Title: Building bridges between Lorentzian and Euclidean formalisms for BV quantization

Date: May 12, 2017 09:00 AM

URL: http://pirsa.org/17050023

Abstract:

Motivation: quantize theories w/ symmetries (Y-M th, gravity).

$\mathcal{F}$ - fields in the theory. Describe the space of

<u>gauge inv. on-shell fields</u> $= H^0(\mathcal{F}, s)$

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s — BV differential

Strategy: 1) Start with classical $\left([\cdot,\cdot] \text{ on } \mathcal{F} \right)$

2) Deform it to a non-commutative prod. $\star$

3) Renormalization

4) Deform the differential to $\hat{s}$ (quantum observ.)

I. pAQFT

From physics:

1) spacetime: glob. hyp. Lorentzian mfd $\mathcal{M} = (M, g)$

↳ has a Cauchy surface Σ. $M = \Sigma \times \mathbb{R}$.

2) What kind of objects? ⇝ $\mathcal{E}(\mathcal{M}) = \Gamma(E \xrightarrow{\pi} M)$

↳ config. space

E → v. bundle.

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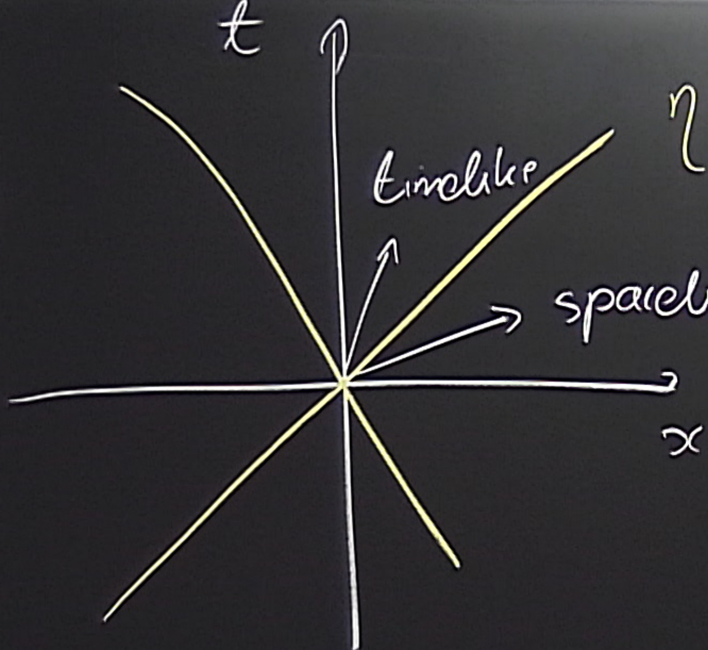
3) Observables: functions on $\mathcal{E}(\mathcal{M})$.

4) Dynamics: Lagrangian formalism

Causal structure: $\mathbb{M} = (\mathbb{R}^4, \eta)$, $\eta = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$



$\eta(v,v) = 0$ lightlike

In general:

$T_x M \cong \mathbb{M}$

Identify the following types of curves:

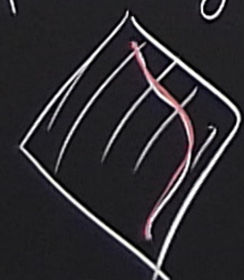
1) spacelike curve γ (i.e. $\dot{\gamma}$ is spacelike)

