

Title: Equivalence of theories in the presence of boundaries: the example of General Relativity

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Abstract:

Equivalence of theories in the presence of boundaries,
the case of General Relativity J. With A.S. Cattaneo

[Do we have a good notion of equivalence of gauge theories?]

Equivalence of theories in the presence of boundaries,
the case of General Relativity

J. With A.S. Cattaneo

[Do we have a good notion of equivalence of gauge theories?]

Sometimes things can go wrong, the case of GR.

Classical equivalence

$\Pi \rightsquigarrow F_\Pi$ space of classical fields

S_Π action functional

D_Π distribution (symmetries)

EL_Π solutions to the E.L. equations

" EL_Π/D_Π "

\mathbb{R}^n/D_n
Equivalence of theories in the presence of boundaries,

$$(F_n, S_n, D_n) \sim (F'_n, S'_n, D'_n)$$

$$EL_n/D_n \sim EL'_n/D'_n$$

Theories are solution equivalent

Cosmological structures coincide.

1D Gravity coupled to matter. $I \subset \mathbb{R}$

$$T(q) = \frac{m}{2} \|\dot{q}\|^2$$

$$S_G^0[q, g] = \int_I \left(\frac{1}{\sqrt{g}} T(\dot{q}) - \sqrt{g} (V - \epsilon) \right) dt$$

$$q: I \rightarrow \mathbb{R}^n$$

$$g: I \rightarrow \mathbb{R}_+$$

$$F_I = C^\infty(I, \mathbb{R}^n) \times C^\infty(I, \mathbb{R}_+)$$

$g(t) dt dt$ metric on I .

1D Gravity coupled to matter. $V(\dot{q})$

$$T(q) = \frac{m}{2} \|\dot{q}\|^2$$

$$S_G^0[q, g] = \int (\dot{q} - \sqrt{g}(V - E)) dt$$

$$q: I \rightarrow \mathbb{R}^n$$

$$g: I \rightarrow \text{Metric on } I$$

$$F_I = C^\infty(I, \mathbb{R}^n) \times C^\infty(I, \mathbb{R}_+)$$

$$g(t) dt dt \quad \text{metric on } I$$

presence of boundaries

S'_n, D'_n

q^i / D'_n

$$\delta S_G \rightarrow EL \left\{ \begin{array}{l} g = \frac{I}{E-V} * \\ \frac{d}{dt} \left(\frac{m}{\sqrt{g}} \dot{q}^i \right) = -\sqrt{g} \partial_i V \end{array} \right.$$

$$S_G[q, g = \frac{I}{E-V}] = \dots = 2 \int \sqrt{I(E-V)} dt = S_J$$

Jacobi action for reparametr. invariant classical mechanics.
 Classically equivalent theories.

CAUTION

CAUTION

Diffeomorphisms act by Lie derivative.

$$Qg = L_{\zeta}g$$

$$\zeta \in \mathfrak{X}(M)(I)$$

$$Qq = L_{\zeta}q$$

$$S_{q|I} = S_{q|I}^0 + \int g^{\dagger} L_{\zeta}g + q^{\dagger} L_{\zeta}q + \zeta^{\dagger} L_{\zeta}\zeta$$

$$Q\zeta = L_{\zeta}\zeta$$

$$S_{\zeta} = S_{\zeta}^0 + \int q^{\dagger} L_{\zeta}q + \zeta^{\dagger} L_{\zeta}\zeta$$

$$T^*L(I)(F_{q/\zeta} \times \mathfrak{X}(M)(I))$$

CAUTION

Apply the BV-BFU analysis

$$\left(\begin{array}{c} \mathcal{F}^{\partial} \\ \mathcal{F}_{\mathcal{G}/\mathcal{J}} \end{array}, \begin{array}{c} \mathcal{S}^{\partial} \\ \mathcal{S}_{\mathcal{G}/\mathcal{J}} \end{array}, \begin{array}{c} \delta \alpha^{\partial} \\ \parallel \\ \mathcal{W}_{\mathcal{G}/\mathcal{J}}^{\partial} \end{array} \right)$$

$$\pi_{\mathcal{G}/\mathcal{J}} : \mathcal{F}_{\mathcal{G}/\mathcal{J}} \rightarrow \mathcal{F}_{\mathcal{G}/\mathcal{J}}^{\partial}$$

$$\mathcal{L}_{\mathcal{Q}/\mathcal{J}}^{\partial} \mathcal{W}^{\partial} = \delta \mathcal{S}^{\partial}$$

$$\boxed{\begin{array}{l} \pi_{*} \mathcal{Q} = \mathcal{Q}^{\partial} \\ \mathcal{L}_{\mathcal{Q}}^{\partial} \Delta \mathcal{L}_{\mathcal{W}} = \delta \mathcal{S} + \pi^{*} \alpha^{\partial} \end{array}}$$

