Title: Equivalence of theories in the presence of boundaries: the example of General Relativity

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Abstract:

Equivalence of theories in the presence of boundaries, the case of General Relativity. J. With A.S. Cattaneo.

[Do we have a good notion of equivalence of gauge theories?]

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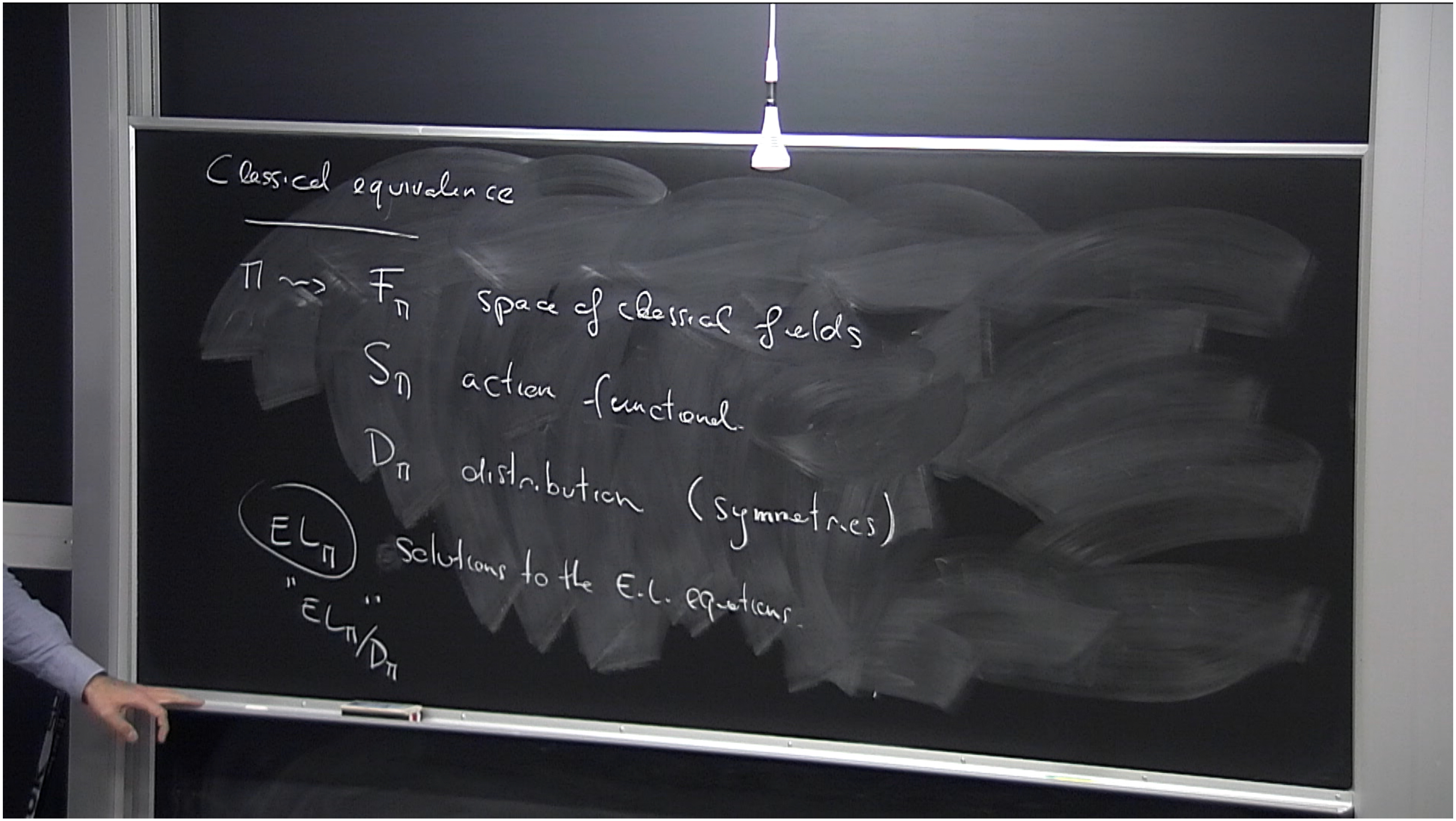
Sometimes things can go wrong, the case of GR.


Classical equivalence
$\Pi \rightsquigarrow F_\Pi$ space of classical fields
S_Π action functional
D_Π distribution (symmetries)
EL_Π solutions to the E.L. equations
"EL_Π / D_Π"

