Title: Virasoro constraints, localization and some comments on BV

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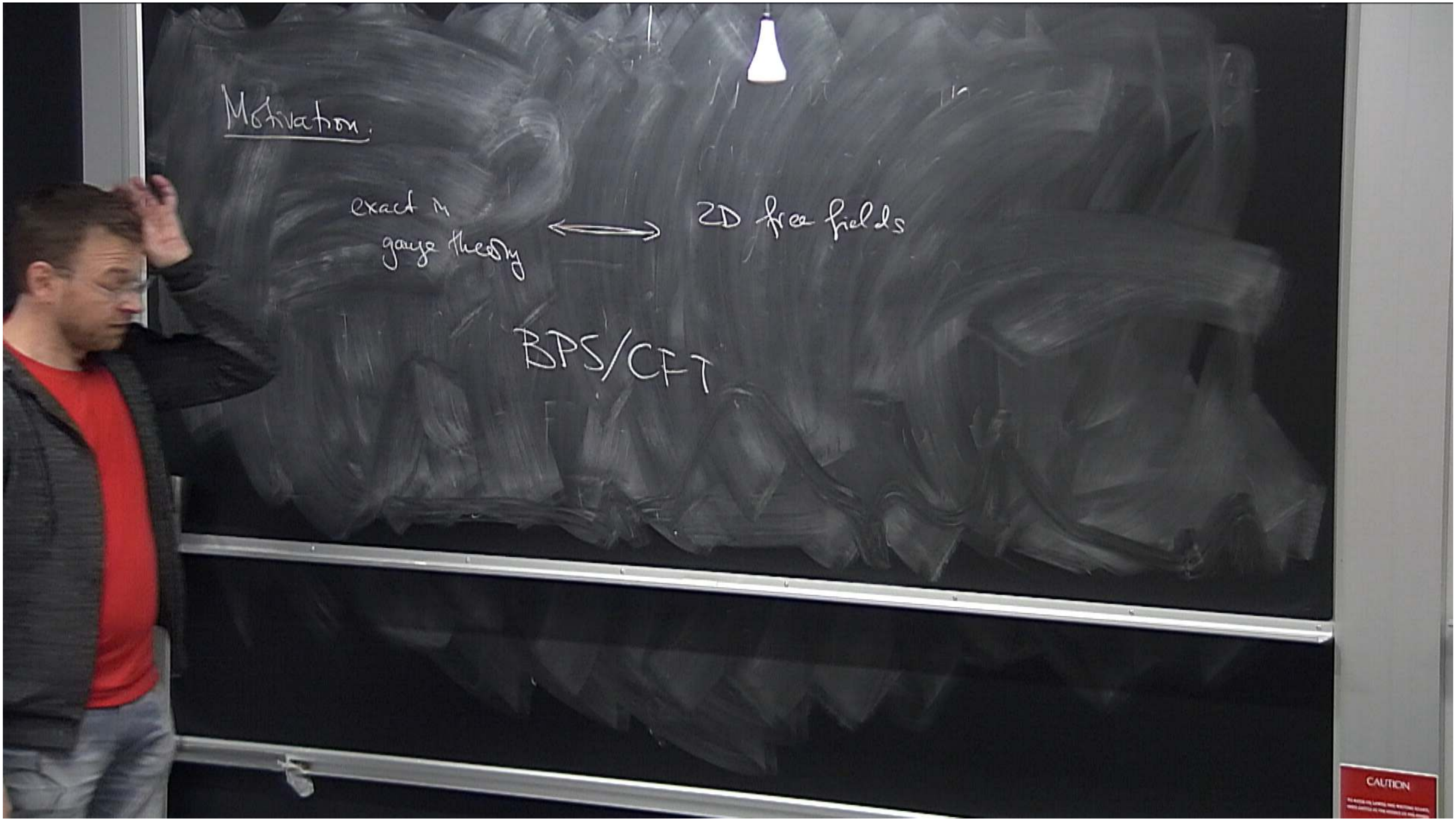
URL: http://pirsa.org/17050024

Abstract:


Motivation.
exact N
gauge theory
2D free fields
BPS/CFT
CAUTION

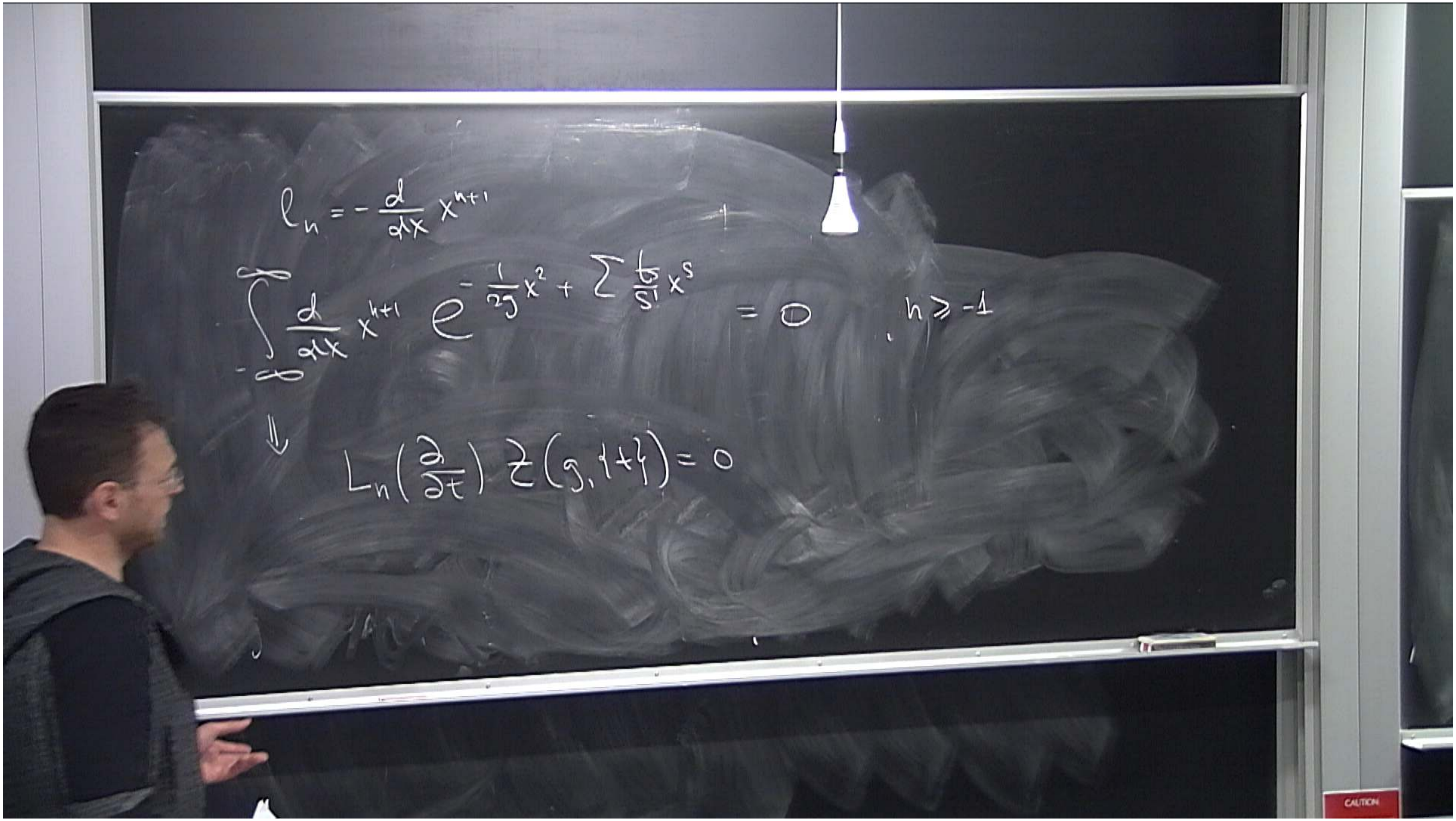
$$\int_{-\infty}^{\infty} f(x)\,dx \qquad \int_{-\infty}^{\infty} \frac{d}{dx} f(x)\,dx = 0$$