2) timelike

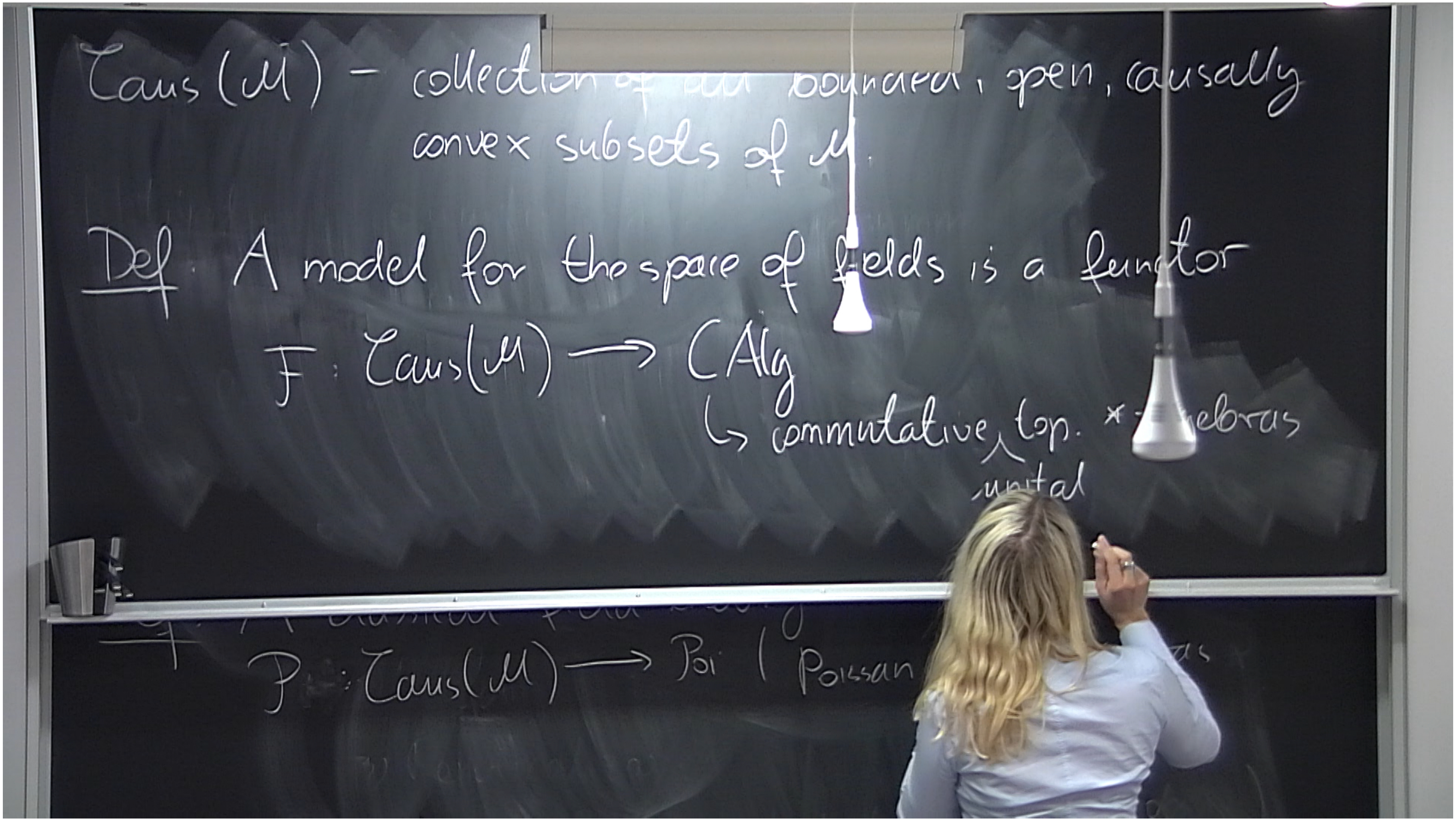
3) null

$\left.\begin{array}{l} \text{2) timelike} \\ \text{3) null} \end{array}\right\}$ causal

Def: A subset $\mathcal{O} \subset \mathcal{M}$ is called causally convex if it contains all causal curves starting and finishing in it.

✓ 

 ✗


Caus(M) - collection
open, causally
convex subsets of M.
Def: A model for the space of fields is a functor
F : Caus(M) ⟶ CAlg
↳ commutative top. *-
unital
P : Caus(M) ⟶ Poi (Poisson

$\mathfrak{F} : \mathfrak{Caus}(M) \longrightarrow CAlg$

↳ commutative, unital top. $*$-algebras

e.g. $\mathfrak{F}$ is regular functionals on $\mathcal{E}(M)$

Def. A classical field theory is a functor

$\mathfrak{P} : \mathfrak{Caus}(M) \longrightarrow \mathfrak{Poi}$ (Poisson, unital top. $*$-algebras)

s.t. $O_1, O_2 \in \mathfrak{Caus}(M)$ are spacelike

$\lfloor \mathfrak{P}(O_1), \mathfrak{P}(O_2) \rfloor = \{0\}$

e.g. $\mathcal{F}$ is regular functionals on $\mathcal{E}(M)$ (g.e. ...)

