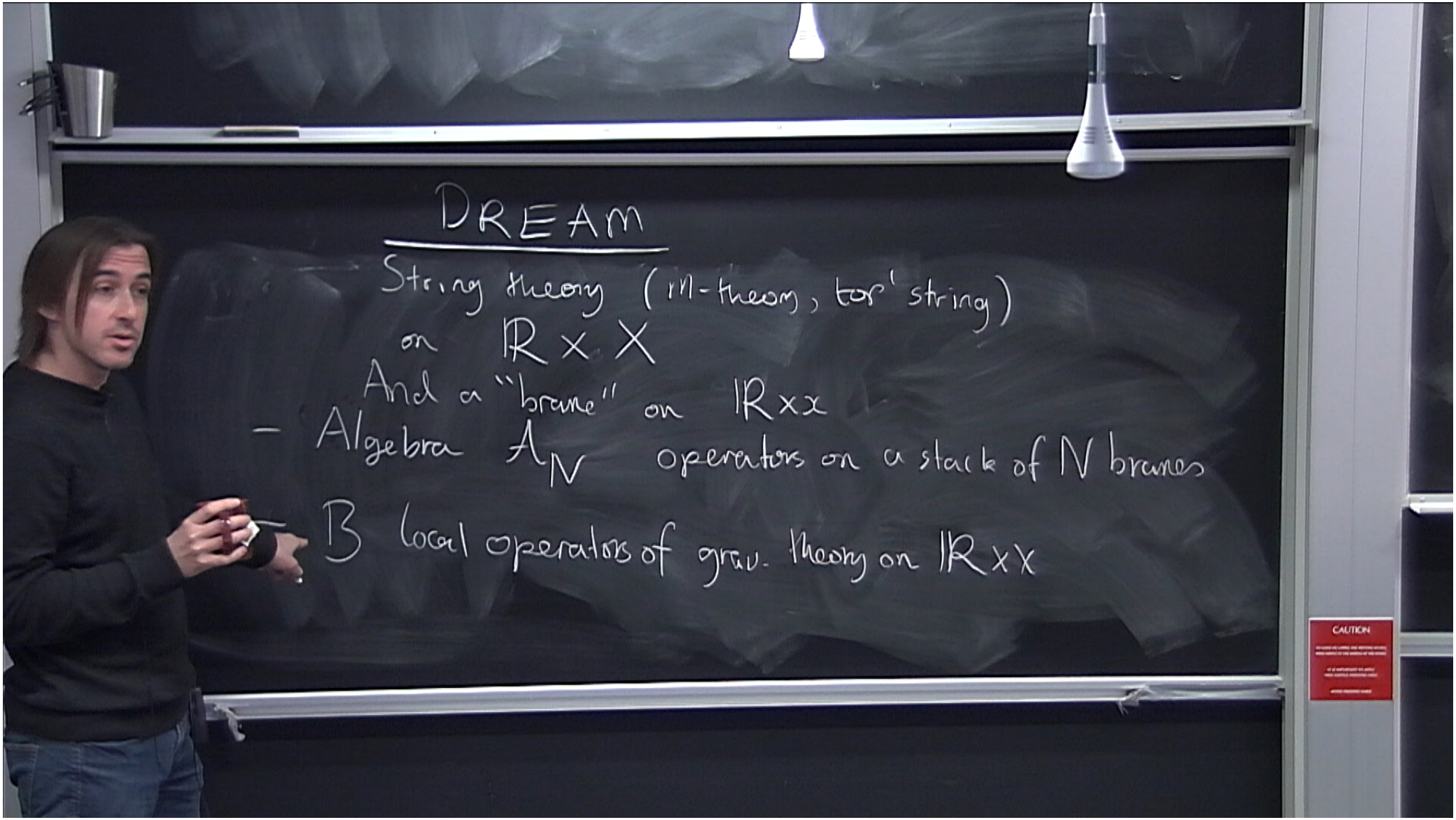
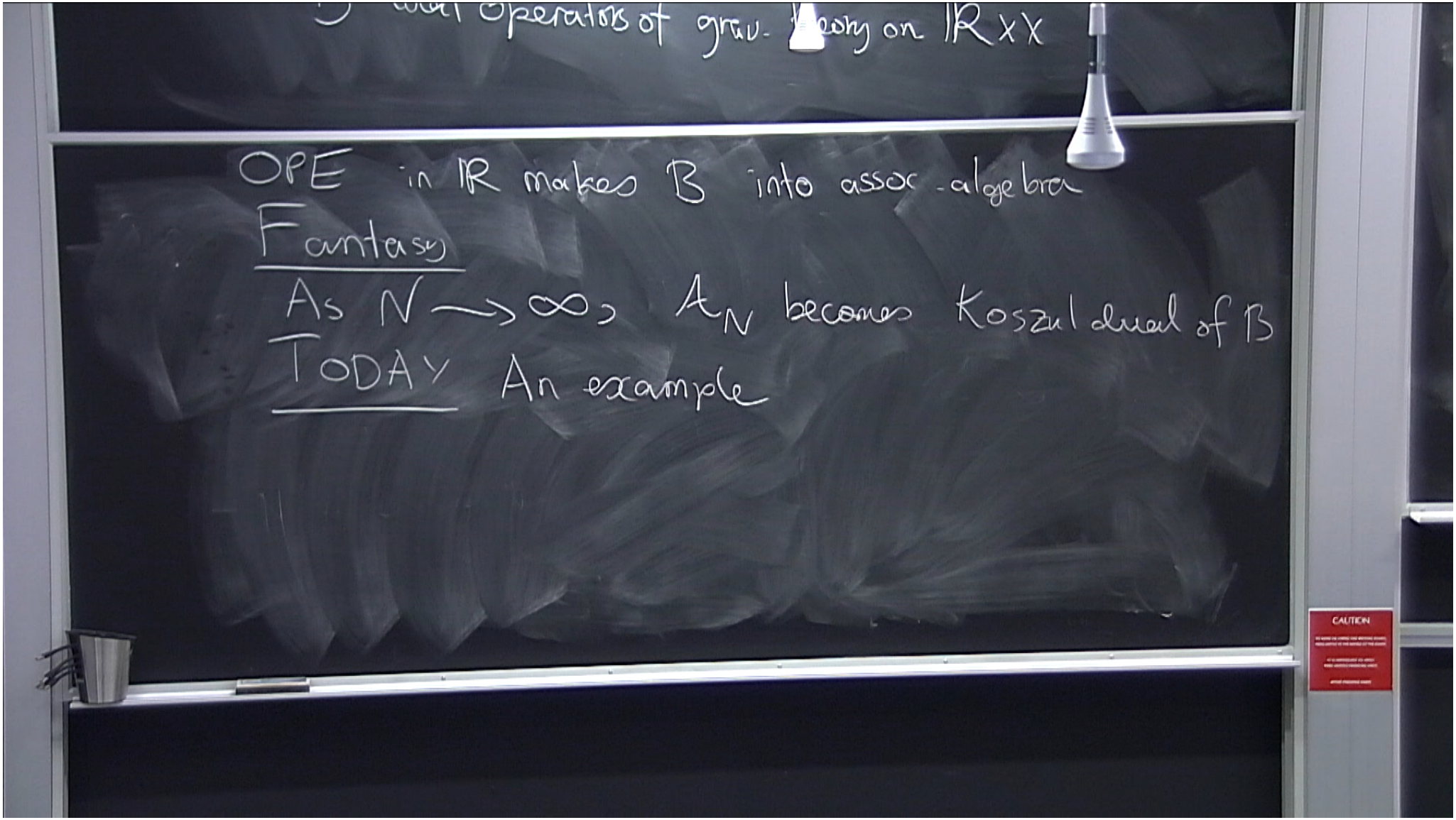
Title: A link between AdS/CFT and Koszul duality

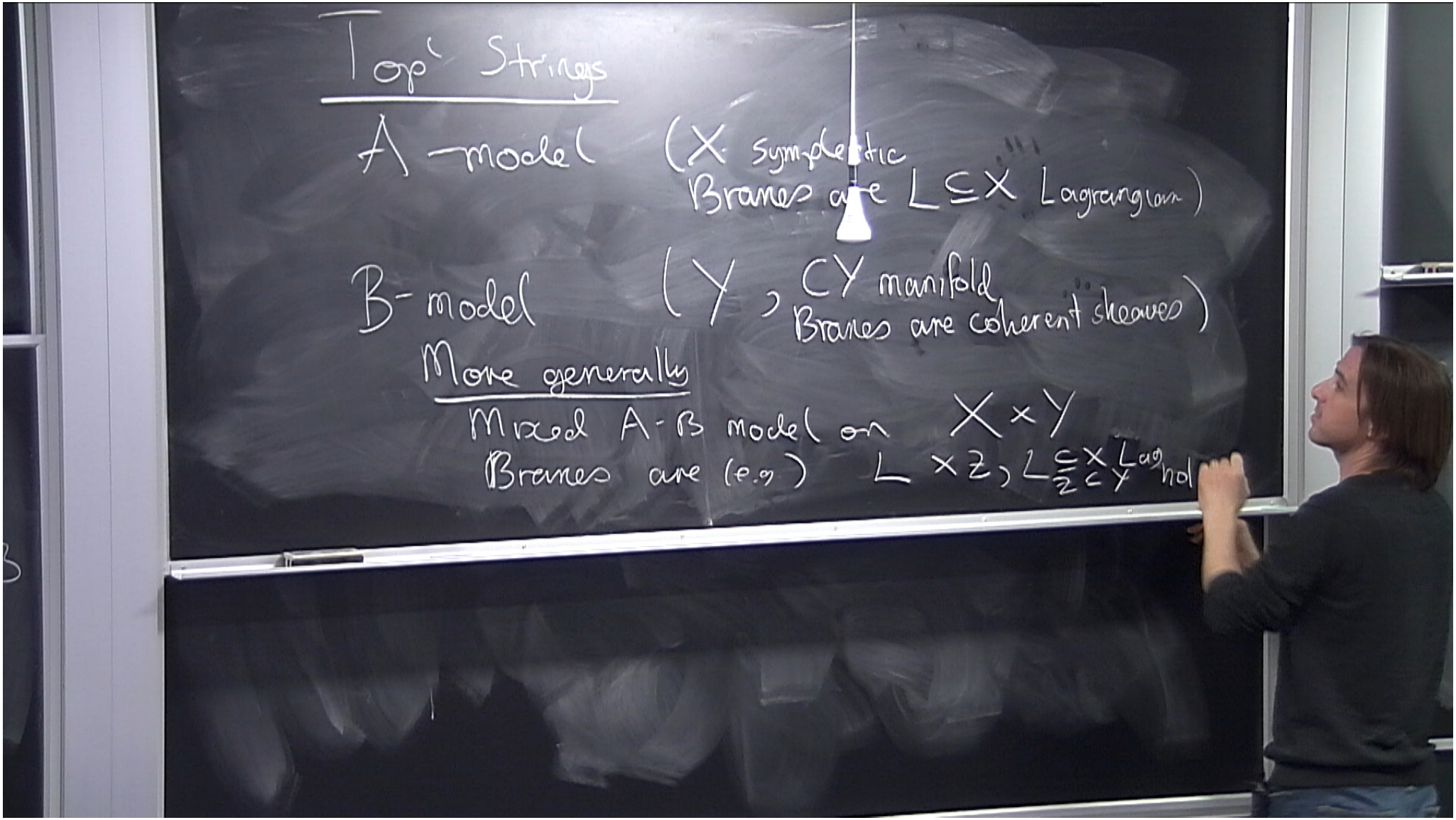
Date: May 10, 2017 11:00 AM

URL: http://pirsa.org/17050021

Abstract:


DREAM
String theory (M-theory, tor' string)
on $\mathbb{R} \times X$
And a "brane" on $\mathbb{R}^{1,x}$
- Algebra $\mathcal{A}_N$ operators on a stack of N branes
- B local operators of grav. theory on $\mathbb{R}^{1,x}$
CAUTION


operators of grav. theory on $\mathbb{R}\times X$
OPE in $\mathbb{R}$ makes B into assoc. algebra
Fantasy
As $N \to \infty$, A_N becomes Koszul dual of B
TODAY An example
CAUTION


Top' Strings
A-model (X: symplectic
Branes are L⊆X Lagrangian)
B-model (Y, CY manifold
Branes are coherent sheaves)
More generally
Mixed A-B model on X×Y
Branes are (e.g.) L×Z, L⊆X Lag

Mixed A-B model on $X \times Y$

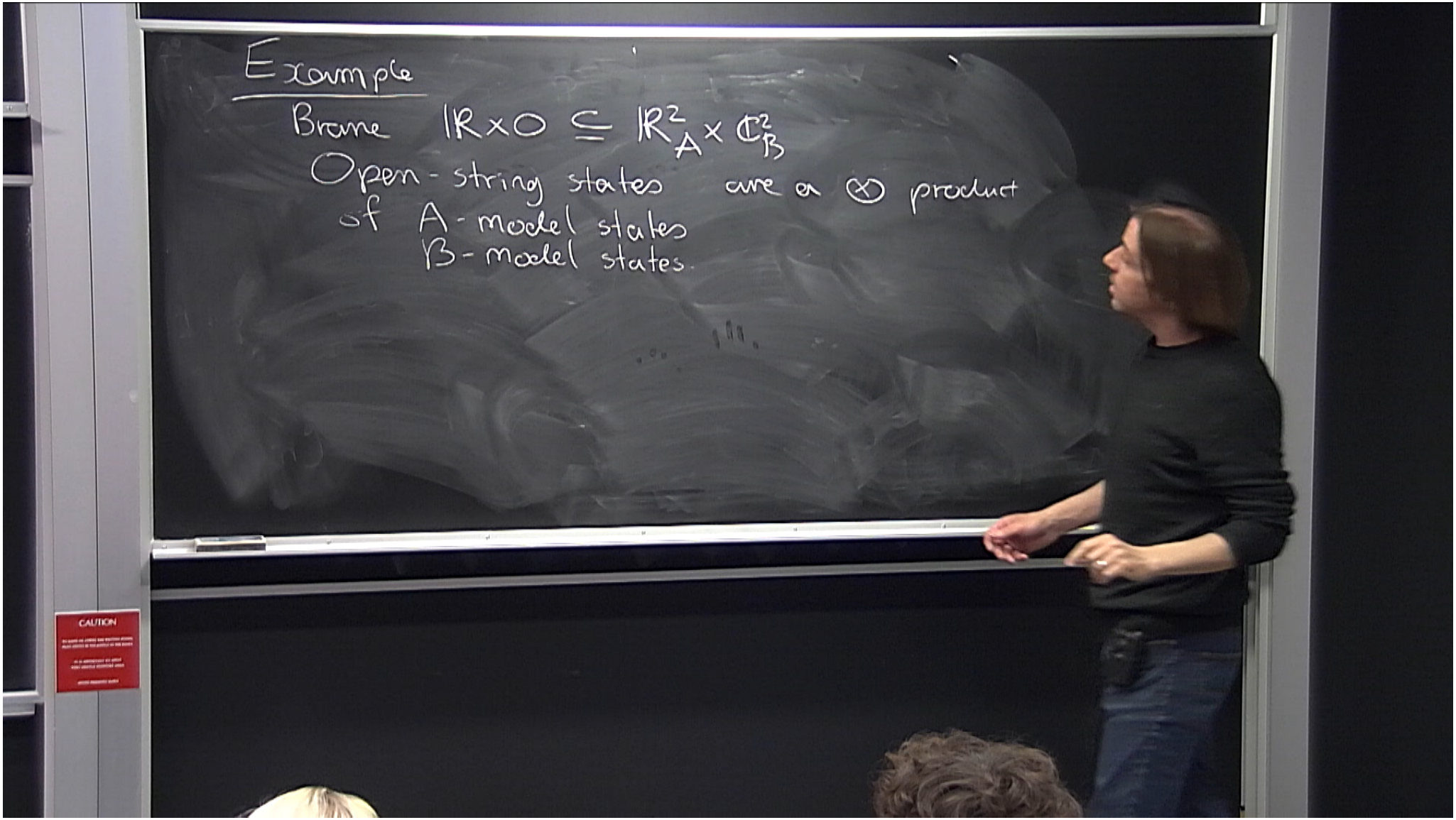
Branes are (e.g.) $L \times Z$, $L \subseteq X$ Lag$_{hol}$

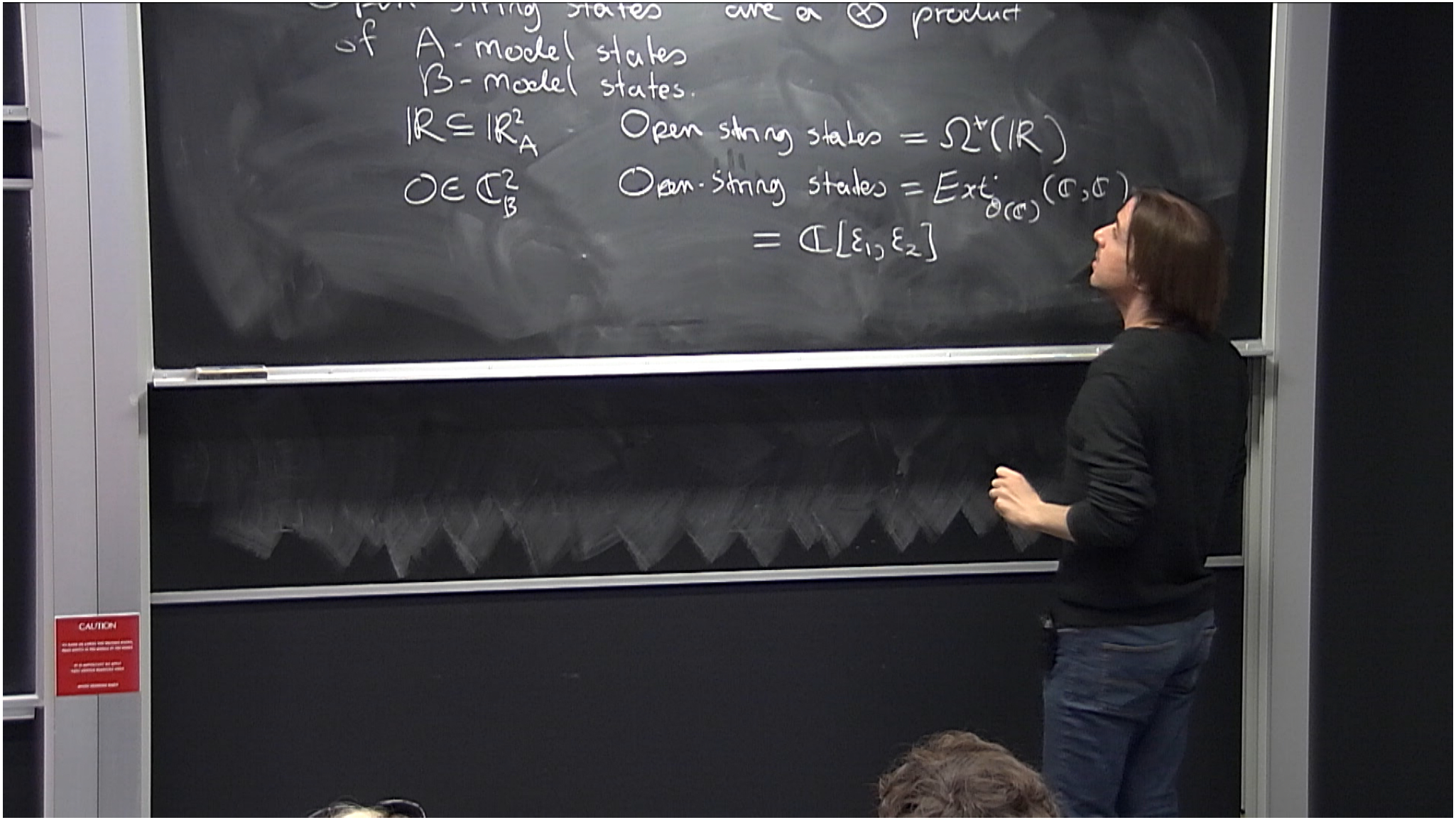
We'll consider $\mathbb{R}^2_A \times \mathbb{C}^2_B$

Open-Strings

If $\mathcal{C}$ is a CY category, $\mathcal{F} \in Ob\,\mathcal{C}$, then we get a field theory whose fields are $\mathbb{R}Hom(\mathcal{F}, \mathcal{F})[1]$ (open-string states)

Action comes from A_∞ str. here


Example
Brane $\mathbb{R} \times 0 \subseteq \mathbb{R}^2_A \times \mathbb{C}^2_B$
Open-string states are a $\otimes$ product
of A-model states
B-model states.
CAUTION

