Title: Through the Big Bang

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Abstract: I will show how the intrinsic definition of observables in relativity through dynamical similiarity (known as Shape Dynamics) leads to the continuation of Einstein's equations classically through the big bang singularity in simple cosmological scenarios. By appealing to general principles I argue that this is a generic feature, and that the singularity can be viewed as an artifact of the redundant description imposed by absolute length scales. I will then lay out some other welcome features of intrinsic relational systems, and discuss the broader questions raised by a theory of physics that is independent of physical dimensions such as mass and length.

Through the Big Bang

(and other adventures in dynamical similiarity)

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Lots of underlying work Henrique Gomes and Sean Gryb
Massive support and initiation from Julian Barbour

Introduction

We will examine homogeneous, anisotropic cosmology in terms of its *shape*.

By removing scale we see a dynamical similarity of the system.

Full dynamics is GR where GR solutions exist.

Shape equations of motion form autonomous system.

Shape Dynamics gives a continuation through the Big Bang.

Continuation is: *unique, deterministic, classical*

Overarching goal: A fully intrinsic, relational description of physics.

Something we all know...

Friedmann's Equation:

$$H^2 = \frac{8\pi}{3}\left(\Lambda + \frac{\Omega_k}{a^2} + \frac{\Omega_d}{a^3} + \frac{\Omega_r}{a^4} + \ldots + \frac{\Omega_w}{a^{3(1+w)}}\right)$$

Dynamical Similarity - system invariant under:

$$\begin{aligned} a &\rightarrow \alpha a \\ \Lambda &\rightarrow \Lambda \\ \Omega_k &\rightarrow \alpha^2 \Omega_k \\ \Omega_d &\rightarrow \alpha^3 \Omega_d \\ \Omega_r &\rightarrow \alpha^4 \Omega_r \\ \Omega_w &\rightarrow \alpha^{3(1+w)} \Omega_w \end{aligned}$$

Often see α written as a_0^{-1}. Scale factor contains an $\mathcal{R}_+$ freedom.

Back in the Elevator, Albert!

How do we know gravity acts? Consider two bodies in a free-falling box

Standard Equivalence: See local gravity through relative motion.

But how do they measure how far apart they are?

Need a third body - measure relative change: Gravity acts aniostropically.

General Principle: Only intrinsic observations exist.

Leads to Dynamical Similarity

The Intrinsic Picture

What do the terms in a Lagrangian represent?
Usual Interpretation:

- $q, \dot{q}$ as measured by lab equipment/frame.
- Problem: Can't describe the equipment.
- So it can not be a theory of 'everything'.

'Intrinsic' Interpretation

- Pick a rod means: Fix $F(\vec{q}) = 1$ to give scale.
- $q, \dot{q}$ over-describe reality
- Shape space mods dynamical similarity $\sim \mathcal{R}^n/\mathcal{R}_+$
- Compact: Finite measures, flow theorems, etc

Claim: This is how we really do physics.

Dynamical Similiarity

All quantities defined intrinsically

Couplings are inferred observationally! - E.g. as terms in a Taylor series.

Example: Central Motion

$$\mathcal{L} = \frac{\dot{r}^2 + r^2\dot{\theta}^2}{2} - \frac{C_1}{r} - \frac{C_2}{r^2} - ...$$

$$\frac{r''}{r} - \frac{2r'^2}{r^2} = C_1 r + (C_2 - J) + ...$$

Intrinsic observations invariant under $r \to \lambda r,\ C_1 \to \frac{C_1}{\lambda}...$
Recall: Use observations to set the C_i

Example: Dependence of Chemistry on α

$$\frac{\hbar^2}{2m}\nabla^2\psi(\overrightarrow{r}) = (\beta V(\overrightarrow{r}) - E)\psi(\overrightarrow{r})$$

Coulumb-like potentials (homogeneous for now - talk to Yuri!) $V = |\overrightarrow{r}_1 - \overrightarrow{r}_2|^n$

