

Title: Shape dynamics in terms of connection variables

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Abstract:

# Shape dynamics with connection variables

Lee Smolin

May 2017

[arXiv:1407.2909](https://arxiv.org/abs/1407.2909)

Thanks to Henrique, Tim

Begin with the Plebanski action:

$$S = \int_{\mathcal{M}} B^i \wedge F_i - \frac{1}{2} \phi_{ij} B^i \wedge B^j \quad \phi_{ii} = -3\Lambda$$

Introduce a scalar field and rescale:

$$B^i \rightarrow e^\psi B^i, \quad A^i \rightarrow e^{-\psi} A^i, \quad \phi_{ij} \rightarrow \phi_{ij}$$

The action becomes:

$$S = \int_{\mathcal{M}} B^i \wedge f_i - \frac{e^{2\psi}}{2} \phi_{ij} B^i \wedge B^j$$
$$f^i = dA^i + \frac{e^{-\psi}}{2} \epsilon^{ijk} A_j \wedge A_k - A^i \wedge d\psi$$

$$S = \int_{\mathcal{M}} B^i \wedge f_i - \frac{e^{2\psi}}{2} \phi_{ij} B^i \wedge B^j$$

$$f^i = dA^i + \frac{e^{-\psi}}{2} \epsilon^{ijk} A_j \wedge A_k - A^i \wedge d\psi$$

**Equations of motion:**

$$\frac{\delta S}{\delta B^i} : \quad f^i = e^{2\psi} \phi^{ij} B_j$$

$$\frac{\delta S}{\delta A^i} : \quad \mathcal{D} \wedge B^i = dB^i - e^{-\psi} \epsilon^{ijk} A_j \wedge B_k - B^i \wedge d\psi = 0$$

$$\frac{\delta S}{\delta \phi^{ij}} : \quad B^i \wedge B^j - \frac{1}{3} \delta^{ij} B^k \wedge B_k = 0$$

$$\frac{\delta S}{\delta \psi} : \quad e^{2\psi} = - \frac{d(B^i \wedge A_i) - \frac{1}{2} e^{-\psi} \epsilon_{ijk} B^i \wedge A^j \wedge A^k}{\phi_{ij} B^i \wedge B^j}$$

**The canonical theory:**

**Canonical momenta:**

$$\pi^{ai} = \epsilon^{abc} B_{bc}^i, \quad \pi_\phi = 0 \quad \pi_\psi = -\pi_i^a A_a^i$$

This gives rise to a new constraint:

$$\mathcal{S} = \pi_\psi + \pi_i^a A_a^i = 0 \quad \delta_\rho \Phi = \{\Phi, \mathcal{S}(\rho)\}$$

$\mathbf{S}$  generates Weyl transformations:

$$\delta_\rho A_a^i = \rho A_a^i, \quad \delta_\rho \pi_i^a = -\rho \pi_i^a, \quad \delta_\rho \psi = \rho, \quad \delta_\rho \pi_\psi = 0$$

Write the action in terms of constraints:

$$S = \int dt \int_\Sigma \left( \pi_i^a \dot{A}_a^i + \pi_\psi \dot{\psi} - A_0^i \mathcal{G}^i - B_{0a}^i \mathcal{J}_i^a - \rho \mathcal{S} \right)$$

Gauss law:  $\mathcal{G}^i = -\partial_a \pi_i^a + e^{-\psi} \epsilon^{ijk} A_{aj} \pi_k^a - \pi^{ai} \partial_a \psi = 0$

$$= -\mathcal{D}_a \pi_i^a$$

**Second class constraints:**  $\mathcal{J}_i^a = \epsilon^{abc} (f_{bc}^i - e^{2\psi} \phi^{ij} B_{bcj}) = 0$

Solve these to eliminate the B's, find first class constraints:

**Hamiltonian constraint:**  $\mathcal{H} = \epsilon^{ijk} (\pi_i^a \pi_j^b f_{abk} - 3\Lambda \epsilon_{abc} \pi_i^a \pi_j^b \pi_k^c) = 0$

**vector constraint:**  $\mathcal{V}_a = \pi_i^b f_{ab}^i$

**Gauss law:**  $\begin{aligned} \mathcal{G}^i &= -\partial_a \pi_i^a + e^{-\psi} \epsilon^{ijk} A_{aj} \pi_k^a - \pi^{ai} \partial_a \psi = 0 \\ &= -\mathcal{D}_a \pi_i^a \end{aligned}$

A combination of V and G generates spatial diffeos:

$$\mathcal{D}_a = \mathcal{V}_a - A_a^i \mathcal{G}_i$$

$$\mathcal{D}(v) = \int_{\Sigma} v^a \mathcal{D}_a = \int_{\Sigma} (\pi_i^a \mathcal{L}_v A_a^i + \pi_{\psi} \mathcal{L}_v \psi)$$

## Understanding the new Gauss law constraint.

It generates a modified gauge transformation:

$$\delta_\lambda A_a^i = \{A_i^a, \mathcal{G}(\rho)\} = -\partial_a \lambda^i + e^{-\psi} \epsilon^{ijk} A_{aj} \lambda_k + \lambda_i \partial_a \psi$$