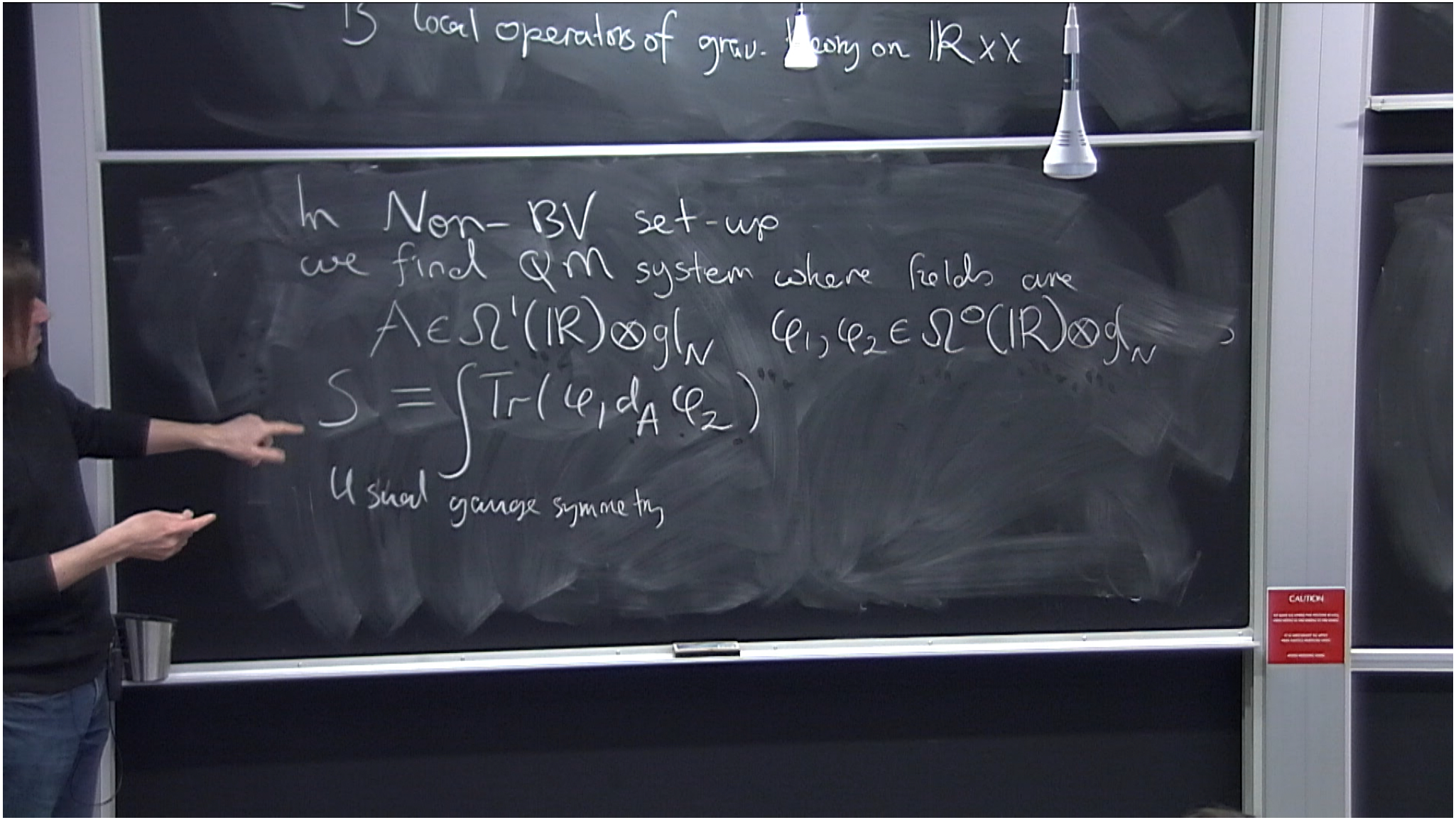

of A-model states
B-model states.
$\mathbb{R} \subset \mathbb{R}^2_A$
Open string states $= \Omega^*(\mathbb{R})$
$O \in \mathbb{C}^2_B$
Open-string states $= Ext^*_{\mathcal{O}(\mathbb{C})}(\mathbb{C},\mathbb{C})$
$= \mathbb{C}[\varepsilon_1, \varepsilon_2]$
CAUTION

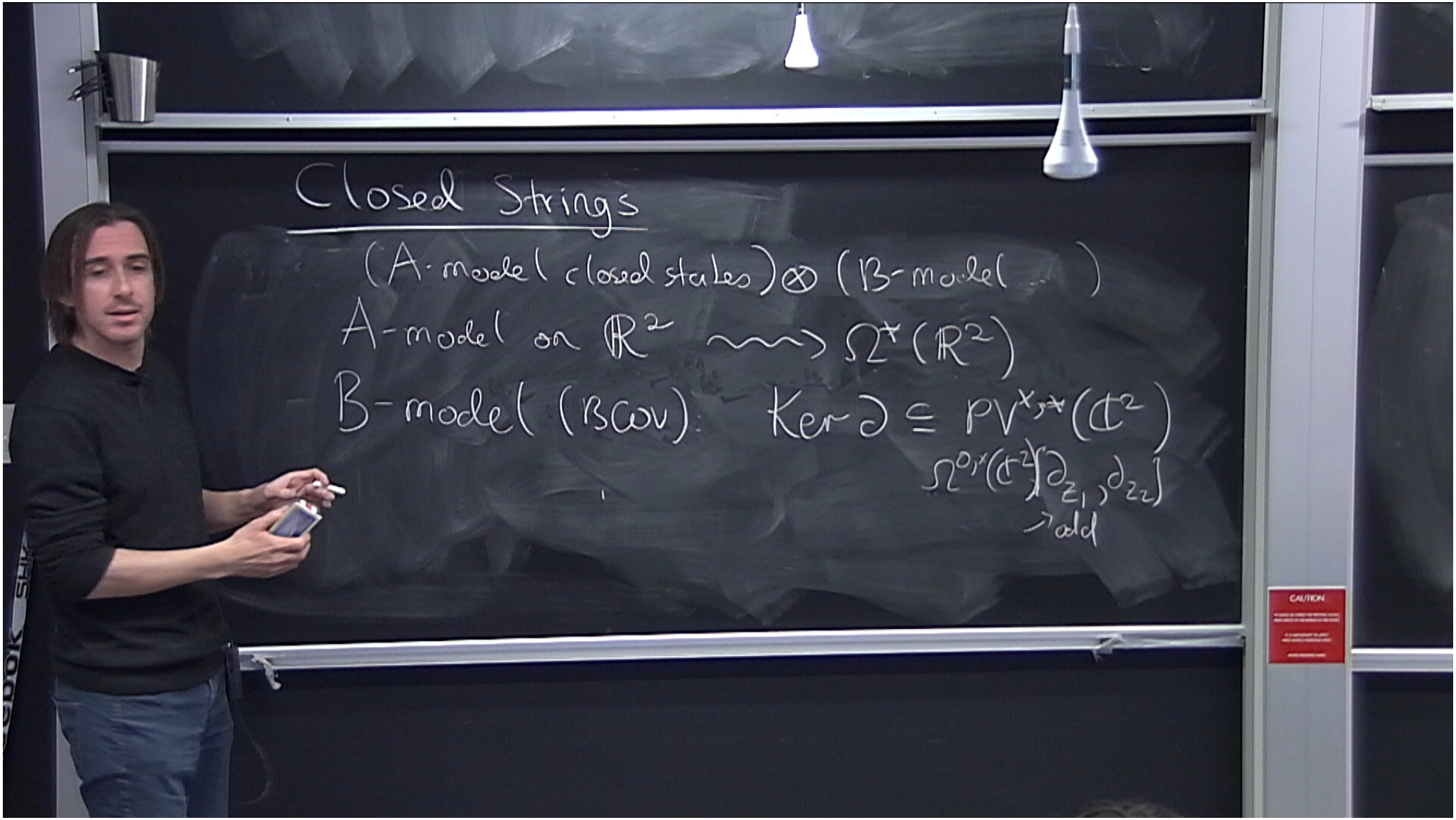
In sum

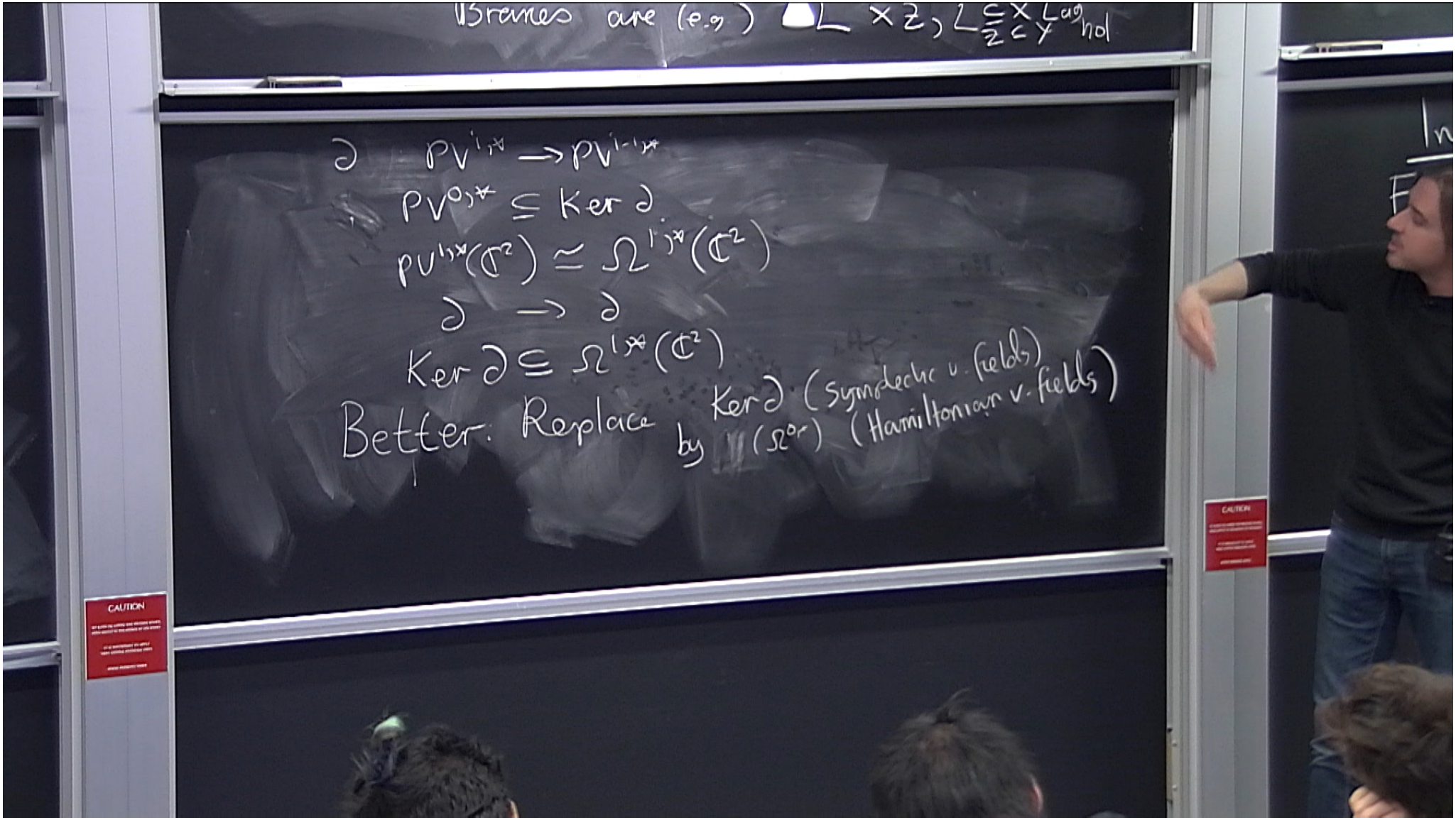
Fields (in BV) of open-string theory are