CAUTION
DO NOT RE-ENTER THE EQUIPMENT AREA,
WHILE WORKING ON THE SYSTEM OR THE WIRING.
IF AN ELECTRICAL SHOCK OCCURS,
IMMEDIATELY REPORT TO THE SUPERVISOR.
AVOID ELECTRICAL SHOCK

CLAIM For 1D gravity "sub-g"
There is a BV-BEV structure

$$S^{\partial} = \int \left(\frac{1}{2m} \Pi^2 + V(q) - E \right) C$$

$$J = \int \left(\frac{1}{2m} \|\dot{q}\|^2 + V(q) - E \right) dt$$

$$\Pi_q \left\{ \begin{array}{l} p = \frac{m \dot{q}}{\sqrt{g}} + q + z \\ b = \frac{1}{\sqrt{g}} (z + z - 2g + g) \\ c = \sqrt{g} \end{array} \right. \quad \left. \begin{array}{l} q^2 \\ T_q = T^* (\mathbb{R}^n \times \mathbb{R}[1]) \end{array} \right.$$

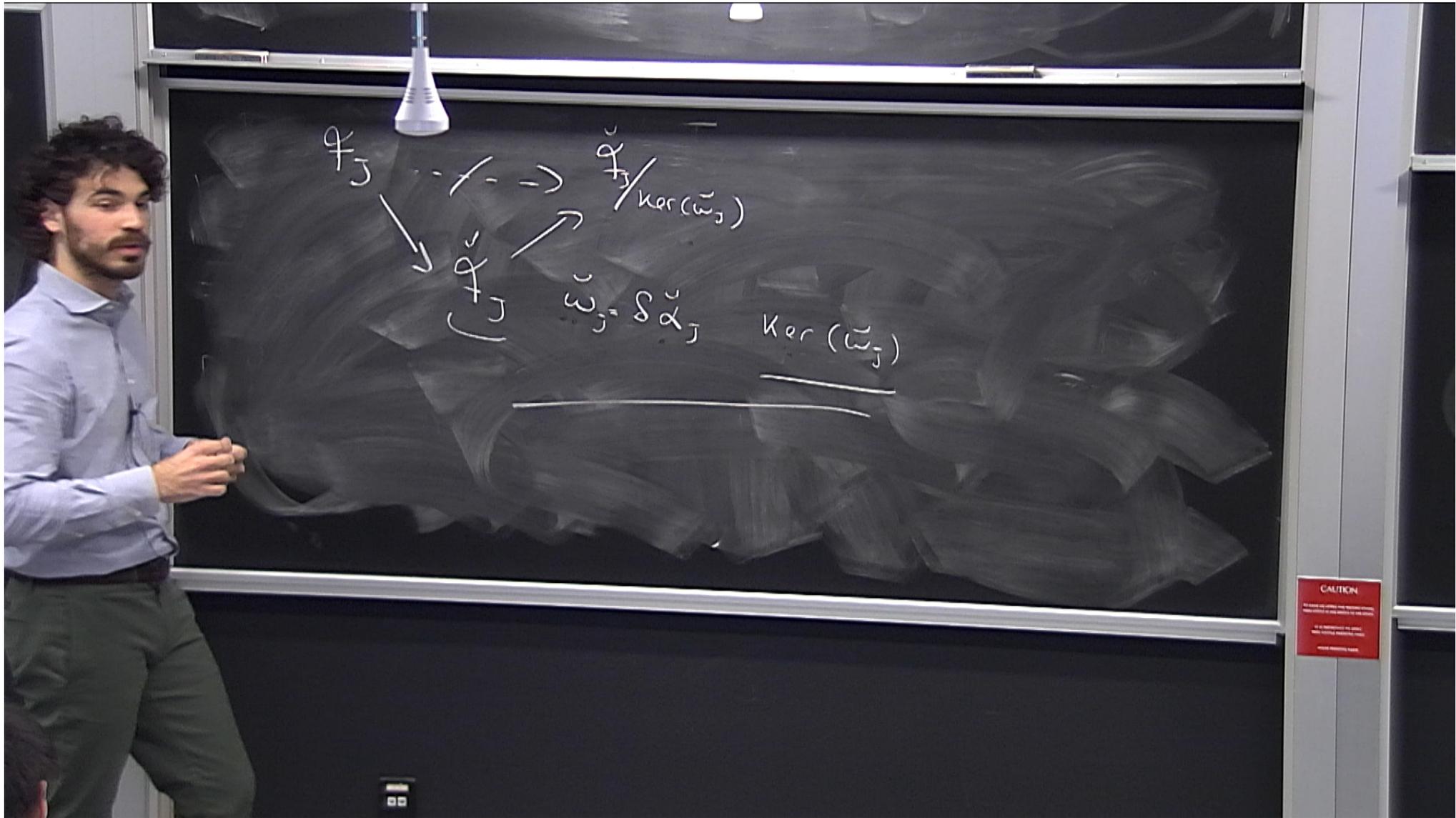
Sometimes things can go wrong, the case of