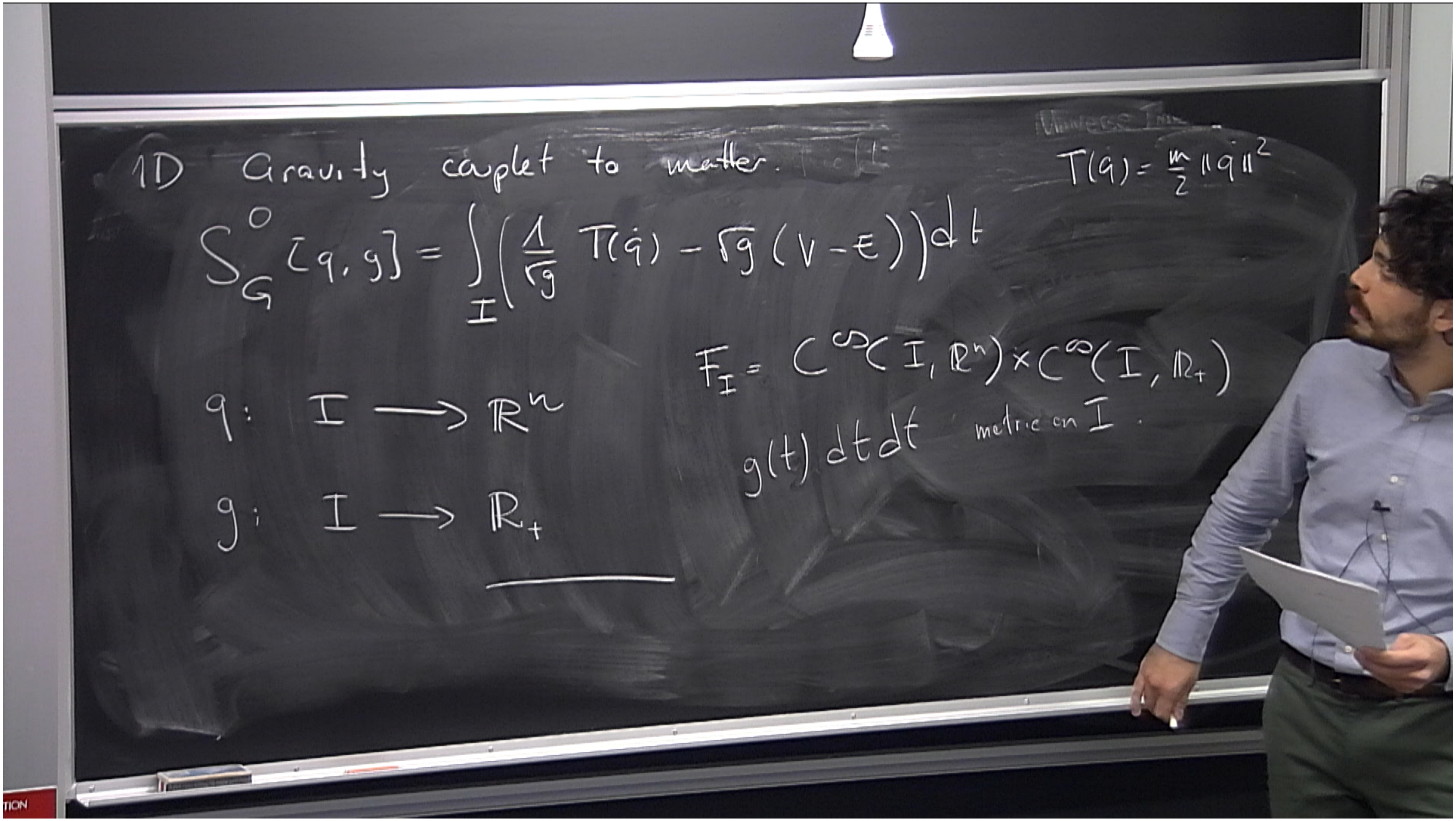
$/D_\pi$

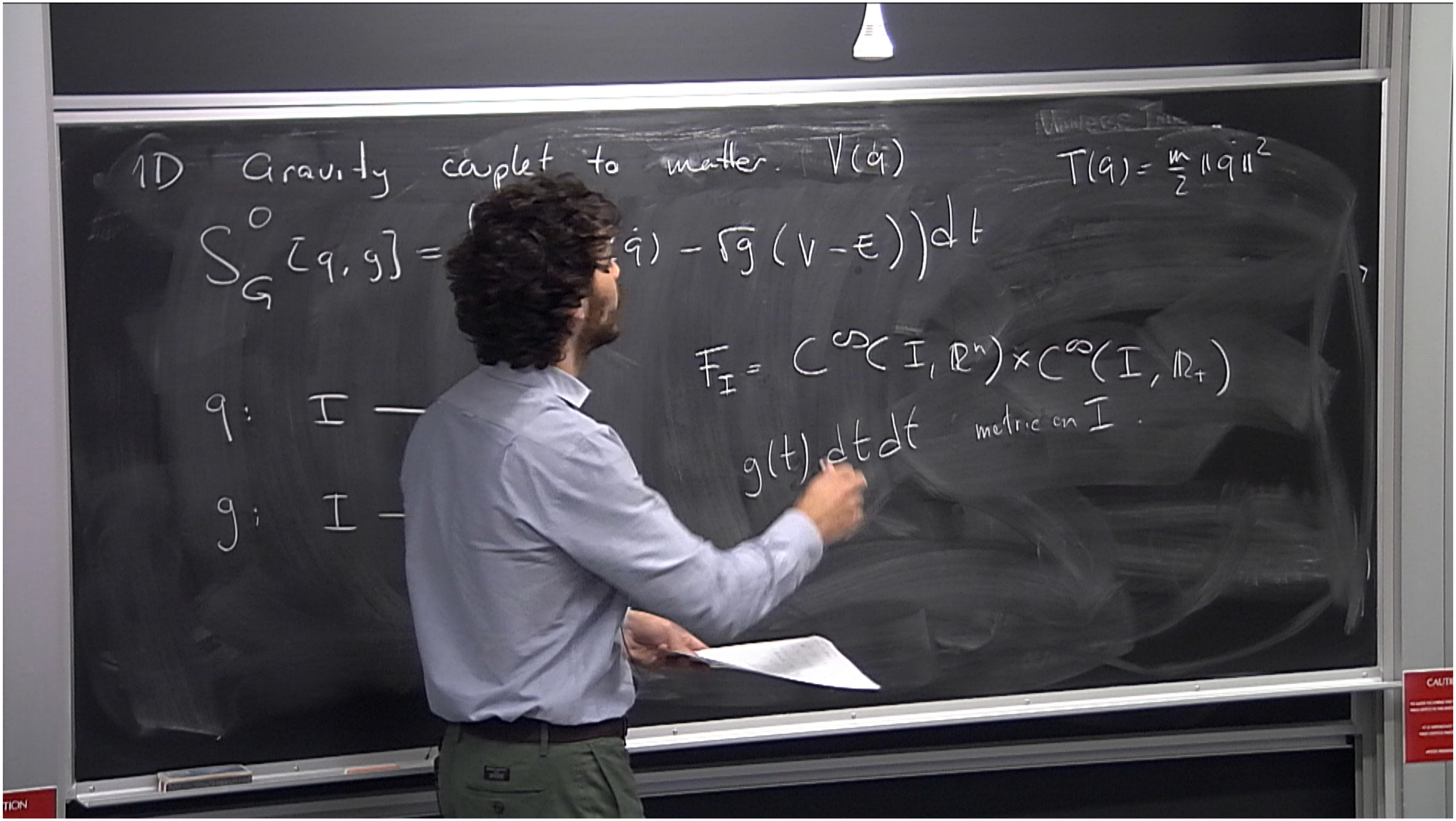
Equivalence of theories in the presence of boundaries.

$$(F_\pi, S_\pi, D_\pi) \sim (F'_\pi, S'_\pi, D'_\pi)$$

$$EL_\pi/D_\pi \sim EL'_\pi/D'_\pi$$

- Theories are solution equivalent
- Canonical structures coincide.


1D Gravity couplet to matter.
$T(q) = \frac{m}{2} \|\dot{q}\|^2$
$S^0_G[q,g] = \int_I \left(\frac{1}{\sqrt{g}} T(\dot{q}) - \sqrt{g}(V - E) \right) dt$
$q: I \longrightarrow \mathbb{R}^n$
$g: I \longrightarrow \mathbb{R}_+$
$F_I = C^\infty(I, \mathbb{R}^n) \times C^\infty(I, \mathbb{R}_+)$
$g(t)\, dt\, dt$ metric on I.


1D Gravity couplet to matter. V(q)
T(q) = m/2 ||q̇||²
S⁰_G [q, g] =
q̇) − √g (V − E))dt
F_I = C^∞(I, ℝⁿ) × C^∞(I, ℝ₊)
q: I —
g: I —
g(t) dt dt
metric on I .

presence of boundaries,

$$\delta S_G \longrightarrow \text{EL} \begin{cases} g = \dfrac{I}{E-V} \quad * \\ \dfrac{d}{dt}\left(\dfrac{m}{\sqrt{g}}\,\dot q^{i}\right) = -\sqrt{g}\,\partial_i V \end{cases}$$

$$S_G\left[q, g = \frac{I}{E-V}\right] = \cdots = 2\int \sqrt{T(E-V)}\,dt = S_J$$

Jacobi action for reparametris. invariant classical mechanics.

Classically equivalent theories.

Diffeomorphisms act by Lie derivative.

$$Q\, g = L_\xi\, g \qquad \xi \in \mathfrak{X}[1](I)$$

$$Q\, q = L_\xi\, q \qquad S_q = S^0_q + \int g^+ L_\xi g + q^+ L_\xi q + \xi^+ L_\xi \xi$$

$$Q\, \xi = \tfrac{1}{2} L_\xi\, \xi \qquad S_J = S^0_J + \int q^+ L_\xi q + \xi^+ L_\xi \xi$$

$$T^*[-1]\left(\mathcal{F}_{q/J} \times \mathfrak{X}[1](I)\right)$$

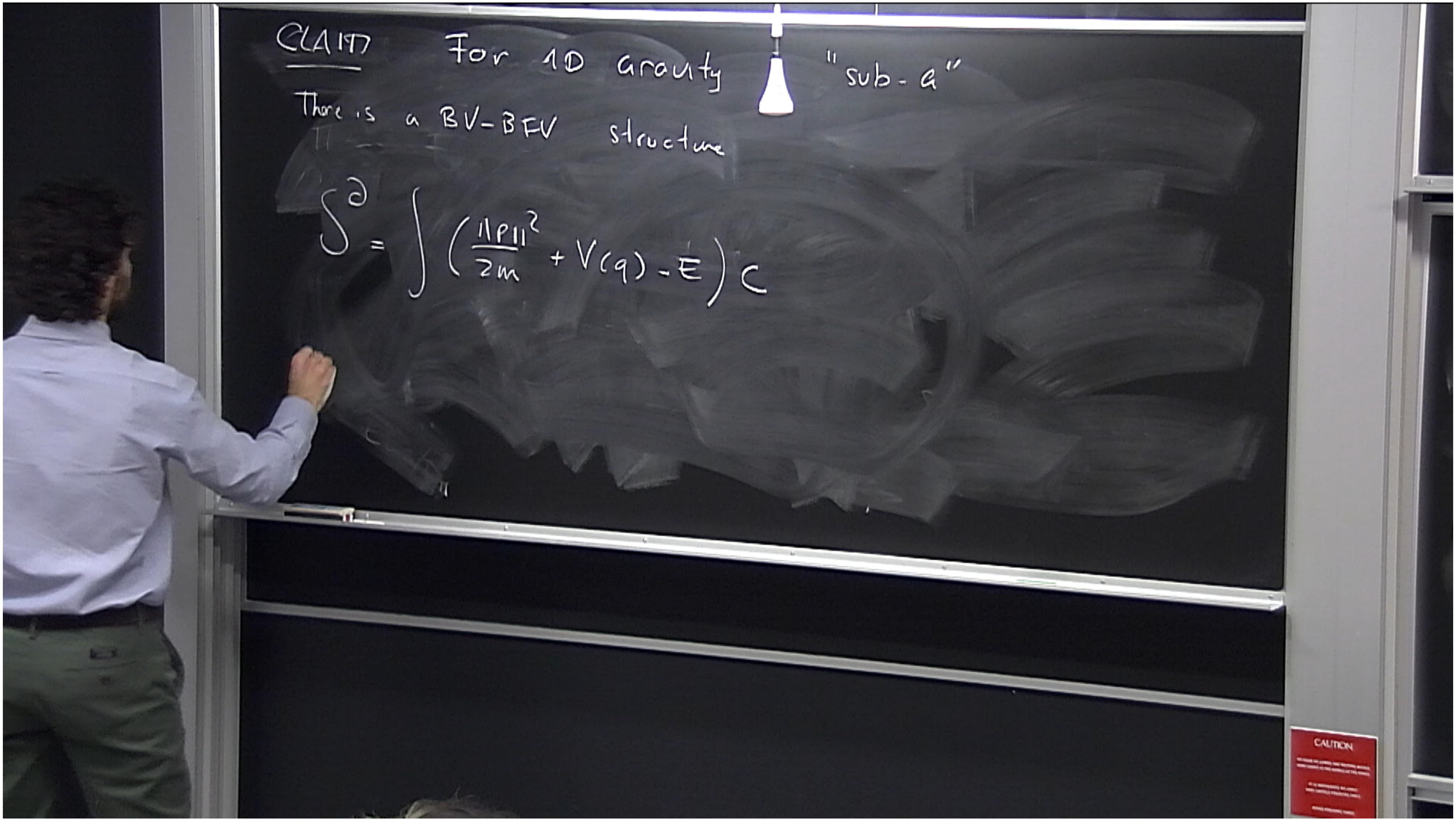
Apply the BV-BFV analysis.