1D Warmup:

$$\frac{d^2 f(z)}{dz^2} = \beta z^n f(z) + \epsilon f(z)$$

Define $g(x) = f(\lambda x) : z = \lambda x$ hence

$$\frac{d^2 g(x)}{dx^2} = \beta \lambda^{n+2} x^n g(x) + \lambda^2 \epsilon g(x)$$

$$\beta \to \beta' : \lambda = \left(\frac{\beta'}{\beta}\right)^{\frac{1}{n+2}}$$

$$\frac{d^2 g(x)}{dx^2} = \beta' x^n g(x) + \epsilon' g(x) \quad \epsilon' = \left(\frac{\beta'}{\beta}\right)^{\frac{2}{n+2}} \epsilon.$$

Example: Dependence of Chemistry on α

3D Version:

$$\nabla_z^2 F(\overrightarrow{z}) = \beta V(\overrightarrow{z}) F(\overrightarrow{z}) + \epsilon F(\overrightarrow{z})$$

$$\nabla_x = \frac{1}{\lambda^2}\nabla_z : G(\overrightarrow{x}) = F(\lambda\overrightarrow{x})$$

$$\begin{aligned} \frac{1}{\lambda^2}\nabla_x^2 F(\lambda\overrightarrow{x}) &= \beta V(\lambda\overrightarrow{x}) F(\lambda\overrightarrow{x}) + \epsilon F(\lambda\overrightarrow{x}) \\ &= \beta\lambda^n V(\overrightarrow{x}) F(\lambda\overrightarrow{x}) + \epsilon F(\lambda\overrightarrow{x}) \end{aligned}$$

$$\nabla_x^2 G(\overrightarrow{x}) = \beta\lambda^{n+2} V(\overrightarrow{x}) G(\overrightarrow{x}) + \lambda^2 \epsilon G(\overrightarrow{x})$$

Changing the coupling changes the eigenvalue but not the eigenvector
Scales change, shapes don't.
Dynamical Similarity: An observer in a single force universe wouldn't see this change.
Multiple Forces: 1 parameter family of couplings that are indistinguishable

Rods and Clocks

Personal Aesthetics:

- Intrinsic 'rods and clocks' construction of cosmology.
- Cosmology is dynamics of everything: Rods and clocks are inside.
  - Kilogram des Archives in Paris
  - Metre defined in terms of krypton wavelength
  - Second determined by oscillations of caesium atom
- We really want to describe some observables in terms of others.
- What we know are numbers - ratios of dimensionful quantities.

Shape Dynamics can play two roles here:

- Fundamental Theory
- Way to see effects of dimensionful units.

Goal: $s_1(s_2)$ or $s_1(u), s_2(u)$.

What a singularity is (and what it isn't)

Singularity: Point beyond which we cannot generically extend geodesics.
A singularity is NOT (just)

- Infinite Energy Density (Vacuum collapse)
- Infinite Observable

These can serve to identify where a singularity might be. Typical classification:
Krolak:

$$\int_0^{\tau} R_{xx} d\tau \to \infty$$

Tipler:

$$\int_0^{\tau} \int_0^{\tau'} R_{xx} d\tau' d\tau \to \infty$$

Physically: Strong singularity cannot be passed by any detector.

Toy Model

Toy Model: Particle in expanding 2D square well.

- Absolute space (Newtonian) description: $x, y, \dot{x}, \dot{y}$.
- Walls expand from center at $x_w = \dot{x_w} t$.
- On collision, $\dot{x} - > -\dot{x} + 2\dot{x_w}$.
- Eventually $|\dot{x}|$ and $|\dot{y}| < \dot{x_w}$. No further reflections.
- Path consists of straight line segments at constant velocity.
- Particle 'ends' at $|x|, |y| \to \infty$.

We want to construct the shape description of this system.
We consider the particle and walls to be everything.
In particular, no external rod or clock.

