Title: Topological Boundary Modes from Quantum Electronics to Classical Mechanics

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Abstract: <p>Over the past several years, our understanding of topological electronic phases of matter has advanced dramatically.   A paradigm that has emerged is that insulating electronic states with an energy gap fall into distinct topological classes.   Interfaces between different topological phases exhibit gapless conducting states that are protected topologically and are impossible to get rid of.   In this talk we will discuss the application of this idea to the quantum Hall effect, topological insulators, topological superconductors and the quest for Majorana fermions in condensed matter.   We will then show that similar ideas arise in a completely different class of problems.   Isostatic lattices are arrays of masses and springs that are at the verge of mechanical instability.   They play an important role in our understanding of granular matter, glasses and other 'soft' systems. Depending on their geometry, they can exhibit zero-frequency 'floppy' modes localized on their boundaries that are insensitive to local perturbations.   The mathematical relation between this classical system and quantum electronic systems reveals an unexpected connection between theories of hard and soft matter.</p>

Topological Boundary Modes from Quantum Electronics to Classical Mechanics

C.L. Kane
University of Pennsylvania

Organizing Principles for Understanding Matter

Symmetry

- What operations leave a system invariant?
- Distinguish phases of matter by symmetries

symmetry group p4

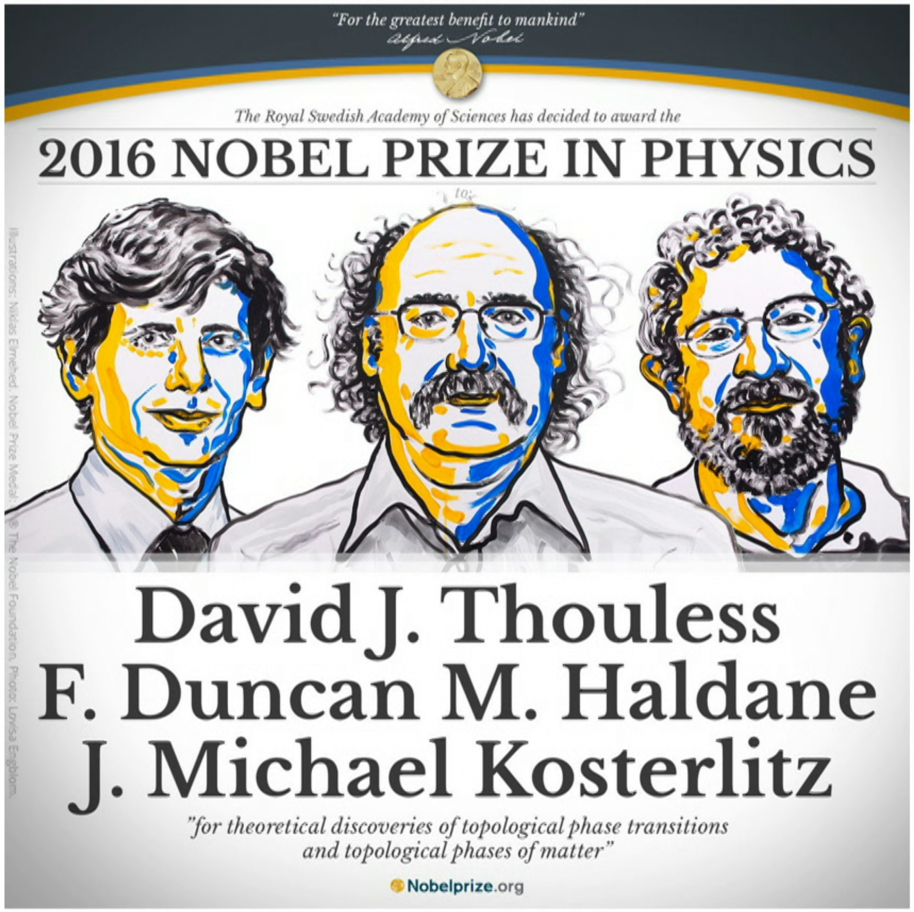
symmetry group p31m

Topology

- What stays the same when a system is deformed?
- Distinguish topological phases of matter

genus = 0

genus = 1

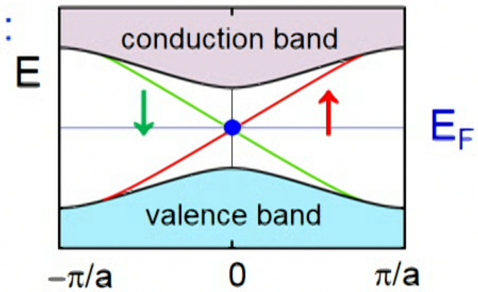


"for theoretical discoveries of topological phase transitions and topological phases of matter"

Topological Boundary Modes from Quantum Electronics to Classical Mechanics

I. Quantum Electronic Topological Phases : Topological Band Theory

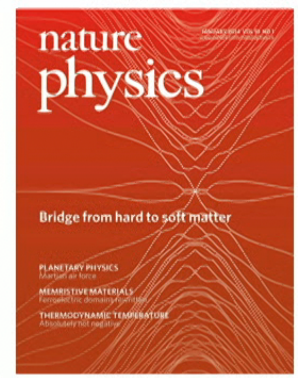
- Quantum Hall Effect
- Topological Insulators
- Topological Superconductors



II. Classical Mechanical Modes in Isostatic Lattices

- Floppy Modes and Maxwell's counting rule
- Topological boundary modes.

III. Synthesis



Kane, Lubensky Nat. Phys. **10**, 39 (2014)

Thanks to: Gene Mele, Liang Fu, Jeffrey Teo, Fan Zhang, Ben Wieder, Saad Zaheer, Andrew Rappe, Steve Young, Youngkuk Kim, Tom Lubensky

