Title: Relationalism and the speed of light: Are we in a relationship?

Date: May 15, 2017 02:45 PM

URL: http://pirsa.org/17050046

Abstract: Most practical studies in Shape Dynamics involve an N-body Newtonian interaction which is described by a homogeneous potential. This property allows one to proof several interesting features like the emergence of an arrow of time. However, more generic interactions are not described by these kind of potentials and introduce additional dimensionful coupling constants. Thus, it is an open question whether more generic interactions can be written in a fully relational manner. By studying the concrete example of the gravitational Weber interaction which is, in a sense, a more realistic theory of gravity, we show that it is possible translate non-Newtonian interactions, which have inhomogeneous potentials and additional coupling constants, into a relational language. This opens the door to study other interactions and may shed light into the relationalization of gravity as described by general relativity.

Relationalism and the speed of light

Are we in a relationship?

Yuri Bonder

Universidad Nacional Autónoma de México
bonder@nucleares.unam.mx

Shape Dynamics Workshop
May 15 – 17, 2017
Perimeter Institute

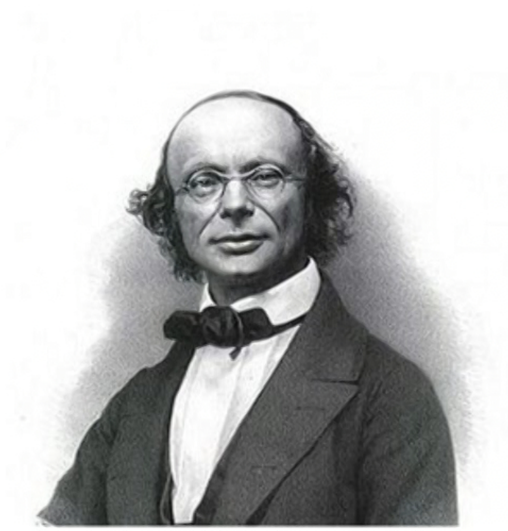
Questions:

- 'Relationalise' an interaction with an extra coupling constant?
- Does it add a shape space dimension?

Weber electrodynamics

Example: Weber electrodynamics (1846).

- No fields, forces and (charged) particles.
- Describes Coulomb, Ampère, Faraday, radiation.
- First link of light (c) and electrodynamics
- Initially Maxwell thought it was correct.
- It has been falsified.



Wilhelm Eduard Weber

Weber gravity

The force on particle 1 by particle 2 is

$$\vec{F}_{12} = \vec{r}_{12}\frac{-Gm_1 m_2}{r_{12}^3}\left(1 - \frac{3\dot{r}_{12}^2}{c^2} + \frac{6r_{12}\ddot{r}_{12}}{c^2}\right),$$

$$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2, \quad r_{12} = |\vec{r}_{12}|, \quad \dot{r}_{12} = \frac{dr_{12}}{dt} \neq \left|\frac{d\vec{r}_{12}}{dt}\right|, \quad \ddot{r}_{12} = \frac{d^2 r_{12}}{dt^2}.$$

- Completely relational (only relative dist/vel/accel).
- On a Newtonian spacetime.
- Additional dimensional object: c = speed of light.
- Homogeneous (invariant under rescalings of $\vec{r}$ and t)?

Weber gravity

The force on particle 1 by particle 2 is

$$\vec{F}_{12} = \vec{r}_{12} \frac{-Gm_1 m_2}{r_{12}^3} \left(1 - \frac{3\dot{r}_{12}^2}{c^2} + \frac{6 r_{12} \ddot{r}_{12}}{c^2} \right),$$

$$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2, \quad r_{12} = |\vec{r}_{12}|, \quad \dot{r}_{12} = \frac{dr_{12}}{dt} \neq \left| \frac{d\vec{r}_{12}}{dt} \right|, \quad \ddot{r}_{12} = \frac{d^2 r_{12}}{dt^2}.$$

- Invariant under global rotations and translations $\Rightarrow$
  N particles in 3 spatial dimensions, $3N - 6$ indep. r_{ij}.
- $\vec{F}_{12} = -\vec{F}_{21}$, central, and conservative $\Rightarrow$ $\vec{P}$, $\vec{L}$, E conserved.
- GR phenomena: perihelion precession, "radiation," UFF.
- Newtonian limit: $c \to \infty$.
- Falsifiable: it does not affect light $\Rightarrow$ toy model.