$$\pi_q \left\{ \begin{array}{l} p = \frac{m q}{\sqrt{g}} + q + z \\ b = \frac{1}{\sqrt{g}} (z + z - 2g + g) \\ c = \sqrt{g} \end{array} \right. \quad \left. \begin{array}{l} \mathcal{F}_q^2 = T^* (\mathbb{R}^n \times \mathbb{R}^{r_1}) \\ \omega_q^2 = \int_{\mathbb{R}^n} \delta p \delta q + \delta b \delta c \end{array} \right.$$

the case General Relativity J. With A.S. Cattaneo

[Do we have a of equivalence of gauge theories?]

Sometimes the case of GR.



Apply the BV-BFV formalism to the Einstein-Hilbert action. ADT

$$\pi \in \mathcal{P}\mathcal{R}^{(d,1)}(\pi) \ni g$$

$d \neq 1$

g has no lightlike directions on $\partial\pi$

$$g = \begin{pmatrix} -2\alpha + \beta_a \beta^a & \beta_a \\ \beta_a & \gamma_{ab} \end{pmatrix} \quad \gamma \in \mathcal{R}(\partial\pi)$$

$$K = \frac{1}{2} \gamma^{ab} \partial_n \gamma_{ab}$$

CAUTION
DO NOT TOUCH THE BOARD
IF IT IS NECESSARY TO CLEAN
THE BOARD, PLEASE CONTACT
THE ASSISTANT

$$\pi \quad PR^{(d,1)}(\pi) \ni g$$

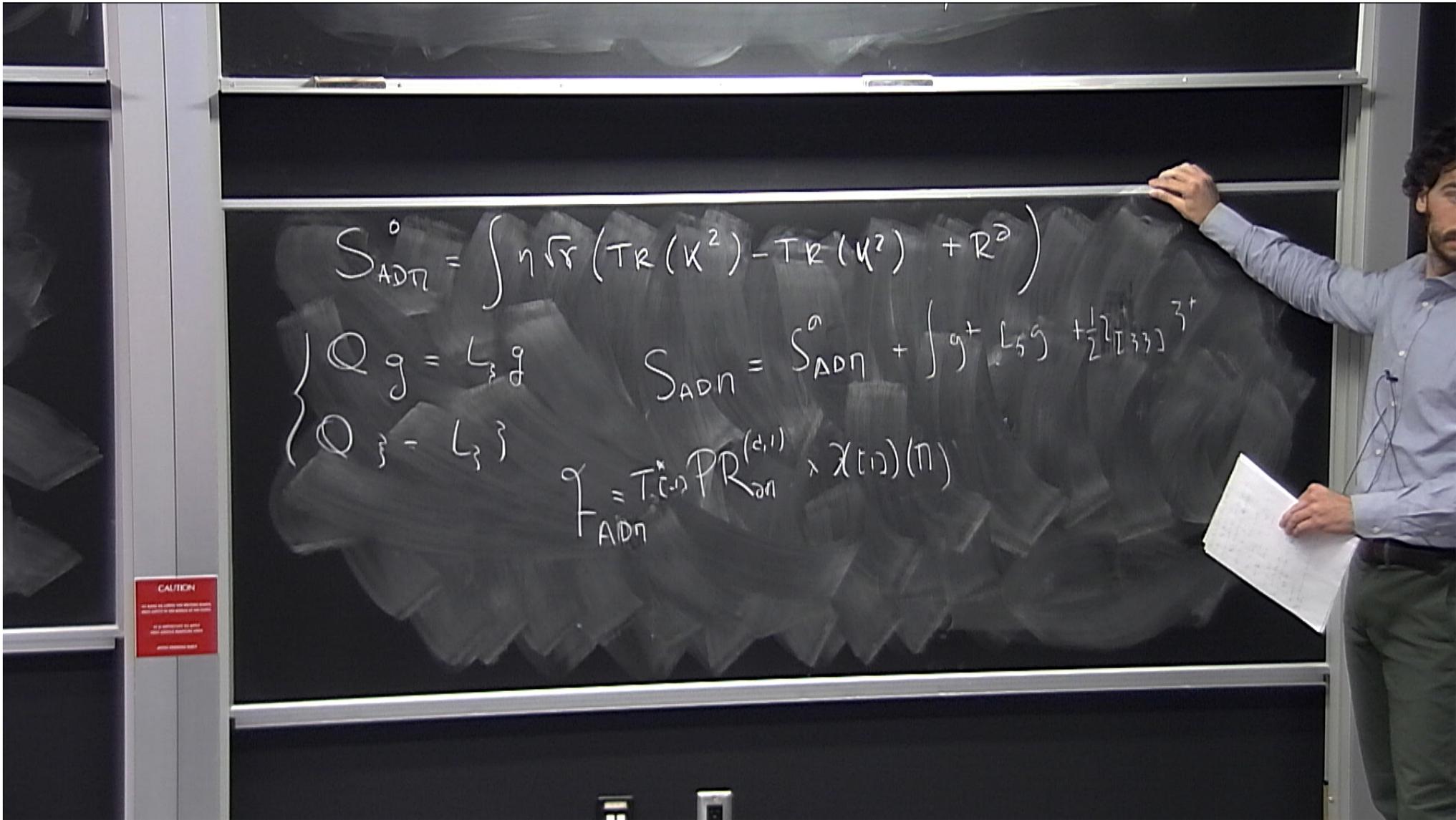
$$d \neq 1$$

g has no lightlike directions on $\partial\pi$

$$g = \begin{pmatrix} -\gamma + \beta_a \beta^a & \beta_a \\ \beta_a & \gamma_{ab} \end{pmatrix} \quad \gamma \in R(\partial\pi)$$

$$K = \frac{1}{2} \gamma^{-1} (2\nabla_a \beta^a - \partial_n \gamma)$$

CAUTION
 AN OPEN OR CLOSED END OF A CABLE
 MAY BECOME A SHOCK HAZARD.
 ALWAYS USE PROPER HANDLING TECHNIQUE.
 ALWAYS WEAR YOUR SAFETY GOGGLES.