Shape of Toy Model

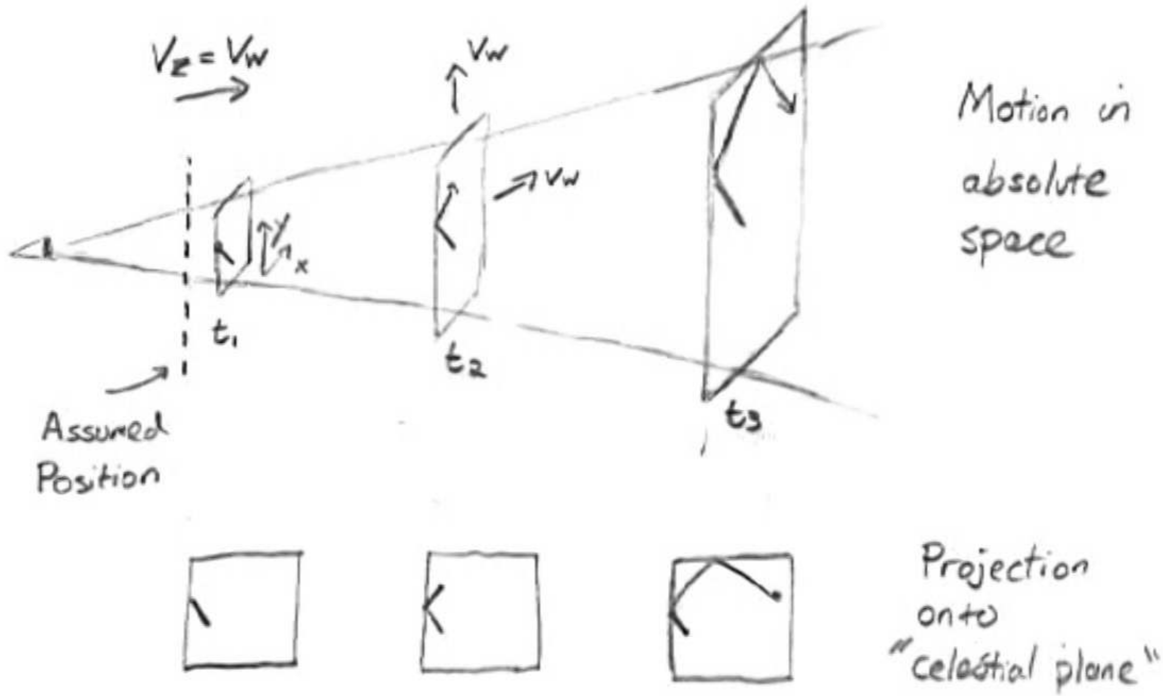
- 'Everything' consists of Particle and walls.
- Only Rod: Fraction of distance between walls.
- No clock! We do not know if particle is slowing down.

Idea: Change point of view so that the walls remain stationary.

If the walls appear not to move, our rod is time invariant.

- Consider an observer moving perpendicularly away from plane
- Let $v_{obs} = \dot{x_w}$
- Project observations onto a 'celestial plane' at a fixed distance

Rescaled Expanding Walls Toy Model



Shape Interpretation of Toy Model

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Our observer sees the walls fixed, and the particle appears to slow.

$$\bar{x'} = \frac{dx}{dt} = \frac{z_o x - x_o \dot{x_w}}{(z_o + \dot{x_w} t)^2} \quad \bar{y'} = \frac{dy'}{dt} = \frac{z_o y - x_o \dot{x_w}}{(z_o + \dot{x_w} t)^2}$$

In particular, note that $\frac{dx'}{dy'}$ is constant - the particle still moves in straight lines.

As $t \to \infty, x', y' \to \frac{\dot{x}}{\dot{x_w}}, \frac{\dot{y}}{\dot{x_w}}$. System determined by point+direction.

But without access to a clock, we do not know the particle is slowing.

So the shape interpretation continues the dynamics through $t \to \infty$

Particle keeps moving in a straight line. Joins with second solution, opposite direction.

Interpretation

Arrow of time: On hitting wall, angle incidence < angle reflection.

Angle opens up with time. What happens after $t = \infty$?

Opposite solution has angle incidence > angle reflection.

Shape Dynamics and Newtonian Mechanics know the same system.

