Title: Bulk-boundary BV quantization for 2-1 theories

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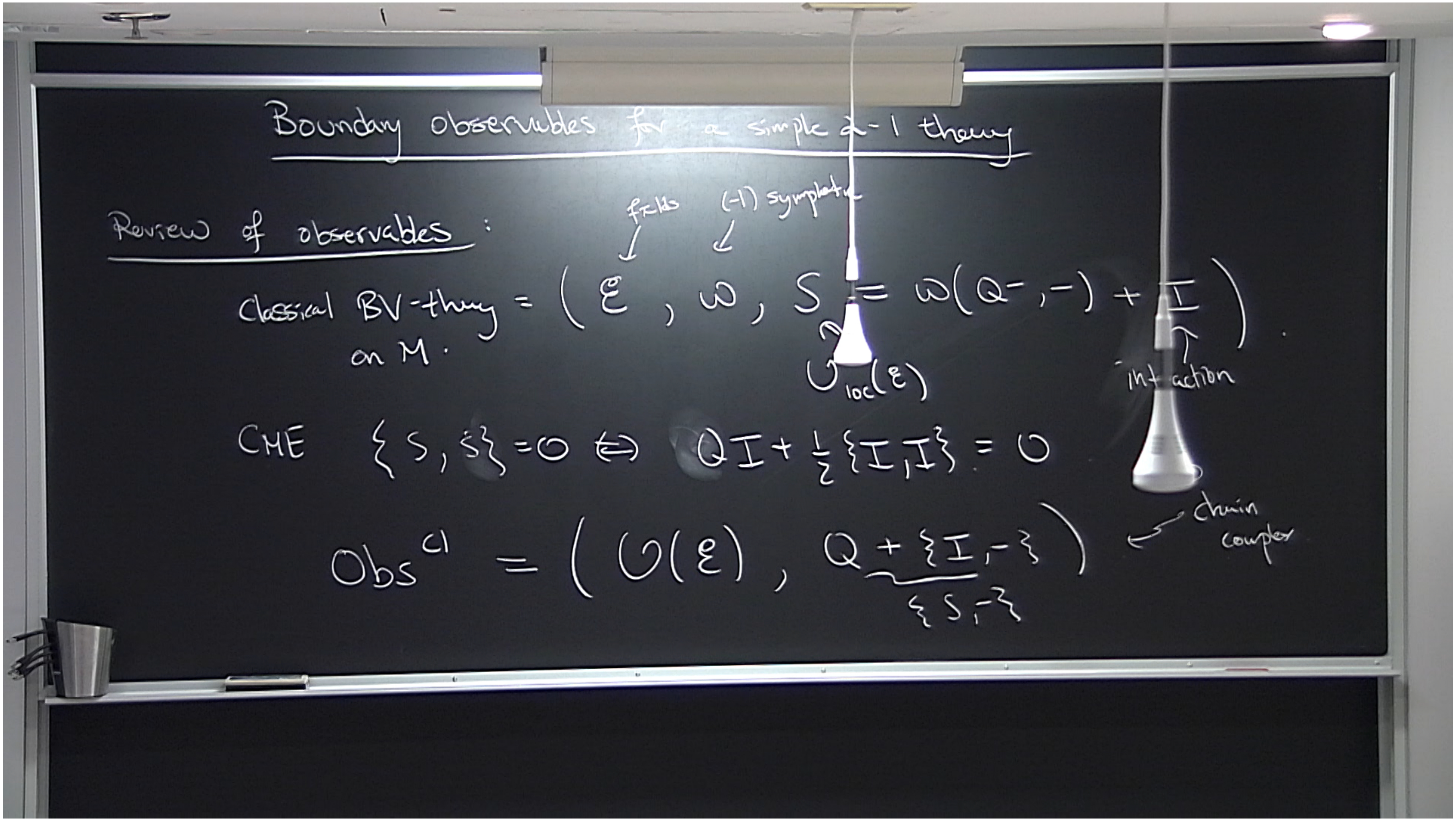
Abstract:



Boundary observables for a simple 2-1 theory
Review of observables:
fields
(-1) symplectic
Classical BV-theory on M. = $(\mathcal{E}, \omega, S = \omega(Q-,-) + I)$.
$\mathcal{O}_{loc}(\mathcal{E})$
interaction
CME $\{S,S\}=0 \Leftrightarrow QI + \frac{1}{2}\{I,I\} = 0$
$Obs^{cl} = (\mathcal{O}(\mathcal{E}), \underbrace{Q + \{I,-\}}_{\{S,-\}})$
chain complex

$\rightsquigarrow$ Obs^{cl} form a factorization algebra.

in fact a P_0- factorization algebra

Quantum observables: Introduce BV Laplacian:

for linear $x, y \in \mathrm{Obs}^{cl}$, $\Delta(x \cdot y) = \{x, y\}$

Quantum observables: Introduce BV-Laplacian:

for linear $x, y \in \mathrm{Obs}^{cl}$, $\Delta(xy) = \{x, y\}$

$\mathcal{E}^\vee \subset$ distributional sections

Problem: cannot define on Obs^{cl}

Extends to observables by the rule $\Delta(fg) = \Delta(f)g \pm f\Delta g \pm \{f, g\}$.

