

Title: Renormalization Group flows of spacetime

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Abstract: I will discuss the use of (functional) Renormalization Group in models of quantum gravity. I will highlight the challenges that occur in continuum approaches to quantum gravity, such as asymptotically safe gravity, as well as challenges in discrete approaches, such as tensor models.

# Renormalization Group flows of spacetime

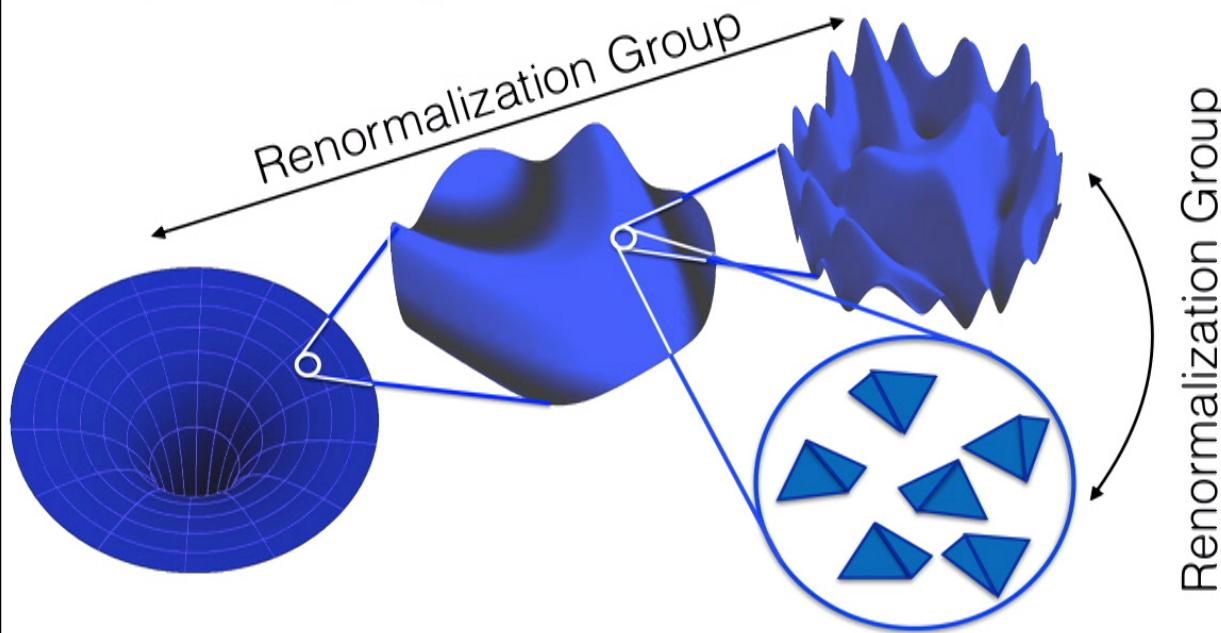
Astrid Eichhorn  
University of Heidelberg



May 16th,  
Perimeter Institute for Theoretical Physics  
Workshop on Shape Dynamics

# Outline

- 1) Functional RG: Probing the scale dependence of QFTs
- 2) RG for spacetime in the continuum:  
Setting a scale in Quantum Gravity
- 3) RG for discrete spacetimes:  
Scales in a pre-geometric setting?



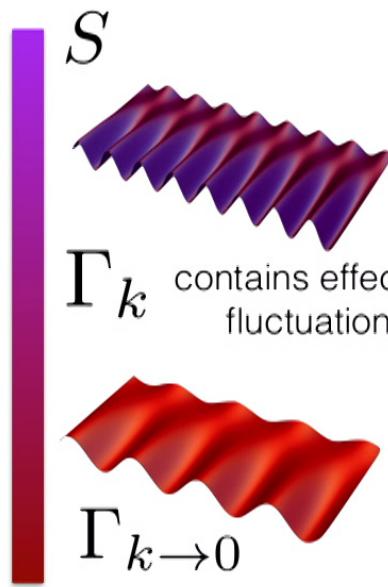
# Functional Renormalization Group

$$e^{-\Gamma_k[\phi]} = \int \mathcal{D}\varphi e^{-S[\varphi] - \frac{1}{2} \int \varphi(-p) R_k(p) \varphi(p)}$$

$\phi = \langle \varphi \rangle$

Wetterich '93; Morris, '94

scale- and momentum-dependent ``mass'':  
dials ``resolution scale''

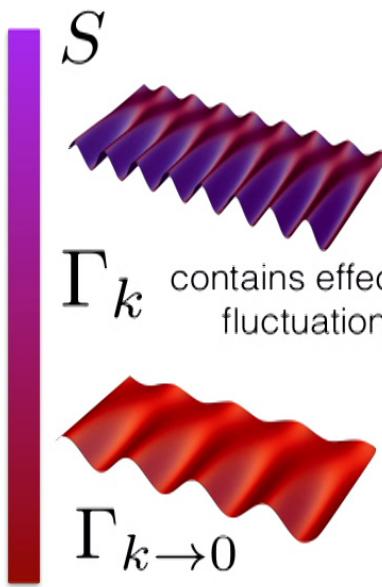
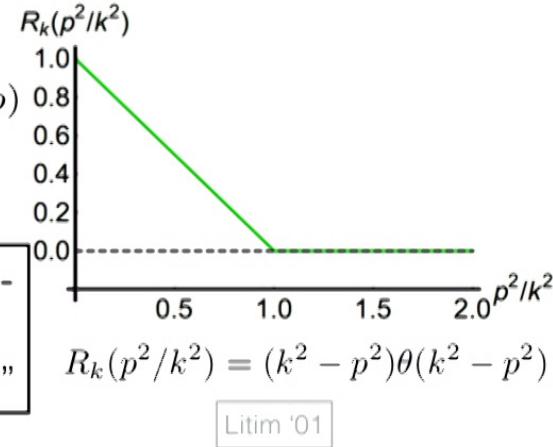


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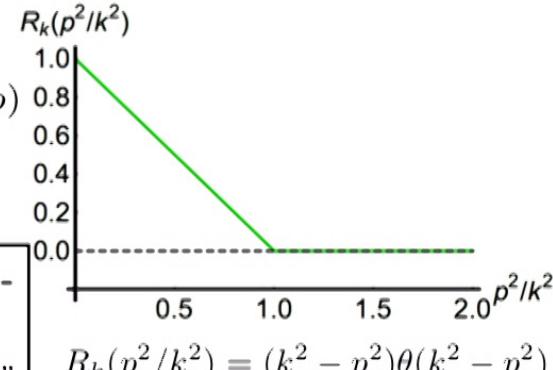


