Title: Towards a general AdS/Ricci-flat correspondence

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Abstract: <p>The AdS/Ricci-flat (AdS/RF) correspondence is a map between families of asymptotically locally AdS solutions on a torus and families of asymptotically flat spacetimes on a sphere. In this talk I will discuss how to relax these restrictions for linearized perturbations around solutions connected via the original AdS/RF correspondence.
To this end we perform a Kaluza-Klein (KK) reduction, keeping all (massive) KK modes, of AdS on torus and of Minkowski on a sphere. We show that in the limit of large dimension of the compact manifolds (torus and sphere), the AdS/RF correspondence maps individual KK modes from one side to the other.
When the dimension is finite, the correspondence maps single modes to infinite superpositions of modes. One may further take appropriate limits so that there is either no torus (AdS side) or no sphere (Minkowski side) to map perturbations of solutions that possess no symmetry, thus completely relaxing the original restrictions.
This correspondence should allow us to develop a detailed holographic dictionary for asymptotically flat spacetimes.</p>

Towards a general AdS/Ricci-flat correspondence

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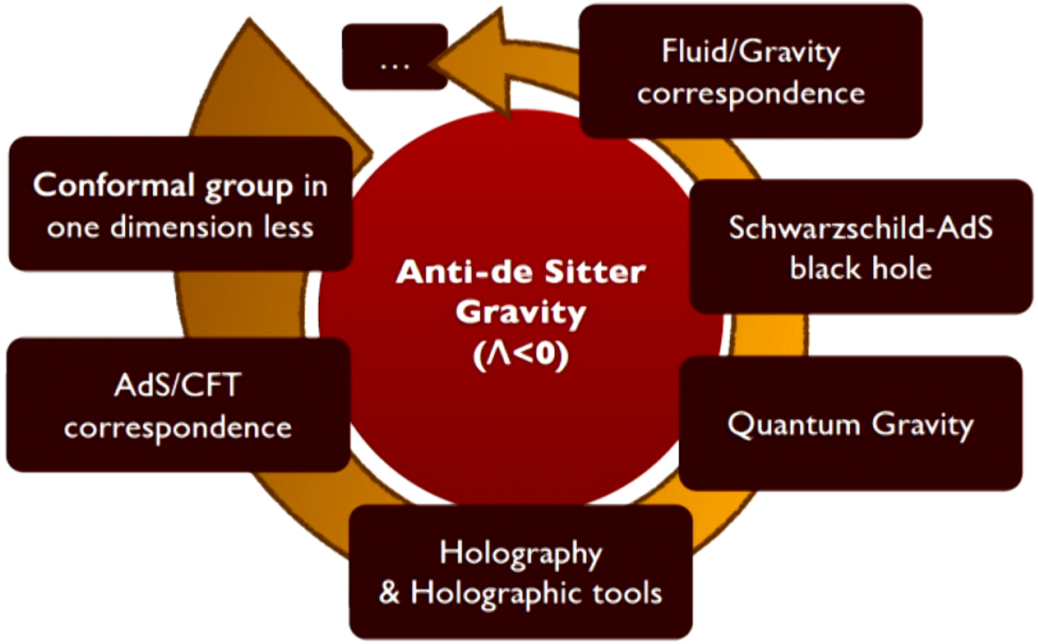
2 May 2017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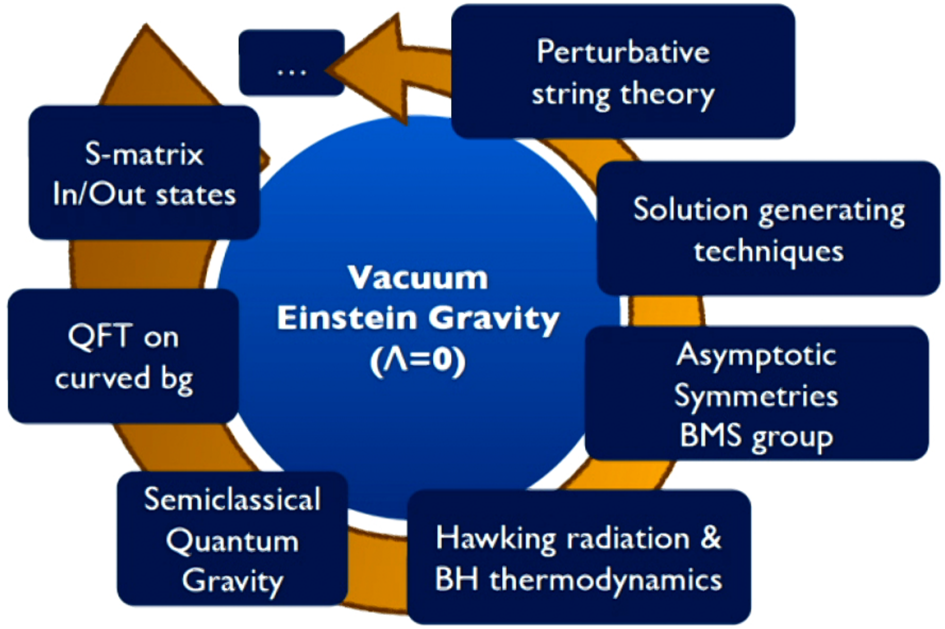
Fluid/Gravity correspondence
Conformal group in one dimension less
Anti-de Sitter Gravity (Λ<0)
Schwarzschild-AdS black hole
AdS/CFT correspondence
Quantum Gravity
Holography & Holographic tools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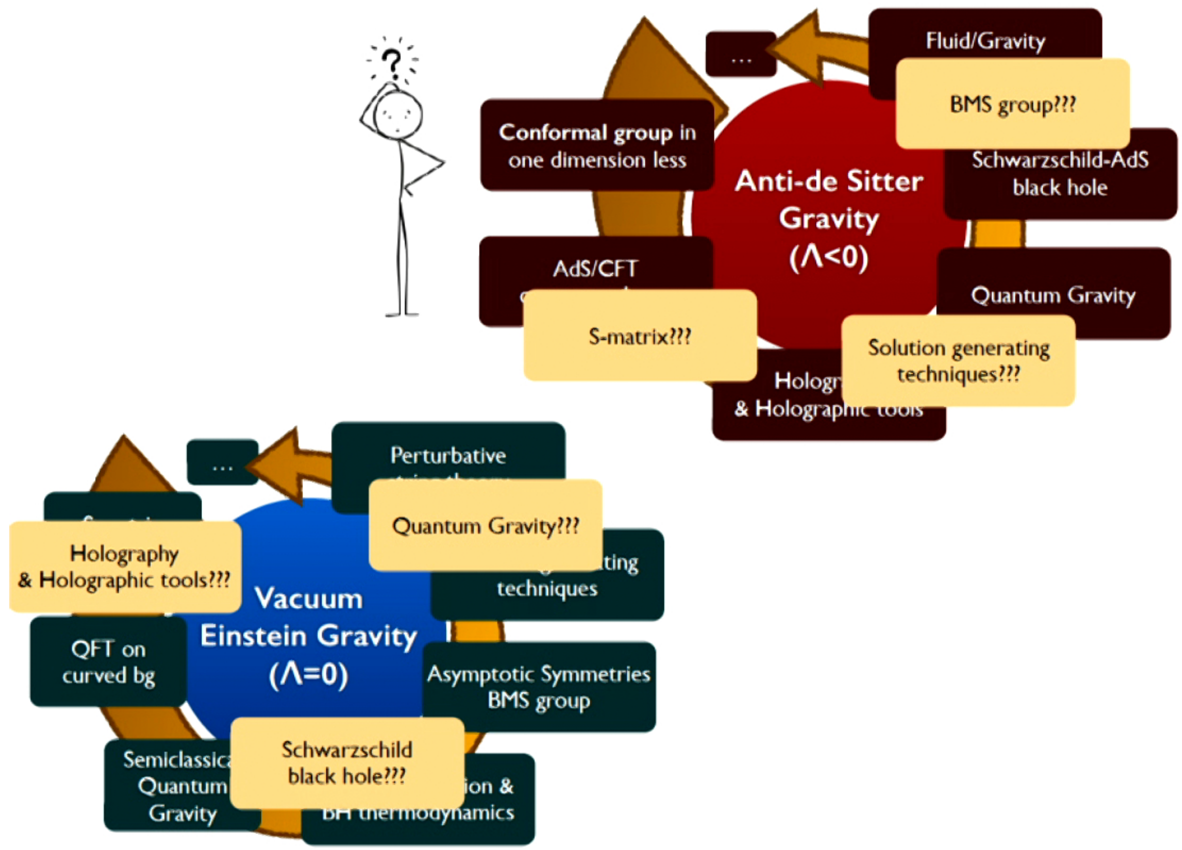
Perturbative string theory
S-matrix In/Out states
Solution generating techniques
Vacuum Einstein Gravity (Λ=0)
QFT on curved bg
Asymptotic Symmetries BMS group
Semiclassical Quantum Gravity
Hawking radiation & BH thermodynamics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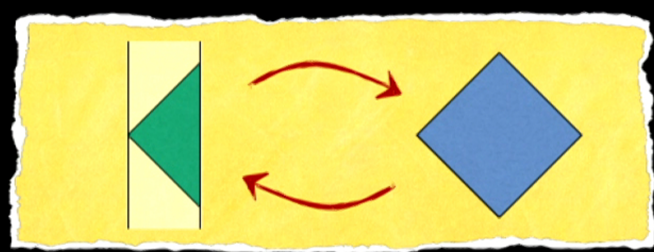
AdS/Ricci-flat correspondence



