Title: Timeless cosmology with records

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Abstract: On the path towards quantum gravity we find friction between temporal relations in quantum mechanics (QM) (where they are fixed and field-independent), and in general relativity (where they are field-dependent and dynamic). In this talk, I will erase that distinction. I encode gravity, along with other types of interactions, in the timeless configuration space of spatial fields, with dynamics obtained through a path integral formulation. The framework demands that boundary conditions for this path integral be uniquely given. Such uniqueness arises if a reduced configuration space can be defined and if it has a profoundly asymmetric fundamental structure. These requirements place strong restrictions on the field and symmetry content of theories encompassed here. When these constraints are met, the emerging theory has no non-unitary measurement process; the Born rule is given merely by a particular volume element built from the path integral in (reduced) configuration space. Time, including space-time, emerges as an effective concept; valid for certain curves in configuration space but not assumed from the start. When some notion of time becomes available, conservation of (positive) probability currents ensues. I will show that, in the appropriate limits, a Schroedinger equation dictates the evolution of weakly coupled source fields on a classical gravitational background. Due to the asymmetry of reduced configuration space, these probabilities and currents avoid a known difficulty of standard WKB approximations for Wheeler DeWitt in minisuperspace: the selection of a unique Hamilton-Jacobi solution to serve as background. I illustrate these constructions with a simple example of a quantum gravitational theory for which the formalism is applicable, and give a formula for calculating gravitational semi-classical relative probabilities in it. Although this simple model gives the same likelihood for the evolution of all TT gravitational modes, there is evidence that a slightly more complicated model would favor modes with the smallest eigenvalues of the Laplacian and thus drive towards homogeneity.

Prelude: the Mott bubble chamber.

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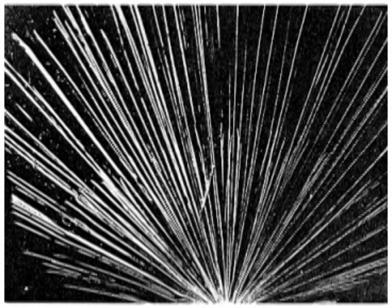
The Wave Mechanics of α-Ray Tracks.

By N. F. Mott, St. John's College, Cambridge.

(Communicated by C. G. Darwin, F.R.S.—Received October 21, 1929).

The present note is suggested by a recent paper by Prof. Darwin,* and is intended to show how one of the most typically particle-like properties of matter can be derived from the wave mechanics. In the theory of radioactive disintegration, as presented by Gamow, the α-particle is represented by a spherical wave which slowly leaks out of the nucleus. On the other hand, the α-particle, once emerged, has particle-like properties, the most striking being the ray tracks that it forms in a Wilson cloud chamber. It is a little difficult to picture how it is that an outgoing spherical wave can produce a straight track; we think intuitively that it should ionise atoms at random throughout



Question: why do spherically symmetric wave-functions produce collinear rays? Let's see.

$(H_o + \lambda H_{\text{int}} - E)|\psi\rangle = 0$. Two-state detectors: $|0\rangle, |1\rangle$

with $H_{\text{int}} = \sum_{j=0}^{J} f_j(q)(a_j + a_j^\dagger)$, and $f_j(q)$ is localized.

We can solve perturbatively:[Halliwell] $|\psi\rangle = \sum_{i=0} \lambda^i |\psi^{(i)}\rangle$

$$\text{with} \quad |\psi^{(0)}\rangle = |\psi\rangle|0\rangle \cdots |0\rangle, \quad |\psi^{(i)}\rangle = \sum_{j_k=\{0,1\}} |\psi^{(i)}_{j_1\cdots j_J}\rangle |j_1\rangle \cdots |j_J\rangle$$

In the end, obtain (for n detectors registering):

$$\langle q_f|\psi_n\rangle \propto \int dq_n \cdots dq_1 f_n(q_n) W(q_f, q_n) \cdots f_2(q_2) W(q_2, q_1)\psi(q_1)$$

where $(H_o - E)W = 1.$*

In a WKB semi-classical approximation:

$W(q_i, q_j) = \Delta^{1/2}(q_i, q_j)e^{iS(q_i,q_j)}$ and

$p_j = \nabla_{q_j} S(q_j, q_i)$, $p_i = -\nabla_{q_i} S(q_j, q_i)$. Now we get:

$$\int dq_n \cdots dq_1 \prod_{k=1}^{n} \Delta^{1/2}(q_{k+1}, q_k) f_k(q_k) \exp\left(i \sum_{k=1}^{n} S(q_{k+1}, q_k)\right)\psi(q_1)$$

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Stationarity means: $\nabla_{q_j} S(q_j, q_i) + \nabla_{q_i} S(q_j, q_i) = 0$.