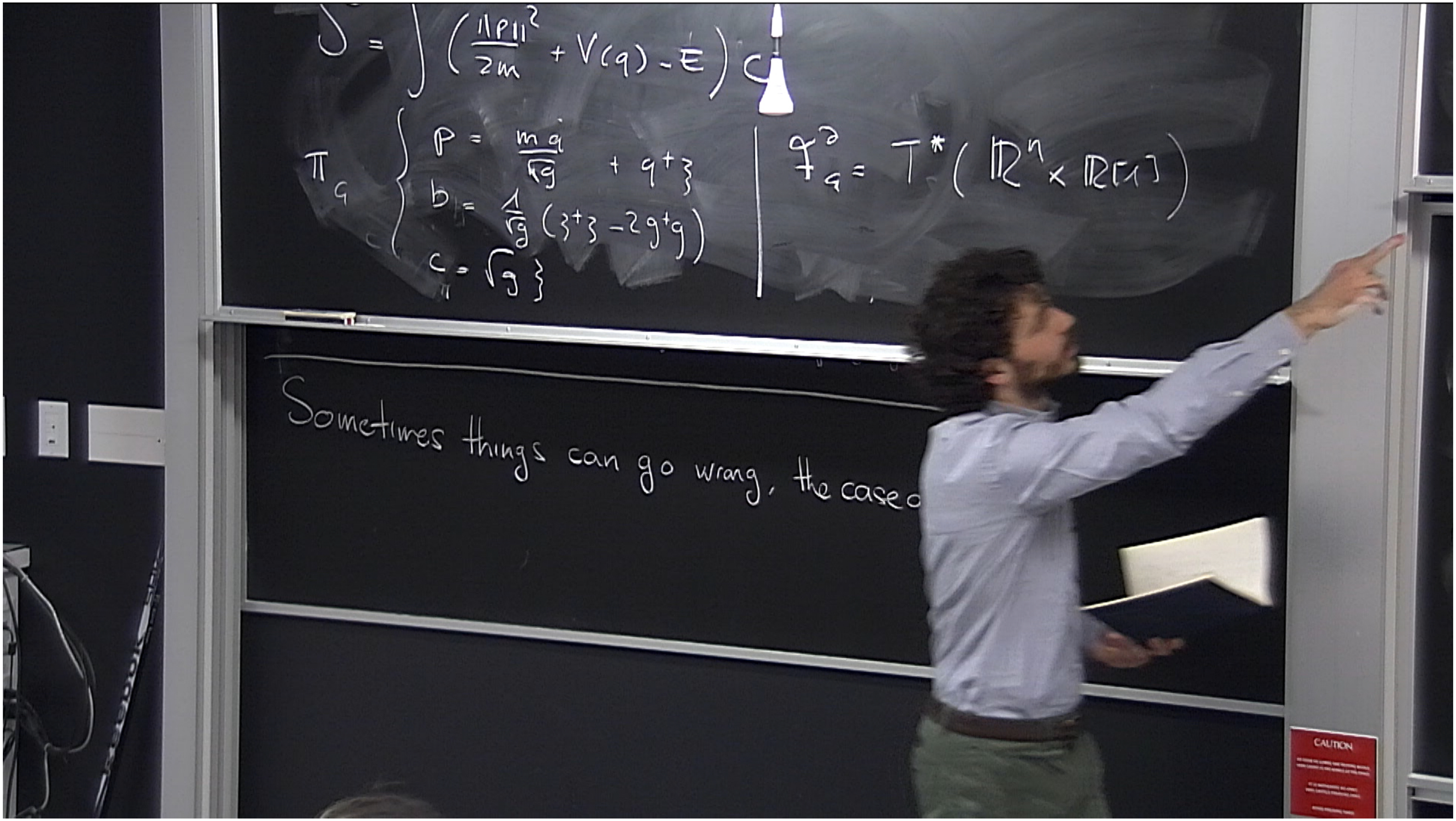
$$\left(\mathcal{F}^\partial_{G/J},\ S^\partial_{G/J},\ \underset{\displaystyle \delta\alpha^\partial}{\overset{\shortparallel}{}}\!\!\omega^\partial_{G/J}\right)$$

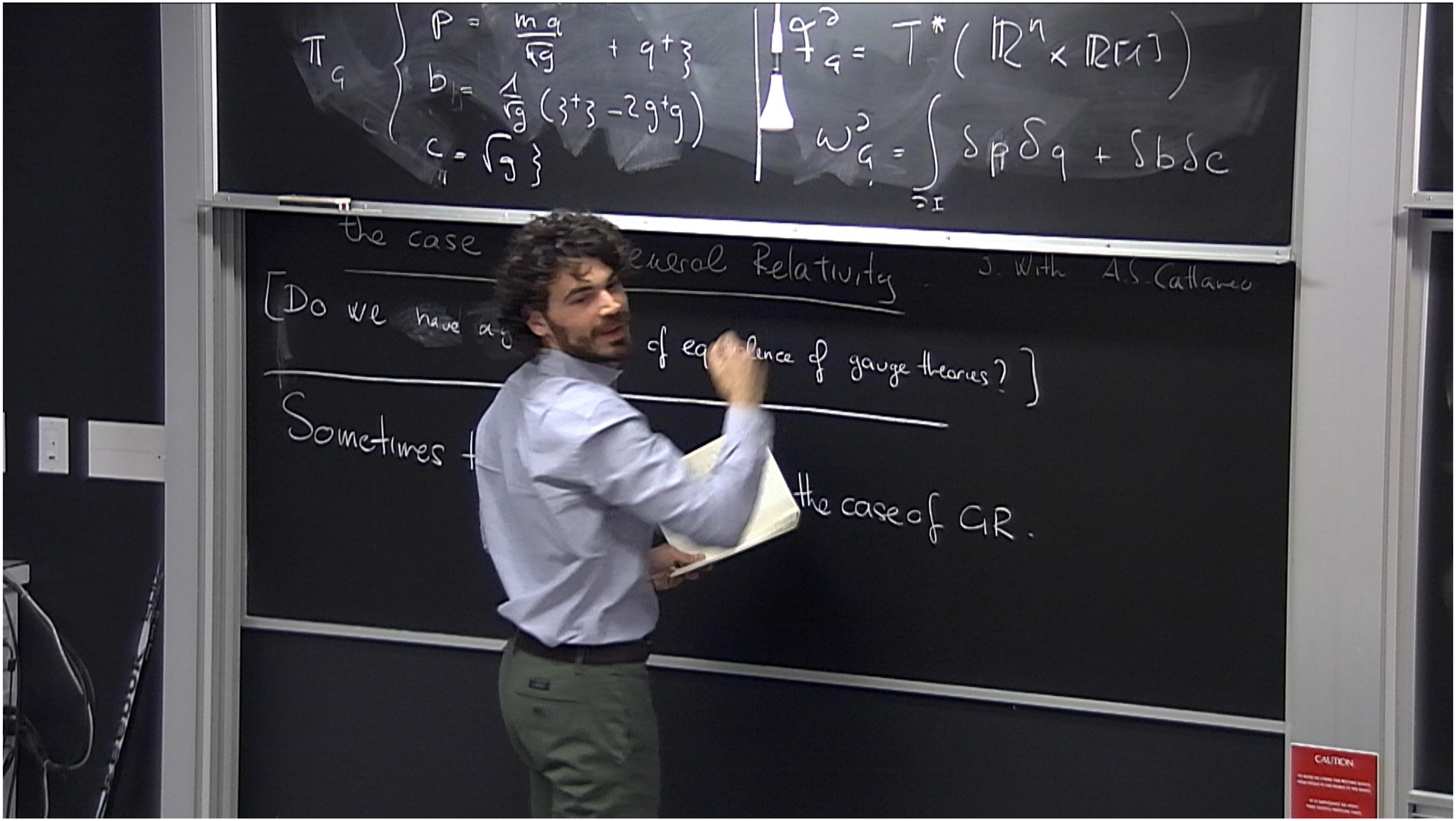
$$\pi_{G/J} : \mathcal{F}_{G/J} \longrightarrow \mathcal{F}^\partial_{G/J}$$

