

Title: Timeless cosmology with records

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Abstract: On the path towards quantum gravity we find friction between temporal relations in quantum mechanics (QM) (where they are fixed and field-independent), and in general relativity (where they are field-dependent and dynamic). In this talk, I will erase that distinction. I encode gravity, along with other types of interactions, in the timeless configuration space of spatial fields, with dynamics obtained through a path integral formulation. The framework demands that boundary conditions for this path integral be uniquely given. Such uniqueness arises if a reduced configuration space can be defined and if it has a profoundly asymmetric fundamental structure. These requirements place strong restrictions on the field and symmetry content of theories encompassed here. When these constraints are met, the emerging theory has no non-unitary measurement process; the Born rule is given merely by a particular volume element built from the path integral in (reduced) configuration space. Time, including space-time, emerges as an effective concept; valid for certain curves in configuration space but not assumed from the start. When some notion of time becomes available, conservation of (positive) probability currents ensues. I will show that, in the appropriate limits, a Schroedinger equation dictates the evolution of weakly coupled source fields on a classical gravitational background. Due to the asymmetry of reduced configuration space, these probabilities and currents avoid a known difficulty of standard WKB approximations for Wheeler DeWitt in minisuperspace: the selection of a unique Hamilton-Jacobi solution to serve as background. I illustrate these constructions with a simple example of a quantum gravitational theory for which the formalism is applicable, and give a formula for calculating gravitational semi-classical relative probabilities in it. Although this simple model gives the same likelihood for the evolution of all TT gravitational modes, there is evidence that a slightly more complicated model would favor modes with the smallest eigenvalues of the Laplacian and thus drive towards homogeneity.

Prelude: the Mott bubble chamber.



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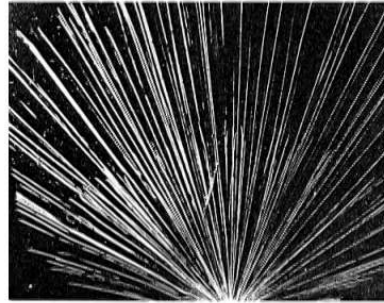
The Wave Mechanics of α -Ray Tracks.

By N. F. MOTT, St. John's College, Cambridge.

(Communicated by C. G. Darwin, F.R.S.—Received October 21, 1929).

The present note is suggested by a recent paper by Prof. Darwin,* and is intended to show how one of the most typically particle-like properties of matter can be derived from the wave mechanics. In the theory of radioactive disintegration, as presented by Gamow, the α -particle is represented by a spherical wave which slowly leaks out of the nucleus. On the other hand, the α -particle, once emerged, has particle-like properties, the most striking being the ray tracks that it forms in a Wilson cloud chamber. It is a little difficult to picture how it is that an outgoing spherical wave can produce a straight track; we think intuitively that it should ionise atoms at random throughout





Question: why do spherically symmetric wave-functions produce collinear rays? Let's see.

$(H_o + \lambda H_{\text{int}} - E)|\psi\rangle = 0$. Two-state detectors: $|0\rangle, |1\rangle$

with $H_{\text{int}} = \sum_{j=0}^J f_j(q)(a_j + a_j^\dagger)$, and $f_j(q)$ is localized.

We can solve perturbatively:^[Halliwell] $|\psi\rangle = \sum_{i=0} \lambda^i |\psi^{(i)}\rangle$

with $|\psi^{(0)}\rangle = |\psi\rangle|0\rangle \cdots |0\rangle$, $|\psi^{(i)}\rangle = \sum_{j_k=\{0,1\}} |\psi_{j_1 \cdots j_j}^{(i)}\rangle |j_1\rangle \cdots |j_j\rangle$



In the end, obtain (for n detectors registering):

$$\langle q_f | \psi_n \rangle \propto \int dq_n \cdots dq_1 f_n(q_n) W(q_f, q_n) \cdots f_2(q_2) W(q_2, q_1) \psi(q_1)$$

where $(H_o - E)W = 1$.*

In a WKB semi-classical approximation:

$$W(q_i, q_j) = \Delta^{1/2}(q_i, q_j) e^{iS(q_i, q_j)} \text{ and}$$

$p_j = \nabla_{q_j} S(q_j, q_i)$, $p_i = -\nabla_{q_i} S(q_j, q_i)$. Now we get:

$$\int dq_n \cdots dq_1 \prod_{k=1}^n \Delta^{1/2}(q_{k+1}, q_k) f_k(q_k) \exp(i \sum_{k=1}^n S(q_{k+1}, q_k)) \psi(q_1)$$

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Stationarity means: $\nabla_{q_j} S(q_j, q_i) + \nabla_{q_i} S(q_j, q_i) = 0$.