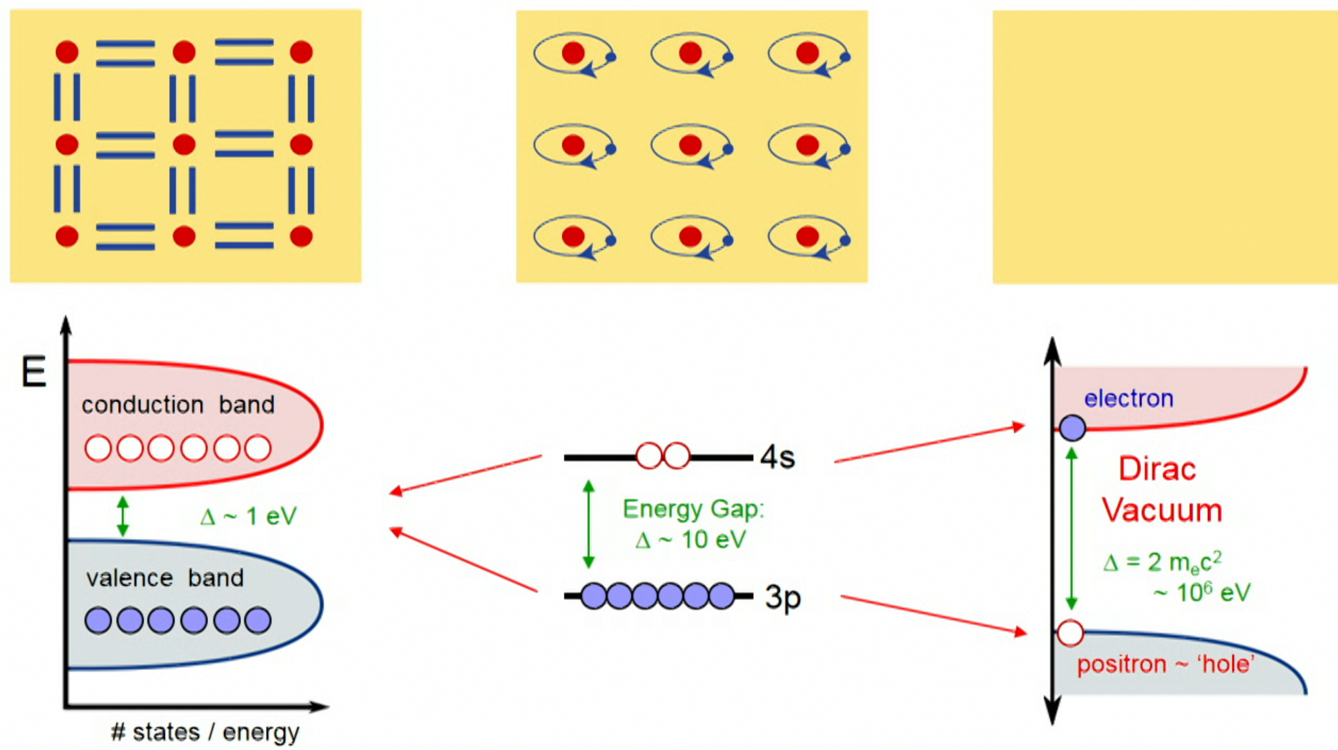
The Insulating State

Characterized by energy gap: absence of low energy electronic excitations

Covalent Insulator — e.g. intrinsic semiconductor

Atomic Insulator — e.g. solid Argon

The vacuum

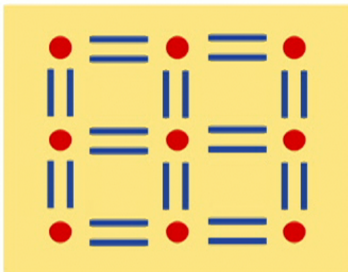


The Insulating State

Characterized by energy gap: absence of low energy electronic excitations

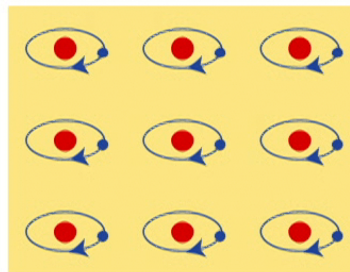
Covalent Insulator

e.g. intrinsic semiconductor

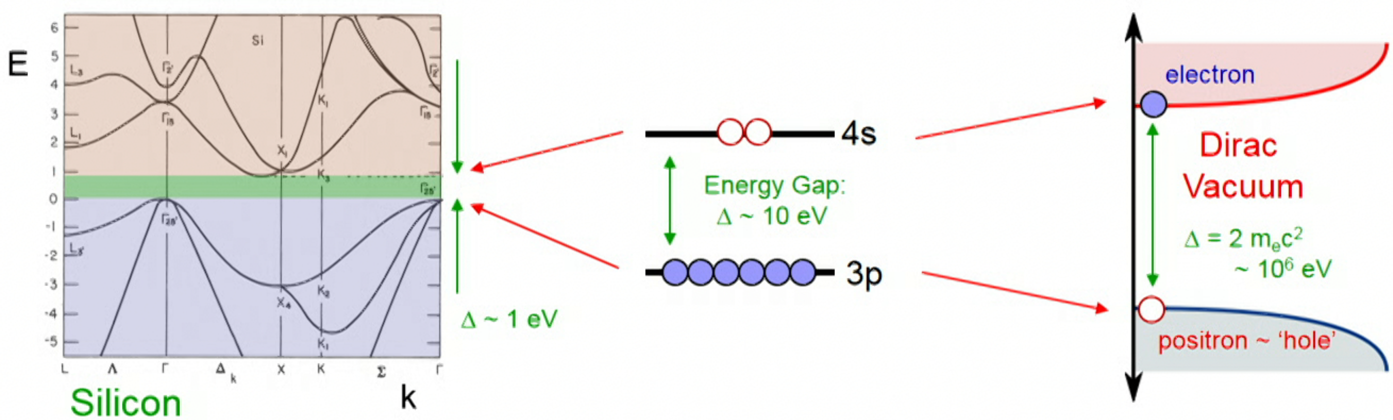


Atomic Insulator

e.g. solid Argon



The vacuum



Topology and Adiabatic Continuity

Insulators are topologically equivalent if they can be continuously deformed into one another without closing the energy gap

genus = 0

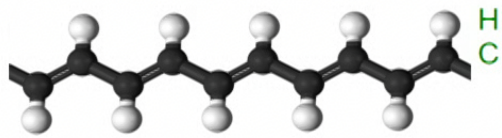
Are there "topological phases" that are not adiabatically connected to the trivial insulator (ie the vacuum) ?

genus = 1

Topological Phases in 1D: Su Schrieffer Heeger Model

Polyacetylene: A 1D conducting polymer

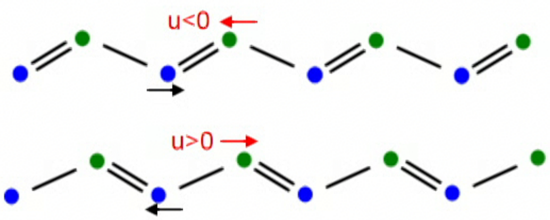
- **Undimerized** :

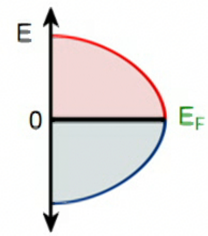


- **Dimerized** : $u = \pm u_0$

"A phase" : u<0

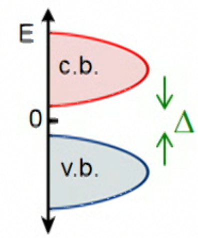
"B phase" : u>0





Conductor

Gap Δ=0



Insulator

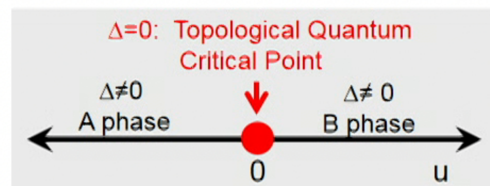
Gap $\Delta \sim |u| \neq 0$

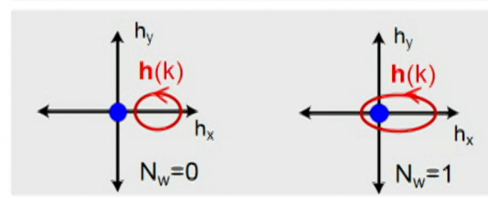
A and B phases are topologically distinct

- Separated by a quantum phase transition
- Distinguished by integer* topological invariant

  N_w = winding number characterizing valence band

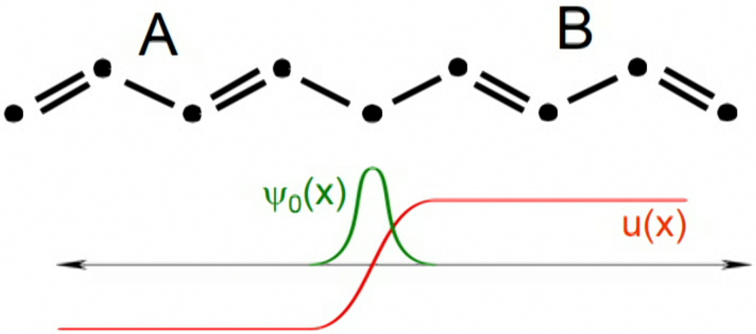
* Assuming band theory with 'particle-hole' symmetry

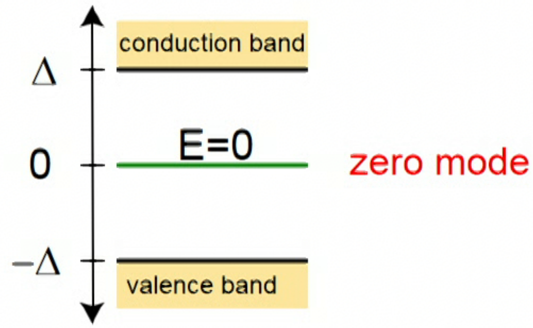




Topological Boundary Modes

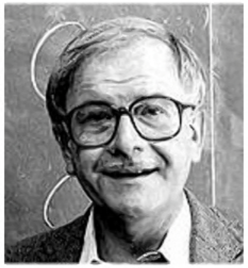
Jackiw and Rebbi 76,
Su Schrieffer, Heeger 79





Bulk Boundary Correspondence :

At the boundary between topologically distinct insulating phases, there exist topologically protected low energy states.