Let's undo the shift

$$\mathcal{A}_a^i = e^{-\psi} A_a^i$$

this transforms normally under shifted gauge transformations

$$\delta_{e^{-\psi} \lambda} \mathcal{A}_a^i = -\partial_a (e^{-\psi} \lambda^i) + \epsilon^{ijk} \mathcal{A}_{aj} (e^{-\psi} \lambda_k)$$

this is generated by a shifted Gauss's law constraint:

$$\tilde{\mathcal{G}}^i = e^{-\psi} \mathcal{G}^i = e^{-\psi} (-\partial_a \pi_i^a + e^{-\psi} \epsilon^{ijk} A_{aj} \pi_k^a + \pi^{ai} \partial_a \psi) = 0$$

this generates shifted gauge transformations

$$\tilde{\delta}_\lambda A_a^i = \{A_i^a, \tilde{\mathcal{G}}(\rho)\} = \{A_i^a, \mathcal{G}(e^{-\psi} \rho)\}$$

We define a new, shifted holonomy:

$$T[\gamma; A, \psi] = Tr[Pe^{\int_{\gamma} e^{-\psi} A}] = Tr[Pe^{\int ds \mathcal{A}_a^i \tau_i \dot{\gamma}^a(s) ds}]$$

Which is invariant under S and the new shifted gauge transformations:

$$\delta_{e^{-\psi} \lambda} \mathcal{A}_a^i = -\partial_a(e^{-\psi} \lambda^i) + \epsilon^{ijk} \mathcal{A}_{aj}(e^{-\psi} \lambda_k)$$

There are shifted flux and area operators

$$\mathcal{F}(S, v^i, \psi) = \int_S v^i e^{\psi} E_i$$

T and F satisfy the regular holonomy-flux algebra.

## The constraint algebra

The algebra of the shifted Gauss's law constraints is business class, ie with structure functions:

$$\{\tilde{\mathcal{G}}(\lambda^i), \tilde{\mathcal{G}}(\mu^i)\} = \tilde{\mathcal{G}}(e^{-\psi} \epsilon^{ijk} \lambda_j \mu_k)$$

The rest of the algebra is first class:

$$\{\mathcal{G}(\lambda^i), \mathcal{S}(\rho)\} = \mathcal{G}(\rho \lambda^i)$$

$$\{\mathcal{S}(\rho), \mathcal{D}(v)\} = \mathcal{S}(\mathcal{L}_v \rho)$$

$$\{\mathcal{S}(\rho), \mathcal{S}(\sigma)\} = 0$$

The constraint algebra

Hence there are two first class systems:

G, D and H      or G,D and S

But H and S are second class:

$$\{\mathcal{H}(N), \mathcal{S}(\rho)\} = \mathcal{H}(N\rho) + \int_{\Sigma} (3\Lambda N \rho \det(\pi) - (N \partial_a \rho) \epsilon^{ijk} \pi_i^a \pi_j^b A_{bk})$$

This completes the definition of the linking theory based on the doubly extended phase space:

$$\Gamma^{ext} = (A_a^i, \psi; \tilde{\pi}_i^a, \tilde{\pi}_{\psi})$$

Gauge fixing back to general relativity:

$$\text{fix } S \text{ by } \psi = 0$$

Then solve  $S=0$  to set  $\pi_\psi = -\pi_i^a A_a^i$

Gauge fixing to shape dynamics:

$$\text{fix } H \text{ by } \pi_\psi = 0$$

Then solve  $S=0$  to set  $\mathcal{S} = \tilde{\pi} = \pi_i^a A_a^i = 0$

This extends maximal  $\text{Tr}K=0$  slicing

Alternatively, fix  $H$  by imposing CMC slicing by

$$\xi = \pi_\psi - \langle \pi_\psi \rangle \sqrt{\det(\tilde{\pi}_i^a)} = 0 \quad \langle \pi_\psi \rangle = \frac{\int \langle \pi_\psi \rangle}{V}$$

$S=0$  implies  $\tilde{\pi} = \pi_i^a A_a^i = -\langle \pi_\psi \rangle \sqrt{\det(\tilde{\pi}_i^a)}$

Next, solve  $H=0$  for  $\psi = F(A_a^i, \pi_i^a)$   $w^a = \epsilon^{ijk} \pi_i^a \pi_j^b A_{bk}$

$$w^a \partial_a e^\psi = e^{-\psi} (\pi_i^a A_a^j \pi_j^b A_b^i - \tilde{\pi}^2) + \epsilon^{ijk} \pi_i^a \pi_j^b \partial_a A_{bk} + \Lambda e^{2\psi} \det(\tilde{\pi}_i^a)$$

Finally, fix  $N$  to preserve the gauge condition:  $\{\xi, H\} = 0$

$$0 = N e^{-\psi} [\tilde{\pi}^2 - \pi_i^a A_a^i \pi_j^b A_b^j] + \partial_b (N A_a^i \tilde{E}^{aj} \tilde{E}^{bk} \epsilon_{ijk}) - \sqrt{\det(\tilde{\pi}_i^a)} \langle N e^{-\psi} [\tilde{\pi}^2 - (\pi_i^a A_a^j \pi_j^b A_b^i)] \rangle - \frac{3}{2} \langle \pi_\psi \rangle e_a^i \mathcal{D}_b (N \tilde{E}^{aj} \tilde{E}^{bk} \epsilon_{ijk})$$

Are there any routes towards quantization?

- Gauge fix back to GR then quantize: quantum GR.
- Quantize the CMC gauge fixed theory: extremely complicated!  
(Scary for me but perhaps not for others.)
- Quantize the linking theory.  $\Psi[A, \psi]$ 
  - There is a scale invariant holonomy-flux algebra
  - $e^\psi$  functions a bit like a dynamical Immirzi parameter.
- We can't keep both H and S first class.
  - Try  $S|\psi\rangle = 0$  and  $\langle\psi|H|\psi\rangle = 0 \rightarrow \langle\psi|[S, H]|\psi\rangle = 0$  etc
  - Or  $\langle\psi|S|\psi\rangle = 0$  and  $\langle\psi|H|\psi\rangle = 0$  no further relations
  - Shifted flux operators satisfy regular holonomy flux operator
    - Area spectra is shifted by  $e^\psi$ .
    - Other scale invariant observables? Angles?

