Title: The quantum equation of state of the universe produces a small cosmological constant

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Abstract: Relationalism is the strict disentanglement of physical law from the definition of physical object. This can be formalized in the shape dynamcis postulate that the objective evolution of the universe is described by an "equation of state of a curve in relational configuration space." The application of this postulate to General Relativity implies that gravity is described by an equation of state of a curve on conformal superspace. It turns out that the naive quantization of these equations of state introduces an undesired preferred time parametrization. However, it turns out that one can still describe the quantum evolution of the system as an equation of state of the Bohmian trajectory which remains manifestly parametrization independent. These quantum systems generically develop quasi-isolated bound states (atoms) that can be used as reference systems. It turns out that the system as a whole expands if described in units defined by these atoms. This produces phenomenological effects that are usually ascribed to the presence of a cosmological constant. This "effective cosmological constant" is however unaffected by vacuum energy. I pesent the formal argument for this statement and show this explicitly by remormalizing a scalar field coupled to shape dynamics.

Pure Shape Dynamics:

The quantum equation of state of the universe produces a small cosmological constant

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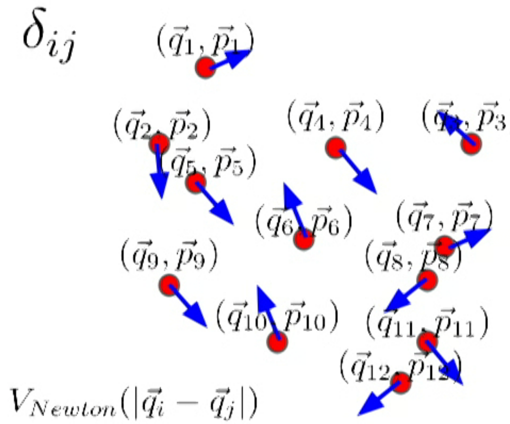

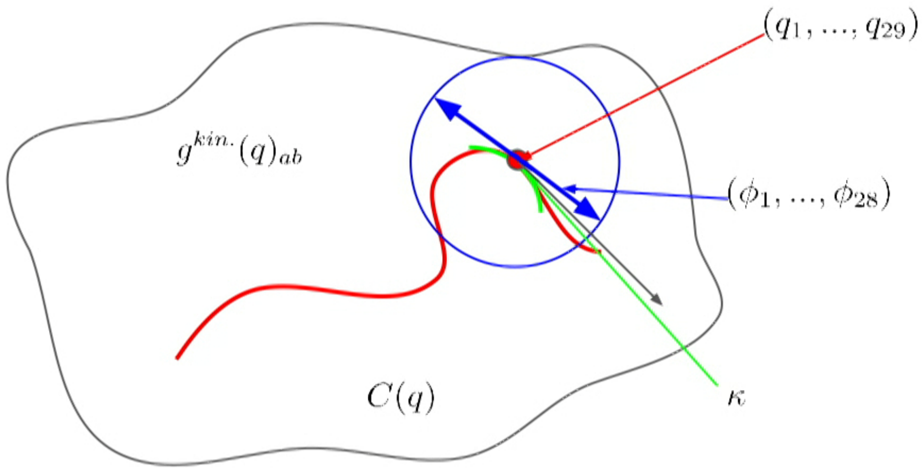

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Outline of the Argument

1. Identify relationalism as disentanglement of physical law from operational definition
   $\Rightarrow$ formalization of equation of state of geometry of curve in relational configuration (=”shape”)
space
2. Simplest application to quantum systems:
   Equation of state of geometric properties of Bohmian trajectory in shape space
3. Providing some technical background:
   a. Classical equation of state for Newtonian N-body problem
   b. Bohmian trajectories on shape space
   c. Simple quantum models, atoms, emergence of units, apparent expansion
   d. Well-defined Bohmian equations $\Rightarrow$ Renormalization
   e. Equation of state for ADM system
4. Cosmological model (Bianchi IX) with matter field (scalar field with local d.o.f.)
   $\Rightarrow$ renormalization of Bohmian e.o.m. on shape space leads constant term + curvature term as opposed to vacuum term + curvature term (as in the ADM representation)
   $\Rightarrow$ *vacuum energy does not gravitate in shape dynamics*

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Relationalism = Disentanglement of Law from Definition

Problem: Phys. objects are defined by their phys. properties, while phys. laws specify phys. properties of systems.
$\Rightarrow$ *"What part of the physical laws are actual predictions?"*

Simple example: Consider free particles and Newton's first law

In an *inertial system* free particles move along straight lines at constant *speed*.