So q_i lies on the classical path from q_{i-1} to q_{i+1} .

Integral will be larger when f_k have support along classical path.

* Importantly, one needs boundary conditions.

Can Mott be a starting point to solve main conceptual problems in QG?*

Analogy has been made before: [Barbour, Halliwell, Kiefer, etc]

New considerations: shape space, and its topology.

*Here are some of the problems that keep me up at night: The *quantum mechanical* properties of

1a) the quantum superposition principle

2a) the non-locality of instantaneous measurements;

clash with the *general relativistic* properties of

1b) a fixed causal structure

2b) space-time covariance

Still leaves out the measurement problem in the foundations of quantum mechanics (which I will address), and the relational philosophy of shape dynamics.

Like in Tim's talk:

- Want a separation between law and the objects the law act on (separation between causal and acausal)
- Relational access to such objects. Shape space. Or rather **The reduced configuration space (under local symmetries)**.
Conceptual and technical obstructions to applying this to refoliation invariant theories.

Unlike Tim:

- In the semi-classical approximation, I want the dynamics in each path to be expressible locally (in some gauge).
Not Bohmian.
- No evolution parameter in-built. Only emergent order (and then emergent duration).

In this talk I will:

- Construct a theory which has no non-unitary measurement process.
- Uniquely construct the Born rule as a volume element in reduced configuration space.
- Time, including space-time, will emerge as an effective concept; valid for certain curves in configuration space but not assumed from the start.
- Show conservation of (positive) probability currents, in the appropriate limits. A Schroedinger equation dictates the evolution of weakly coupled source fields on a classical gravitational background.
- Illustrate these constructions with a simple example of a quantum gravitational theory for which the formalism is applicable.

Pre-requisites

Let M be a closed topological manifold encoding a weak notion of locality (neighborhoods). E.g.: $M = S^3$.

Given a configuration space \mathcal{Q} . E.g.:

$\mathcal{Q} = \text{Riem} := \{g \in C_+^\infty(T^*M \otimes_S T^*M)\}$ I will require

- ① Local gauge group \mathcal{G} : has to act locally in M , and pointwise on configuration space: $\mathcal{G} \times \mathcal{Q} \rightarrow \mathcal{Q}$,¹ and be such that
 - There exists a unique orbit $[q_0] \in \mathcal{Q}/\mathcal{G}$ corresponding to the “most homogeneous element”. (i.e. the one with the highest dimensional isotropy subgroup of \mathcal{G}).

¹For metric variables, already very restrictive!

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- ② A pre-probability function, $F : \mathbb{C} \rightarrow \mathbb{R}_+$, such that

$$F(z_1 z_2) = F(z_1) F(z_2). \text{ E.g. } F(z) = |z|^2.$$

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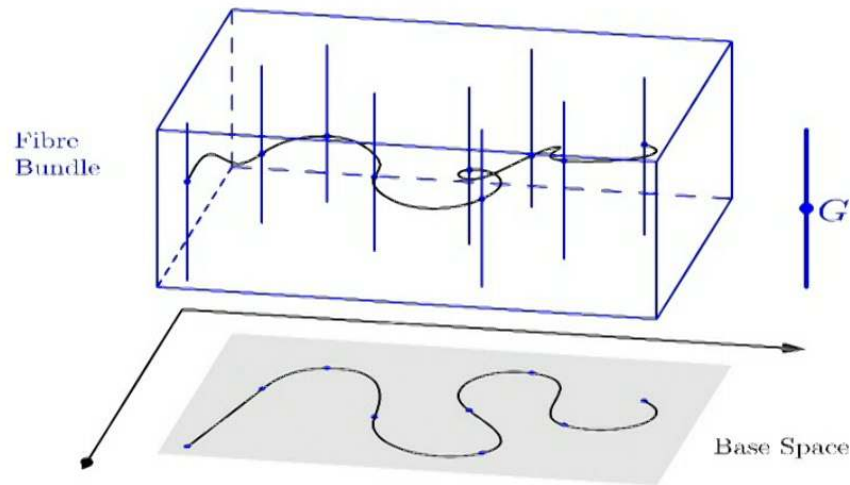
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Element 1, allows me to form a fibration over configuration space:



Also allows one to have a gauge-connection 1-form:

$$T_q Q \simeq H_q \oplus V_q. \text{ (not necessarily a global gauge section).}$$

For $\gamma : I \rightarrow Q$, have $\dot{\gamma} \mapsto \hat{H}(\dot{\gamma}) =: \dot{\gamma}_H$. (analogous to Julian's 'best-matching'. It is a lifting prescription).