ADM  $\longrightarrow$  Connection variables  $\rightarrow$  LQG

$$g_{ab}, \pi^{ab} \sim K_{ab}$$

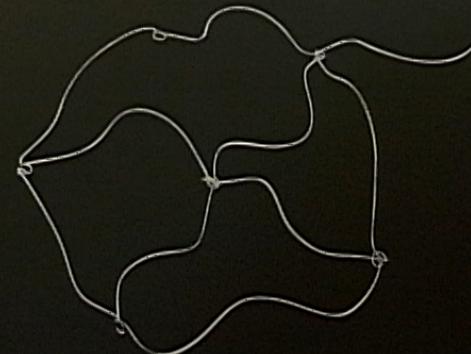


$$g^i, E^i \sim e^i, A_a^i = \cancel{\partial} K_a^i + P_a^i$$

$$\begin{matrix} S = \int \mathcal{L} \\ \downarrow \\ C^\phi \end{matrix}$$

$$\rho e^{i\sigma A} = h$$
$$E^a_i \sim$$

$\rightarrow$  AL - regions

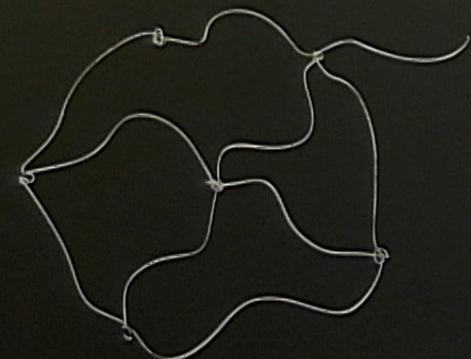


$n$  variables  $\rightarrow$  LQG

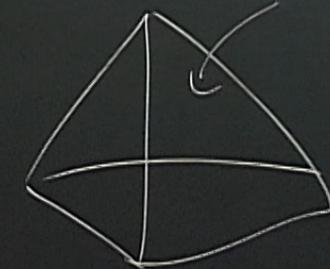
$$\mathcal{P} e^{i \int A} = h$$

$$E_i^a \rightsquigarrow \int_S E d^2x$$

$\rightarrow$  AL-repres



discrete geometry interpretations

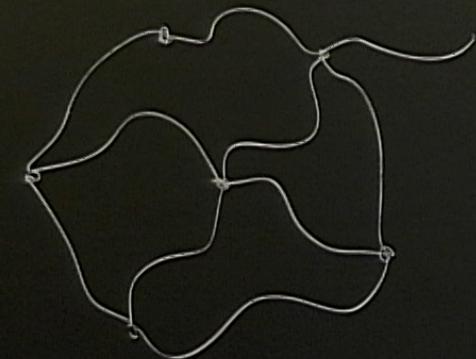


$n$  variables  $\rightarrow$  LQG

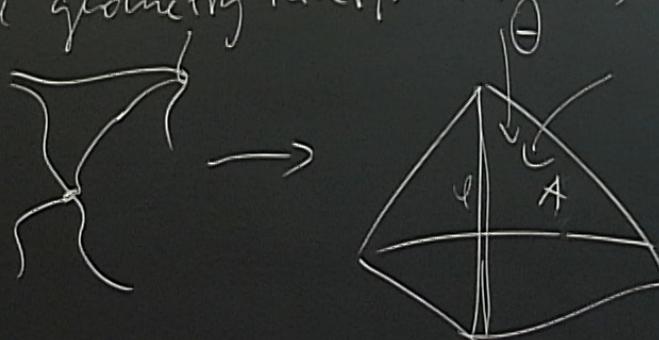
$$p_e \circ t = h$$

$$E_i^a \rightsquigarrow \int_S E d^2x$$

$\rightarrow$  AL-reps



discrete geometry interpretations



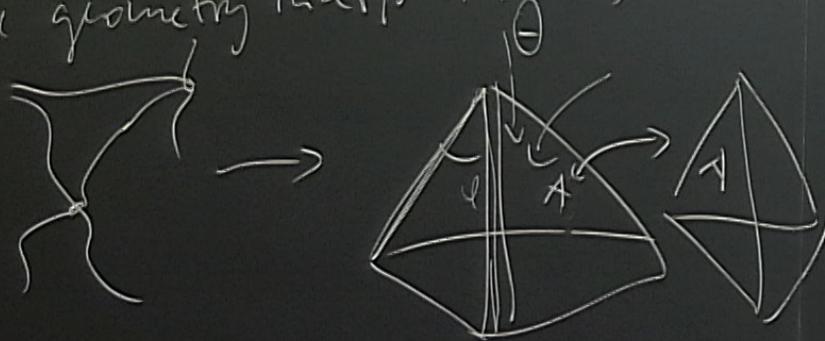
$h$

$$\rightarrow \int_S E d^2x$$

- $\text{spec}(A) \sim \underline{\gamma}$

- $\{\psi_1, \psi_2\} = \gamma \frac{\sin \alpha}{A}$

discrete geometry interpretations



- $\text{spec}(A) \sim \underline{\underline{\gamma}}$

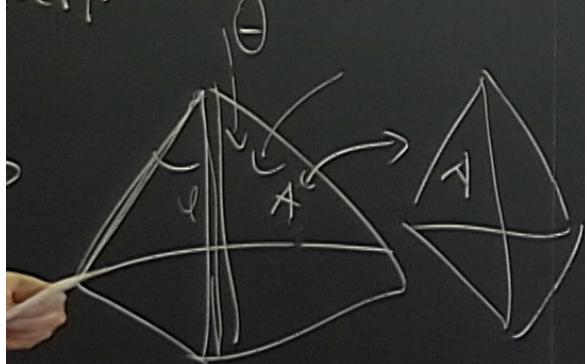
- $\{\varphi_1, \varphi_2\} = \gamma \frac{\sin \alpha}{A}$