Required definitions:
1. *Inertial system:* A system in which free particles move along straight lines.
2. *Speed:* requires a notion of intrinsic clock and rod

$\Rightarrow$ conventions: $\odot$ Coordinate origin = centre of mass
$\odot$ Frame = particle 1 defines direction x; orthog. comp. of particle 2 direction y
$\odot$ use motion of particle 3 to define unit speed
(due to dynamical similarity there is no independent notion of clock and rod)

$\Rightarrow$ Once we have conventions for all undefined quantities, we obtain the "true physical predictions" of the physical law

A relational description extracts the true predictions of physics.

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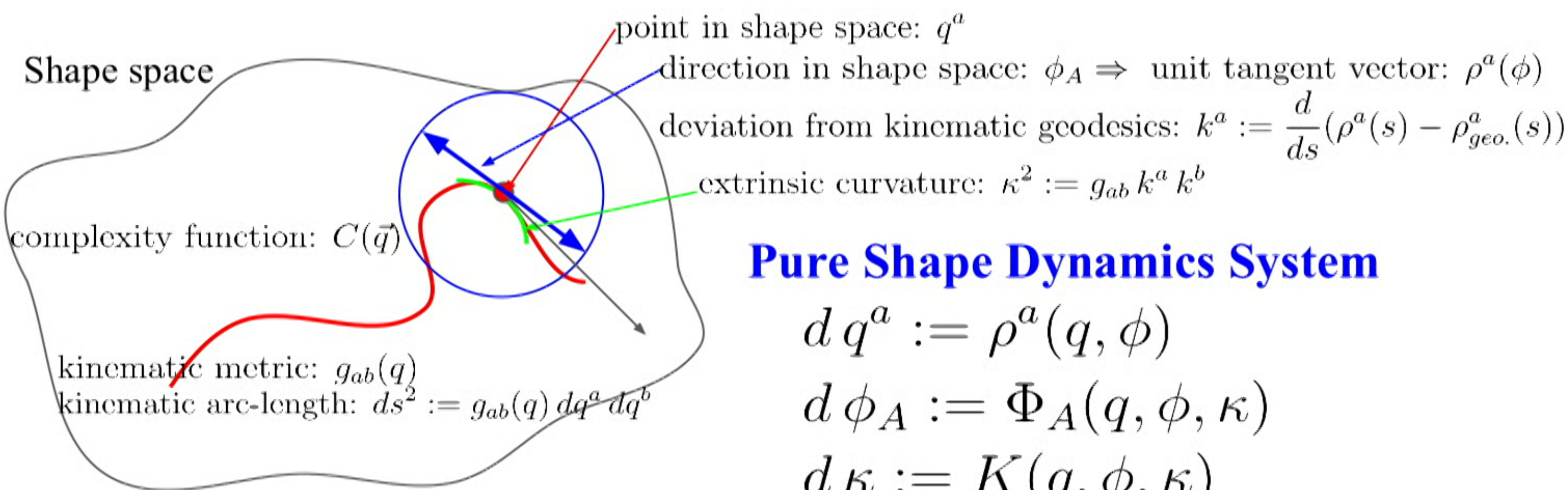
Objective Dynamics = Geom. of curve of relational config.

