Title: Poisson Sigma Model with symplectic target

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Abstract:

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POISSON SIGMA MODEL
WITH
SYMPLECTIC TARGET

(F. BONECHI, INFN)

PSM has a well understood
perturbative theory;

Meaningful results on manifolds
with boundary

Kontsevich's formality

No nonperturbative resuets
there are two cases that have
non perturbative contributions

A-model
$\Updownarrow$
PSM with
symplectic target

BF-theory
(or 2d YM)
PSM with
$\mathfrak{g}^*$-target

AKSZ FORMULATION of PSM

source Σ $\left(\dim \Sigma = 2\right)$

target (M, π) Poisson manifold

$$\mathcal{F} = \mathrm{Maps}\left(T[1]\Sigma, T^*[1]M\right)$$

↳ symplectic of degree -1

superfields

$$\mathbb{X}^\mu = \underset{0}{X^\mu} + \underset{-1}{\eta^{+\mu}_{\alpha}}\,\theta^\alpha + \frac{1}{2}\,\underset{-2}{b^{+\mu}_{\alpha\beta}}\,\theta^\alpha\theta^\beta$$

$$\underset{0}{\mathbb{X}^\mu}$$

$$\eta_\mu = \underset{1}{b_\mu} + \underset{0}{\eta_{\mu\alpha}}\,\theta^\alpha + \frac{1}{2}\,\underset{-1}{X^+_{\mu,\alpha\beta}}\,\theta^\alpha\theta^\beta$$

action

$$S = \int_{T[1]\Sigma} d\mathbb{X}^\mu \, \eta_\mu + \frac{1}{2} \alpha^{\mu\nu}(\mathbb{X}) \eta_\mu \eta_\nu$$

$$\{S, S\} = 0$$ CLASSICAL MASTER EQUATION

AKSZ hierarchy of observables

$$P \in C^\infty(T^*[1]M) = \Gamma(\Lambda TM)$$

$$\mathcal{O}_P = P(\mathbb{X}, \eta)$$

$$\{S, \mathcal{O}_P\} = \mathcal{O}_{d_{LP}(P)} + D\mathcal{O}_P$$

$$d_{LP}(P) = [\pi, P]$$ Lichnerowicz Poisson differential

Expand $\mathcal{O}_P$ in form degree

$$\mathcal{O}_P = \mathcal{O}_P^{(0)} + \mathcal{O}^{(1)}_{P,\alpha}\,\theta^\alpha + \frac{1}{2}\,\mathcal{O}^{(2)}_{P,\alpha\beta}\,\theta^\alpha\theta^\beta$$

Evaluate components with k-cycles of Σ

If $d_{LP}(P) = 0$

$$\mathcal{O}_P^{(0)}(u_0), \quad \int_\gamma \mathcal{O}_P^{(1)}, \quad \int_\Sigma \mathcal{O}_P^{(2)}$$

are BV-observables

$$\{ S, \mathcal{O}_{P,\gamma_k} \} = 0$$

$$\gamma_k = u_0, \gamma, \Sigma$$

$$\Sigma = S^2$$

$$\mathbb{X}^\mu = X^\mu + s\, b^{+\mu}$$

$$\eta_\mu = b_\mu + s\, b^+_\mu$$

$$|s| = 2$$

$$T^*[-1]\, T^*[1]\, M$$

$$x,\ b$$

PARTITION FUNCTION on the TORUS (ZERO MODES)

$$\Sigma = \mathbb{T}^2 \qquad X_\Sigma = \mathbb{R}^2[1]$$

$$\mathbb{X}^\mu = x^\mu + e^1 \eta_1^{+\mu} + e^2 \eta_2^{+\mu} - s\, b^{+\mu}$$

$$\boldsymbol{\eta}_\nu = b_\mu + e^1 \eta_{\mu 1} + e^2 \eta_{\mu 2} + s\, x^+_\mu$$

$$s = e^1 e^2 \qquad \int s = 1$$

$$S^{(0)} = \int \pi^{\mu\nu}(\mathbb{X}) \boldsymbol{\eta}_\mu \boldsymbol{\eta}_\nu$$

there exist a canonical berezinian

$$v = dx\, dx^+ d\eta_1 d\eta_1^+ d\eta_2 d\eta_2^+ db\, db^+$$

$$\Delta e^{\frac{i}{\hbar} S^{(0)}} = 0$$

Let π be symplectic, J compatible complex structure

$$\eta_z^{+\mu} = \eta_2^{+\mu} + i \eta_1^{+\mu} \qquad \eta_{z\mu} = \eta_{2\mu} + i \eta_{1\mu}$$