Equivalently: $\mathrm{id} \in \mathcal{E}^\vee \otimes \mathcal{E}$, via symplectic pairing ω

$\rightsquigarrow K_0 \in \bar{\mathcal{E}} \otimes \bar{\mathcal{E}}$

Δ = Contraction with K_0

Soln's: 1) "Smeared observables"

$$\tilde{\mathcal{O}}(\mathcal{E}) \overset{\simeq}{\hookrightarrow} \mathcal{O}(\mathcal{E})$$

think: $(\Omega^\bullet_c \hookrightarrow \bar{\Omega}^\bullet_c)$

$\int \simeq$

$\rightsquigarrow [Q, Q^{GF}] = D$ generalized laplacian

$\mathcal{E} \otimes \mathcal{E} \Rightarrow K_L$

$$DK_t - \frac{\partial K_t}{\partial t} = 0.$$

Propagator: $P_{\varepsilon<L} = \int_{t=\varepsilon}^{L} Q^{GF} K_t \, dt.$

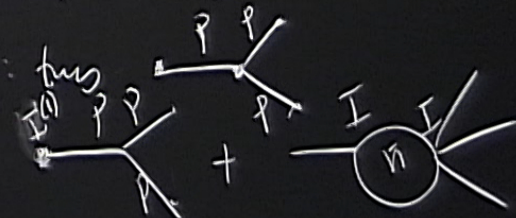
Goal: Define a quantization $\{I[L]\} \subset \mathcal{O}(\mathcal{E})[[\hbar]]$

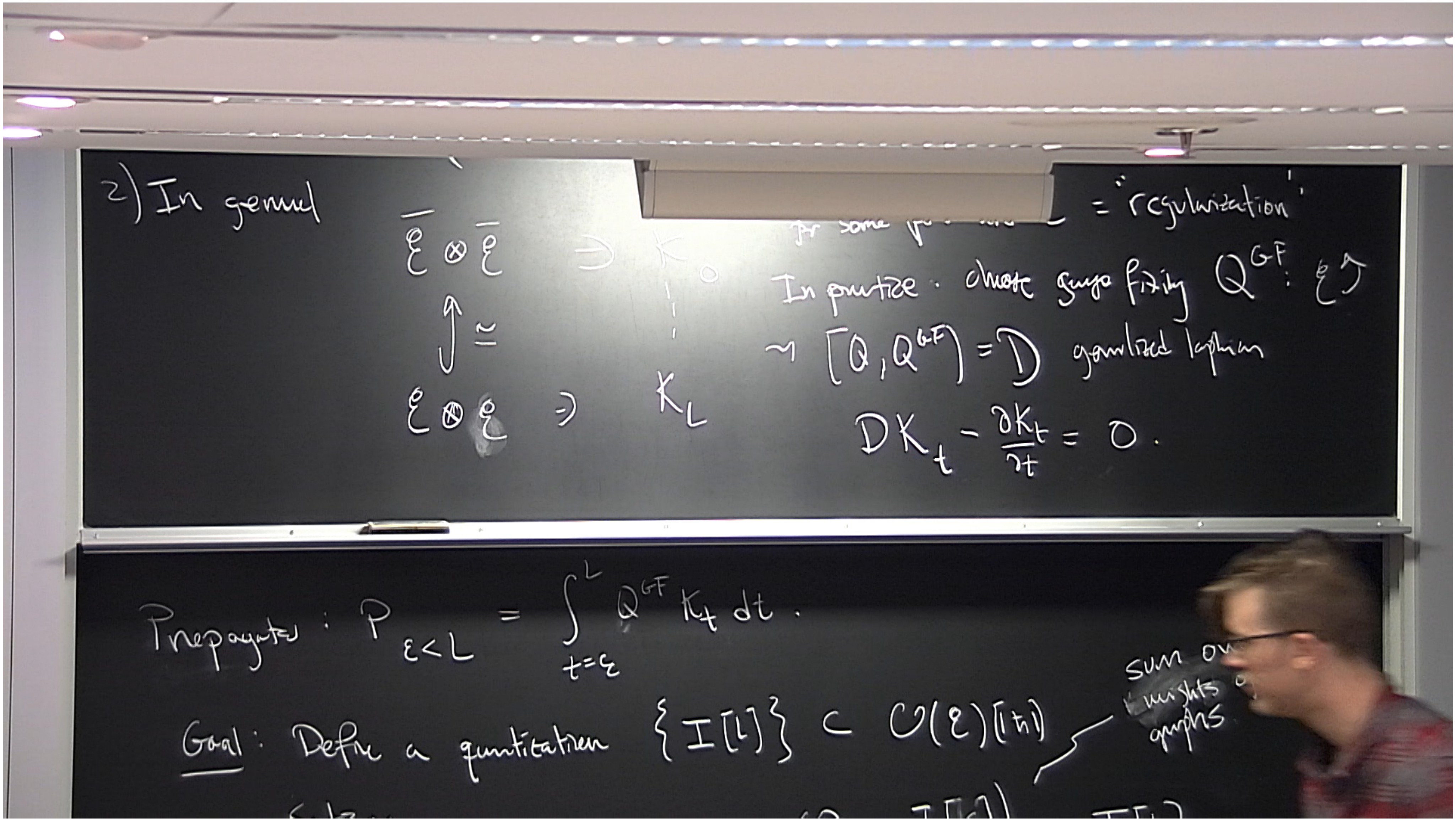
sum over weights of graphs

satisfying homotopy RG flow $W(P_{\varepsilon<L}, I[\varepsilon]) = I[L]$.