"All physical objects (including the reference objects used in conventions) are contained in the universe" $\Rightarrow$

1. only relational configurations exist, but no absolute configurations
2. only change exists, but no time parametrization

Theoretical dynamics of the universe is the *mathematical* description of an **un**-parametrized curve in **shape space**

Equation of state of geometry of a pure (i.e. unparametrized) curve in shape space



Pure Shape Dynamics System

$$d\, q^a := \rho^a(q, \phi)$$
$$d\, \phi_A := \Phi_A(q, \phi, \kappa)$$
$$d\, \kappa := K(q, \phi, \kappa)$$

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Example: Newtonian N-body universe (I)

We start with Newton's equation for N particles with Newtonian potential and E=0, (P=0, L=0):

$$\frac{d\,r_I^a}{d\,t} = \frac{\pi_I^a}{m} \quad \text{(assuming equal masses)} \quad \text{and} \quad \frac{d\,\pi_I^a}{d\,t} = -\frac{\partial V(r)}{\partial\, r_I^a} \quad \text{(assuming homogeneity of deg. -1)}$$

Coordinate change to decouple

size and duration from pre-shapes and their momenta:

$$R := \sqrt{\sum_I \vec{r}_I^{\,2}} \qquad D := \sum_I \vec{p}^I . \vec{r}_I \qquad q^a := \frac{r_I^j}{R} \qquad p_a := \left(\frac{\partial q^a}{\partial r_I^j}\right)^{-1} \pi_j^I$$

Consider an intrinsic parametrization (e.g. arc-length param. In shape space):

$$ds^2 = g_{ab}(q)\, dq^a\, dq^b$$

⇨ The evolution of the shape becomes a kinematic statement

$\frac{dq^a}{ds} = u^a(\phi)$ unit tangent vector in direction ϕ $\qquad \Phi_A(q,p) :=$ direction defined through momenta p_a

(q^a, ϕ_A) local coordinates on unit tangent bundle

⇨ The equations for the change of direction along the curve become

$$\frac{d\phi_A}{ds} = \frac{\partial \Phi_A(q,p)}{\partial q^a} u^a(q,\phi) - \frac{\partial \Phi_A(q,p)}{\partial p_a}\left(g_{bc,a} u^b(q,\phi) u^c(q,\phi) + \frac{1}{p^2}\frac{\partial V(R,q)}{\partial q}\right)$$

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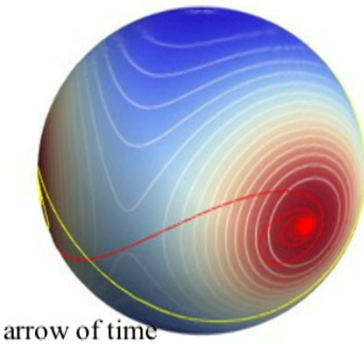
Example: Newtonian N-body universe (II)

Using $V(R,q) = -\alpha\, C(q)/R$ together with the dim.less $\kappa := ||\pi||/(\alpha R)$ we find

$$\frac{d\phi_A}{ds} = \frac{\partial \Phi_A}{\partial q^a}(q,\phi)\, u^a(q,\phi) + \frac{\partial \Phi_A}{\partial p_a}(q,\phi)\left(\frac{1}{\kappa}\frac{\partial C(q)}{\partial q^a} - \frac{1}{2} g_{bc,a}(q) u^b(q,\phi) u^c(q,\phi)\right)$$



and calculating the evolution of $\kappa := ||\pi||/(\alpha R)$ using Newton's equations

$$\frac{d\kappa}{d\,s} = \kappa \frac{D}{||\pi||} + 2\frac{\partial C(q)}{\partial q^a}\, u^a(q,\phi)$$ using E=0 we find $$\frac{d\kappa}{d\,s} = 2\frac{\partial C(q)}{\partial q^a}\, u^a(q,\phi) \pm \kappa\sqrt{|2\frac{C}{\kappa} - 1|}$$

sign from gravitational arrow of time

⇨ Equation of state of the curve in shape space:

$$\frac{dq^a}{ds} = u^a(q,\phi)$$

$$\frac{d\phi_A}{ds} = \frac{\partial \Phi_A}{\partial q^a}(q,\phi)\, u^a(q,\phi) + \frac{\partial \Phi_A}{\partial p_a}(q,\phi)\left(\frac{1}{\kappa}\frac{\partial C(q)}{\partial q^a} - \frac{1}{2} g_{bc,a}(q) u^b(q,\phi) u^c(q,\phi)\right)$$

$$\frac{d\kappa}{d\,s} = 2\frac{\partial C(q)}{\partial q^a}\, u^a(q,\phi) \pm \kappa\sqrt{|2\frac{C}{\kappa} - 1|}$$

Interpretation in terms of extrinsic curvature K of the curve:

$$K^2 = ||\nabla^g_u u||^2_g = g_{ab}\left(\frac{d}{ds}(u^a - u^a_{geo})(u^b - u^b_{geo})\right) = \frac{g^{ab} C_{,a} C_{,b}}{\kappa^2}$$

Note: the scale and duration decouple due to homogeneity in dimension and energy conservation (always!)

