Title: Compact spherically symmetric solutions and gravitational collapse in SD

Date: May 15, 2017 04:00 PM

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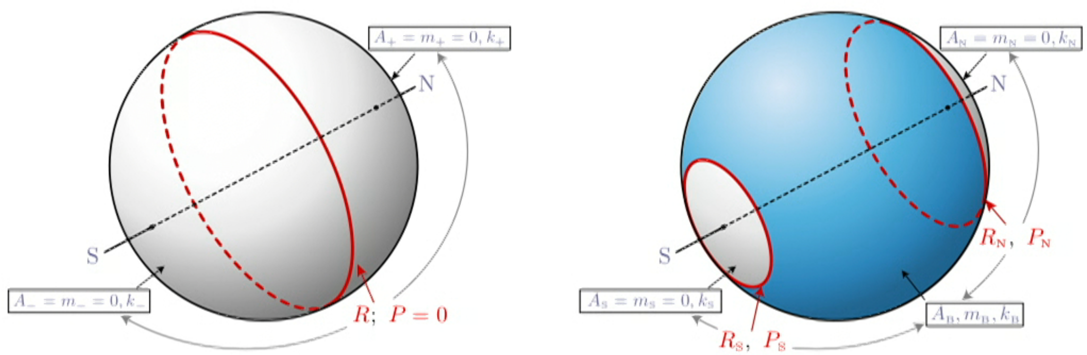
Abstract: I will review the current status of our understanding of spherically symmetric compact solutions of Shape Dynamics, which have nontrivial degrees of freedom when matter is present. I will show some new solutions of GR in a CMC foliation: a single thin spherical shell of matter in equilibrium in a compact foliation of de Sitter, and the simplest possible model of a black hole or compact star. This is provided by a universe with the topology of a 3-sphere with two thin spherical shells of dust. One of the shells models the `fixed starsâ€™, or the `rest of the universeâ€™, while the other shell models collapsing matter. Both are needed for a truly relational description of gravitational collapse. It turns out that such a solution of GR cannot be evolved past a point at which the foliationceases to be admissible, but it still makes sense past that point as a solution of Shape Dynamics, because the shape degrees of freedom seem to be unaffected. My conjecture is that we have found another example of departure between GR and SD, and this departure happens whenever ordinary matter undergoes gravitational collapse.

Compact spherically symmetric solutions and gravitational collapse in SD



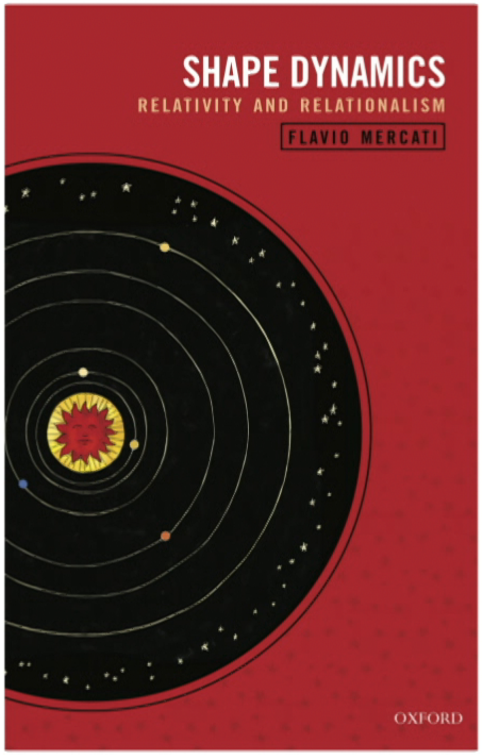
Shape Dynamics Workshop

Perimeter Institute

May 2017

Flavio Mercati

What I mean with Shape Dynamics



F.M. “A Shape Dynamics Tutorial” **arXiv: 1409.0105 v2**

What I mean with Shape Dynamics

- Theory of classical evolution of 3D conformal geometry.

- York's Hamiltonian gives one possible choice of such an evolution, which is equivalent to ADM in CMC foliation ($\Rightarrow$ well constrained experimentally).

$$\mathrm{tr}K \propto \frac{p^{ij} g_{ij}}{\sqrt{g}} = \textit{const.} \qquad \begin{cases} [g] = \{g'_{ij} \sim g_{ij} \, , \, g'_{ij} = e^{\varphi} g_{ij}\} \\ \pi^{ij} g_{ij} = \nabla_j \pi^{ij} = 0 \, , \quad p^{ij} = \pi^{ij} + \frac{1}{3}\sqrt{g}\langle p \rangle g^{ij} \end{cases}$$

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- Quantum construction principle: I'd like to study the class of 3D conformal theories that are asymptotically safe, and see if York's belongs to it.

- In the meantime, we need to learn to do (classical) physics with 3D conformal geometry: this is not trivial, because we have to renounce the assumption of a smooth (modulo singularities) spacetime, and focus on the **continuity of the shape degrees of freedom**,

- This exercise delivers interesting results: we can **continue cosmological solutions through the Big Bang** (see David's talk) - *i.e.* past the point where the ADM evolution cannot be continued.

- For black holes, I will show a similar **failure of ADM evolution** which **does not affect the shape degrees of freedom**.

Early attempts at SD black holes

∗ Henrique Gomes, "A Birkhoff theorem for Shape Dynamics," **CQG 31 (2014) 085008, arXiv:1305.0310.**
Maximal foliation of Schwarzschild does not cover the singularity. Should be interpreted as a wormhole?

- H. Gomes, Gabriel Herczeg, "A Rotating Black Hole Solution for Shape Dynamics," **CQG 31 (2014) 175014, arXiv:1310.6095.**
G. Herczeg, "Parity Horizons, Black Holes, and Chronology Protection in Shape Dynamics," **CQG 33 (2016) 225002, arXiv:1508.06704.**
F. M. Daniel Guariento, "Self-gravitating fluid solutions of Shape Dynamics" **PRD 94, (2016), arXiv:1606.01215**
Extensions of ∗ to charged/rotating/w.fluid cases.

- F. M., H. Gomes, Tim Koslowski, Andrea Napoletano, "Gravitational collapse of thin shells of dust in asymptotically flat Shape Dynamics," **PRD 95 (2017) 044013, arXiv:1509.00833.**
Generation of the solution of ∗ as result of gravitational collapse of matter.