Weber gravity

The force on particle 1 by particle 2 is

$$\vec{F}_{12} = \vec{r}_{12}\frac{-Gm_1 m_2}{r_{12}^3}\left(1 - \frac{3\dot{r}_{12}^2}{c^2} + \frac{6r_{12}\ddot{r}_{12}}{c^2}\right),$$

$$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2, \quad r_{12} = |\vec{r}_{12}|, \quad \dot{r}_{12} = \frac{dr_{12}}{dt} \neq \left|\frac{d\vec{r}_{12}}{dt}\right|, \quad \ddot{r}_{12} = \frac{d^2 r_{12}}{dt^2}.$$

- Invariant under global rotations and translations $\Rightarrow$
  N particles in 3 spatial dimensions, $3N - 6$ indep. r_{ij}.
- $\vec{F}_{12} = -\vec{F}_{21}$, central, and conservative $\Rightarrow$ $\vec{P}$, $\vec{L}$, E conserved.
- GR phenomena: perihelion precession, "radiation," UFF.
- Newtonian limit: $c \to \infty$.
- Falsifiable: it does not affect light $\Rightarrow$ toy model.

Getting relational: the algorithm

Basic steps (that minimize the calculations):

1. Lagrangian/Hamiltonian
2. (First order) EOM and configuration space metric
3. Length scale and intrinsic parametrization
4. EOM for intrinsic parameter
5. Normalization (dimensionless coordinates)
6. Choose an extra dimensionless variable
7. Find EOM for this quantity, check if system closes
8. If not, promote its derivative as a new DOF and return to 7

Weber Lagrangian and metric

- Weber Lagrangian

$$L = \frac{1}{2}\sum_i m_i \dot{\vec{r}}_i^{\,2} - U, \quad U = \frac{-G}{2}\sum_{i,j|i\neq j}\frac{m_i m_j}{r_{ij}}\left(1 + \frac{3\dot{r}_{ij}^2}{c^2}\right).$$

- In term of the Newtonian potential U_{N} (and $\alpha, \beta = 1, 2, 3$):

$$\begin{aligned} L &= \frac{1}{2}\sum_{i,j,\alpha,\beta} g_{ij\alpha\beta}(x)\dot{x}_i^\alpha \dot{x}_j^\beta - U_{\mathrm{N}}(x), \\ g_{i\alpha i\beta} &= m_i\delta_{\alpha\beta} + \sum_{k|k\neq i}\frac{6Gm_i m_k}{c^2 r_{ik}^3}(x_i^\alpha - x_k^\alpha)(x_i^\beta - x_k^\beta), \\ g_{i\alpha j\beta} &= -\frac{6Gm_i m_j}{c^2 r_{ij}^3}(x_i^\alpha - x_j^\alpha)(x_i^\beta - x_j^\beta), \quad i \neq j. \end{aligned}$$

- $g_{i\alpha j\beta}$ is not homogeneous!

Weber Lagrangian and metric

- Weber Lagrangian

$$L = \frac{1}{2}\sum_i m_i \dot{\vec{r}}_i^2 - U, \quad U = \frac{-G}{2}\sum_{i,j|i\neq j} \frac{m_i m_j}{r_{ij}}\left(1 + \frac{3\dot{r}_{ij}^2}{c^2}\right).$$

- In term of the Newtonian potential $U_{\rm N}$ (and $\alpha, \beta = 1, 2, 3$):

$$\begin{aligned} L &= \frac{1}{2}\sum_{i,j,\alpha,\beta} g_{ij\alpha\beta}(x)\dot{x}_i^\alpha \dot{x}_j^\beta - U_{\rm N}(x), \\ g_{i\alpha i\beta} &= m_i\delta_{\alpha\beta} + \sum_{k|k\neq i} \frac{6Gm_i m_k}{c^2 r_{ik}^3}(x_i^\alpha - x_k^\alpha)(x_i^\beta - x_k^\beta), \\ g_{i\alpha j\beta} &= -\frac{6Gm_i m_j}{c^2 r_{ij}^3}(x_i^\alpha - x_j^\alpha)(x_i^\beta - x_j^\beta), \quad i \neq j. \end{aligned}$$

- $g_{i\alpha j\beta}$ is not homogeneous!

