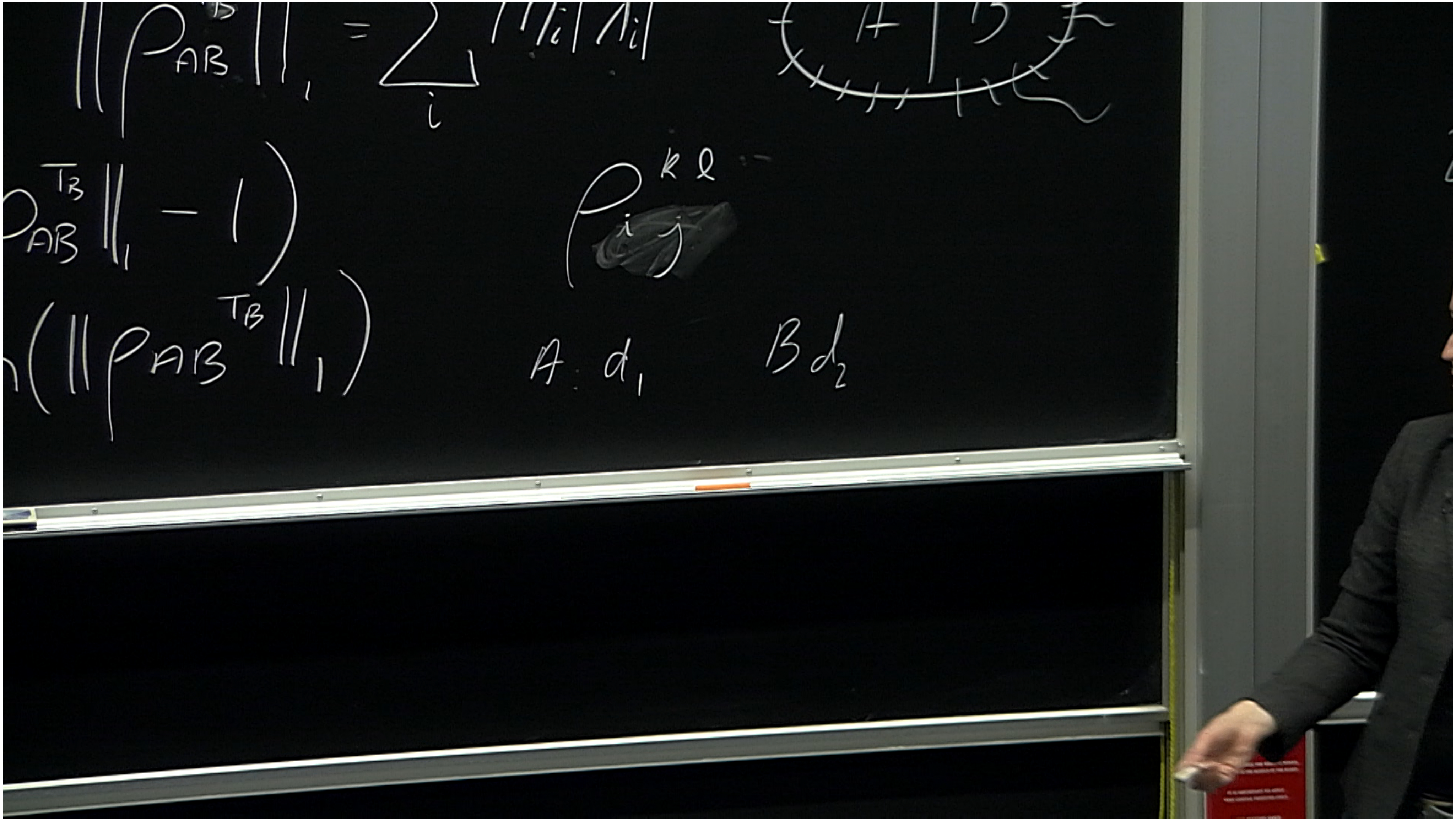

A
B
A
B
C
A
B
C
$\rho_{AB}^{T_B}$
$\|\rho_{AB}^{T_B}\|_1 = \sum_i m_i |\lambda_i|$
A | B
$\mathcal{N} = \frac{1}{2}\left(\|\rho_{AB}^{T_B}\|_1 - 1\right)$
$\mathcal{E}(\rho) = \ln\left(\|\rho_{AB}^{T_B}\|_1\right)$