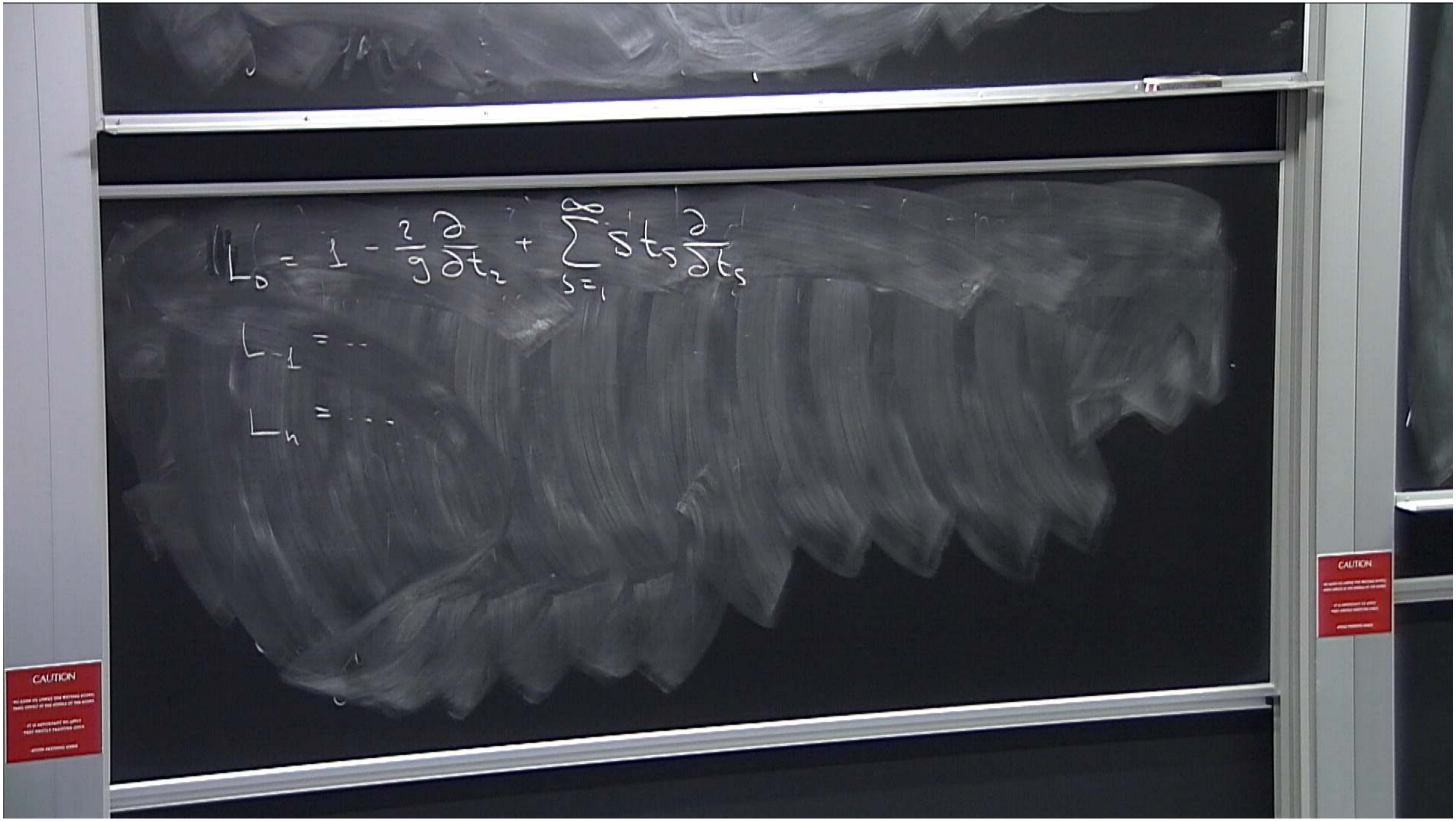
$$\Downarrow$$

$$Z(g,\{t\}) = \int_{-\infty}^{\infty} dx\, e^{-\frac{1}{2g}x^2 + \sum_{n=3}^{\infty} \frac{t_n}{n!} x^n}$$

Motivation.

exact


$l_n = -\frac{d}{dx} x^{n+1}$
$\int_{-\infty}^{\infty} \frac{d}{dx} x^{n+1} e^{-\frac{1}{2g}x^2 + \sum \frac{t_s}{s!} x^s} = 0$, $n \geqslant -1$
$\Downarrow$
$L_n\left(\frac{\partial}{\partial t}\right) Z(g, \{t\}) = 0$
CAUTION


$L_0 = 1 - \frac{2}{g}\frac{\partial}{\partial t_2} + \sum_{s=1}^{\infty} s t_s \frac{\partial}{\partial t_s}$
$L_{-1} = \dots$
$L_n = \dots$
CAUTION
CAUTION

$$Z(g,\{t\}) = e^{t_0}\sqrt{2\pi g}\sum_{p=0}^{\infty}\frac{1}{2^p p!}\underbrace{B_{2p}(t_4,\dots t_{2p})}_{\text{Bell polyn.}}g^p$$