Weber Lagrangian and metric

- For simplicity, all particles have mass m.
- More general: m average mass; relative weights in g_{ab}.
- The Lagrangian can be written as (a, b config. space indices)

$$L = T - U_{\mathrm{N}}, \qquad T = \frac{m}{2} g_{ab}(x) \dot{x}^a \dot{x}^b,$$

and U_{N} is the total Newtonian potential.

- g_{ab} is dimensionless.
- Formally, trivial to get the Hamiltonian (no constraints)

$$H = \frac{1}{2m} g^{ab}(x) p_a p_b + U_{\mathrm{N}}, \qquad p_a = \frac{\partial L}{\partial \dot{x}^a}.$$

- The energy $E = H[x, p(x, \dot{x})] = T + U_{\mathrm{N}}$ is conserved.

Equations of motion

- As expected, the EOM are

$$\ddot{x}^a + \Gamma^a_{bc}\dot{x}^b\dot{x}^c = \frac{F^a}{m}, \qquad \Gamma^a_{bc} = \frac{1}{2}g^{ad}\left(\frac{\partial g_{bd}}{\partial x^c} + \frac{\partial g_{dc}}{\partial x^b} - \frac{\partial g_{bc}}{\partial x^d}\right),$$

$$F^a = -g^{ab}\frac{\partial U_{\mathrm{N}}}{\partial x^b},$$

- Let $v^a = \dot{x}^a$, then

$$\begin{aligned} \dot{v}^a &= -\Gamma^a_{bc}v^b v^c + \frac{F^a}{m}, \\ \dot{x}^a &= v^a. \end{aligned}$$

Length scale and intrinsic parametrization

- Length scale: $R^2 = g_{ab}x^a x^b$.
- Dimensionless parameter:

$$R^2 ds^2 = g_{ab}dx^a dx^b = g_{ab}v^a v^b dt^2.$$

then

$$\frac{dt}{dv} = \frac{R}{|v|}, \qquad |v| = \sqrt{g_{ab}v^a v^b}.$$

where sign is chosen s.t. s grows in the direction t does.

- In terms of s

$$\begin{aligned} \frac{dv^a}{ds} &= \frac{R}{|v|}\left(-\Gamma^a_{bc}v^b v^c + \frac{F^a}{m}\right), \\ \frac{dx^a}{ds} &= R\frac{v^a}{|v|}, \end{aligned}$$

Normalization

- Let $u^a = v^a/|v|$ and $q^a = x^a/R$, then

$$\begin{aligned}
\frac{du^a}{ds} &= \frac{RF^a}{m|v|^2} - R\Gamma^a_{bc}u^b u^c - \frac{u^a}{|v|}\frac{d|v|}{ds}, \\
\frac{dq^a}{ds} &= u^a - \frac{q^a}{R}\frac{dR}{ds}.
\end{aligned}$$

- Is the q^a equation purely kinematical?
- Need another dimensionless quantity.

Additional dimensionless variable

- The new dimensionless quantity must be a quotient of

$$G,\ m,\ c,\ E,\ R,\ |v|.$$

- Convenient not to use R and $|v|$ at the same time (we have dR/ds and $d|v|/ds$). We use R.
- We do not use E to single out the $E = 0$ case.
- Use the ratio of gravitational and rest energies.
- Let $\kappa = Gm/Rc^2$, then

$$\frac{d\kappa}{ds} = -\frac{Gm}{R^2c^2}\frac{dR}{ds} = -\frac{\kappa}{R}\frac{dR}{ds}.$$

- Explicitly

$$\frac{1}{R}\frac{dR}{ds} \quad = \quad g_{da}R\Gamma^d_{bc}q^a q^b u^c + g_{ab}q^a u^b.$$