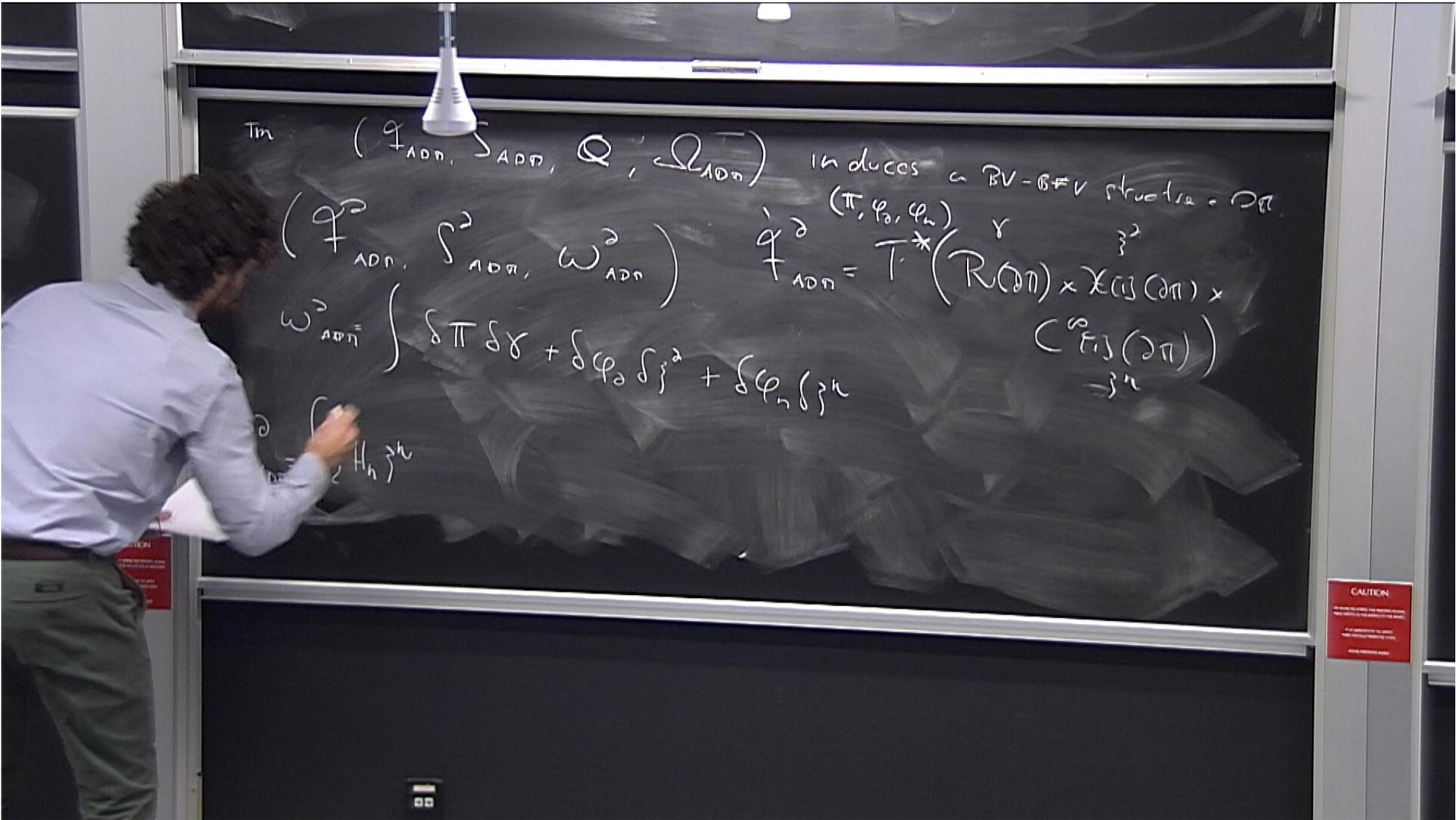


$$S_{AD\eta}^0 = \int \eta \sqrt{\gamma} (\text{TR}(K^2) - \text{TR}(K^2) + R^2)$$

$$\left\{ \begin{array}{l} Qg = L_3 g \\ Q\beta = L_3 \beta \end{array} \right. \quad S_{AD\eta} = S_{AD\eta}^0 + \int g^+ L_3 g + \frac{1}{2} [\dots]^2$$

$$T_{AD\eta} = T_{(d,1)} P R_{(d,1)} \times \chi(t,1)(\eta)$$

CAUTION
 ATTENTION
 ATTENTION



$$(q_{AD\pi}^a, S_{AD\pi}^a, \omega_{AD\pi}^a)$$

$$q_{AD\pi}^a = T^* \left(R(\partial\pi) \times X(\partial\pi) \times \left(\begin{matrix} \gamma \\ \{ \} \\ \{ \} \end{matrix} \right) \right)$$

$$\omega_{AD\pi}^a = \int \delta\pi \delta\gamma + \delta\varphi_a \delta\dot{\gamma}^a + \delta\varphi_n \delta\dot{\gamma}^n$$

$$S_{AD\pi}^a = \int \left\{ \begin{array}{l} H_n \dot{\gamma}^n + \lambda_3 H^3 + \varphi_n L_3 \dot{\gamma}^n - \gamma^{-1} (\varphi_a \dot{\gamma}^a) \dot{\gamma}^n + (\varphi_a L_3 \dot{\gamma}^a) \dot{\gamma}^n \\ \{ H_n, H^a \} \end{array} \right\} \text{Canonical constraints}$$

CAUTION
Do not touch the electrical parts
of the board in the absence of the teacher.

CAUTION
Do not touch the electrical parts
of the board in the absence of the teacher.