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Coordinate change to decouple

size and duration from pre-shapes and their momenta:

$$R := \sqrt{\sum_I \vec{r}_I^{\,2}} \qquad D := \sum_I \vec{p}^{\,I}.\vec{r}_I \qquad q^a := \frac{r_I^j}{R} \qquad p_a := \left(\frac{\partial q^a}{\partial r_I^j}\right)^{-1} \pi_j^I$$

Consider an intrinsic parametrization (e.g. arc-length param. In shape space):

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Quantum Theory of the Universe

Motivation for Shape Dynamics: Find relational foundations for Quantum Gravity

What is Quantum Gravity?

1. A well defined physical system (i.e. mathematical model with consistent physical interpretation)
2. Contains a classical limit that is well described by Einstein's equations
3. The acquisition of information of one practically isolated subsystem about another practically isolated subsystem by weak interactions is described by the quantum formalism

Problems:

1. Technical: Quantization produces time parametrization anomaly $i\hbar\partial_t\,\Psi = \hat{H}(t/\hbar)\,\Psi \;\Rightarrow\; i\partial_\lambda\,\Psi = \hat{H}(\lambda)\,\Psi$
2. Conceptual: Outcome problem of quantum mechanics ("measurement" problem)
3. Practical: How should the relational postulate be implemented in quantum theories?

$\Rightarrow$ *These problems can be solved by considering Bohmian trajectories on shape space as the history of the universe*

(i..e. quantum mechanics and spacetime emerge as effective descriptions)

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Bohmian Mechanics (complicated model)

Note: the curve has only a direction, while the gradient of the phase of the wave function has also a length!

Introduce Bohmian "shape of the universe" Q^a in addition to wave function on shape space $\Psi(q) = R(q)\, e^{i\,S(q)}$ to define

$$d\,Q^a = \frac{g^{ab}(Q) S_{,b}(Q)}{\sqrt{g^{ab}(Q) S_{,a}(Q) S_{,b}(Q)}}$$ (guidance equation in arc-length param.)

$$d\Psi(q) = \frac{\hat{H}(Q,\kappa;S)\Psi(q)}{\sqrt{g^{ab}(Q) S_{,a}(Q) S_{,b}(Q)}}$$ (modified Schrödinger equation)

$$d\kappa = 2\frac{g^{bc}(Q) S_{,a}}{\sqrt{g^{ab}(Q) S_{,a}(Q) S_{,b}(Q)}} C_{,a} \pm \kappa \sqrt{\left|1 - \frac{2C(q)}{\kappa}\right|}$$ (curvature equation)

This system traces out the same curve as the classical Newtonian N-body system on shape space if

1. the Hamiltonian is $\hat{H}(Q,\kappa;S) = -\frac{1}{2}\Delta_g - \frac{\sqrt{g^{ab}(Q) S_{,a}(Q) S_{,b}(Q)}}{\kappa} C(q)$
2. The gradient of the quantum potential $V_{quant}(q) = -\frac{1}{2}\frac{\Delta_g\, R(q)}{R(q)}$ is negligible.

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Bohmian Mechanics (simple model)

Use standard Bohmian mechanics on shape space

$$\dot{Q}^a = g^{ab}(Q)\, S_{,a}(Q) \qquad \text{(standard guidance equation)}$$

$$-i\dot{\Psi}(q) = \left(-\frac{1}{2}\Delta_g - C(q)\right)\Psi(q) \qquad \text{(standard Schrödinger equation)}$$