- "enlarged phase space"

$$\{\theta_1, \theta_2\} = \frac{1}{\gamma} \frac{f(\phi)}{A}$$

$$\{\varphi, \theta\} = \frac{\hat{f}(\phi)}{A}$$

interpretations



$$rad(A) \sim \underline{\underline{r}}$$

$$\{\psi_1, \psi_2\} = r \frac{\sin \alpha}{A}$$

"enlarged phase space"

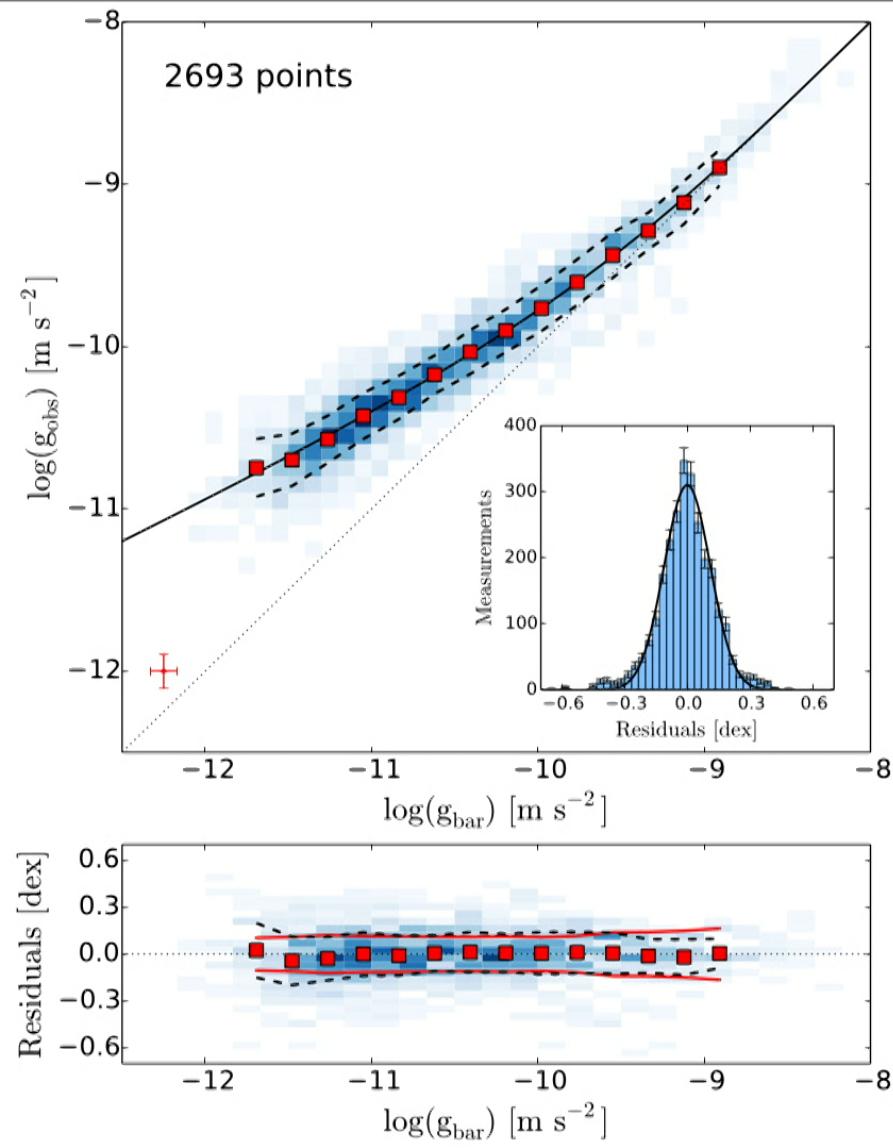
$$\{\theta_1, \theta_2\} = \frac{1}{r} \frac{f(\phi)}{A}$$