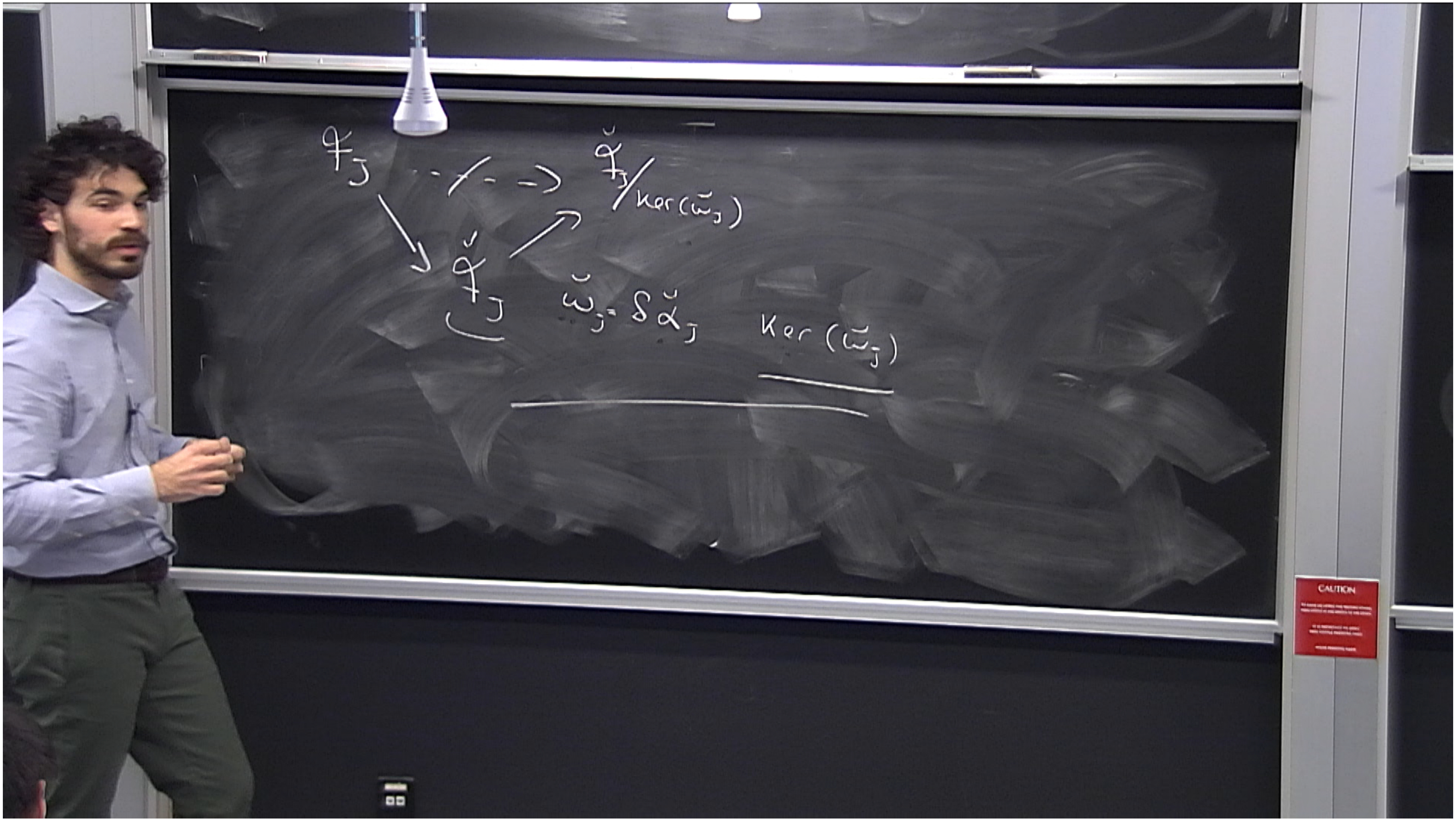
$$\iota_{Q^\partial_{G/J}}\, \omega^\partial = \delta S^\partial$$

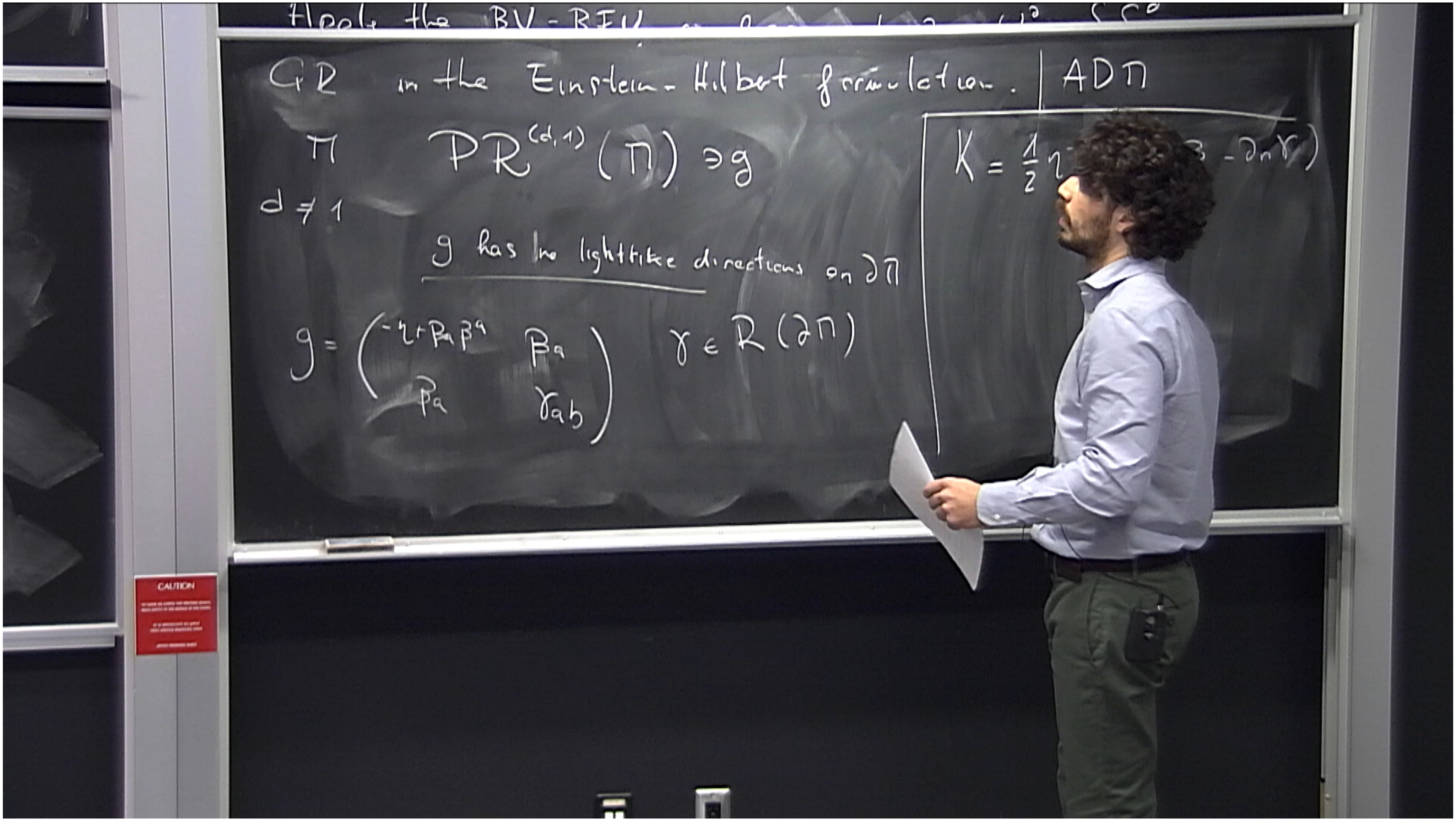
$$\boxed{\begin{aligned} \pi_* Q &= Q^\partial \\ \iota_Q \Omega_{G/J} &= \delta S + \pi^* \alpha^\partial \end{aligned}}$$