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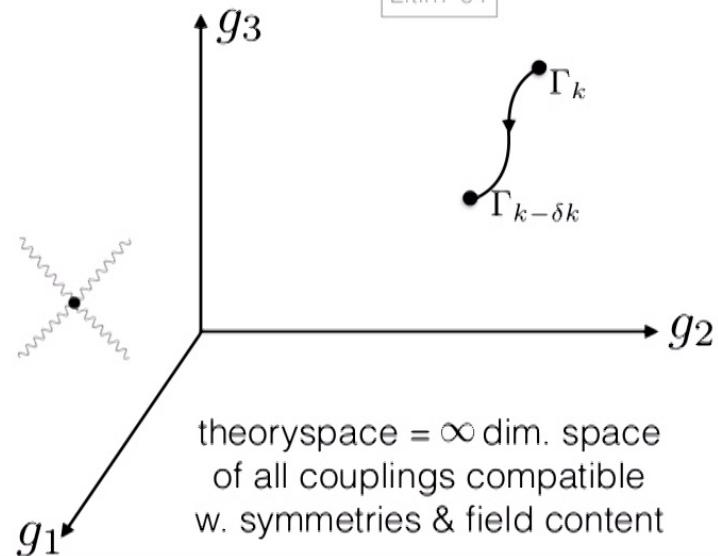
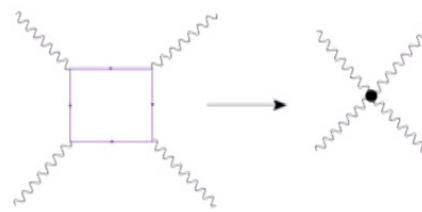
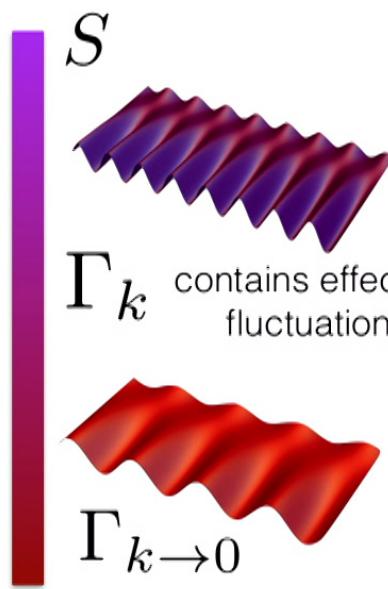
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Litim '01

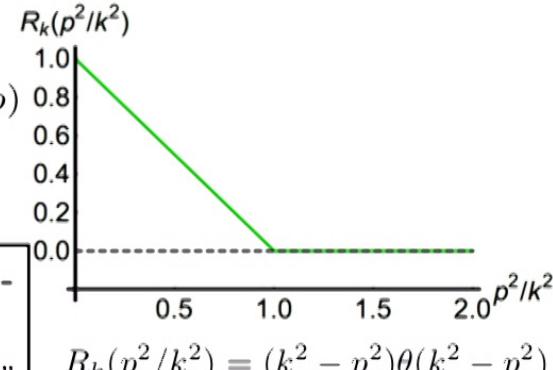


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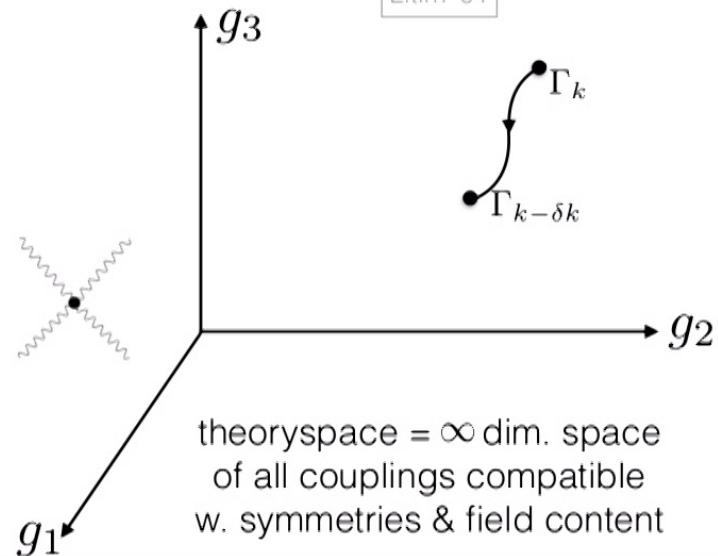
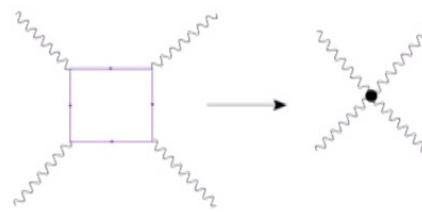
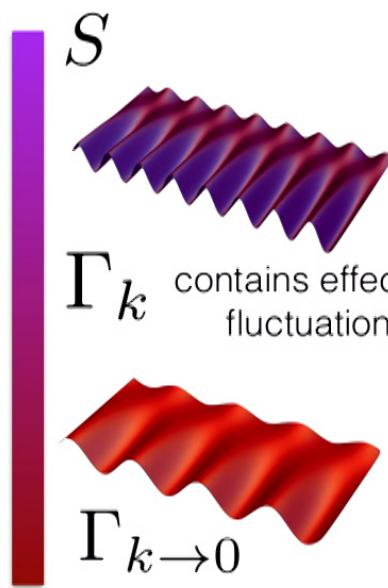
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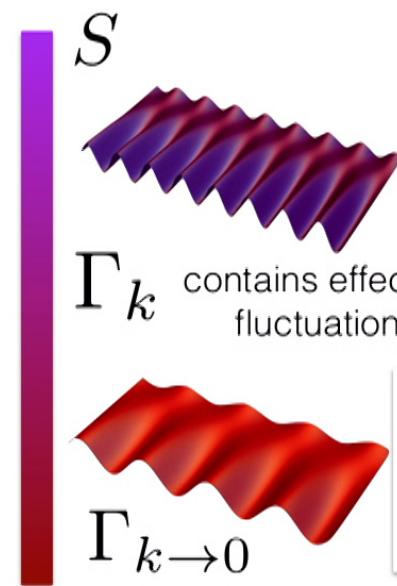


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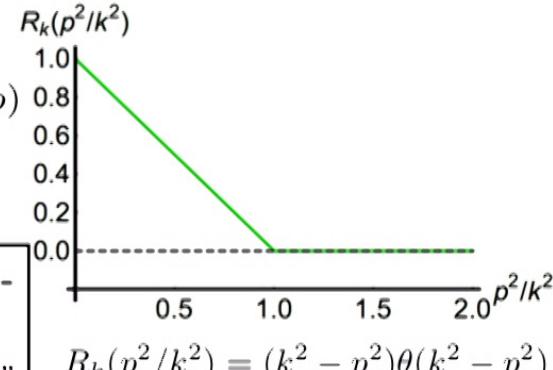
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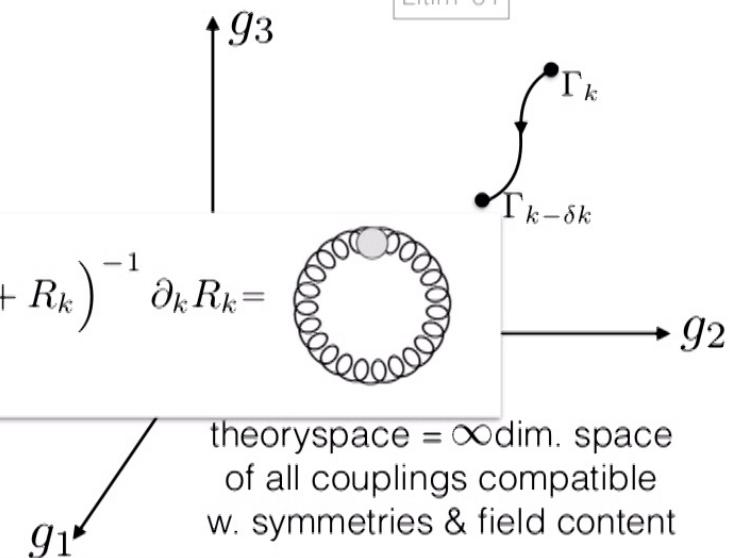
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$$\partial_k \Gamma_k = \frac{1}{2} S \text{Tr} \left( \Gamma_k^{(2)} + R_k \right)^{-1} \partial_k R_k =$$