~ a bridge between AdS gravity and vacuum Einstein gravity ~

Outline

1. Review of holography
   - AdS holography
   - Non-conformal brane holography
2. AdS/Ricci-flat correspondence
3. General AdS/RF correspondence
4. Conclusions

Holography

- Any gravitational theory is expected to be holographic, *i.e.* it should have a description in terms of a non-gravitational theory in one dimension less.
- The current formulation of holographic dualities depends sensitively on the detailed structure of asymptotics: the case we understand best is that with AdS asymptotics.

Outline

AdS holography

- The best understood holographic dualities are those obtained in string theory via decoupling limits of branes.
- The prototype example is the near-horizon limit of D3 brane, which yields the duality between string theory on $AdS_5 \times S^5$ and $N = 4$ SYM at the conformal boundary of AdS_5.
- CFT's contain a universal sector describing the correlators of the energy-momentum tensor $T_{\mu\nu}$.
- This universal sector is described in AdS/CFT correspondence by *AdS gravity*,

$$S = \int d^{d+1}x\sqrt{G}(R - 2\Lambda)$$

AdS asymptotics and holography

- The AdS solution,

$$ds^2 = \frac{1}{r^2}(dr^2 + \eta_{ij} dx^i dx^j)$$

represents the vacuum of the CFT.

- The spacetime has a conformal boundary at $r = 0$.
- We need to impose boundary conditions there. The Dirichlet problem in AdS is to fix a conformal class:

$$g_{(0)ij}(x) \sim e^{2\sigma(x)} g_{(0)ij}(x)$$

- The corresponding bulk metric has the following form [Fefferman-Graham (1985)]

$$ds^2 = \frac{1}{r^2}\left(dr^2 + (g_{(0)ij} + r^2 g_{(2)ij} + \cdots + r^d g_{(d)ij} + \cdots) dx^i dx^j\right)$$

Correlation functions

In gauge/gravity duality:

- $g_{(0)ij}$ is identified with the source for T_{ij}.
- The expectation value of the T_{ij} in the presence of sources is [de Haro, Solodukhin, KS (2000)]

$$\langle T_{ij} \rangle \sim g_{(d)ij}$$

- To compute correlation functions we consider

$$g_{(0)ij} = \eta_{ij} + h_{ij}$$

and compute exact bulk solutions perturbatively in h_{ij}.
- Regularity in the interior results in

$$g_{(d)ij} = g^B_{(d)ij} + \mathcal{T}_{ijkl} h^{kl} + \frac{1}{2} \mathcal{T}_{ijklmn} h^{kl} h^{mn} + \cdots$$

- $\mathcal{T}_{ijkl} \sim \langle T_{ij}(k) T_{kl}(-k) \rangle$,
$\mathcal{T}_{ijklmn} \sim \langle T_{ij}(k_1) T_{kl}(k_2) T_{mn}(-k_1 - k_2) \rangle$, etc.

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2-point function

- To compute 2-point function one needs a solution of the linearized equation.
- Regular linear perturbations around AdS are given by

$$h_{ij}(k) = h_{(0)ij}(k)\frac{1}{2^{d/2-1}\Gamma(d/2)}(kr)^{d/2}K_{d/2}(kr)$$

- ... and using the asymptotic expansion

$$\frac{1}{2^{d/2-1}\Gamma(d/2)}(kr)^{d/2}K_{d/2}(kr) = 1 + \cdots + r^d k^d + \cdots$$

- we find

$$\langle T_{ij}(k)T_{mn}(-k)\rangle = \Pi_{ijmn}k^d$$

  where Π_{ijmn} is a projection to transverse traceless tensors.
- This is precisely the correct 2-point function for the stress energy tensor of a d-dimensional CFT (when d even $k^d \to k^d \log k$)..

Outline

Non-conformal branes

- A decoupling limit for non-conformal branes (D0, D1, D2, D4), similar to the one leading to the AdS/CFT conjecture was presented in [Itzhaki et al (1998)].
- For these branes the geometry is *conformal* to $AdS_{p+2} \times S^{8-p}$ and after reducing over the sphere one obtains an asymptotically power-law geometry with a running dilaton [Boonstra, KS, Townsend (1998)].
- A precise holographic framework for these cases was established much later [Kanitscheider, KS, Taylor (2008)] [Wiseman, Withers (2008)].

Holographic dictionary: non-conformal branes

- For the non-conformal branes, the starting point is the Lagrangian obtained by consistent reduction over the transverse sphere S^{8-p}:

$$S = \int d^{p+2}x \sqrt{-g} e^{\phi} \left[R - \frac{2\sigma - p - 2}{2\sigma - p - 1} (\partial\phi)^2 - 2\sigma(2\sigma - 1) \right]$$

The parameter σ takes the values:

$$\{D0, D1, D2, D4\} = \{7/5, 3/2, 5/3, 3\}$$

- The dual theory is not conformal: it has a dimensionful coupling constant and ϕ encodes this coupling.
- This action captures holographically correlators of $T_{\mu\nu}, \mathcal{O}$. This is the universal sector of the corresponding QFTs.