This system can be described as $dQ^a = u^a(Q,\phi)$

$$d\phi_A = \frac{\partial \Phi_A}{\partial Q^a} u^a(Q,\phi) - \frac{\partial \Phi_A}{\partial u^a}\left(g^{bc}_{,a}(Q) u_b(Q,\phi) u_c(Q,\phi) - \frac{\tilde{C}_{,a}(Q)}{\kappa}\right)$$

$$d\kappa = 2u^a(Q,\phi)\, C_{,a}(Q) + \kappa\, \sigma$$

using the effective direction $u^a(Q,\phi) = \dfrac{g^{ab}(Q) S_{,b}(Q)}{\sqrt{g^{cd}(Q) S_{,c}(Q) S_{,d}(Q)}}$ and effective potential $\tilde{C}(q) = C(q) + V_{quant}(q)$

This system expands apparently like $R(t) = \sqrt{R_o^2 + E_o t^2}$

whenever the longitudinal quantum force $\sigma := \dfrac{-1}{\kappa} u^a(Q,\phi) \left(\dfrac{\Delta_g R}{R}\right)_{,a}(Q)$ is negligible.

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Simple Model (atoms define units)

Consider an isolated pair of particles, i.e. $|(q^1, q^2, q^3) - \pi/2| \ll 1$ with $M(q^3, ..., q^{3N-4})$ bounded, then

$$C(q) = \frac{1}{\sqrt{c^2 q^1 + s^2 q^1 (c^2 q^2 + s^2 q^2 c^2 q^3)}} + \frac{M(q^3, ..., q^{3N-4})}{sq^1\, sq^2\, sq^3} \qquad \Rightarrow \qquad \frac{1}{||\vec{q}||} + M(q^3, ..., q^{3N-4})$$

Hamiltonian becomes $H = -\frac{1}{2}\Delta_q - \frac{c}{||\vec{q}||} + H_{rest} + H_{int}$ where H_{int} can be treated perturbatively

so the effective wave function $\Psi_{eff}(\vec{q}) = \Psi(\vec{q}, q_{rest} = Q_{rest})$ satisfies Hydrogen Schrödinger equation with small perturb.

$\Rightarrow$ effective unit of length by Bohr radius $r_o = \frac{1}{2c}$

effective unit of time from Rhydberg $t_o = \frac{1}{\omega_o}$ where $P_{nm}(t) = \left| \int_0^t ds e^{i\omega_o/n^2 - \omega_o/m^2} \left\langle n | \hat{H}_{int}(Q_{rest}; t) | m \right\rangle \right|^2$

assume accessible Bohr radius (equilibrium position between quantum force and classical force)

$\Rightarrow$ size of the universe in Bohr units $R = r_o \sqrt{\sum_{I<J} (f_{12}^{IJ})(Q^a)^2} = r_o\, R(q)$

(this size evolves classically when the quantum force is negligible for the rest)

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Equation of state for ADM (I)

Start ADM equations of motion (analogous to Newtonian eom) using metric $g_{ab}(x)$ and metric momenta $\pi^{ab}(x)$

Dynamics on conformal Riemann (not conformal superspace) $\Rightarrow$ Change of coordinates to separate

Local scale and expansion *from* conformal metric and momenta

$$\omega(x) := \sqrt{|g|(x)} \qquad \pi(x) := g_{ab}(x)\pi^{ab}(x) \qquad \rho_{ab(x)} := \frac{g_{ab}(x)}{|g|^{1/3}(x)} \qquad \sigma^a_b(x) := \left(g_{bc}(x)\delta^a_d - \frac{1}{3}\delta^a_b g_{cd}(x)\right)\pi^{cd}(x)$$

Impose: 1. "scale-decoupling" slicing condition $\frac{d}{dt}\ln(\omega(x,t)) = f(t) \quad \Rightarrow \quad N(x) = \frac{\sqrt{g}(x)}{\pi(x)}$

2. "intrinsic" parametrization condition, e.g. $\left(\frac{ds}{dt}\right)^2 = \int d^3x\, \omega_0\, \dot\rho_{ab}\rho^{ac}\rho^{bd}\dot\rho_{cd}$

Decoupling: 1. Local part of $\omega(x)$ decouples due to slicing condition $\Rightarrow$ only $V(t) := \int d^3x\, \sqrt{|g|(x,t)}$ remains

2. expansion $\pi(x)$ decouples completely due to local Hamilton constraint $\pi^2(x) = \left(6\,\sigma^a_b\sigma^b_a - R(\omega,\rho)\omega^2 + \text{matter}\right)(x)$