$$\{\varphi_1, \theta\} = \frac{\hat{r}(0)}{A}$$

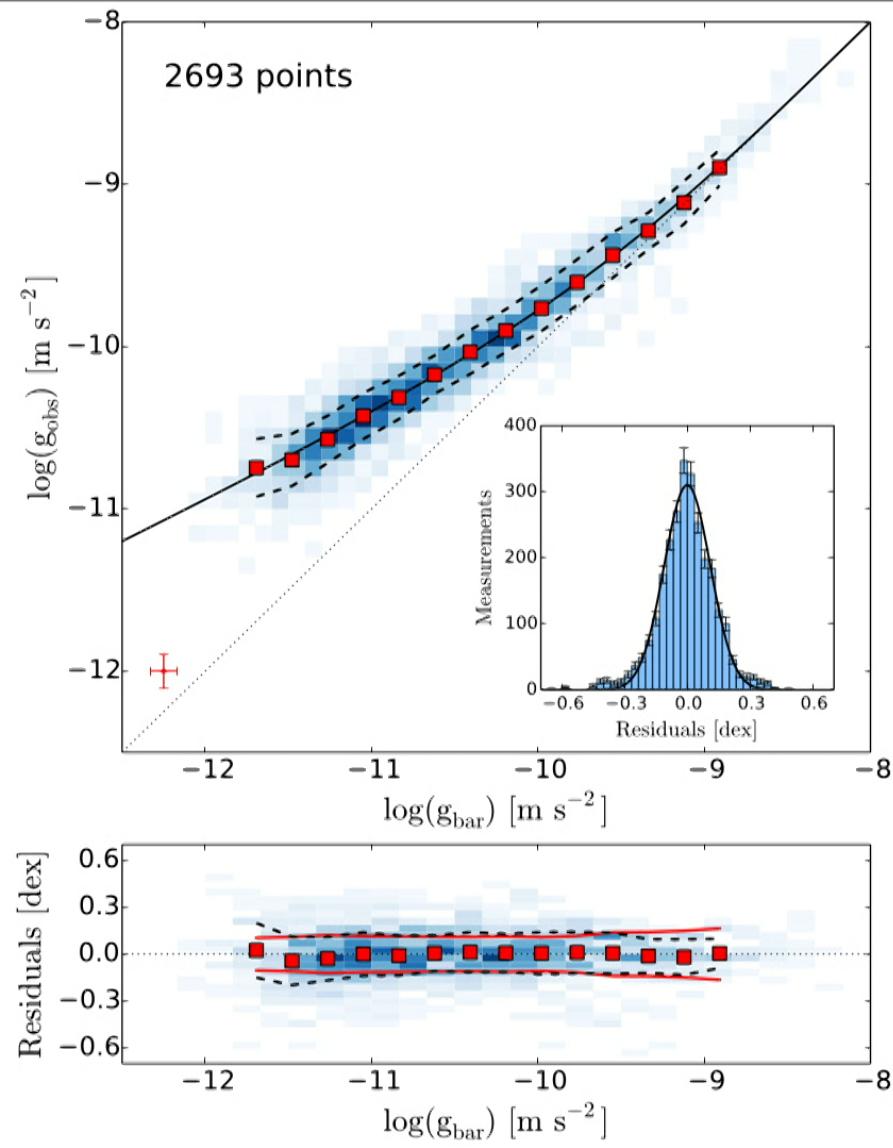
$$\left\{ \frac{1}{r} E \right.$$

$$\left\{ \bar{E}, A \right\}$$

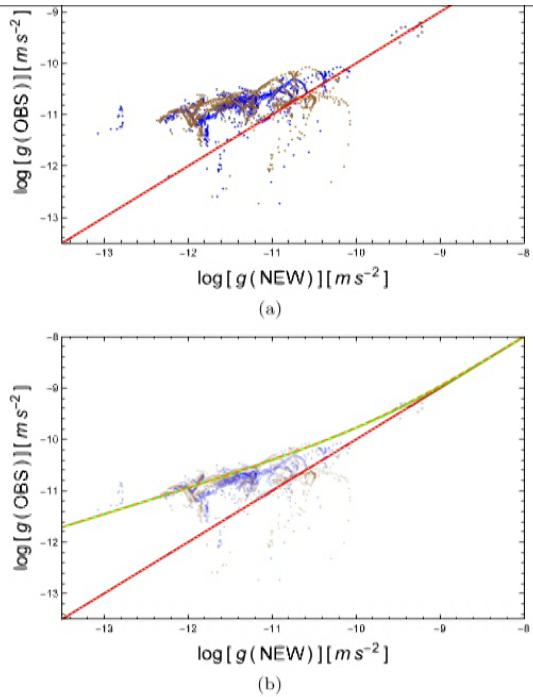
$$\left\{ \bar{E}, \bar{E} \right\}$$



McGaugh, Lelli,  
Schombert,  
arXiv:1609.05917v1



McGaugh, Lelli,  
Schombert,  
arXiv:1609.05917v1



Similar behavior for a sample of dwarf galaxies.

**Figure 3.** Fig. 3(a) shows the point by point comparison of  $(g_{NEW}, g_{OBS})$ , with the blue points using the mass predicted by CG and the brown points using the mass predicted by MOND. Fig. 3(b) shows the same points with  $g_{MON}$  shown in the dashed orange line and  $g_{MLS}$  shown in the green line. The line of unity is shown in red in both plots.

#### ALTERNATIVE GRAVITY ROTATION CURVES FOR THE LITTLE THINGS SURVEY

JAMES G. O'BRIEN,<sup>1</sup> THOMAS L. CHIARELLI,<sup>1</sup> JEREMY DENTICO,<sup>1</sup> MODESTAS STULGE,<sup>1</sup> BRIAN STEFANSKI,<sup>1</sup> ROBERT MOSS,<sup>2</sup> AND SPASEN CHAYKOV<sup>3</sup>

<sup>1</sup> Wentworth Institute of Technology

The low end gives clear evidence for a new scale in very low energy astrophysics:

The Tully Fischer Relation:

- Galaxies have flat rotation curves, with velocity V.

- Total luminosity L

[astro-ph/0204521](#)

$$C L = V^a \quad a=3.9 \pm 0.2$$

- $K = L/M$  ( $M$ -total mass)