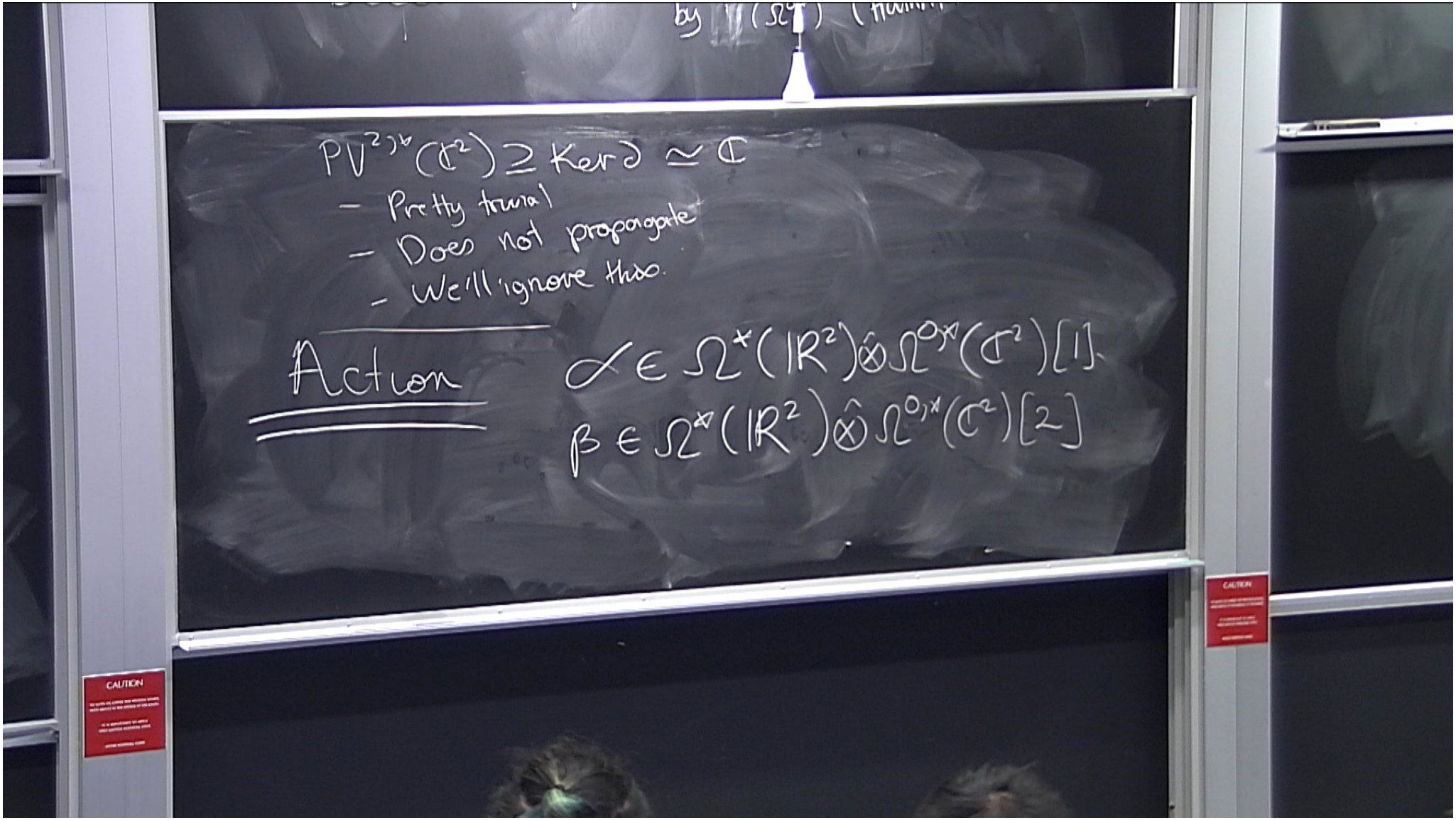
$\alpha \in \Omega^*(\mathbb{R})[\epsilon_1, \epsilon_2] \otimes \mathfrak{gl}_N[1]$

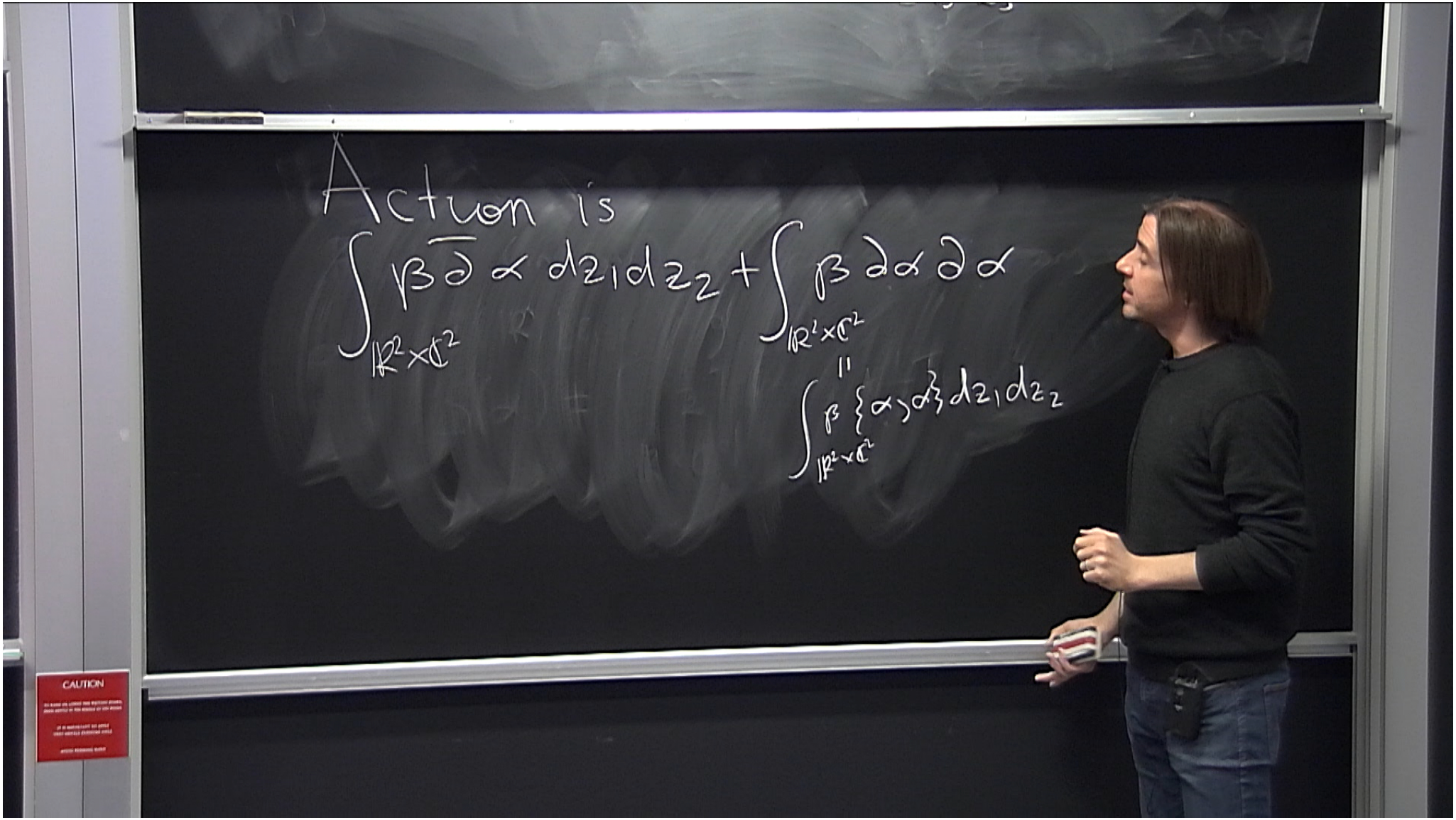
$$S(\alpha) = \int_{\mathbb{R} \times \mathbb{R}^{0|2}} \frac{1}{2} \mathrm{Tr}(\alpha d \alpha) + \frac{1}{3} \mathrm{Tr} \alpha^3$$