Order $\hbar^0$: trees

Order $\hbar^1$:




2) In general
$\bar{\mathcal{E}} \otimes \bar{\mathcal{E}} \ni K_0$
$\mathcal{E} \otimes \mathcal{E} \ni K_L$
= "regularization"
In practice: choose gauge fixing $Q^{GF} : \mathcal{E}$
$\rightsquigarrow [Q, Q^{GF}] = D$ generalized Laplacian
$D K_t - \frac{\partial K_t}{\partial t} = 0$.
Propagator: $P_{\varepsilon < L} = \int_{t=\varepsilon}^{L} Q^{GF} K_t \, dt$.
Goal: Define a quantization $\{I[L]\} \subset \mathcal{O}(\mathcal{E})[[\hbar]]$
sum over weights of graphs

$\mathcal{E}\otimes\mathcal{E} \Rightarrow K_L$ $\qquad DK_t - \frac{\partial K_t}{\partial t} = 0.$

Propagator: $P_{\varepsilon<L} = \int_{t=\varepsilon}^{L} Q^{GF} K_t \, dt.$

Goal: Define a quantization $\{I[L]\} \subset \mathcal{O}(\mathcal{E})[[\hbar]]$ — sum over weights of graphs

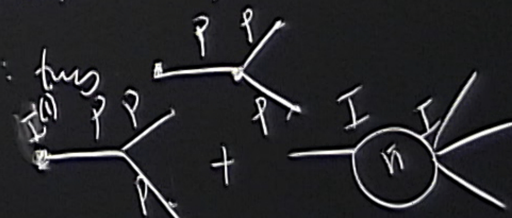
Satisfying 1) homotopy RG flow $W(P_{\varepsilon<L}, I[\varepsilon]) = I[L]$.

2) QME

$QI[L] + \hbar \Delta_L I[L] + \frac{1}{2}\{I[L], I[L]\}_L = 0$

Order $\hbar^0$: trees

Order $\hbar^1$:



2) In general

$$\bar{\mathcal{E}} \otimes \bar{\mathcal{E}} \ni K_0$$

$$\uparrow \cong$$

$$\mathcal{E} \otimes \mathcal{E} \ni K_L$$

for some = "regularization"

In practice: choose gauge fixing $Q^{GF} : \mathcal{E} \circlearrowleft$

$\rightsquigarrow [Q, Q^{GF}] = D$ generalized laplacian

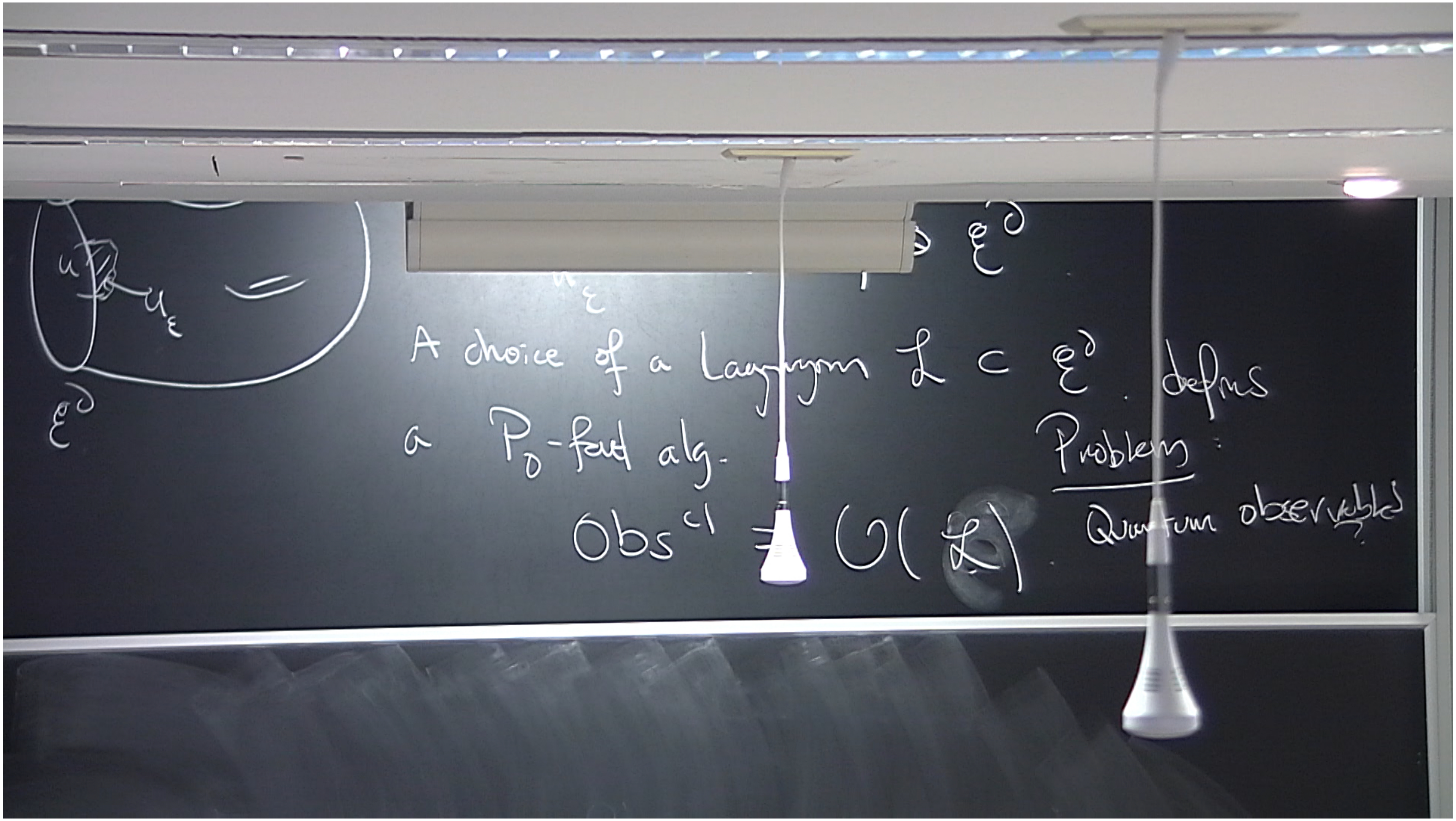
$$D K_t - \frac{\partial K_t}{\partial t} = 0.$$