The static wavefunction

Given an action functional on curves on \mathcal{Q} , $S(\gamma)$, respecting the given gauge symmetry group \mathcal{G} . Defines $\tilde{S}([\gamma])$, for $[\gamma] : I \rightarrow \mathcal{Q}/\mathcal{G}$.
Can define a propagator. e.g.: for $\mathcal{Q} = \text{Riem}$, and $\mathcal{G} = \text{Diff} \times \mathcal{C}$,

$$W(g_1, [g_2]) := A \int_{g_1}^{[g_2]} \mathcal{D}[\gamma] \mathcal{D}f \exp [iS[\gamma_H(g_1, f^* g_2)]/\kappa]$$

Measure is projected Liouville,^[Barvinsky '91] with Jacobian for \hat{H} .

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Using element 2, define static volume-form in \mathcal{Q}/\mathcal{G} , $P([q])d^n[q]$:

$$P([q]) = F(W(q_o, [q])) : \mathcal{Q}/\mathcal{G} \rightarrow \mathbb{R}_+$$

Where $[q_o]$ is “most homogeneous configuration”^[HG '16]

$$q_o = \text{Arg}(\text{Sup}_{q \in \mathcal{Q}} \dim(\text{Iso}_q(\mathcal{G})))$$

The fundamental asymmetry of gravitational shape space.

$[g_o]$ is unique for $M = S^3$ and $\mathcal{G} = \text{Diff} \times \mathcal{C}$. Namely, $g_o = d\Omega_o^3$.

Start off from g_o and follow a horizontal curve ($g^{ab}\dot{g}_{ab} = 0$).
Volume-form doesn't change, $\det g$ can't go to zero. \Rightarrow
degenerate metrics are not reachable from \mathcal{Q}/\mathcal{C} .

Apart from degenerate metrics, round spheres have the largest isotropy subgroup of $\text{Diff}(S^3)$.

In fact, $\mathcal{Q}/\text{Diff}\times\mathcal{C}$ is a 'stratified', or nested, union of manifolds of increasing dimension, like a cube. Here there is a unique least dimensional corner. I take this as the 'origin' of shape space.

(Relation to "a complexity functional"? The Yamabe invariant's unique saddle point also $d\Omega_o^3$.)

Semi-classical approx. for oscillatory path integrals

For extremum paths $\gamma_{\text{cl}}^\alpha$, between q_i, q_f , the Van Vleck determinant is:

$$\Delta_\alpha := \det \left(\frac{\delta^2 S_{\gamma_{\text{cl}}^\alpha}(q_i, q_f)}{\delta q_i \delta q_f} \right) = \det \left(\frac{\delta q_f}{\delta p_i^\alpha} \right)^{-1}$$

Semi-classical approximation is then:²

$$W_{\text{cl}}(q_i, q_f) = \sum_\alpha (\Delta_\alpha)^{1/2} \exp(i S_{\gamma_{\text{cl}}^\alpha}(q_i, q_f)/\kappa)$$

$$|W_{\text{cl}}|^2 = \sum_\alpha \Delta_\alpha + \underbrace{2 \sum_{\alpha \neq \nu} |\Delta_\alpha \Delta_\nu|^{1/2} \cos \left(\frac{S_{\gamma_{\text{cl}}^\alpha} - S_{\gamma_{\text{cl}}^\nu}}{\kappa} \right)}_{\text{Interference effects}}$$

(although haven't yet justified taking $F(z) = |z|^2$)

²For field space version see [Barvinsky'93]

We are trying to find the form of $F : \mathbb{C} \rightarrow \mathbb{R}_+$.