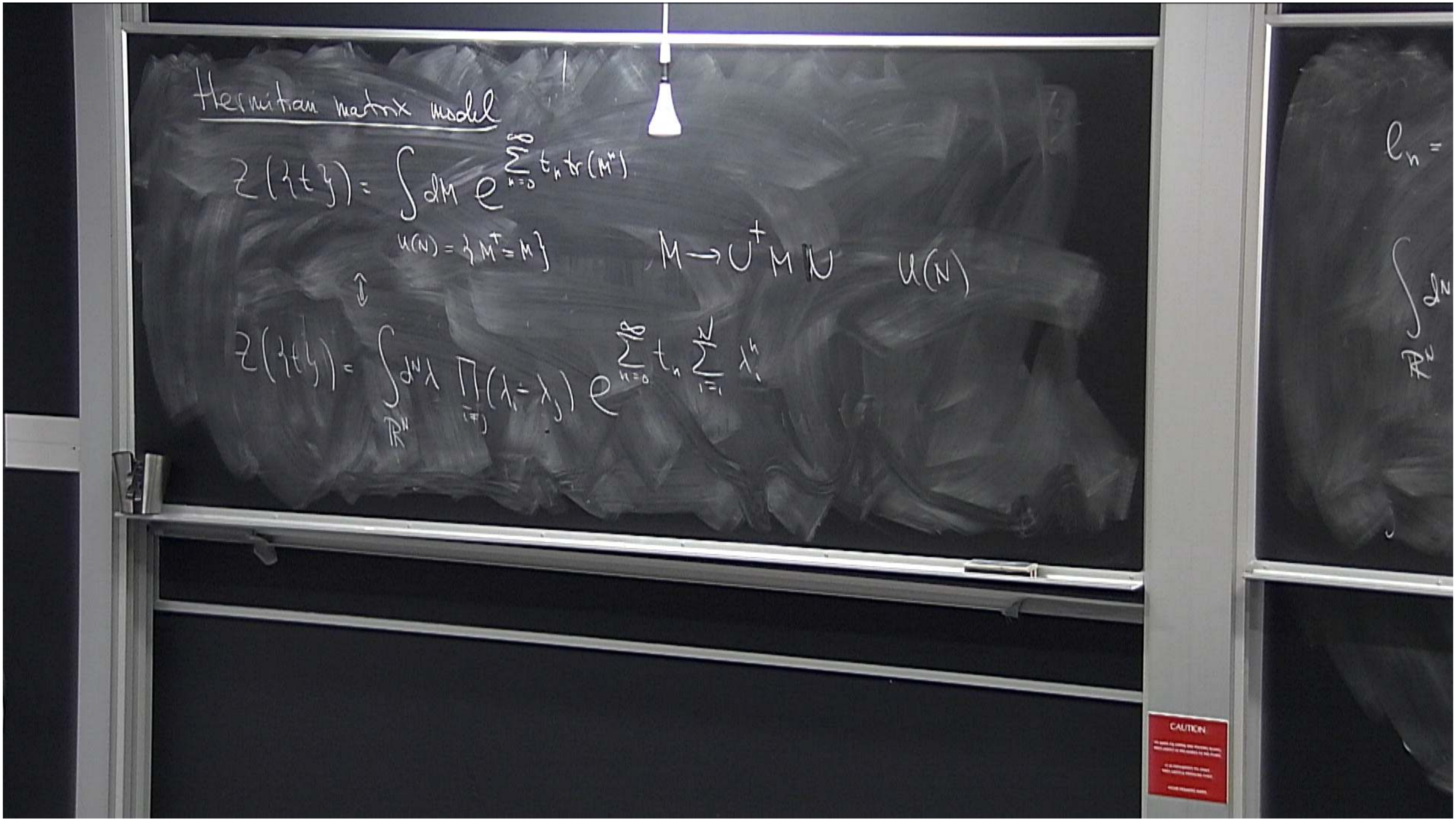
Hermitian matrix model

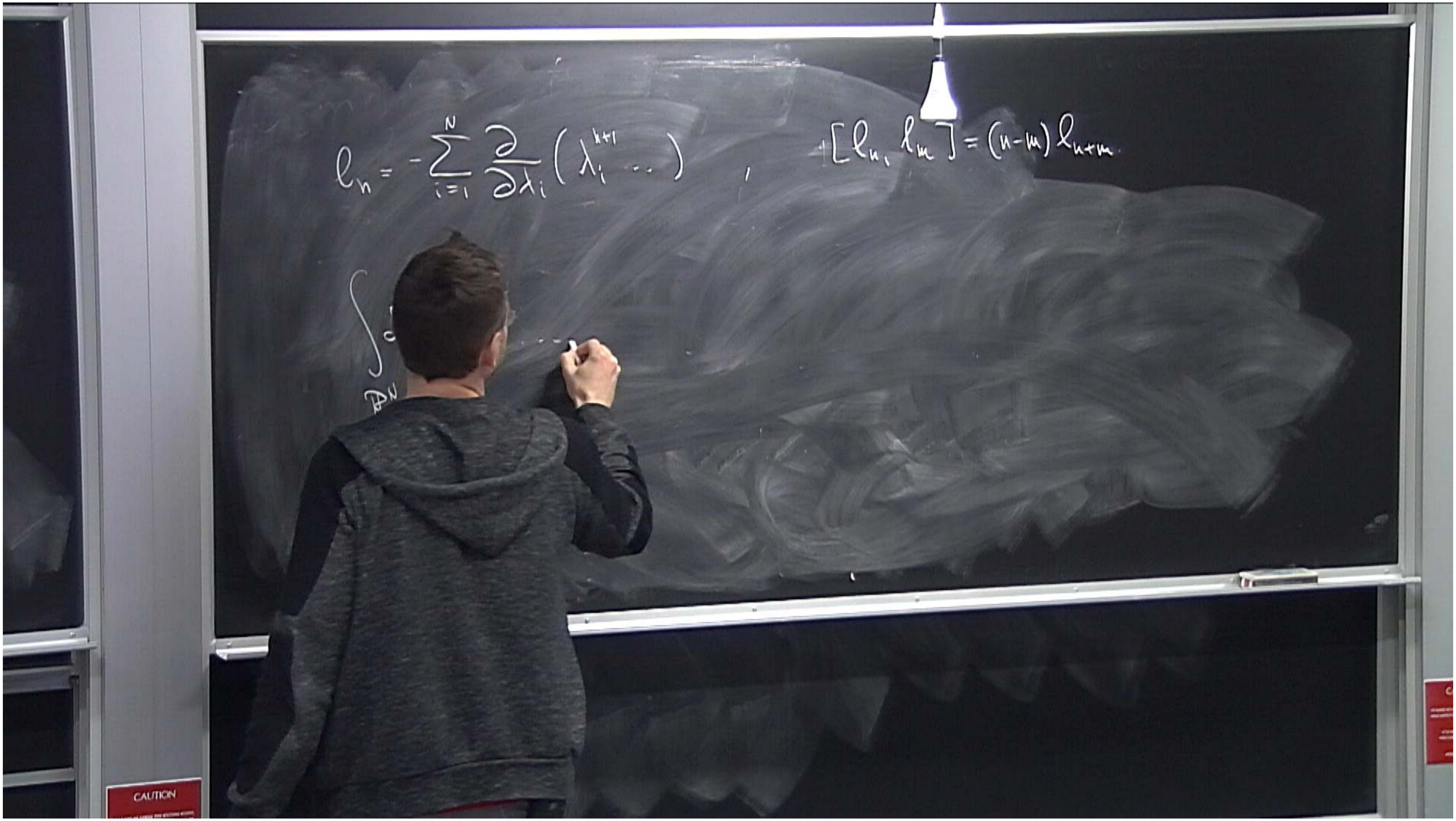
$$Z(\{t\}) = \int_{\mathcal{U}(N) = \{M^\dagger = M\}} dM \, e^{\sum_{n=0}^{\infty} t_n \operatorname{tr}(M^n)}$$

$M \to U^\dagger M U \qquad \mathcal{U}(N)$

$$\Updownarrow$$

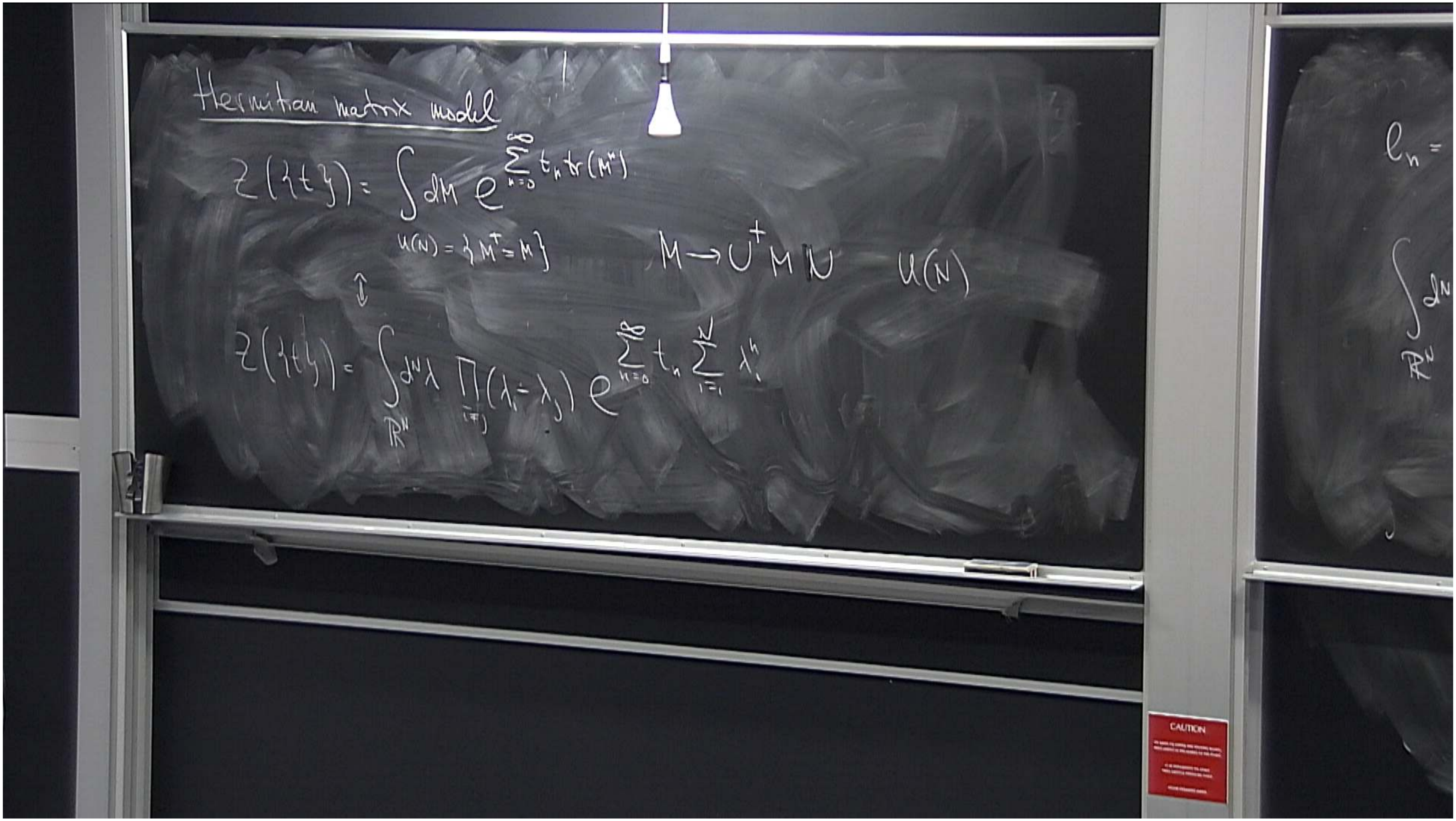
$$Z(\{t\}) = \int_{\mathbb{R}^N} d^N\lambda \prod_{i \neq j} (\lambda_i - \lambda_j) \, e^{\sum_{n=0}^{\infty} t_n \sum_{i=1}^{N} \lambda_i^n}$$