$$CK M = V^4$$

- $CK$  should be prop to  $G$

- $CK = Ga_0$

$$a_0 = 1.2 \cdot 10^{-8} \text{ cm/sec}^2$$

$$= \sqrt{\Lambda} \cdot c^2/6$$

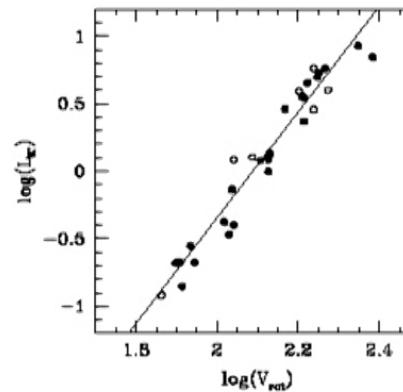
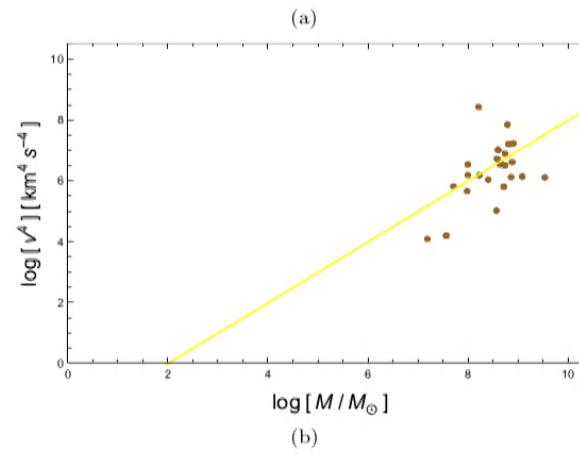
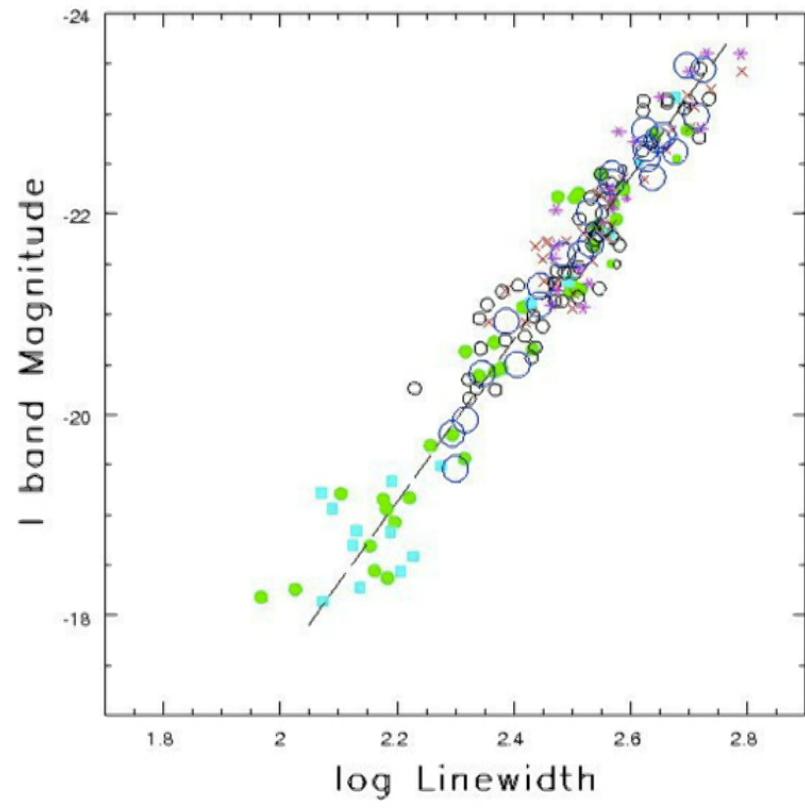


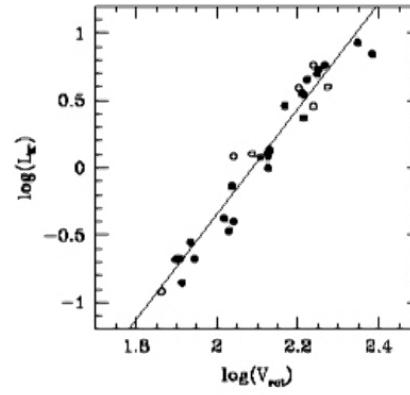
Figure 2: The near-infrared Tully-Fisher relation of Ursa Major spirals ((Sanders & Verheijen 1998)). The rotation velocity is the asymptotically constant value. The velocity is in units of kilometers/second and luminosity in  $10^{10} L_\odot$ . The unshaded points are galaxies with disturbed kinematics. The line is a least-square fit to the data and has a slope of  $3.9 \pm 0.2$ .

$$a = \sqrt{a_N g_0} \sim \sqrt{\frac{GM}{r}} = \frac{V}{r}$$

$$\sqrt{4} = GM g_0$$



**Figure 1.** Fig. 1(a) shows the Tully Fisher plot for CG while 1(b) shows the plot for MOND, where in both figures the last observed velocity was used. The yellow solid line



There is a phenomenological model that describes the rotation curves:

Assume that in the low, low acceleration limit there is a modification of Newton's law of gravitational acceleration

Newtonian gravitational acceleration:  $a_N = GM/r^2$

**New Law:**

(Milgrom, 1982 MOND)