So q_i lies on the classical path from q_{i-1} to q_{i+1}.

Integral will be larger when f_k have support along classical path.

* Importantly, one needs boundary conditions.

Can Mott be a starting point to solve main conceptual problems in QG?*

Analogy has been made before: [Barbour, Halliwell, Kiefer, etc]

New considerations: shape space, and its topology.

*Here are some of the problems that keep me up at night: The *quantum mechanical* properties of

1a) the quantum superposition principle

2a) the non-locality of instantaneous measurements;

clash with the *general relativistic* properties of

1b) a fixed causal structure

2b) space-time covariance

Still leaves out the measurement problem in the foundations of quantum mechanics (which I will address), and the relational philosophy of shape dynamics.

Like in Tim's talk:

- Want a separation between law and the objects the law act on (separation between causal and acausal)
- Relational access to such objects. Shape space. Or rather
  The reduced configuration space (under local symmetries).
  Conceptual and technical obstructions to applying this to refoliation invariant theories.

Unlike Tim:

- In the semi-classical approximation, I want the dynamics in each path to be expressible locally (in some gauge).
  Not Bohmian.
- No evolution parameter in-built. Only emergent order (and then emergent duration).

In this talk I will:

- Construct a theory which has no non-unitary measurement process.
- Uniquely construct the Born rule as a volume element in reduced configuration space.
- Time, including space-time, will emerge as an effective concept; valid for certain curves in configuration space but not assumed from the start.
- Show conservation of (positive) probability currents, in the appropriate limits. A Schroedinger equation dictates the evolution of weakly coupled source fields on a classical gravitational background.
- Illustrate these constructions with a simple example of a quantum gravitational theory for which the formalism is applicable.

Pre-requisites

Let M be a closed topological manifold encoding a weak notion of locality (neighborhoods). E.g.: $M = S^3$.

Given a configuration space $\mathcal{Q}$. E.g.:
$\mathcal{Q} = \text{Riem} := \{g \in C^\infty_+(T^*M \otimes_S T^*M)\}$ I will require

1. Local gauge group $\mathcal{G}$: has to act locally in M, and pointwise on configuration space: $\mathcal{G} \times \mathcal{Q} \to \mathcal{Q}$,[1] and be such that
   - There exists a unique orbit $[q_o] \in \mathcal{Q}/\mathcal{G}$ corresponding to the "most homogeneous element". (i.e. the one with the highest dimensional isotropy subgroup of $\mathcal{G}$).

[1] For metric variables, already very restrictive!

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2. A pre-probability function, $F : \mathbb{C} \to \mathbb{R}_+$, such that

   $F(z_1 z_2) = F(z_1)F(z_2)$. E.g. $F(z) = |z|^2$.

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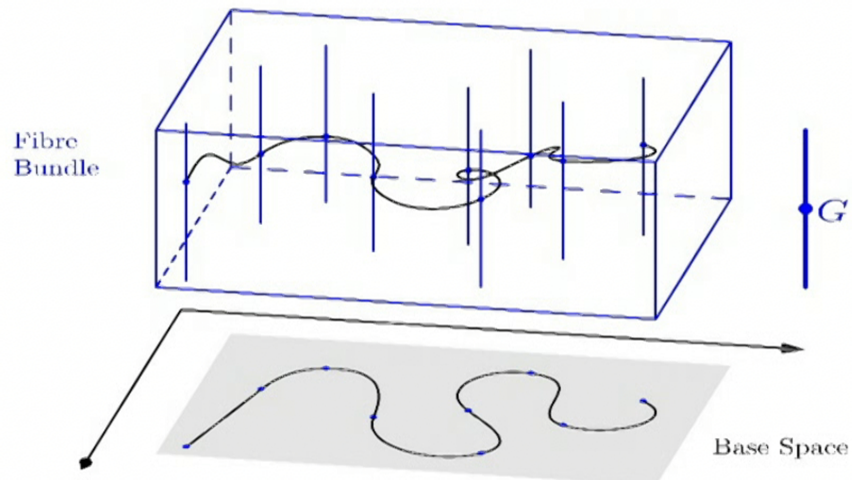
Element 1, allows me to form a fibration over configuration space:



Also allows one to have a gauge-connection 1-form:
$T_q\mathcal{Q} \simeq H_q \oplus V_q$. (not necessarily a global gauge section).