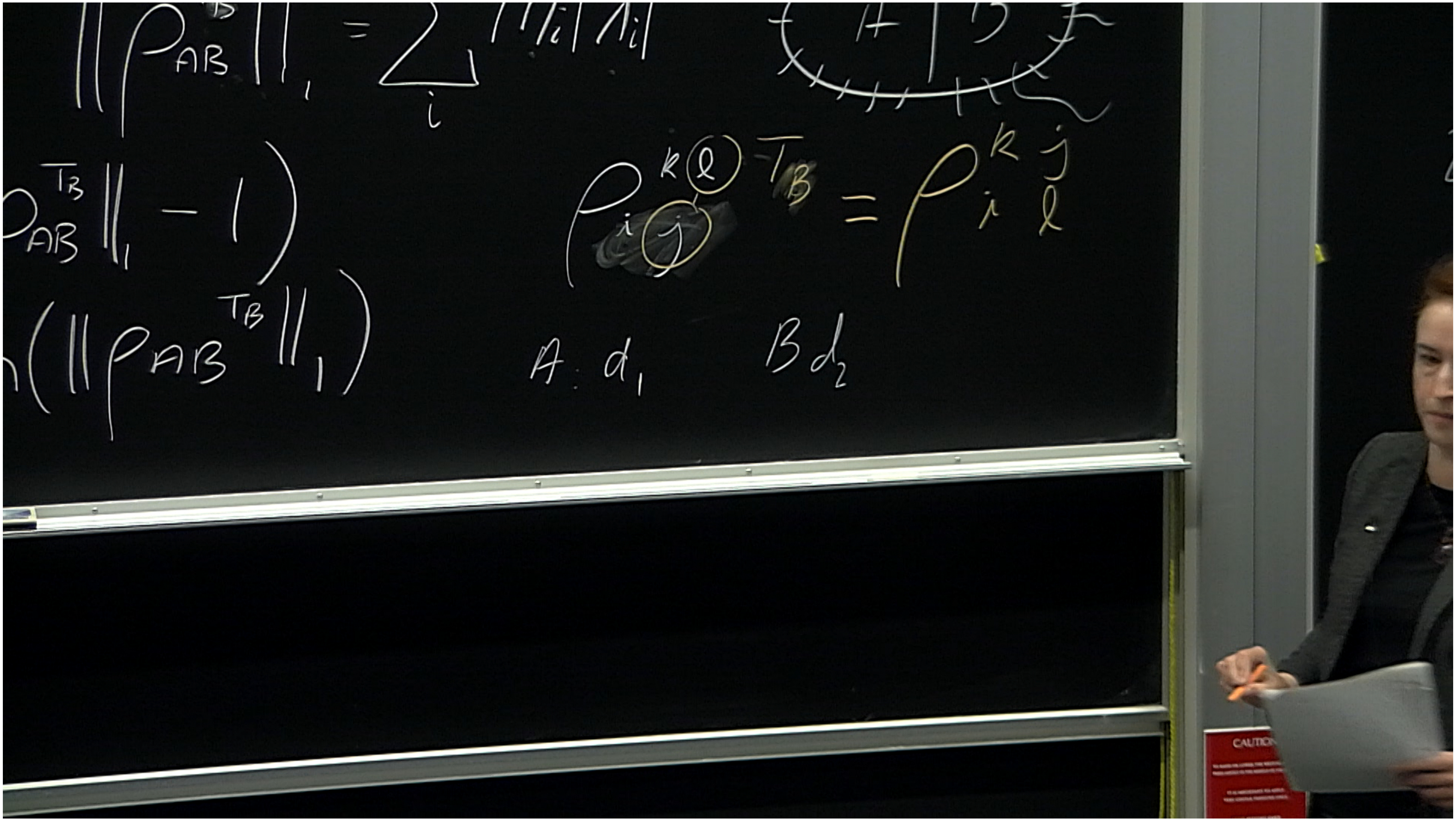

A
C
B

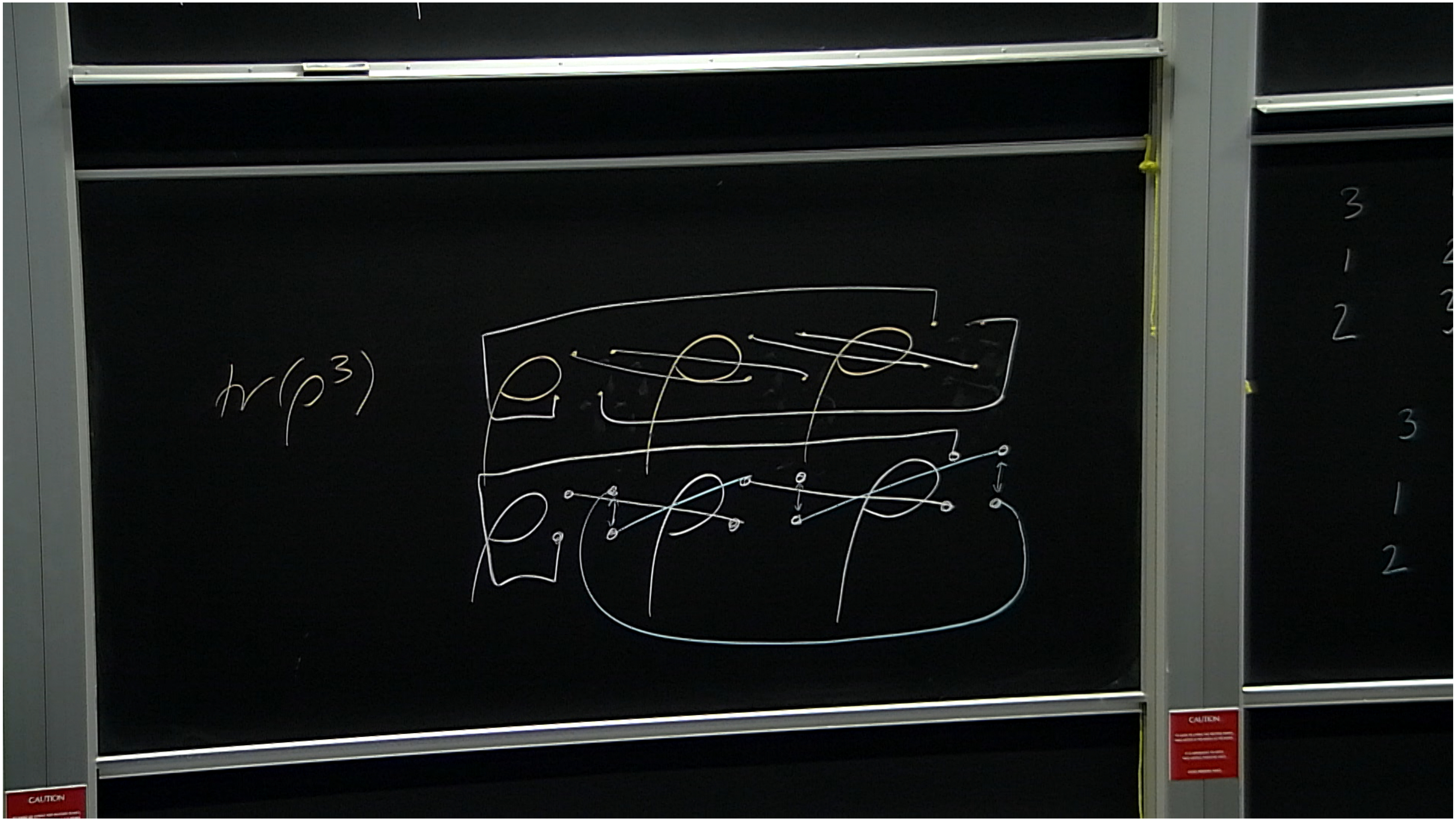


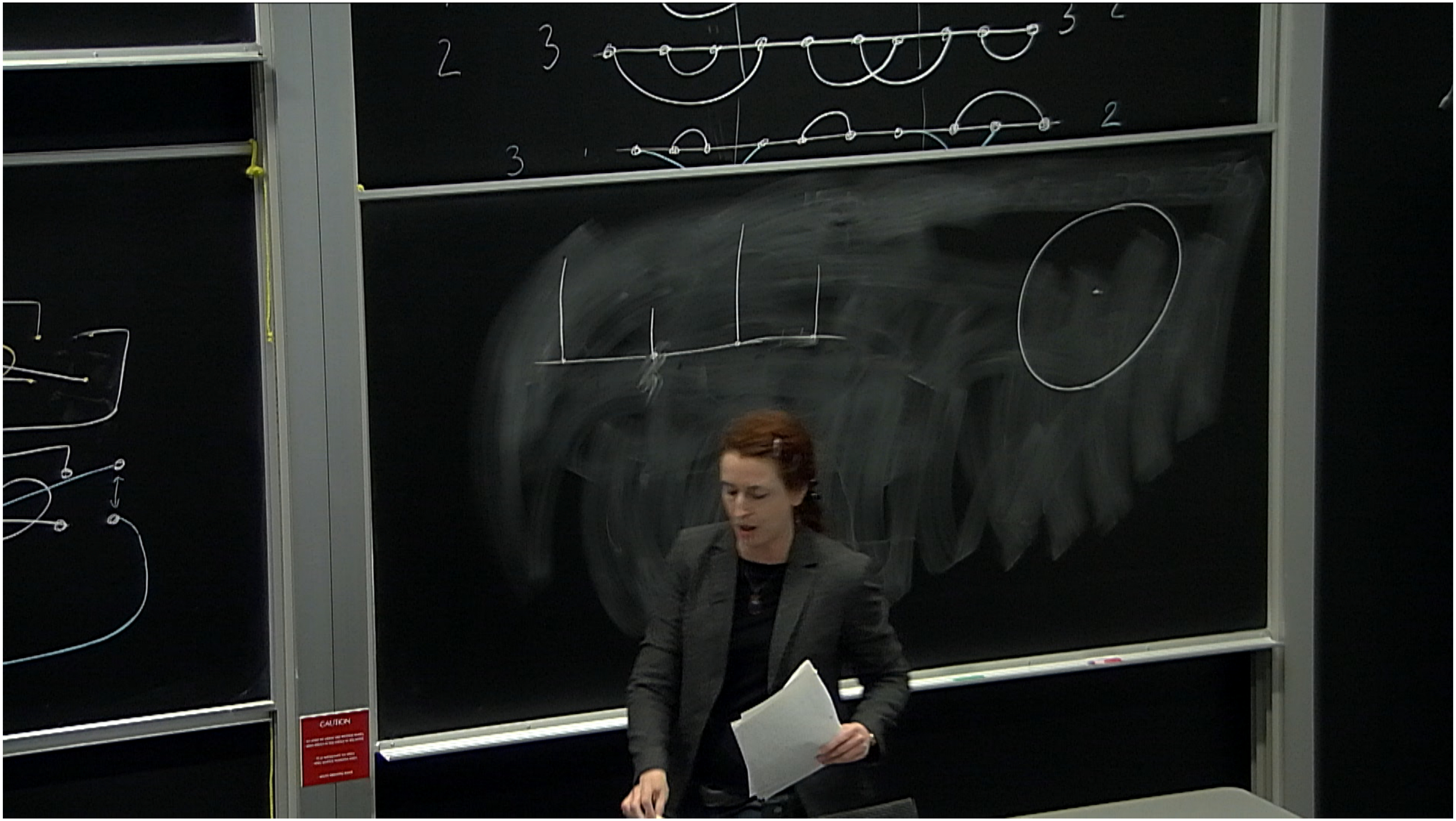
$tr((\rho_{AB}^{T_3})^2)$
$= tr(\rho_{AB}^2)$