$$a > a_0$$

$$a = a_N$$

$$a_0 = 1.2 \cdot 10^{-8} \text{ cm/sec}^2 = \sqrt{\Lambda} c^2/6$$

$$a < a_0$$

$$a = (a_N a_0)^{1/2}$$

*Far enough out from the center of a galaxy  $a < a_0$  so*

$$v^2/r = a = (a_N a_0)^{1/2} = (GM a_0)^{1/2} / r \rightarrow v^4 = GM a_0$$

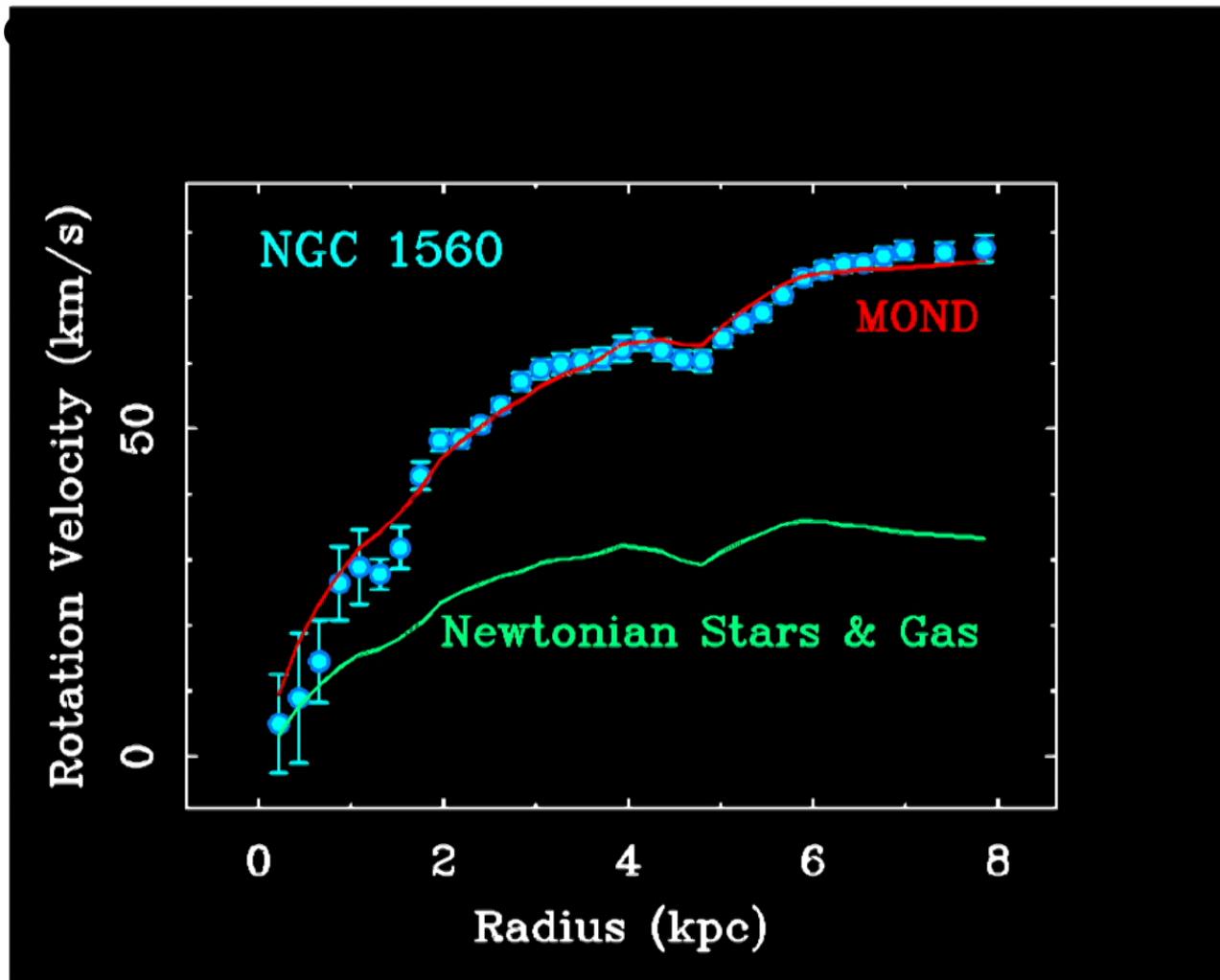
Fine, but...

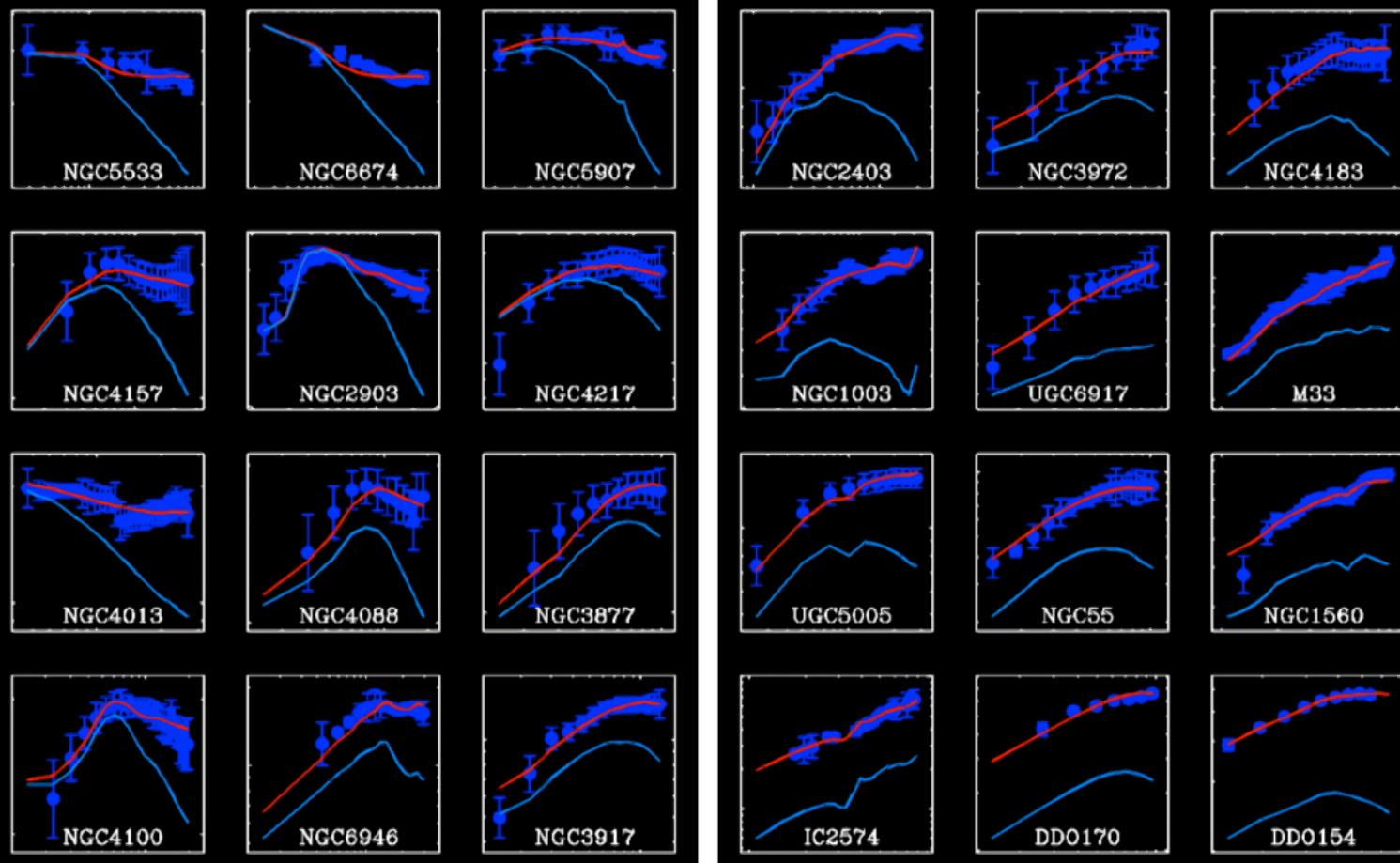
- How well does this explain the details of galaxy rotation curves?
- How well does this compare with the hypothesized dark matter?