Holography and asymptotics

- Since the solutions are not asymptotically AdS one needs to develop the dictionary from scratch.
- Extending the results of Fefferman-Graham to this system one finds similar asymptotic expansions but with unusual powers,

$$ds^2 = \frac{1}{r^2}\left(dr^2 + (g_{(0)ij} + r^2 g_{(2)ij} + \cdots + r^\sigma g_{(\sigma)ij} + \cdots)dx^i dx^j\right)$$

  and similar for the scalar field.
- Holographic renormalization leads to holographic formulae:

$$\langle T_{ij}\rangle \sim g_{(\sigma)ij}$$

  and similar for $\langle \mathcal{O}\rangle$.
- One can also work out linear fluctuations

$$h_{ij}(k) = h_{(0)ij}(k)\frac{1}{2^{\sigma/2-1}\Gamma(\sigma/2)}(kr)^{\sigma/2}K_{\sigma/2}(kr)$$

  and from here the corresponding 2-point functions ....
  [Kanitscheider, KS, Taylor (2008)]

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Generalized dimensional reduction [Kanitscheider, KS (2009)]

The starting action for the non-conformal branes can be obtained from $AdS_{2\sigma+1}$ gravity by

1. reducing over $T^{2\sigma-p-1}$ torus, keeping only the overall size of the torus,
2. analytically continuing on σ

$$S = \int d^{2\sigma+1}x \sqrt{-g}\,[R - 2\Lambda] \quad \text{with} \quad ds^2_\Lambda = ds^2_{p+2}(r,x) + e^{\frac{2\phi(r,x)}{2\sigma-p-1}} d\vec{y}^2$$

$$\to \qquad S = \int d^{p+2}x \sqrt{-\hat{g}} e^{\phi} \left[\hat{R} + \frac{2\sigma-p-2}{2\sigma-p-1} (\partial\phi)^2 - 2\sigma(2\sigma-1) \right].$$

- The reduced action and equations of motion depend smoothly on σ, provided $2\sigma - p - 1 > 0$.
- This is a consistent reduction: all solutions of the reduced theory originate from solutions of AdS gravity.

Holographic dictionary: non-conformal branes

- For the non-conformal branes, the starting point is the Lagrangian obtained by consistent reduction over the transverse sphere S^{8-p}:

$$S = \int d^{p+2}x \sqrt{-g} e^{\phi} \left[R - \frac{2\sigma - p - 2}{2\sigma - p - 1} (\partial\phi)^2 - 2\sigma(2\sigma - 1) \right]$$

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Holography

- All results needed for establishing a holographic dictionary are inherited from the corresponding AdS results via the generalized dimensional reduction:
  - Asymptotic solutions
  - Counterterms
  - Renormalized 1-point functions in the presence of sources.
- This links also black hole solutions and the nearby hydrodynamic regime and explains their conserved charges, thermodynamics and the values of the transport coefficients:
  - Equation of state: $P = \epsilon/(2\sigma - 1)$
  - Transport coefficients: $\eta/s = 1/4\pi$,
    $\zeta/\eta = 2(1/(d-1) - c_s^2)$

Holography for asymptotically flat spacetimes?

In the two classes of holographic spacetimes we have just reviewed:

- The fields that parametrize the boundary conditions at infinity act as QFT sources.
- The gravitational on-shell action is the generating functional of QFT correlation functions.

This requires that

- The fields that parametrize the boundary conditions are unconstrained.
- The infinities in the on-shell action are local in these fields.

➠ Both of these conditions fail for asymptotically flat spacetimes [de Haro, Solodukhin, KS (2001)]

➠ A straightforward extension of the holographic methodology to AF spacetimes does not work.

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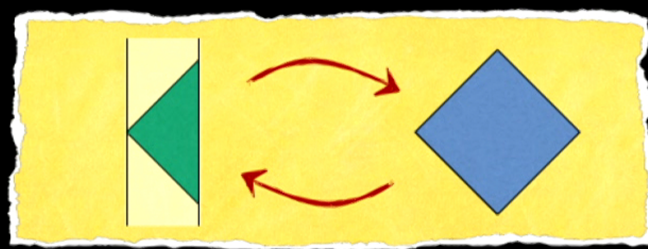
AdS/Ricci-flat correspondence



~ a bridge between AdS gravity and vacuum Einstein gravity ~

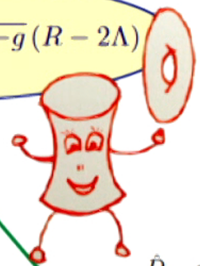
AdS/Ricci-flat correspondence

Trading curvatures: AdS ↔ Ricci-flat

AdS gravity

$D = d+1$

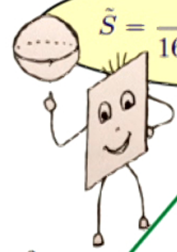
$$S = \frac{1}{16\pi G_N}\int_{\mathcal{M}} d^{d+1}x\,\sqrt{-g}\,(R - 2\Lambda)$$



Vacuum Einstein gravity

$\tilde{D} = n+p+3$

$$\tilde{S} = \frac{1}{16\pi \tilde{G}_N}\int_{\mathcal{M}} d^{n+p+3}x\,\sqrt{-\tilde{g}}\,\tilde{R}$$



Reduction on $\mathcal{T}^{d-p-1}$

$$ds^2_\Lambda = d\hat{s}^2_{p+2} + e^{\frac{2\phi(x)}{d-p-1}}\,d\vec{y}^2_{d-p-1}$$

Reduction on $\mathcal{S}^{n+1}$

$$ds^2_0 = e^{\frac{2\phi(x)}{n+p+1}}\left(d\hat{s}^2_{p+2} + \ell^2\, d\Omega^2_{n+1}\right)$$

$\hat{D} = p+2$

$$\hat{S} = \frac{1}{16\pi \hat{G}_N}\int_{\mathcal{M}} d^{p+2}x\,\sqrt{-\hat{g}}\,e^{\phi}\left(\hat{R} + \alpha\,(\partial\phi)^2 + \beta\right)$$

$$\alpha = \frac{d-p-2}{d-p-1} \qquad \beta = -2\Lambda$$

$$\alpha = \frac{n+p+2}{n+p+1} \qquad \beta = \mathcal{R}_{\mathcal{S}^{n+1}}$$

AdS/Ricci-flat correspondence

A map relating AdS and Ricci-flat solutions

Caldarelli, Camps, Goutéraux & KS '12, '13

1. Solutions to **AdS gravity** in d+1 dimensions of the form:

$$ds_\Lambda^2 = d\hat{s}_{p+2}^2(x) + e^{\frac{2\phi(x)}{d-p-1}}\, d\vec{y}_{d-p-1}^2$$