- ALL THESE ISSUES SOLVED BY **COMPACT BOUNDARY CONDITIONS:**
  - If topology of spatial slice is S^3, no boundary charges and variational problem well-posed.
  - A can be arbitrary if we put some matter around the poles (and a positive cosmological constant) to ensure compact boundary conditions.
  - A has interpretation of dilatational momentum. In a compact space this has to be **relational:** if some matter expands, the rest of the universe has to collapse around it.

arXiv: 1704.04196 - F.M. "Thin shells of dust in a compact universe"

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Spherical symmetry in SD

arXiv: 1704.04196 - F.M. "Thin shells of dust in a compact universe"

ASSUMPTIONS:

- Spherical Symmetry $\Rightarrow$ local conformal flatness (no metric shape DOFs)

$$ds^2 \propto dr^2 + \sin^2 r(d\theta^2 + \sin^2\theta d\phi^2)\,.$$

- Compactness (S^3 topology) $\Rightarrow$ no arbitrary boundary conditions

$$\int \sqrt{|g|}\, d^3x < \infty\,.$$

- Matter sources (to have dynamical DOFs). Simplest possibility:

infinitely thin shells of dust.

Spherically Symmetric Ansatz

$$g_{ij} = \begin{pmatrix} \mu^2(r) & 0 & 0 \\ 0 & \sigma(r) & 0 \\ 0 & 0 & \sigma(r)\,\sin^2\theta \end{pmatrix}, \quad p^{ij} = \sin\theta \begin{pmatrix} \frac{f(r)}{\mu(r)} & 0 & 0 \\ 0 & \frac{1}{2}s(r) & 0 \\ 0 & 0 & \frac{\frac{1}{2}s(r)}{\sin^2\theta} \end{pmatrix},$$

Vacuum ADM-CMC constraints

$$\mathcal{H} = \frac{1}{\sqrt{g}}\left(p^{ij}p_{ij} - \frac{1}{2}p^2\right) + \sqrt{g}(2\Lambda - R), \quad \mathcal{H}_i = -2\,\nabla_j p^j{}_i, \quad \mathcal{C} = p - \langle p \rangle \sqrt{g},$$

$$\langle p \rangle = \frac{\int p^{ij} g_{ij} d^3x}{\int \sqrt{g} d^3x} = \text{"York time"}.$$

Exact solution of the vacuum constraints

$$g_{rr} = \mu^2(r) = \frac{(\sigma')^2}{\frac{A^2}{\sigma} + \left(\frac{2}{3}\langle p\rangle A - 8\,m\right)\sqrt{\sigma} + 4\,\sigma - \frac{1}{9}\left(12\,\Lambda - \langle p\rangle^2\right)\sigma^2}$$

$$g_{\theta\theta} = \sigma(r)\,, \qquad p^{rr} = \frac{\sin\theta}{\mu}\left(\frac{1}{3}\langle p\rangle\,\sigma + \frac{A}{\sqrt{\sigma}}\right)\,, \qquad p^{\theta\theta} = \frac{\sin\theta}{2}\left(\frac{2}{3}\langle p\rangle - \frac{A}{\sigma^{\frac{3}{2}}}\right)\mu\,.$$

Two integration constants:

- m = Misner-Sharp mass $= \frac{\sqrt{\sigma}}{2}\left(1 - {}^{(4)}g^{\mu\nu}\partial_\mu(\sqrt{\sigma})\partial_\nu(\sqrt{\sigma})\right)$,
- A = "dilatational momentum". Related to integral of radial momentum of matter sources $A \propto \int P_r dr$

The fundamental polynomial

If we require the metric to be Euclidean ($\mu^2 > 0$ and $\sigma > 0$), $\sigma(r)$ has to take values where the denominator of the following is positive:

$$\mu^2 = \frac{\sigma\,(\sigma')^2}{A^2 + \left(\frac{2}{3}\langle p\rangle A - 8\,m\right)\sigma^{\frac{3}{2}} + 4\,\sigma^2 - \frac{1}{9}\left(12\,\Lambda - \langle p\rangle^2\right)\sigma^3},$$

making all quantities dimensionless by rescaling with the M–S mass:

$$z = \frac{\sqrt{\sigma}}{|m|}, \quad C = \frac{A}{2\,m^2}, \quad \tau = |m|\,\langle p\rangle\,, \quad \lambda = m^2\Lambda\,.$$

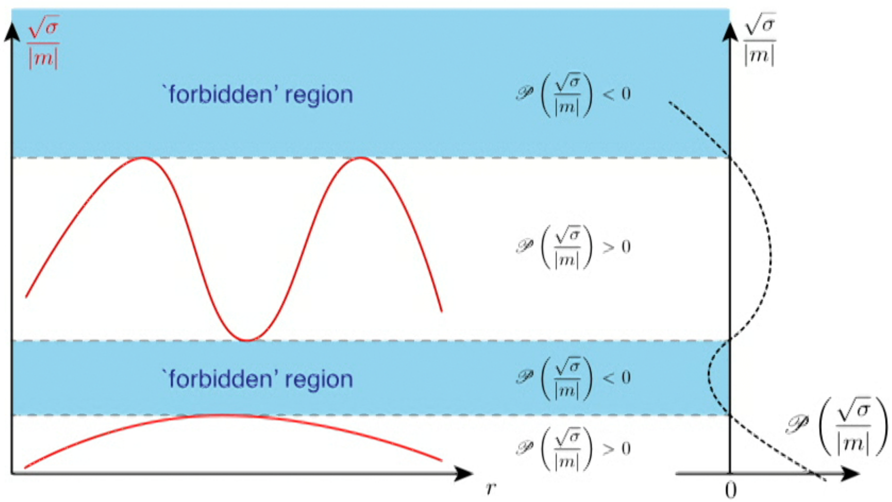
the denominator turns into the following polynomial:

$$\boxed{\mathscr{P}[z] = \frac{1}{36}\left(6C + \tau z^3\right)^2 - (\pm 2\,z^3) - \frac{1}{3}\lambda\,z^6 + z^4}$$

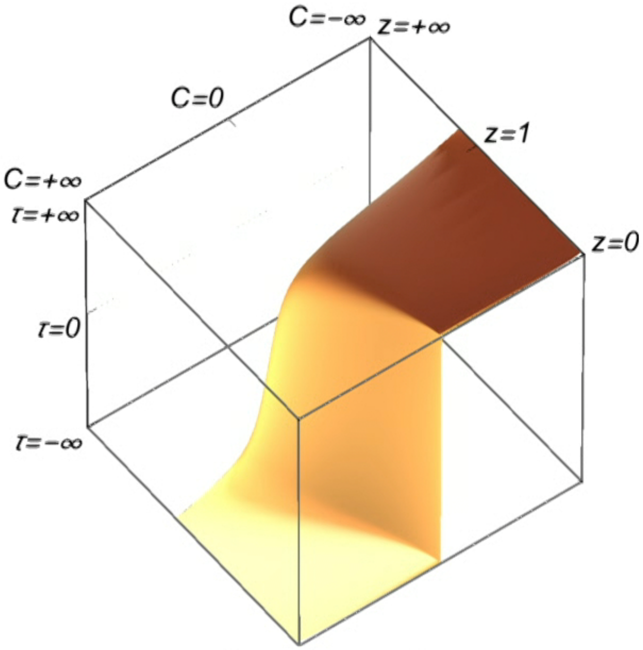
where $+$ if $m > 0$ and $-$ if $m < 0$.