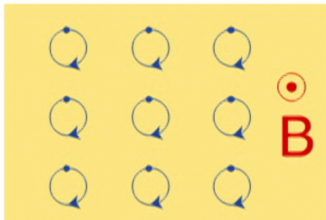


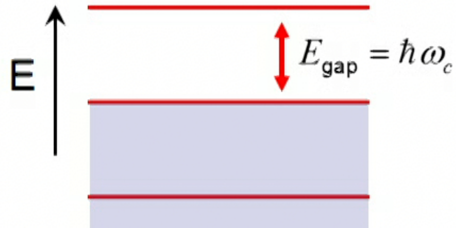
Roman Jackiw

Robert Schrieffer

Integer Quantum Hall Effect

2D Cyclotron Motion, Landau Levels





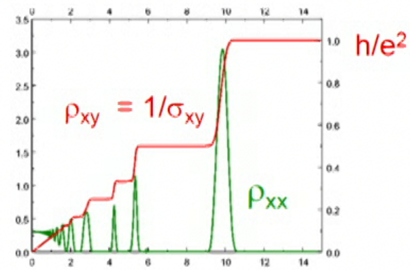
David Thouless

Energy gap, but not insulator

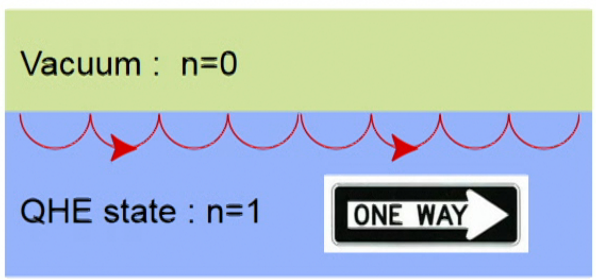
Quantized Hall conductivity : $\sigma_{xy} = n\frac{e^2}{h}$

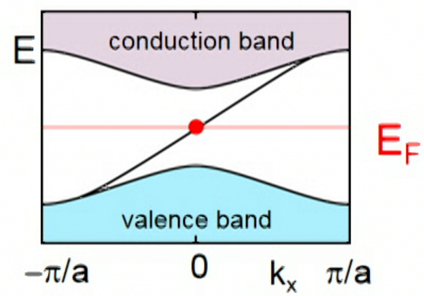
Topological invariant : $n \in Z$: "Chern number"

Thouless, Kohomoto, Nightengale and den Nijs '84



Edge States : Topologically protected 1D chiral Dirac fermions



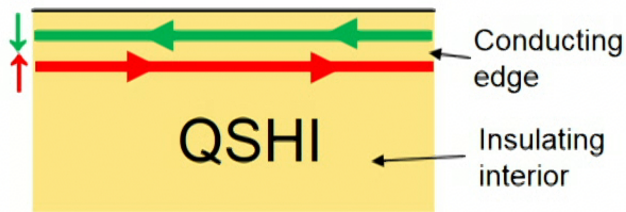


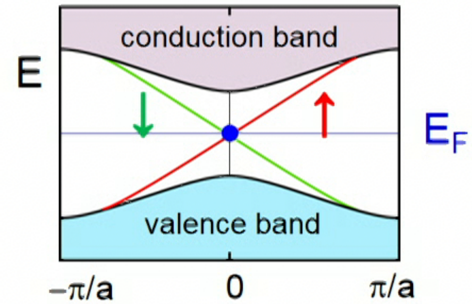
Z_2 Topological Insulator

Insulators in 2D and 3D with time reversal symmetry have two topological classes.
Distinguished by Z_2 topological invariant $\nu = 0, 1$

Two Dimensions: Quantum spin Hall insulator

Kane and Mele '05
Bernevig and Zhang '06

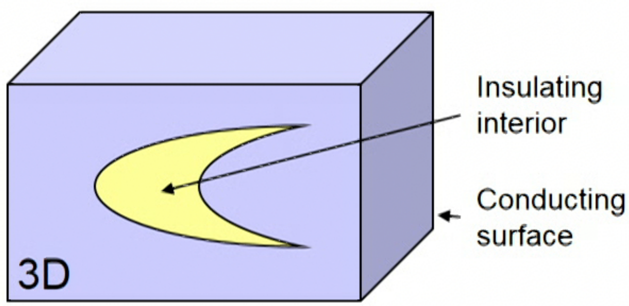


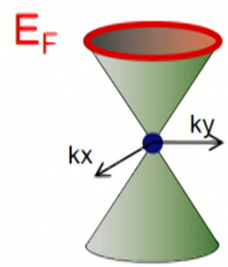


Experiment : HgTe/CdTe quantum wells
Bernevig, Hughes, Zhang '06 ; Konig et al. (Molenkamp) '07

Three Dimensions: 3D TI

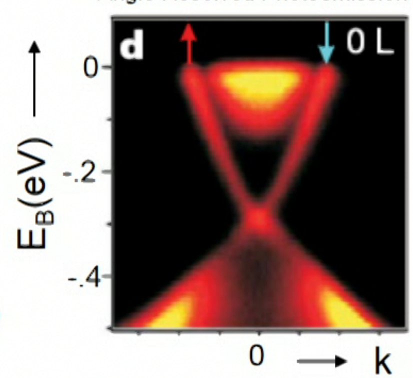
Fu & Kane '06; Moore & Balents '06; Roy '06





"Surface Dirac Cone"

Angle Resolved Photoemission

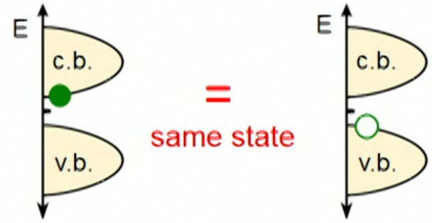


Experiment : Bi_xSb_{1-x}, Bi_2Se_3, ….
Hsieh et al. (Hasan) '08

Topological Superconductivity

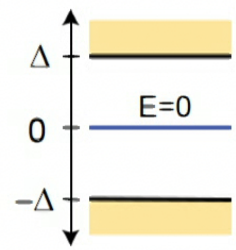
Key ingredients of BCS model of superconductivity :

- Similar to insulator: energy gap for quasparticle excitations
- Intrinsic Particle – Hole symmetry



1D Topological Superconductor (Kitaev 2000)

- Two topological classes
- Protected zero energy end state



Majorana Fermion

- Particle = Anti-Particle
- Application to quantum information : (Kitaev)
  - 2 Majorana bound states store 1 qubit of quantum information nonlocally
  - Immune from local sources of decoherence
  - "Braiding" can perform quantum operations

Ettore Majorana
1906–1938?

Alexei Kitaev

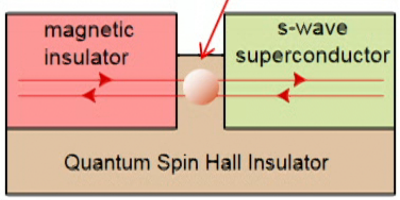
Quest for Majorana in Condensed Matter

Superconducting Proximity Effect: Use ordinary superconductors and topological materials to engineer topological superconductivity

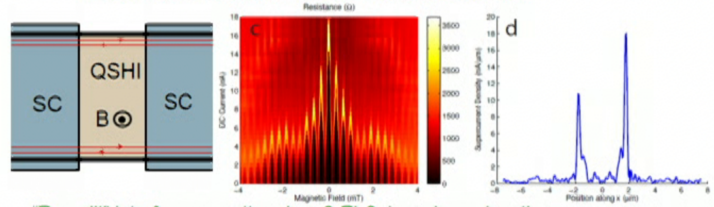
Superconductor - Topological Insulator Devices

Theory: Fu, Kane '07, '08

Majorana Boundary Mode



Expt: Hart, ... Yacoby '14 (HgTe);
Pribiag, ... Kouwenhoven '14 (InAs/GaSb)

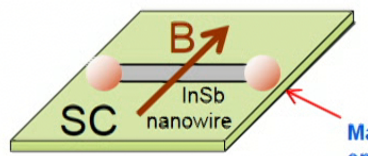


"Two slit" interference pattern in a S-TI-S Josephson Junction
Demonstrates edge superconductivity

Superconductor - Semiconductor Nanowire Devices

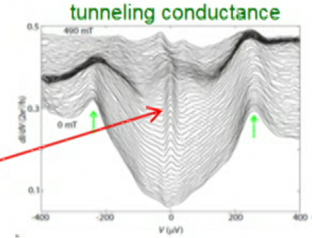
Theory:
Lutchyn, Sau , Das Sarma '10
Oreg, Refael, von Oppen '10