Propagator: $P_{\varepsilon < L} = \int_{t=\varepsilon}^{L} Q^{GF} K_t \, dt$.

Goal: Define a quantization $\{I[L]\} \subset \mathcal{O}(\mathcal{E})[[\hbar]]$

sum over weights of graphs

Satisfying $W(P \quad I[\varepsilon]) = I[L]$



u_ε
$\mathcal{E}^\partial$
A choice of a Lagrangian $\mathcal{L} \subset \mathcal{E}^\partial$. defines
a P_0-fact alg.
$\mathrm{Obs}^{cl} = \mathcal{O}(\mathcal{L})$.
Problem:
Quantum observables?

$\mathcal{E}$ a P_0-fact alg.

$Obs^{cl} = \mathcal{O}(\mathcal{E})$

Problem: Quantum observables

• Example: BF-theory on a 2-manifold Σ. Lie $\mathfrak{g}$ ...

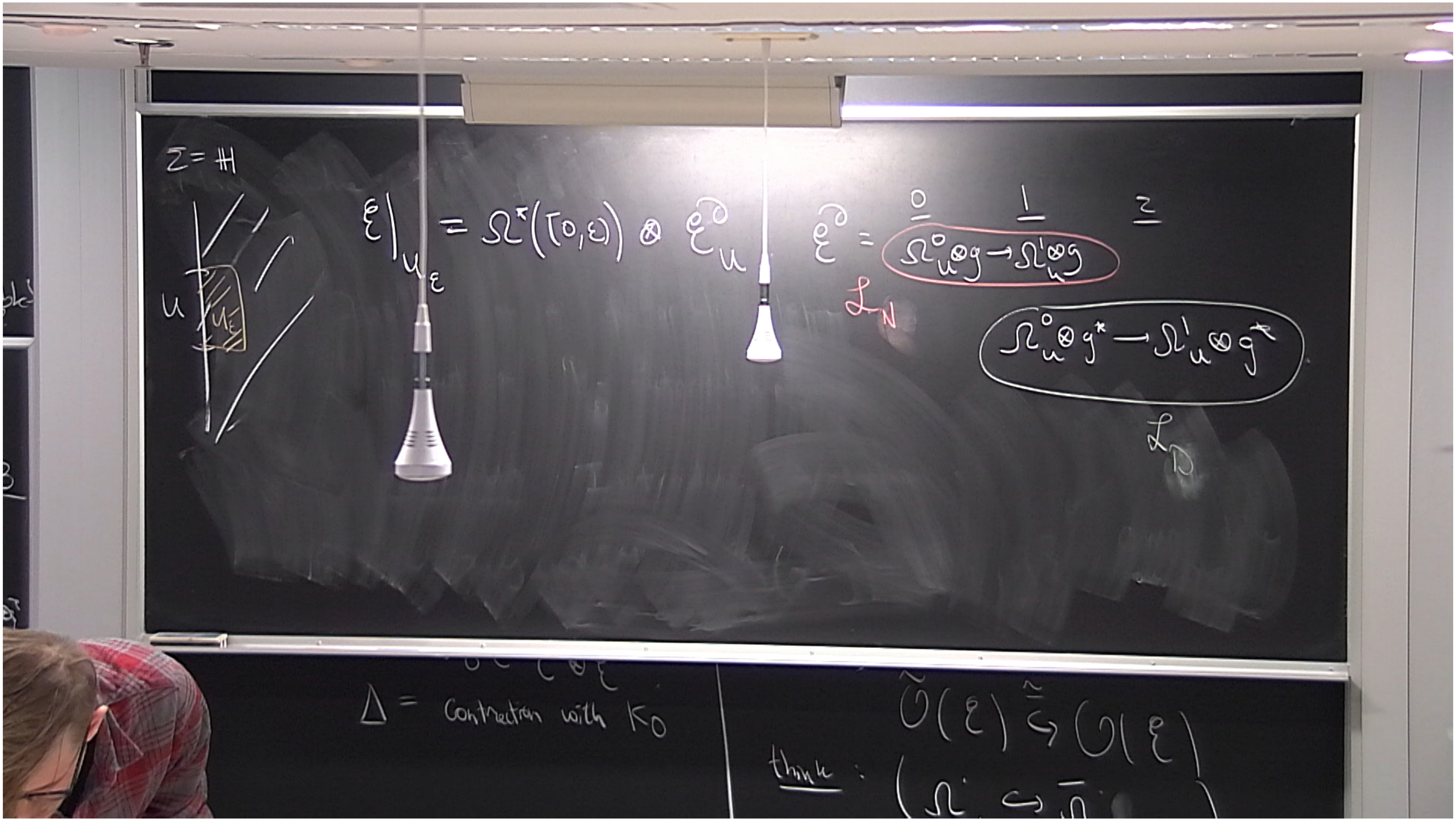
$$\underline{\mathrm{Map}}\left(\Sigma_{dR}, T^*[-1]B\mathfrak{g}\right) = \begin{array}{cccc} \underline{0} & \underline{1} & \underline{2} & \underline{3} \\ A \in \Omega^0_\Sigma \otimes \mathfrak{g} \to & \Omega^1_\Sigma \otimes \mathfrak{g} \to & \Omega^2_\Sigma \otimes \mathfrak{g} & \\ B \in & \Omega^0_\Sigma \otimes \mathfrak{g}^* \to & \Omega^1_\Sigma \otimes \mathfrak{g}^* \to & \Omega^2_\Sigma \otimes \mathfrak{g}^* \end{array}$$