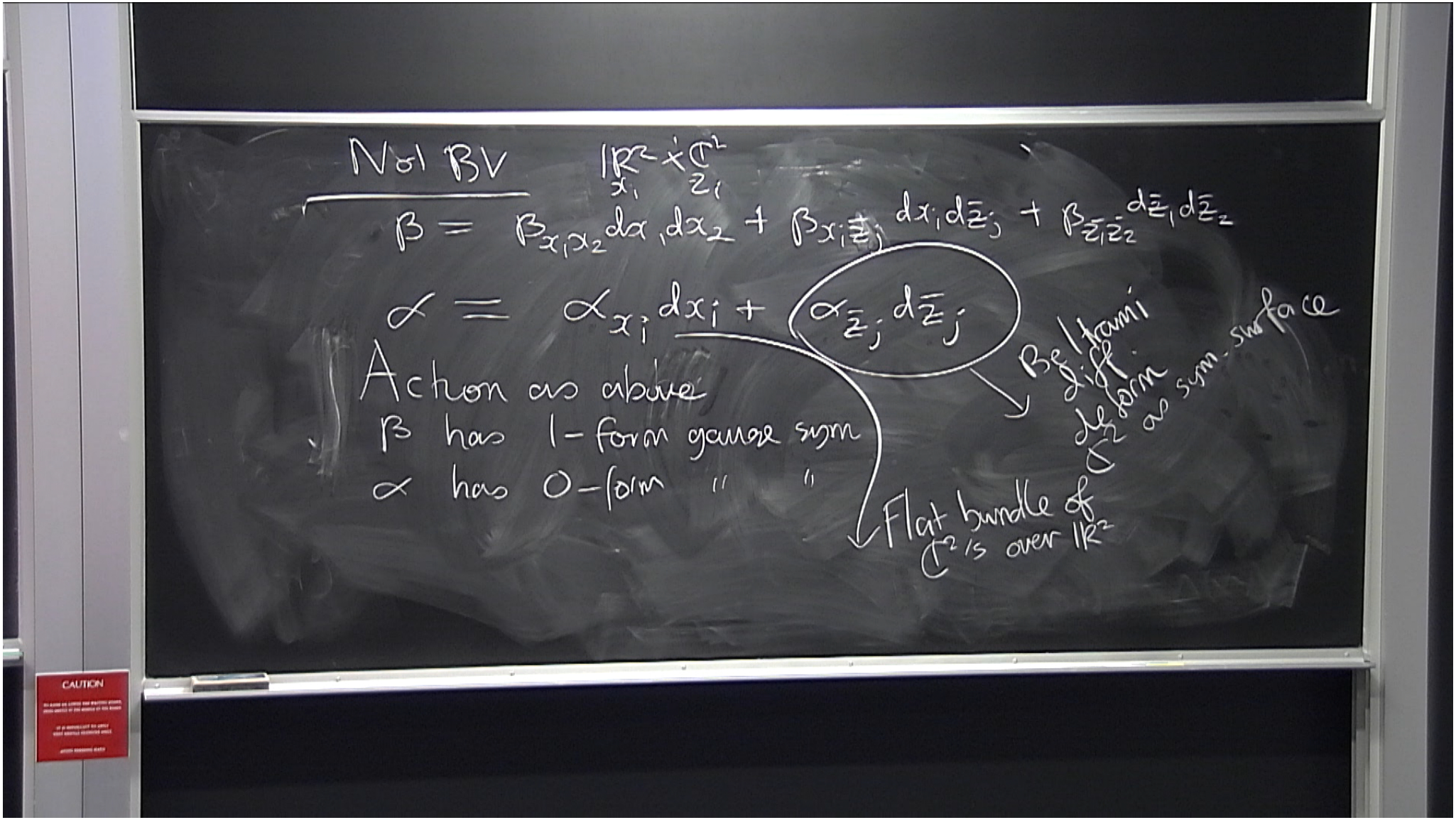

IS local operators of grav. theory on $\mathbb{R}\times X$
In Non-BV set-up
we find QM system where fields are
$A \in \Omega^1(\mathbb{R}) \otimes \mathfrak{gl}_N$ $\varphi_1, \varphi_2 \in \Omega^0(\mathbb{R}) \otimes \mathfrak{gl}_N$
$S = \int \mathrm{Tr}(\varphi_1 d_A \varphi_2)$
U shaul gauge symmetry
CAUTION


Closed Strings
(A-model closed states) ⊗ (B-model ...)
A-model on $\mathbb{R}^2$ ⇝ $\Omega^*(\mathbb{R}^2)$
B-model ((BCOV): Ker ∂ ⊆ $PV^{*,*}(\mathbb{C}^2)$
$\Omega^{0,*}(\mathbb{C}^2)[\partial_{z_1}, \partial_{z_2}]$
→ odd
CAUTION


Branes are (e.g.)
$\partial \quad PV^{i,*} \to PV^{i-1,*}$
$PV^{0,*} \subseteq \ker \partial$
$PV^{i,*}(\mathbb{C}^2) \simeq \Omega^{i,*}(\mathbb{C}^2)$
$\partial \to \partial$
$\ker \partial \subseteq \Omega^{i,*}(\mathbb{C}^2)$
Better: Replace $\ker \partial$ (symplectic v. fields) by $(\Omega^{0,*})$ (Hamiltonian v. fields)
CAUTION


$PV^{2,b}(\mathbb{C}^2) \supseteq \ker \partial \simeq \mathbb{C}$
- Pretty trivial
- Does not propagate
- We'll ignore this.
Action
$\alpha \in \Omega^*(\mathbb{R}^2) \hat{\otimes} \Omega^{0,*}(\mathbb{C}^2)[1]$
$\beta \in \Omega^*(\mathbb{R}^2) \hat{\otimes} \Omega^{0,*}(\mathbb{C}^2)[2]$

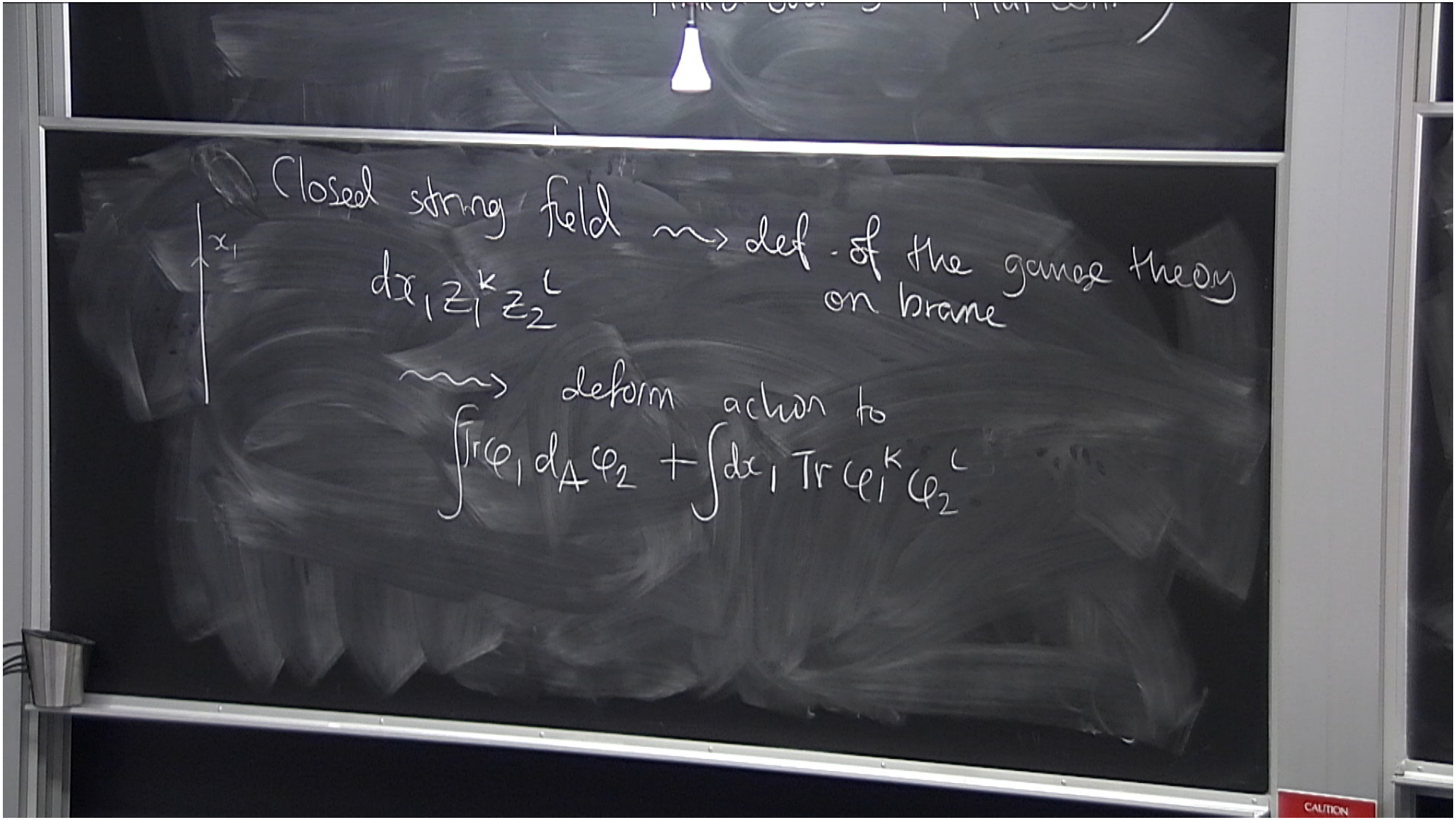

Action is
$\int_{\mathbb{R}^2\times\mathbb{C}^2} \beta\bar{\partial}\alpha\, dz_1 dz_2 + \int_{\mathbb{R}^2\times\mathbb{C}^2} \beta\partial\alpha\,\partial\alpha$
$= \int_{\mathbb{R}^2\times\mathbb{C}^2} \beta\{\alpha,\alpha\}\, dz_1 dz_2$
CAUTION

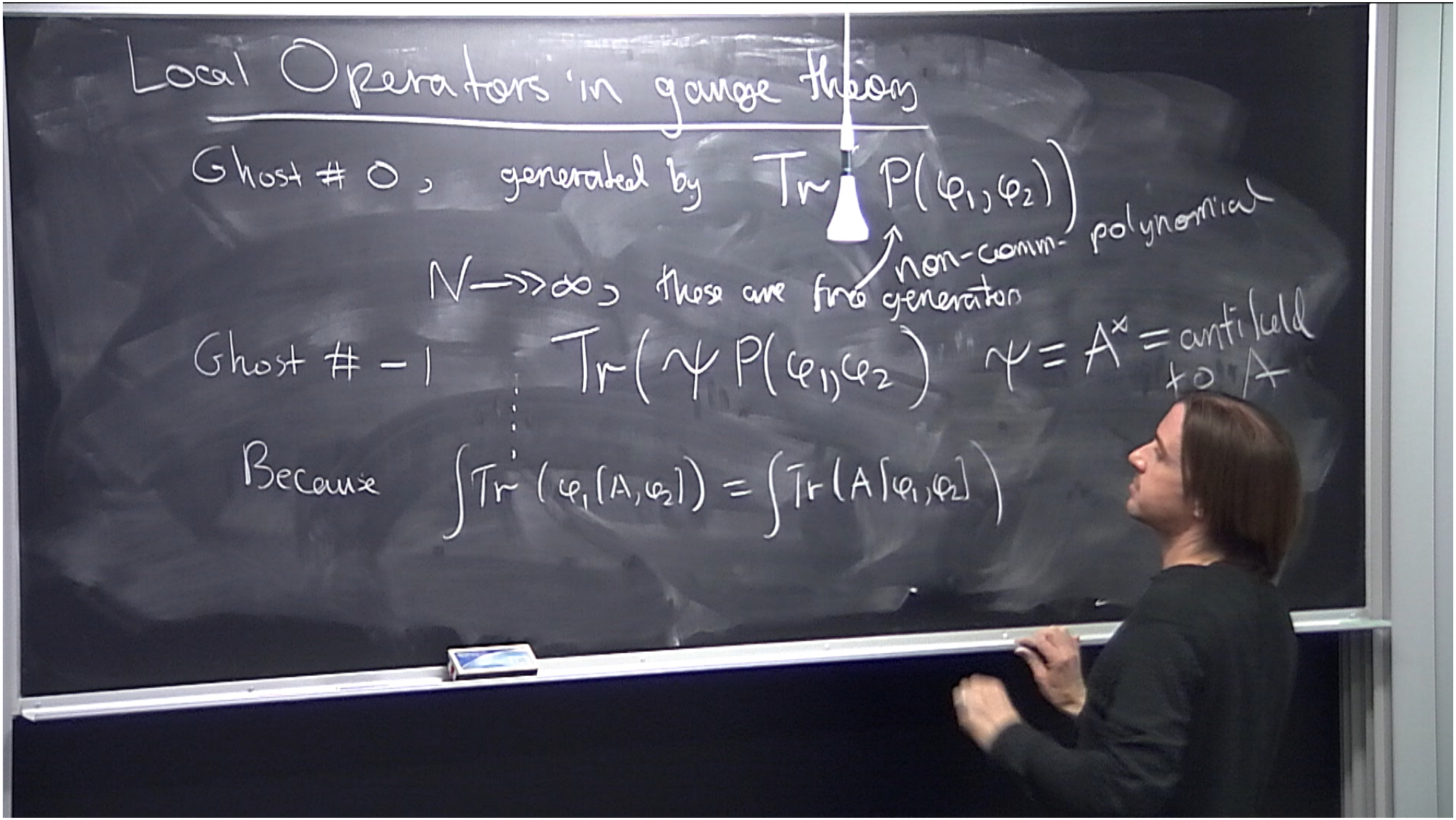

Nol BV
$\mathbb{R}^2_{x_i} \times \mathbb{C}^2_{z_i}$
$\beta = \beta_{x_1x_2} dx_1 dx_2 + \beta_{x_i\bar{z}_j} dx_i d\bar{z}_j + \beta_{\bar{z}_1\bar{z}_2} d\bar{z}_1 d\bar{z}_2$
$\alpha = \alpha_{x_i} dx_i + \alpha_{\bar{z}_j} d\bar{z}_j$
Action as above
β has 1-form gauge sym
α has 0-form " "
Beltrami diff. form as sym. surface
Flat bundle of $\mathbb{C}^2$ is over $\mathbb{R}^2$