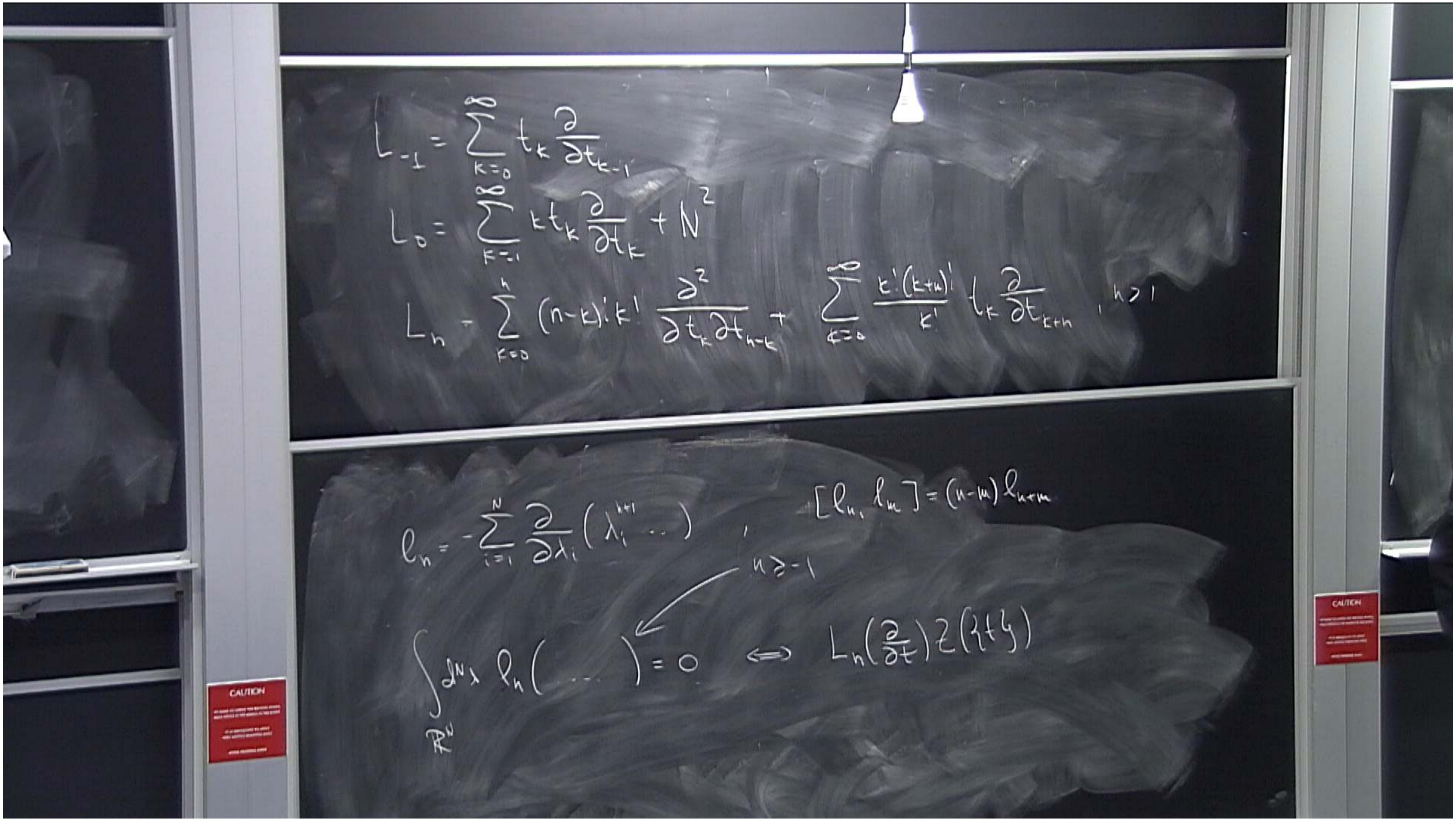

Hermitian matrix model
$Z(\{t\}) = \int_{\mathcal{U}(N)=\{M^\dagger=M\}} dM \, e^{\sum_{n=0}^{\infty} t_n \operatorname{tr}(M^n)}$
$M \to U^\dagger M U$
$\mathcal{U}(N)$
$\Updownarrow$
$Z(\{t\}) = \int_{\mathbb{R}^N} d^N\lambda \prod_{(i\neq j)} (\lambda_i - \lambda_j) \, e^{\sum_{n=0}^{\infty} t_n \sum_{i=1}^{N} \lambda_i^n}$
$\ell_n =$
$\int_{\mathbb{R}^N} d^N$
CAUTION


$\ell_n = -\sum_{i=1}^{N} \frac{\partial}{\partial \lambda_i} \left(\lambda_i^{n+1} \dots \right)$
$[\ell_n, \ell_m] = (n-m)\,\ell_{n+m}$
CAUTION

$$\ell_n = -\sum_{i=1}^{N} \frac{\partial}{\partial \lambda_i} \left(\lambda_i^{n+1} \cdots \right) \quad , \quad [\ell_n, \ell_m] = (n-m)\, \ell_{n+m}$$

$$\int_{\mathbb{R}^N} d^N\lambda \; \ell_n \left(\cdots \right) = 0$$


Hermitian matrix model
$Z(\{t\}) = \int dM \, e^{\sum_{n=0}^{\infty} t_n \operatorname{tr}(M^n)}$
$U(N) = \{M^\dagger = M\}$
$M \to U^\dagger M U$
$U(N)$
$\Updownarrow$
$Z(\{t\}) = \int_{\mathbb{R}^N} d^N\lambda \prod_{i \neq j} (\lambda_i - \lambda_j) \, e^{\sum_{n=0}^{\infty} t_n \sum_{i=1}^{N} \lambda_i^n}$
$\ell_n =$
CAUTION


$L_{-1} = \sum_{k=0}^{\infty} t_k \frac{\partial}{\partial t_{k-1}}$
$L_0 = \sum_{k=1}^{\infty} k t_k \frac{\partial}{\partial t_k} + N^2$
$L_n = \sum_{k=0}^{n} (n-k)!\,k!\, \frac{\partial^2}{\partial t_k \partial t_{n-k}} + \sum_{k=0}^{\infty} \frac{k!\,(k+n)!}{k!} t_k \frac{\partial}{\partial t_{k+n}}, \; n \geq 1$
$\ell_n = -\sum_{i=1}^{N} \frac{\partial}{\partial \lambda_i} \left(\lambda_i^{n+1} \ldots \right), \; n \geq -1$
$[\ell_n, \ell_m] = (n-m)\, \ell_{n+m}$
$\int_{\mathbb{R}^N} d^N\lambda \; \ell_n(\ldots) = 0 \iff L_n\left(\frac{\partial}{\partial t}\right) Z(\{t\})$
CAUTION
CAUTION

$t_0, t_1, t_2 \ldots\ldots$ Heisenberg alg.

$\rightsquigarrow$ $a_n = \frac{n!}{\sqrt{2}} \frac{\partial}{\partial t_n}$, $a_{-n} = \frac{n!}{\sqrt{2}} t_n$

$n > 0$

$L_n = \frac{1}{2} \sum_{m=-\infty}^{\infty} :a_{n-m} a_m:$ $\longleftarrow$ Virasoro for free chiral boson

$c = 1$

$$L_n = \frac{1}{2}\sum_{m=-\infty} :a_{n-m}\, a_m: \quad \longleftarrow \text{Virasoro for free chiral boson}$$

$$c = 1$$

$$\varphi(\lambda) = \sum_n a_{-n}\,\lambda^n$$

$$\prod_{i\neq j}(\lambda_i - \lambda_j) = e^{\sum_{i\neq j}\log(\lambda_i-\lambda_j)}\ , \qquad S(\lambda) = :e^{\varphi(\lambda)}:$$

$$Z(\{t\}) = \int d\lambda_1\, S(\lambda_1)\cdots\int d\lambda_N\, S(\lambda_N)\,|0\rangle \ ; \qquad \left[L_n, \int S(\lambda)\,d\lambda\right] = 0$$

$U(N)$

$\mathcal{N}=2$ gauge theory in 3D with adj chiral multiplet of mass M

$$Z = \oint \prod_{i=1}^{N} \frac{dw_i}{2\pi i w_i} \prod_{i\neq j} \frac{(w_i/w_j; q)_\infty}{(t w_i/w_j; q)_\infty} \, e^{\sum_{n=0}^{\infty} t_n \sum_{i=1}^{N} w_i^n} \qquad \mathbb{R}^2_\varepsilon \times S^1$$