Def. A classical field theory is a functor

$\mathcal{P}: \mathrm{Caus}(\mathcal{M}) \longrightarrow \mathrm{Poi}$ (Poisson, top. unital $*$-algebras)

s.t. $\mathcal{O}_1, \mathcal{O}_2 \in \mathrm{Caus}(\mathcal{M})$ are spacelike

$$\left[\mathcal{P}(\mathcal{O}_1), \mathcal{P}(\mathcal{O}_2) \right] = \{0\}$$

(morphisms are injective)

$$\mathcal{P}(\mathcal{M}) = \lim_{\mathcal{O} \in \mathrm{Caus}(\mathcal{M})} \mathcal{P}(\mathcal{O})$$

Def: These are said to be on-shell if for $\mathcal{O}$ neigh of Σ (Cauchy surface), I have:
$\mathrm{Caus}(\mathcal{M})$

$$\mathfrak{A}(\mathcal{O}) \cong \mathfrak{A}(\mathcal{M}).$$

Interaction (perturbative)

Dyson series:

"time-ordered product"

$$U_I(t,s) = 1 + \sum_{n=1}^{\infty} \frac{i^n \lambda^n}{n!\,\hbar^n} \int_{[s,t]\times\mathbb{R}^3} T\left(:L_I(x_1):, \dots, :L_I(x_n): \right) d^4x_1 \cdots d^4x_n$$

"Wick polynomials"

Def: Time-ordered product is given as $(\mathfrak{A}_T, \xi, \mathcal{T})$

1) $\mathfrak{A}_T : \mathrm{Caus}(\mathcal{M}) \longrightarrow \mathrm{CAlg}$

2) ξ natural iso $\mathfrak{v}\circ\mathfrak{A}_T \overset{\cdot}{\longrightarrow} \mathfrak{v}\circ\mathfrak{A}$

3) $\mathcal{T} : \mathcal{F}[[\hbar]] \overset{\cdot}{\longrightarrow} \mathfrak{A}_T$, for $F \in \mathcal{F}(\mathcal{O})$
$\mathcal{T}F$ is a Wick ordered obs.

2) $\mathcal{O}_1 < \mathcal{O}_2$

$F \cdot_T G = G \star F$ introduces time-ordering

Formal S-matrix: $\mathcal{S}(V) = e_T^{\frac{i :V:}{\hbar}}$, $:V: \equiv TV$

$V \in \mathcal{F}(\mathcal{M})$

$dS \in \Gamma(T^*\mathcal{E}(M))$

EOM's: $dS(\varphi) = 0$

$dS(\varphi) = -(\Box + m^2)\varphi = P\varphi$

hyperbolic

$dS(\varphi) = \underbrace{(\Delta + m^2)}_{P}\varphi = 0$

elliptic

$\delta = -\iota_{dS}$

$PV_{reg} \subset \Gamma\left(\overset{\bullet}{\Lambda}\,\Gamma(T\mathcal{E}(M))\right)$

↳ regular polyvector fields

$\delta = \iota_{dS}$

PV polyvector fields

$T\mathrm{Obs}_s^{cl}(\mathcal{O}) = (PV_{reg}(\mathcal{O}), \delta)$

Propagators

G_R, G_A retarded & advanced Green functions for P

$G_R f$

f

$G_A f$

3)

JF is a Wick ordered obs.

G_R, G_A retarded & advan
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$G_R f$

f

$G_A f$

Dirac propagator:

$$G_D = \frac{1}{2}(G_R + G_A)$$

Def: Time-ordered product is given as $(\mathfrak{A}_T, \xi, T)$

1) $\mathfrak{A}_T : \mathrm{Caus}(\mathcal{M}) \longrightarrow \mathrm{CAlg}$

ξ natural iso $\nu \circ \mathfrak{A}_T \longrightarrow \nu \circ \mathfrak{A}$

2) $F \in \mathcal{F}(O)$

Nick ordering

$$T = e^{\frac{i\hbar}{2}\mathcal{D}_D}$$

$$\mathcal{D}_D(F) = \left\langle G_D, \frac{\delta^2 F}{\delta\varphi^2} \right\rangle$$

Wick ordering

"$W = e^{\hbar \partial_G}$" $\searrow$ bilinear operation.

$$Obs^q(\mathcal{O}) = (\mathcal{P}V_{reg}(\mathcal{O})[[\hbar]], \hat{s}, \triangle)$$

$\triangle \to$ BV Laplacian

$$\hat{s} = \delta - i\hbar \triangle$$

V

$J : \mathrm{Obs}^{q'}(\mathcal{O}) \to \mathrm{Obs}^{cl}(\mathcal{O})[[\hbar]]$

intertwines the differentials

& deforms the product.

W intertwines the differentials

& deforms the factorisation product