Title: N=1 from (1,0)

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Abstract: <p>We will discuss some expectations regarding properties of N=1 SCFTs in four dimensions obtained by compactifying (1,0) theories in six dimensions on a Riemann surface. We will illustrate in detail how these properties come about in the special case of compactifications of two M5 branes probing Z_2 singularity. In particular, we will obtain a large class of strongly coupled N=1 theories in four dimensions obtained in such compactifications. We will derive some of their robust properties, such as anomalies and supersymmetric indices.</p>

N=1 QFTs in four dimensions from (1,0) in six

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Technion

SSR, Vafa, Zafrir 1610.09178
Bah, Hanany, Maruyoshi, SSR, Tachikawa, Zafrir 1702.04740
Gaiotto and SSR 1503.05159
Gadde, SSR, Willett 1505.05834

Fields/Strings seminar

May 9, 2017 - Perimeter.

Spaces of QFTs from dimensional reduction

- Reduce D dimensional SCFTs to D' $(< D)$ dimensions in different ways
- Example, $6d$ $(1,0)$ theories reduced on Riemann surface $\mathcal{C}_{g,\mathbf{s}}$
- We concentrate on N M5 branes probing A_{k-1} singularity
- Theories in *4D*, $T_{N,k;g,\mathbf{s}|\mathbf{F}}$, labeled by (N,k) and $(g,\mathbf{s})$ *and a choice of background gauge fields F*
- $k = 1$ is well studied

  [$\mathcal{N} = 2$ Gaiotto 09, $\mathcal{N} = 1$ Benini-Tachikawa-Wecht 10, Beem-Bah-Bobev-Wecht 11]
- Today, crank up k

Two routes to the jungle

- The six dimensional route: Can compute the expected symmetries, anomalies, and structure of the conformal manifold for each choice of theory in six dimensions and compactification.
- The arguments are very general and robust.
- The four dimensional route: There are very few theories which are known (through Lagrangian constructions) that can be matched to six dimensional expectations.
- We will start from this particular civilized corner in the theory space and force our way into the wild jungle.

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6d
Lagrangian-ville
4d

The strategy

- Start from reviewing general expectations from six dimensions
- Concentrate on case of two M5 branes probing $\mathbb{Z}_2$ singularity
- Will go to four dimensions and discuss theories $T_{2,2;g\geq 2,\mathbf{s}|\mathbf{F}}$
- Will encounter new beasts and constructions in four dimensions
- Discuss $T_{N,k;g=1,\mathbf{s}|\mathbf{F}}$ and relation to toric quivers

Six dimensions

- We have variety of six dimensional superconformal theories with $(1,0)$ supersymmetry.
- In general $(1,0)$ theories have some global symmetry G.
- Eg: N M5 branes probing $\mathbb{Z}_k$ singularity.
- The flavor symmetry is

$$\text{general } N,\ k: \qquad G = su(k)su(k)u(1)$$

$$N=2, \text{ general } k: \qquad G = su(2k)$$

$$\text{general } N,\ k=2: \qquad G = su(2)su(2)su(2)$$

$$N=2 \text{ and } k=2: \qquad G = so(7)$$

Six dimensions on $\mathcal{C}_g$

- We consider the $(1,0)$ theory on a Riemann surface $\mathcal{C}_g$.
  (*Do not discuss punctures since they add a layer of complication.*)

- To preserve $\mathcal{N}=1$ supersymmetry in 4d need to twist

$$so(2)_{\mathcal{C}_g} \quad \to \quad so(2)_{\mathcal{C}_g} - so(2)_R$$

  where $so(2)_R$ is the Cartan of $so(3)_R$.

- In addition can turn on background configuration for flavor symmetry G
  - flat G bundles, continuous parameters
    (holonomies around the cycles)
  - flux for abelian subgroup $\mathbf{L}$ of G through $\mathcal{C}_g$, discrete parameters
    $(c_1(L))$

Six dimensions on $\mathcal{C}_g$: symmetries and parameters

- Compactification is specified by choice of $rank(G)$ dimensional vector of fluxes $\mathcal{F}$.
- Flux for abelian subgroup **L** breaks the symmetry down to

$$G_{max} \subset G$$

- The flat bundles then can be turned on to G_{max}
- The moduli space of flat G_{max} bundles becomes part of the parameter space, conformal manifold, of the four dimensional theory

Six dimensions on $\mathcal{C}_g$: Predictions for 4d

- Compactifications labeled by choice of $\mathcal{C}_g$ and choice of flux, $\mathcal{F}$: discrete choices.
- The symmetries are given by a subgroup of G, G_{max} (commuting with the fluxes).
- The conformal manifold consists of complex structure moduli for $\mathcal{C}_g$ and moduli space of flat connections for G_{max} *(L is number of $u(1)$ factors in G_{max})*: continuous choices.

