Title: Quantum singularity resolution in homogeneous cosmology and the implications for shape dynamics

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Abstract: I will present results on the quantization of an FRLW model that utilises a Schrodinger-type evolution equation. In contrast to standard Wheeler--DeWitt-type quantisations, the quantum model resolves the classical singularity, exhibits a quantum bounce, and displays novel early-universe phenomenology. A global scale emerges because of a scale anomaly, and suggests an interesting scenario for quantum shape dynamics. I will give the details of the quantization procedure and show how these techniques can be used more generally for anisotropic models. I will end by speculating about how these techniques might be applicable to a genuine quantum shape model of the universe.

Quantum singularity resolution in homogeneous cosmology and the implications for shape dynamics

Sean Gryb with **Karim Thébault**

University of BRISTOL

Department of Philosophy

Shape Dynamics Workshop

Perimeter Institute

17 May 2017

Context

A biased, distorted, and selective history of shapes:

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- York's conformal method (72).

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- Problem: volume preserving condition $\Rightarrow$ a big mess!
- How to deal with the global mode? $\Rightarrow$ dynamical similarity?

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Dynamical Similarity

- Perhaps the missing insight?
- A 'new' kind of symmetry.
- Must consider families of constant energy hypersurfaces!

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In this talk

- I will consider a quantum cosmological model that allows for superpositions of energy eigenstates.

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- I will consider a quantum cosmological model that allows for superpositions of energy eigenstates.
- I will **not** quotient by dynamical similarity!

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Singularity resolution through dynamical quantum gravity

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What goes wrong?

$\Rightarrow$ We believe it is the treatment of time

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- Qualitative features carry over to Bianchi I and IX.
- Natural way to quotient by dynamical similarity at the quantum level.

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The classical model

A simple FRWL cosmology

Homogeneous and isotropic mini-superspace model with massless scalar field and vanishing spatial curvature.

Hamiltonian:

$$H = N\left[-\frac{\kappa}{12V_0 a}\pi_a^2 + \frac{1}{2V_0 a^3}\pi_\phi^2 + \frac{V_0 a^3}{\kappa}\Lambda\right]$$

Variables

- a - scale factor
- ϕ - scalar field

Parameters

- Λ - cosmological constant
- π_ϕ - momentum of scalar field

(V_0 - fiducial volume; κ - Newton constant)

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Geometry of configuration space

The Hamiltonian can be written as

$$H = N\left(g^{ab}p_a p_b - \frac{\Lambda}{2}\right) \quad (1)$$

$g^{ab} \Rightarrow$ inverse Minkowski metric on configuration space $\mathcal{C}$!

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Solutions $\Rightarrow$ geodesics on $(\mathcal{C}, g)$

Restriction

Geometrically, $a > 0$ puts us in Rindler spacetime.

Boundary $\Rightarrow$ geodesic incompleteness on $(\mathcal{C}, g)$ (classical singularity and quantum complications)

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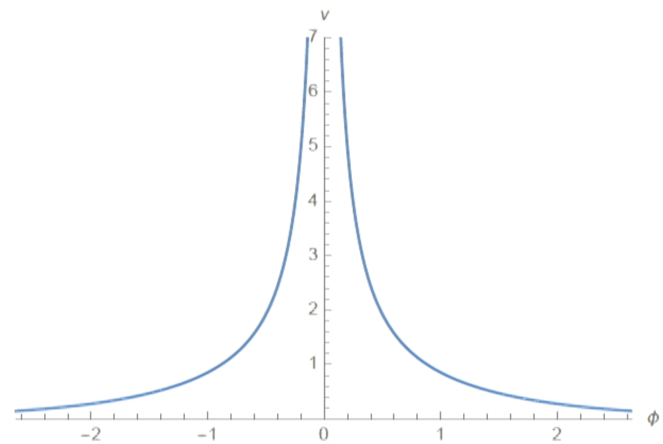
Solutions

Boost symmetry suggests Rindler coordinates: $v \propto a^3$, $\varphi \propto \phi$

$$v = v_0 + \sqrt{t - 1}$$

$$\phi = \phi_\infty + \text{arctanh}\left(t^{-1}\right)$$



Parameters

- v_0 - initial time
- φ_∞ - asymptotic φ
- Λ - total energy
- π_φ - conserved p

Symmetries

- Time translations
- Boost invariance
- Time units
- Space units

Symmetries $\Rightarrow$ no physically relevant parameters.!