Expt: Mourik, ...Kouwenhoven '12



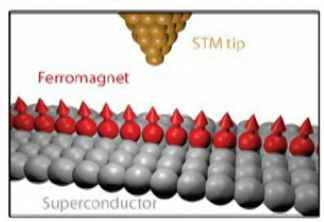
Majorana end mode

tunneling conductance

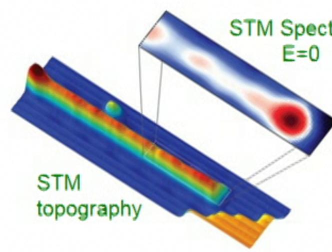


Ferromagnetic Atomic Chains on Superconductors

Nadj-Perg, ..., Yazdani '14 (Fe on Pb)



STM Spectroscopy
E=0

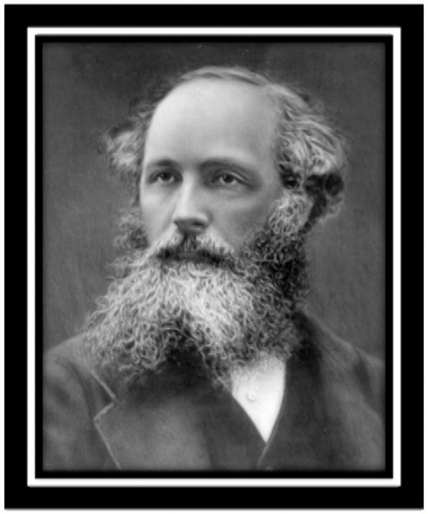


STM topography

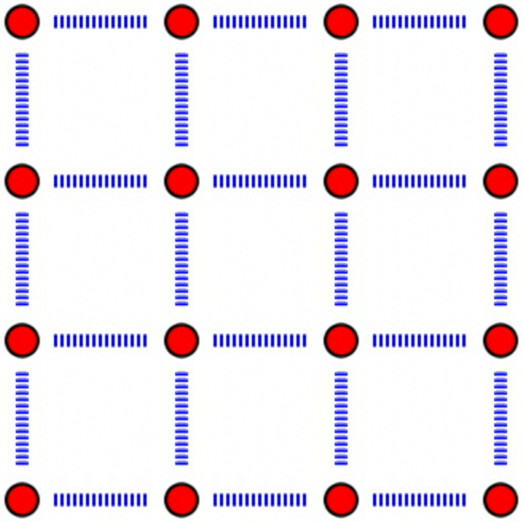
Majorana end mode

Maxwell Problem

JC Maxwell 1865



Is a “frame” or configuration of masses and springs mechanically stable ?



Maxwell Counting Rule:

$$N_{fm} \geq d\, n_s - n_b$$

n_s = # sites

n_b = # bonds, d = dimension

N_{fm} = # zero frequency “floppy modes”

Maxwell Problem

JC Maxwell 1865

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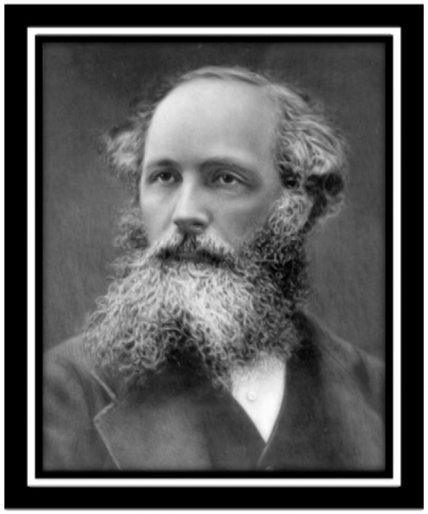

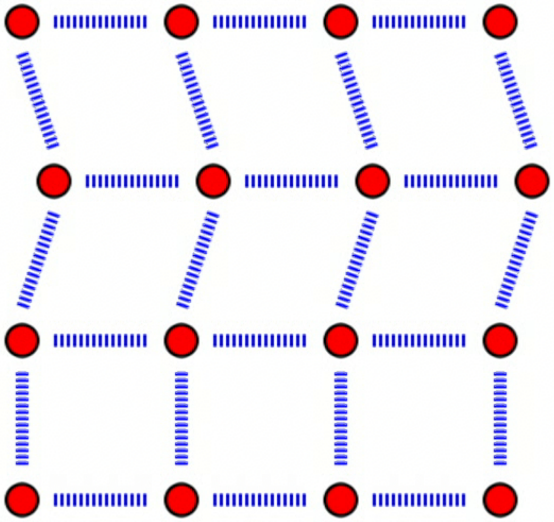

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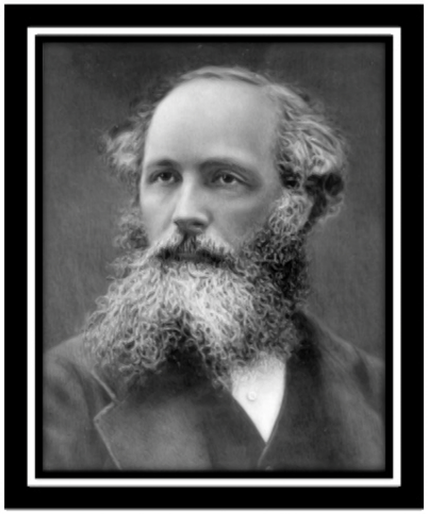

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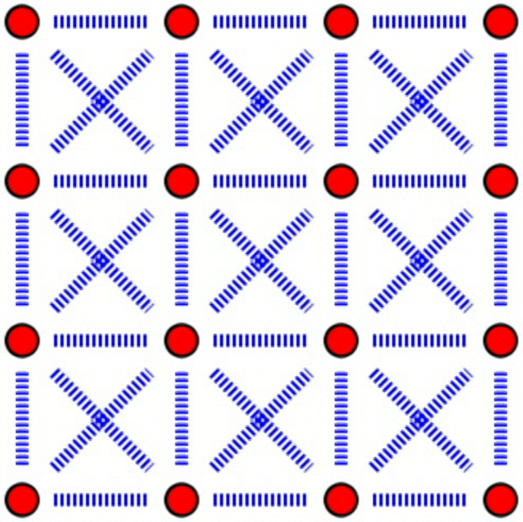

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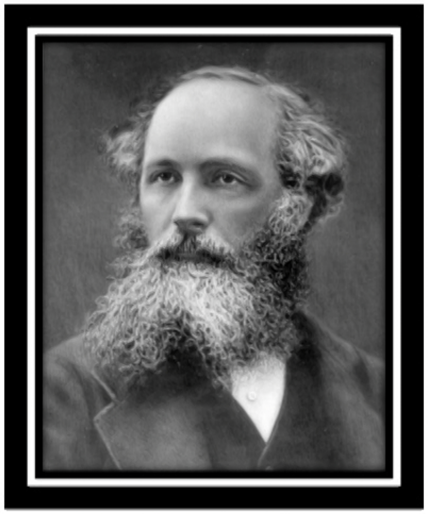

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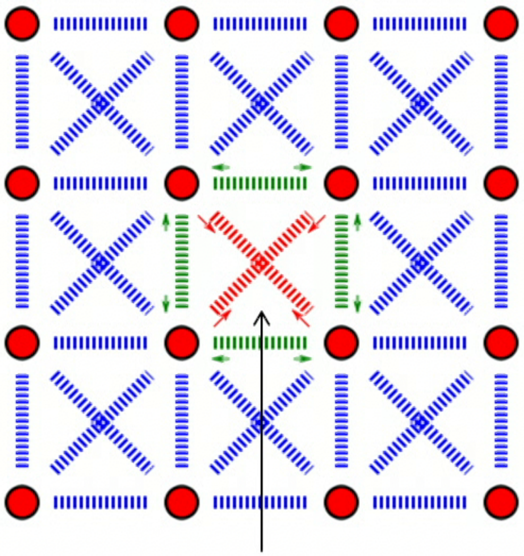