$\Rightarrow$ equation of motion for conformal metric $d\rho_{ab}(x) = N(\rho,\omega,\hat\sigma)\frac{\rho_{bc}\hat\sigma^c_a}{\omega_o}$

where: $\hat\sigma^a_b(x) := \frac{\sigma^a_b(x)}{\int d^3y\, \sqrt{\sigma^c_d(y)\sigma^d_c(y)}}$

Equation of state for ADM (II)

Use ADM equ. of motion with slicing and parametrization condition to derive a long equation of motion of the form

$$d\,\hat{\sigma}^a_b(x) := \hat{\Sigma}^a_b[\rho, \omega(t=0), \hat{\sigma}; \kappa; x)$$

where

$$\kappa = \frac{\int d^3x\,\pi(x)}{\int d^3x\,\sqrt{\sigma^a_b(x)\,\sigma^b_a(x)}}$$

Decoupling (continued):

and note that ω(t=0) is **not** dynamical !

3. *V(t)* and $\sigma := \int d^3x\,\sqrt{\sigma^a_b(x)\sigma^b_a(x)}$ decouple b/c dimensional homogeneity and dynamical similarity

$\Rightarrow$ Decoupled equations of motion:

$$d\,\rho_{ab}(x) = \frac{\rho_{bc}(x)\,\hat{\sigma}^c_a(x)}{\omega(t=0)}$$

$$d\,\hat{\sigma}^a_b(x) := \hat{\Sigma}^a_b[\rho, \omega(t=0), \hat{\sigma}; \kappa; x)$$

$$d\kappa = K[\rho, \omega_o, \sigma; \kappa)$$

These equations describe the intrinsic equation of state of a curve of conformal metrics.

Note: equations are spatially covariant $\Rightarrow$ equation of state of curve on conformal superspace

Symmetry reduction ⇒ Bianchi IX model

use Misner anisotropy parameters q^a and direction in shape space $u^a(\phi) = (\sin\phi, \cos\phi)$

and ADM Hamiltonian $H_{ADM} = \vec{p}^2 - \left(\frac{3}{8}\tau^2 - 2\Lambda\right) v^2 - v^{4/3}\, C(q)$

$$\Rightarrow \begin{aligned} dq^a &= u^a(\phi) \\ d\phi &= \frac{\partial\phi}{\partial u^a(\phi)} \frac{C_{,a}(q)}{2\,\kappa} \\ d\kappa &= u^a(\phi)\, C_{,a}(q) + \frac{1}{2}\sqrt{\kappa}\sigma \\ d\sigma &= \frac{1}{\sqrt{\kappa}}\left(\frac{\sigma^2}{4} + \frac{2}{3}C(q)\right) \end{aligned}$$

(where σ is related to the “jerk” of the curve, due to cosmological constant)

(so, σ plays the same role as the longitudinal part of the quantum force)

⇒ Bohmian model: $\dot{Q}^a = S_{,a}(Q)$

$$\hat{H} = -\Delta + C(q)$$

has Bianchi IX dynamics as semi-classical limit with cosmological constant given by longitudinal quantum force

$$\sigma := \frac{-1}{\kappa} u^a(Q,\phi) \left(\frac{\Delta_g\, R}{R}\right)_{,a} (Q)$$

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Well defined Bohmian equations ⇒ Renormalization

Consider Bohmian guidance equation for field a field theory $\dot{\Phi}(x) = \left.\frac{\delta S[\phi]}{\delta\phi(x)}\right|_{\phi=\Phi}$

⇒ is well defined on 1-particle Hilbert H_1 space when $S[\phi] = F\left(\langle f_1, \phi\rangle, ..., \langle f_n, \phi\rangle\right)$ for n finite, F smooth and f_i in H_1

$$\dot{S}[\phi] = -\frac{1}{2}\underbrace{\left\langle \frac{\delta S[\phi]}{\delta\phi(x)}, \frac{\delta S[\phi]}{\delta\phi(x)} \right\rangle}_{\text{fine}} - \underbrace{V[\phi]}_{\text{problem}} + \frac{1}{2\,R[\phi]} \int d^3x \underbrace{\frac{\delta^2 R[\phi]}{\delta\phi(x)^2}}_{\text{problem}}$$

for quadratic potential $V[\phi] = \langle\phi|E^2|\phi\rangle$ the two problems solve one another if one imposes