The fundamental polynomial

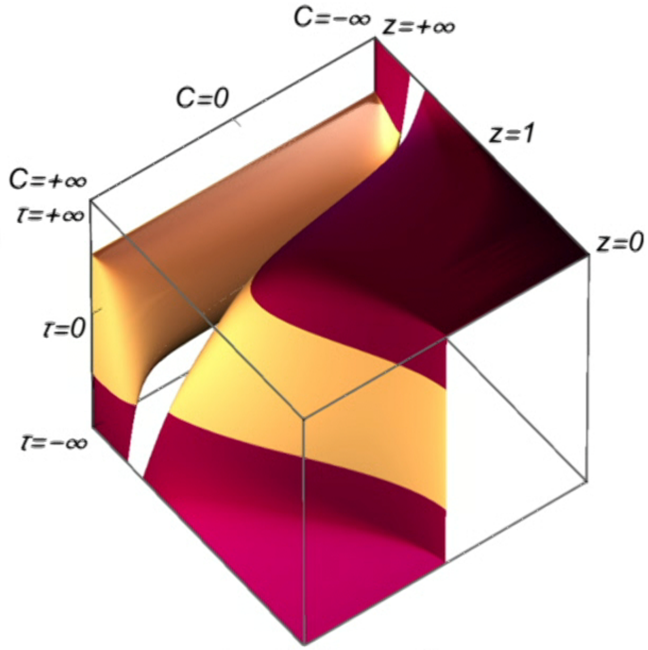
$$\mu^2 = \frac{\sigma(\sigma')^2}{\mathscr{P}[\sqrt{\sigma/m^2}]} \quad \Rightarrow \quad \sigma' \neq 0 \;\; \text{unless} \;\; \mathscr{P}[\sqrt{\sigma/m^2}] = 0$$



The 'forbidden region'

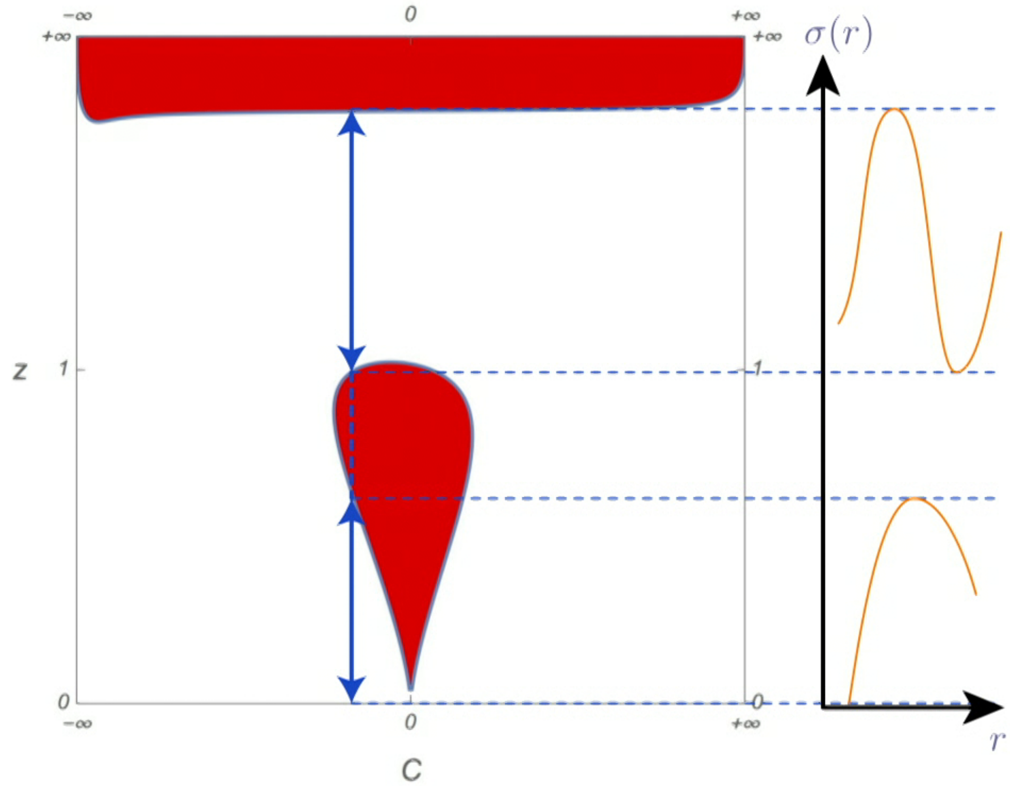


λ=−1, m>0



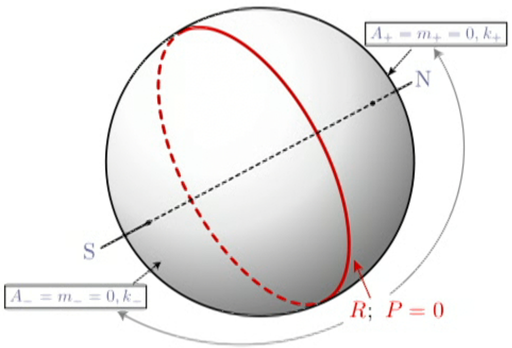
λ=0.1, m>0

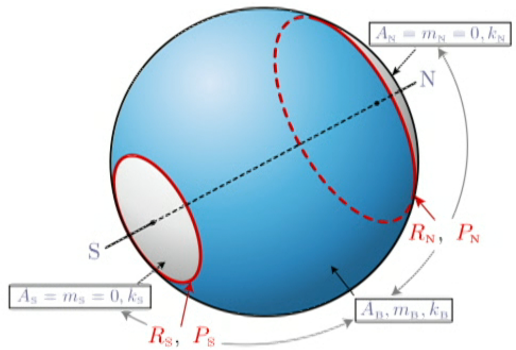
$$z = \frac{\sqrt{\sigma}}{|m|}, \qquad C = \frac{A}{2\,m^2}, \qquad \tau = |m|\,\langle p\rangle\,, \qquad \lambda = m^2\Lambda\,.$$



$\lambda = 0.1 \quad \tau = 0.3 \quad m > 0$

Introduction of matter: thin shells of dust





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Sources for the Hamiltonian and Diffeomorphism constraints:

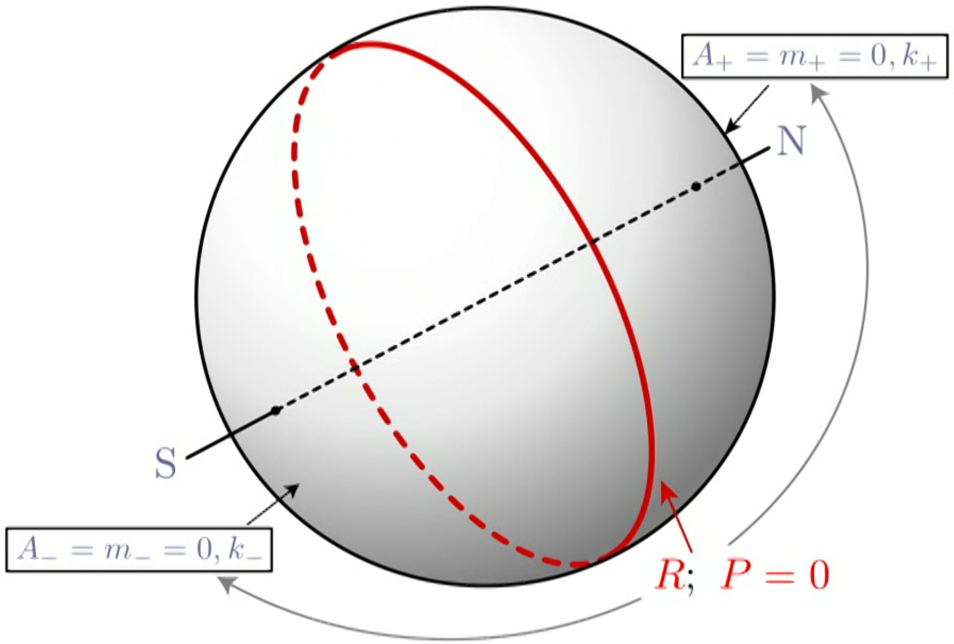
$$\int \mathcal{H}\, d\theta d\phi + 4\pi\, \delta(r-R)\sqrt{g^{rr}\, P^2 + M^2} \approx 0\,,$$
$$\int \mathcal{H}_i\, d\theta d\phi + 4\pi\, \delta^r{}_i \delta(r-R)\, P \approx 0\,,$$

imply two 'jump conditions':

$$\lim_{r\to R^+} \sigma'(r) - \lim_{r\to R^-} \sigma'(r) = -\tfrac{1}{2}\sqrt{P^2 + M^2\mu^2(R)}\,,$$

$$A_+ - A_- = -\frac{\sigma^{\frac{1}{2}}(R)}{2\mu(R)} P\,,$$

Single Shell



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Single-shell universe - II

We only have two regions, call them $+$ $(r > R)$ and $-$ $(r < R)$.

Regularity at the poles implies $m_\pm = A_\pm = 0$.

Then the diffeo jump condition $A_+ - A_- \propto P = 0$ kills P,

and the fundamental equation is identical on the two sides:

$$\mu(r) = \begin{cases} \dfrac{|\sigma'|}{\sqrt{4\,\sigma - \frac{1}{9}(12\,\Lambda - \langle p\rangle^2)\sigma^2}} & r < R \\ \dfrac{|\sigma'|}{\sqrt{4\,\sigma - \frac{1}{9}(12\,\Lambda - \langle p\rangle^2)\sigma^2}} & r > R \end{cases}$$

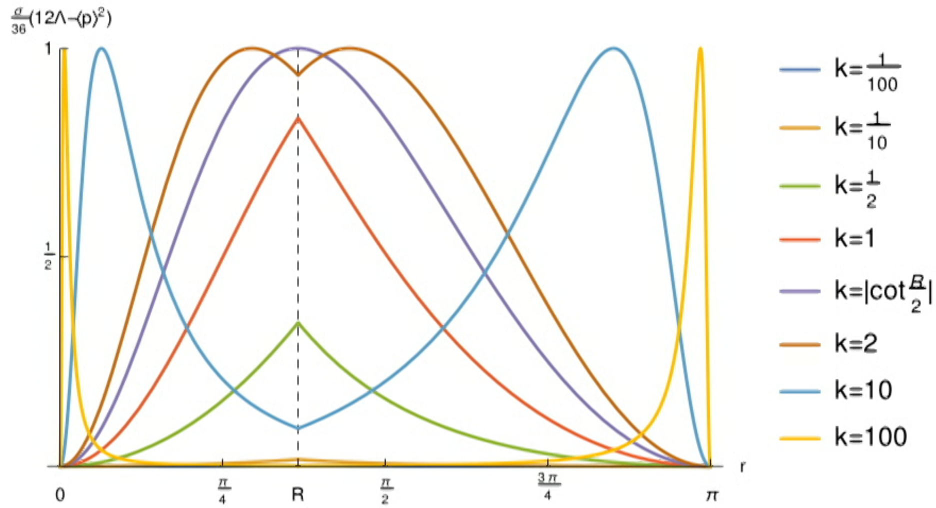
so to have μ continuous: $|\lim_{r\to R^+} \sigma(r)| = |\lim_{r\to R^-} \sigma(r)|$.

Only possibility: $\lim_{r\to R^+} \sigma'(r) = -\lim_{r\to R^-} \sigma'(r)$.

Single-shell universe - III

Exact solution of fundamental Eq. in isotropic gauge $\sigma = \mu^2 \sin^2 r$:

$$\sigma = \frac{36}{12\Lambda - \langle p \rangle^2} \times \begin{cases} 1 - \left(\frac{1 - k^2 \tan^2 \frac{r}{2}}{1 + k^2 \tan^2 \frac{r}{2}} \right)^2 & \textit{for} \quad r < R \\ 1 - \left(\frac{k^2 - \cot^4 \frac{R}{2} \tan^2 \frac{r}{2}}{k^2 + \cot^4 \frac{R}{2} \tan^2 \frac{r}{2}} \right)^2 & \textit{for} \quad r > R \end{cases}$$



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Single-shell universe - IV

Four dynamical variables: $\langle p \rangle$, P, R and k and **two constraints:**

$$P \approx 0 \quad \text{and} \quad h(R,k,\langle p \rangle) = |\sigma'(R)| - \tfrac{1}{4}\sqrt{P^2 + M^2 \mu^2(R)}\,,$$

Isotropic-gauge symplectic potential: $\theta = \int p^{ij}\delta g_{ij} \approx -\frac{2}{3} V \delta \langle p \rangle - 4\pi R \delta P$,

$$V = \frac{1728\,\pi}{(12\Lambda - \langle p \rangle^2)^{3/2}} \left\{ \tan^{-1}\left[k \tan\left(\tfrac{R}{2}\right)\right] - \frac{k\tan\left(\frac{R}{2}\right)\left(1 - k^2 \tan^2\left(\frac{R}{2}\right)\right)}{\left(k^2\tan^2\left(\frac{R}{2}\right) + 1\right)^2} \right\}.$$

The symplectic form can be calculated exactly:
$\{f, g\} = \partial_i f\, \omega^{-1}(R,k)^{ij}\, \partial_j g$, where

And **the two constraints are first-class:**

$$\{h(R,k,\langle p \rangle), P\} = -\frac{\cot R}{2\pi} h(R,k,\langle p \rangle) \approx 0\,.$$

Single-shell universe - V

4D phase space with 2 first-class constraints. One needs to be **gauge-fixed** and the other **generates dynamical evolution**. An obvious gauge-fixing for $P \approx 0$ is $R = \bar{R} =$ *const.* Moreover h is first-class wrt this gauge-fixing.