TETRADIC PALATINI FORMULATION OF GR

in 4D

$$e: \mathbb{T}\mathbb{T} \xrightarrow{\sim} \mathcal{U} \quad (V, \eta) \equiv V_x \quad \mathcal{P} \rightarrow \mathbb{T} \quad \text{SO}(3,1) \text{ bundle}$$

$$\searrow \quad \swarrow$$

$$\mathcal{A}_\mathcal{P} \ni A \quad \text{so}(3,1) \ni \Lambda^2 V$$

$$S_{\text{PAL}} = \int_{\mathcal{P}} \text{TR} (e \wedge e \wedge F_\star) \quad \rightsquigarrow \quad \begin{cases} e \wedge F_A = 0 \\ e \wedge d_A e = 0 \end{cases} \rightsquigarrow e \wedge F_{A(e)} = 0 \Leftrightarrow \text{Einstein}$$

$$\Leftrightarrow d_A e = 0 \Leftrightarrow A = A(e)$$

CAUTION
DO NOT TOUCH THE BOARD OR THE MARKERS
DO NOT TOUCH THE BOARD OR THE MARKERS



$$Q_e = L_3^A e - [c, e]$$

$$Q_A = L_3 F_A - d_A c$$

$$Q_c = \frac{1}{2} (\gamma_1^3, F_A - [c, c])$$

$$Q_3 = L_3^3$$

$$Q^2 = 0 \quad \mathcal{F}_{PAL} = T_{[1]}^{(1)}(\dots)$$

$$c \in \Omega^0(\pi, \Lambda^2 V) [1]_x$$

$$e \in \Omega^1(\pi, V) \quad x$$

$$z \in \mathcal{X}^{[1]}(\pi) \quad x$$

$$A \in \mathcal{A}_p$$

CAUTION
Do not lean against the chalkboard
Do not touch the chalkboard
Do not touch the chalkboard

• Connected structures coincide.

CLAIM $(\mathcal{F}_{\text{PAL}}, \mathcal{S}_{\text{PAL}}, \Omega_{\text{PAL}}, \mathcal{C}_{\text{PAL}})$

DOES NOT induce BV-BFV structure

In degree zero

$$F_{2\pi}^{\partial} = T^*(\Omega_{\mathcal{U}}^1(\partial\pi, \mathcal{U}))$$

$e|_{\partial\pi}$

$$A \in \mathcal{A}_{2\pi}$$

$$W_{2\pi} = \int e f e \delta A$$

$$\hat{A} \wedge e = 0$$

$$A = \tilde{A} + \hat{A}$$

$A \in \Omega^1(\partial\pi, \mathbb{R}^V)$

CAUTION

$$S_{\text{AD}\pi}^{\partial} = \int \left\{ \begin{array}{l} H_n \\ \dots \\ H_n \end{array} \right.$$

$$S_4^*(T^*I)$$

$$\epsilon_L^{\text{PAL}} / \partial \pi$$

$$H_1, H_2$$

isotropic subm.