For $\gamma : I \to \mathcal{Q}$, have $\dot{\gamma} \mapsto \hat{H}(\dot{\gamma}) =: \dot{\gamma}_H$. (analogous to Julian's 'best-matching'. It is a lifting prescription).

The static wavefunction

Given an action functional on curves on $\mathcal{Q}$, $S(\gamma)$, respecting the given gauge symmetry group $\mathcal{G}$. Defines $\tilde{S}([\gamma])$, for $[\gamma] : I \to \mathcal{Q}/\mathcal{G}$. Can define a propagator. e.g.: for $\mathcal{Q} = \mathit{Riem}$, and $\mathcal{G} = \mathrm{Diff} \ltimes \mathcal{C}$,

$$W(g_1, [g_2]) := A \int_{g_1}^{[g_2]} \mathcal{D}[\gamma]\mathcal{D}f \exp\left[iS[\gamma_H(g_1, f^* g_2)]/\kappa\right]$$

Measure is projected Liouville,[Barvinsky '91] with Jacobian for $\hat{H}$.

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Using element 2, define static volume-form in $\mathcal{Q}/\mathcal{G}$, $P([q])d^n[q]$:

$$P([q]) = F(W(q_o, [q])) : \mathcal{Q}/\mathcal{G} \rightarrow \mathbb{R}_+$$

Where $[q_o]$ is "most homogeneous configuration" [HG '16]

$$q_o = \text{Arg}(\text{Sup}_{q\in\mathcal{Q}} \dim(\text{Iso}_q(\mathcal{G})))$$

The fundamental asymmetry of gravitational shape space.

$[g_o]$ is unique for $M = S^3$ and $\mathcal{G} = \text{Diff} \ltimes \mathcal{C}$. Namely, $g_o = d\Omega_o^3$.

Start off from g_o and follow a horizontal curve ($g^{ab}\dot{g}_{ab} = 0$). Volume-form doesn't change, $\det g$ can't go to zero. $\Rightarrow$ degenerate metrics are not reachable from $\mathcal{Q}/\mathcal{C}$.

Apart from degenerate metrics, round spheres have the largest isotropy subgroup of Diff(S^3).

In fact, $\mathcal{Q}/\text{Diff}\ltimes\mathcal{C}$ is a 'stratified', or nested, union of manifolds of increasing dimension, like a cube. Here there is a unique least dimensional corner. I take this as the 'origin" of shape space.

(Relation to "a complexity functional"? The Yamabe invariant's unique saddle point also $d\Omega_o^3$.)

Semi-classical approx. for oscillatory path integrals

For extremum paths $\gamma_{\rm cl}^{\alpha}$, between q_i, q_f, the Van Vleck determinant is:

$$\Delta_\alpha \ := \ \det\left(\frac{\delta^2 S_{\gamma^\alpha_{\rm cl}}(q_i,q_f)}{\delta q_i \delta q_f}\right) = \det\left(\frac{\delta q_f}{\delta p_i^\alpha}\right)^{-1}$$

Semi-classical approximation is then:[2]

$$W_{\rm cl}(q_i,q_f) \ = \ \textstyle\sum_\alpha (\Delta_\alpha)^{1/2} \exp\left(i\, S_{\gamma^\alpha_{\rm cl}}(q_i,q_f)/\kappa\right)$$

$$|W_{\rm cl}|^2 \ = \ \textstyle\sum_\alpha \Delta_\alpha + \underbrace{2\sum_{\alpha\neq\nu} |\Delta_\alpha \Delta_\nu|^{1/2} \cos\left(\frac{S_{\gamma^\alpha_{\rm cl}} - S_{\gamma^\nu_{\rm cl}}}{\kappa}\right)}_{\text{Interference effects}}$$

(although haven't yet justified taking $F(z) = |z|^2$)

[2] For field space version see [Barvinsky'93]

We are trying to find the form of $F : \mathbb{C} \to \mathbb{R}_+$.