unit $\mathcal{T}^{d-p-1}$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

$$\Lambda = -\frac{d(d-1)}{2\ell^2}$$

2. Extract (p+2)-dim metric $\hat{g}(x)$ and the scalar $\phi(x)$

3. Substitute $d \to -n$ in $\hat{g}(x)$ and $\phi(x)$

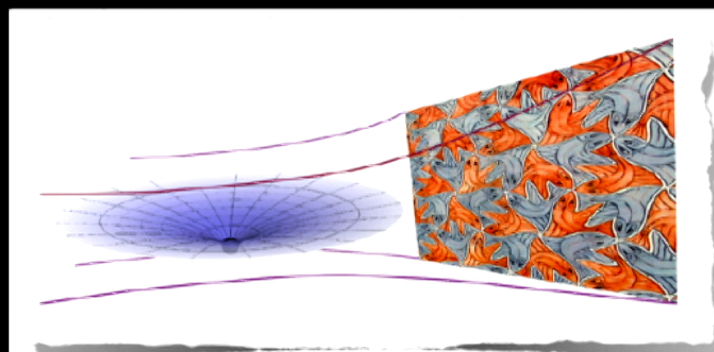
unit $\mathcal{S}^{n+1}$

4. Insert back in $ds_0^2 = e^{\frac{2\phi(x)}{n+p+1}} \left(d\hat{s}_{p+2}^2(x) + \ell^2\, d\Omega_{n+1}^2\right)$

Then, the metric ds_0^2 is **Ricci-flat** $\tilde{R}_{\mu\nu} = 0$

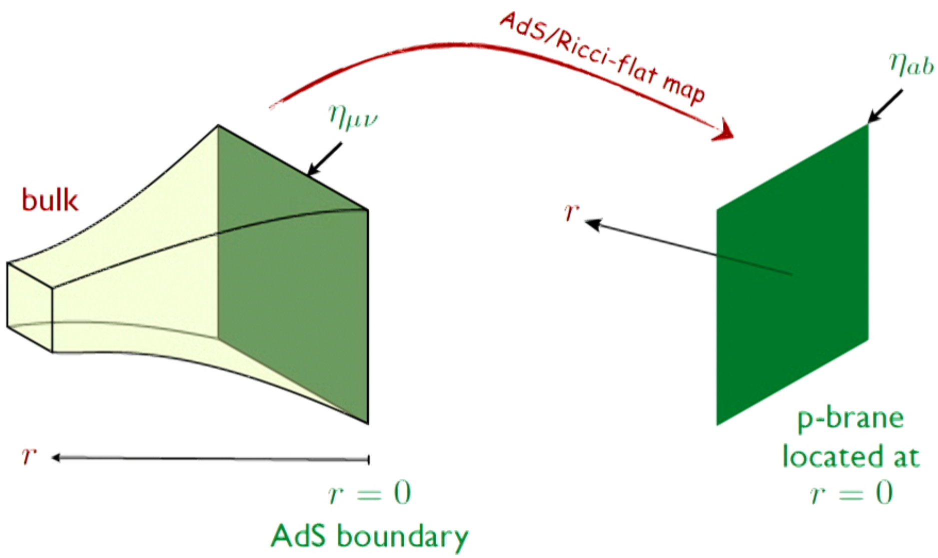
(it solves **vacuum Einstein** equations in (n+p+3) dimensions)

Some simple applications



~ what can we learn using this map? ~

AdS on a torus $\leftrightarrow$ Minkowski



AdS spacetime with $(d-p-1)$ of the boundary directions compactified on a torus is mapped to Minkowski spacetime

$$ds^2_\Lambda = \frac{\ell^2}{r^2}\left(dr^2 + \eta_{ab}dx^a dx^b + d\vec{y}^2\right) \leftrightarrow ds^2_0 = (dr^2 + r^2 d\Omega^2_{n+1}) + \eta_{ab}dx^a dx^b,$$

Holographic dictionary 1

- On AdS, the boundary condition was to choose a metric at $r = 0$. In this example we chose $g_{(0)ab} = \eta_{ab}$

- This translates on the Ricci-flat side into a choice of a metric at the location of a p-brane.

Correlation functions

- As discussed earlier, to compute correlation functions we set $g_{(0)\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.
- We have seen that the linear transverse-traceless fluctuations are given by

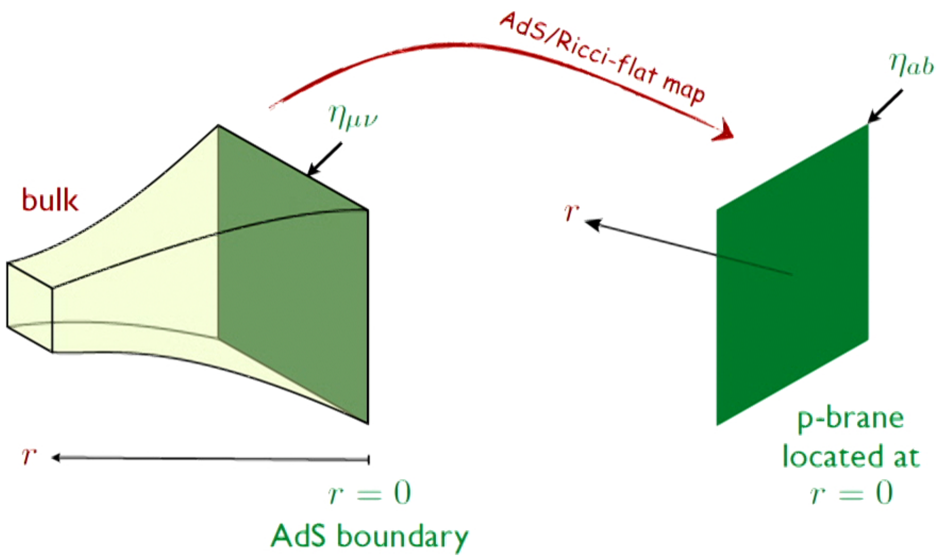
$$h^{\Lambda}_{ij}(k) = h_{(0)ij}(k)\frac{1}{2^{d/2-1}\Gamma(d/2)}(kr)^{d/2}K_{d/2}(kr)$$

- Applying to AdS/Ricci-flat flat correspondence, $d \to -n$, leads to

$$h^{0}_{ij}(k) = h_{(0)ij}(k)\frac{2^{n/2+1}}{\Gamma(-n/2)}\frac{K_{n/2}(kr)}{(kr)^{n/2}}$$

- This perturbation falls-off at infinity, so the metric is asymptotically flat (transverse to the p-brane).
- Regularity in the interior of AdS is mapped to asymptotic flatness.

AdS on a torus ↔ Minkowski



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- ⇒ Regularity in the interior of AdS is mapped to asymptotic flatness.

Black branes and the AdS/Ricci-flat correspondence

- The Planar AdS black brane

$$ds^2_\Lambda = \frac{1}{r^2}(-f(r)d\tau^2 + d\vec{x}^2 + d\vec{y}^2) + \frac{dr^2}{r^2 f(r)}, \qquad f = 1 - (r/b)^d$$

- ... is mapped to the Schwarzschild black p-brane

$$ds^2_0 = -f(r)d\tau^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\Omega^2_{n+1} + d\vec{x}^2, \qquad f = 1 - (b/r)^n$$

- ... and the special case $n = -1$ (no sphere) is just Rindler space

$$ds^2_0 = -r d\tau^2 + 2d\tau dr + d\vec{x}^2$$

$$(\tau \to \tau/b + \log f(r),\, r \to b^2 f(r))$$

Implications

- On the AdS side conformal invariance dictates the equation of state:

$$T^i_i = 0 \qquad \Rightarrow \qquad \varepsilon = (d-1)P$$

- Under the map this becomes:

$$\varepsilon = -(n+1)P \quad \Rightarrow \quad c_s^2 = \frac{\partial P}{\partial \varepsilon} = -\frac{1}{n+1},$$

  There is an instability for the sound modes. This is the Gregory-Laflamme instability. [Emparan, Harmark, Niarchos, Obers (2009)]

- When $n = -1$:

$$\varepsilon = 0, \qquad P > 0.$$

  This is the Rindler fluid [G.Compère, P. McFadden, KS, M. Taylor (2011)].