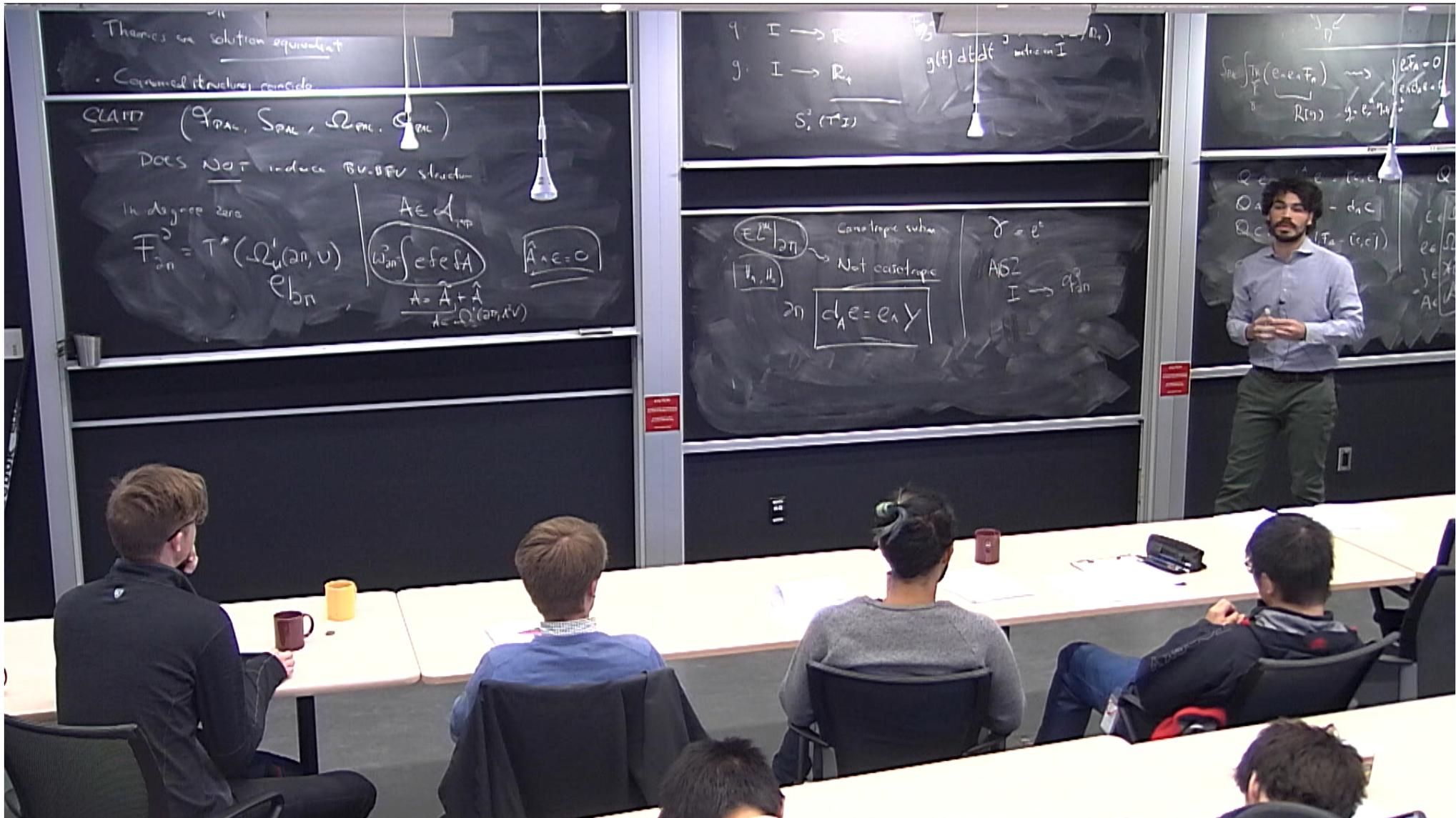
Not isotropic

∂n

$$d_A e = e_{\perp} \gamma$$

CAUTION

CAUTION



Thematics are solution equivalent

• Canonical structures coincide

CLAIM $(\mathcal{F}_{\text{PAC}}, \mathcal{S}_{\text{PAC}}, \mathcal{Q}_{\text{PAC}}, \mathcal{G}_{\text{PAC}})$

DOES NOT induce BU-BFU structure

In degree zero

$F_{\text{an}}^2 = T^*(\mathcal{Q}_{\text{an}}(2n, U))$

e_{bn}

$A \in \mathcal{A}_{\text{an}}$

$\int_{\text{an}} \langle e, e \rangle \langle A, A \rangle$

$\hat{A} \in \mathcal{E} = 0$

$A = \hat{A} + \tilde{A}$

$\tilde{A} \in \mathcal{Q}'(2n, U)$

$\gamma: I \rightarrow \mathbb{R}^n$

$g(t) dt dt^T$ metric on I

$S^1(1,1)$

$E \subset \mathcal{A}_{\text{an}}$ Casotrape subset

(γ, γ) Not casotrape

$\gamma = e^t$

ASZ

$I \rightarrow \mathcal{A}_{\text{an}}$

$d_A e = e_A \gamma$

$\int_{\text{an}} \langle e, e \rangle \langle F_{\text{an}}, F_{\text{an}} \rangle$

$R(n)$

$e_{\text{bn}} = 0$

$e_{\text{bn}} = e_{\text{bn}}$

$Q_{\text{an}} = e_{\text{bn}} - \langle e, e \rangle$

$Q_{\text{an}} = -d_A e$

$Q_{\text{an}} = (F_{\text{an}} - \langle e, e \rangle)$

$e_{\text{bn}} \in \mathcal{Q}_{\text{an}}$

$e_{\text{bn}} \in \mathcal{Q}_{\text{an}}$

$A \in \mathcal{A}_{\text{an}}$