State of self-stress

Maxwell * Counting Rule: (Calladine '78)

$$N_{fm} - N_{ss} = d\, n_s - n_b$$

n_s = # sites

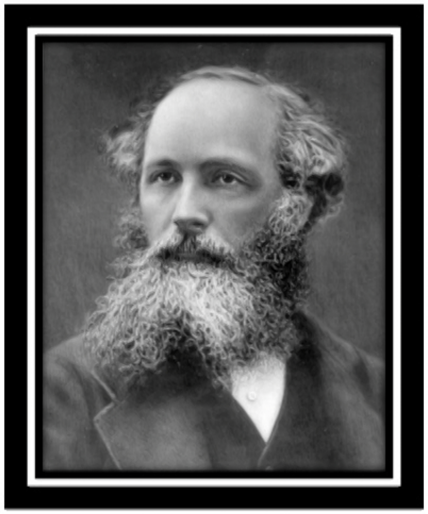
n_b = # bonds, d = dimension

N_{fm} = # zero frequency "floppy modes"

N_{ss} = # states of self-stress

Maxwell Problem

JC Maxwell 1865



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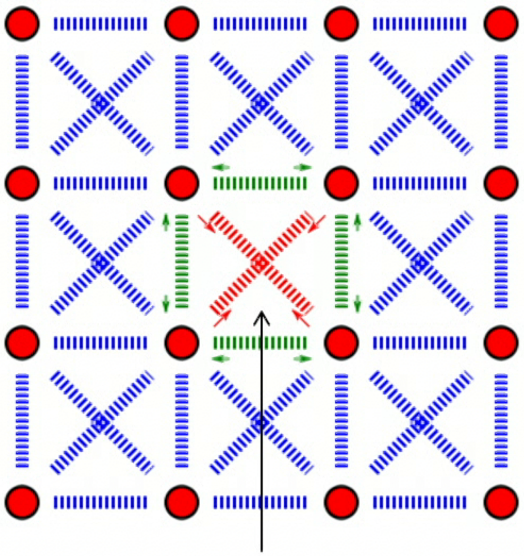

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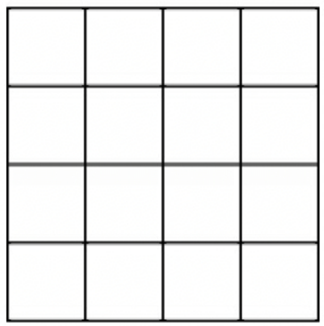
N_{ss} = # states of self-stress

Periodic Isostatic Lattice

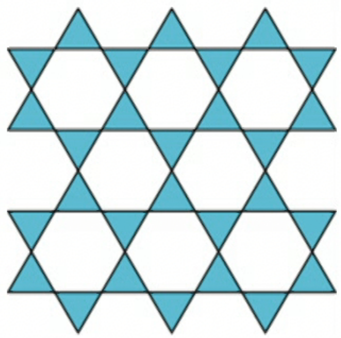
A periodic structure with $dn_s - n_b = 0$

Coordination number (# neighbors): $z = 2d$

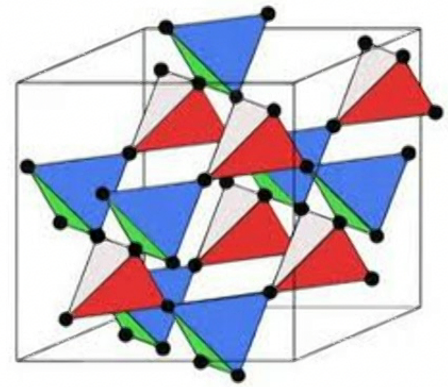
On the verge of mechanical instability



d=2 square lattice (z=4)



d=2 kagome lattice (z=4)



d=3 pyrochlore (z=6)

A model system for problems in soft matter and statistical physics

- Rigidity percolation
- Random closed packing, Jamming
- Network glasses

isostatic on the average

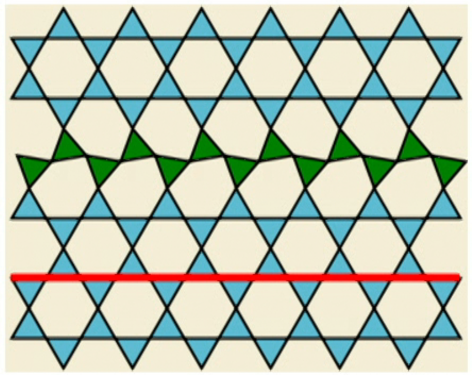
Kagome Lattice Model

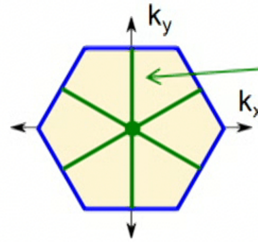
Sun, Souslov, Mao and Lubensky 2012

Untwisted

floppy
mode (fm)

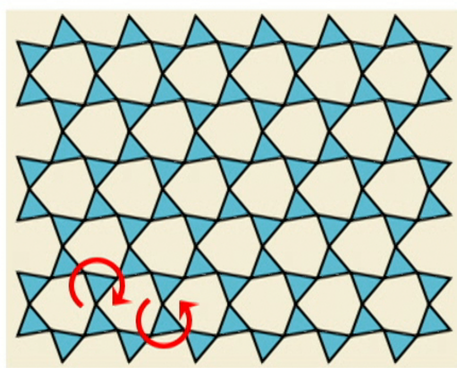
state of
self stress
(ss)

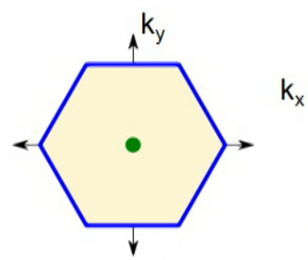




floppy modes
$\omega(\mathbf{k}) = 0$

Twisted





twisting eliminates
both FM and SS
(except at $\mathbf{k}$=0)

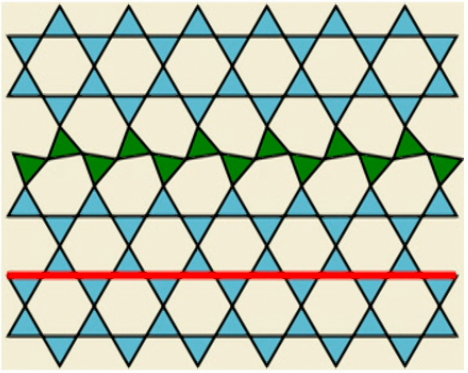
Kagome Lattice Model

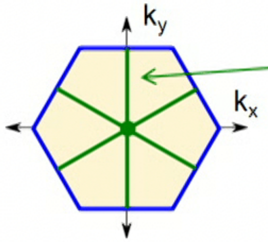
Sun, Souslov, Mao and Lubensky 2012

Untwisted

floppy mode (fm)

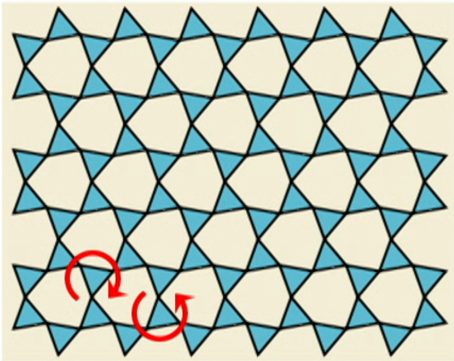
state of self stress (ss)

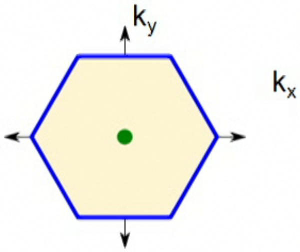




floppy modes $\omega(\mathbf{k}) = 0$

Twisted





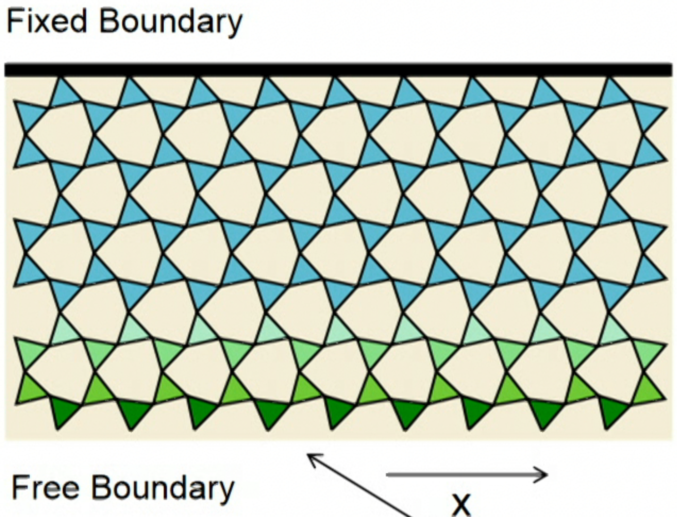
twisting eliminates both FM and SS (except at $\mathbf{k}$=0)