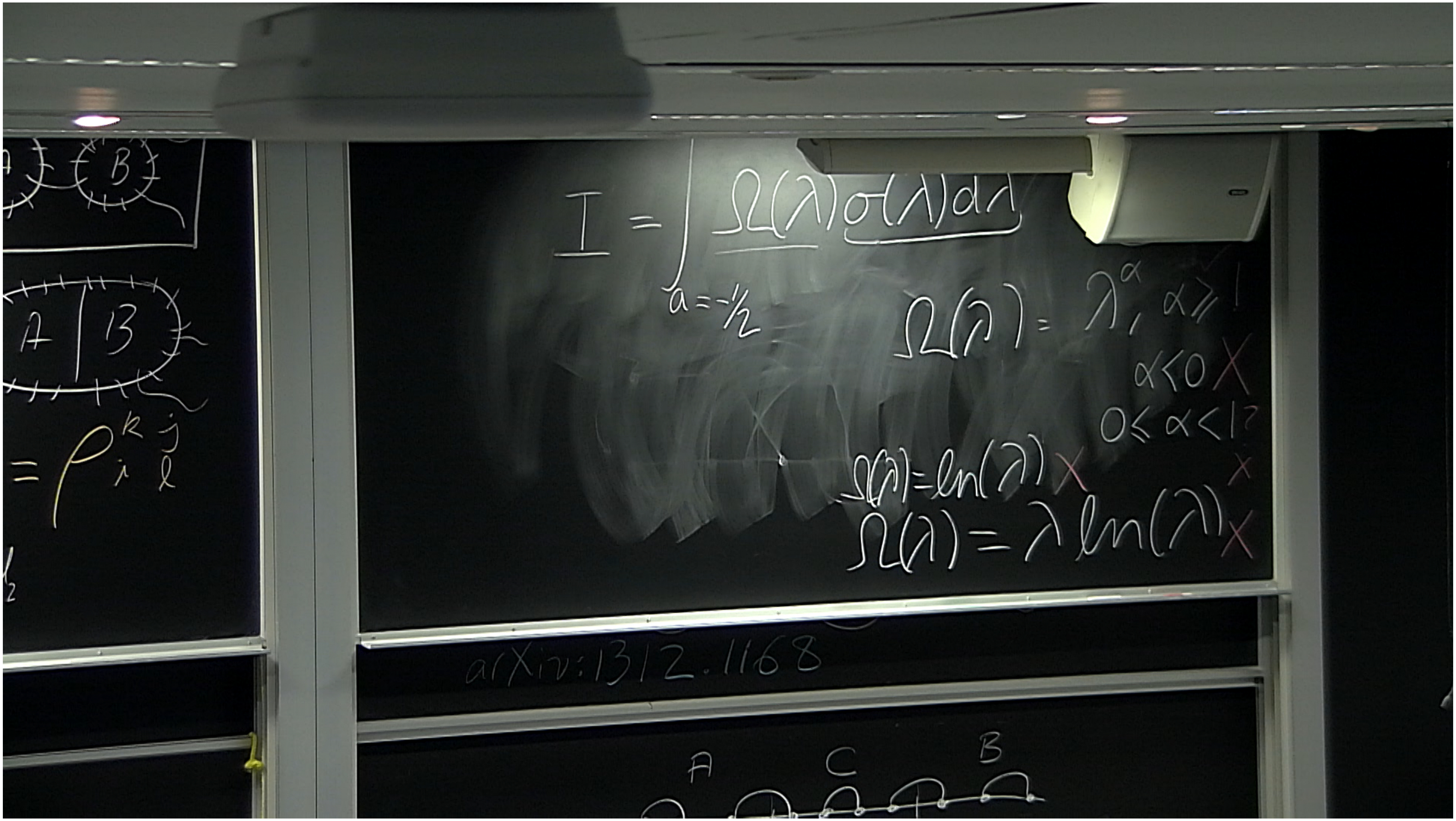

A
C
B
3 1
1 2
2 3
1 3
2 1
3 2
3 1
1 2
2 3
2
3
1

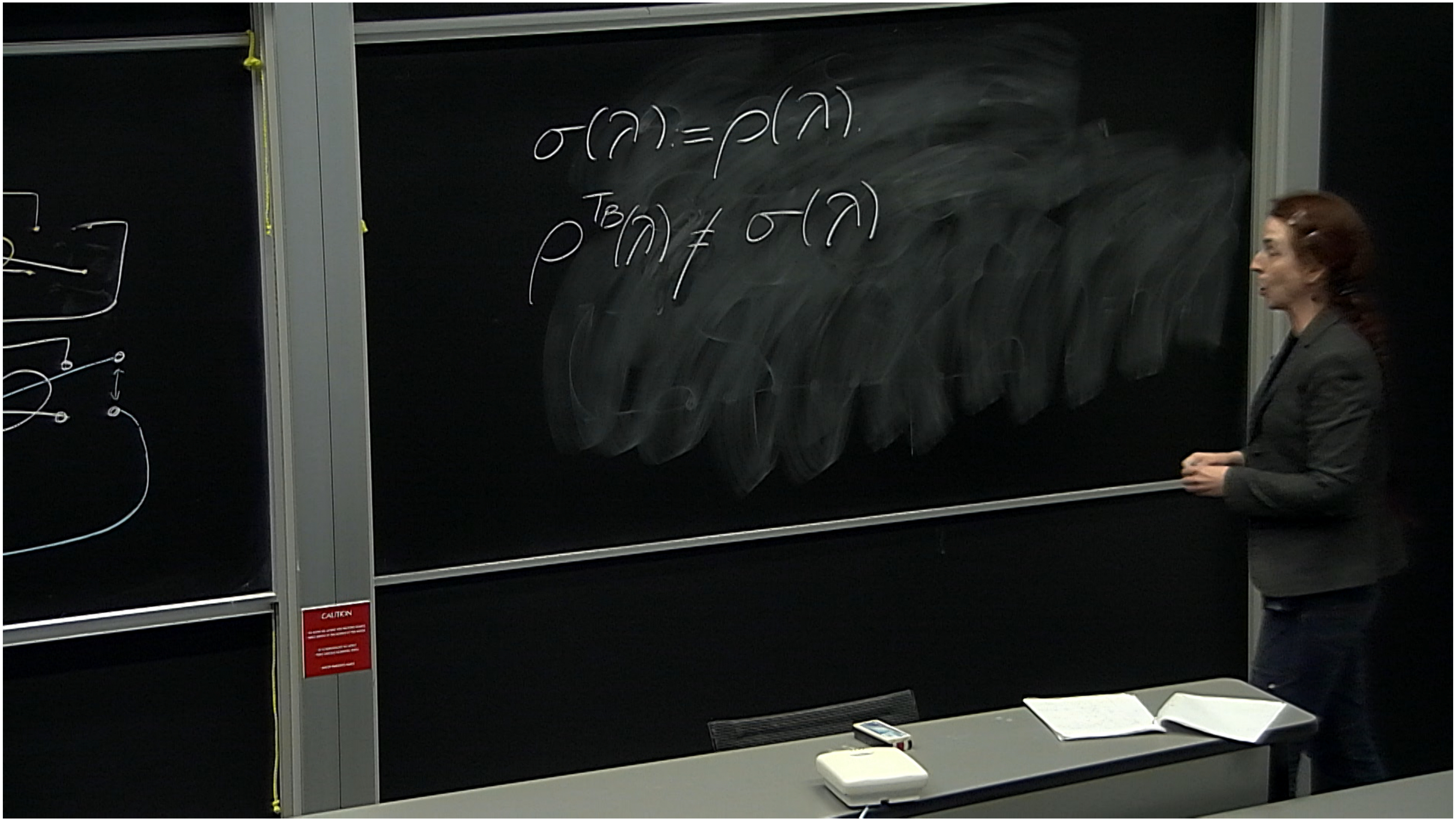

A : d1
B d2

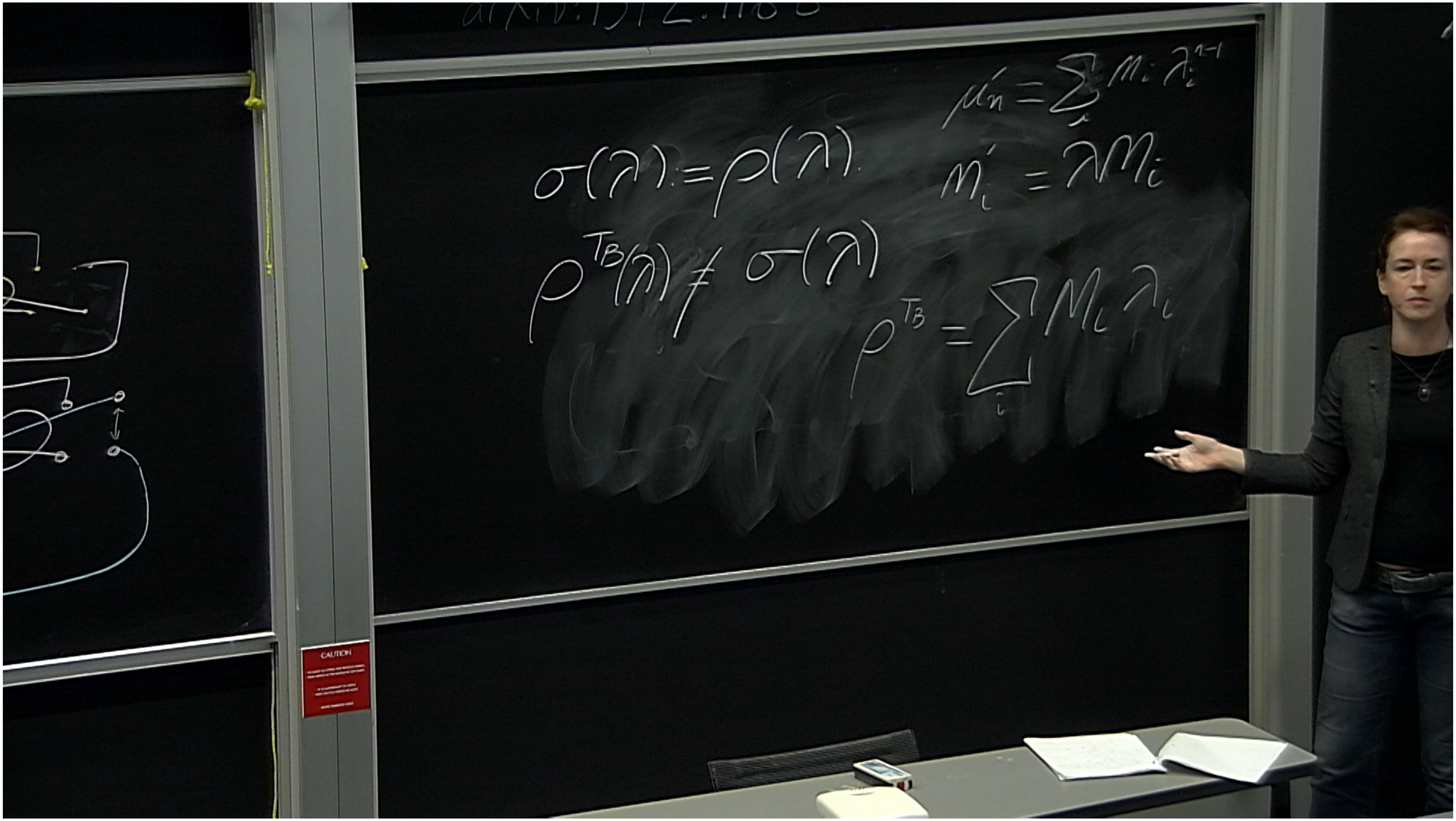

A : d1
B d2
CAUTION

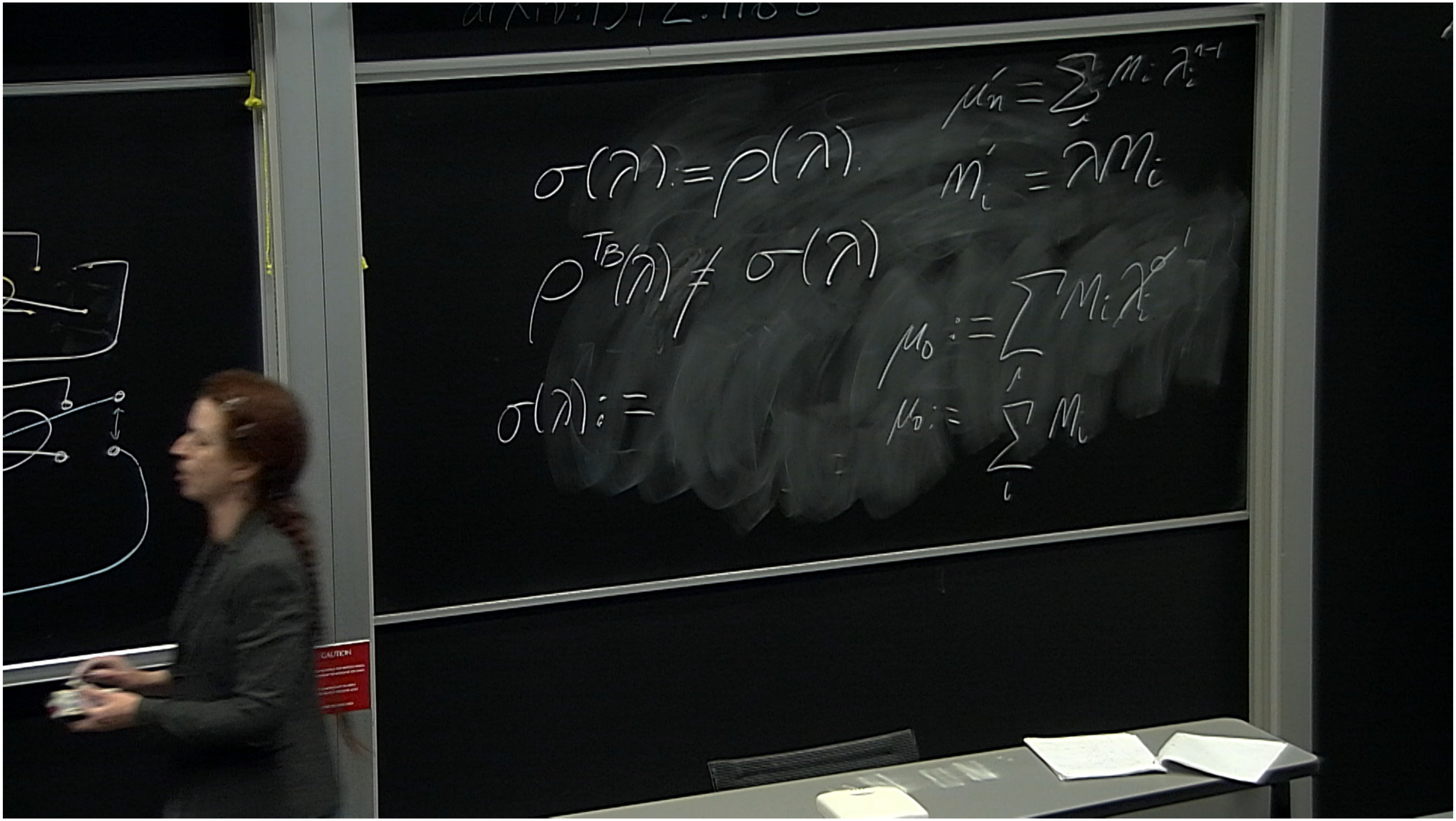

tr(ρ³)
3
1
2
3
1
2

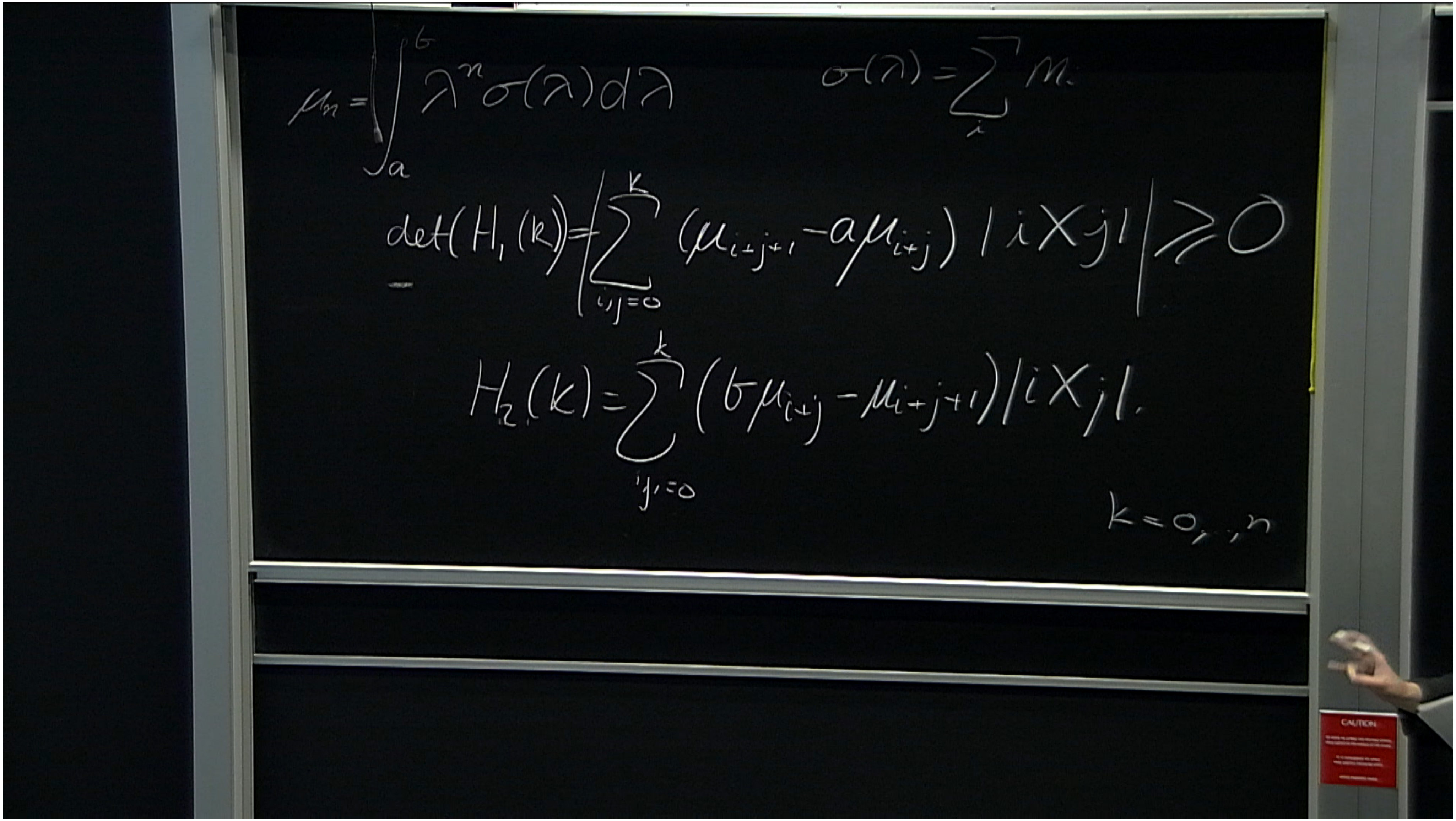

2
3
3
3
2
2
CAUTION


I = ∫ Ω(λ) σ(λ) dλ
a = -1/2
Ω(λ) = λ^α, α ≥ 1
α < 0
0 ≤ α < 1 ?
Ω(λ) = ln(λ)