Replacing $P = 0$ and $R = \bar{R}$ in the leftover Hamiltonian constraint $h \approx 0$:

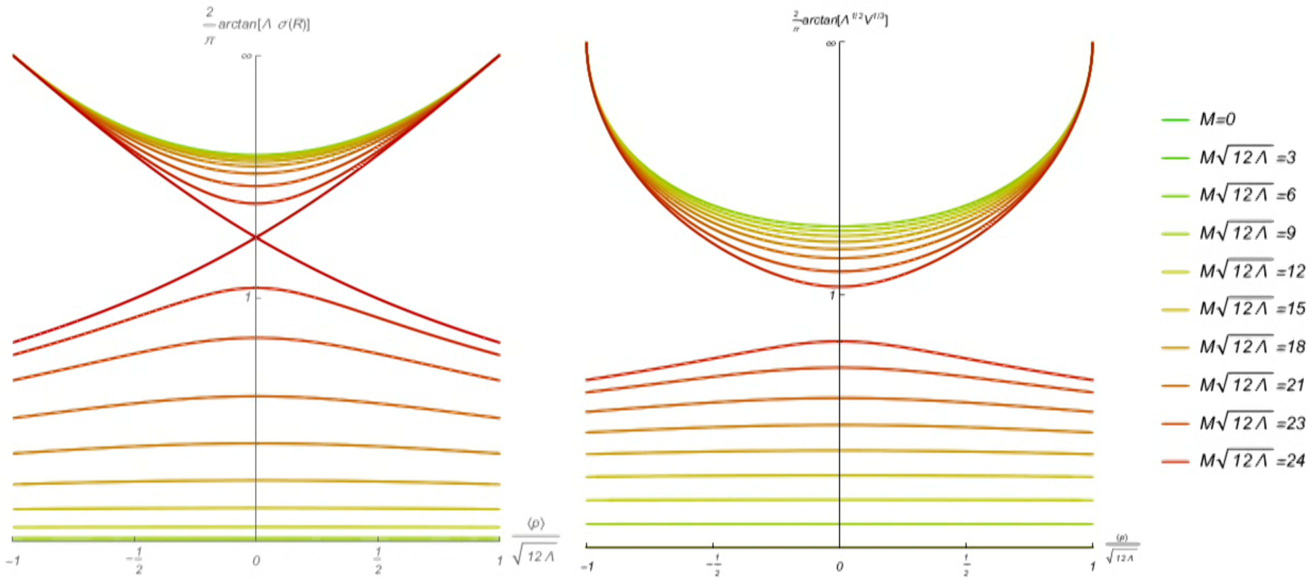
$$6\sqrt{\frac{k^2}{12\Lambda - \langle p \rangle^2}}\left(\frac{M}{\sqrt{2\cos^4\frac{\bar{R}}{2}\left(k^2\tan^2\frac{\bar{R}}{2}+1\right)^2}} - 48\sqrt{\frac{k^2}{12\Lambda - \langle p \rangle^2}}\frac{\cos\frac{\bar{R}}{2}\left(k^2-\cot^2\frac{\bar{R}}{2}\right)}{\sin^3\frac{\bar{R}}{2}\left(k^2+\cot^2\frac{\bar{R}}{2}\right)^3}\right) \approx 0,$$

the above equation has real roots only if

$$M^2 < \frac{24^2}{12\Lambda - \langle p \rangle^2}.$$

Single-shell universe - VI

Plot of $\sigma(\bar{R})$ and the spatial volume V vs. York time $\langle p \rangle$:



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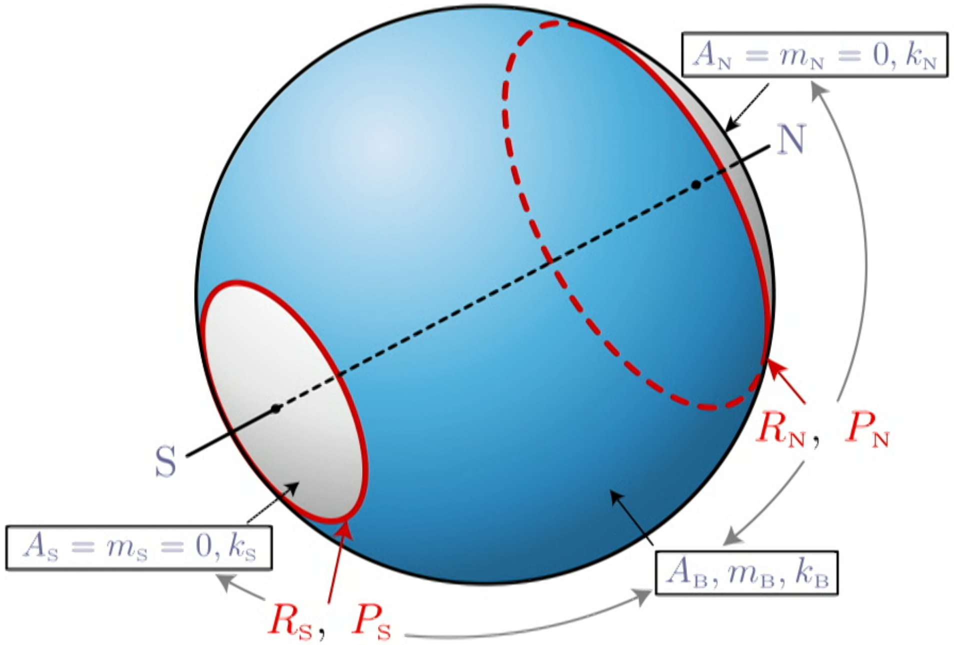
Two Shells



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Twin Shell universe - II

The following two jump conditions ($a = S, N$):

$$\lim_{r \to R_a^+} \sigma'(r) - \lim_{r \to R_a^-} \sigma'(r) = -\tfrac{1}{2}\sqrt{P_a^2 + M_a^2 \mu^2(R_a)}\,,$$

together with the equation relating μ^2, σ and σ' calculated at $r = R_a$, give us two constraints:

$$\boxed{\frac{M_a^4}{16} + 4A_{\mathsf{B}}^2\left(T\rho_a^2 - 4\right) + M_a^2\rho_a\left(T\rho_a^3 - 4\rho_a - 2X\right) + 16X^2\rho_a^2 \approx 0}$$

where $T = \frac{1}{9}\left(12\Lambda - \langle p \rangle^2\right)$, $X = \frac{1}{6}\langle p \rangle A_{\mathsf{B}} - 2\,m_{\mathsf{B}}$ and $\rho_a^2 = \sigma(R_a)$.