Litim '01



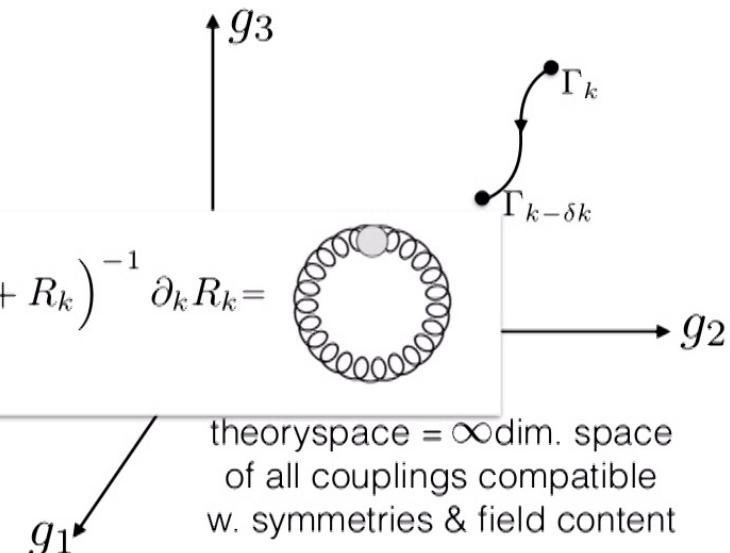
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- Path integral & Wetterich equation: equivalent
- Wetterich equation: UV & IR finite through regulator

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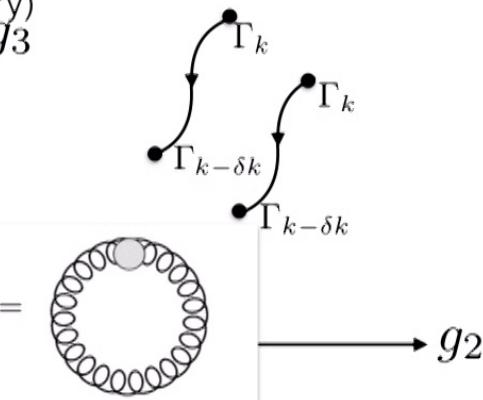
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Wetterich '93; Morris, '94

- Path integral & Wetterich equation: equivalent
- Wetterich equation: UV & IR finite through regulator
  - microscopic/classical action: initial condition to integrate the flow (= path integral for all possible micr. actions with given field content, symmetry)

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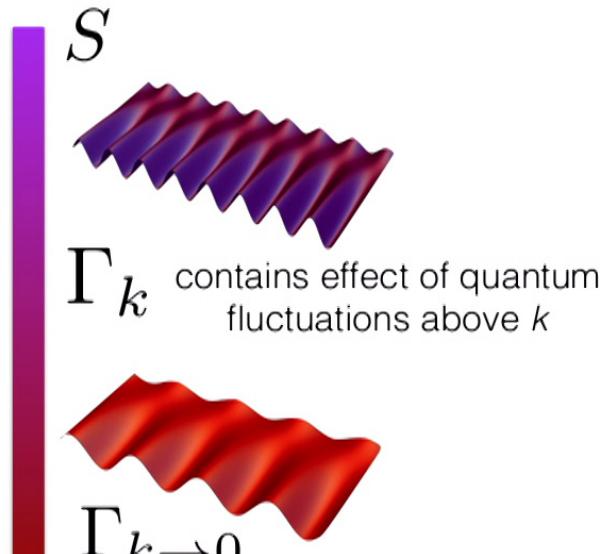


$g_1$

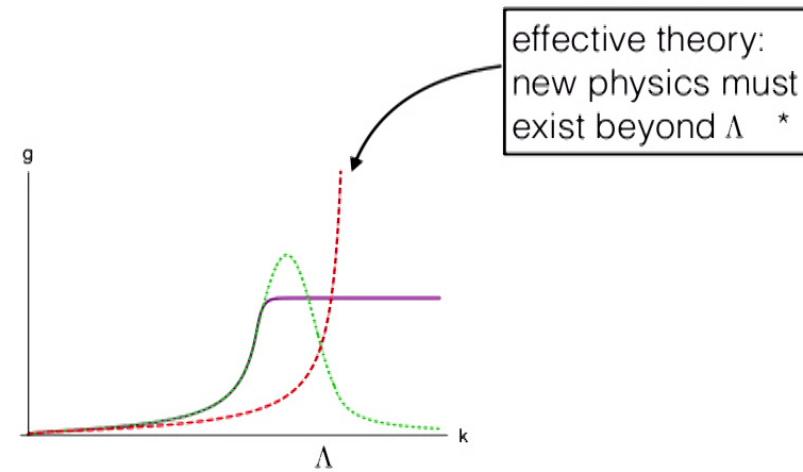
theoryspace =  $\infty$ dim. space  
of all couplings compatible  
w. symmetries & field content

# Search for UV completion in quantum gravity

given a set of fields and symmetries  
can explore whether model is  
fundamental (RG fixed point)  
or effective (Landau pole/triviality)