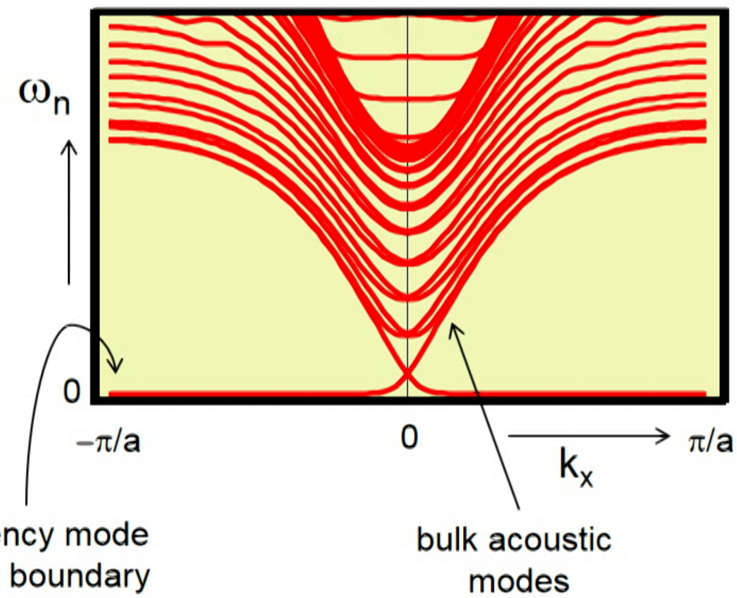
Floppy Modes on a Free Boundary

For twisted Kagome, floppy modes required by Maxwell's count are localized on boundary

Strip Geometry

Normal Mode Spectrum





zero frequency mode
localized at boundary

Tom Lubensky

2012 Tom: Are my boundary modes related to your boundary modes?

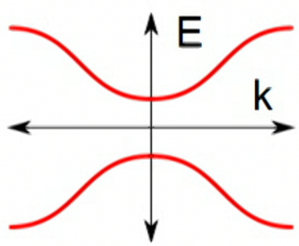
CLK: I don't think so

2013 Tom: Are you sure ?

Schrodinger Equation

$$i\hbar\dot{\psi}_i = H_{ij}\psi_j$$

1st order in time
Hamiltonian H has positive or negative eigenvalues E

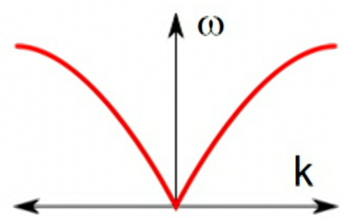


Topologically classify valence band

Newton's Laws

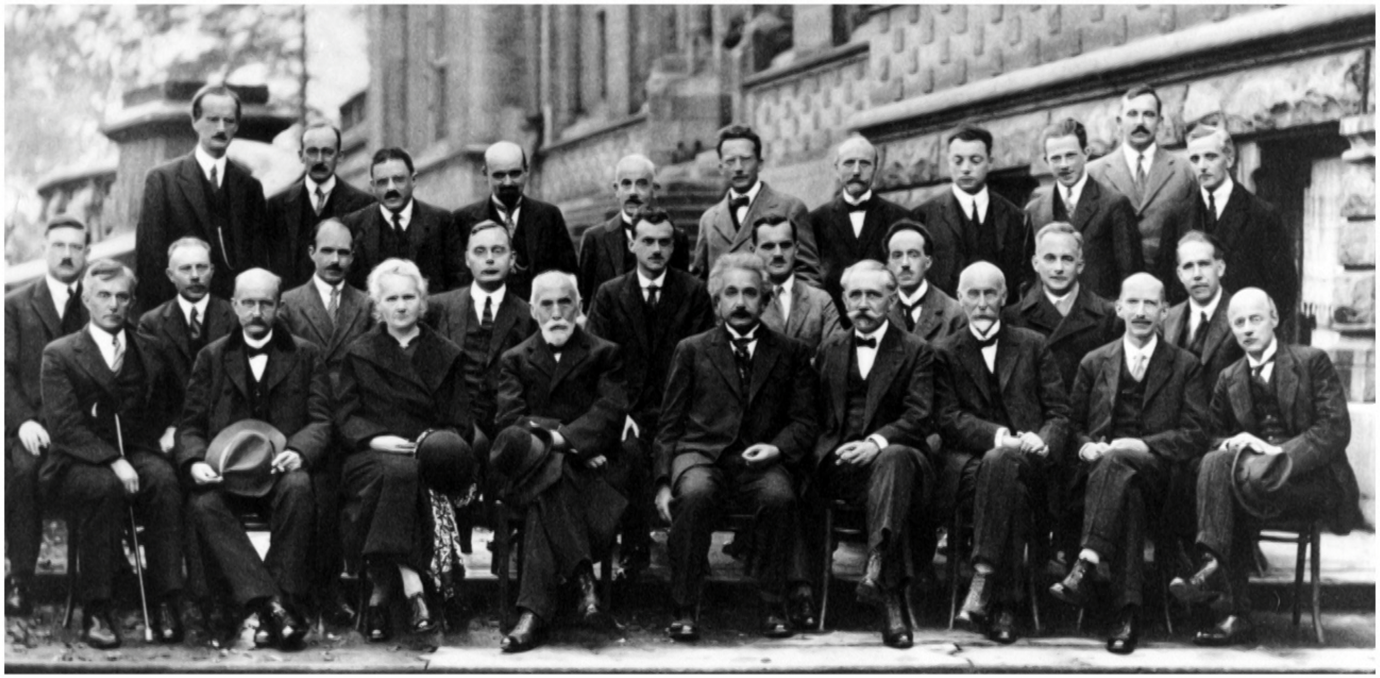
$$m\ddot{u}_i = -D_{ij}u_j$$

2nd order in time
Dynamical matrix D has only positive eigenvalues $m\omega^2$



No "valence band"

Solvay 1927



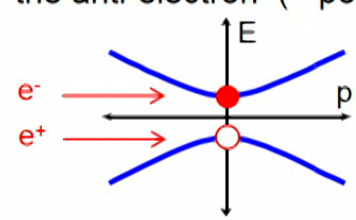
Solvay 1927

Solvay 1927

$$\begin{pmatrix} p_x^2+p_y^2+m^2 & 0 \\ 0 & p_x^2+p_y^2+m^2 \end{pmatrix} = \begin{pmatrix} m & p_x-ip_y \\ p_x+ip_y & -m \end{pmatrix}\begin{pmatrix} m & p_x-ip_y \\ p_x+ip_y & -m \end{pmatrix}$$

$$\sqrt{(p_x^2+p_y^2+m^2)I} = p_x\sigma_x + p_y\sigma_y + m\sigma_z$$

Dirac's Square Root predicted the anti-electron (= positron)



Square Root of Dynamical Matrix

Elastic Energy :

$$U = \frac{1}{2} u \cdot D \cdot u = \frac{1}{2} k \sum_n x_n^2 = \frac{1}{2} u \cdot QQ^T \cdot u$$