Van-Vleck relates initial infinitesimal volume around q_i, and final volume around q_f, as transported by classical paths: $\Delta_\alpha = \frac{\rho_f^\alpha}{\rho_i}$

For $\rho_o = 1$, if there is a single classical curve between q_o, q_f, $\Delta_\alpha = \Delta$,

$$F(W(q_o, q_f)) =: \rho_f = \Delta = |W_{\text{cl}}|^2$$

which establishes that on some region, $F(z) \approx |z|^2$. But

$F(z) = \sum_i a_i |z\bar{z}|^i$, thus

$F(z_1 z_2) = \sum_i a_i (z_1 z_2 \bar{z}_1 \bar{z}_2)^i \quad (*)$ and,

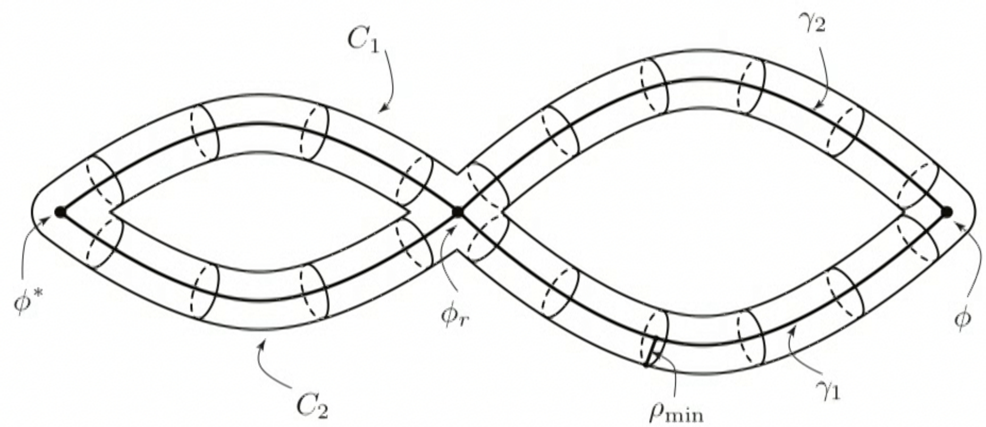
$F(z_1)F(z_2) = \sum_i a_i (z_1 \bar{z}_1)^i \sum_j a_j (z_2 \bar{z}_2)^j \quad (**)$

Only diagonal terms of (*) can match the polynomials of (**) . Thus $a_i = a\delta_{i,i_o}$ and $a = 1$.

By the semi-classical limit above, $i_o = 1 \Rightarrow F(z) = |z|^2$. $\square$

Extremal coarse-grainings and '(pre-)records'.

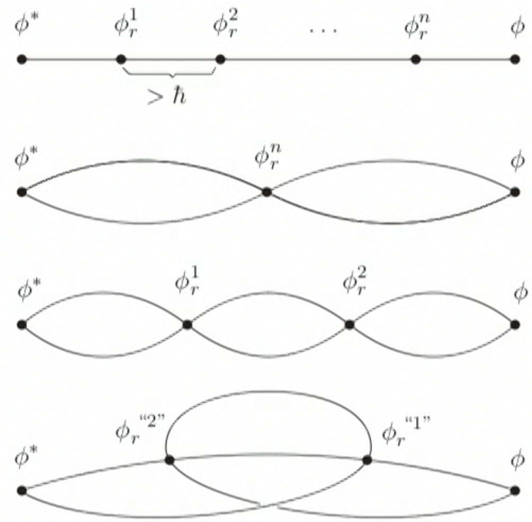
Assume many extremal paths between q_o and q, but they all go through q_r:



Then, can show: $W_{\text{cl}}(q_o, q) \approx W_{\text{cl}}(q_o, q_r) W_{\text{cl}}(q_r, q)$

thus $P(q) = P(q_r)P(q|q_r)$ where $P(q|q_r) = |W(q_r, q)|^2$.

For multiple such records,



For first 3 cases: $P(q_o, q) \approx P(q_o, q_r^1)P(q_r^1, q_r^2)\cdots P(q_r^n, q)$

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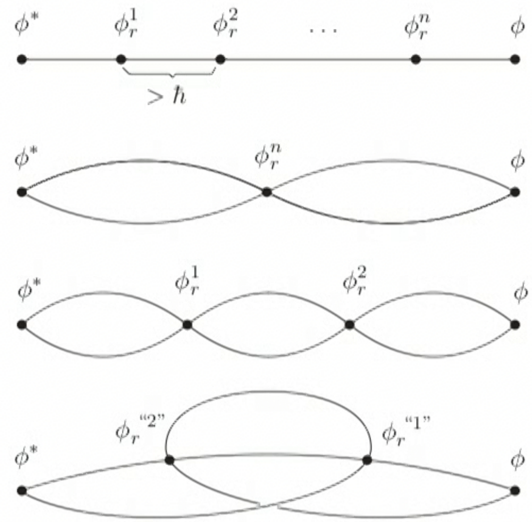

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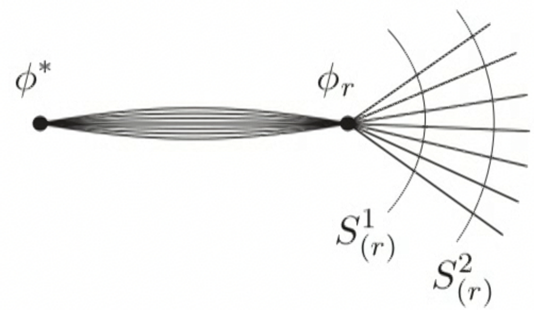
Like in Mott!