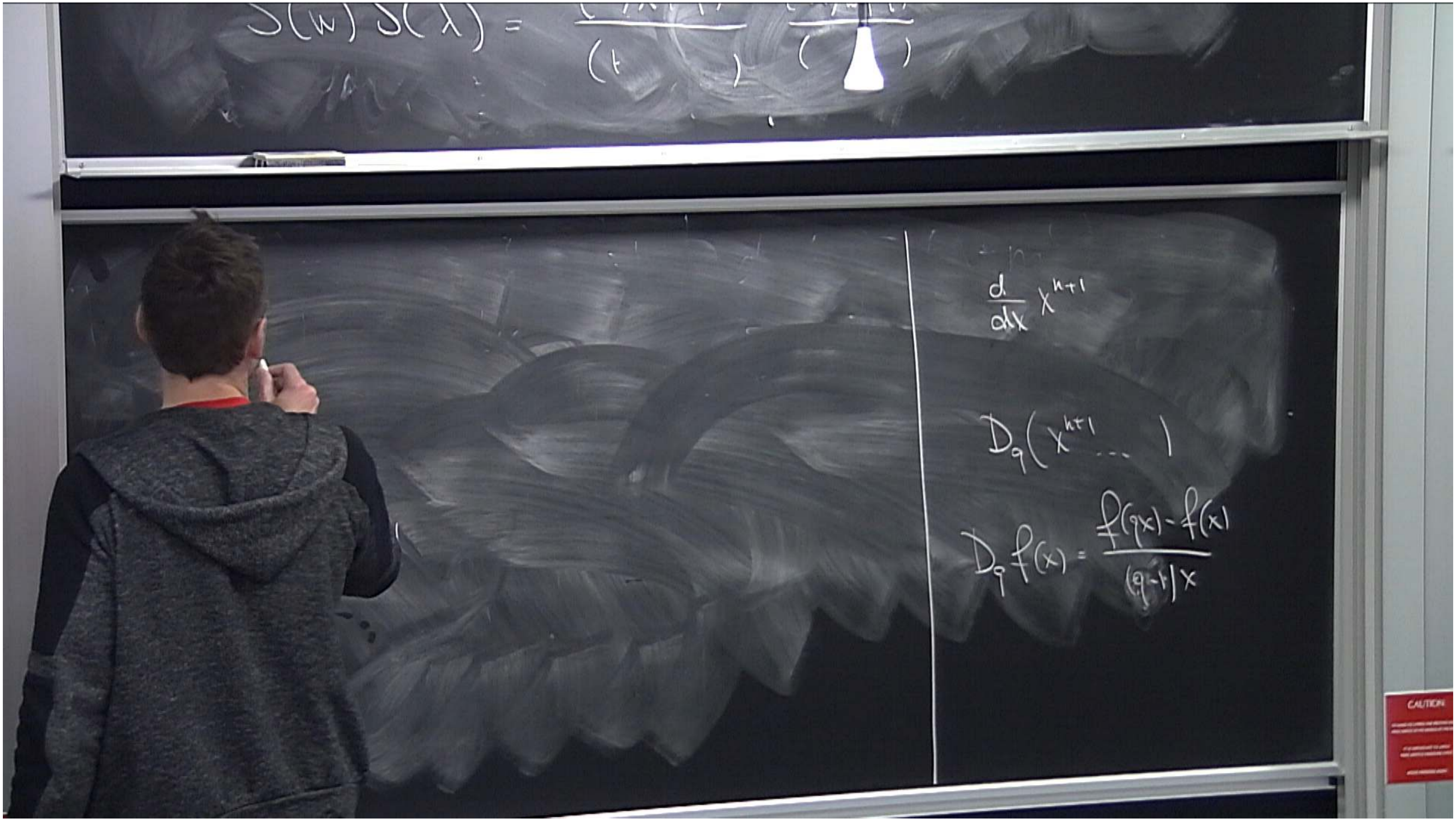
$$(z;q)_\infty = \prod_{n=0}^{\infty} (1 - z q^n), \quad |q|<1, \quad q = e^{\varepsilon}, \quad t = e^{M}$$

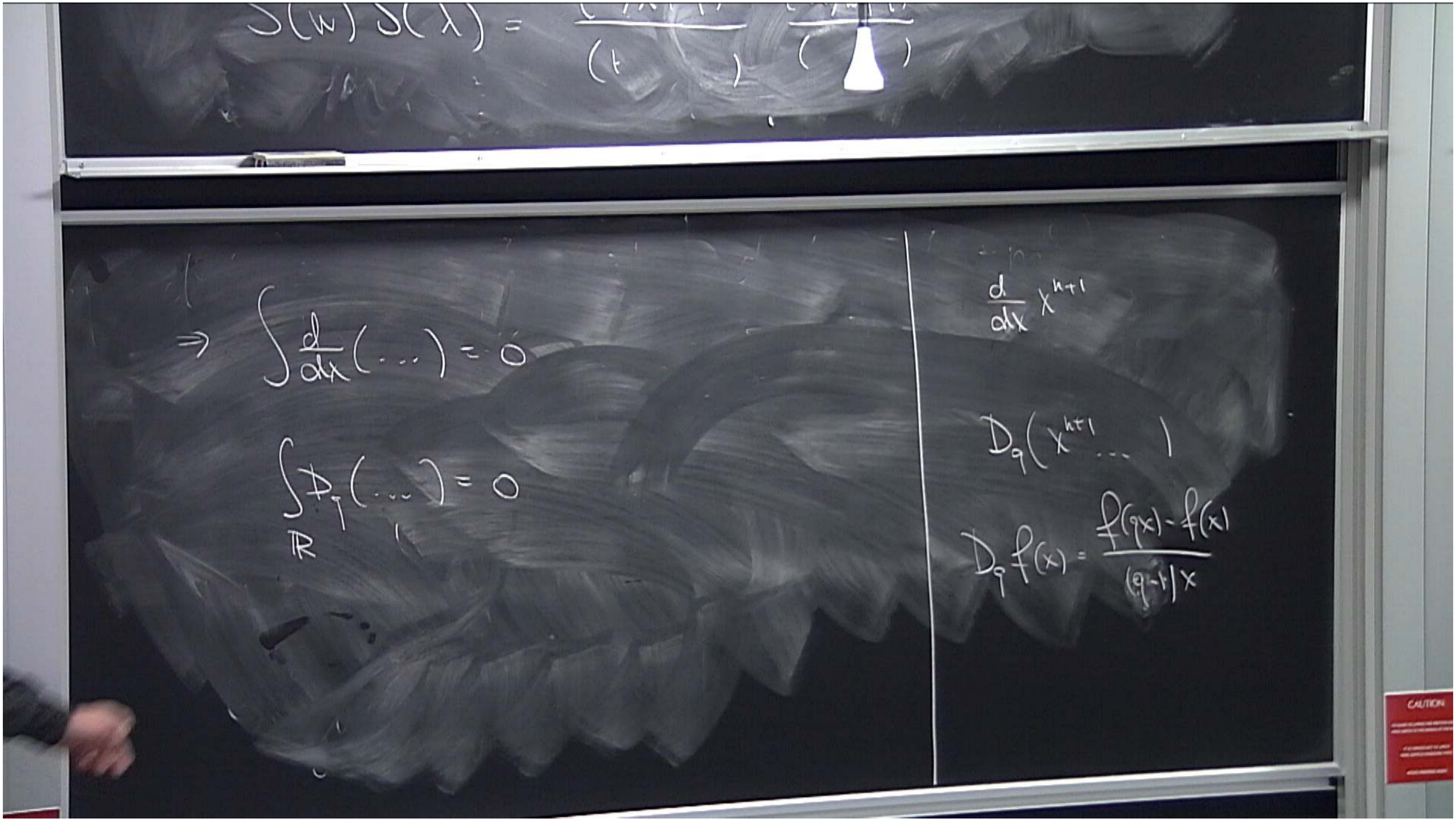
$q, t \qquad a_n = (q^{n/2} - q^{-n/2})\, t_n \,, \quad a_n = \frac{1}{n}(t^{n/2} - t^{-n/2})(p^{n/2} + p^{-n/2}) \frac{\partial}{\partial t_n}$