Define the Lagrangian submanifold $\mathcal{L}$ of $\mathrm{Map}(\mathbb{R}^2[1], T^*[1]M)$

$$\eta^{+i}_{z} = \eta^{+\bar{i}}_{\bar{z}} = \eta_{zi} = \eta_{\bar{z}\bar{i}} = x^{+} = b^{+} = 0$$

$$\mathcal{L} \underset{\Gamma}{=} T^*[1]M \oplus T^*M \oplus T[-1]M$$

$$\int_{\mathcal{L}} \sqrt{\nu}\, e^{\frac{i}{\hbar} S^{(0)}} = \chi(M)$$ Euler characteristic

QUANTUM CORRECTIONS to ZEROMODES

EFFECTIVE ACTION

Suppose $M = V$ a vector space

$$\mathcal{F}_\Sigma = \Omega(\Sigma) \otimes \left(V \oplus V^*[1]\right)$$

$$P : \Omega(\Sigma) \longrightarrow H_{dR}(M)$$

$$K : \Omega^i(\Sigma) \longrightarrow \Omega^{i-1}(\Sigma)$$

$$dK + Kd = id - P$$

$$PK = KP = 0 \quad K^T = K \quad K^2 = 0$$

$$\mathcal{L}_K = \ker P \cap \ker K \subset \Omega(\Sigma)$$

$$\mathcal{L} = \mathcal{L}_K \otimes V \oplus V^*[1]$$

↳ gauge fixing

Define by the BV push-forward

the effective action

$$e^{\frac{i}{\hbar} S_{eff}} = \int_{\mathcal{L}} e^{\frac{i}{\hbar} S} \in C(\mathcal{F}_{\Sigma}^{(0)})$$

PROPOSITION [B, Mnev, Cattaneo 2012]

For any closed Σ,

if π is regular and unimodular

there are not quantum corrections

$S_{eff} = S^{(0)}$ the BV action of the zero modes

[globalization is included]

COMPARING A-model and PSM

$\phi : \Sigma \longrightarrow M$ bosonic map $|\phi| = 0$

$\chi \in \Gamma(\phi^* TM)$ fermionic $|\chi| = 1$

$\rho \in \Gamma(T^*\Sigma \otimes \phi^* TM)$ fermionic $|\rho| = -1$

$$\rho^{\alpha\mu} = \varepsilon^{\alpha}{}_{\beta} J^{\mu}{}_{\nu} \rho^{\beta\nu}$$

$H \in \Gamma(T^*\Sigma \otimes \phi^* TM)$ bosonic $|H| = 0$

$$H^{\alpha\mu} = \varepsilon^{\alpha}{}_{\beta} J^{\mu}{}_{\nu} H^{\beta\nu}$$

$$S = \int_{\Sigma} d^2\sigma \Big[-\frac{1}{4} H^{\alpha\mu} H_{\alpha\mu} + H^{\alpha}{}_{\mu} \partial_{\alpha} \phi^{\mu} - i \rho^{\alpha}{}_{\mu} D_{\alpha} \chi^{\mu} - \frac{1}{8} \rho^{\alpha}{}_{\mu} \rho_{\alpha\nu} \chi^{\rho} \chi^{\sigma} R_{\rho\sigma}{}^{\mu\nu} \Big]$$

A-MODEL ODD SYMMETRY

$$Q(\phi^\mu) = \chi^\mu \qquad Q(\chi^\mu) = 0$$

$$Q(\rho^\mu_\alpha) = H^\mu_\alpha - i\Gamma^\mu_{\nu\sigma}\chi^\nu\rho^\sigma_\alpha$$

$$Q(H^{\alpha\mu}) = -\frac{1}{4}\chi^\nu\chi^\sigma\left(R_{\nu\sigma}{}^\mu{}_\tau + R_{\nu\sigma\mu'\tau'}J^{\mu'\mu}J^{\tau'}_\tau\right)\rho^{\alpha\tau}$$
$$- i\Gamma^\mu_{\nu\sigma}\chi^\nu H^{\alpha\sigma}$$