Van-Vleck relates initial infinitesimal volume around q_i , and final volume around q_f , as transported by classical paths: $\Delta_\alpha = \frac{\rho_f^\alpha}{\rho_i}$

For $\rho_o = 1$, if there is a single classical curve between q_o, q_f ,
 $\Delta_\alpha = \Delta$,

$$F(W(q_o, q_f)) =: \rho_f = \Delta = |W_{cl}|^2$$

which establishes that on some region, $F(z) \approx |z|^2$. But

$F(z) = \sum_i a_i |z\bar{z}|^i$, thus

$$F(z_1 z_2) = \sum_i a_i (z_1 z_2 \bar{z}_1 \bar{z}_2)^i \quad (*) \text{ and,}$$

$$F(z_1) F(z_2) = \sum_i a_i (z_1 \bar{z}_1)^i \sum_j a_j (z_2 \bar{z}_2)^j \quad (**)$$

Only diagonal terms of (*) can match the polynomials of (**).

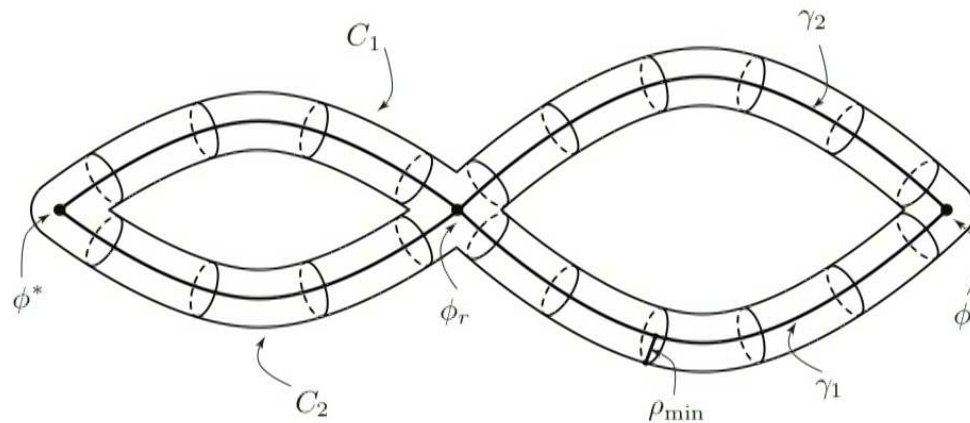
Thus $a_i = a \delta_{i, i_o}$ and $a = 1$.

By the semi-classical limit above, $i_o = 1 \Rightarrow F(z) = |z|^2$. \square



Extremal coarse-grainings and '(pre-)records'.

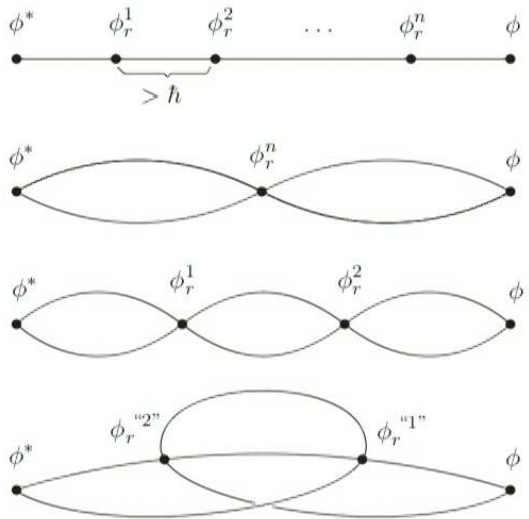
Assume many extremal paths between q_o and q , but they all go through q_r :



Then, can show: $W_{cl}(q_o, q) \approx W_{cl}(q_o, q_r)W_{cl}(q_r, q)$

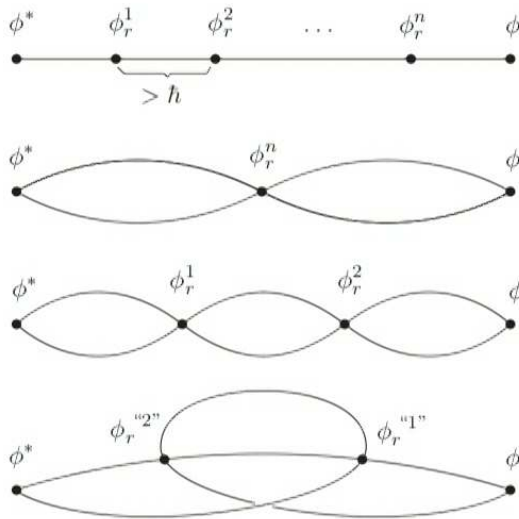
thus $P(q) = P(q_r)P(q|q_r)$ where $P(q|q_r) = |W(q_r, q)|^2$.