$$dim_{\mathbb{C}}\mathcal{M}_{g,\mathcal{F}} = (3g-3) + (g-1)dimG_{max} + L$$

- On general point of the conformal manifold the symmetry is $u(1)^L$
- The 't Hooft anomalies for all the symmetries in 4d can be inferred from 6d by integrating the anomaly polynomial over the Riemann surface

Six dimensions on $\mathcal{C}_g$: Two M5 branes on $\mathbb{Z}_2$ (A)

- Take two M5 branes on $\mathbb{Z}_2$ singularity, $G = so(7)$
- There is rather rich variety for G_{max}

G_{max}	$u(1)^3$	$su(2)_{diag}u(1)^2$	$su(2)u(1)^2$	$su(2)su(2)u(1)$
$\mathcal{F}$	(a,b,c)	$(b,\pm b,c)$	$(a,0,b)$	$(b,0,0)$

G_{max}	$su(3)u(1)$	$so(5)u(1)$	$so(5)u(1)$	$so(7)$
$\mathcal{F}$	$(b,0,\pm b)$	$(0,0,b)$	$(a,\pm a,0)$	$(0,0,0)$

- For every choice of g and $\mathcal{F}$ we obtain in general different symmetries in four dimensions and different conformal manifolds.

Six dimensions on $\mathcal{C}_g$: Two M5 branes on $\mathbb{Z}_2$ (B)

- The 't Hooft anomalies of the four dimensional theory can be obtained by integrating the anomaly eightform of the six dimensional theory over the Riemann surface, $\int_{\mathcal{C}_g} \mathcal{I}_8(\mathcal{F}) = \mathcal{I}_6^{(g,\mathcal{F})}$

- Example, $G_{max} = so(7)$, $\mathcal{F} = (0,0,0)$

$$a = \frac{51}{8}(g-1)\,, \qquad c = \frac{52}{8}(g-1)$$

- Example, $G_{max} = so(5)u(1)$, $\mathcal{F} = (0,0,2g-2)$

$$a = \frac{187}{24}(g-1)\,, \qquad c = \frac{194}{24}(g-1)$$

- Example, $G_{max} = su(3)u(1)$, $\mathcal{F} = (2,0,2)$

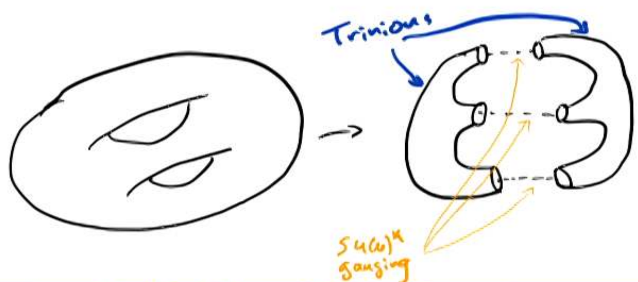
$$a = \frac{35}{2560}(267+11\sqrt{385})(g-1)\,, \quad c = \frac{1}{2560}(9425+401\sqrt{385})(g-1$$

Four dimensions

- How can we find the theories in four dimensions arising in the compactifications?
- Based on experience with $k = 1$ the theories might not have a description using standard Lagrangian
- We will discuss the construction for two M5 branes on $\mathbb{Z}_2$ singularity.
- The derivation turns out to be much more involved than the final result
- We will state the result in couple of case and then allude to the derivation

Trinions

- The basic building block of theories corresponding to Riemann surface is a trinion
- Trinion theory corresponds to compactification of the six dimensional theory on sphere with three maximal punctures
- It has $su(N)^k$ symmetry associated to each puncture and some sub-group of G_{max}
- Given the trinions any Riemann surface is constructed by gluing the trinions through gauging the puncture symmetries