$$Q^2 = 0$$

$$\boxed{Q(S) = 0}$$

A-MODEL OBSERVABLES

$$\omega \in \Omega^k_{dR}(M) \quad d\omega = 0$$

$$A^{(0)}_\omega = \omega_{\mu_1 \dots \mu_k} \chi^{\mu_1} \dots \chi^{\mu_k}$$

$$A^{(1)}_\omega = i\, k\, \omega_{\mu_1 \dots \mu_k} d\phi^{\mu_1} \chi^{\mu_2} \dots \chi^{\mu_k}$$

$$A^{(2)}_\omega = -\frac{k(k-1)}{2} \omega_{\mu_1 \dots \mu_k} d\phi^{\mu_1} d\phi^{\mu_2} \chi^{\mu_3} \dots \chi^{\mu_k}$$

$$Q A^{(i)}_\omega + i\, d A^{(i-1)}_\omega = 0$$

$$A^{(0)}_\omega(u), \quad \oint_\gamma A^{(1)}_\omega, \quad \int_\Sigma A^{(2)}_\omega$$

are Q-closed

FROM PSM TO A-MODEL [B. Cattaneo Iraso]

COMPLEX GAUGE FIXING

introduce a complex structure J on M

ε on Σ

Define $\mathcal{L}_{\varepsilon,J} \subset \mathcal{F}_\Sigma$

$$\left\{ b^+ = X^+ = \eta_{zi} = \eta_{\bar{z}\bar{i}} = \eta^{+i}_{z} = \eta^{+\bar{i}}_{\bar{z}} = 0 \right\}$$

introduce the Levi-Civita connection for the Kahler metric

$$p_{\bar{z}i} = \eta_{\bar{z}i} - \Gamma^{e}_{ij}\, \eta^{+j}_{\bar{z}}\, b_e$$

the gauge fixed action reads

$$S = \int dz d\bar{z} \Big(i p_{z\bar{j}} \partial_{\bar{z}} X^{\bar{j}} - i p_{\bar{z} j} \partial_z X^j + i \eta^{+i}_{\bar{z}} D_z b_i$$
$$- i \eta^{+\bar{j}}_z D_{\bar{z}} b_{\bar{j}} + g^{k\bar{r}} R^{e}_{k\bar{j}i} \eta^{+i}_{\bar{z}} \eta^{+\bar{j}}_z b_e b_{\bar{r}} +$$
$$g^{i\bar{j}} p_{\bar{z}i} p_{z\bar{j}} \Big)$$

the dependence on p is quadratic, we can integrate it out and recover the action of the A-model (after integrating the auxiliary field)

OBSERVABLES

PSM hierarchy depends on a class of $H_{LP}(M,\pi)$. When π is nondegenerate

$$H_{LP}(M,\pi) = H_{dR}(M)$$

so $\mathcal{O}_P = \mathcal{O}_{\#(\omega)}$ $\qquad \underset{\pi}{\#}(\omega) = P$

but $\mathcal{O}^{(k)}_{\#(\omega)} \neq A^{(k)}_{\omega} \equiv$ A-model observable !!

CHANGING THE PSM OBSERVABLES

this construction is valid for any π

$$\#_\pi : \Omega M \longrightarrow \Gamma(\Lambda TM)$$

$$\#_\pi \circ d = d_{LP} \circ \#_\pi$$

Define $A^{(k)}_\omega(x, b) \quad \forall \omega \in \Omega^k M$

$$A^{(0)}_\omega = (-)^k \omega_{\mu_1 \ldots \mu_k} b^{\mu_1} \ldots b^{\mu_k} \qquad b^\mu \equiv \pi^{\mu\nu} b_\nu$$

$$A^{(1)}_\omega = i(-i)^{k-1} k\, \omega_{\mu_1 \ldots \mu_k} dX^{\mu_1} b^{\mu_2} \ldots b^{\mu_k}$$

$$A^{(2)}_\omega = \frac{k(k-1)}{2} \omega_{\mu_1 \ldots \mu_k} dX^{\mu_1} dX^{\mu_2} b^{\mu_3} \ldots b^{\mu_k}$$