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Quantization Preliminaries

Hamilton–Jacobi equation:

$$\mathcal{H}\left(q, \frac{\partial S}{\partial p}\right) = \frac{\Lambda}{2}$$

Quantization ambiguity

$$\hat{\mathcal{H}}\Psi = \frac{\Lambda}{2}\Psi \qquad \text{or} \qquad \hat{\mathcal{H}}\Psi = i\frac{\partial \Psi}{\partial \tau}$$

$\Rightarrow$ Timeless (Wheeler–DeWitt) vs genuine evolution (unimodular gravity-like)

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Relational quantization:

- Take approach with genuine evolution.
- τ - unobservable parameter ordering successive states.

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Failure of timeless (Wheeler–DeWitt) approach

Wheeler–DeWitt equation is timeless.

$\Rightarrow$ deparametrize wrt internal time. (e.g., φ)

Problem (example)

$< v >$ problematic when $\varphi \to \infty$ (takes definite values)

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Relation quantization $\Rightarrow$ $\hat{\varphi}$ does not take definite values.

$< v >, < \varphi >$ protected by uncertainty relations!

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Unitarity

Boundary of $\mathcal{C} \Rightarrow \hat{\mathcal{H}}$ not essentially self-adjoint!

$$< \Phi, \hat{\mathcal{H}}\Psi > = < \hat{\mathcal{H}}\Phi, \Psi > + \text{ boundary}$$

(Boundary term $\Rightarrow$ quantum imprint of classical singularity.)

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Self-adjoint Extensions

- Guaranteed by von Neumann. ($\hat{\mathcal{H}}$ - symmetric and real)
- Trick: the equation (boundary $= 0$) is conformally invariant.
- Use eigenfunctions on conformal completion $\tilde{g} = v^2 g$ (maps $\mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}^2$) to anchor solutions in Minkowski.
- Put phase between solutions to remove Λ-(scale) dependence.

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Relation quantization $\Rightarrow$ $\hat{\varphi}$ does not take definite values.

$< v >, < \varphi >$ protected by uncertainty relations!

We will use genuine evolution: $\hat{\mathcal{H}}\Psi = \Box\Psi = i\frac{\partial\Psi}{\partial\tau}$.

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'Bound' states ($\Lambda < 0$)

Spectrum of Klein–Gordan operator on Rindler for $\Lambda < 0$:

- AdS-like states exit.
- Spectrum is discrete but unbounded: (k - momentum of φ)

$$\Lambda_n = \Lambda_{\text{ref}} e^{2\pi n/k} \quad (n \in \mathbb{Z})$$

- Conformal tower of states with accumulation point at $\Lambda = 0$.
- Analogues of atomic systems: *Efimov effect*.
- Eigenstates behave like $\psi_\Lambda \propto K_{ik}(\sqrt{\Lambda} v) \sim e^{-v}$.

$\Rightarrow$ no late-time semi-classical limit!

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'Unbound' states ($\Lambda > 0$)

- dS-like states exist and persist to late-time, semi-classical limit.
- Continuous spectrum given by phase-shifted Bessel functions:

$$\psi_\Lambda \sim J_{ik}(\sqrt{\Lambda}v) + \tan\theta I_{ik}(\sqrt{\Lambda}v)$$

  $\theta \propto k \log \Lambda/\Lambda_{\text{ref}}$ ($U(1)$ s.a. ext. parameter).
- Gaussian states lead to semi-classical physics.
- New dimensionful parameter $\rightarrow \Lambda_{\text{ref}}$ gives meaning to size of quantum effects ($\hbar$).
- Self-adjoint extension parameter: phase between in- and out-state.

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Analogue model

- System is mathematically equivalent to N-particle system with $1/r^2$ potential (in a particular regime).
- Regime in question is a good effective model for some 3-body atomic systems.
- Known as *Efimov effect* $\Rightarrow$ vast literature.
- Unbound states can be scattered off bound states.
- Self-adjoint extension parameter gives scattering length via phase shift, θ.
- θ is macro parameter encoding micro-physics.

How seriously can we take the analogue model or simulator of the early universe?