Also note: $\frac{P(q_o,q_1)}{P(q_o,q_2)} \approx \frac{P(q_r,q_1)}{P(q_r,q_2)}$.

Here there is no time, but we still want to define 'conservation of probability'.

Problem: for one q_r there can be many redundant record relations. Define a 'screen' in shape space:



By constant action-distance (arc-length for a Jacobi metric).

Possible to show that 'flux' through q_r greater than through $S_{(r)}$.

For a Jacobi action functional of the form:

$S[\gamma] = \int dt \left(V(q) G^{ab}\dot{\gamma}_a\dot{\gamma}_b\right)^{1/2}$, can show that path integral satisfies:

$$\kappa^2 \left(G^{ab}\frac{\delta^2\Psi(q)}{\delta q^a \delta q^b}\right) - V(q)\Psi(q) = 0$$

(not WdW, hidden integration in a, b). For metric + source:

$$\left(-\frac{1}{2m_p^2}\nabla^2 + m_p^2 V[g] + H_{\mathrm{mat}}(g,\varphi)\right)\Psi[g,\varphi] = 0$$

where $\nabla^2 := \int d^3x\, d^3y \left(G_{abcd}(x,y)\frac{\delta^2}{\delta g_{ab}(x)\delta g_{cd}(y)}\right)$

with ansatz:

$$\Psi(g,\varphi) = \exp\left(i m_p^2 S[g]\right) A[g]\psi[g,\varphi] + \mathcal{O}(m_p^{-2})$$

We obtain:

$$i \int d^3x\, d^3y\, G_{abcd}(x,y) \frac{\delta S[g]}{\delta g_{ab}(x)} \frac{\delta \psi[g,\varphi]}{\delta g_{cd}(y)} = H_{\text{mat}}(g,\varphi)\psi[g,\varphi]$$

and defining:

$$\frac{\partial}{\partial T} = \int d^3x\, d^3y\, G_{abcd}(x,y) \frac{\delta S[g]}{\delta g_{ab}(x)} \frac{\delta}{\delta g_{cd}(y)}$$

we get: $i \frac{\partial}{\partial T} \psi[g,\varphi] = H_{\text{mat}}(g,\varphi)\psi[g,\varphi]$

Since $(dT)^{ab}(x) = \frac{1}{V[g]} \frac{\delta S[\phi]}{\delta g_{ab}(x)}$

and $J_{cd}(x) = \int d^3y\, G_{abcd}(x,y) \left(\frac{\delta S[g]}{\delta g_{ab}(x)} A^2[g] \right)$

infinitesimal flux through constant T screen $\mathbf{dT} \cdot \vec{J} \approx A^2$.

None of this can be derived if e.g.: $\Psi(g,\varphi) \approx$
$\exp(im_p^2 S_1[g]) A_1[g]\psi_1[g,\varphi] + \exp(im_p^2 S_2[g]) A_2[g]\psi_2[g,\varphi]$

Big problem for WdW! we are safe because of g_o.

Quick remark

Like Hartle-Hawking:

- Impose (there controversial) boundary conditions in superspace.
- Derives space-times from WKB approx IN minisuperspace.

Unlike Hartle-Hawking:

- There is a reduced config space outside of minisuperspace approx.
- Has natural boundary conditions on shape space, which naturally correspond to a most homogeneous state (which is also a corner of reduced configuration space).