## MODIFIED NEWTONIAN DYNAMICS AS AN ALTERNATIVE TO DARK MATTER

*Robert H. Sanders<sup>1</sup> & Stacy S. McGaugh<sup>2</sup>*

astro-ph/020452





## **Galaxies well fit by MOND 84 listed at present**

UGC 2885 NGC 5533 NGC 6674 NGC 7331  
NGC 5907 NGC 2998 NGC 801 NGC 5371  
NGC 5033 NGC 2903 NGC 3521 NGC 2683  
NGC 3198 NGC 6946 NGC 2403 NGC 6503  
NGC 1003 NGC 247 NGC 7739 NGC 300  
NGC 5585 NGC 55 NGC 1560 NGC 3109  
UGC 128 UGC 2259 M 33 IC 2574  
DDO 170 DDO 168 NGC 3726 NGC 3769  
NGC 3877 NGC 3893 NGC 3917 NGC 3949  
NGC 3953 NGC 3972 NGC 3992 NGC 4010  
NGC 4013 NGC 4051 NGC 4085 NGC 4088  
NGC 4100 NGC 4138 NGC 4157 NGC 4183  
NGC 4217 NGC 4389 UGC 6399 UGC 6446  
UGC 6667 UGC 6818 UGC 6917 UGC 6923  
UGC 6930 UGC 6973 UGC 6983 UGC 7089  
NGC 1024 NGC 3593 NGC 4698 NGC 5879  
IC 724 F563-1 F563-V2 F568-1  
F568-3 F568-V1 F571-V1 F574-1  
F583-1 F583-4 UGC 1230 UGC 5005  
UGC 5999 Carina Fornax Leo I  
Leo II Sculptor Sextans Sgr

**Galaxies not fit by MOND:**  
**none**

**Galaxies for which MOND fit is dubious: 11**

### **Galaxy                    problem**

NGC 2841 distance discrepant from Hubble  
flow value  
NGC 2915 distance uncertain  
DDO 154 last few points dropping  
(no Newtonian fit, either)  
IC 1613 very uncertain inclination and  
asymmetric drift  
F565-V2 inclination very uncertain  
UGC 5750 inclination very uncertain  
UGC 6446 distance uncertain  
UGC 6818 interaction?  
UGC 6973 very dusty - does light trace mass?  
Ursa Minor very sensitive to Milky Way parameters  
Draco sensitive to Milky Way parameters

MOND from quantum gravity?

A uniform acceleration  $a$  is associated to a temperature and a length scale  $l_a$ , which is both the distance to their acceleration horizon and the peak wavelength of the Unruh radiation.

$$T_U = \frac{\hbar a}{2\pi c}$$

$$l_a = \frac{c^2}{a} = \frac{\hbar c}{T_U}$$

In the presence of a positive cosmological constant, the temperature experienced by a static, accelerating detector is given by Deser and Levine.

$$T_{DL} = \sqrt{T_{dS}^2 + \left(\frac{\hbar a}{2\pi c}\right)^2} \quad T_{dS} = \frac{\hbar c^2}{2\pi} \sqrt{\frac{\Lambda}{3}}$$

So the peak wavelength is always less than the distance to the cosmological horizon.

$$\lambda_{DL} = \frac{1}{\sqrt{\frac{1}{l_a^2} + \frac{1}{R^2}}} \leq R \quad \frac{\Lambda}{3} = \frac{1}{R^2}$$

$$x \rightarrow \lambda x$$

$$t \rightarrow \lambda t$$

$$a = \sqrt{a_N g_b} \sim \sqrt{\frac{GM_{10}}{r}} = \frac{V}{r}$$

6.4  $\frac{GMm}{r^2}$

# Shape dynamics with connection variables

Lee Smolin

May 2017

[arXiv:1407.2909](https://arxiv.org/abs/1407.2909)

Thanks to Henrique, Tim