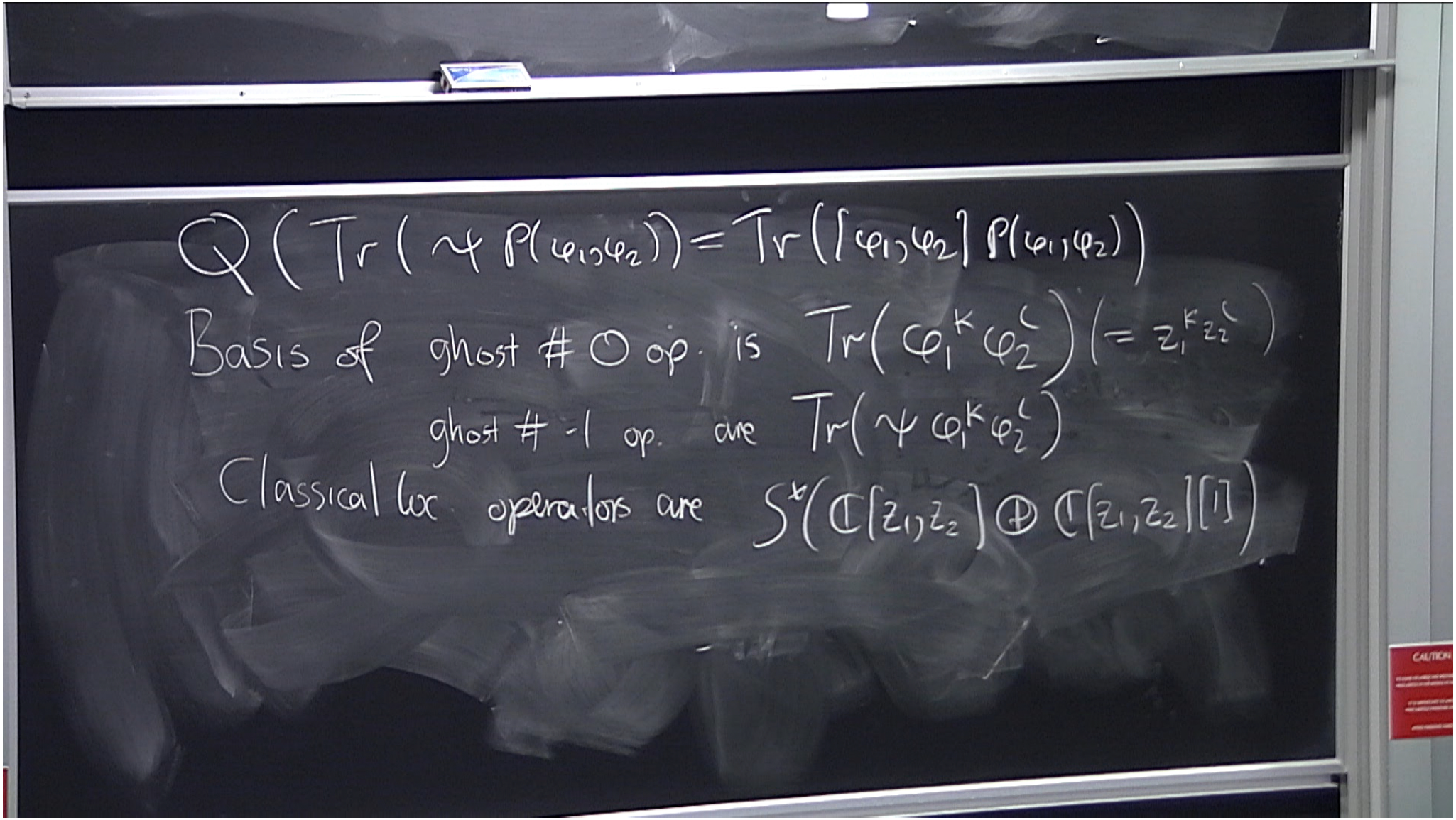
If we put this on $\mathbb{R} \times S^1 \times X$ $\times$ hol. symplectic