$R[\phi] = G\left(\langle f_1, \phi\rangle, ..., \langle f_m, \phi\rangle\right) \exp\left(-\frac{1}{2}\langle\phi|E|\phi\rangle\right)$ and adds counter term $\frac{1}{2}\mathrm{Tr}\,(E)$ to the potential

this form of $R[\phi]$ is then preserved by the radial part of the Schrödinger equation

$$\dot{R}[\phi] = -\left\langle \frac{\delta R[\phi]}{\delta\phi(x)}, \frac{\delta R[\phi]}{\delta\phi(x)} \right\rangle - \frac{R[\phi]}{2} \int d^3x \frac{\delta^2 S[\phi]}{\delta\phi(x)^2}$$

Generally: use background field ansatz $\Phi(x) = \bar{\phi}(x) + F(x)$ and impose that fluctuation field $F(x)$ has well defined e.o.m.

⇒ implement splitting symmetry $(\bar{\phi}(x), F(x)) \to (\bar{\phi}(x) - \epsilon(x), F(x) + \epsilon(x))$

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Bohmian Renormalization (III)

Free scalar field Hamiltonian $H = -\frac{1}{2}\int d^3x d^3y\, \delta(x,y) \frac{\delta^2}{\delta\phi(x)\,\delta\phi(y)} + \frac{1}{2}\int d^3x\, \phi(x)\left(-\Delta + m^2\right)\phi(x)$

$\Rightarrow$ expansion of potential to second order $V[\bar\phi, f] = \frac{1}{2}\int d^3x\, \bar\phi(x)\left(-\Delta + m^2\right)\bar\phi(x) + \int d^3x\, \bar\phi(x)\left(-\Delta + m^2\right) f(x) + \frac{1}{2}\int d^3x\, f(x)\left(-\Delta + m^2\right) f(x)$

replace all coincidence limits with heat kernel $\delta(x,y) = \lim_{\Lambda\to\infty} H_\Lambda(x,y) = \lim_{\Lambda\to\infty}\left\langle x \left| \exp\left(\frac{\Delta}{\Lambda^2}\right)\right| y\right\rangle$ to regularize

identify quadratic term $Q(x,y) = \langle x|\sqrt{m^2 - \Delta}|y\rangle$

use $\lim_{\Lambda\to\infty}\left[\frac{\Lambda}{2} - \frac{1}{\sqrt{4\pi}}\int_0^\infty \frac{ds}{s^{3/2}}\, e^{-s\,K + \frac{1}{s\,\Lambda^2}}\right] = \lim_{\Lambda\to\infty}\frac{\Lambda}{2}\left(1 - e^{-2\frac{\sqrt{K}}{\Lambda}}\right) = \lim_{\Lambda\to\infty}\frac{\Lambda}{2}\left(\frac{2}{\Lambda}\sqrt{K} + \mathcal{O}(\Lambda^{-2})\right)$ to calculate operator sq. root

using $\int_0^\infty \frac{ds}{s^{a+1}} e^{-(s\,K + \frac{1}{s\,\Lambda^2})} = 2\left(\sqrt{K}\,\Lambda\right)^a K_a\left(2\frac{\sqrt{K}}{\Lambda}\right)$ where $K = m^2 - \Delta$

$\Rightarrow$ regularized quadratic term $Q_\Lambda = \frac{\Lambda}{2}\, e^{-\frac{K}{\Lambda^2}} - \frac{1}{4\pi}\int_0^\infty \frac{ds}{s^{3/2}} e^{-(Ks + \frac{1}{s\Lambda^2})}$