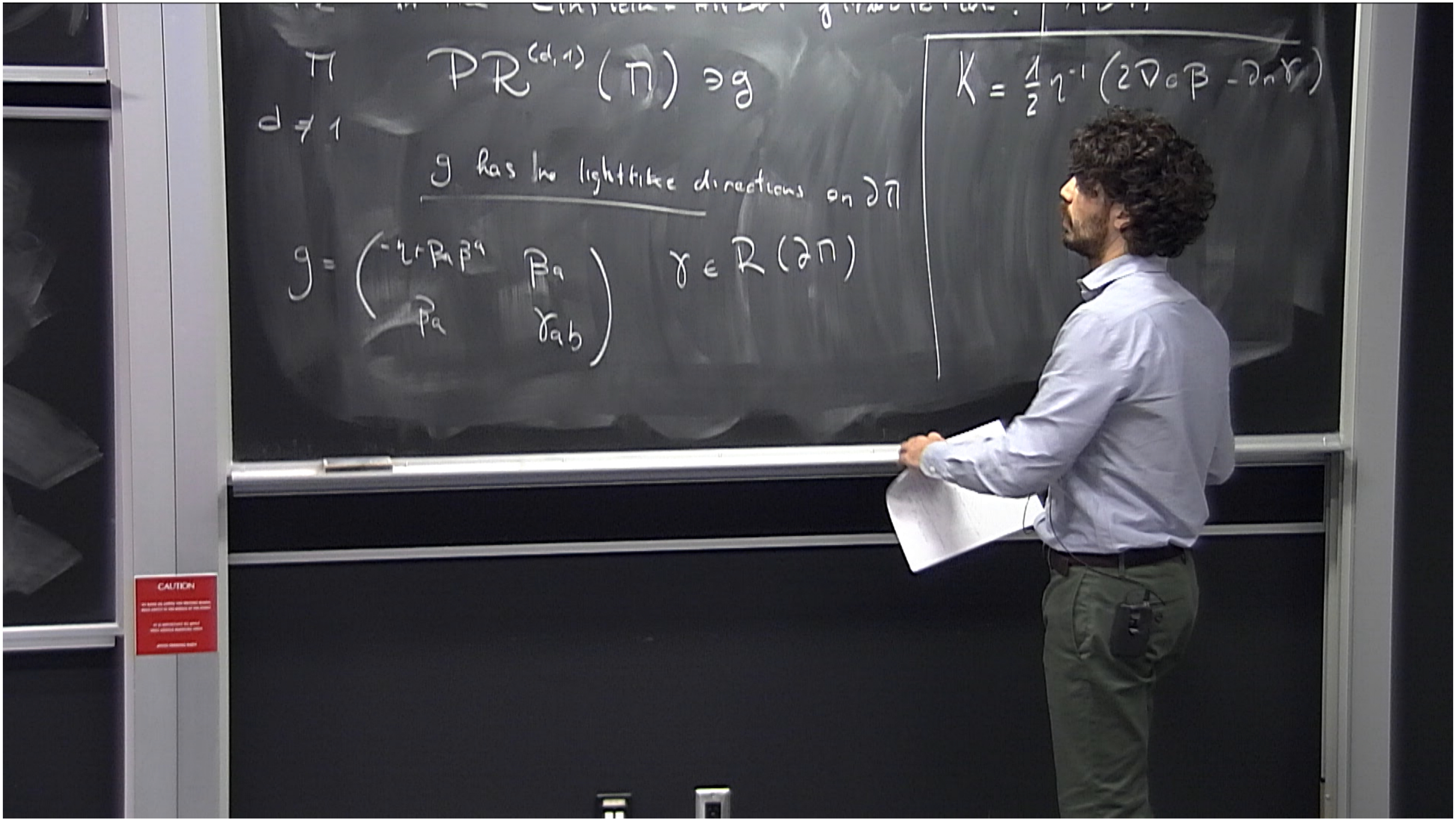

CLAIM For 1D gravity "sub-a"
There is a BV-BFV structure
$S^\partial = \int \left(\frac{\|p\|^2}{2m} + V(q) - E \right) c$