Can interpret the result two ways:

- Newtonian Mechanics Fundamental
  - SD lets us count solutions, new principle at ∞ (ekpyrosis)
- Shape Dynamics Fundamental
  - NM arises as effective description of SD, bad 'coordinate' at ∞.


Simplest Cosmological System
We want to examine the simplest cosmological models first.
Homogeneous, Anisotropic cosmology: Bianchi models
Generic (BKL Conjecture)
Simplest systems with shape
Isotropic cosmology has no shape evolution: Akin to equilateral triangle, when scale is removed, nothing changes. Cannot distinguish inertial motion from any forces.
Here we present Bianchi IX, but results generalize.
(Classification based on structure of symmetry group)

Simplest Cosmological System

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- Generic (BKL Conjecture)
- Simplest systems with shape

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(Classification based on structure of symmetry group)

Dynamics

$$ds^2 = -dt^2 + v^{2/3}\left(\exp[-2b]\sigma_1^2 + \exp[b + \frac{a}{\sqrt{3}}]\sigma_2^2 + \exp[b - \frac{a}{\sqrt{3}}]\sigma_3^2\right)$$

a and b are shape and v is scale.
Dynamics of our system come from the Lagrangian:

$$\mathcal{L} = v\left(-\frac{\dot{v}^2}{v^2} + \frac{\dot{a}^2}{2} + \frac{\dot{b}^2}{2} + \frac{\dot{\phi}^2}{2} + \mathcal{R}(v, a, b)\right)$$

We want to describe the *shape* of our system: a and b.

Anisotropies behave like scalar fields on potential given by Ricci scalar $\mathcal{R}$.

Complete behaviour complicated. Infinite number of Taub transitions.

'Cosmological billiards' (Damour) - similar to reflections in toy model.

The Shape Potential

The Ricci scalar splits into scale and shape terms:

$$\mathcal{R} = v^{4/3} V_s(a, b)$$

Where we call V_s the 'Shape Potential'

$$V_s = F(2b) + F(\sqrt{3}a - b) + F(-\sqrt{3}a + b)$$

$$F(x) = \exp(-\frac{x}{\sqrt{6}}) - \frac{1}{2}\exp(\frac{2x}{\sqrt{6}})$$

For large x the second term will dominate.

So, at large radius the shape potential approaches

$$V_s \to \exp(\frac{4b}{\sqrt{6}}) + \exp(\frac{\sqrt{3}a - b}{\sqrt{6}}) + \exp(-\frac{\sqrt{3}a + b}{\sqrt{6}})$$

Asymptotic Behaviour

We find the Hamiltonian in shape variables:

$$\mathcal{H} = v^2\tau^2 - \frac{{k_a}^2}{2} + \frac{{k_b}^2}{2} + \frac{\pi_\phi^2}{2} + v^{4/3}V_s[a, b]$$

We will evolve in volume until the singularity ($v \to 0$).
Quiescence: Eventually k_a and k_b become less than $\frac{4}{3}$ - no more reflections.
$v^{4/3}$ term dominates shape potential.

$$\begin{aligned} a &= a_o + k_a \log(v) + \ldots \\ b &= b_o + k_b \log(v) + \ldots \end{aligned}$$

As we go to the singularity, $v^{4/3}V_s \to 0$, but $a, b \to \infty$.

Zooming Out of the Shape Potential

- Want to use intuition gained from toy model.
- But shape potential isn't simple like walls.
- Idea: Pick a point on potential to have constant 'size'.
- Move plane of motion from observer to fix magnitude.