Twin Shell universe - III

Rescaling everything with the Misner–Sharp mass of the 'belt' region:

$$C = \frac{A_{\mathsf{B}}}{2\, m_{\mathsf{B}}^2}\,, \quad \tau = |m_{\mathsf{B}}|\, \langle p \rangle\,, \quad \lambda = m_{\mathsf{B}}^2\, \Lambda\,, \quad z_a = \frac{\rho_a}{|m_{\mathsf{B}}|}\,, \quad M_a = |m_{\mathsf{B}}|\, \mu_a\,.$$

we get a polynomial equation similar to the 'fundamental polynomial':

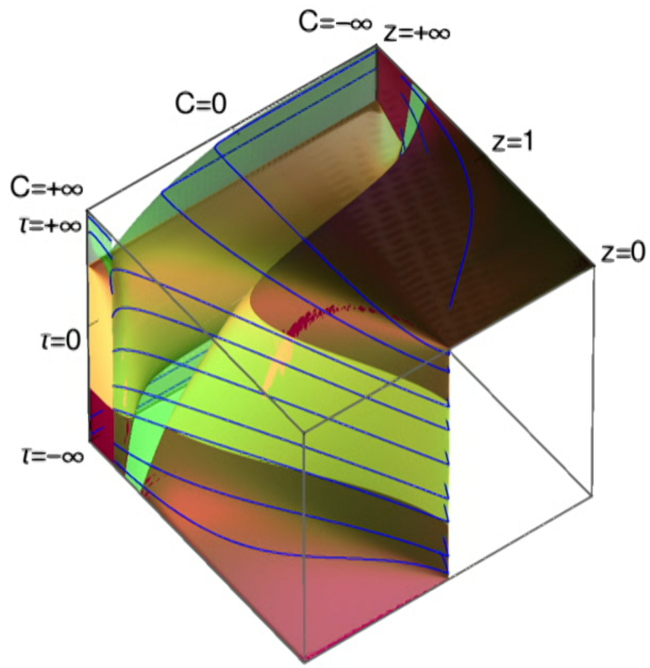
$$\frac{\mu_a^4}{16} + \mu_a^2 z_a \left[\pm 4 - \tfrac{2}{3} C\tau - \tfrac{1}{9} z_a^3 \left(\tau^2 - 12\lambda\right) - 4 z_a\right]$$

$$-\frac{64}{3}\left[\pm C\tau z_a^2 - C^2\left(\lambda z_a^2 - 3\right) - 3 z_a^2\right] = 0\,,$$

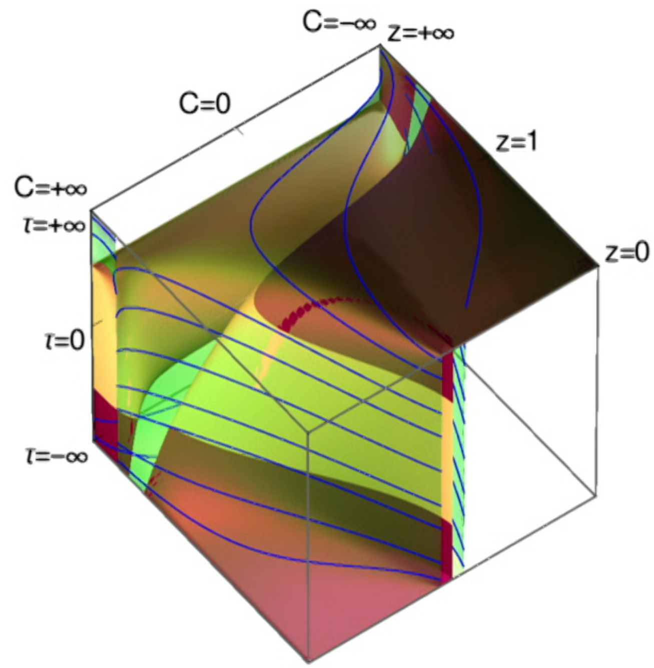
Its solutions can be represented in the same 3D space (z, C, τ) as the 'forbidden region'. I call them the **'on-shell surfaces'.**

REMARKABLY, THE ON-SHELL SURFACES NEVER ENTER THE 'FORBIDDEN REGION'.