$p = t q^{-1}$

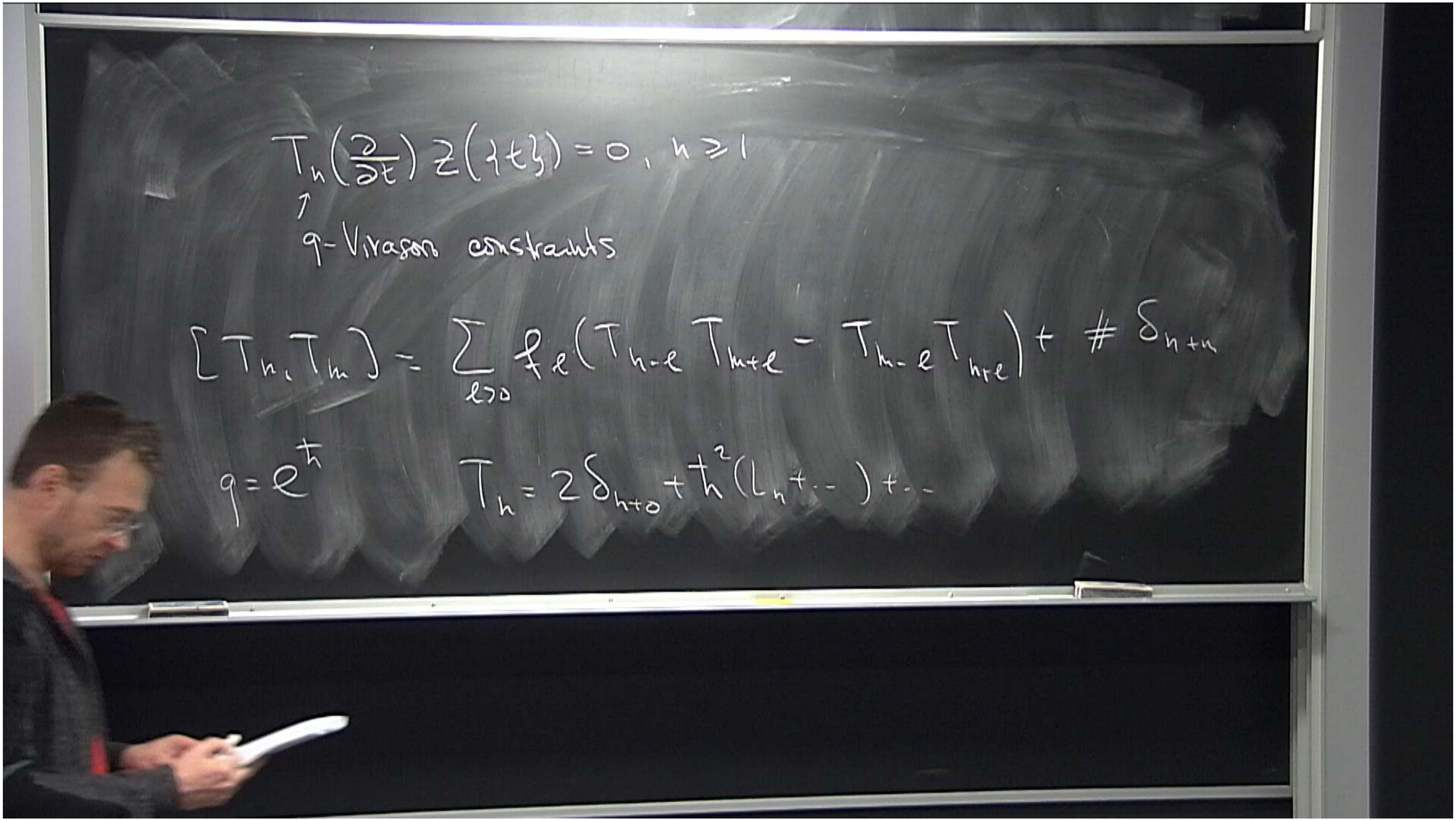
$S(w) = :e^{\sum_{n \neq 0} \frac{w^{-n}}{q^{n/2} - q^{-n/2}} a_n}:$ up to zero modes

$S(w)\, S(\lambda) = \frac{(w/\lambda ; q)}{(t \qquad)} \; \frac{(\lambda / w ; q)}{(\qquad)}$

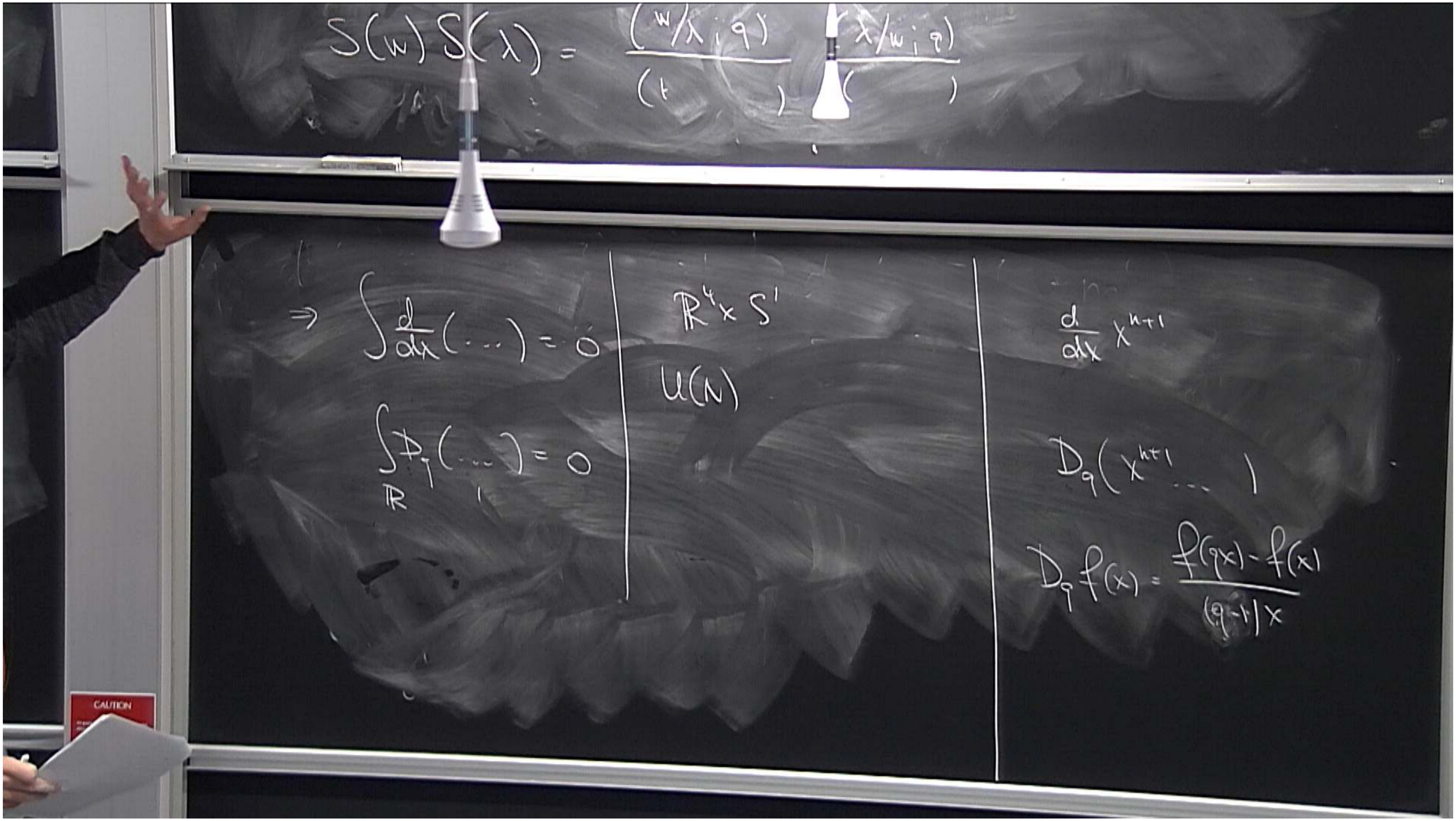

S(w) S(λ) =
d/dx x^{n+1}
D_q(x^{n+1} ...)
D_q f(x) = (f(qx) - f(x)) / ((q-1)x)