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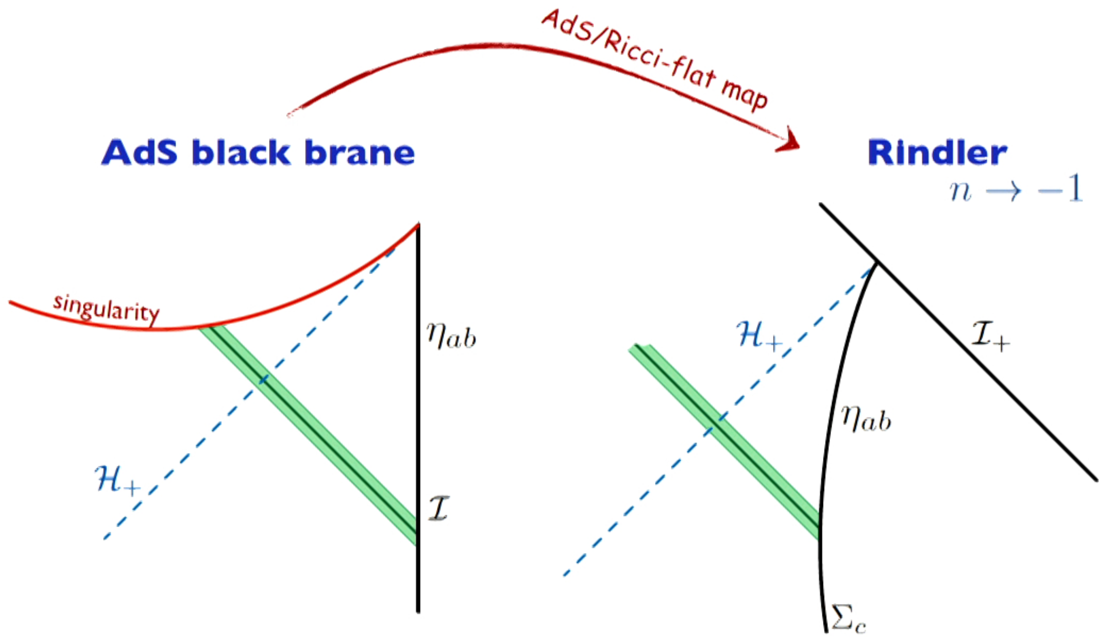
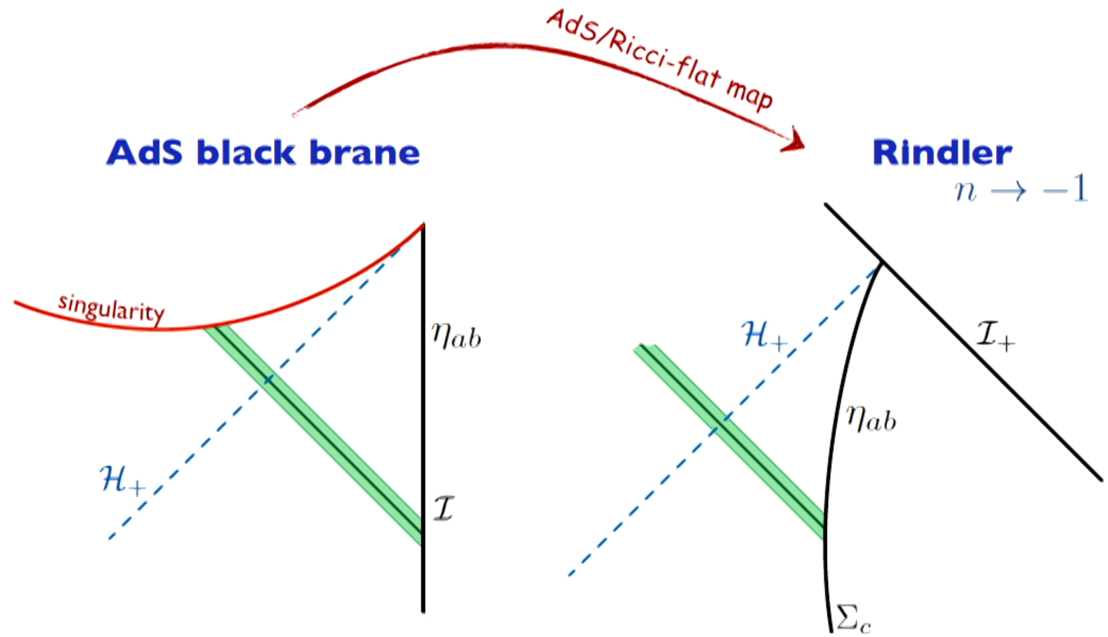
Black brane/Rindler correspondence



Dirichlet b.c. on AdS boundary

Dirichlet b.c. on timelike surface Σ_c

Black brane/Rindler correspondence



Dirichlet b.c. on AdS boundary → Dirichlet b.c. on timelike surface Σ_c

Black branes and the AdS/Ricci-flat correspondence

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$$(\tau \to \tau/b + \log f(r),\, r \to b^2 f(r))$$

Summary

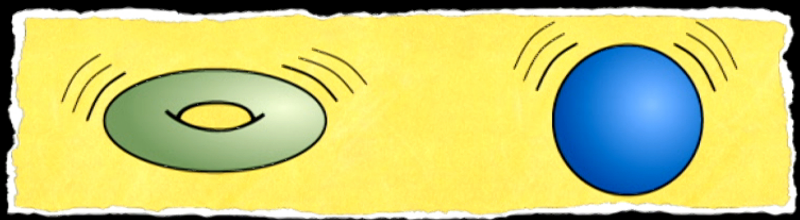
- The AdS/RF correspondence maps asymptotically locally AdS solutions to Ricci-flat spacetimes.

  The Ricci-flat spacetimes inherit holographic properties of AdS.

Holography for asymptotically flat spacetimes?

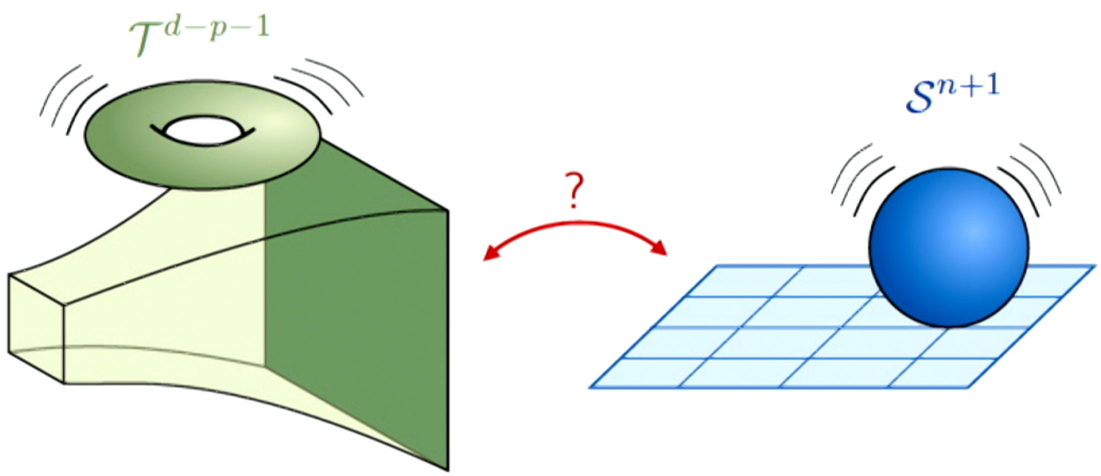
- The map captures the dynamics along the extended directions along the branes (correlation functions, hydro behavior, transport coefficents)
- The transverse sphere is FROZEN. These modes are critical to understand asymptotically flat holography (for example Schwarzschild black holes).