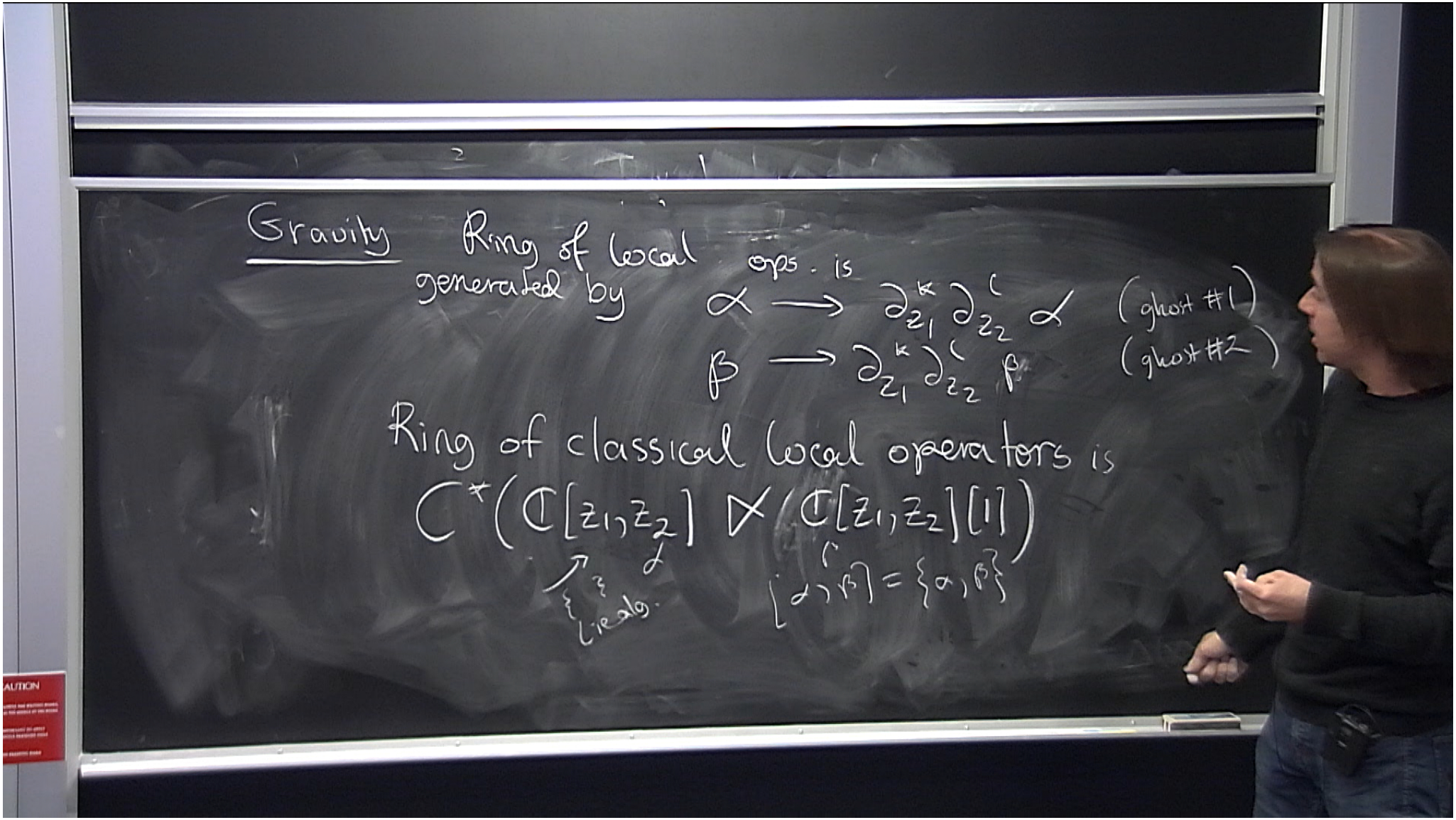
Phase space is

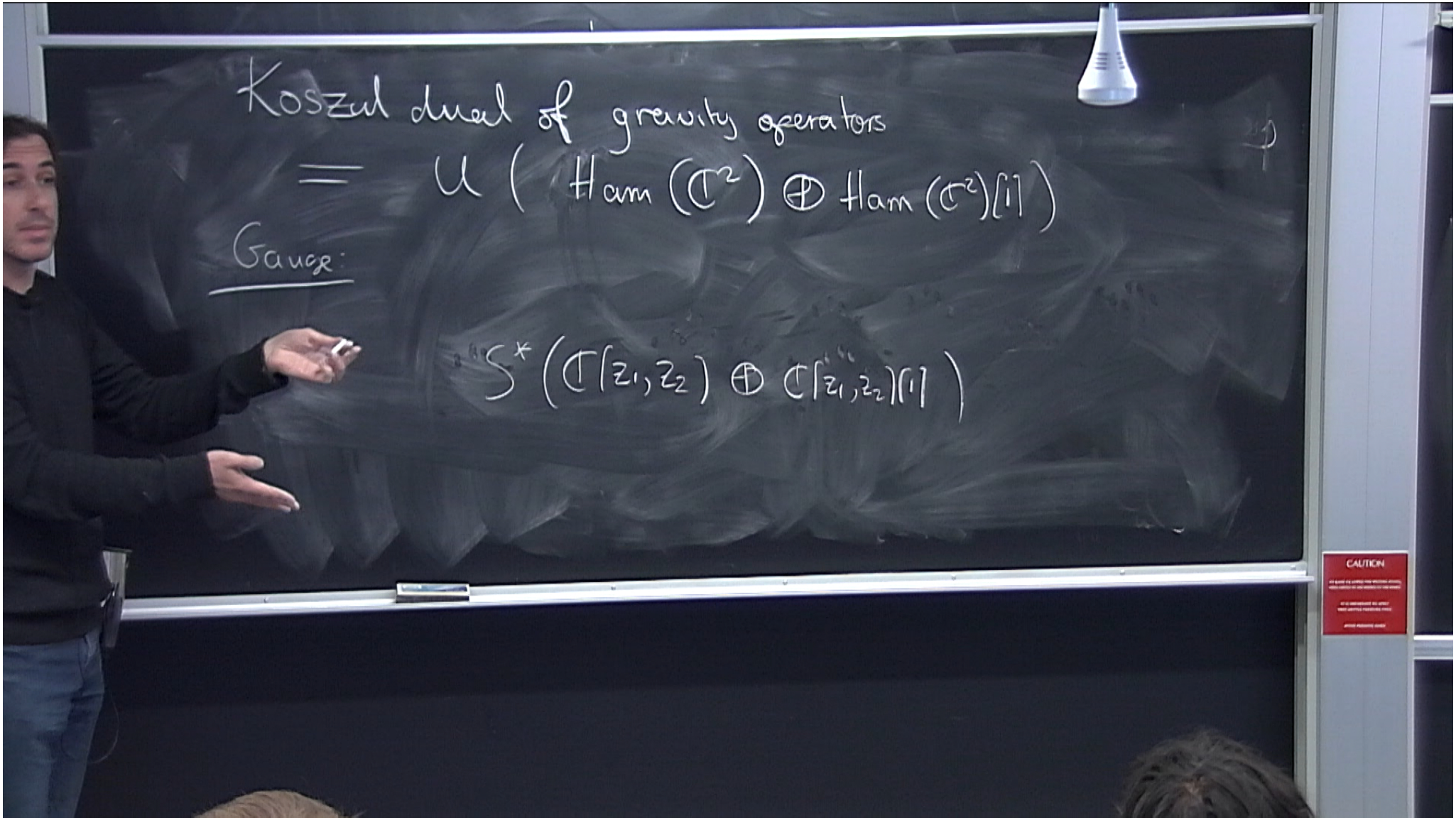
T^* (Moduli of hol-sym. surfaces fibred over S^1 + flat conn.)

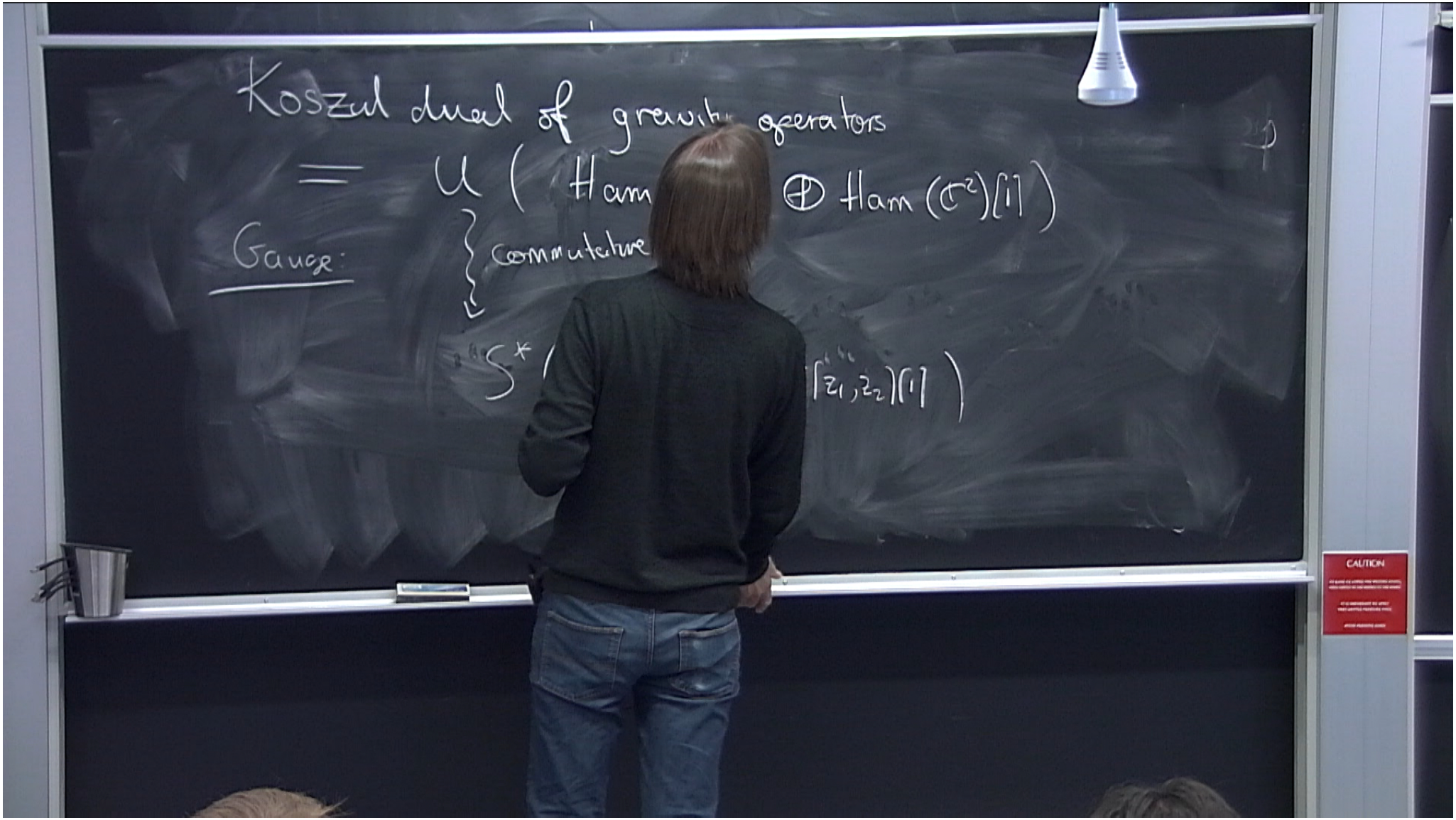

Closed string field ⇝ def. of the gauge theory on brane
x_1
$dx_1 z_1^k z_2^l$
⇝ deform action to
$\int \mathrm{Tr}\, \varphi_1 d_A \varphi_2 + \int dx_1 \mathrm{Tr}\, \varphi_1^k \varphi_2^l$
CAUTION

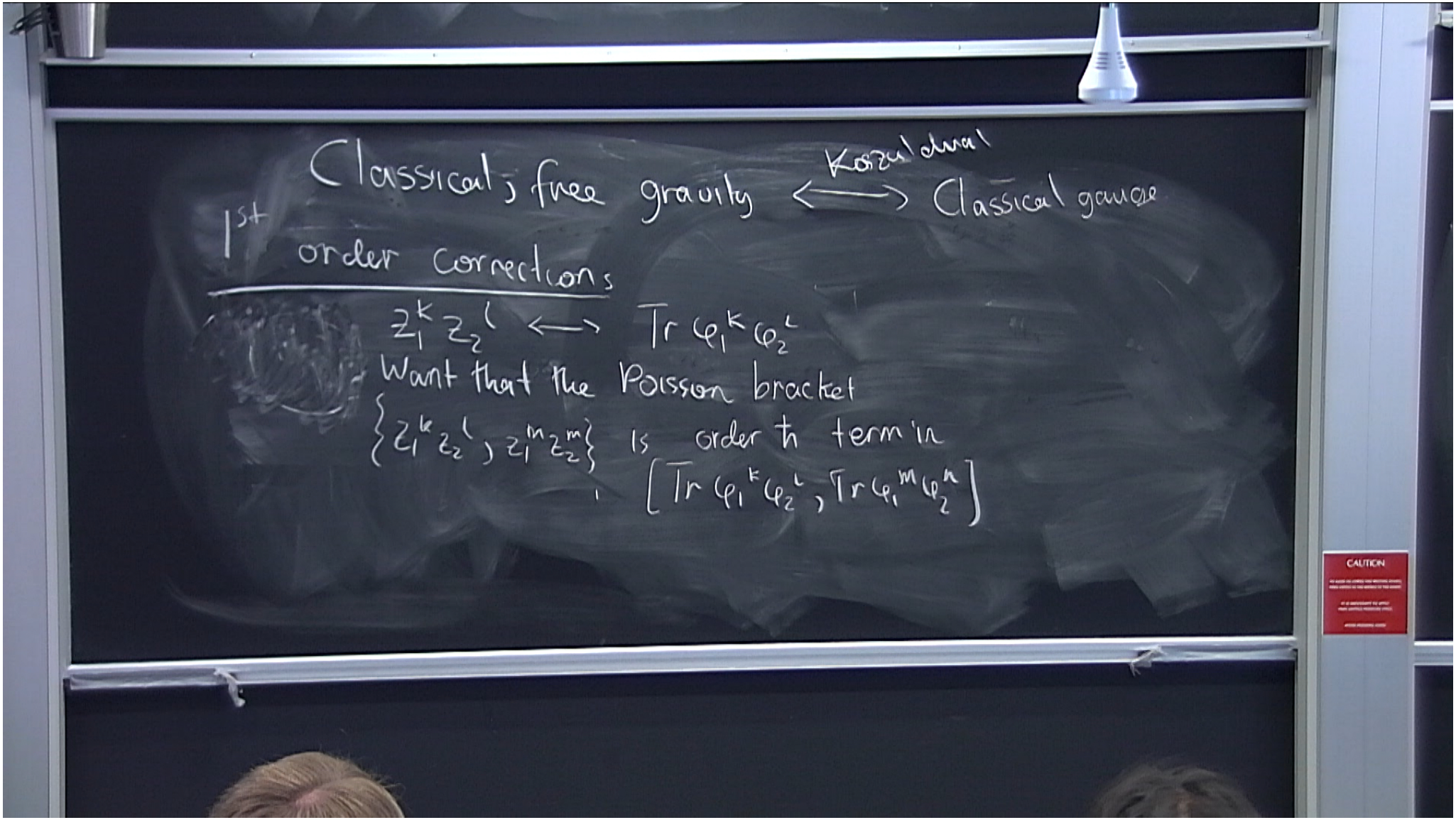

Local Operators in gauge theory
Ghost # 0, generated by Tr P(φ1, φ2)
non-comm- polynomial
N →> ∞, those are free generators
Ghost # -1 Tr(ψ P(φ1, φ2)
ψ = A* = antifield to A
Because ∫Tr(φ1[A, φ2]) = ∫Tr(A[φ1, φ2])