$d\, n_s$ x n_b "equilibrium matrix" Q

Extension of spring n $\quad x_n = Q^T_{ni} u_i \quad$ Displacement of site i

Force on site i $\quad f_i = Q_{in} t_n \quad$ Tension in spring n

"Supersymmetric partners"

$$D = QQ^T$$

$$\tilde{D} = Q^T Q$$

D and $\tilde{D}$ have same eigenvalues: ω_n^2 except zero modes

$D \cdot u = 0$ floppy mode

$\tilde{D} \cdot t = 0$ state of self stress

Equivalent "Quantum Hamiltonian"

$$H = \begin{bmatrix} 0 & Q \\ Q^T & 0 \end{bmatrix} \quad ; \quad H^2 = \begin{bmatrix} QQ^T & 0 \\ 0 & Q^T Q \end{bmatrix}$$

eigenvalues of H :

$$E_n = \pm \omega_n$$

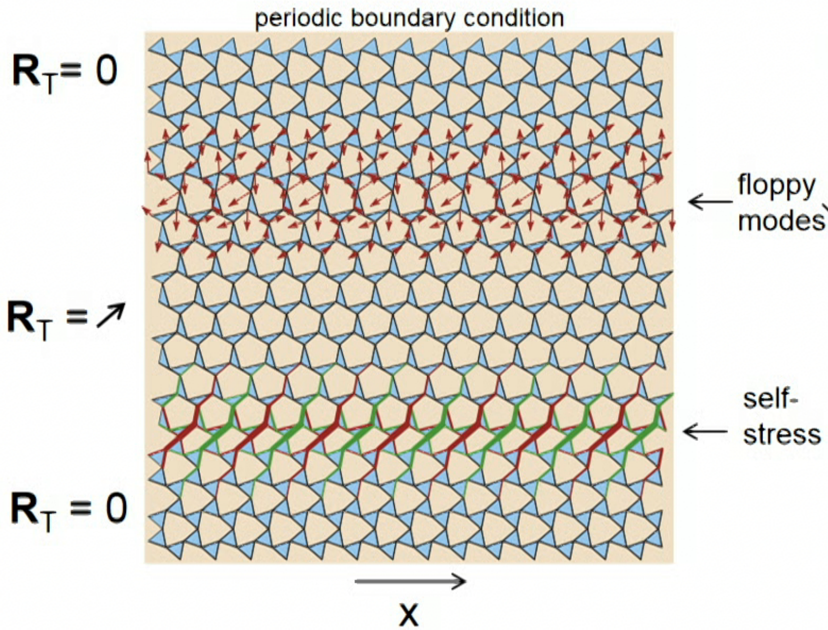
Symmetries

Time reversal (H=H*)
Particle – Hole (H τ^z = -τ^z H)
} Class "BDI" (same as SSH model)

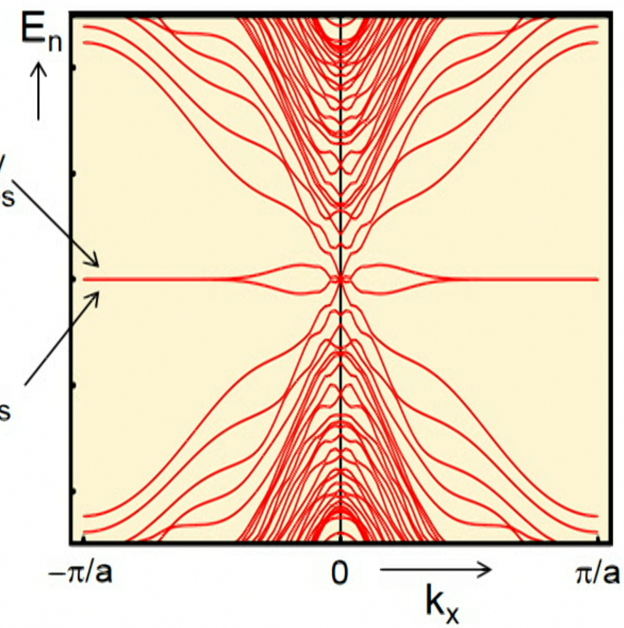
New Topological Phases and Domain Walls

Z x Z topological invariant: $\mathbf{R}_T = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2$ (lattice vector)

"Deformed" Kagome lattice model can have : $\mathbf{R}_T \neq 0$



Spectrum of $H \sim \sqrt{D}$



Index Theorem

A "local" generalization of Maxwell's counting rule

Variant of a famous theorem in mathematics

Attiyah and Singer '63
Callias, Bott and Seeley '78

floppy modes and states of self stress in region S

$$N^S_{\text{fm}} - N^S_{\text{ss}} = \nu^S_L + \nu^S_T$$

"Local count" of sites and bonds in S

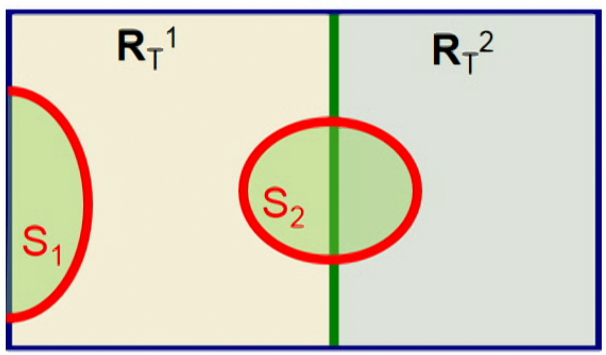
$$\nu^S_L = d n^S_s - n^S_b$$

Depends on edge termination

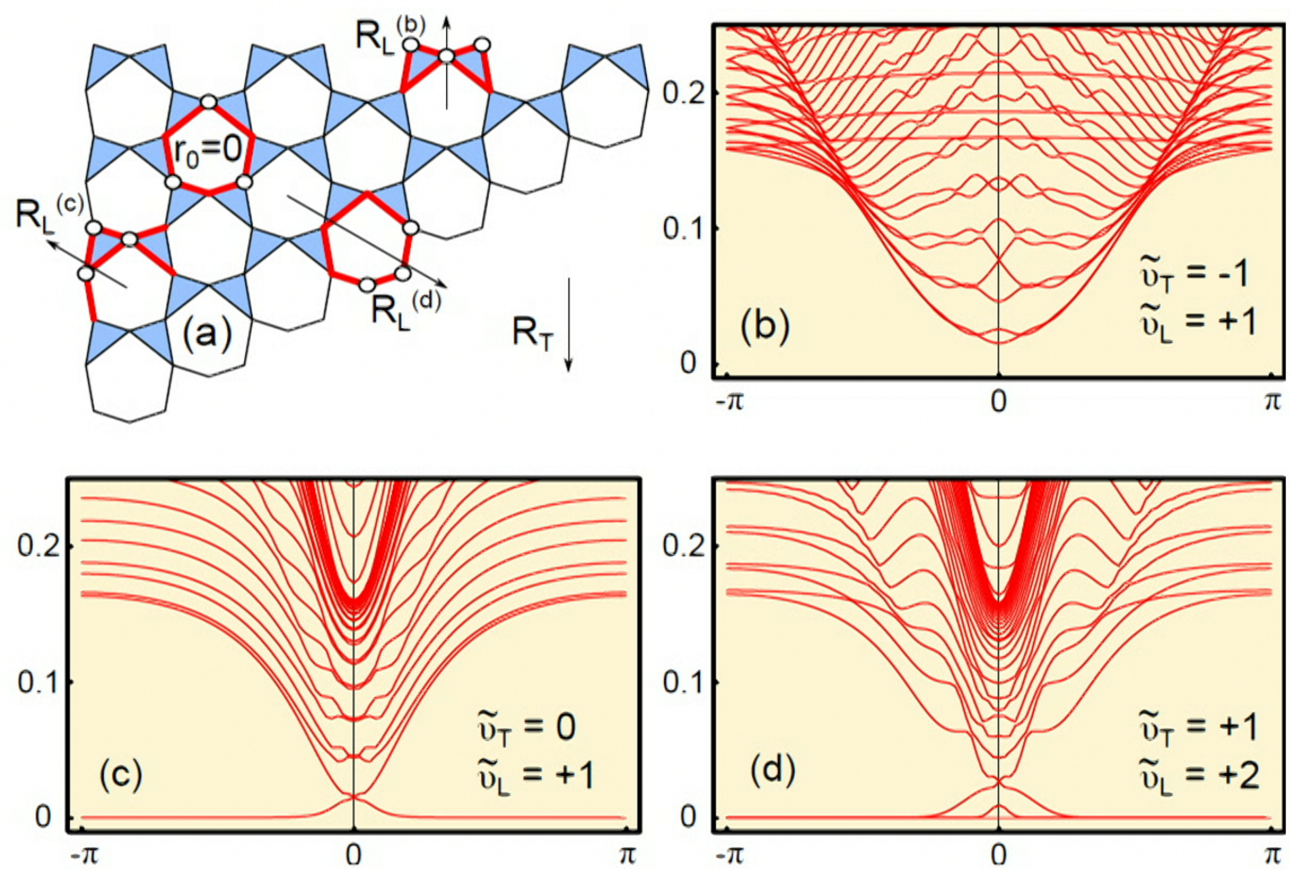
"Topological count" on boundary of S

$$\nu^S_T = \int_{\partial S} \frac{d^{d-1}S}{V_{\text{cell}}} \hat{n} \cdot \mathbf{R}_T$$