For multiple such records,



For first 3 cases: $P(q_o, q) \approx P(q_o, q_r^1)P(q_r^1, q_r^2) \cdots P(q_r^n, q)$

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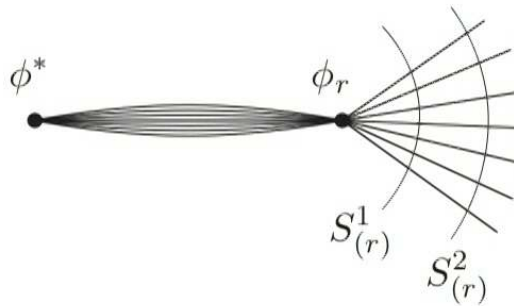
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Like in Mott!

Also note: $\frac{P(q_o, q_1)}{P(q_o, q_2)} \approx \frac{P(q_r, q_1)}{P(q_r, q_2)}$.

Here there is no time, but we still want to define 'conservation of probability'.

Problem: for one q_r there can be many redundant record relations. Define a 'screen' in shape space:



By constant action-distance (arc-length for a Jacobi metric).

Possible to show that 'flux' through q_r greater than through $S_{(r)}$.

For a Jacobi action functional of the form:

$S[\gamma] = \int dt (V(q)G^{ab}\dot{\gamma}_a\dot{\gamma}_b)^{1/2}$, can show that path integral satisfies:

$$\kappa^2 \left(G^{ab} \frac{\delta^2 \Psi(q)}{\delta q^a \delta q^b} \right) - V(q) \Psi(q) = 0$$

(not WdW, hidden integration in a, b). For metric + source:

$$\left(-\frac{1}{2m_p^2} \nabla^2 + m_p^2 V[g] + H_{\text{mat}}(g, \varphi) \right) \Psi[g, \varphi] = 0$$

where $\nabla^2 := \int d^3x d^3y \left(G_{abcd}(x, y) \frac{\delta^2}{\delta g_{ab}(x) \delta g_{cd}(y)} \right)$

with ansatz:

$$\Psi(g, \varphi) = \exp(im_p^2 S[g]) A[g] \psi[g, \varphi] + \mathcal{O}(m_p^{-2})$$

We obtain:

$$i \int d^3x d^3y G_{abcd}(x, y) \frac{\delta S[g]}{\delta g_{ab}(x)} \frac{\delta \psi[g, \varphi]}{\delta g_{cd}(y)} = H_{\text{mat}}(g, \varphi) \psi[g, \varphi]$$

and defining:

$$\frac{\partial}{\partial T} = \int d^3x d^3y G_{abcd}(x, y) \frac{\delta S[g]}{\delta g_{ab}(x)} \frac{\delta}{\delta g_{cd}(y)}$$

$$\text{we get: } i \frac{\partial}{\partial T} \psi[g, \varphi] = H_{\text{mat}}(g, \varphi) \psi[g, \varphi]$$

$$\text{Since } (dT)^{ab}(x) = \frac{1}{V[g]} \frac{\delta S[\phi]}{\delta g_{ab}(x)}$$

$$\text{and } J_{cd}(x) = \int d^3y G_{abcd}(x, y) \left(\frac{\delta S[g]}{\delta g_{ab}(x)} A^2[g] \right)$$

infinitesimal flux through constant T screen $d\mathbf{T} \cdot \vec{J} \approx A^2$.

None of this can be derived if e.g.: $\Psi(g, \varphi) \approx \exp(im_p^2 S_1[g]) A_1[g] \psi_1[g, \varphi] + \exp(im_p^2 S_2[g]) A_2[g] \psi_2[g, \varphi]$

Big problem for WdW! we are safe because of g_o .

Quick remark

Like Hartle-Hawking:

- Impose (there controversial) boundary conditions in superspace.
- Derives space-times from WKB approx IN minisuperspace.

Unlike Hartle-Hawking:

- There is a reduced config space outside of minisuperspace approx.
- Has natural boundary conditions on shape space, which naturally correspond to a most homogeneous state (which is also a corner of reduced configuration space).