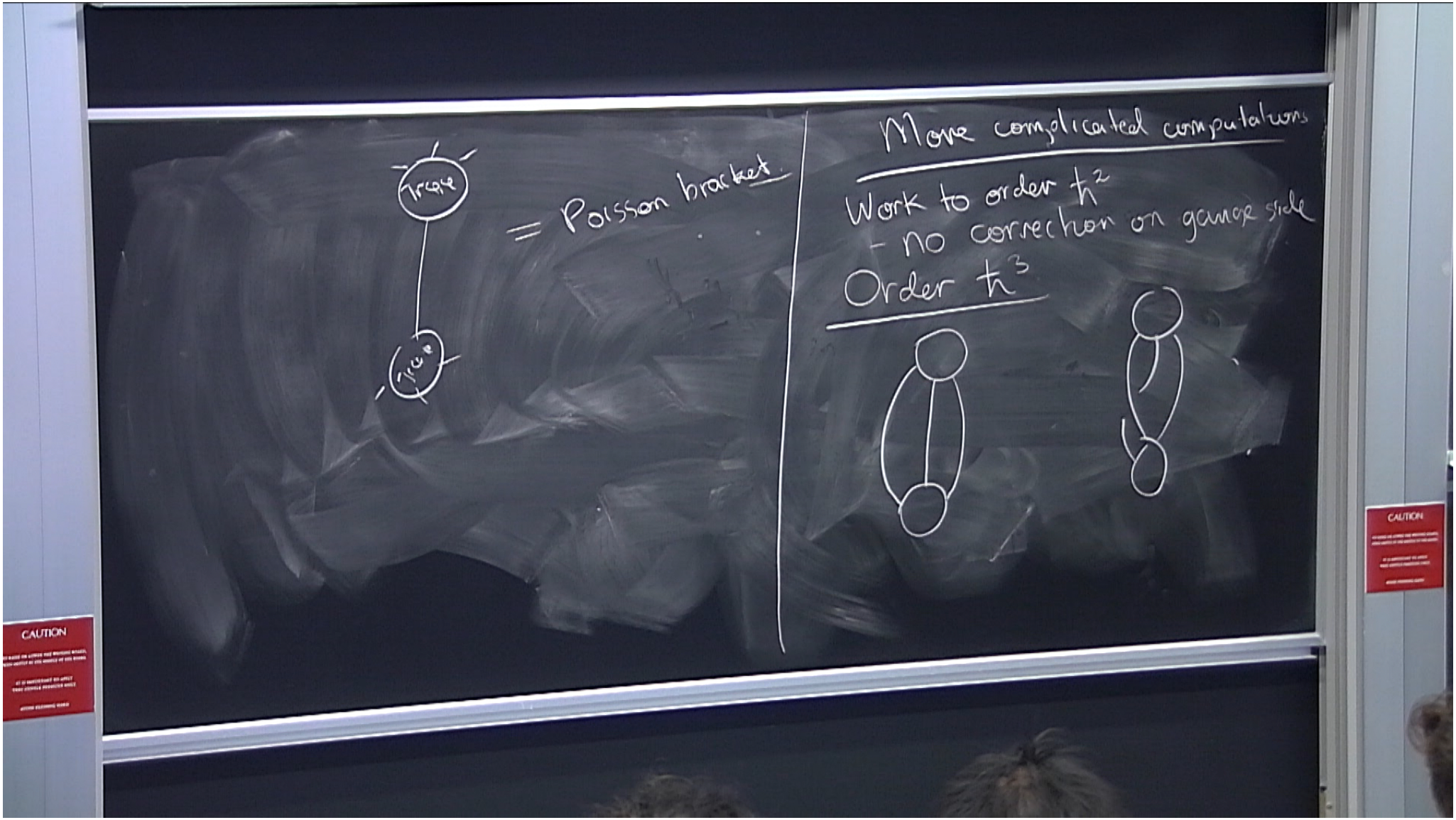

$Q(\mathrm{Tr}(\psi P(\varphi_1,\varphi_2)) = \mathrm{Tr}([\varphi_1,\varphi_2] P(\varphi_1,\varphi_2))$
Basis of ghost # 0 op. is $\mathrm{Tr}(\varphi_1^k \varphi_2^l)\ (= z_1^k z_2^l)$
ghost # -1 op. are $\mathrm{Tr}(\psi \varphi_1^k \varphi_2^l)$
Classical loc operators are $S^\flat(\mathbb{C}[z_1,z_2] \oplus \mathbb{C}[z_1,z_2][1])$

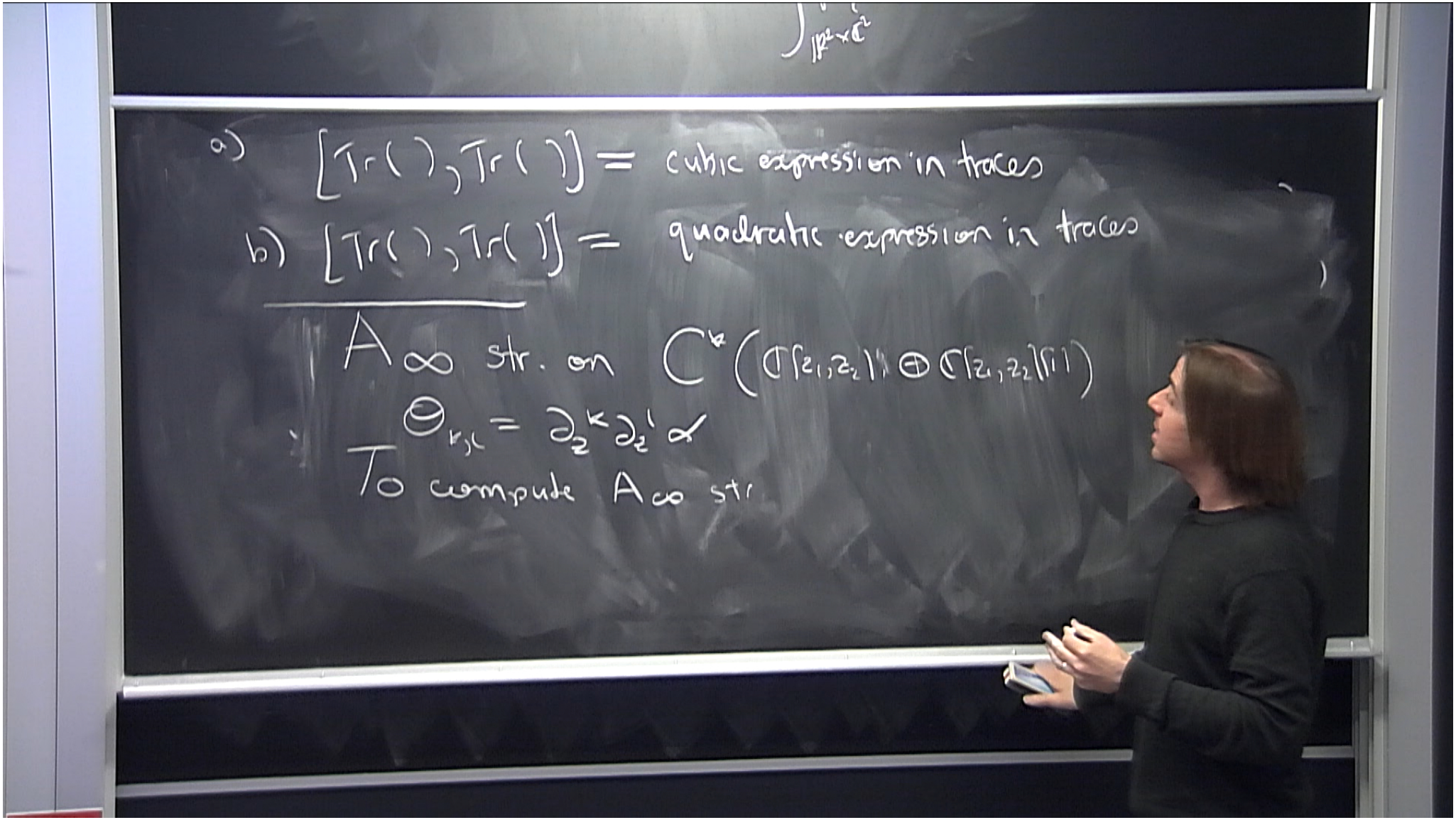

Gravity
Ring of local ops. is generated by
$\alpha \longrightarrow \partial_{z_1}^{k} \partial_{z_2}^{\ell} \alpha$ (ghost #1)
$\beta \longrightarrow \partial_{z_1}^{k} \partial_{z_2}^{\ell} \beta$ (ghost #2)
Ring of classical local operators is
$C^*(\mathbb{C}[z_1, z_2] \ltimes (\mathbb{C}[z_1, z_2][1]))$
α
{ Lie alg.
$[\alpha, \beta] = \{\alpha, \beta\}$


Koszul dual of gravity operators
= U(Ham(ℂ²) ⊕ Ham(ℂ²)[1])
Gauge:
S*(ℂ[z₁,z₂] ⊕ ℂ[z₁,z₂][1])
CAUTION


Koszul dual of
operators
Gauge:
commutative
CAUTION


Classical; free gravity
Koszul dual
Classical gauge
1st order corrections
$z_1^k z_2^l \longleftrightarrow \mathrm{Tr}\,\varphi_1^k \varphi_2^l$
Want that the Poisson bracket
$\{z_1^k z_2^l, z_1^m z_2^n\}$ is order th term in
$[\mathrm{Tr}\,\varphi_1^k \varphi_2^l, \mathrm{Tr}\,\varphi_1^m \varphi_2^n]$
CAUTION

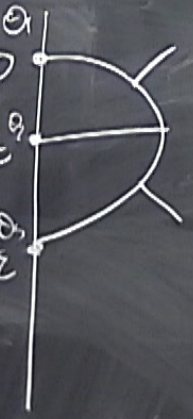

Trace
Trace
= Poisson bracket
More complicated computations
Work to order $\hbar^2$
- no correction on gauge side
Order $\hbar^3$
CAUTION
CAUTION


a) $[\mathrm{Tr}(\ ),\mathrm{Tr}(\ )] =$ cubic expression in traces
b) $[\mathrm{Tr}(\ ),\mathrm{Tr}(\ )] =$ quadratic expression in traces
A_∞ str. on $C^*(\mathbb{C}[z_1,z_2,1] \oplus \mathbb{C}[z_1,z_2][1])$
$\Theta_{k,l} = \partial_z^k \partial_{\bar{z}}^l \alpha$
To compute A_∞ str

To compute A_∞ str.

m_3 First non-trivial one is

$\lim_{\varepsilon \to 0} \int_{t=0}^{\varepsilon}$



$3 \to 2$ A_∞ operation

$\Leftrightarrow$ $2 \to 3$ Koszul dual side

m_2 $\lim_{\varepsilon \to 0}$