$\parallel$

$\mathcal{E}$

symp pairing $\omega(A,B) = \int \langle A, B \rangle$

Action: $S(A,B) = \int \langle F_A, B \rangle$


$\Sigma = \mathbb{H}$
U
U_ε
$\mathcal{E}|_{U_\varepsilon} = \Omega^*(\Gamma(0,\varepsilon)) \otimes \mathcal{E}^\partial_U$
$\mathcal{E}^\partial = $ 0 1 2
$\Omega^0_U \otimes \mathfrak{g} \to \Omega^1_U \otimes \mathfrak{g}$
$\mathcal{L}_N$
$\Omega^0_U \otimes \mathfrak{g}^* \to \Omega^1_U \otimes \mathfrak{g}^*$
$\mathcal{L}_D$
$\Delta =$ Contraction with K_0
$\tilde{\mathcal{O}}(\mathcal{E}) \xrightarrow{\simeq} \mathcal{O}(\mathcal{E})$
think :

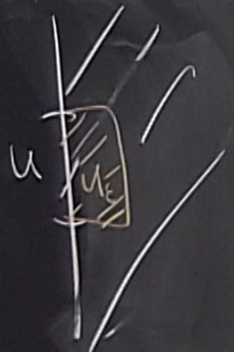
$\underline{a_N} \simeq \mathrm{Obs}_{\partial,N} = \mathcal{O}(\mathcal{L}_N[1])$

<u>Prop</u>: For each $I \overset{\text{interval}}{\subset} \mathbb{R} = \partial\mathbb{H}$, there is a quasi-isomorphism

$$\mathrm{Obs}^{cl}_{\partial,N}(I) \simeq C^{\bullet}_{Lie}(\mathfrak{g})$$

Moreover, the induced P_0-structure is trivial.

$\Sigma = \mathbb{H}$



$$\mathcal{E}\big|_{U_\varepsilon} = \Omega^*([0,\varepsilon)) \otimes \mathcal{E}^\partial_U, \qquad \mathcal{E}^\partial = \begin{array}{ccc} \underline{0} & & \underline{1} & & \underline{2} \\ \Omega^0_U\otimes\mathfrak{g} & \to & \Omega^1_U\otimes\mathfrak{g} & & \end{array}$$

$\mathcal{L}_N$

$$\Omega^0_U\otimes\mathfrak{g}^* \to \Omega^1_U\otimes\mathfrak{g}^*$$

$\mathcal{L}_D$

$$\mathrm{Obs}^{cl}_{\partial,N}(I) = \Big(\mathrm{Sym}\big(\underset{-1}{\Omega^0_I \otimes \mathfrak{g}} \to \underset{0}{\Omega^1_I \otimes \mathfrak{g}} \big)^\vee ,\ d \Big).$$

$$\simeq \Big(\mathrm{Sym}(\mathfrak{g}[1])^* ,\ d = d_{CE} \Big)$$

$$= C^\bullet_{Lie}(\mathfrak{g})$$

$\mathrm{Obs}^q_{\partial,N}(I) = C_{Lie}(\mathfrak{g})[[\hbar]]$.

- $\mathcal{L}_D = \overset{1}{\Omega^0 \otimes \mathfrak{g}^*} \to \overset{2}{\Omega^1 \otimes \mathfrak{g}^*}$

Trivial Lie structure.