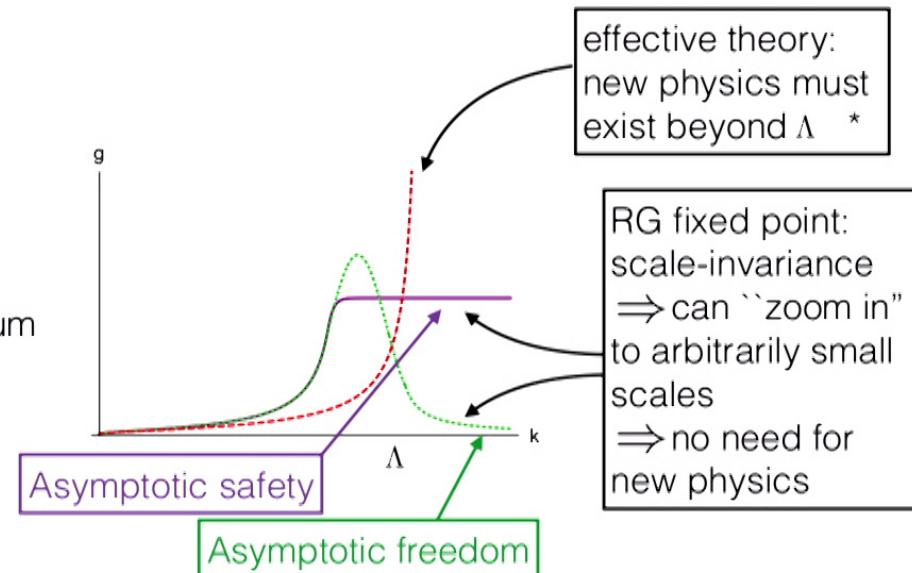
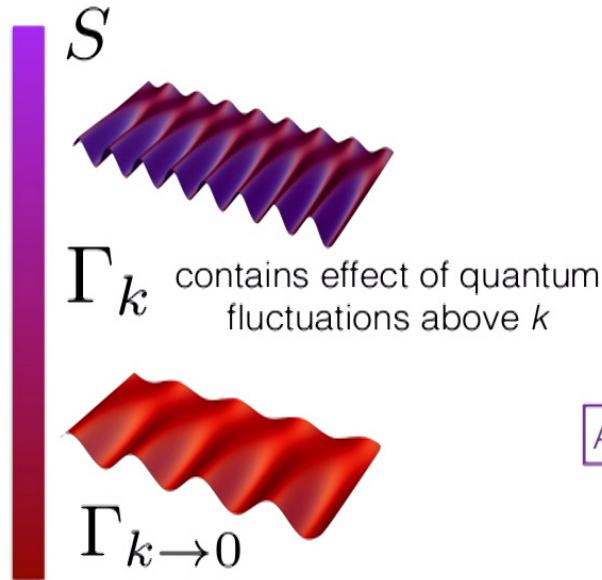


\* note: perturbative renormalizability not sufficient for fundamental theory



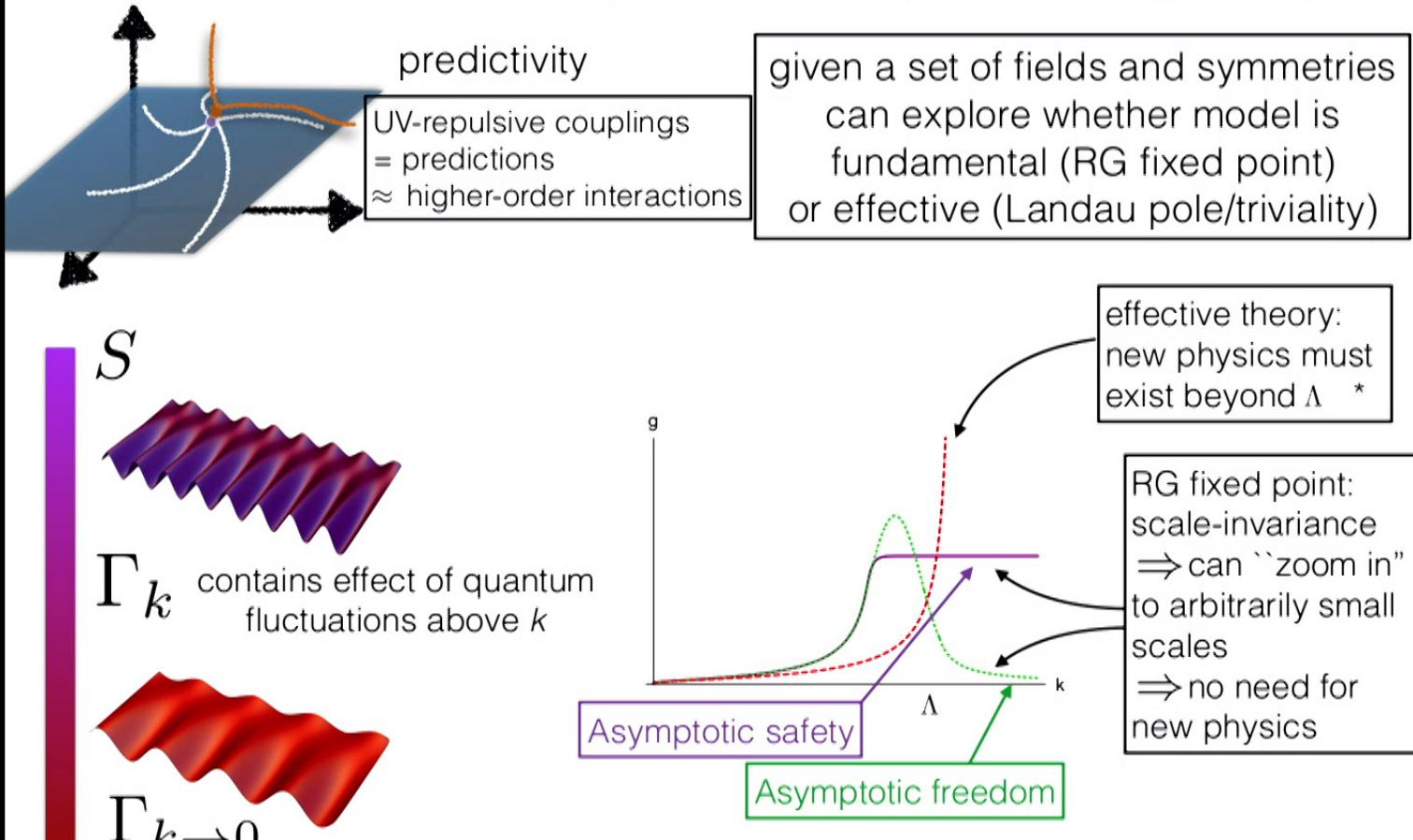
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Setting a scale in a geometric setting  $\int \mathcal{D}g_{\mu\nu}$   
covariant generalization of momentum: (eigenvalues of) Laplacian

$g_{\mu\nu} R_k^{\mu\nu\kappa\lambda} (-D^2) g_{\kappa\lambda} \rightarrow$  one-loop structure of flow equation 

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$g_{\mu\nu} R_k^{\mu\nu\kappa\lambda} (-D^2) g_{\kappa\lambda} \rightarrow$  one-loop structure of flow equation 

→ background field method

linear split:  $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$  Reuter '96, ...

exponential split:  $g_{\mu\nu} = \bar{g}_{\mu\nu} (\exp(h_{..}))_\nu^\kappa$   
A.E. '13, Percacci, Vacca '15, ...  
...

$h_{\mu\nu} R_k^{\mu\nu\kappa\lambda} (-\bar{D}^2) h_{\kappa\lambda} \rightarrow$  one-loop structure of flow equation intact

## No free lunch

$h_{\mu\nu} R_k^{\mu\nu\kappa\lambda} (-\bar{D}^2) h_{\kappa\lambda} \rightarrow$  one-loop structure of flow equation intact

shift symmetry: ("there is only one metric")

$$h_{\mu\nu} \rightarrow h_{\mu\nu} - \gamma_{\mu\nu} \quad \bar{g}_{\mu\nu} \rightarrow \bar{g}_{\mu\nu} + \gamma_{\mu\nu}$$

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broken by regulator

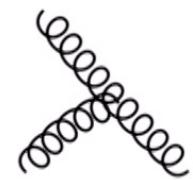
→ if the regulator breaks a symmetry, the flow no longer preserves the symmetry, even if the initial condition does