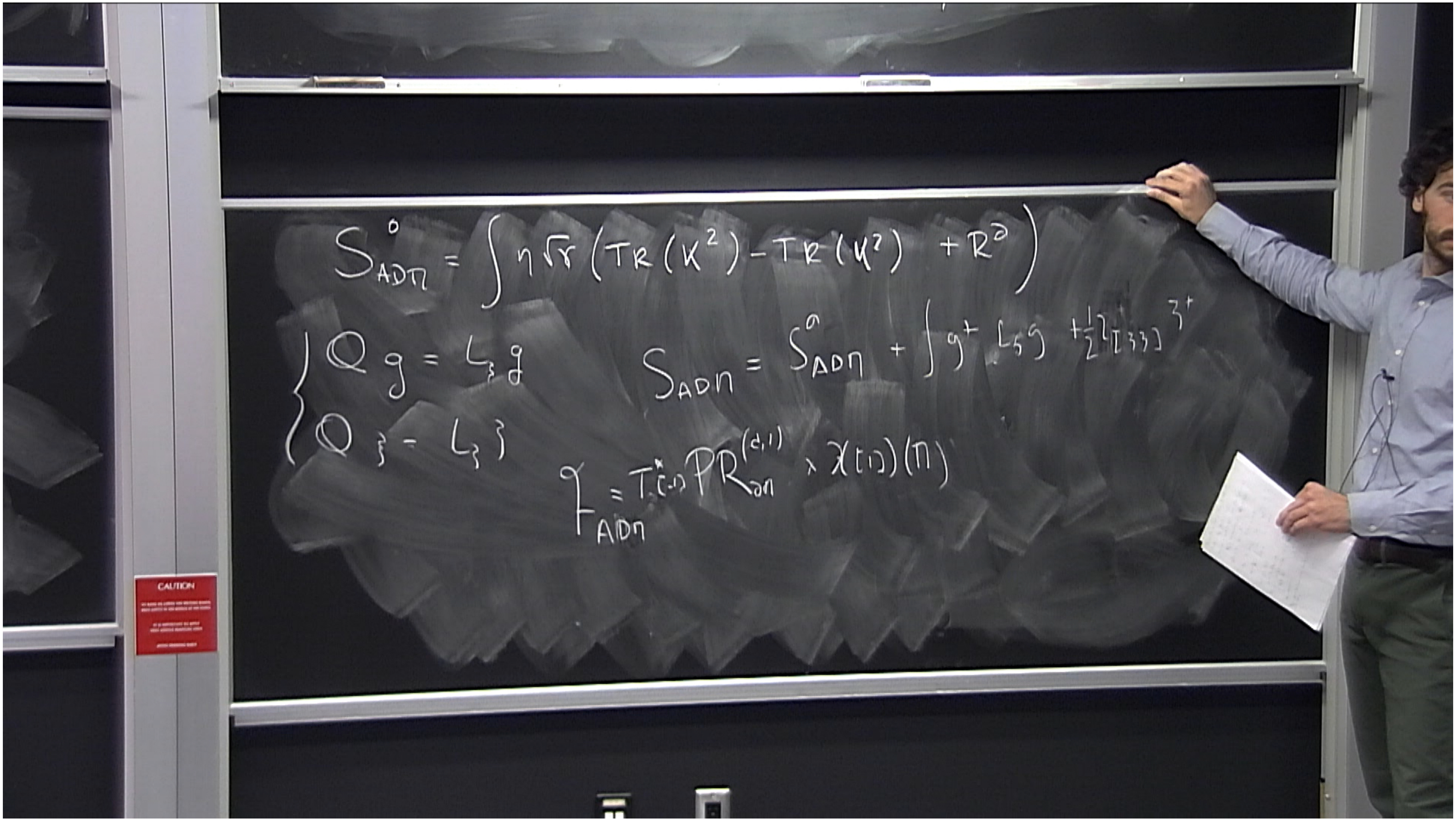

Sometimes things can go wrong, the case

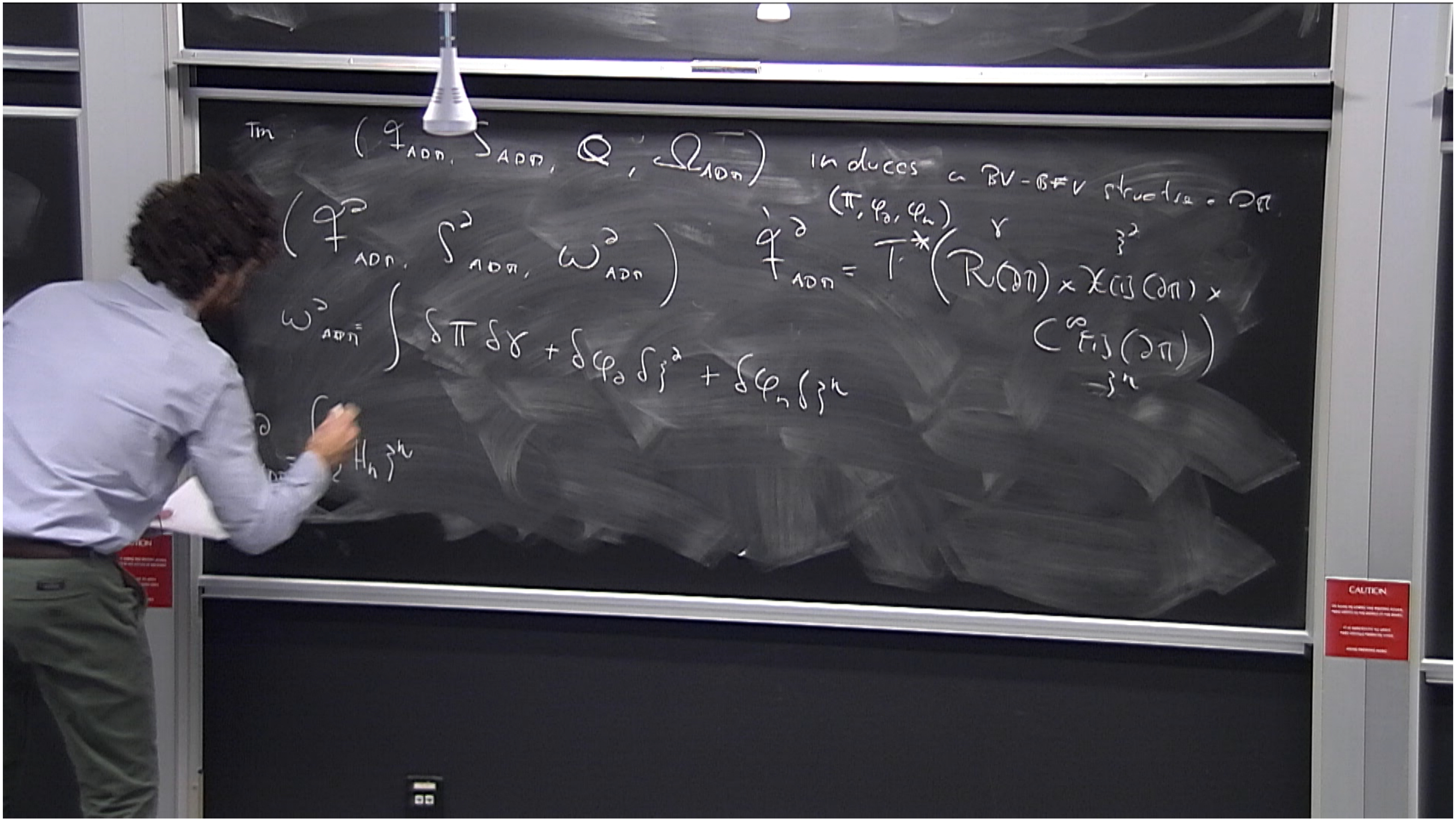

the case
Relativity
With A.S. Cattaneo
Do we have
of gauge theories?
Sometimes
the case of GR.
CAUTION


$\mathfrak{g}_J$
$\check{\mathfrak{g}}_J / \ker(\check{\omega}_J)$
$\check{\mathfrak{g}}_J$
$\check{\omega}_J = \delta \check{\alpha}_J$
$\ker(\check{\omega}_J)$


GR in the Einstein-Hilbert formulation.
ADM
M PR^(d,1)(M) ∋ g
d ≠ 1
g has no lightlike directions on ∂M
g = (-η+β_a β^a β_a ; β_a γ_ab)
γ ∈ R(∂M)
CAUTION


Π $PR^{(d,1)}(\Pi) \ni g$
$d \neq 1$
g has no lightlike directions on $\partial\Pi$
$g = \begin{pmatrix} -\eta + \beta_a\beta^a & \beta_a \\ \beta_a & \gamma_{ab} \end{pmatrix}$
$\gamma \in R(\partial\Pi)$
$K = \frac{1}{2}\eta^{-1}(2\nabla_{(a}\beta_{b)} - \partial_n\gamma)$
CAUTION


S°ADM = ∫ N √γ (TR(K²) − TR(K²) + R∂)
Q g = Lξ g
Q ξ = Lξ ξ
SADM = S°ADM + ∫ g⁺ Lξ g + ½ ξ[ξ,ξ] ξ⁺
γADM = T*[−1] PR(d,1)∂M × X[1](M)


Thm: (F_ADπ, S_ADπ, Q, Ω_ADπ) induces a BV-BFV structure on ∂π
(π, φ_∂, φ_n)
(F^∂_ADπ, S^∂_ADπ, ω^∂_ADπ)
F^∂_ADπ = T*(R(∂π) × X[1](∂π) × C^∞[1](∂π))
ω^∂_ADπ = ∫ δπ δγ + δφ_∂ δξ^∂ + δφ_n δξ^n