$$\mathcal{O}^{(0)}_{\#\omega} = \frac{i^k}{k!} A^{(0)}_\omega$$

$$\mathcal{O}^{(1)}_{\#\omega} = \frac{i^k}{k!} A^{(1)}_\omega + Q_{BV}\left(C^{(1)}_{\#\omega}\right)$$

$$\mathcal{O}^{(2)}_{\#\omega} = \frac{i^k}{k!} A^{(2)}_\omega + Q_{BV}\left(C^{(2)}_{\#\omega}\right) - d\, C^{(1)}_{\#\omega}$$

$$\Sigma = S^2$$

$$\mathbb{X}^\mu = X^\mu + s\, b^{+\mu} \qquad |s| = 2$$

$$\eta_\mu = b_\mu + s\, b^+_\mu$$

$$T^*[-1]\, T^*[1] M$$

$$x, b$$

$$\mathbb{X}^\mu$$

$$\eta_\mu = \underset{1}{b_\mu} + \eta_{\mu\alpha}\theta^\alpha + \dots$$

$$\eta_\mu = b_\mu + \theta\, b_\mu$$

$$\mathbb{X}^\mu \qquad\qquad T^*[-1]\, T^*[1]\, M$$

$$\eta_\mu = \underset{1}{b_\mu} + \eta_{\mu\alpha}\theta^\alpha + \dots \qquad\qquad x, b$$

$$\#\omega\left(\mathbb{X}, \eta\right)$$

$$\#\omega \to p$$

when π is Kahler

$$A_{\omega}^{(k)}\Big|_{\mathcal{L}_{J,J}} = A_{\omega}^{(k)}$$

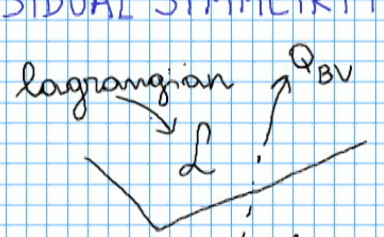
the A-model observables coincide with PSM hierarchy

(we can think to this fact as the BV proof that GW invariants do not depend on the choice of the complex structures)

BV ORIGIN of A-model supersymmetry

In Witten's argument of localization the essential ingredient is given by the odd symmetry. Has this symmetry a BV origin?

RESIDUAL SYMMETRY IN BV-FORMALISM



$i: \mathcal{L} \longrightarrow \mathcal{F}$

choose a symplectic tubular neighborhood

$$\begin{array}{c} \mathcal{F} \simeq T^*[-1]\mathcal{L} \\ \big\downarrow \pi \\ \mathcal{L} \end{array}$$

$$Q^{\pi}_{\mathcal{L}}(f) = i^*\left(Q_{BV}(\pi^* f)\right) \qquad f \in C(\mathcal{L})$$

$\cap\, C(\mathcal{L})$

we can think of it as an atlas of canonical coordinates $\{x^A, x^+_A\}$ adapted to $\mathcal{L}$ s.t. transition functions are

$$y = y(x), \quad y^+ = \frac{\partial x}{\partial y} x^+$$

so that

$$Q^\pi_{\mathcal{L}} = - \frac{\partial S}{\partial x^+_A}(x,0) \frac{\partial}{\partial x^A}$$

1) As a consequence of the CME

$$Q^\pi_{\mathcal{L}}(S|_{\mathcal{L}}) = 0$$

2) $Q^\pi_{\mathcal{L}}$ squares to zero only on-shell

$$(Q^\pi_{\mathcal{L}})^2 = \sigma^{AB} \partial_A S|_{\mathcal{L}} \partial_B$$

where $S = S_{\mathcal{L}} - Q^{\pi A}_{\mathcal{L}} x^+_A + \frac{1}{2} x^+_A \sigma^{AB} x^+_B + \dots$

BACK TO COMPLEX GAUGE FIXING of PSM

Depending on the existence of

$$\Lambda : TM \longrightarrow T\bar{M}$$

1) invertible

2) orthogonal

3) $\Lambda J + J \Lambda = 0$

we can choose a tubular neighborhood such that $\left(Q_{\mathcal{L}}^{\pi}\right)^2 = 0$

Moreover it is possible to redefine fields so that the gauge fixed action coincides with A-model action with auxiliary field H

Is this Λ necessary? If yes what is the meaning of this obstruction?