Ω(λ) = λ ln(λ)
arXiv:1312.1168
A
C
B


$\sigma(\lambda) := \rho(\lambda)$
$\rho^{T_B}(\lambda) \neq \sigma(\lambda)$
CAUTION


$\mu_n' = \sum_i m_i \lambda_i^{n-1}$
$\sigma(\lambda) = \rho(\lambda)$
$m_i' = \lambda m_i$
$\rho^{T_B}(\lambda) \neq \sigma(\lambda)$
$\rho^{T_B} = \sum_i m_i \lambda_i$
CAUTION


$\sigma(\lambda) := \rho(\lambda)$
$\rho^{T_B}(\lambda) \neq \sigma(\lambda)$
$\sigma(\lambda) :=$
$\mu_n = \sum_i m_i \lambda_i^{n-1}$
$m_i' = \lambda m_i$
$\mu_0 := \sum_i m_i$

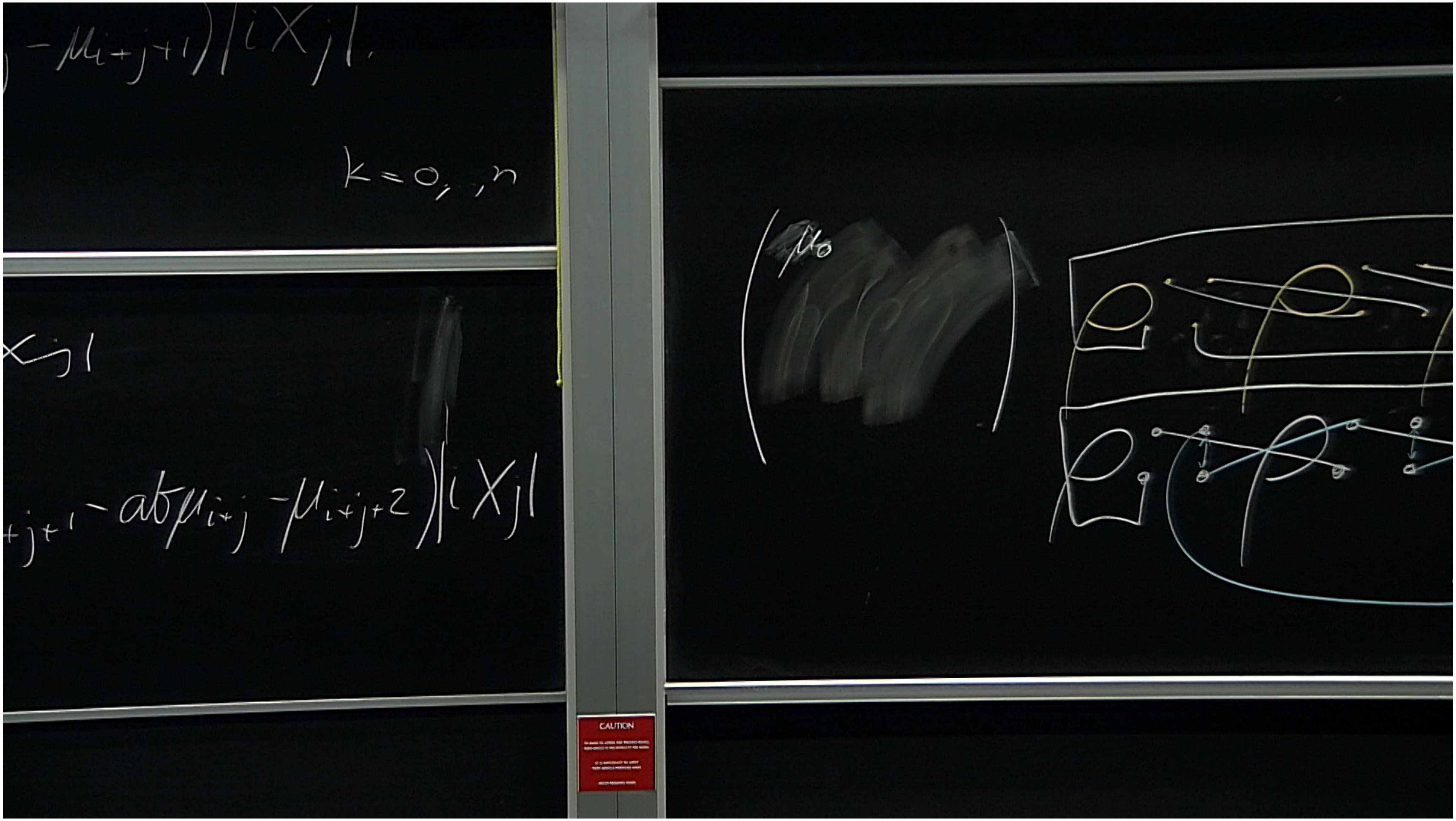

$\mu_n = \int_a^b \lambda^n \sigma(\lambda) d\lambda$
$\sigma(\lambda) = \sum_i m_i$
$\det(H_1(k)) = \left| \sum_{i,j=0}^{k} (\mu_{i+j+1} - a\mu_{i+j}) |i\rangle\langle j| \right| \geq 0$
$H_2(k) = \sum_{i,j=0}^{k} (b\mu_{i+j} - \mu_{i+j+1}) |i\rangle\langle j|.$
$k = 0, .., n$