The on-shell surfaces

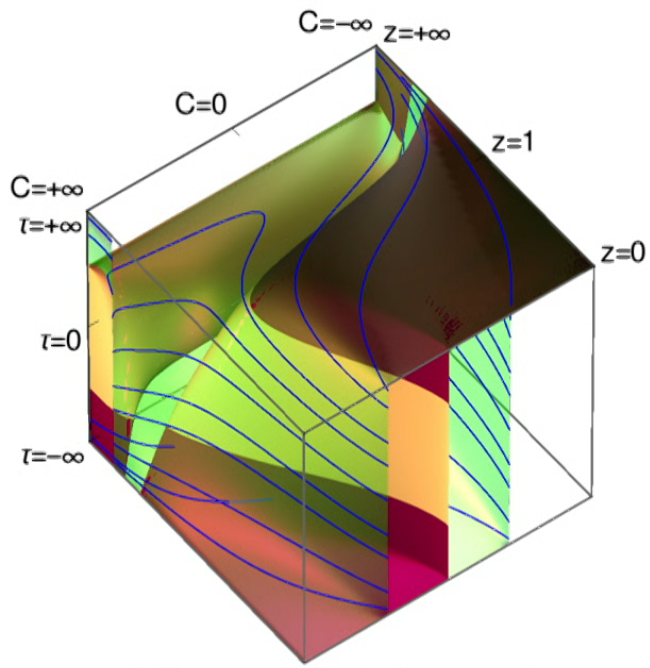


M/m$_B$=0.01, λ=0.1, m$_B$>0

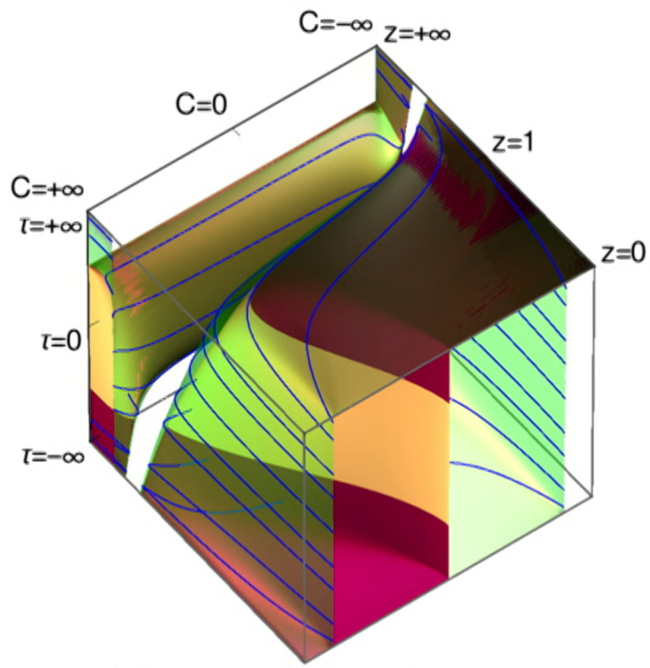


M/m$_B$=2, λ=0.1, m$_B$>0

The on-shell surfaces

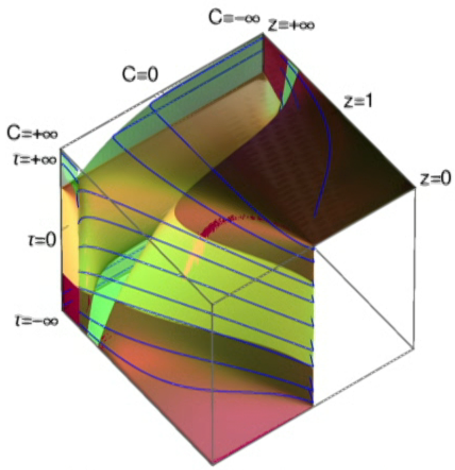


M/m_B=5, λ=0.1, m_B>0

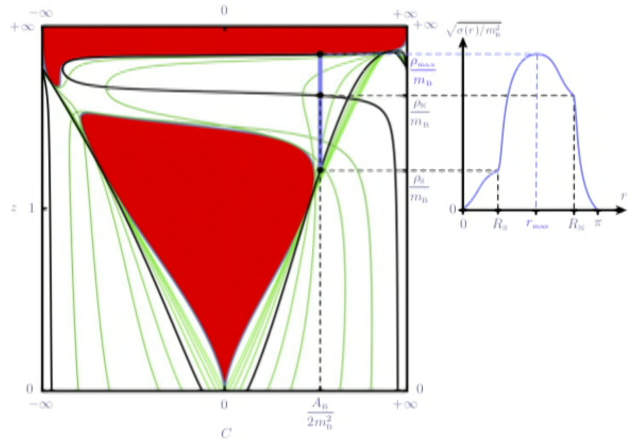


M/m_B=10, λ=0.1, m_B>0

How to use the on-shell surface diagram - I



M/m_B=0.01, λ=0.1, m_B>0



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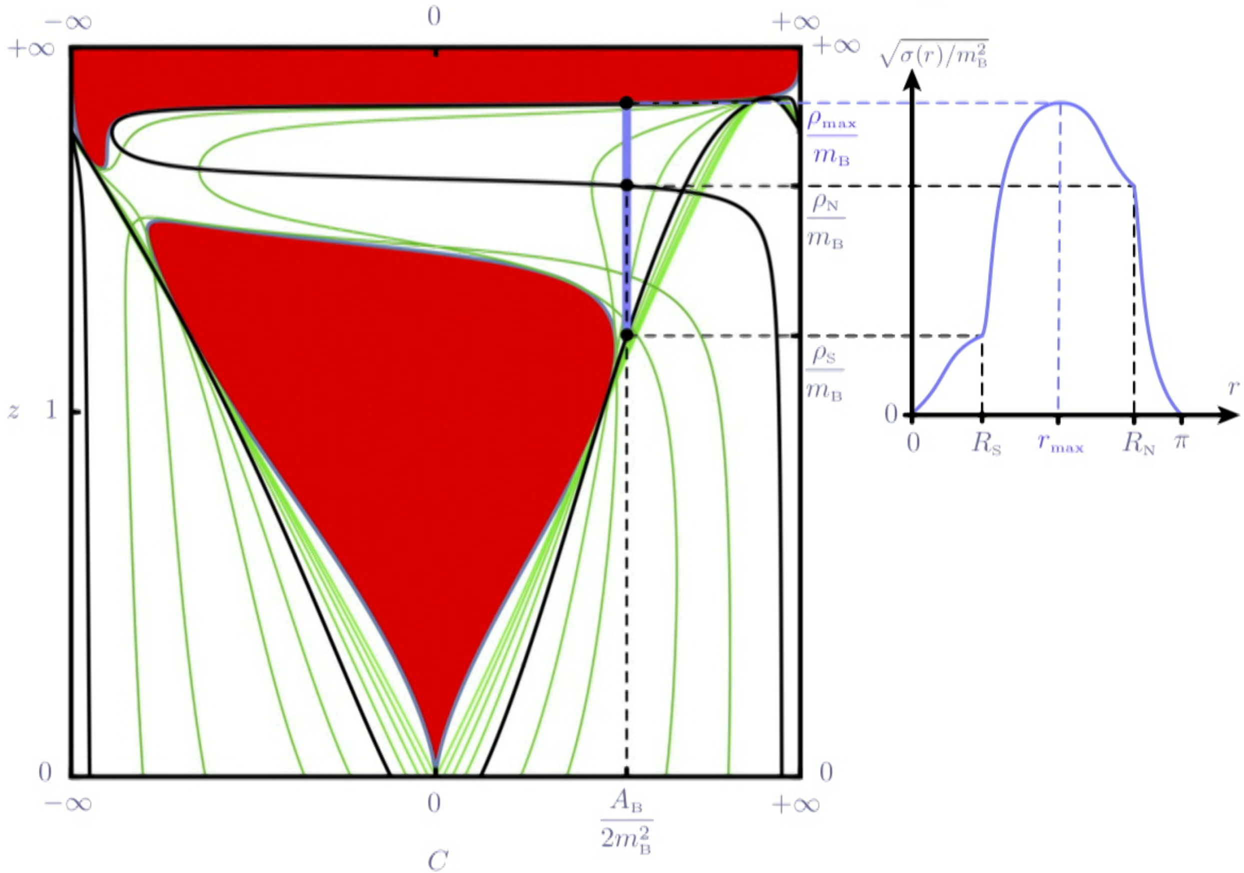
How to use the on-shell surface diagram - II