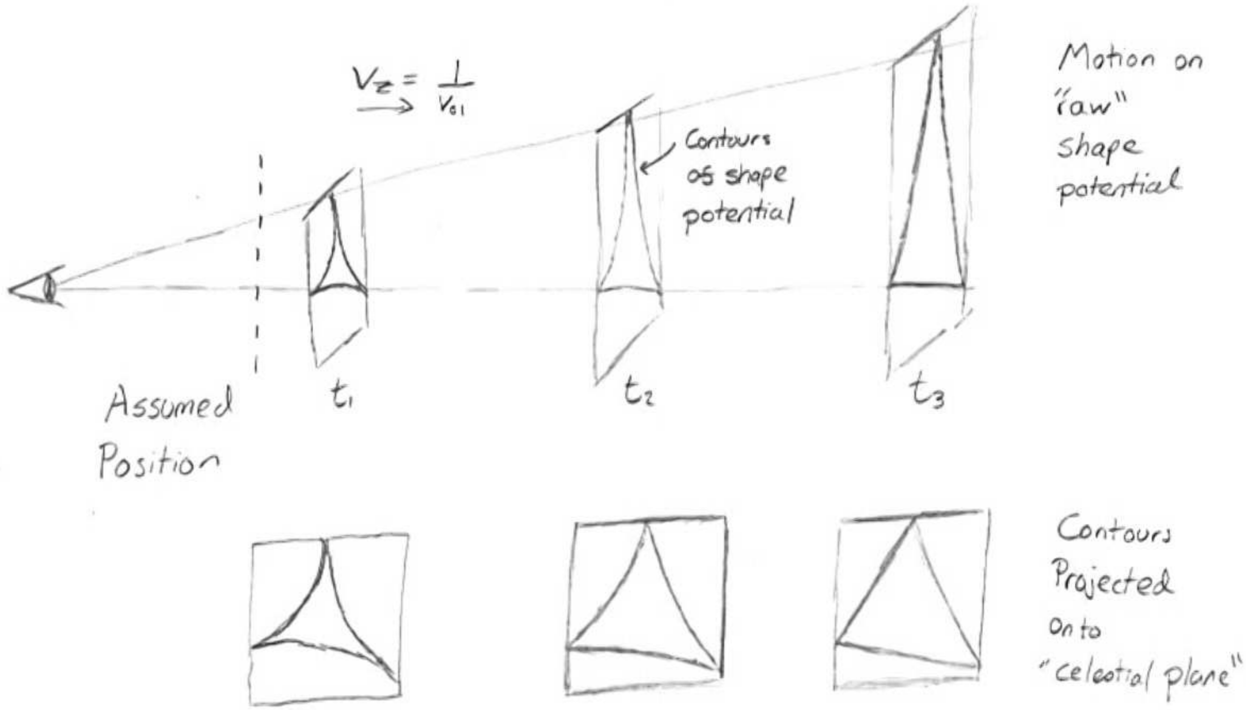
$$0 = \frac{dPE(\theta)}{dv} = \frac{\partial PE(\theta)}{\partial v} + \frac{\partial PE(\theta)}{\partial x}\frac{dx}{dv}$$

Hence

$$\frac{dz}{dv} = -\frac{4V_s[z]}{3vV_s'[z]}$$

Only need this to work asymptotically - can set $\frac{dz}{dv} = \frac{4}{3v\sqrt{2}}$
Shape potential becomes triangular well.

Illustration of Rescaled Shape Potential



Through the Singularity

At GR's singularity, our system approaches:

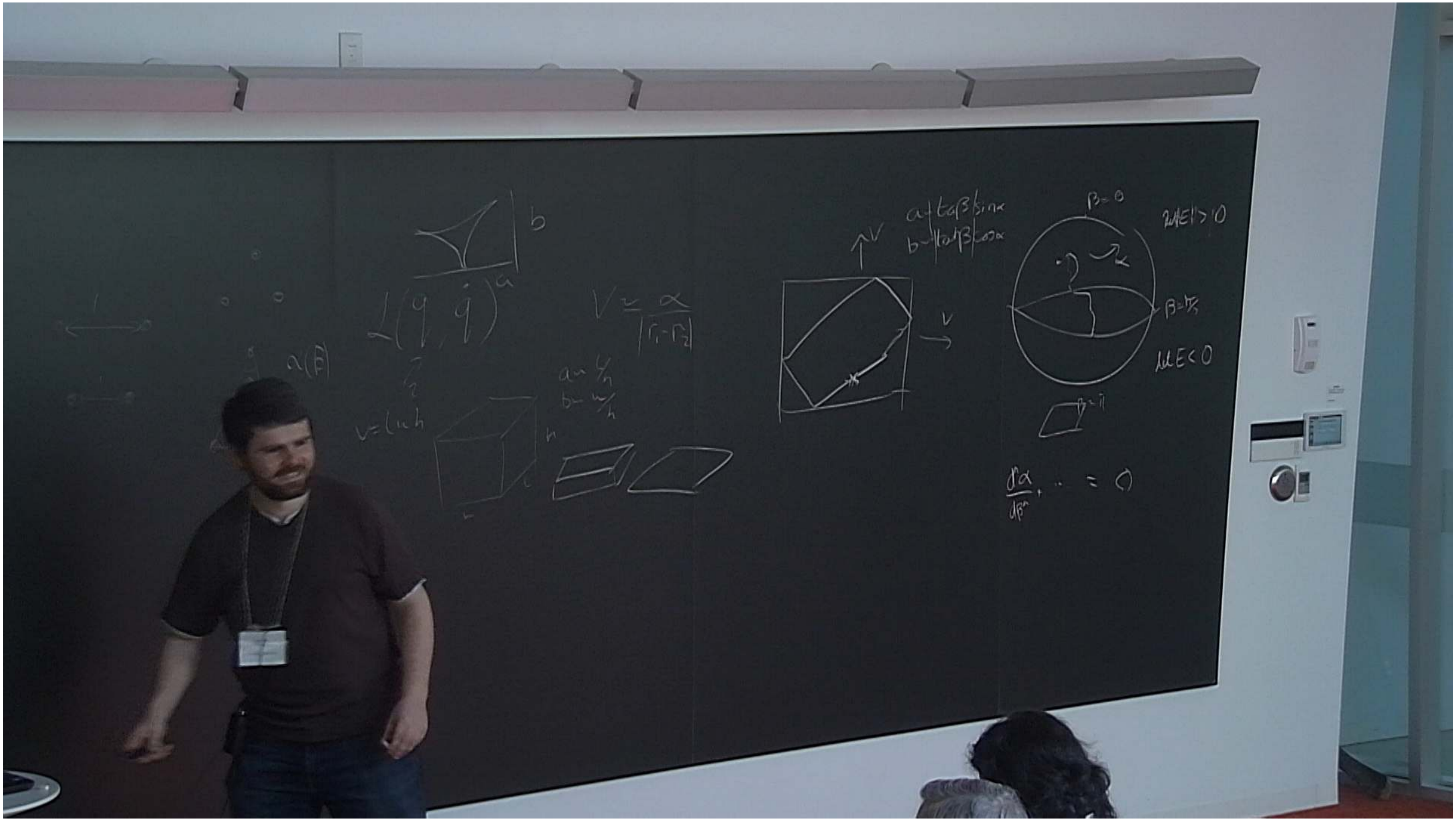
$$\{a', b'\} = \{k_a, k_b\}$$

Since the rescaled shape potential becomes zero, motion becomes linear.
To continue through the singularity, continue the linear motion!
Solution connects to a second solution with opposite time arrow.

From shape perspective, we conserve $k_a, k_b, ak_b - bk_a, \pi_\phi$.

Again two interpretations:

- General Relativity Fundamental
  - SD lets us count solutions, new principle at singularity (ekpyrosis)
- Shape Dynamics Fundamental
  - GR arises as effective description of SD, bad 'coordinate' at 'Janus point'.



More Precisely

Equations of motion are:

$$\dot\alpha = 2\,p\,\gamma\,\cot^2\beta, \qquad \dot\beta = 2\,p\,\cos^2\beta\,\sqrt{1-\frac{\gamma^2}{\tan^2\beta}},$$

$$\dot\gamma = p\,\sigma^2\sqrt{1-\frac{\gamma^2}{\tan^2\beta}}\left(\sqrt{1-\frac{\gamma^2}{\tan^2\beta}}\frac{\partial C}{\partial\alpha}-\gamma\cos^2\beta\frac{\partial C}{\partial\beta}\right)e^{\frac{\sigma}{2}\left(\omega-\sqrt{\tan^2\beta-\gamma^2}\right)},$$