→ enlarged theory space

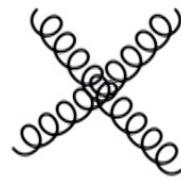
$$\begin{aligned} S[g_{\mu\nu}] &= -\frac{1}{16\pi G_N} \int d^4x \sqrt{g} R \\ &= -\frac{1}{16\pi \bar{G}_N} \int d^4x \sqrt{\bar{g}} \bar{R} \quad \beta_{\bar{G}_N} \neq \beta_{G_1} \neq \beta_{G_2} \dots \\ &\quad -\frac{1}{16\pi G_1} \int d^4x \sqrt{\bar{g}} h_{\mu\nu} K^{\mu\nu}[\bar{g}] + \frac{1}{16\pi G_2} \int d^4x \sqrt{\bar{g}} h_{\mu\nu} h_{\kappa\lambda} K^{\mu\nu\kappa\lambda}[\bar{g}] \\ &\quad + \frac{1}{16\pi G_3} \int d^4x \sqrt{\bar{g}} h_{\mu\nu} h_{\kappa\lambda} h_{\rho\sigma} K^{\mu\nu\kappa\lambda\rho\sigma}[\bar{g}] + \dots \end{aligned}$$

# How expensive is it?

"Newton couplings" in gravity-matter systems



$$\sqrt{G_{(3,0)}}$$



$$G_{(4,0)}$$



$$\sqrt{G_{(1,2)}}$$



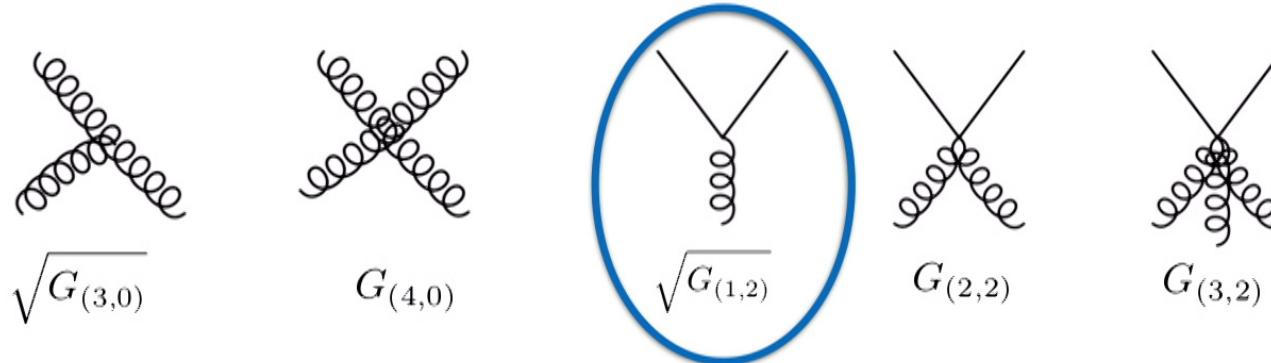
$$G_{(2,2)}$$



$$G_{(3,2)}$$

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background Newton coupling:

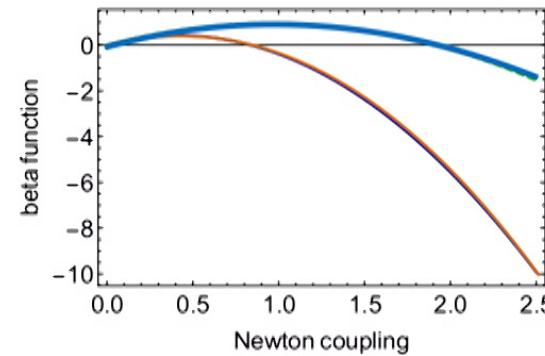
$$\beta_G = 2G + \frac{G^2}{6\pi} (-46 + N_S) + \dots$$

[Dona, A.E., Percacci, '13]

``graviton''-scalar coupling:

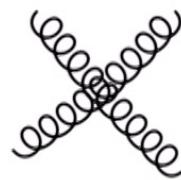
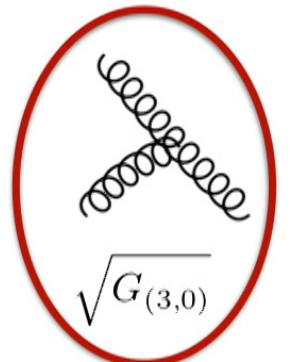
$$\beta_{G_{(1,2)}} = 2G_{(1,2)} + \frac{G_{(1,2)}^2}{3\pi} \left( -10 + \frac{N_S}{8} \right) + \dots$$

[Dona, A.E., Labus, Percacci, '15]

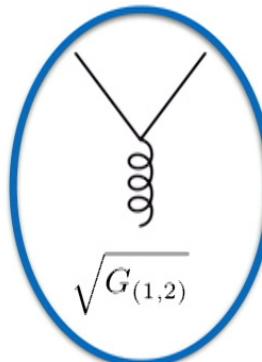


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quantitative difference but qualitative agreement

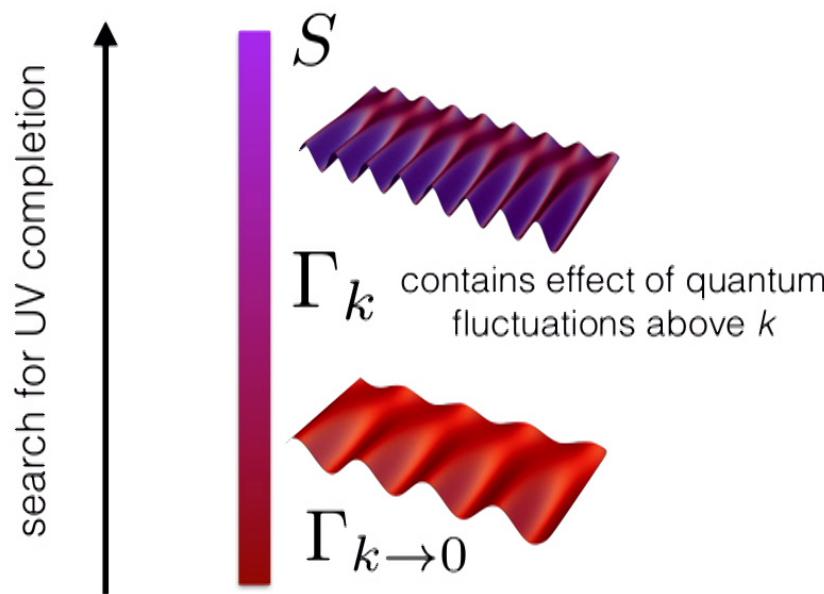
scalars drive gravity into  
the ``strong-gravity'' regime  
⇒ violation of weak-gravity bound

[A.E., Held, Pawłowski '16; Christiansen, A.E. '17, A.E., Held '17]

⇒ asymptotic safety only for  $N_S < N_{S\text{ crit}}$   
→ contact of QG to BSM pheno.  
→ what about qm shape dynamics?