- P_0 structure is identified w/ $\wedge \otimes [\,,\,]_{\mathfrak{g}}$

$$\Gamma(\mathcal{L}_D ; \mathrm{Sym}^2 \mathcal{L}_D).$$

linear P_0-structure

$$\mathcal{L}_D \longrightarrow \mathrm{Sym}^2 \mathcal{L}_D \Leftrightarrow \mathcal{L}_D \otimes \mathcal{L}_N \xrightarrow{\wedge \otimes [\,,\,]} \mathcal{L}_D.$$

Upshot: for each $I \subset \mathbb{R} =$

$$\mathrm{Obs}^{f}_{\partial, D}(I) = \left(\widetilde{\mathcal{O}}(\mathcal{L}_D),\ d + \hbar\Delta\right)$$

Implies: the fact alg on $\mathbb{R} = \partial\mathbb{H}$ is the ass. alg

$$\simeq \left(\mathrm{Sym}\left(\overset{-1}{\Omega^0_c \otimes \mathfrak{g}} \to \overset{0}{\Omega^1_c \otimes \mathfrak{g}}\right)[\hbar],\ d + \hbar\Delta\right).$$

$$U_\hbar \mathfrak{g} = \mathrm{Tens}(\mathfrak{g} \oplus \mathbb{C}\hbar) \Big/ \begin{matrix} xy - yx - \hbar[x,y] \\ x\hbar = \hbar x \end{matrix} \simeq C^{\mathrm{Lie}}_*\left(\Omega^*_c(I) \otimes \mathfrak{g} \oplus \mathbb{C}\hbar[-1]\right)$$

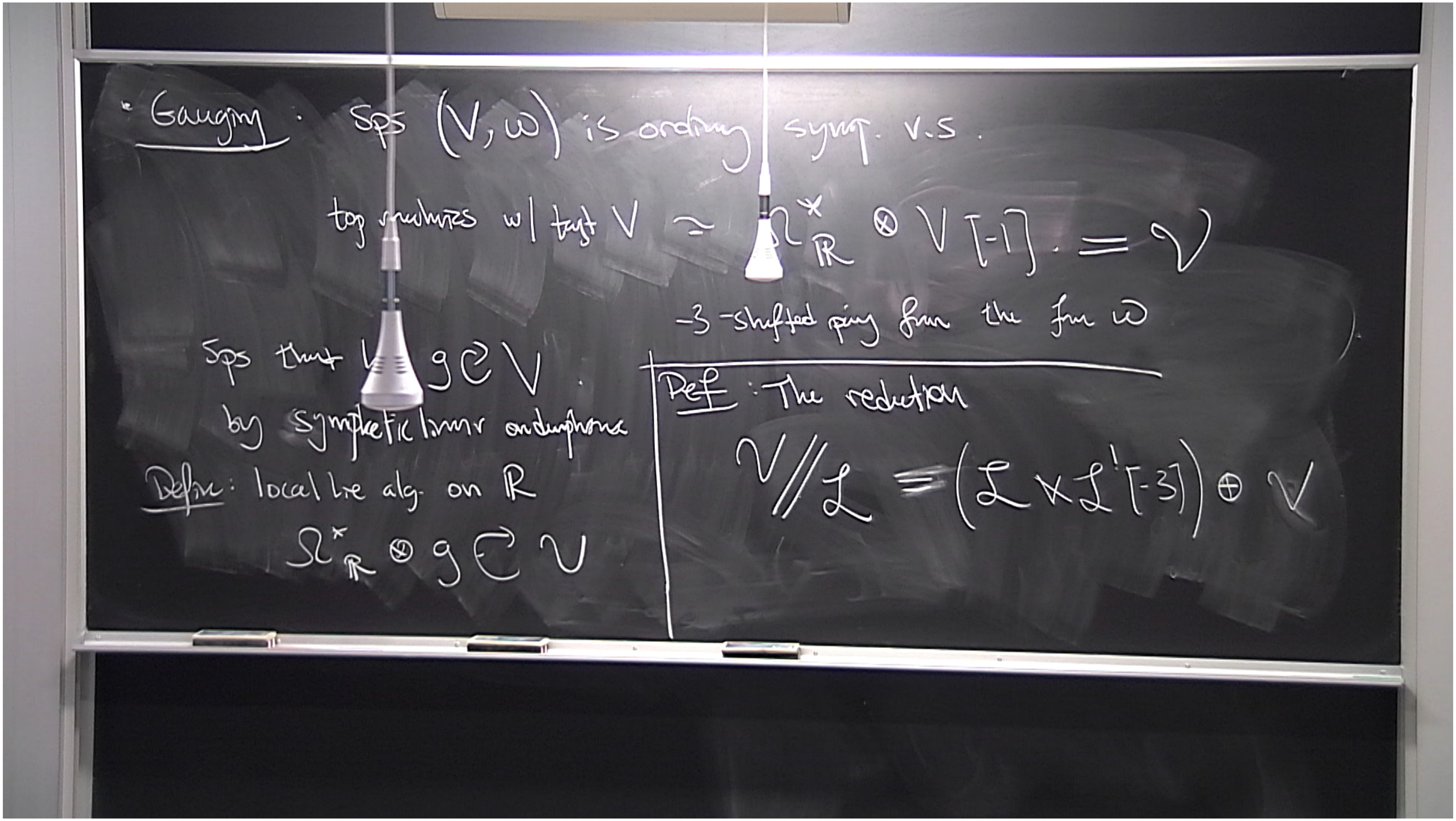
$$d_{CE}(xy) = \hbar\Delta(xy).$$

∂, D

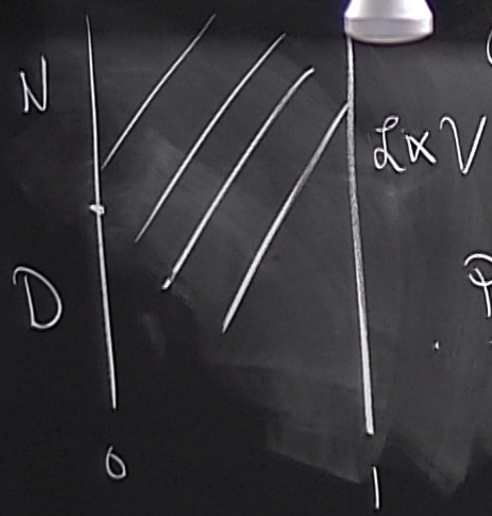
Δ

On gens

$$\Delta : \mathcal{L}_D^\vee \otimes \mathcal{L}_D^\vee \xrightarrow{[,]} \mathcal{L}_D^\vee.$$