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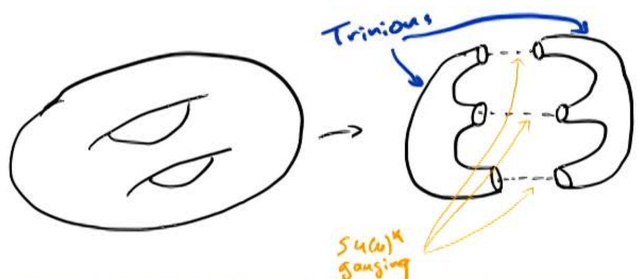
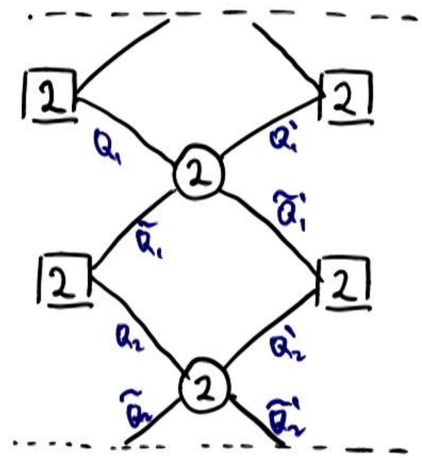


Four dimensions: trinion for $so(7)$ models

Consider coupling two copies of $su(2)$ SQCD with four flavors through superpotentials coupling their mesons.



- The model has manifestly $su(2)^2 su(2)^2$ symmetry and a bunch of $u(1)$ factors.
- Studying supersymmetric partition functions one can find evidence that somewhere on the conformal manifold the symmetry is enhanced to three copies of $su(2)^2$ appearing symmetrically. (Dimofte, Gaiotto 12)

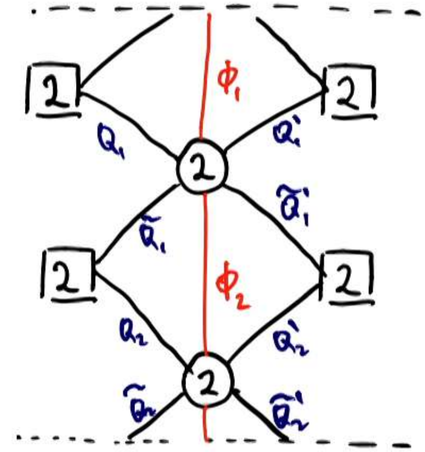
Tuning to the symmetry enhanced point this theory is a candidate for a trinion theory for compactification of two M5 branes on $\mathbb{Z}_2$ singularity.

Four dimensions: $so(7)$ models

- Consider gluing the trinions by gauging $su(2)^2$ symmetries to form general Riemann surfaces
- The symmetry we gauge is not apparent in the Lagrangian and we expect it to appear only in strong coupling
- However, we can compute robust quantities from the Lagrangian describing the trinion away from the symmetry enhanced locus
- Anomalies: $a = \frac{51}{8}(g-1)\,, \qquad c = \frac{52}{8}(g-1)$
- Index indicates the symmetry of the theory is consistent with $so(7)$
- Dimension of conformal manifold $dim\mathcal{M} = (3g-3) + (g-1)\mathbf{21}$
- This matches exactly the expectations for $G_{max} = so(7)$ models!!!!!

Four dimensions: trinion for $so(5)u(1)$ models

Consider coupling two copies of $su(2)$ SQCD with six flavors together.



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Four dimensions: $so(5)u(1)$ models

- Consider gluing the trinions by gauging $su(2)^2$ symmetries to form general Riemann surfaces
- The symmetry we gauge is not apparent in the Lagrangian and we expect it to appear only in strong coupling
- However, we can compute robust quantities from the Lagrangian describing the trinion away from the symmetry enhanced locus

- Anomalies: $a = \frac{187}{24}(g-1)\,, \qquad c = \frac{194}{24}(g-1)$

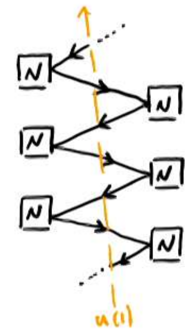
- Index indicates symmetry is consistent with $so(5)u(1)$

- $dim\mathcal{M} = (3g-3) + (g-1)\mathbf{10} + g$

- This matches exactly the expectations for $G_{max} = so(5)u(1)$ $\mathcal{F} = (0, 0, 2g-2)$ models!!!!!

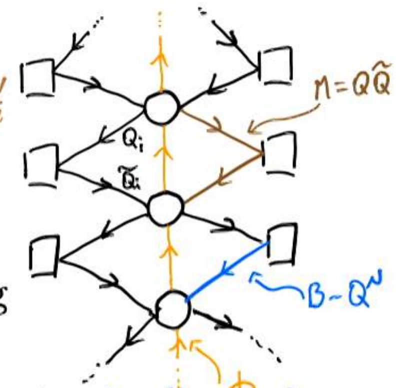
Lagrangians: Free trinions and gauge theories

- $\mathbb{Z}_k$ orbifold of hypermultiplets, free trinion
- $su(N)^k \times su(N)^k \times u(1)$ puncture symmetry, two maximal and one minimal
- $u(1)^{k-1} \times u(1)^{k-1} \times u(1)_t$ internal symmetry, Cartan of $su(k) \times su(k) \times u(1)$