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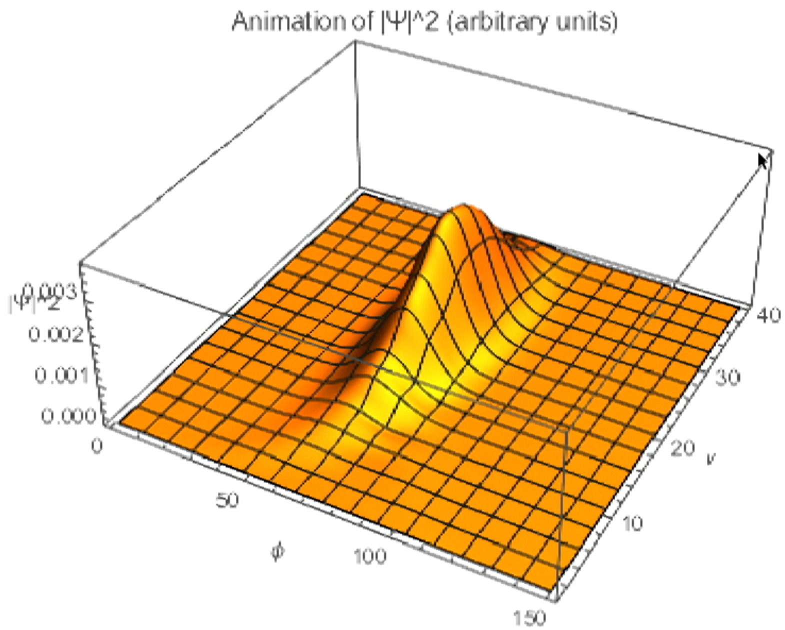
Bounce Solution



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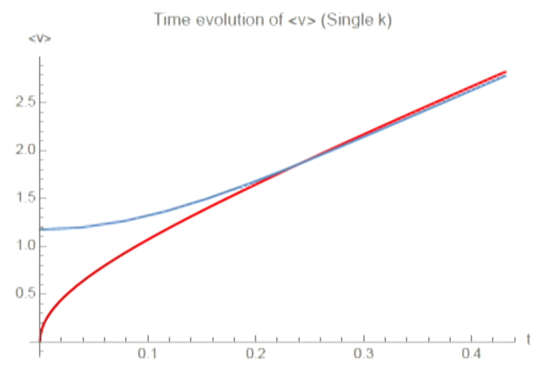
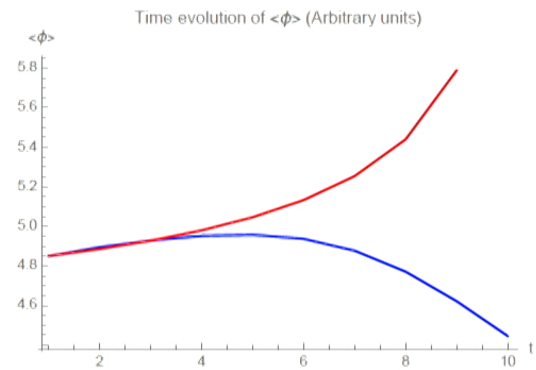
Expectation values





$< v >$ has a min when $v_{\text{class}} = 0$.

$< \phi >$ is finite when $\phi_{\text{class}} \to \infty$.

Singularity resolved and classical limit recovered!

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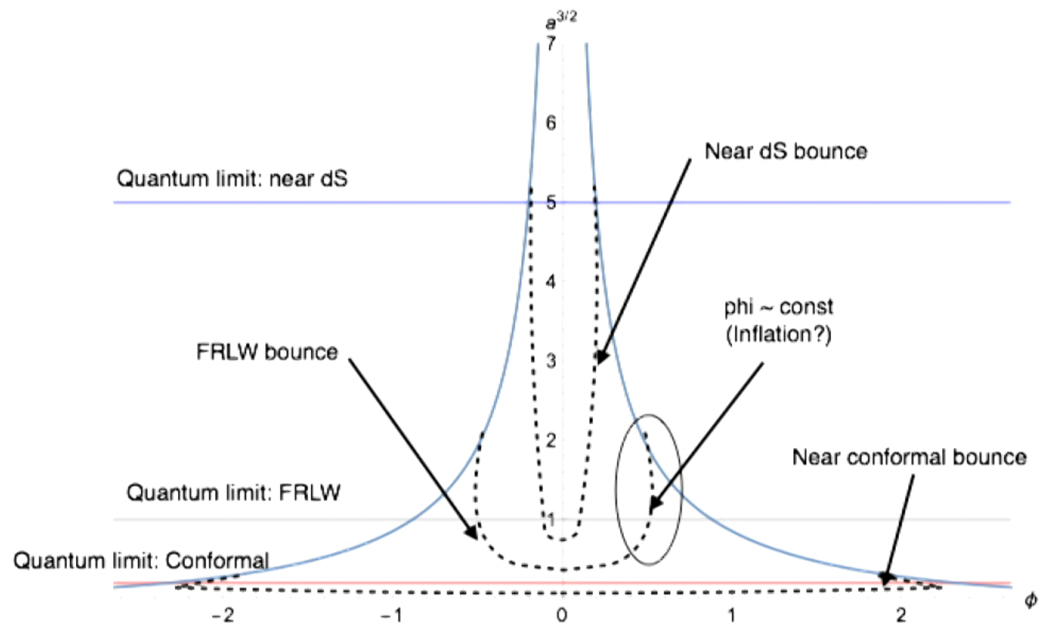
General Classical/Quantum Regimes (Cartoon)



New parameter, Λ_{ref}, gives meaning to $\hbar$ and relative size of quantum effects.