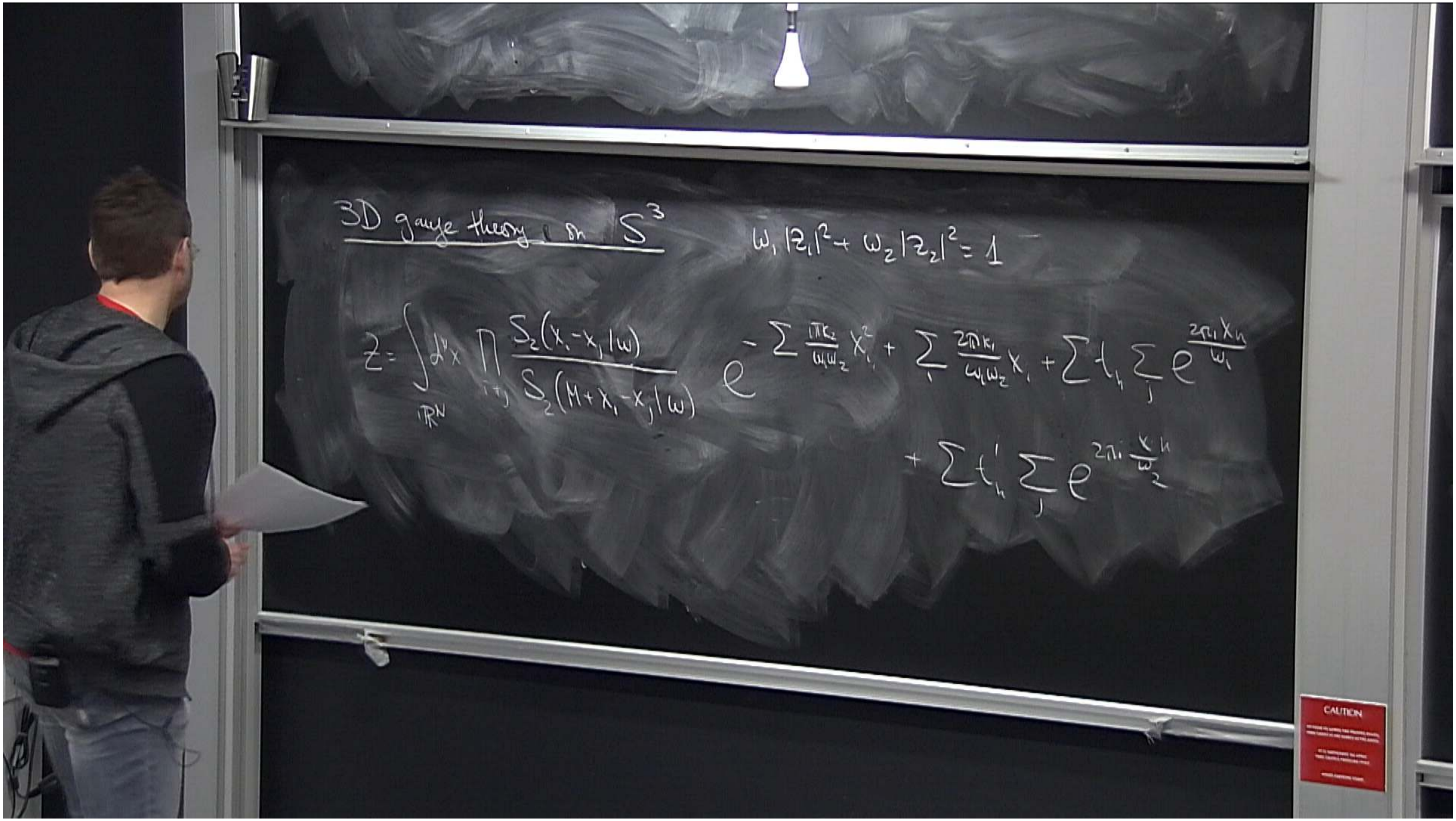


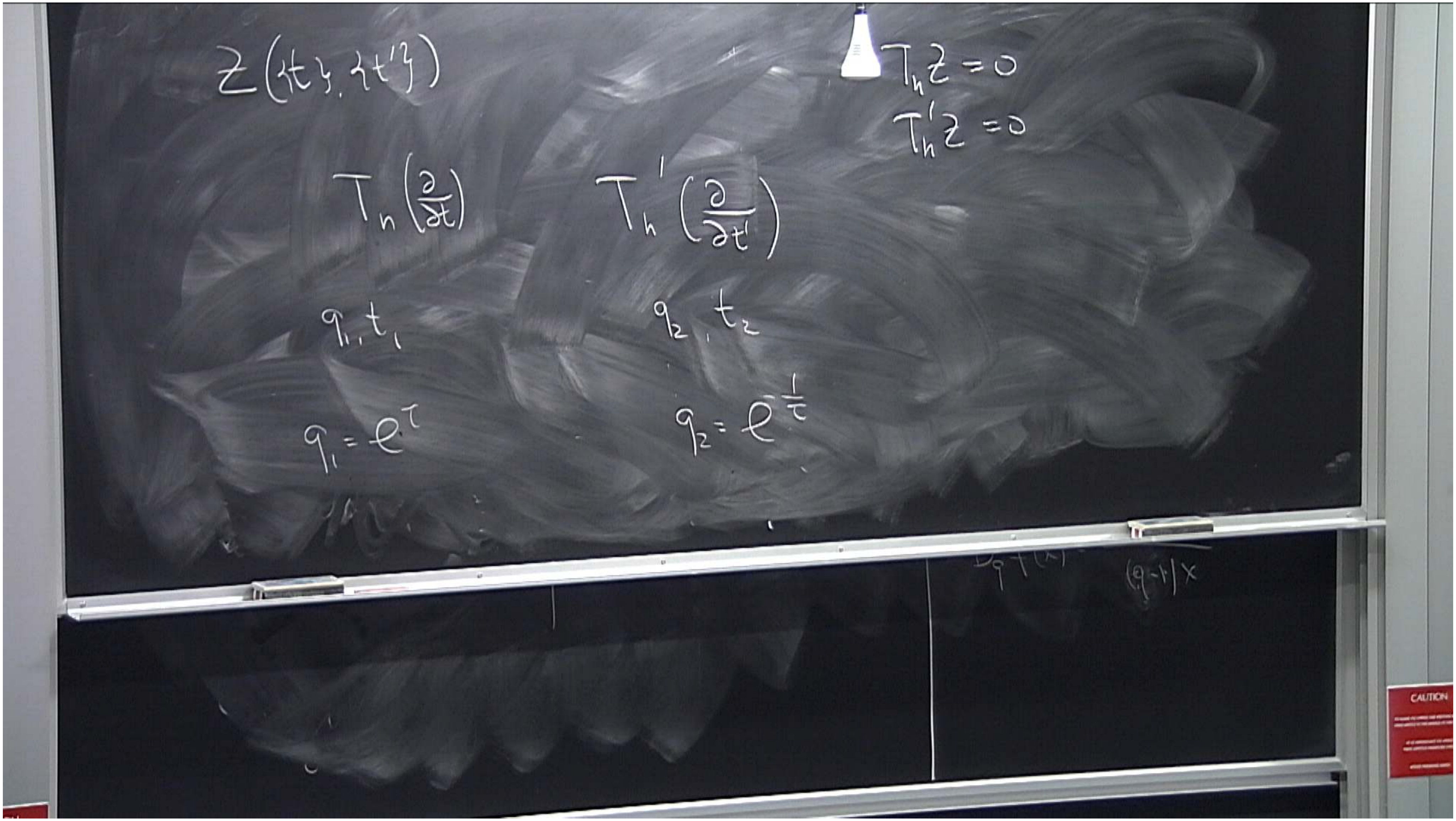
$\Rightarrow \int \frac{d}{dx}(\ldots) = 0$
$\int_{\mathbb{R}} D_q(\ldots) = 0$
$\frac{d}{dx} x^{n+1}$
$D_q(x^{n+1} \ldots)$
$D_q f(x) = \frac{f(qx) - f(x)}{(q-1)x}$