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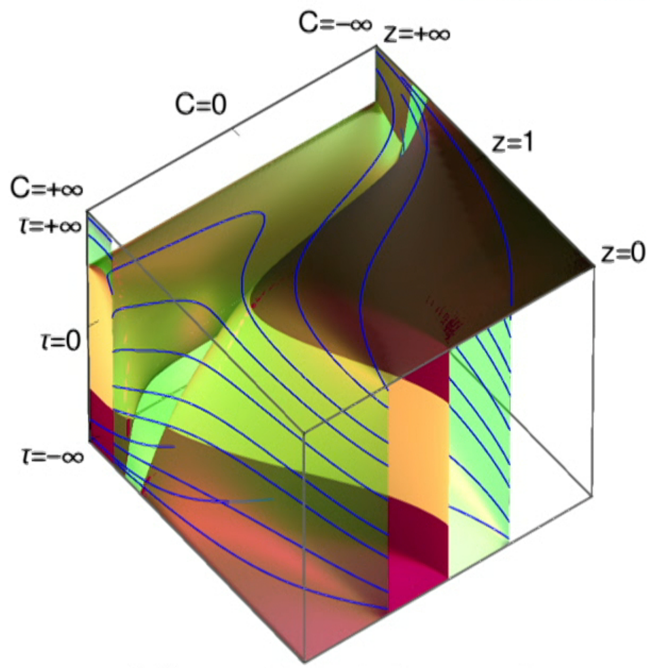
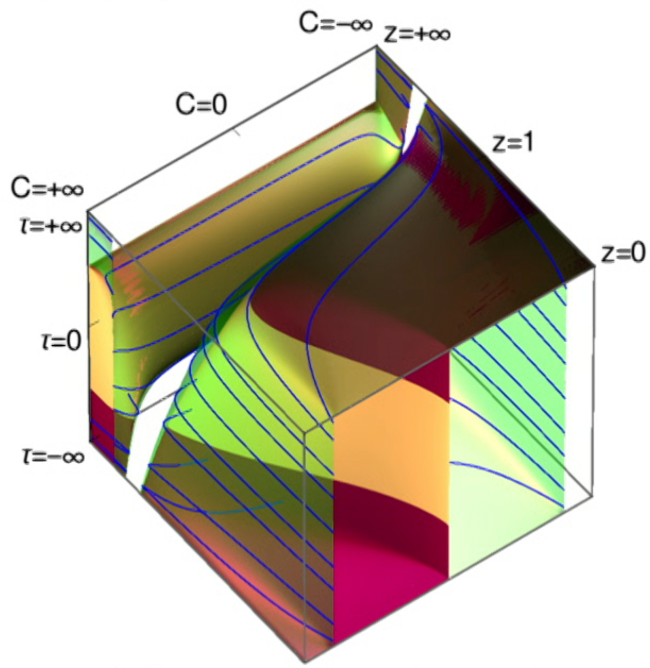
The on-shell surfaces

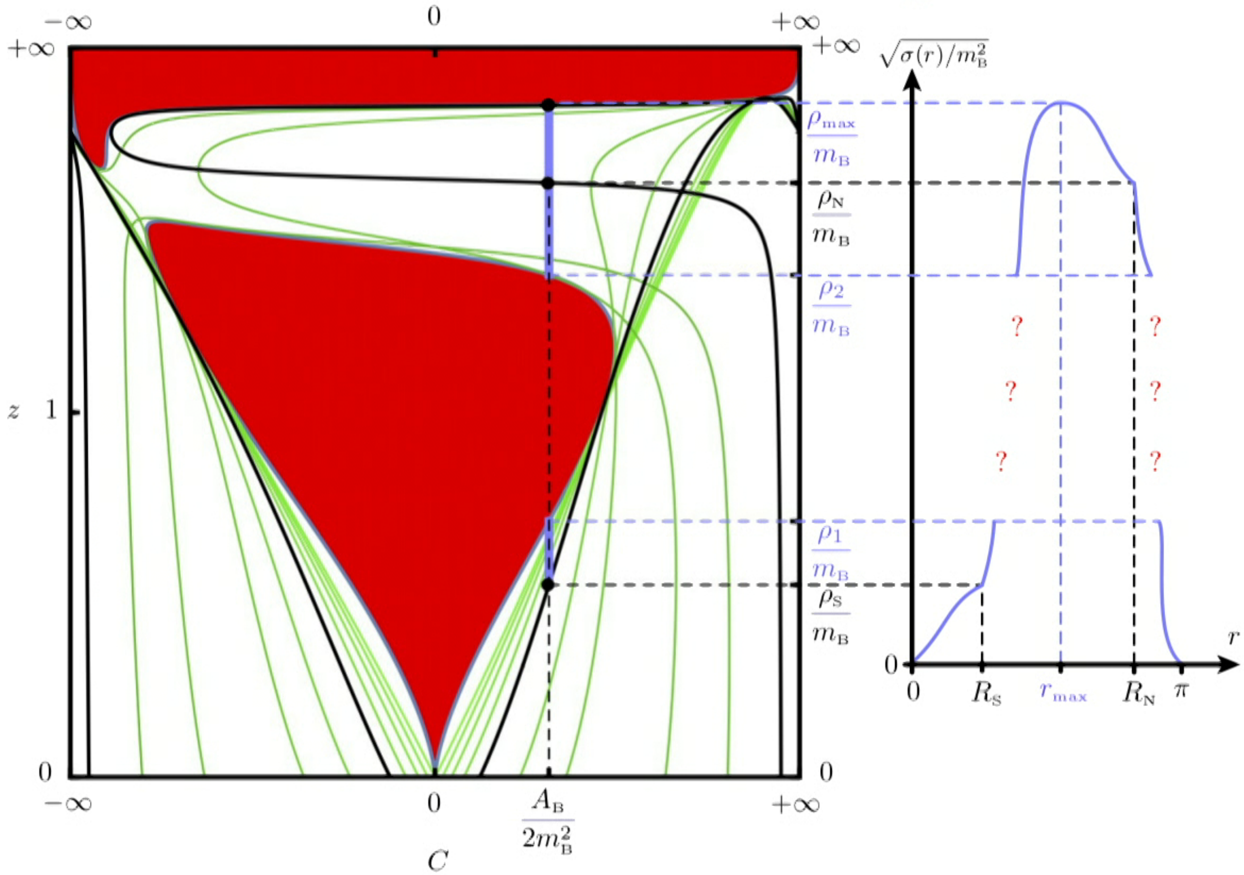


M/m_B



$M/m_B=10, \lambda=0.1, m_B>0$

How to use the on-shell surface diagram - III



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