# Renormalization Group: uses in Quantum Gravity

discovering the continuum limit in discrete models  
connection to (observable) IR physics



# Setting a scale in a pre- geometric setting

RG flow: from many degrees of freedom to  
fewer (coarser) degrees of freedom

example:  $N \times \dots \times N$  matrix/tensor models

$$\int dT_{i_1 \dots i_d} d\bar{T}_{i_1 \dots i_d} e^{-S[\bar{T}, T]}$$

↑  
trace invariants:

$$T_{i_1 \dots i_d} \bar{T}_{i_1 \dots i_d} + T_{i_1 \dots i_d} \bar{T}_{i_1 j_2 \dots j_d} T_{j_1 \dots j_d} \bar{T}_{j_1 i_2 \dots i_d} + \dots$$

[Weingarten, Ambjorn, Durhuus, Frohlich,  
Kazakov, Migdal, Boulatov, 1980's]

[Sasakura '91, Gross '91, Ambjorn, Durhuus, Jonsson '91,  
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physics: discrete path integral over spacetime configurations



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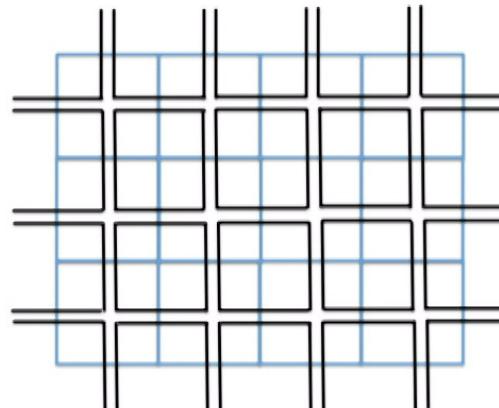
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Gurau '11, Gurau, Rivasseau '11, Gurau '12 ...]

physics: discrete path integral over spacetime configurations

matrix model:

$$S = \text{Tr}T^2 + g \text{ Tr}T^4$$

Feynman-diag. expansion  
= all possible tessellations



continuum limit =  
double-scaling limit:  
 $N \rightarrow \infty \quad g \rightarrow g_c$   
 $N (g - g_c)^{-\frac{1}{\theta}} = \text{const}$

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double-scaling limit:

$$N \rightarrow \infty, g \rightarrow g_c$$

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universal scaling near RG fixed points:

$$\beta_g = \underbrace{\frac{\partial \beta_g}{\partial g} \Big|_{g=g_*}}_{= -\theta} (g - g_*) + \dots$$
$$\rightarrow g(k) - g_* = \left( \frac{k}{k_0} \right)^{-\theta}$$

double-scaling limit = universal scaling near RG fixed point in  $N$

[Brezin, Zinn-Justin, '92]

# No free lunch

example:  $N' \times \dots \times N'$  matrix/tensor models

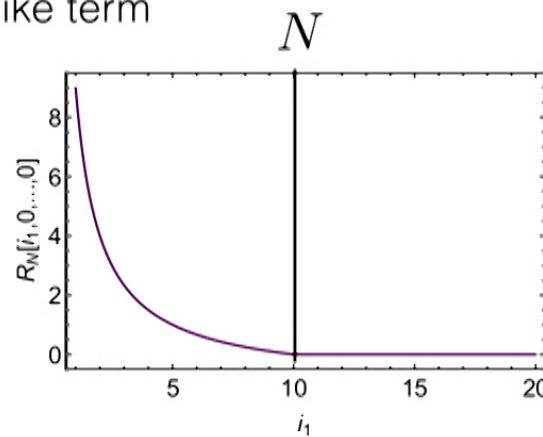
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has  $U(N') \otimes \dots \otimes U(N')$  invariance

$$\text{Regulator: } R_N[i_1, \dots, i_d] = \left( \frac{N}{i_1 + \dots + i_d} - 1 \right) \theta(N - (i_1 + \dots + i_d))$$

index- dependent mass-like term



A.E., Koslowski '13, '14, '17

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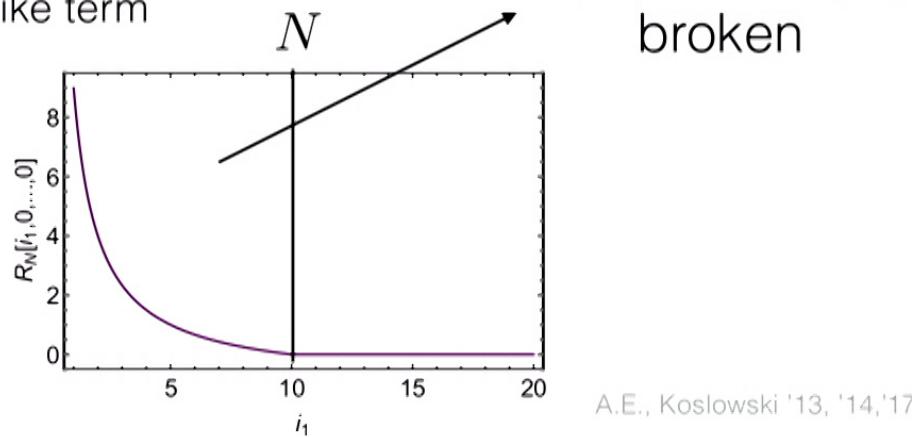
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index- dependent mass-like term

⇒ RG flow in larger theory space containing symmetry-breaking terms



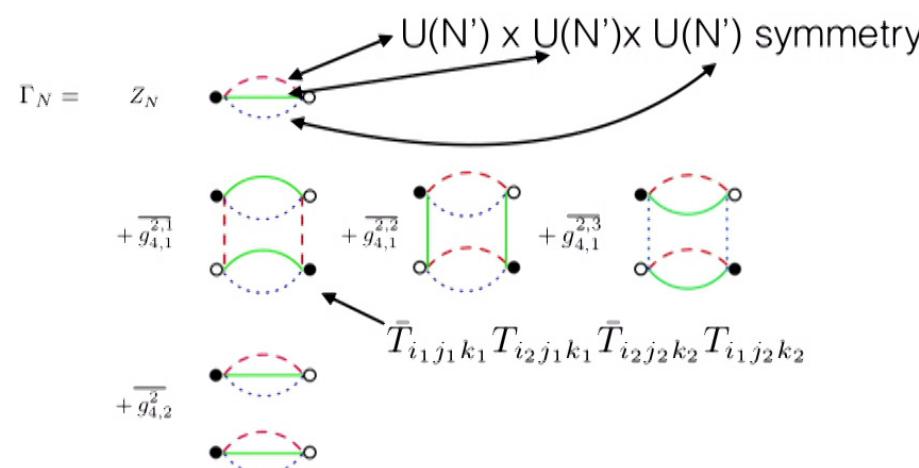
A.E., Koslowski '13, '14, '17

# RG fixed points in rank 3 hermitian tensor models

the model

$T : \circ$

$\bar{T} : \bullet$



$$\text{scaling dimensionality: } \bar{g}_i = N^{d_{\bar{g}_i}} g_i$$

consistently determined by RG equation (not just in tensor models)

$$\beta_{g_i} = -d_{\bar{g}_i} g_i + \#_1(N) g_i g_4 + \#_2(N) g_{i+2} + \dots$$

$$\beta_{g_{i+2}} = -d_{\bar{g}_{i+2}} g_{i+2} + \#_3(N) g_{i+2} g_4 + \#_4(N) g_{i+4} + \dots$$