Additional dimensionless variable

- The new dimensionless quantity must be a quotient of

$$G,\ m,\ c,\ E,\ R,\ |v|.$$

- Convenient not to use R and $|v|$ at the same time (we have dR/ds and $d|v|/ds$). We use R.
- We do not use E to single out the $E = 0$ case.
- Use the ratio of gravitational and rest energies.
- Let $\kappa = Gm/Rc^2$, then

$$\frac{d\kappa}{ds} = -\frac{Gm}{R^2c^2}\frac{dR}{ds} = -\frac{\kappa}{R}\frac{dR}{ds}.$$

- Explicitly

$$\frac{1}{R}\frac{dR}{ds} = g_{da}R\Gamma^d_{bc}q^a q^b u^c + g_{ab}q^a u^b.$$

Normalization

- Let $u^a = v^a/|v|$ and $q^a = x^a/R$, then

$$\begin{aligned} \frac{du^a}{ds} &= \frac{RF^a}{m|v|^2} - R\Gamma^a_{bc}u^b u^c - \frac{u^a}{|v|}\frac{d|v|}{ds}, \\ \frac{dq^a}{ds} &= u^a - \frac{q^a}{R}\frac{dR}{ds}. \end{aligned}$$

- Is the q^a equation purely kinematical?
- Need another dimensionless quantity.

- Still need to deal with $d|v|/ds$ and $R\Gamma^c_{ab}$.
- Since $g_{ab} = \delta_{ab} + \kappa\delta g_{ab}(q)$, then

$$R\Gamma^c_{ab} = \kappa\gamma^c_{ab}(q).$$

- $V = V(q)$ dimensionless and s.t.

$$U_{\mathrm{N}} = \frac{-Gm^2 V}{R} = -\kappa mc^2 V \;\Rightarrow\; RF^a = \kappa mc^2 f^a(q).$$

- Using $T = m|v|^2/2$:

$$\begin{aligned} |v|^2 &= \frac{2T}{m} = \frac{2(E + \kappa mc^2 V)}{m}, \\ \frac{1}{|v|}\frac{d|v|}{ds} &= \frac{-1}{m|v|^2}\frac{dU_{\mathrm{N}}}{ds} = \frac{g_{ab} f^a u^b}{2[(E/\kappa mc^2) + V]}. \end{aligned}$$

- $V(q)$, $f^a(q)$ and $\gamma^c_{ab}(q)$ are known and dimensionless.

EOM: Does the system close?

$$
\begin{aligned}
\frac{du^a}{ds} &= \frac{f^b(\delta^a_b - g_{bc}u^a u^c)}{2[(E/\kappa mc^2) + V]} - \kappa\gamma^a_{bc}u^b u^c, \\
\frac{dq^a}{ds} &= u^b(\delta^a_b - g_{bc}q^a q^c) - \kappa g_{be}\gamma^e_{cd}q^a q^b q^c u^d, \\
\frac{d\kappa}{ds} &= -\kappa g_{ab}q^a u^b - \kappa^2 g_{da}\gamma^d_{bc}q^a q^b u^c.
\end{aligned}
$$

- The system closes!!!
- No need additional variables.
- Same number of DOFs than the Newtonian case.
- DOFs: $\{q, p, \kappa\}$ minus 2 constraints $|q| = 1 = |u|$.