BV-BFV structure ∂M

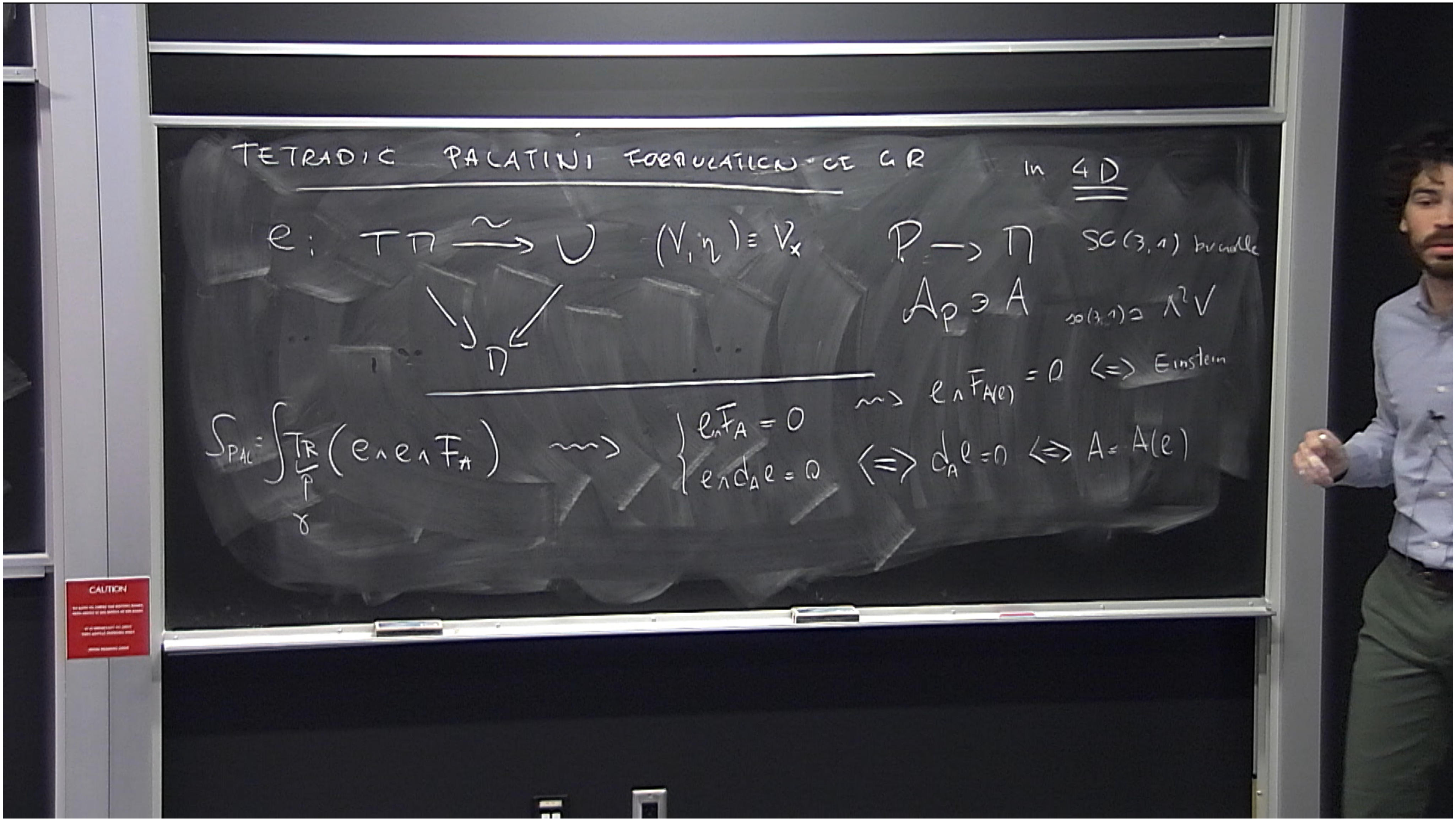
$(\pi, \varphi_\partial, \varphi_n)$

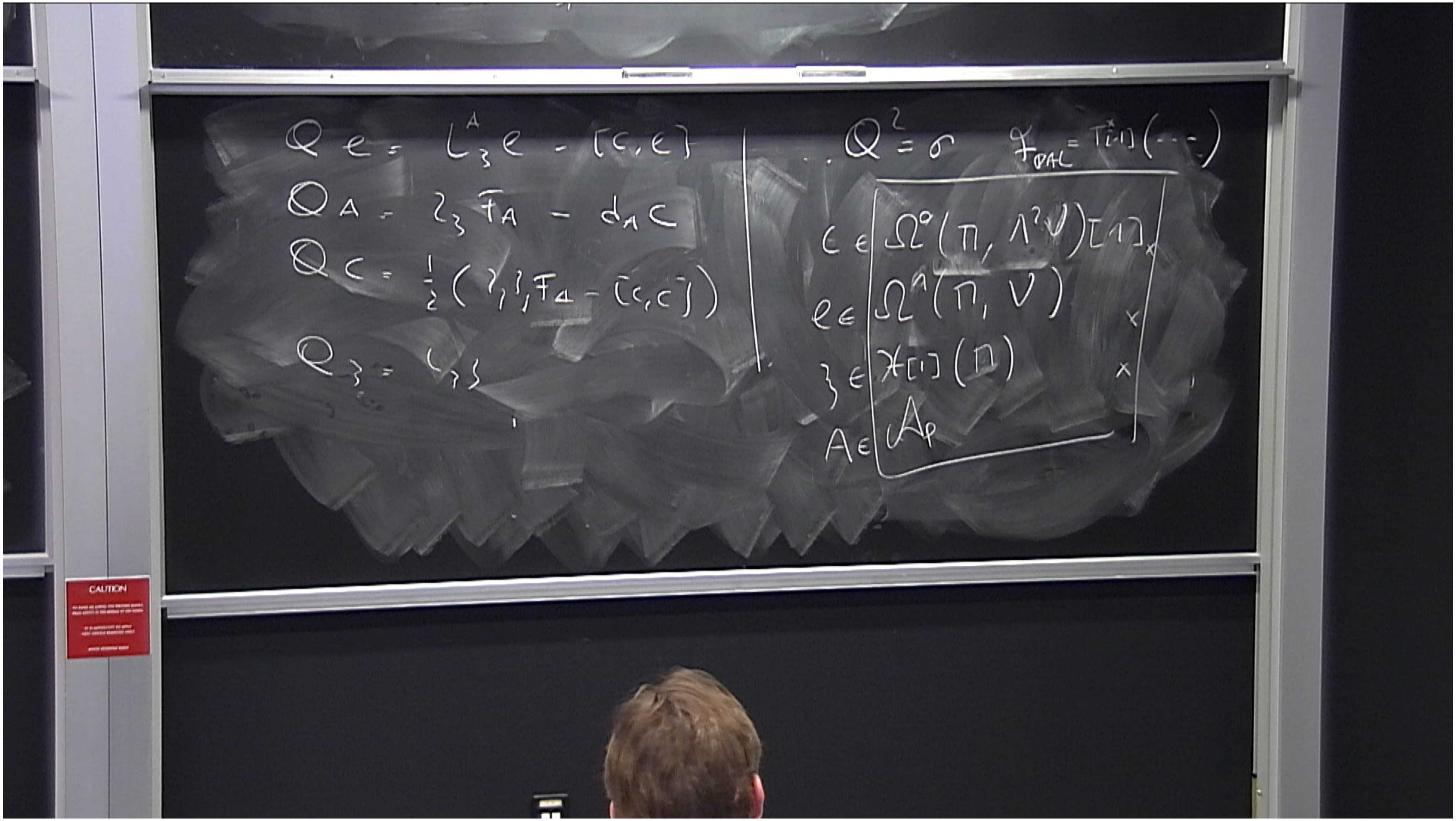
$$\left(\mathcal{F}^\partial_{ADM}, S^\partial_{ADM}, \omega^\partial_{ADM}\right) \qquad \mathcal{F}^\partial_{ADM} = T^*\Big(\overset{\gamma}{\mathcal{R}(\partial M)} \times \overset{\xi^\partial}{\mathfrak{X}[1](\partial M)} \times \underset{\xi^n}{C^\infty[1](\partial M)}\Big)$$

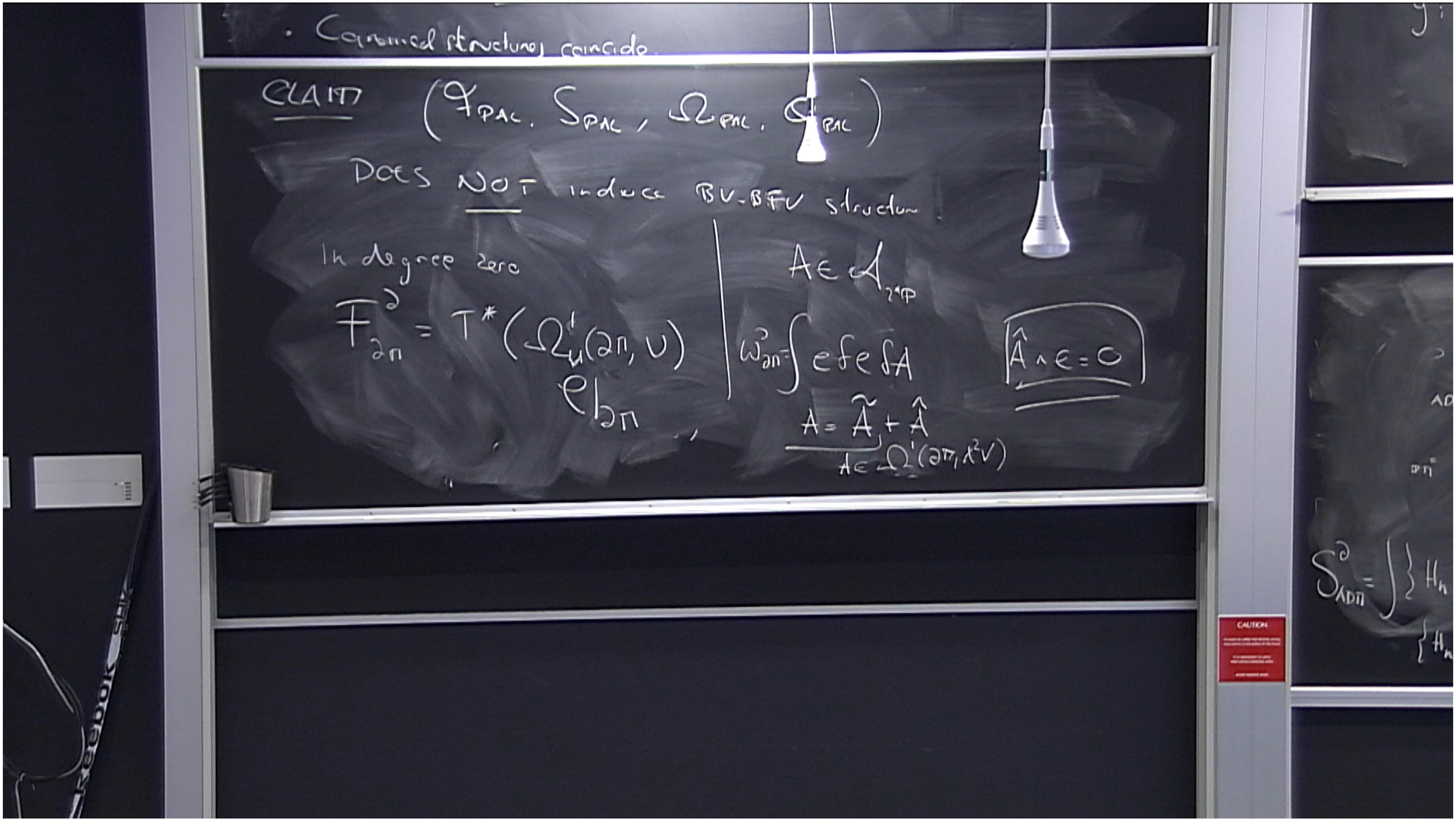
$$\omega^\partial_{ADM} = \int \delta\pi\,\delta\gamma + \delta\varphi_\partial\,\delta\xi^\partial + \delta\varphi_n\,\delta\xi^n$$