Towards a general formulation of the AdS/RF correspondence



~ Unfreezing the torus & the sphere ~

Beyond AdS/RF: linearized perturbations



- General linearized perturbation of AdS and Minkowski
- Full Kaluza-Klein reduction down to p+2 dimensions
- Compare the resulting modes

Linearized perturbations of Minkowski

$$ds^2_{n+p+3} = \underbrace{\eta_{\mu\nu}\, dx^\mu dx^\nu + dr^2}_{p+2} + r^2 \underbrace{\sigma_{ij}\, d\theta^i d\theta^j}_{S^{n+1}} + \underbrace{h_{AB}\, dX^A dX^B}_{\text{perturbation}}$$

Field expansion in $SO(n+2)$ representations:

$$h_{ab} = h^{I_s}_{ab}(x,r)\mathbb{S}^{I_s}(\theta),$$

$$h_{ai} = B^{I_v}_{(v)a}(x,r)\mathbb{V}^{I_v}_i(\theta) + B^{I_s}_{(s)a}(x,r)\mathcal{D}_i\mathbb{S}^{I_s}(\theta),$$

$$h_{(ij)} = \hat{\phi}^{I_t}_t(x,r)\mathbb{T}^{I_t}_{(ij)}(\theta) + \phi^{I_v}_v(x,r)\mathcal{D}_{(i}\mathbb{V}^{I_v}_{j)}(\theta) + \phi^{I_s}_s(x,r)\mathcal{D}_{(i}\mathcal{D}_{j)}\mathbb{S}^{I_s}(\theta),$$

$$h^i{}_i \equiv \sigma^{ij}h_{ij} = \pi^{I_s}(x,r)\mathbb{S}^{I_s}(\theta)$$

	Fields				Spherical harmonics	
Scalars	$h^{I_s}_{ab}$,	$B^{I_s}_{(s)a}$,	$\phi^{I_s}_s$,	π^{I_s}	$\mathbb{S}^{I_s}(\theta)$	Λ^{I_s}
Vectors	$B^{I_v}_{(v)a}$,	$\phi^{I_v}_v$			$\mathbb{V}^{I_v}_i(\theta)$	Λ^{I_v}
Tensor	$\hat{\phi}^{I_t}_t$				$\mathbb{T}^{I_t}_{(ij)}(\theta)$	Λ^{I_t}

KK reduction

- Combine the perturbations in gauge invariant combinations
- Decouple the equations
- Solve them

Results

The solutions take the form

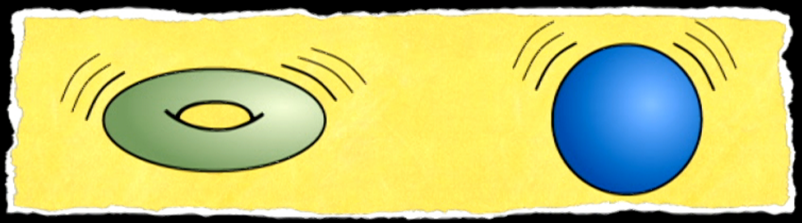
$$r^a\left(c_1 J_b(k_r r) + c_2 Y_b(k_r r)\right) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{cubic terms in } J, Y$$

(times appropriate polarization tensors/vectors)

	Minkowski	AdS
Tensor	$a = 2 - n/2, \quad b = l + n/2$	$a = d/2 - 2, \quad b = d/2$
Vectors	$a = 2 - n/2, \quad b = l + n/2$	$a = d/2 - 2, \quad b = d/2$
Scalars	$a = -n/2, \quad b = l + n/2$ $a = 1 - n/2, \quad b = l + n/2$	$a = d/2 - 2, b = d/2 - 2$ $a = d/2 - 2, b = d/2$ $a = d/2 - 2, b = d/2 - 1$

- The cubic terms are only present in some of the scalar Minkowski modes.
- In the limit $n \to \infty, d \to \infty$ the cubic terms vanish and all modes match under the AdS/RF correspondence.

What about finite n/d?



~ Unfreezing the torus & the sphere ~

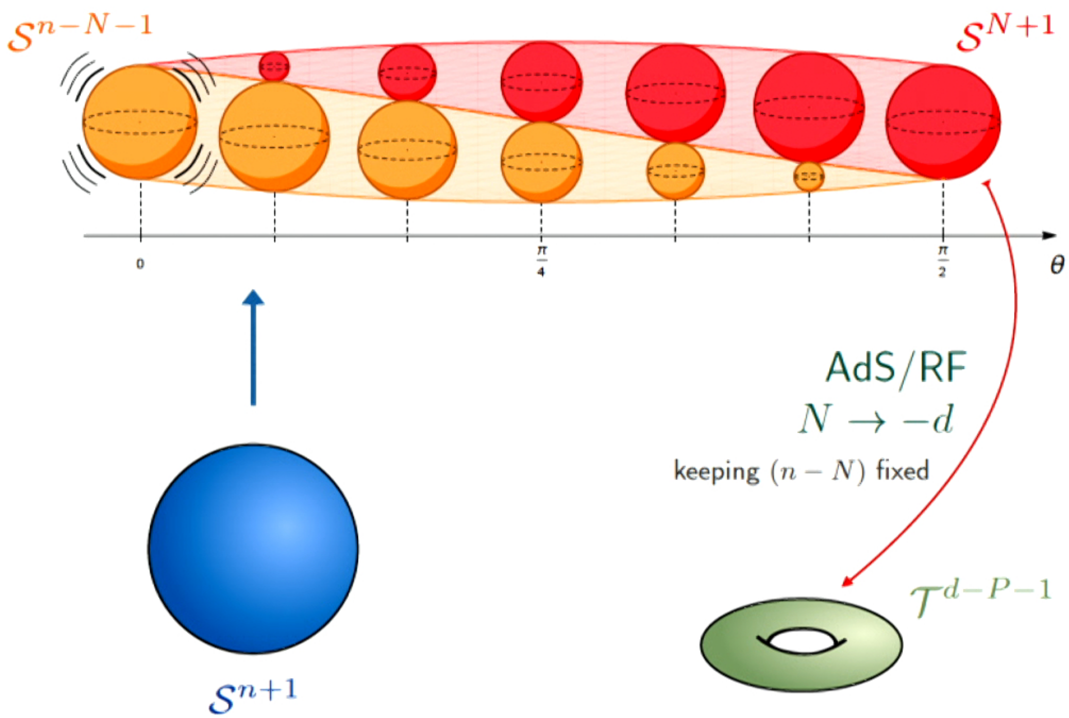
Strategy

- Consider the AdS/RF pair: AdS on $\mathcal{T}^{d-p-1}$/Minkowski on $\mathcal{S}^{n+1}$.

$$s_0^2 = ds_{p+1}^2 + d\rho^2 + \rho^2 d\Omega_{n+1}^2 \Leftrightarrow ds_\Lambda^2 = \frac{1}{r^2}\left(dr^2 + ds_{p+1}^2 + d\chi_{d-p-1}^2\right)$$

- We want to turn on a Minkowski perturbation that depends on $\mathcal{S}^{n+1}$.
- Let us first consider a perturbation that preserves an $SO(N+2)$ invariance of $SO(n+2)$ with $N < n$.
- This can be mapped to AdS using the AdS/RF correspondence, now applied to Minkowski on $\mathcal{S}^{N+1}$.