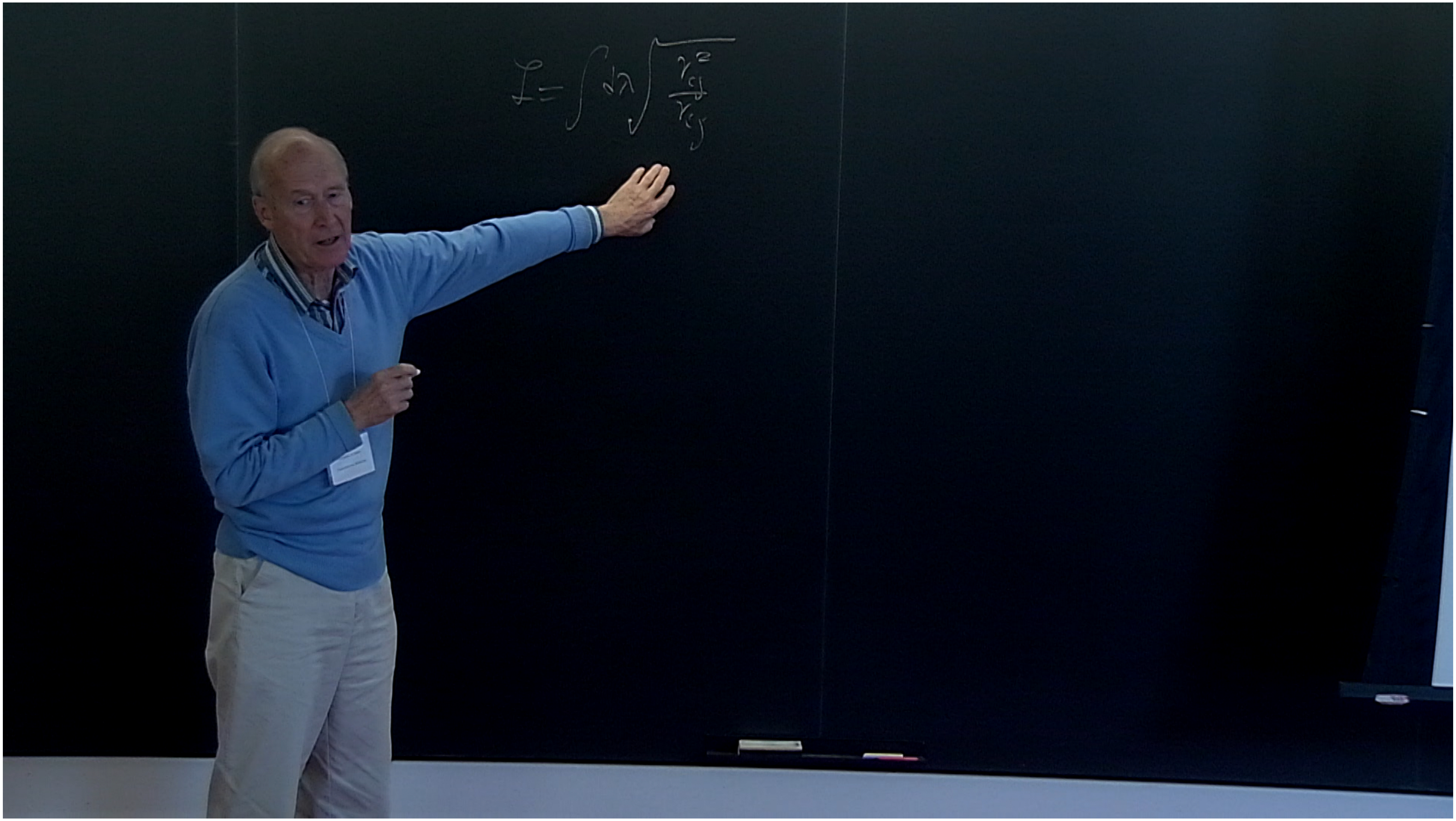
EOM: Further comments

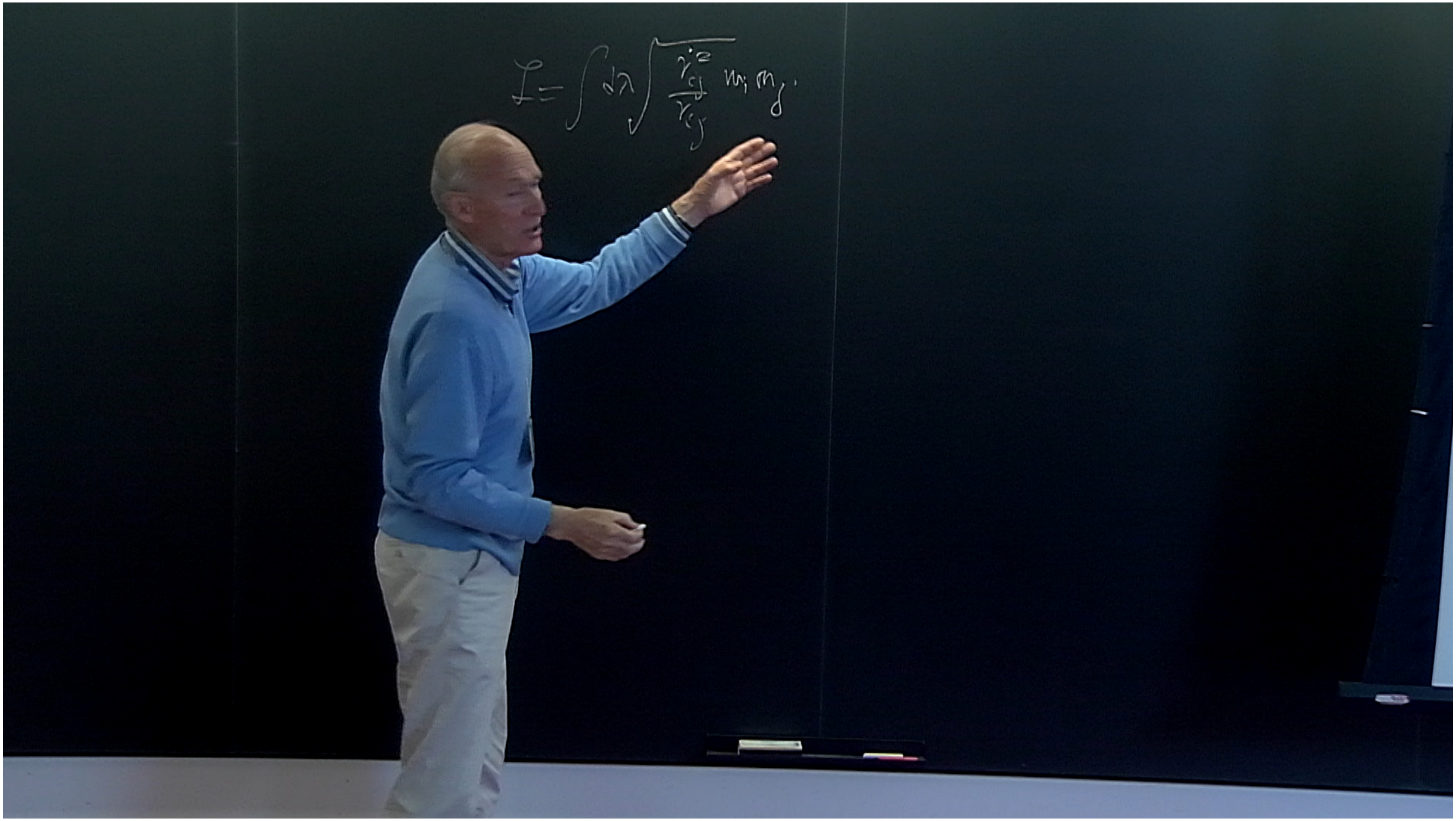
$$
\begin{aligned}
\frac{du^a}{ds} &= \frac{f^b(\delta^a_b - g_{bc}u^a u^c)}{2[(E/\kappa mc^2) + V]} - \kappa\gamma^a_{bc}u^b u^c, \\
\frac{dq^a}{ds} &= u^b(\delta^a_b - g_{bc}q^a q^c) - \kappa g_{be}\gamma^e_{cd}q^a q^b q^c u^d, \\
\frac{d\kappa}{ds} &= -\kappa g_{ab}q^a u^b - \kappa^2 g_{da}\gamma^d_{bc}q^a q^b u^c.
\end{aligned}
$$

- Nothing special when $E = 0$.
- The first terms in the u^a and q^a equations have projectors.
- The last terms in these equations keep the normalization.
- After all, the q^a equation is purely kinematical.

Lessons

- There is an algorithm to put (pre-relational) theories in a relational language.
- The effects of c can be described in a fully relational way.
- New dimensional coupling constant do not change the relational equations structure.
- In fact, having c was useful (κ definition).
- Other interactions may be studied, e.g., a relational MOND.


$\mathcal{L} = \int d\lambda \sqrt{\frac{r_{ij}^2}{r_{ij}}}$



$\mathcal{L} = \int d\lambda \sqrt{\frac{\dot{r}_{ij}^2}{r_{ij}}} \, m_i m_j \, .$