$$\begin{aligned}\dot\omega = &\left\{\frac{4}{3}s\,p\left[\sigma\left(\sqrt{\tan^2\beta-\gamma^2}-\omega\right)-4\right]C(\alpha,\beta)\right.\\ &+p\,\sigma^2\,|\cos\beta|^3\left[(2\gamma^2-\tan^2\beta)+\omega\sqrt{\tan^2\beta-\gamma^2}\right]\frac{\partial C}{\partial\beta}\\ &\left.+\frac{p\,\gamma\,\sigma^2}{\tan^2\beta}\left(2\sqrt{\tan^2\beta-\gamma^2}-\omega\right)\frac{\partial C}{\partial\alpha}\right\}e^{\frac{\sigma}{2}\left(\omega-\sqrt{\tan^2\beta-\gamma^2}\right)},\end{aligned} \qquad (406$$

$$\begin{aligned}\dot\sigma = p\,\sigma^2&\left[-\frac{\sigma}{\tan\beta}\left(\frac{\gamma}{\tan\beta}\,\frac{\partial C}{\partial\alpha}+s\,\cos^2\beta\sqrt{\tan^2\beta-\gamma^2}\frac{\partial C}{\partial\beta}\right)\right.\\ &\left.\tfrac{4}{3}s\,C(\alpha,\beta)\right]e^{\frac{\sigma}{2}\left(\omega-\sqrt{\tan^2\beta-\gamma^2}\right)},\end{aligned}$$

$$\dot p = \frac{p^2}{\tan^2\beta}\left(\gamma\,\frac{\partial C}{\partial\alpha}+s\,\sin(2\beta)\sqrt{\tan^2\beta-\gamma^2}\frac{\partial C}{\partial\beta}\right)e^{\frac{\sigma}{2}\left(\omega-\sqrt{\tan^2\beta-\gamma^2}\right)}.$$

Shape variables autonomous: $\frac{dq_1}{dq_2}$ independent of scale (p).

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Equations of motion asymptote to:

$$\alpha(\beta) \xrightarrow[\beta\to\frac{\pi}{2}^-]{} \varphi_0 \xleftarrow[\beta\to\frac{\pi}{2}^+]{} \alpha(\beta)\,,$$

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$$\frac{\mathrm{d}^4\alpha(\beta)}{d\beta^4} \xrightarrow[\beta\to\frac{\pi}{2}^-]{} 0 \xleftarrow[\beta\to\frac{\pi}{2}^+]{} \frac{\mathrm{d}^4\alpha(\beta)}{d\beta^4}\,.$$

Can express evolution as $\alpha^{(4)} = F(\alpha^{(3)}, \alpha'', \alpha', \alpha, \beta)$.

What happened?

We continued solutions *through* the big bang.

- Singularity not avoided
- Classical dynamics at all times
- GR emerges on both sides

So, what happened to the singularity theorems?

Hawking-Penrose in a nutshell:
Trapped null surfaces + Energy Conditions = Incomplete Geodesics.
(See also Geroch - null congruences of Raychaudhuri eqn.)

All geodesics become orthogonal to a constant extrinsic curvature surface.

Meet with geodesics of spacetime on other side of Janus point.

Discrete Symmetries

Let's consider CPT symmetries.