• Gauging: Sps (V, ω) is ordinary symp. v.s.
top structures w/ tgt $V \simeq \Omega^*_{\mathbb{R}} \otimes V[-1] = \mathcal{V}$
-3-shifted pairing from the form ω
Sps that $\mathfrak{g} \circlearrowright V$
by symplectic linear endomorphisms
Define: local Lie alg. on $\mathbb{R}$
$\Omega^*_{\mathbb{R}} \otimes \mathfrak{g} \circlearrowright \mathcal{V}$
Def: The reduction
$\mathcal{V}/\!/\mathcal{L} = (\mathcal{L} \ltimes \mathcal{L}^![-3]) \oplus \mathcal{V}$

$\mathcal{L} = \Omega^*_{\mathbb{R}} \otimes \mathfrak{g} \circlearrowright V$



Consider bulk thry

$$\Omega^*_{[0,1]} \otimes \left(\mathcal{L} \ltimes \mathcal{L}^![-2]\right) \simeq \text{BF thry.}$$

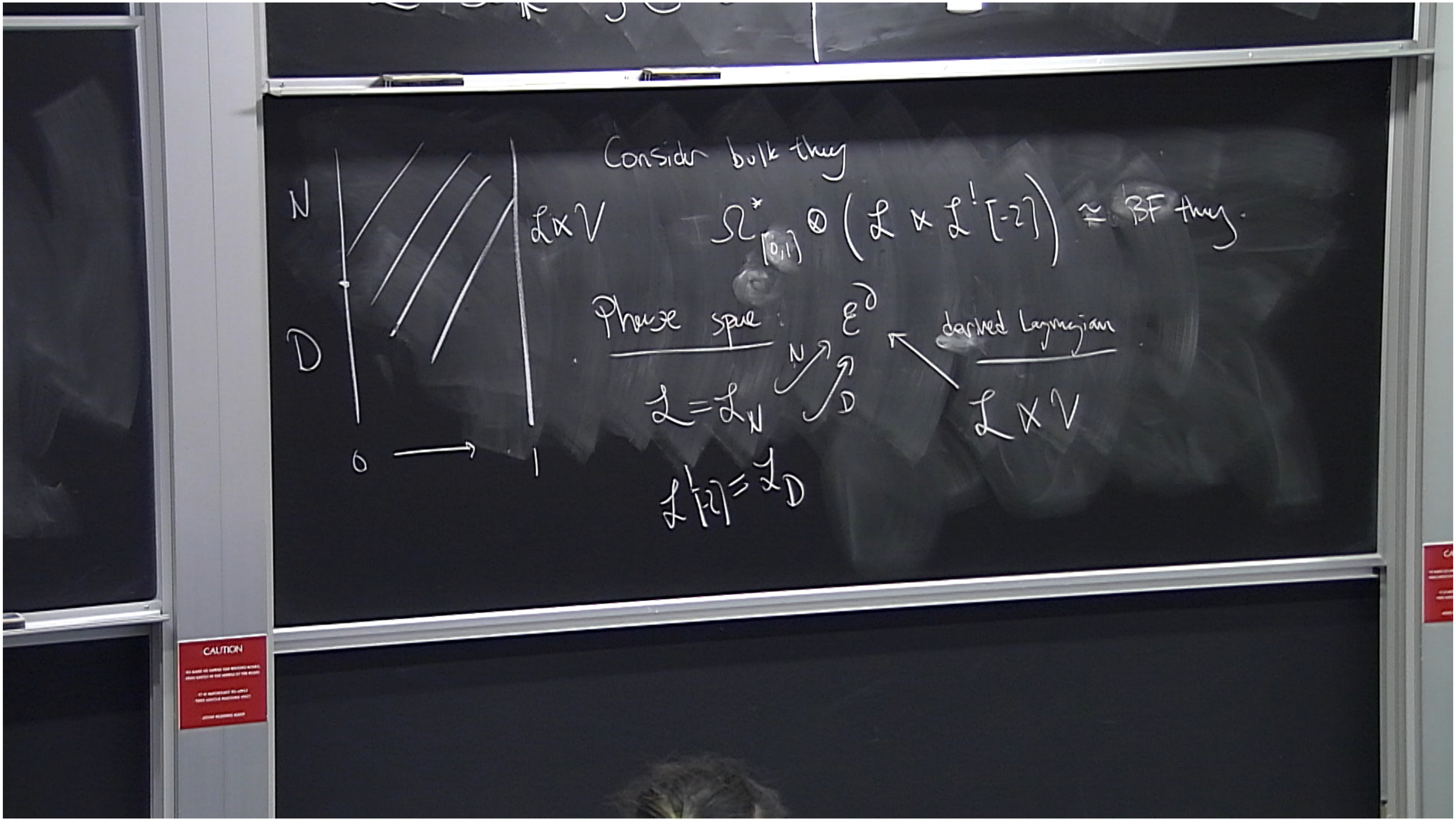
Phase space: $\mathcal{E}^\partial$

$\mathcal{L} = \mathcal{L}_N \xrightarrow{N} \mathcal{E}^\partial$