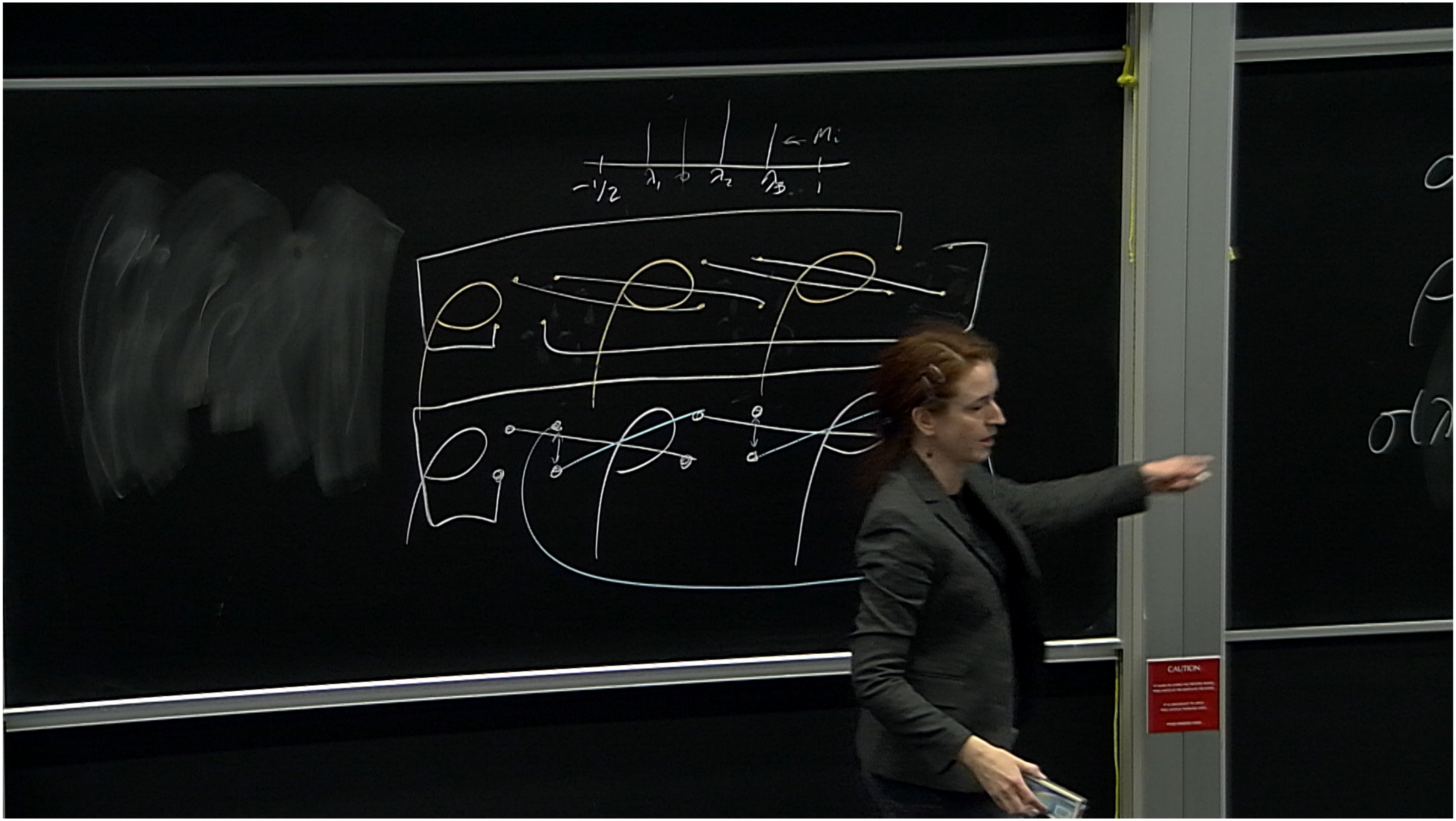
$$H_2(k) = \sum_{i,j=0}^{k} \left(b\mu_{i+j} - \mu_{i+j+1}\right) |i\rangle\langle j|.$$