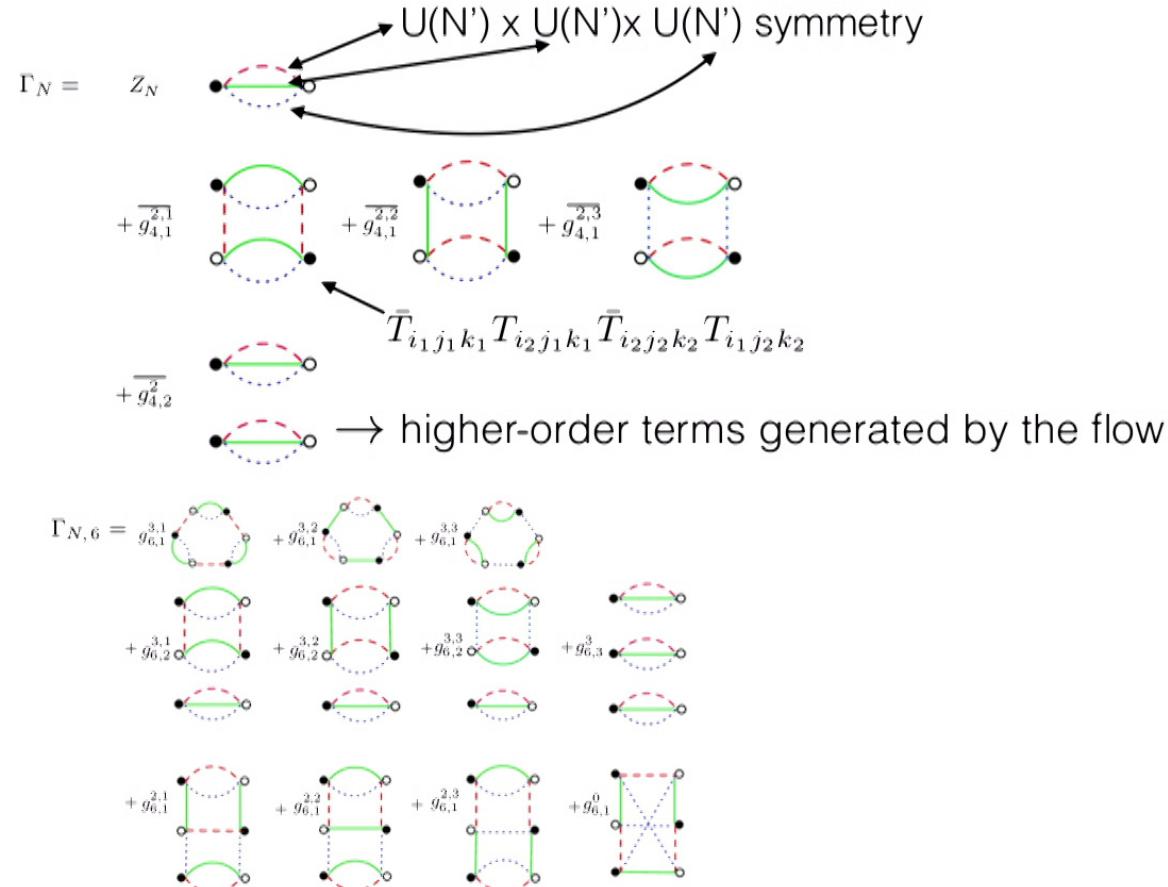
[A.E., Koslowski '17, accepted in Annales de l'Institut Henri Poincaré D]

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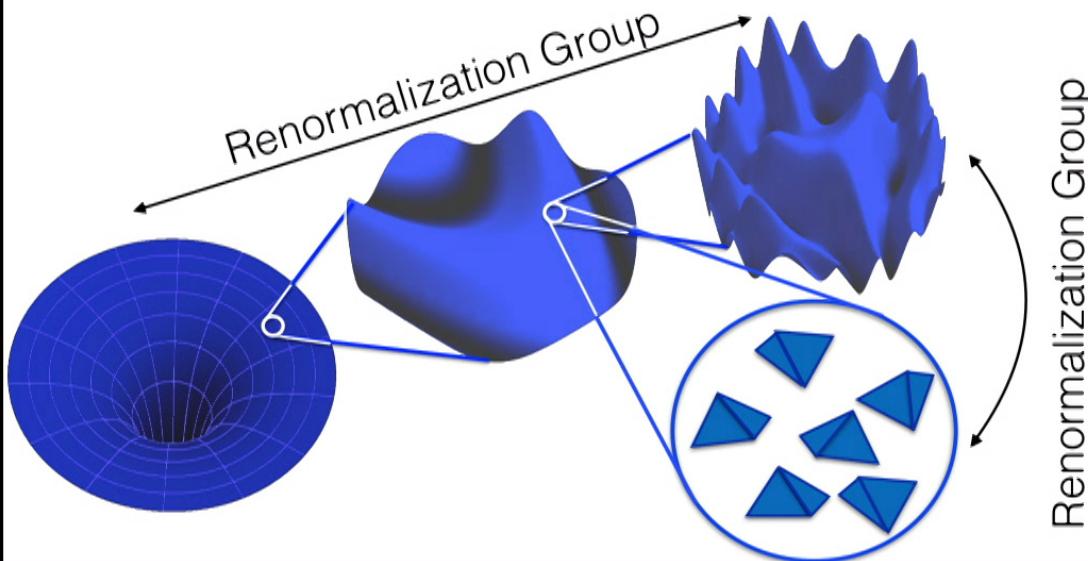
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# Summary

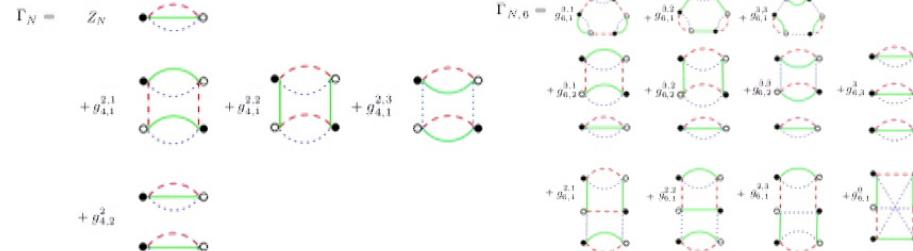
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The price for setting a scale in Quantum Gravity
- 3) RG for discrete spacetimes:  
The price for setting a scale in Quantum Gravity

→ effective universality  
in asymptotically  
safe qg

→ reason behind  
large error in  
estimate of  
scaling exponent  
in tensor models?