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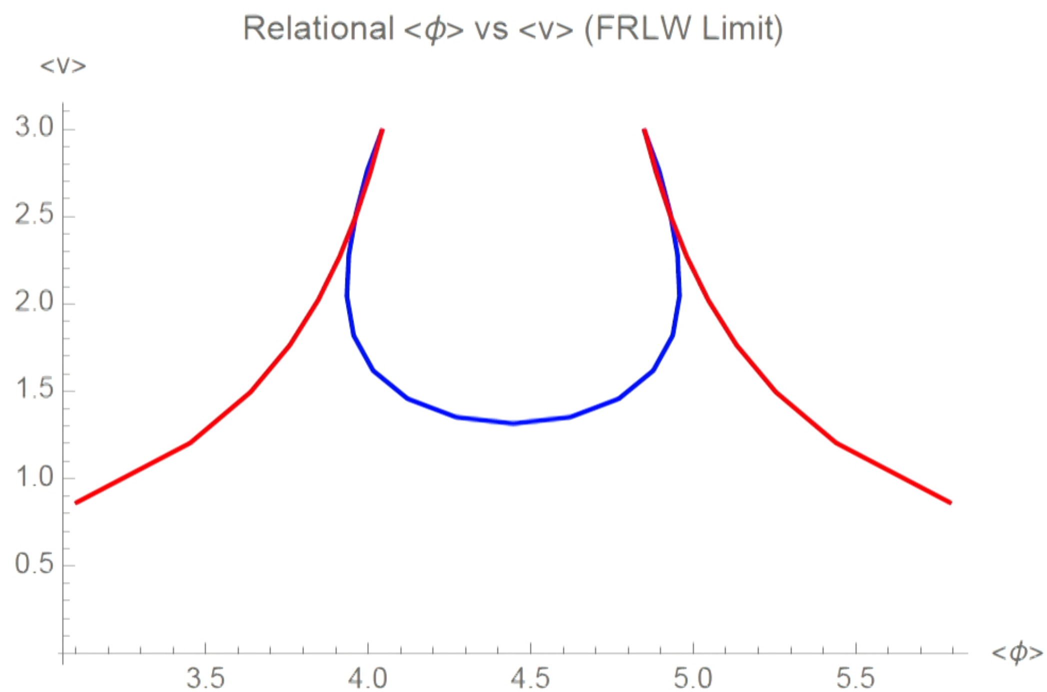
General classical/quantum regimes (simulation)



- Simple example in $\hbar \sim 1$ regime.
- Note inflationary period where $\frac{\partial \langle\phi\rangle}{\partial \langle v\rangle} \approx 0$.

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Features/Summary

We have given a quantization of a simple FRLW cosmology that

- Resolves singularity (improvement over WDW).
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We have given a quantization of a simple FRLW cosmology that

- Resolves singularity (improvement over WDW).
- Recovers semi-classical dS limit.
- Suggest novel phenomenology in quantum regime.
- Provides unique macro-parameter, Λ_{ref}, encoding micro-physics.
- New platform for condensed matter quantum simulation of the early universe.

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Generalizations

- Bianchi I: v wavefunction is identical.
- Bianchi IX: Bessel functions $\Rightarrow$ non-analytic with same asymptotic properties.
- BKL $\Rightarrow$ more general mechanism for singularity resolution via quantum effects.

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- Dynamical similarity quotient $\Rightarrow$ integrate over all Λ_{ref}??

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Generalizations

- Bianchi I: v wavefunction is identical.
- Bianchi IX: Bessel functions $\Rightarrow$ non-analytic with same asymptotic properties.
- BKL $\Rightarrow$ more general mechanism for singularity resolution via quantum effects.
- Dynamical similarity quotient $\Rightarrow$ integrate over all Λ_{ref}??
- Conformal anomaly and holographic renormalization.
- Absorption/Decay: AdS-'bound' state $\Rightarrow$ dS-'emission' (need non-linear corrections).
- Self-adjoint extension parameter $\Rightarrow$ UV-completion required (conformal?).

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