Geometrical toy model ('free-particle in Riem')

For now only with $\mathcal{G} = \text{Diff}(M)$, but still $g_o = d\Omega^3$.

$$S[g] = \int dt \langle \dot{g}, \dot{g} \rangle_{g(t)}^{1/2} = \int dt \left(\int d^3x \dot{g}_{ab} g^{ac} g^{bd} \dot{g}_{cd} \sqrt{g} \right)^{1/2}$$

Horizontal lift given by orthogonality wrt fibers (not a gauge-section). [Vilkowisky '77, DeWitt '94, HG '10]

Geodesics explicit, in closed form. [Freed, Groisser '89; Michor, Medrano '91]

Super-Riemman curvature:

$$R(\mathbf{h}_1, \mathbf{h}_2)\mathbf{h}_3 = -\frac{1}{4}[[\mathbf{h}_1, \mathbf{h}_2], \mathbf{h}_3] + \frac{3}{16}(\text{tr}_g(\mathbf{h}_1\mathbf{h}_3)\mathbf{h}_2 - \text{tr}_g(\mathbf{h}_2\mathbf{h}_3)\mathbf{h}_1)$$

For $h_{ab}g^{ab} = 0$ (otherwise zero).

Geodesics don't reconverge \Rightarrow no interference terms in semi-classical approximation.

For finite-dimensions and geodesic action, can calculate a geometrical Van Vleck: ^[Visser '93]

$$\Delta_{\gamma}^{\text{small d}}(x, y) = 1 + \frac{1}{6}(R_{ab}\dot{\gamma}^a\dot{\gamma}^b S[\gamma]^2 + \mathcal{O}(S[\gamma])^3)$$

Long story about how to translate this to infinite-d. ^[Michor et al '01, HG '16]
Requires regularization.

For two different ranges of initial transverse traceless initial directions, H_1, H_2 at $d\Omega^3$, s.t. $\int_{H_1} \mathcal{D}h^1 = \int_{H_2} \mathcal{D}h^2$

$$\frac{\int_{H_1} \mathcal{D}h^1 J_{TT}(g^0) P(g(\epsilon))}{\int_{H_2} \mathcal{D}h^2 J_{TT}(g^0) P(g(\epsilon))} = 1 + \frac{1}{4} \frac{\int_{H_2} \mathcal{D}h^2 \left(\int d^3x \sqrt{g^*} h_{ab} g_o^{ac} g_o^{bd} h_{cd} \right) - \int_{H_1} \mathcal{D}h^1 \left(\int d^3x \sqrt{g_o} h_{ab} g_o^{ac} g_o^{bd} h_{cd} \right)}{V_H} \epsilon^2 + \dots$$

Relative probability for two screens defined by same arc-length distance.

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Relative probability for two screens defined by same arc-length distance.

Using orthonormal eigenbasis of TT tensors on g_o ,

$\prod_x dh_{ij}(x) \rightarrow \prod_{n=1}^{\infty} d\lambda_{ij}^n$. But then:

$$\int_{H_1} \mathcal{D}h = \int_{H_2} \mathcal{D}h \Rightarrow \int_{H_2} \mathcal{D}h^2 \left(\int d^3x \sqrt{g^0} h_{Tab} g_0^{ac} g_0^{bd} h_{cd} \right) = \int_{H_1} \mathcal{D}h^1 \left(\int d^3x \sqrt{g^0} h_{Tab} g_0^{ac} g_0^{bd} h_{cd} \right).$$


I.e., the measure does not care about the eigenvalues of the TT-modes. But

$$S[g] = \int dt \langle \dot{g}, \dot{g} \rangle_{g(t)}^{1/2} = \int dt \left(\int d^3y f(R) \sqrt{g} \right) \left(\int d^3x \dot{g}_{ab} g^{ac} g^{bd} \dot{g}_{cd} \sqrt{g} \right)^{1/2}$$

changes terms in VV, e.g.: $h_{Tab} g_0^{ac} g_0^{bd} h_{cd} \rightarrow h_{Tab} g_0^{ac} g_0^{bd} \nabla^{2n} h_{cd}$

In such case, higher eigenvalues of H_1 in comparison to H_2 , mean *its relative probability flux is smaller*.

This would mean indeed that more homogeneous modes would be favored.³

³Eigenvalues of TT in S^3 are $\nabla^2 \tau_{ij}^{(n)} = -(n^2 - 3) \tau_{ij}^{(n)}$ 

How did we fare?

Mix better:

- Quantum mechanical notions
 - 1 No collapse of the wavefunction (static prob density).
 - 2 True (fake) evolution in the semi-classical approx.
 - 3 Quantum superposition principle.
- Conformal geometrodynamics
 - 1 Refoliation not fundamental (recovered relationally).
 - 2 A Hamiltonian which separates local gauge symmetries and global evolution.
 - 3 Causal structures corresponding to different extremal curves (can interfere).

Challenge: recover standard GR.