$SO(N+2)$ invariant perturbations of $\mathcal{S}^{n+1}$



Strategy

- The resulting AdS perturbation now depends on coordinates of the torus.
- This maps a single Minkowski mode to an infinite superposition of the AdS modes.
- To consider the general case, take the limit $N \to -1$.

Sketch

Consider an $SO(N+2)$ invariant perturbation of Minkowski

$$\begin{aligned} ds_0^2 &= ds_{p+1}^2 + d\rho^2 + \rho^2 d\Omega_{n+1}^2 + h_{MN}dx^M dx^N \\ &= ds_{p+1}^2 + d\rho^2 + \rho^2(d\psi^2 + \sin^2\psi d\Omega_{N+1}^2 + \cos\psi^2 d\Omega_q^2) + h_{MN}dx^M dx^N \end{aligned}$$

Applying the AdS/RF map using the $\mathcal{S}^{N+2}$ leads to

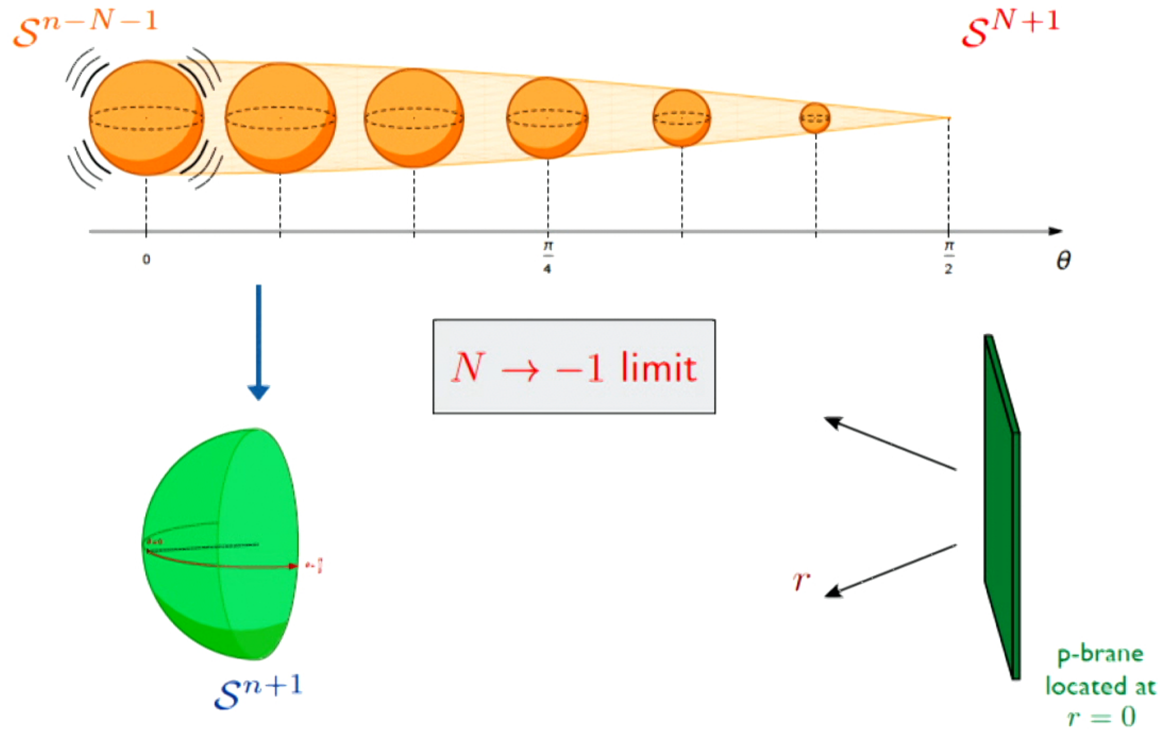
$$ds_\Lambda^2 = \frac{1}{r^2}\left(dr^2 + ds_{p+1}^2 + (d\sigma^2 + \sigma^2 d\Omega_q^2) + d\chi_{d-P-1}^2 + h_{MN}(r,\sigma,\vartheta^\alpha)dx^M dx^N\right.$$

where $r = \rho\sin\psi, \quad \sigma = \rho\cos\psi$.
Converting to cartesian coordinates: $(\sigma, \vartheta^\alpha) \to y^i$

$$ds_\Lambda^2 = \frac{1}{r^2}\left(dr^2 + ds_{p+1}^2 + d\chi_{d-p-1}^2 + h_{MN}(r,y^i)dx^M dx^N\right)$$

We may now Fourier expand h_{MN} in y^i to express the original mode in terms of infinite superposition of modes of AdS on $\mathcal{T}^{d-p-1}$.

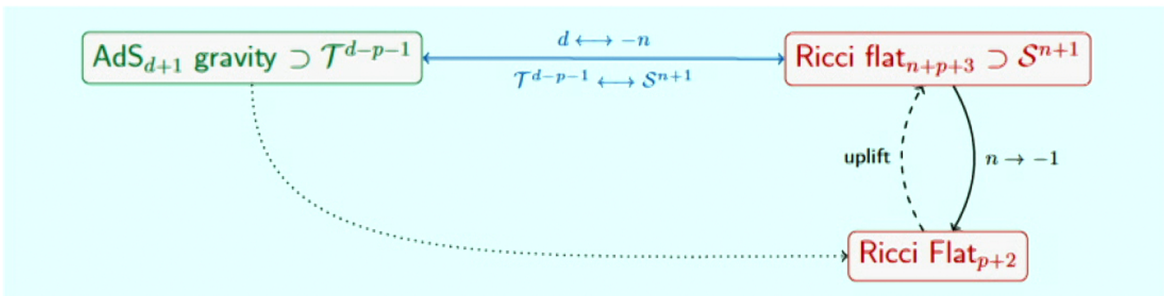
Collapsing the extra $\mathcal{S}^{N+1}$

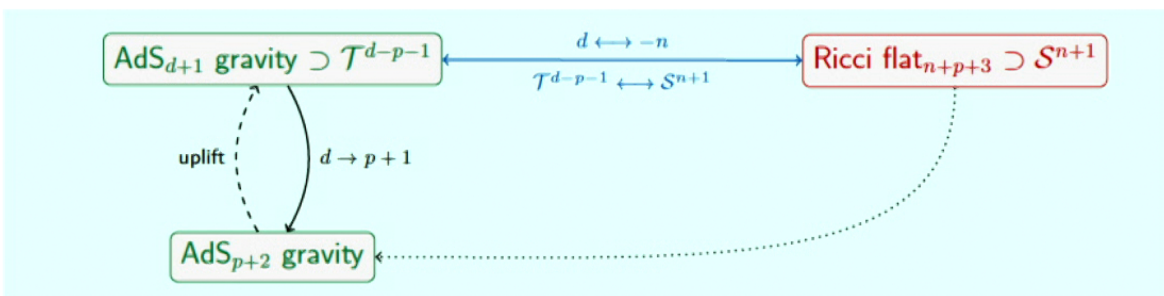


Remarks

- One can also start from a single mode in AdS on $\mathcal{T}^{d-p-1}$ that depends on some of the torus directions and map this to infinite superposition of modes on Minkowski on $\mathcal{S}^{n+1}$.
- Scalar/Vector/Tensor perturbations stay S/V/T perturbations.
- Same procedure works also for a class of non-linear solutions (plane waves).