"Mesons" M charged under $su(N)^k$, "Baryons" B charged under $u(1)$

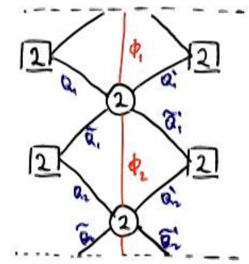
- "Glue" trinions together – gauge $su(N)^k$, add fields Φ_i and couple them as $W = M_i\Phi_i - \Phi_i M_i'$
- Vevs close punctures/shift $\mathcal{F}$
- mesons/maximal, baryons/minimal
- Procedure of closing punctures involves flipping operators; $W \to W + \Phi_{\mathcal{O}}\,\mathcal{O}$



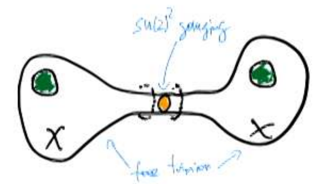
Flippings are important to have the right connection to six dimensions

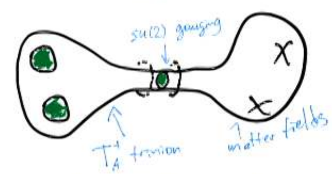
Four dimensions: a construction (A)

There is a way to derive the relevance of the models considered here to compactifications.



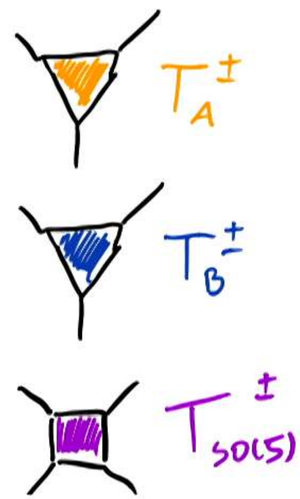
- This model is $\mathbb{Z}_2$ orbifold of $\mathcal{N} = 2$ $SU(2)$ theory with four flavors
- Natural to associate it to some sphere compactification with two maximal and two $u(1)$ punctures
- Consider same model in different pair of pants decomposition with two maximal punctures sitting together
- In this description the theory is a trinion T_A^+ coupled to some matter fields through $su(2)$ gauging
- Similar to Argyres-Seiberg duality





Four dimensions: a construction (B)

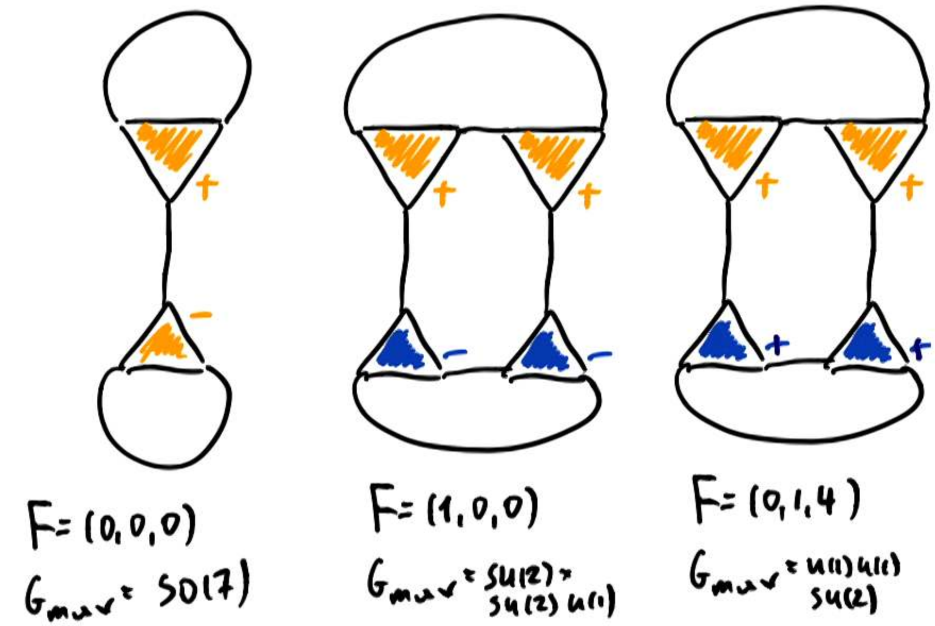
Studying such dualities and some RG flows triggered by vacuum expectation values to operators charged under puncture symmetries we can derive a variety of building block theories.