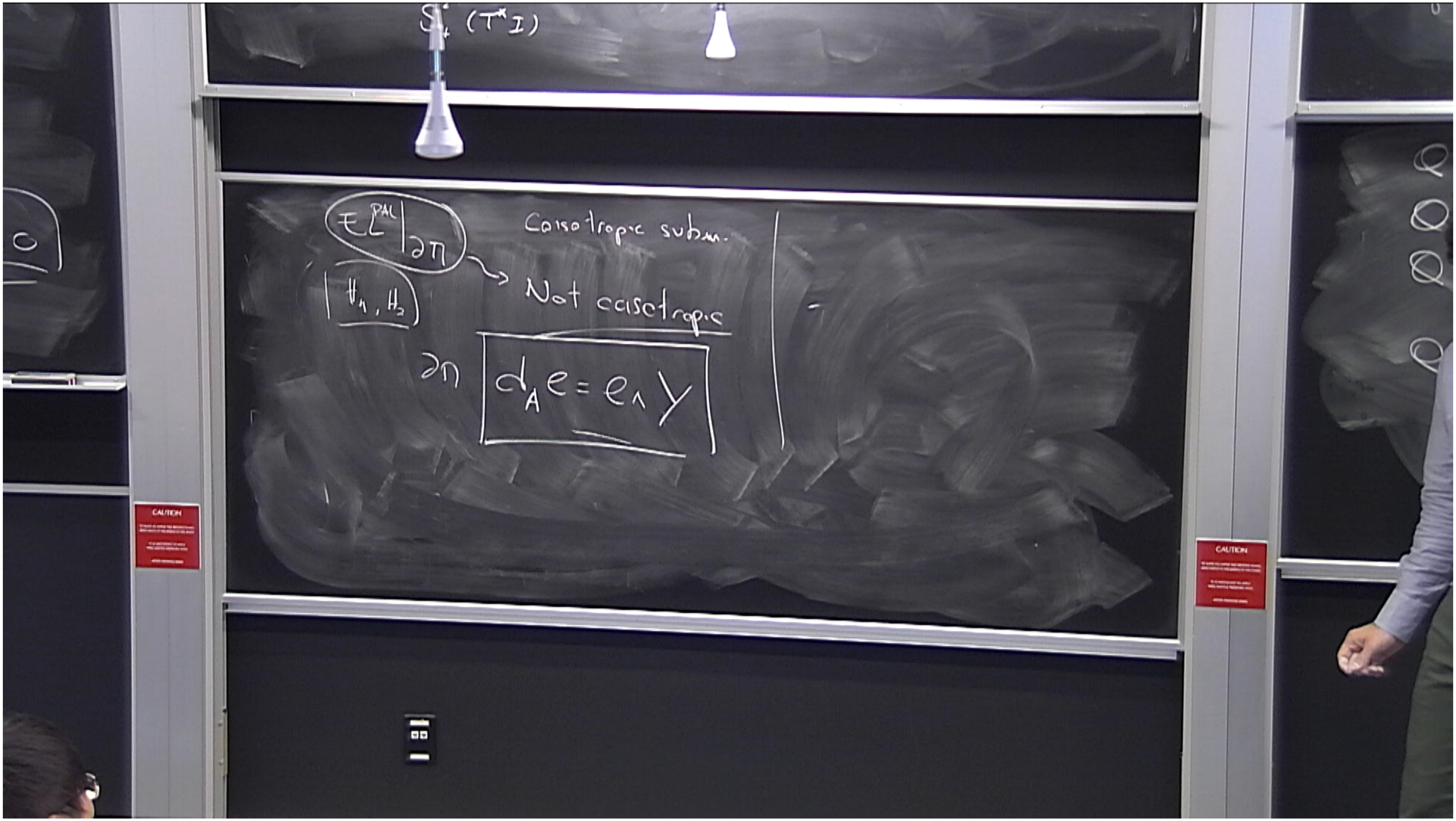
$$S^\partial_{ADM} = \int \Big\{ H_n \xi^n + \iota_{\xi^\partial} H^\partial + \varphi_n L_{\xi^\partial}\xi^n - \gamma^{-1}(\varphi_\partial, d\xi^n)\xi^n + (\varphi_\partial, L_{\xi^\partial}\xi^\partial) \Big\}$$

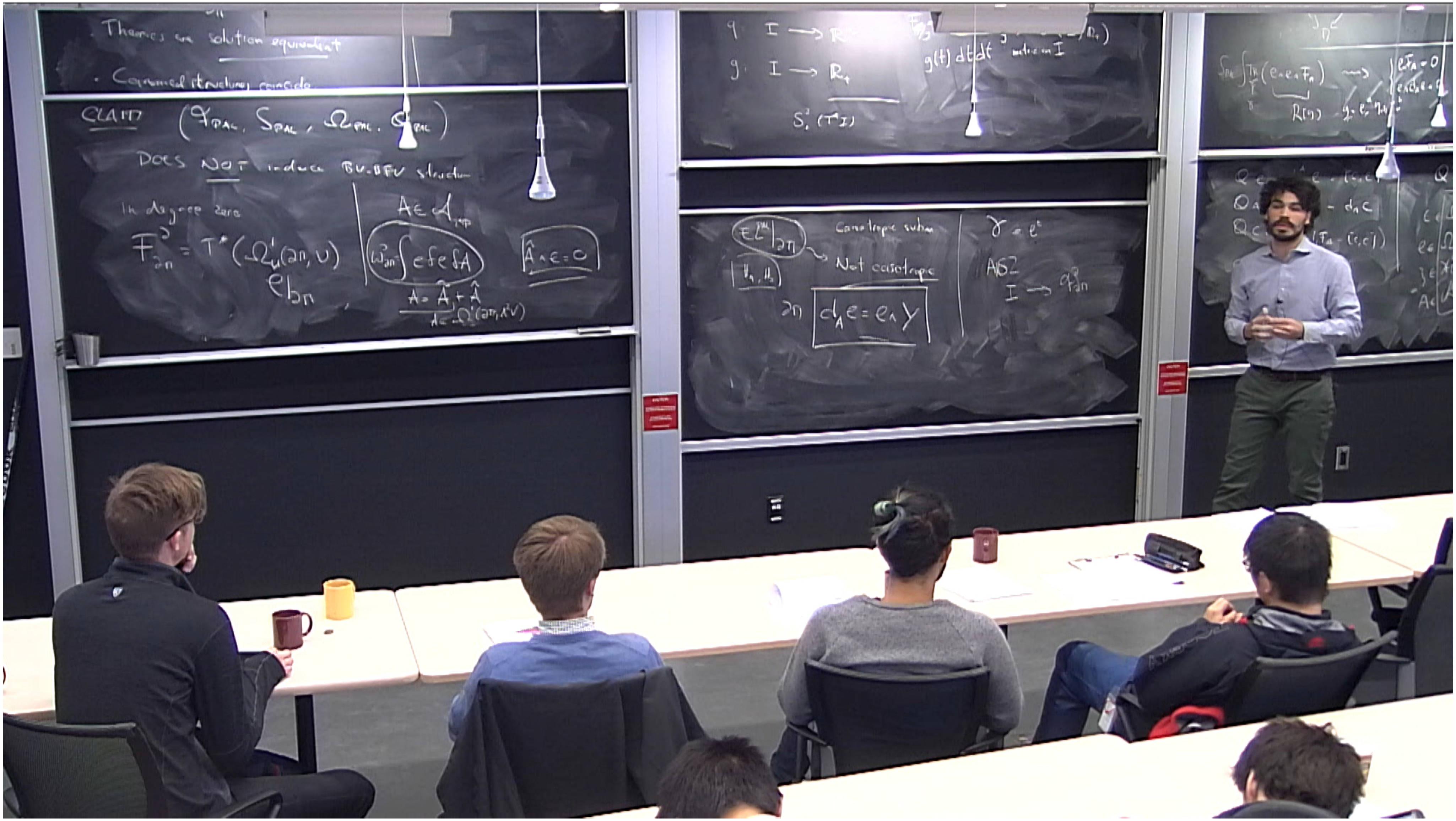
$\{H_n, H^\partial\}$ canonical constraints.


TETRADIC PALATINI FORMULATION OF GR
in 4D
e : TM ~→ V
(V, η) ≡ V_x
P → M
SO(3,1) bundle
A_P ∋ A
so(3,1) ≅ Λ²V
e ∧ F_A(e) = 0 ⟺ Einstein
S_PAL = ∫ Tr (e ∧ e ∧ F_A)
e ∧ F_A = 0
e ∧ d_A e = 0
⟺ d_A e = 0 ⟺ A = A(e)


$Q e = \iota^A_\xi e - [c, e]$
$Q A = \iota_\xi F_A - d_A c$
$Q c = \frac{1}{2}(\iota_\xi \iota_\xi F_A - [c, c])$
$Q \xi = \iota_\xi \xi$
$Q^2 = 0$ $\mathcal{F}_{PAC} = T^*[-1](\ldots)$
$c \in \Omega^0(\Pi, \Lambda^2 V)[1]$ $\times$
$e \in \Omega^1(\Pi, V)$ $\times$
$\xi \in \mathfrak{X}[1](\Pi)$ $\times$
$A \in \mathcal{A}_P$
CAUTION


· Canonical structures coincide.
CLAIM ($\mathcal{F}_{PAC}, S_{PAC}, \Omega_{PAC}, Q_{PAC}$)
DOES NOT induce BV-BFV structure.
In degree zero
$F^{\partial}_{\partial\pi} = T^*(\Omega^1_{V}(\partial\pi, V))$
$e|_{\partial\pi}$
$A \in \mathcal{A}_{\partial\pi}$
$\omega^{\partial}_{\partial\pi} = \int e f e \delta A$
$A = \tilde{A} + \hat{A}$
$A \in \Omega^1(\partial\pi, \Lambda^2 V)$
$\hat{A} \wedge e = 0$


(T^*I)
$EL^{PAL}|_{\partial\Pi}$
Coisotropic subm.
θ_1, θ_2
Not coisotropic
$\partial\Pi$
$d_A e = e \wedge Y$


Theories are solution equivalent
CLAIM
Does NOT induce BV-BFV structure
In degree zero
Not coisotropic