Geometrical toy model ('free-particle in Riem')

For now only with $\mathcal{G}$ =Diff(M), but still $g_o = d\Omega^3$.

$$S[g] = \int dt \langle \dot{g}, \dot{g} \rangle_{g(t)}^{1/2} = \int dt \left(\int d^3x \, \dot{g}_{ab} \, g^{ac} \, g^{bd} \, \dot{g}_{cd} \sqrt{g} \right)^{1/2}$$

Horizontal lift given by orthogonality wrt fibers (not a gauge-section).[Vilkowisky '77, DeWitt '94, HG '10]

Geodesics explicit, in closed form.[Freed, Groisser '89; Michor, Medrano '91]

Super-Riemman curvature:

$$R(\mathbf{h}_1, \mathbf{h}_2)\mathbf{h}_3 = -\tfrac{1}{4}[[\mathbf{h}_1, \mathbf{h}_2], \mathbf{h}_3] + \tfrac{3}{16}\left(\mathrm{tr}_g(\mathbf{h}_1\mathbf{h}_3)\mathbf{h}_2 - \mathrm{tr}_g(\mathbf{h}_2\mathbf{h}_3)\mathbf{h}_1\right)$$

For $h_{ab}g^{ab} = 0$ (otherwise zero).

Geodesics don't reconverge $\Rightarrow$ no interference terms in semi-classical approximation.

For finite-dimensions and geodesic action, can calculate a geometrical Van Vleck:[Visser '93]

$$\Delta_\gamma^{\text{small d}}(x,y) = 1 + \frac{1}{6}(R_{ab}\dot{\gamma}^a\dot{\gamma}^b S[\gamma]^2 + \mathcal{O}(S[\gamma])^3)$$

Long story about how to translate this to infinite-d.[Michor et al '01, HG '16] Requires regularization.

For two different ranges of initial transverse traceless initial directions, H_1, H_2 at $d\Omega^3$, s.t. $\int_{H_1}\mathcal{D}h^1 = \int_{H_2}\mathcal{D}h^2$

$$\frac{\int_{H_1}\mathcal{D}h^1 J_{TT}(g^0)P(g(\epsilon))}{\int_{H_2}\mathcal{D}h^2 J_{TT}(g^0)P(g(\epsilon))} = 1+$$

$$\frac{1}{4}\frac{\int_{H_2}\mathcal{D}h^2\left(\int d^3x\,\sqrt{g^*}\,h_{ab}g_o^{ac}g_o^{bd}h_{cd}\right) - \int_{H_1}\mathcal{D}h^1\left(\int d^3x\,\sqrt{g_o}\,h_{ab}g_o^{ac}g_o^{bd}h_{cd}\right)}{V_H}\epsilon^2 + \ldots$$

Relative probability for two screens defined by same arc-length distance.

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Relative probability for two screens defined by same arc-length distance.

Using orthonormal eigenbasis of TT tensors on g_o,
$\prod_x dh_{ij}(x) \to \prod_{n=1}^{\infty} d\lambda_{ij}^n$. But then:
$\int_{H_1} \mathcal{D}h = \int_{H_2} \mathcal{D}h \Rightarrow \int_{H_2} \mathcal{D}h^2 \left(\int d^3x \sqrt{g^0}\, h_{Tab} g_0^{ac} g_0^{bd} h_{cd} \right) =$
$\int_{H_1} \mathcal{D}h^1 \left(\int d^3x \sqrt{g^0}\, h_{Tab} g_0^{ac} g_0^{bd} h_{cd} \right)$. I.e., the measure does not care about the eigenvalues of the TT-modes. But
$S[g] = \int dt \langle \dot{g}, \dot{g} \rangle_{g(t)}^{1/2} =$
$\int dt \left(\int d^3y\, f(R) \sqrt{g} \right) \left(\int d^3x\, \dot{g}_{ab}\, g^{ac}\, g^{bd}\, \dot{g}_{cd} \sqrt{g} \right)^{1/2}$
changes terms in VV, e.g.: $h_{Tab} g_0^{ac} g_0^{bd} h_{cd} \to h_{Tab} g_0^{ac} g_0^{bd} \nabla^{2n} h_{cd}$

In such case, higher eigenvalues of H_1 in comparison to H_2, *mean its relative probability flux is smaller.*

This would mean indeed that more homogeneous modes would be favored.[3]

[3]Eigenvalues of TT in S^3 are $\nabla^2 \tau_{ij}^{(n)} = -(n^2 - 3)\tau_{ij}^{(n)}$.

How did we fare?

Mix better:

- Quantum mechanical notions
  1. No collapse of the wavefunction (static prob density).
  2. True (fake) evolution in the semi-classical approx.
  3. Quantum superposition principle.
- Conformal geometrodynamics
  1. Refoliation not fundamental (recovered relationally).
  2. A Hamiltonian which separates local gauge symmetries and global evolution.
  3. Causal structures corresponding to different extremal curves (can interfere).

Challenge: recover standard GR.