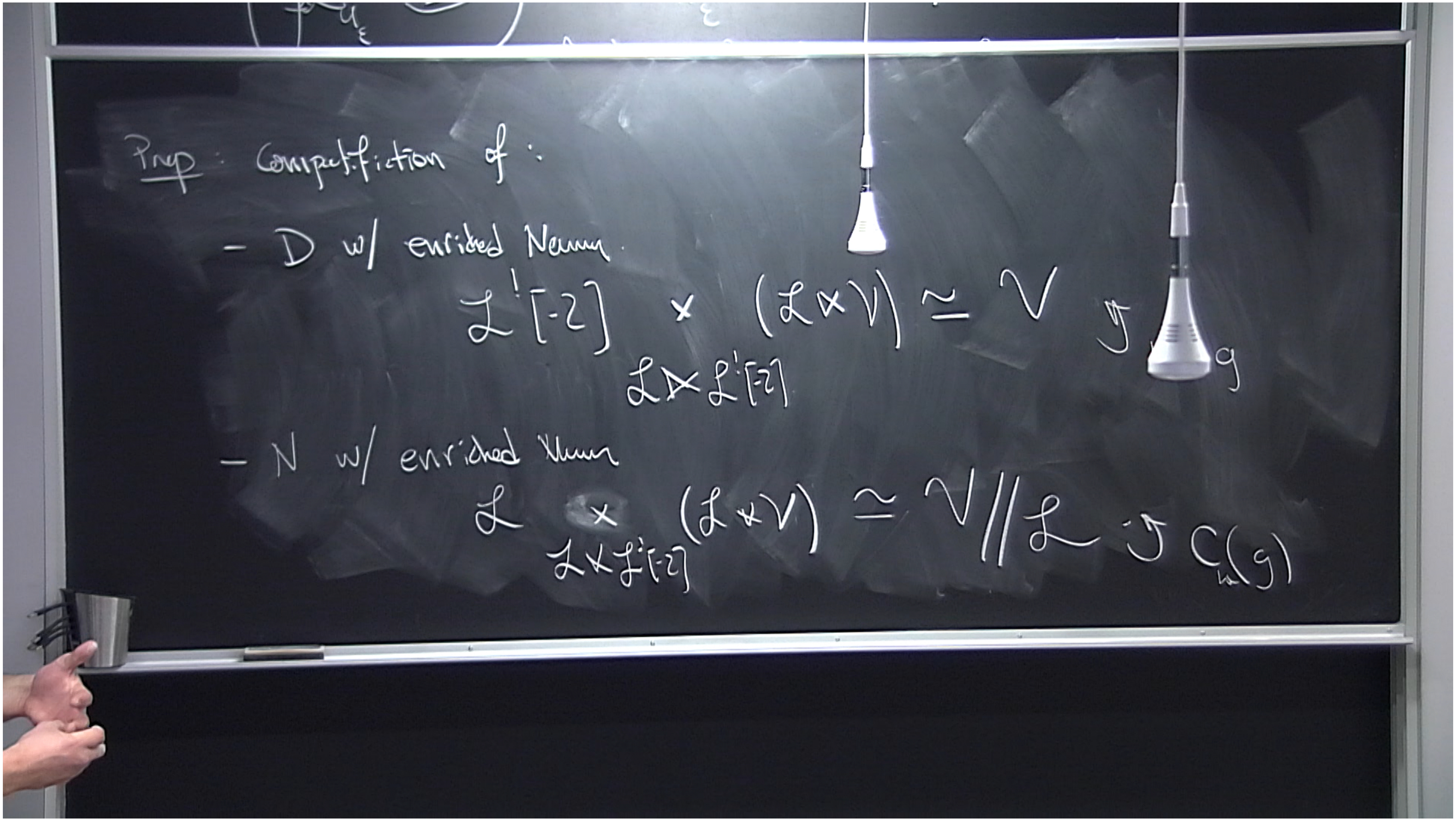
$\mathcal{L}^![-2] = \mathcal{L}_D \xrightarrow{D} \mathcal{E}^\partial$

derived Lagrangian

$\mathcal{L} \ltimes V \to \mathcal{E}^\partial$



Consider bulk thry
N
D
0 → 1
$\mathcal{L} \ltimes V$
$\Omega^*_{[0,1]} \otimes (\mathcal{L} \ltimes \mathcal{L}^![-2]) \simeq$ BF thry.
Phase space: $\mathcal{E}^\partial$
$\mathcal{L} = \mathcal{L}_N$
$\mathcal{L}^![-2] = \mathcal{L}_D$
derived Lagrangian
$\mathcal{L} \ltimes V$
CAUTION


Prop: Compactification of:
- D w/ enriched Neumann
$\mathcal{L}^![-2] \times_{\mathcal{L}\times\mathcal{L}^![-2]} (\mathcal{L}\times V) \simeq V \circlearrowleft \mathfrak{g}$
- N w/ enriched Neumann
$\mathcal{L} \times_{\mathcal{L}\times\mathcal{L}^![-2]} (\mathcal{L}\times V) \simeq V//\mathcal{L} \circlearrowleft C_{\bullet}(\mathfrak{g})$