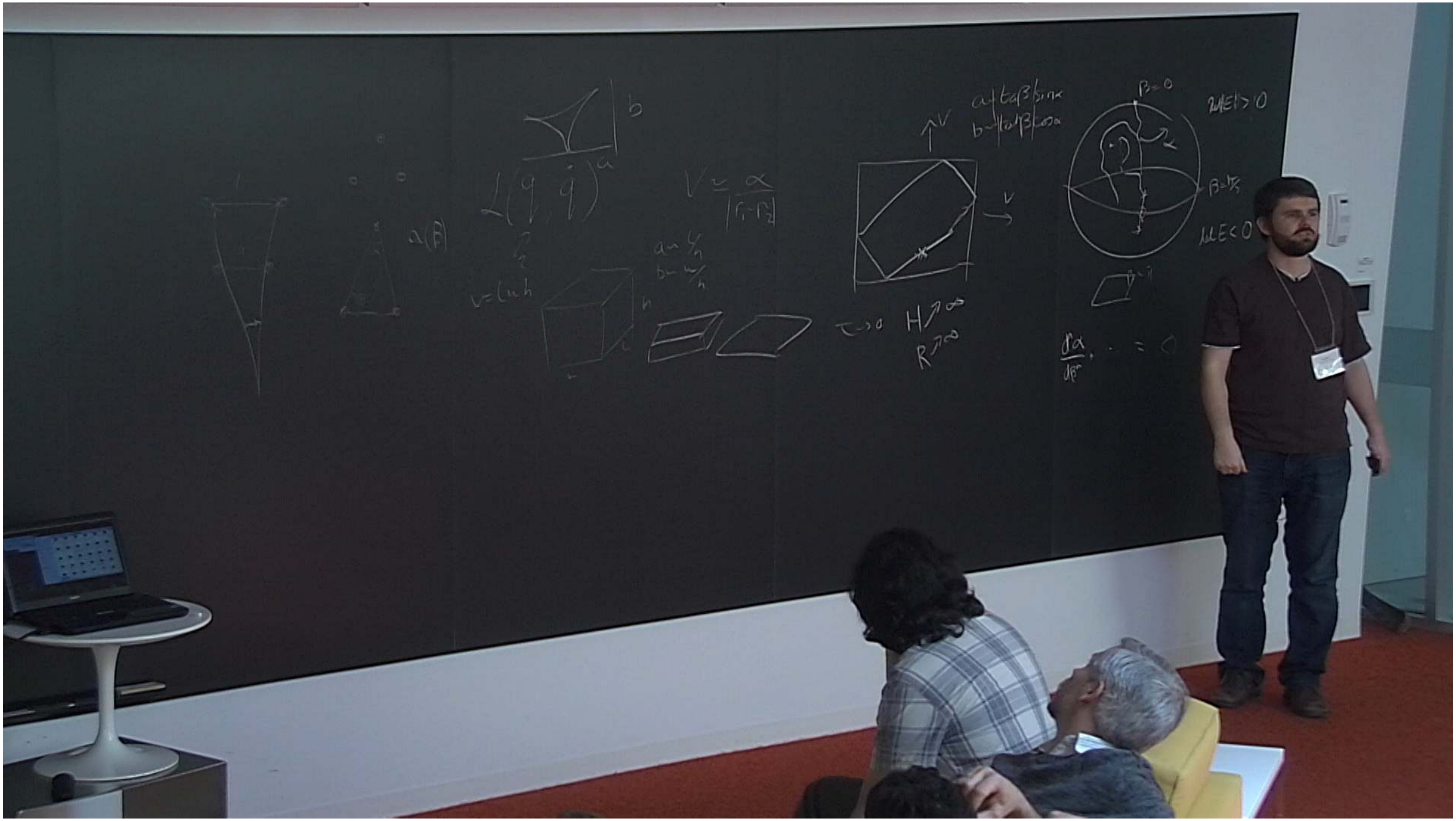
We split a shape solution into two at the Janus point (or joint two at singularity).

'Arrow of time' (Barbour, Koslowski, Mercati) points in opposite directions.

What about other discrete transformations?

Consider evolutions of oriented triad (frame field) in space. At singularity each 'length' flips sign: Parity changes at the Janus point. (Equivalent: signed volume crosses zero).

What about charge? Naive observation: A^{μ} is dual to a one-form - flips.



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What happened to the Friedmann Equation?

OK, this is all very nice, but what about usual cosmology?

It's still there! $\frac{dp}{d\phi} = p$

Reproduces the standard FLRW cosmology with scalar field for isotropy ($\alpha = \beta = 0$) once you pick an a_o away from singularity.

$$H = \frac{\Omega_\phi}{a^6}$$

It's just not observable intrinsically.
Only see motion in FLRW with anisotropy/inhomogeneity/multiple fields.
This equation alone does not work at singularity: scale is meaningless there.

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