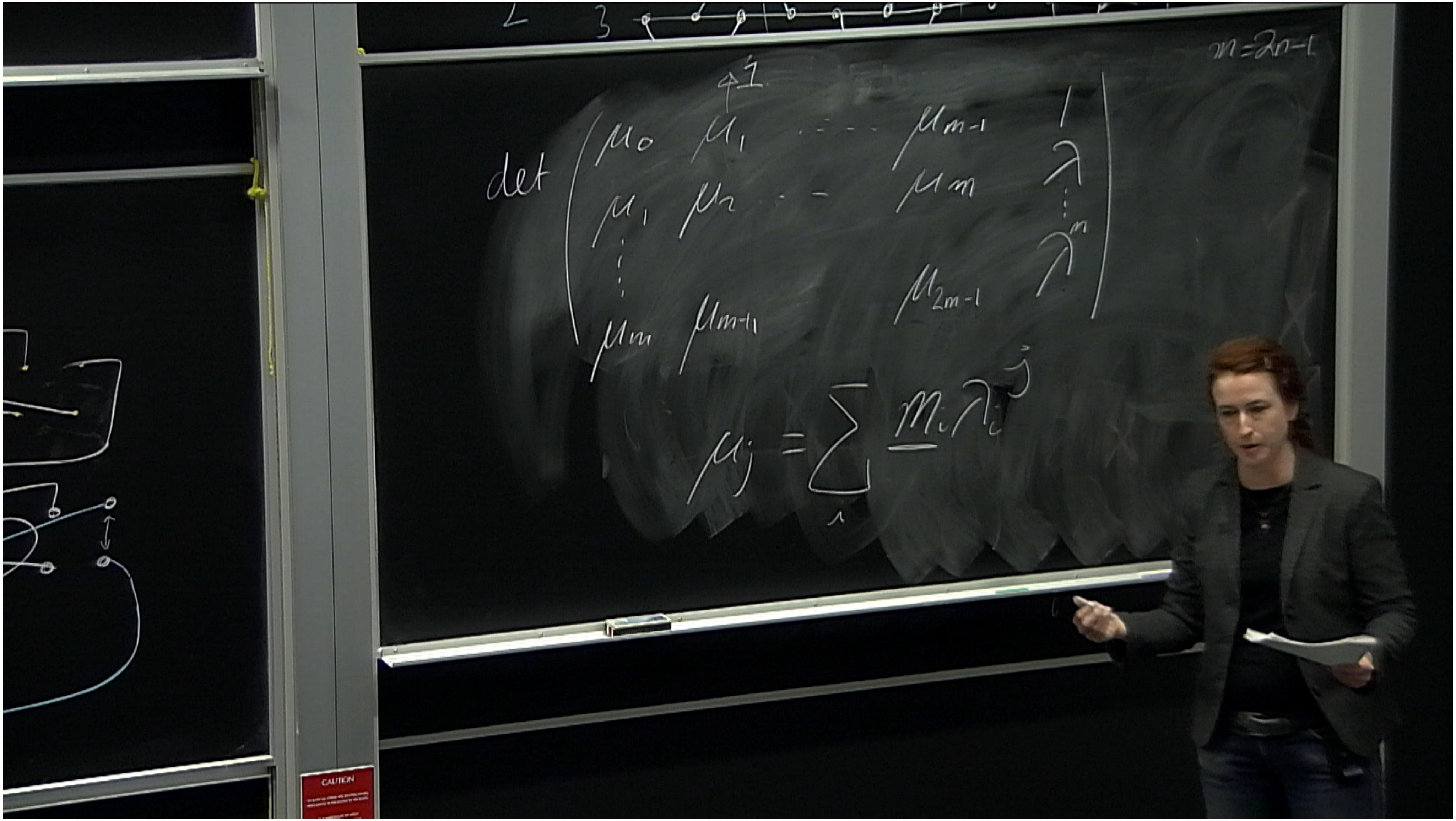
$k = 0, \dots, n$

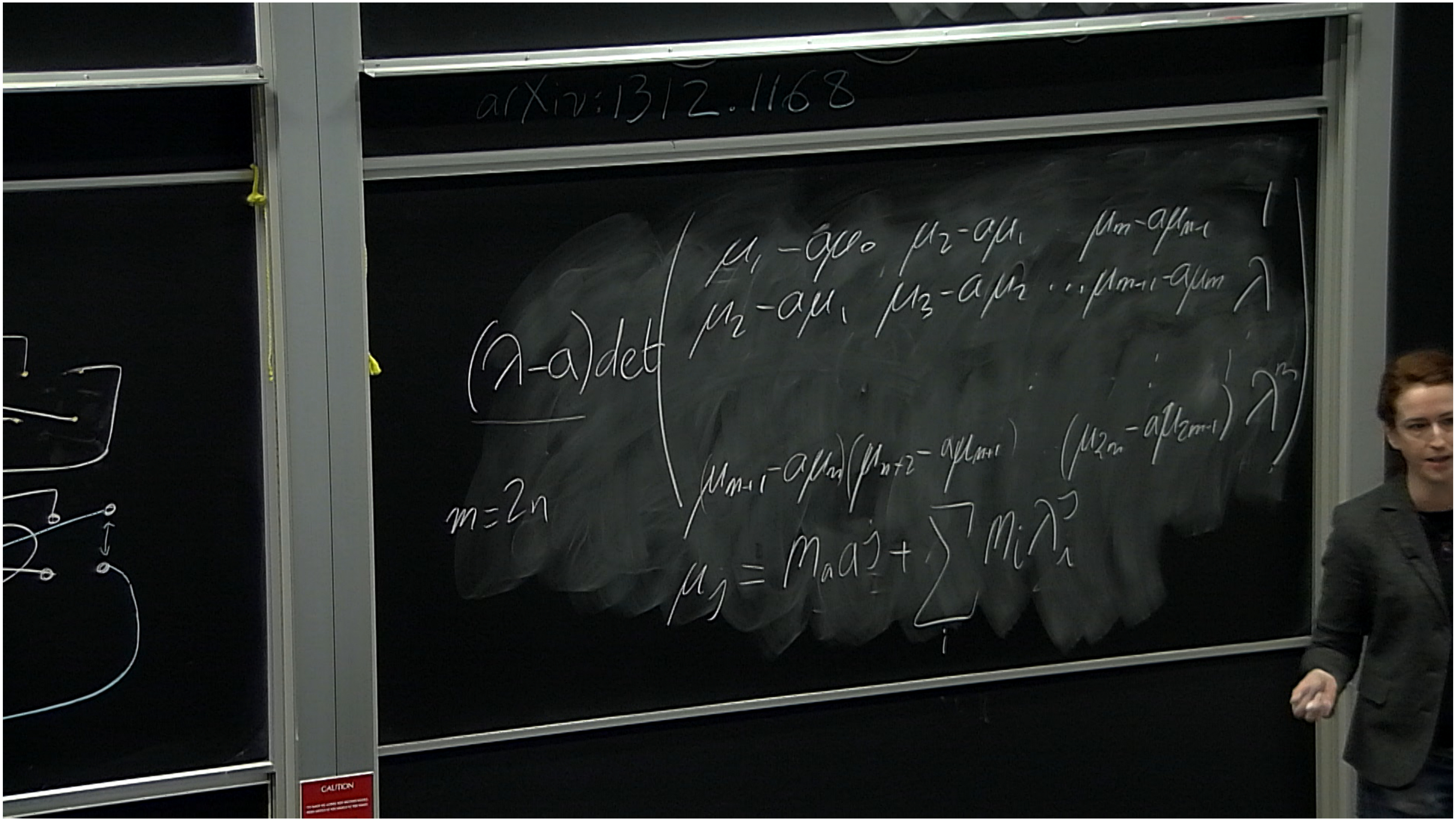
$$H_1(k) = \sum_{i,j}^{k} \mu_{i+j} \, |i\rangle\langle j|$$

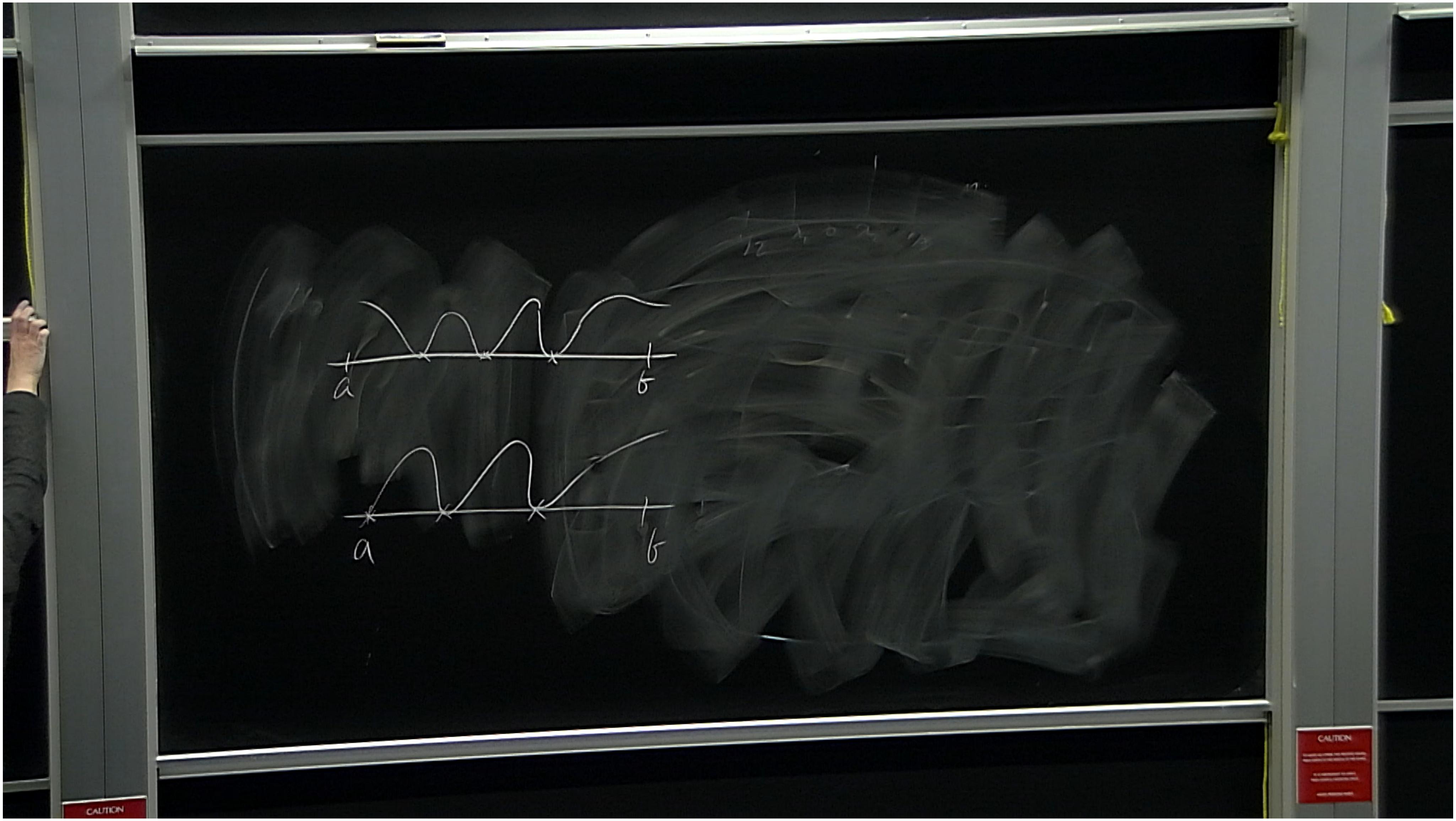
$$H_2(k) = \sum_{i,j=0} \left((a+b)\mu_{i+j+1} - ab\mu_{i+j} - \mu_{i+j+2}\right) |i\rangle\langle j|$$