$\Rightarrow$ counter term potential $U_\Lambda = \mathrm{Tr}\left[\frac{\Lambda}{2}\, e^{-\frac{K}{\Lambda^2}} - \frac{1}{4\pi}\int_0^\infty \frac{ds}{s^{3/2}} e^{-(Ks + \frac{1}{s\Lambda^2})}\right] = \frac{\Lambda}{2}\mathrm{Tr}\left(e^{-\frac{K}{\Lambda^2}}\right) - \frac{1}{4\pi}\int \frac{ds}{s^{3/2}} e^{-\frac{1}{\Lambda^2 s}}\mathrm{Tr}\left(e^{-s\,K}\right)$

is calculated using heat trace $\int d^3x\,\sqrt{g}\left(\frac{\Lambda^4 + m^2\Lambda^2}{2} + \frac{\Lambda^2}{12} R\right) + \mathcal{O}(\Lambda^0)$

SD description of Bianchi IX with scalar field

This program can be applied to a scalar field coupled to gravity, e.g. an evolving Bianchi IX geometry.

$\Rightarrow$ usually $\int d^3x \sqrt{g} \left(\frac{\Lambda^4 + m^2 \Lambda^2}{2} + \frac{\Lambda^2}{12} R \right) + \mathcal{O}(\Lambda^0)$ becomes a fine tuning problem, because the cosmological constant (measured in Planck units) is incredibly small

However, in Shape dynamics, the perceived expansion is not due to a cosmological constant, but due to longitudinal quantum force $\sigma := \frac{-1}{\kappa} u^a(Q, \phi) \left(\frac{\Delta_g R}{R} \right)_{,a} (Q)$ which pretends a time-dependent cosmological constant.

$\Rightarrow$ How is the perceived expansion affected by the renormalization of the SD system?

Quantum SD Hamiltonian $H = -\left(\frac{\partial^2}{\partial (q^1)^2} + \frac{\partial^2}{\partial (q^2)^2} \right) - \frac{1}{2} \int d^3x \sqrt{\omega_o} \frac{\delta^2}{\delta (\phi(x))^2} + C(\vec{q}) + \frac{1}{2} \int d^3x \sqrt{\omega_o} \phi \left(m^2 - \Delta_q \right) \phi$

$\Rightarrow$ semi-classical limit is Bianchi IX with scalar field and effective cosmological constant

$\Rightarrow$ counter term potential $\int d^3x \sqrt{\omega_o} \left(\frac{\Lambda^4 + m^2 \Lambda^2}{2} + \frac{\Lambda^2}{12} C(\vec{q}) \right) + \mathcal{O}(\Lambda^0)$

does not affect longitudinal quantum force

$\Rightarrow$ Vacuum energy does not affect SD equations of motion!

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and ADM Hamiltonian $H_{ADM} = \vec{p}^2 - \left(\frac{3}{8}\tau^2 - 2\Lambda\right) v^2 - v^{4/3}\, C(q)$

$$\Rightarrow \begin{aligned} dq^a &= u^a(\phi) \\ d\phi &= \frac{\partial\phi}{\partial u^a(\phi)} \frac{C_{,a}(q)}{2\,\kappa} \\ d\kappa &= u^a(\phi)\, C_{,a}(q) + \frac{1}{2}\sqrt{\kappa}\sigma \\ d\sigma &= \frac{1}{\sqrt{\kappa}}\left(\frac{\sigma^2}{4} + \frac{2}{3}C(q)\right) \end{aligned}$$

(where σ is related to the “jerk” of the curve, due to cosmological constant)

(so, σ plays the same role as the longitudinal part of the quantum force)

⇒ Bohmian model: $\dot{Q}^a = S_{,a}(Q)$

$$\hat{H} = -\Delta + C(q)$$

has Bianchi IX dynamics as semi-classical limit with cosmological constant given by longitudinal quantum force

$$\sigma := \frac{-1}{\kappa} u^a(Q,\phi) \left(\frac{\Delta_g\, R}{R}\right)_{,a} (Q)$$

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Conclusions

1. Relational considerations require that the universe is described as an equation of geometry of state of an (**un**parametrized) curve on shape space
2. One can find Pure Shape Dynamics systems that are empirically indistinguishable from General Relativity
3. Relational principles of shape dynamics can be applied to Bohmian trajectories
4. Bohmian equations require renormalization
5. Bohmian systems "expand" due to existence of length of gradient of phase
6. This expansion is not affected by vacuum energy
7. There is a rigorous quantization program for shape dynamics

"SD=An equation of state of the geometry of the curve on shape space"

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Thank you !