# Fixed-point results



scheme	$g_{4,1}^{2,1}$	$g_{4,2}^2$	$g_{6,1}^{3,1}$	$g_{6,1}^{2,1}$	$g_{6,2}^{3,1}$	$g_{6,3}^3$	*
full	-1.94	0	—	—	—	—	—
semi-pert	-2.14	0	—	—	—	—	—
pert	-4.62	0	—	—	—	—	—
full	-1.37	0	-2.14	0	0	0	0
semi-pert	-1.47	0	-2.46	0	0	0	0
pert	-2.14	0	-6.12	0	0	0	0

$$\theta = 1$$

[Bonzom, Gurau, Ryan, Tanasa '14  
Dartois, Gurau, Rivasseau '13]

anomalous dimension

$$\eta = -\partial_t \ln Z_N$$

$$\theta : \beta_g = \beta_g(g, \eta(g))$$

$$\theta' : \beta_g = \beta_g(g, \eta(g)_* = \text{const})$$

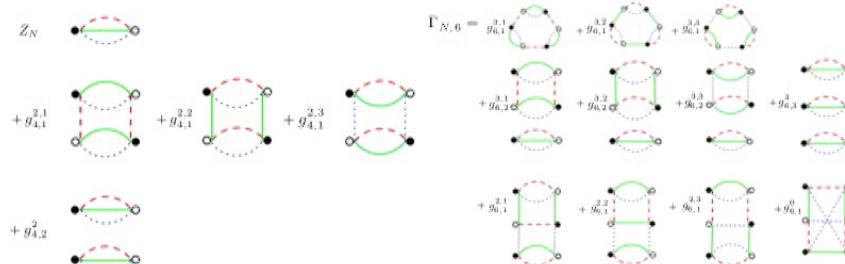
towards convergence in extended truncations?

scheme	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\theta_6$	$\theta_7$	$\theta_8$	$\theta_9$	$\theta_{10}$	$\theta_{11}$	$\theta_{12}$	$\theta_{13}$	$\theta_{14}$	$\eta$
full	2.21	-0.24	-0.93	-0.93	—	—	—	—	—	—	—	—	—	—	-0.54
semi-pert	2	-0.21	-0.68	-0.68	—	—	—	—	—	—	—	—	—	—	-0.54
pert	2	0.69	-2	-2	—	—	—	—	—	—	—	—	—	—	-0.54
full	2.14	-0.59	-1.26	-1.26	-2.24	-2.54	-2.89	-2.89	-3.15	-3.26	-3.26	-3.31	-3.31	-3.89	-0.37
semi-pert	2	-0.58	-1.26	-1.26	-2.24	-2.54	-2.90	-2.90	-3.13	-3.26	-3.26	-3.31	-3.31	-3.90	-0.37
pert	2	-0.35	-2	-2	-3.23	-3.37	-3.43	-4	-4	-4.07	-4.07	-4.14	-4.14	-5	0
scheme	$\theta'_1$	$\theta'_2$	$\theta'_3$	$\theta'_4$	$\theta'_5$	$\theta'_6$	$\theta'_7$	$\theta'_8$	$\theta'_9$	$\theta'_{10}$	$\theta'_{11}$	$\theta'_{12}$	$\theta'_{13}$	$\theta'_{14}$	$\eta$
full	0.93	-0.24	-0.93	-0.93	—	—	—	—	—	—	—	—	—	—	-0.54
semi-pert	0.93	-0.21	-0.93	-0.93	—	—	—	—	—	—	—	—	—	—	-0.54
full	1.33	-0.59	-1.26	-1.26	-2.30	-2.54	-2.89	-2.89	-3.15	-3.26	-3.26	-3.31	-3.31	-3.89	-0.37
semi-pert	1.33	-0.58	-1.26	-1.26	-2.31	-2.54	-2.90	-2.90	-3.13	-3.26	-3.26	-3.31	-3.31	-3.90	-0.37

[A.E., Koslowski '17, accepted in Annales de l'Institut Henri Poincaré D]

# Beyond double scaling?

$$\Gamma_N = \dots Z_N \dots$$



scheme	$g_{4,1}^{2,1}$	$g_{4,2}^2$	$g_{6,1}^{3,1}$	$g_{6,1}^{2,1}$	$g_{6,2}^{3,1}$	$g_{6,3}^3$
full	-1.05	-1.33	-	-	-	-
semi-pert	-1.58	-1.05	-	-	-	-
pert	-4.62	1.73	-	-	-	-
full	-0.53	-2.11	-0.14	0	-0.39	-
semi-pert	-0.63	-2.35	-0.20	0	-0.57	-
pert	-2.01	-1.64	-4.43	0	-4.84	-
full	-1.04	-0.84	-0.97	0	-0.64	-0.99
semi-pert	-1.15	-0.89	-1.21	0	-0.78	-1.14
pert	-2.10	-0.54	-5.54	0	-1.68	-0.47

scheme	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\theta_6$	$\theta_7$	$\theta_8$	$\theta_9$	$\theta_{10}$	$\theta_{11}$	$\theta_{12}$	$\theta_{13}$	$\theta_{14}$	$\eta$
full	2.64	0.14	-0.65	-0.65	-	-	-	-	-	-	-	-	-	-	-0.67
semi-pert	2.28	0.14	-0.68	-0.68	-	-	-	-	-	-	-	-	-	-	-0.66
pert	2	-0.69	-2	-2	-	-	-	-	-	-	-	-	-	-	0
full	3.10	0.25	-0.48	-0.48	$-1.22 \pm i 0.44$	-1.53	-1.53	-1.72	-1.72	-2.46	-2.46	-2.72	-	-	-0.76
semi-pert	2.68	0.26	-0.51	-0.51	$-1.24 \pm i 0.45$	-1.57	-1.57	-1.76	-1.76	-2.49	-2.49	-2.76	-	-	-0.75
pert	2.11	0.34	-2	-2	-2.99	-2.99	-3.54	-3.54	-4	-4	-4.13	-4.13	-5	-	0
full	2.56	0.44	-0.97	-0.97	$-1.80 \pm i 0.42$	-2.45	-2.45	-2.63	-2.63	-2.95	-2.95	-3.06	-3.45	-	-0.52
semi-pert	2.30	0.44	-0.98	-0.98	$-1.81 \pm i 0.42$	-2.47	-2.47	-2.65	-2.65	-2.97	-2.97	-3.06	-3.47	-	-0.51
pert	2.04	0.37	-2	-2	-2.68	-3.16	-3.87	-3.95	-3.95	-4	-4	-4.09	-4.09	-5	0

scheme	$\theta'_1$	$\theta'_2$	$\theta'_3$	$\theta'_4$	$\theta'_5$	$\theta'_6$	$\theta'_7$	$\theta'_8$	$\theta'_9$	$\theta'_{10}$	$\theta'_{11}$	$\theta'_{12}$	$\theta'_{13}$	$\theta'_{14}$	$\eta$
full	0.65	0.47	-0.65	-0.65	-	-	-	-	-	-	-	-	-	-	-0.67
semi-pert	0.68	0.42	-0.68	-0.68	-	-	-	-	-	-	-	-	-	-	-0.66
full	1.03	0.43	-0.48	-0.48	$-1.23 \pm i 0.46$	-1.53	-1.53	-1.72	-1.72	-2.46	-2.46	-2.72	-	-	-0.76
semi-pert	1.04	0.44	-0.51	-0.51	$-1.25 \pm i 0.47$	-1.57	-1.57	-1.76	-1.76	-2.49	-2.49	-2.76	-	-	-0.75
full	1.17	0.64	-0.97	-0.97	$-1.82 \pm i 0.42$	-2.45	-2.45	-2.63	-2.63	-2.95	-2.95	-3.10	-3.45	-	-0.52
semi-pert	1.17	0.63	-0.98	-0.98	$-1.83 \pm i 0.42$	-2.47	-2.47	-2.65	-2.65	-2.97	-2.97	-3.10	-3.47	-	-0.51

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