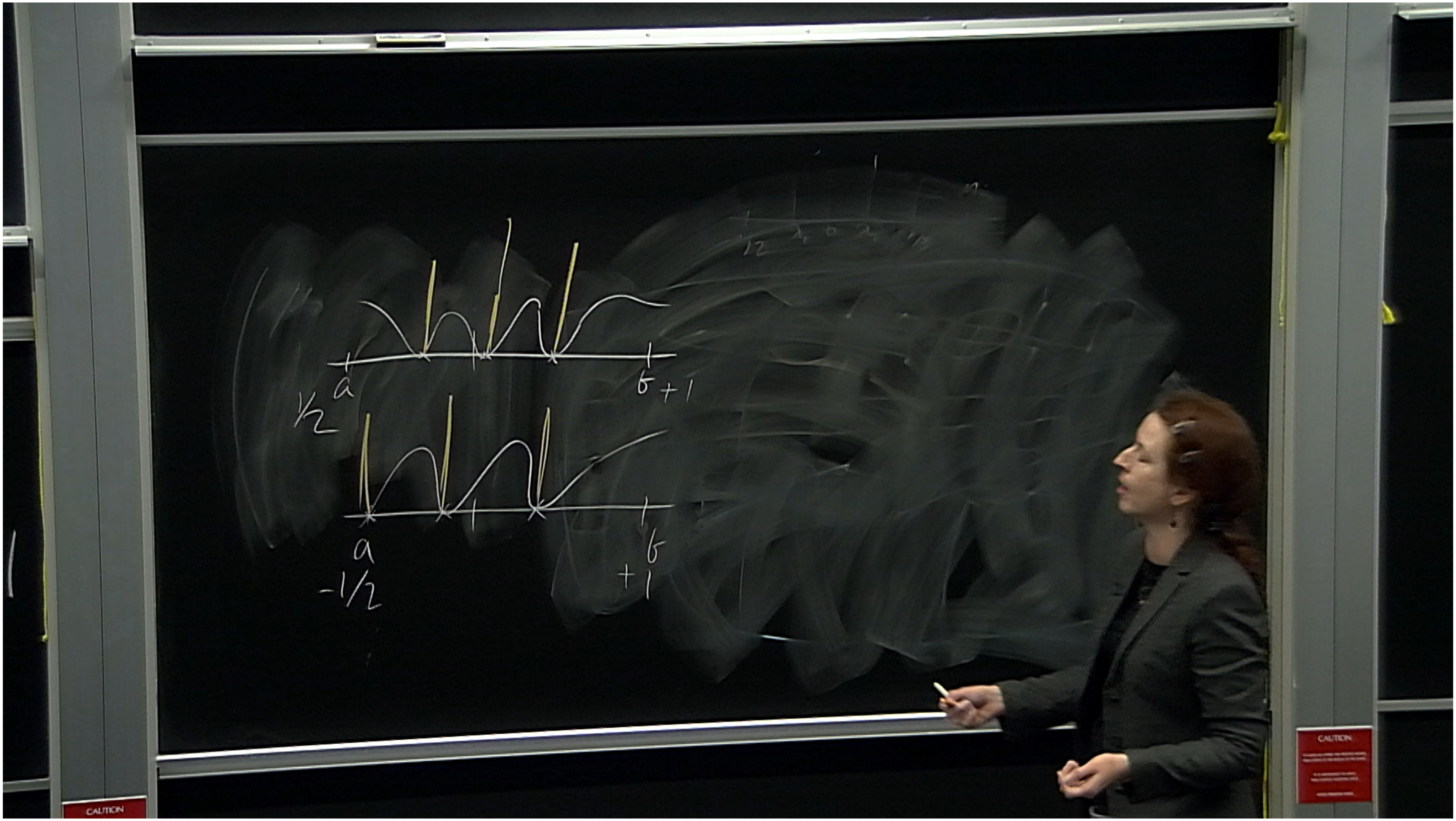

k=0, ,n
CAUTION

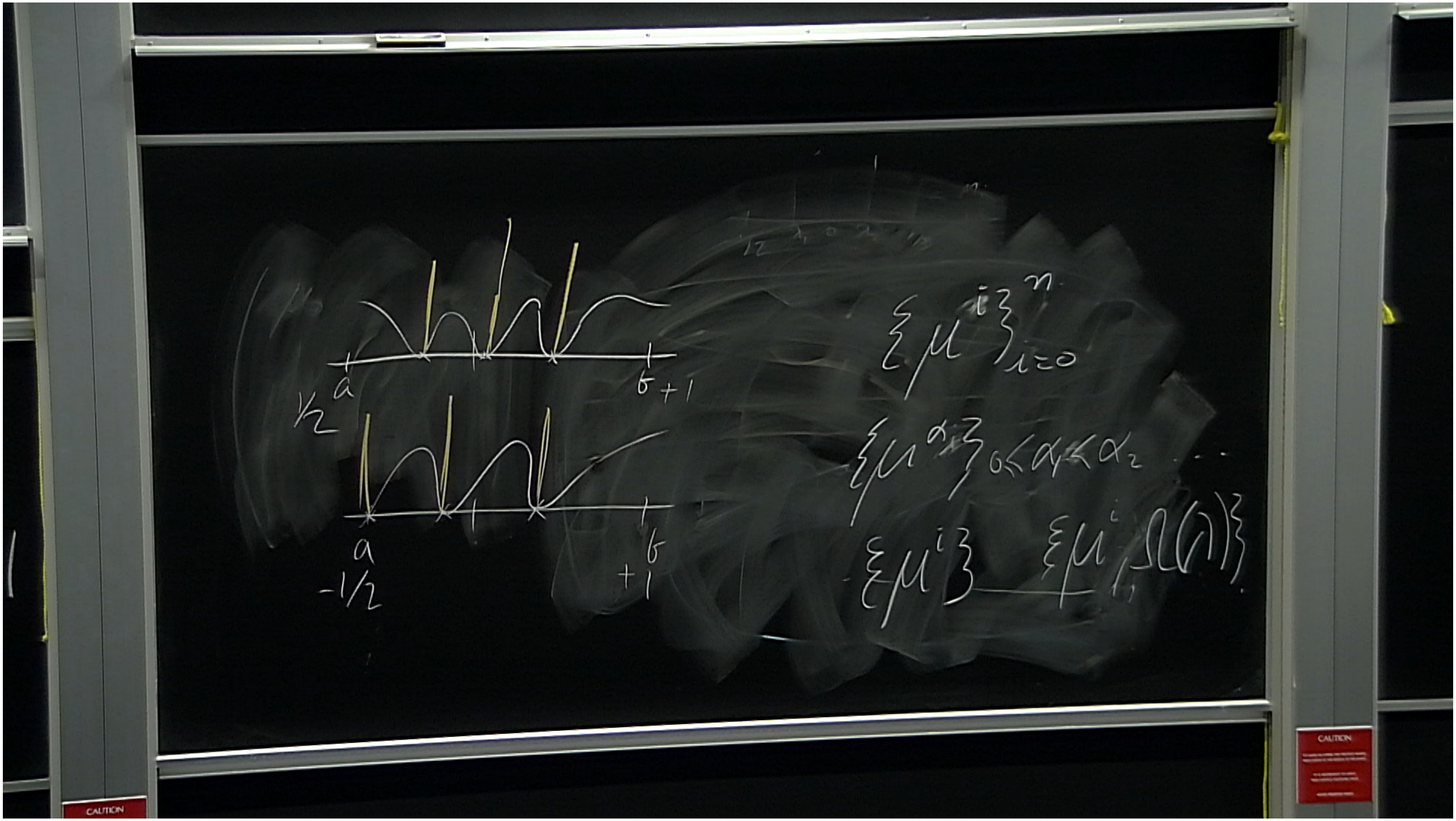

-1/2
λ1
0
λ2
λ3
1
← mi


m = 2n-1
det
μ0 μ1 . . . μm-1
μ1 μ2 . . . μm
μm μm+1 μ2m-1
μj = Σ mi λi^j
CAUTION


arXiv:1312.1168
(λ-a)det
μ1 - aμ0 μ2 - aμ1 μn - aμn-1 1
μ2 - aμ1 μ3 - aμ2 ... μn+1 - aμn λ
(μn+1 - aμn) (μn+2 - aμn+1) (μ2n - aμ2n-1) λ^n
m=2n
μj = ma a^j + Σi mi λi^j


a
b
a
b


a
b +1
1/2
a
b
-1/2
+1
CAUTION



a
b +1
1/2
a
-1/2
b
+1