- $T_A{}^{\pm}$ with $\mathcal{F} = \pm(\frac{1}{4}, \frac{1}{4}, 1)$
  $G_{max} = su(2)_{diag} u(1) u(1)$
- $T_B{}^{\pm}$ with $\mathcal{F} = \pm(-\frac{1}{4}, \frac{1}{4}, 1)$
  $G_{max} = su(2)_{diag} u(1) u(1)$
- $T_{so(5)}{}^{\pm}$ with $\mathcal{F} = \pm(0, 0, 2)$
  $G_{max} = so(5) u(1)$

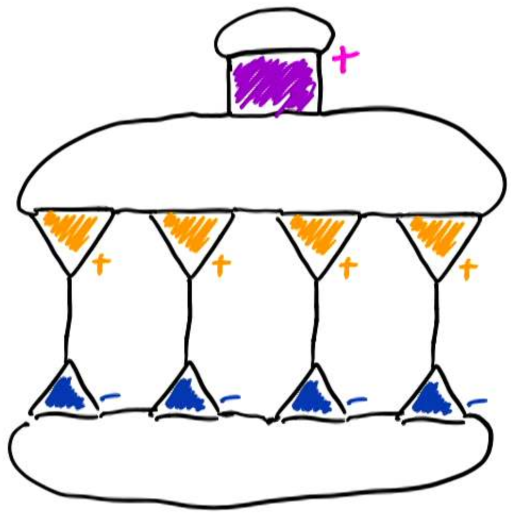
Using these block examples of theories with any G_{max} and a variety of fluxes are constructed

Four dimensions: some models



Symmetries, anomalies, conformal manifolds agree with six dimensions

Four dimensions: $su(3)u(1)$ model



$$a = \frac{35}{512}(267 + 11\sqrt{385}) \ , \qquad c = \frac{1}{512}(9425 + 401\sqrt{385})$$

Symmetries, conformal manifold, and anomalies match six dimensions

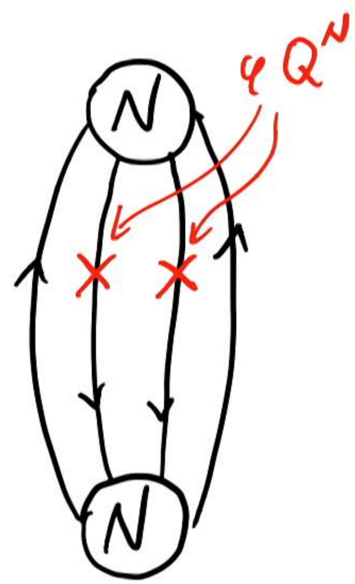
Tori theories

- Restricting to theories with genus one and fluxes only for subgroups of $su(k) \times su(k)$ the theories are Lagrangian.
- Start from F free trinions glued together to form a torus. This is a torus with F minimal punctures
- Close the minimal punctures by giving Vevs to baryons.
- There are $2k$ choices of baryons for each trinion. The rest of the baryons are flipped.
- The choices of the baryons map to different fluxes for $su(k) \times su(k)$
- Anomalies and symmetries can be matched to six dimensions

Tori example

Consider $k = 2$ and general N

- Take torus composed from two free trinions and give Vev to one Q^N baryon and one $\widetilde{Q}^N$ baryon
- The flipping is irrelevant for $N > 2$
- For N two the flipping is relevant. The flux is $(\frac{1}{2}, \frac{1}{2}, 0)$ leading to expected $so(5) \times u(1)$ symmetry
- The symmetry is not apparent in the Lagrangian; contributions of the flipping fields and the composite operators lead to $so(5)$ representations in the index



Four dimensions: observation

- We can trigger flows from models with some G_{max} to other G_{max} by relevant deformations

$$\mathcal{M}_{u(1)^3} \to \mathcal{M}_{su(2)_{diag}u(1)^2} \to \mathcal{M}_{so(5)u(1)} \to \mathcal{M}_{so(7)}$$

- Theories with $G_{max} = so(7)$ have no relevant deformations

Summary

- One can derive a precise dictionary between field theoretic constructions in four dimensions and compactifications of two M5 branes on A_1 singularity for general genus, or N M5 barnes A_k and genus one and no $u(1)$ flux

- The field theories are non Lagrangian in the sense that to define them one needs to conjecture that somewhere on the conformal manifold of a Lagrangian theory certain symmetry is enhanced and then it is possible to gauge this

- The construction undergoes very large variety of checks

Open questions

- Constructions on the field theory side for more general compactifications
- Three dimensional analogues
- Exploring 4d to 2d correspondences of the type found for $k = 1$ case (for example indices)

Thank You!!