Generalized AdS/Ricci-flat correspondence





Rindler/Fluid from Gravity/Fluid

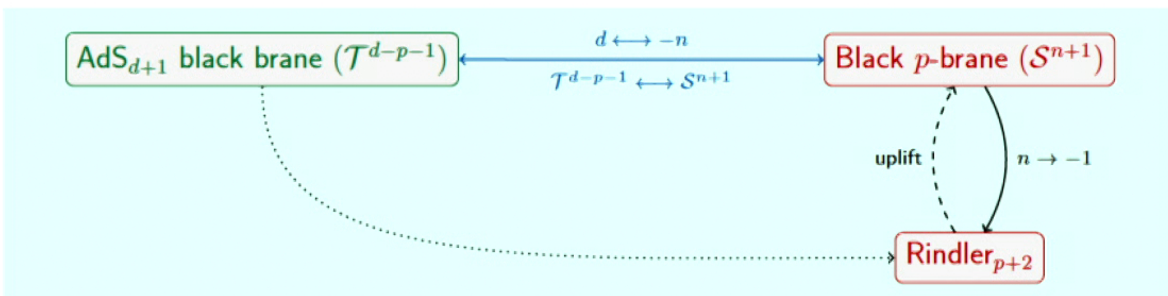
Black p-brane:

$$ds_0^2 = -f(r)d\tau^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{n+1}^2 + d\vec{x}^2, \qquad f(r) = 1 - \frac{r_0^n}{r^n}$$

$n \to -1$

Rindler spacetime in p+2 dimensions:

$$ds_0^2 = -\frac{\rho^2}{4r_0^2}d\tau^2 + d\rho^2 + d\vec{x}^2, \qquad \rho^2 = 4r_0^2(1 - r/r_0)$$



Rindler/fluid metric to second order in a derivative expansion and the corresponding transport coefficients Bredberg et al '12, Compere et al '12, Eling et al '12

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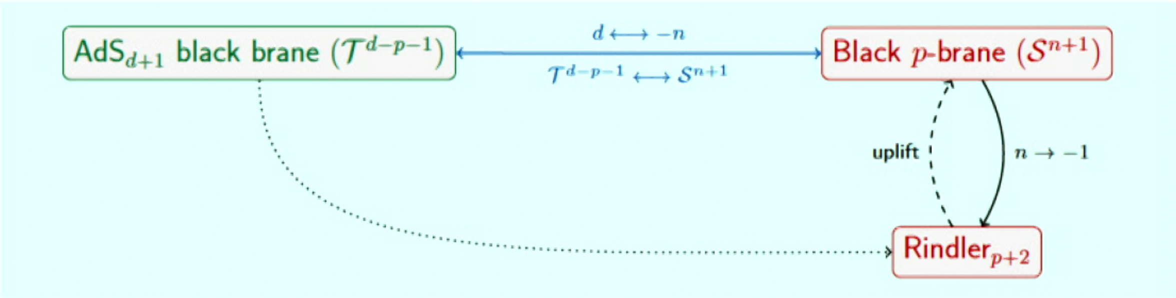

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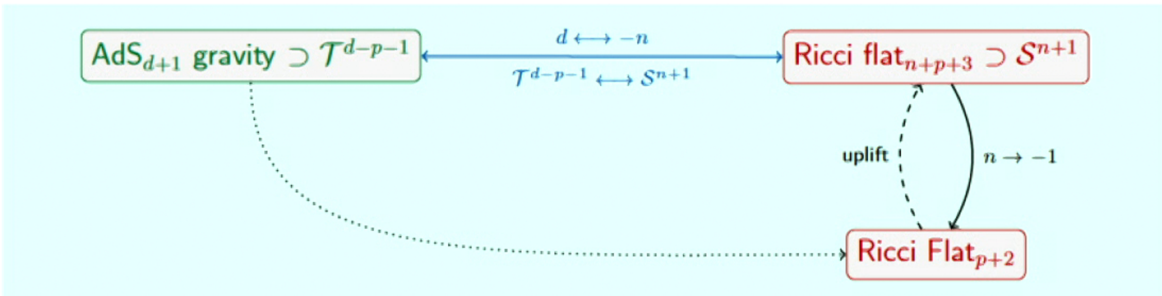
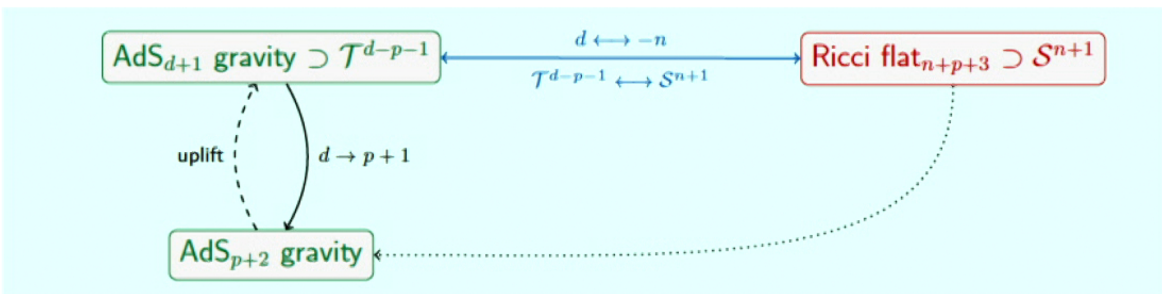
Generalized AdS/Ricci-flat correspondence





Outline

1. Review of holography
   - AdS holography
   - Non-conformal brane holography
2. AdS/Ricci-flat correspondence
3. General AdS/RF correspondence
4. Conclusions

Conclusions/Outlook

- Perturbations of Minkowski spacetime can be mapped to perturbations of AdS
- Unfreezing the sphere and torus.
  - In the large d, n limit individual modes are mapped to each other.
  - At finite n, d, single modes are mapped to superposition of modes.
- Asymptotically flat spacetimes inherit the holographic properties of AdS. Holographic data are encoded in an unexpected way
  - The source for dual operators are located at a position of a p-brane.
  - The stress energy tensor due to this p-brane is holographic.
  - Asymptotic flatness is the analogue of regularity in the interior.
- Future directions
  - Ricci-flat spacetimes inherit a generalized conformal structure from AdS. Extract the implications of this *hidden conformal invariance*.
  - Apply these methods to Schwarzschild.
  - Develop holography for general Ricci-flat spacetimes.