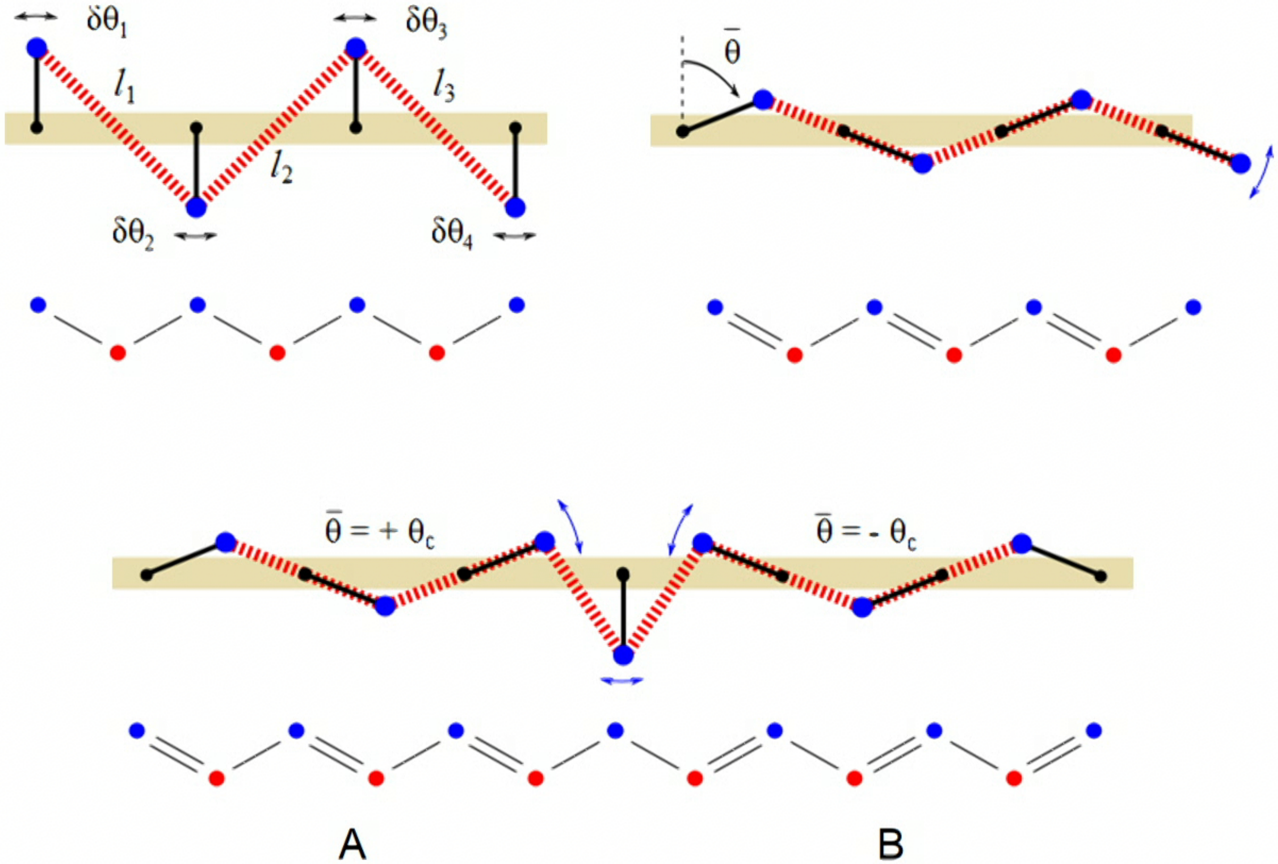
Depends on topological class(es) of bulk



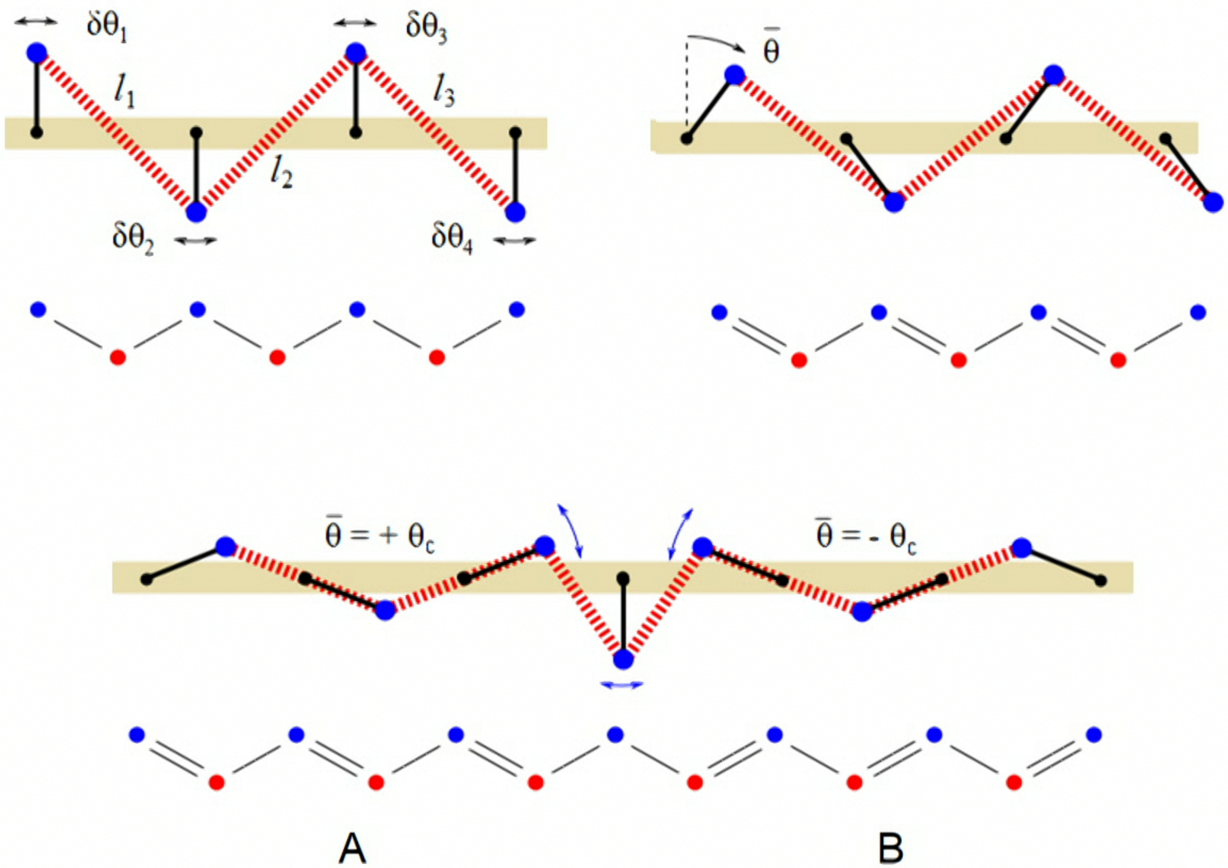
Boundary modes for different edge terminations



Mechanical Analog of SSH Model



Mechanical Analog of SSH Model



A *model* of the model

B. Chen, N. Upadhyaya, V. Vitelli, PNAS 111, 13004 (2014).

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University of Leiden

Conclusion

Topological boundary modes are an elegant consequence of a mathematical structure that has applications in diverse areas

- Topological Electronic Phases
- Mechanical Modes of isostatic systems

Much more to do:

- New materials and experiments on electronic systems
- Experiments on metamaterials?
  - mechanical systems
  - optical, electronic, plasmonic systems?
- Role of interactions and nonlinearities