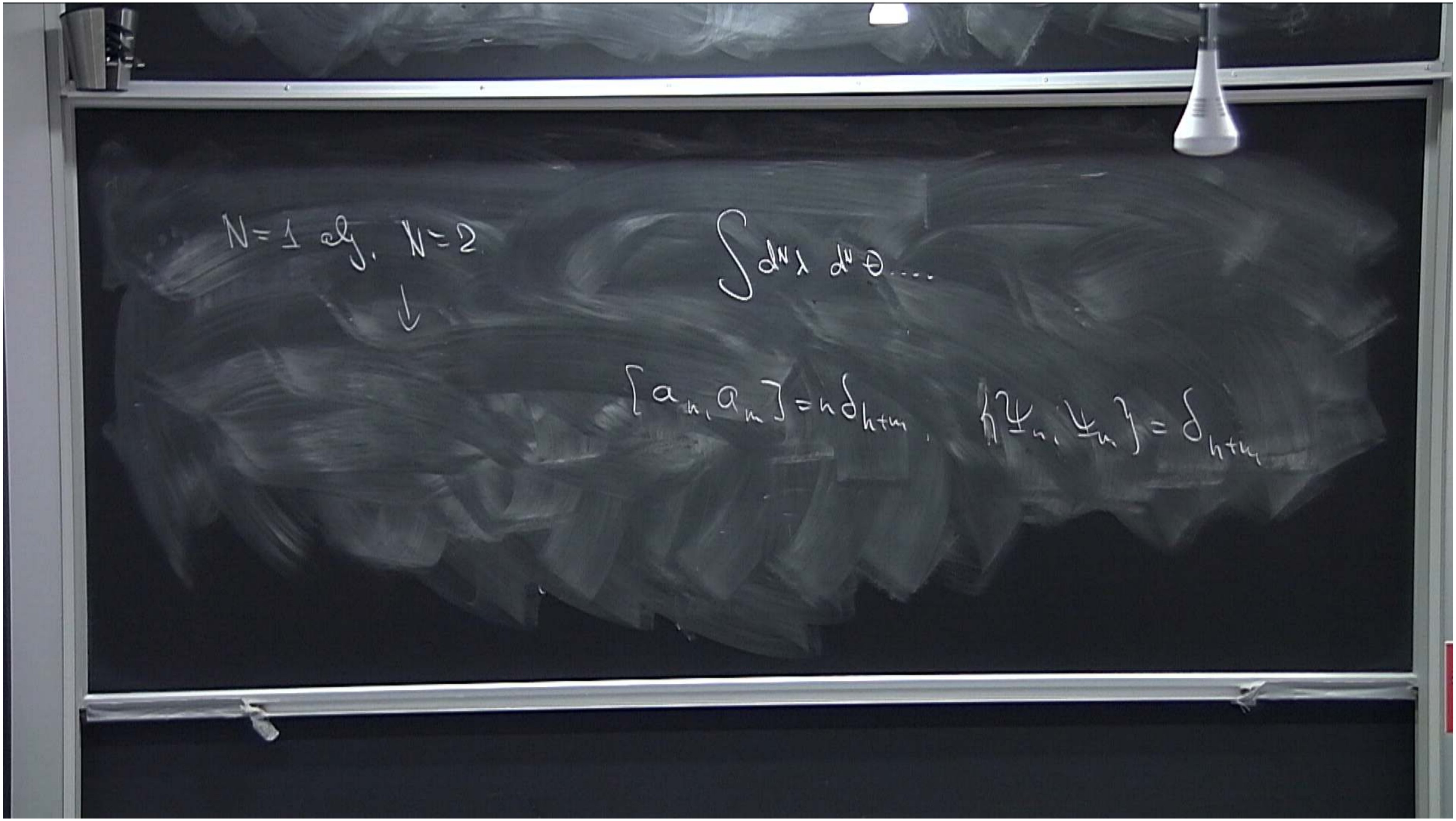


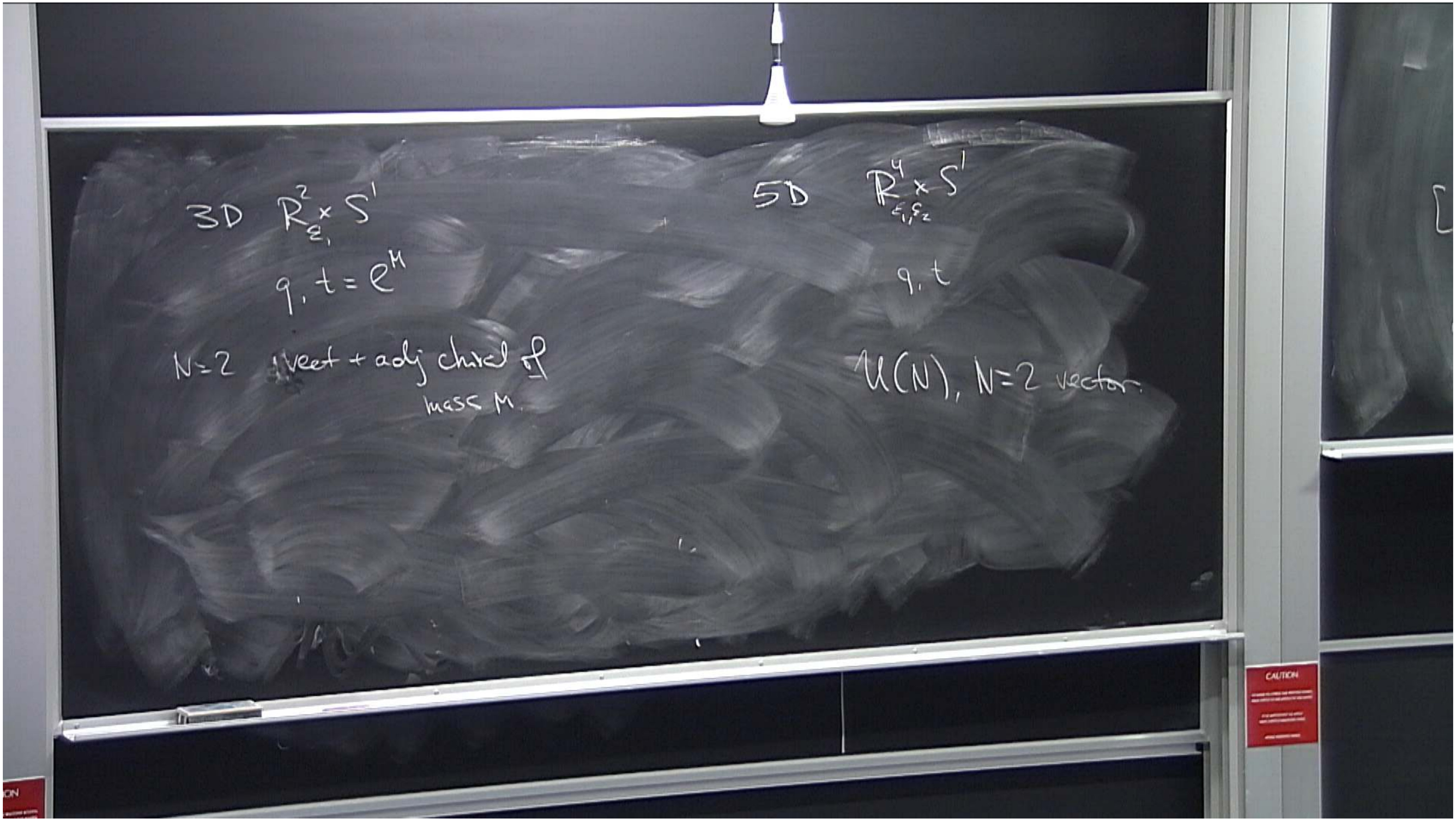
$T_n(\frac{\partial}{\partial t}) Z(\{t\}) = 0, \quad n \geq 1$
q-Virasoro constraints
$[T_n, T_m] = \sum_{l>0} f_l (T_{n-l} T_{m+l} - T_{m-l} T_{n+l}) + \# \delta_{n+m}$
$q = e^{\hbar}$
$T_n = 2\delta_{n+0} + \hbar^2 (L_n + \dots) + \dots$


$S(w)S(\lambda) = \frac{(w/\lambda;q)}{(t \quad)} \frac{(\lambda/w;q)}{(\quad)}$
$\Rightarrow \int \frac{d}{dx}(\ldots) = 0$
$\int_{\mathbb{R}} D_q(\ldots) = 0$
$\mathbb{R}^4 \times S^1$
$U(N)$
$\frac{d}{dx} x^{n+1}$
$D_q(x^{n+1} \ldots)$
$D_q f(x) = \frac{f(qx) - f(x)}{(q-1)x}$
CAUTION


3D gauge theory on S^3
$\omega_1|z_1|^2 + \omega_2|z_2|^2 = 1$
$Z = \int_{\mathbb{R}^N} d^N x \prod_{i\neq j} \frac{S_2(x_i - x_j|\omega)}{S_2(M + x_i - x_j|\omega)} e^{-\sum \frac{i\pi k_2}{\omega_1\omega_2} x_i^2 + \sum_i \frac{2\pi k_1}{\omega_1\omega_2} x_i + \sum t_n \sum_j e^{\frac{2\pi i x_n}{\omega_1}} + \sum t'_n \sum_j e^{2\pi i \frac{x_j n}{\omega_2}}}$
CAUTION


$Z(\{t\}, \{t'\})$
$T_n Z = 0$
$T'_n Z = 0$
$T_n\left(\frac{\partial}{\partial t}\right)$
$T'_n\left(\frac{\partial}{\partial t'}\right)$
q_1, t_1
q_2, t_2
$q_1 = e^{\tau}$
$q_2 = e^{-\frac{1}{\tau}}$


$N=1$ alg. $N=2$
$\int d^N\lambda \; d^N\theta \dots$
$[a_n, a_m] = n\delta_{n+m}$, $\{\psi_n, \psi_m\} = \delta_{n+m}$


3D $\mathbb{R}^2_{\varepsilon_1} \times S^1$
$q, t = e^M$
N=2 vect + adj chiral of mass M.
5D $\mathbb{R}^4_{\varepsilon_1,\varepsilon_2} \times S^1$
q, t
U(N), N=2 vector.