Title: Bulk-boundary BV quantization for 2-1 theories

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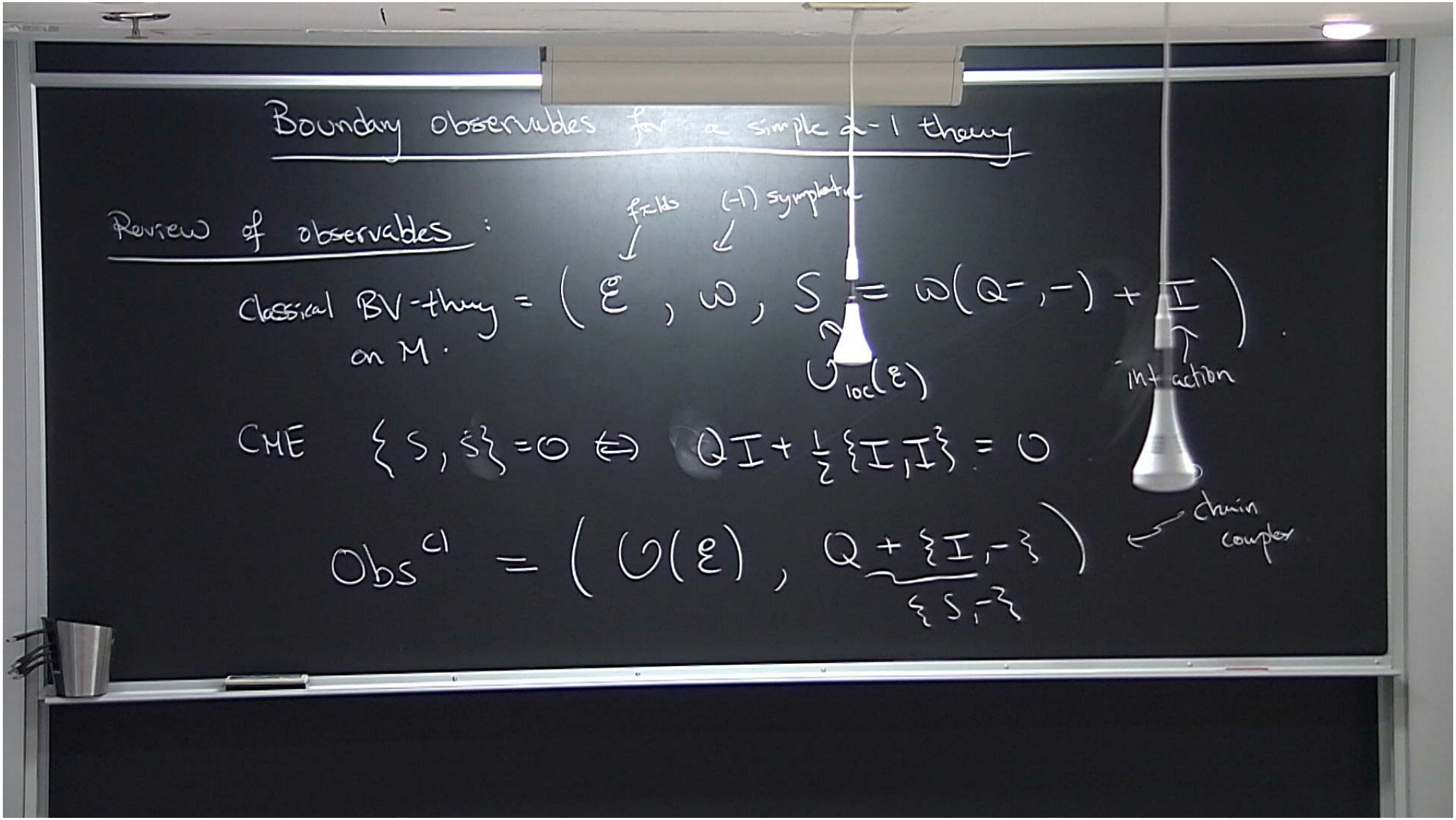
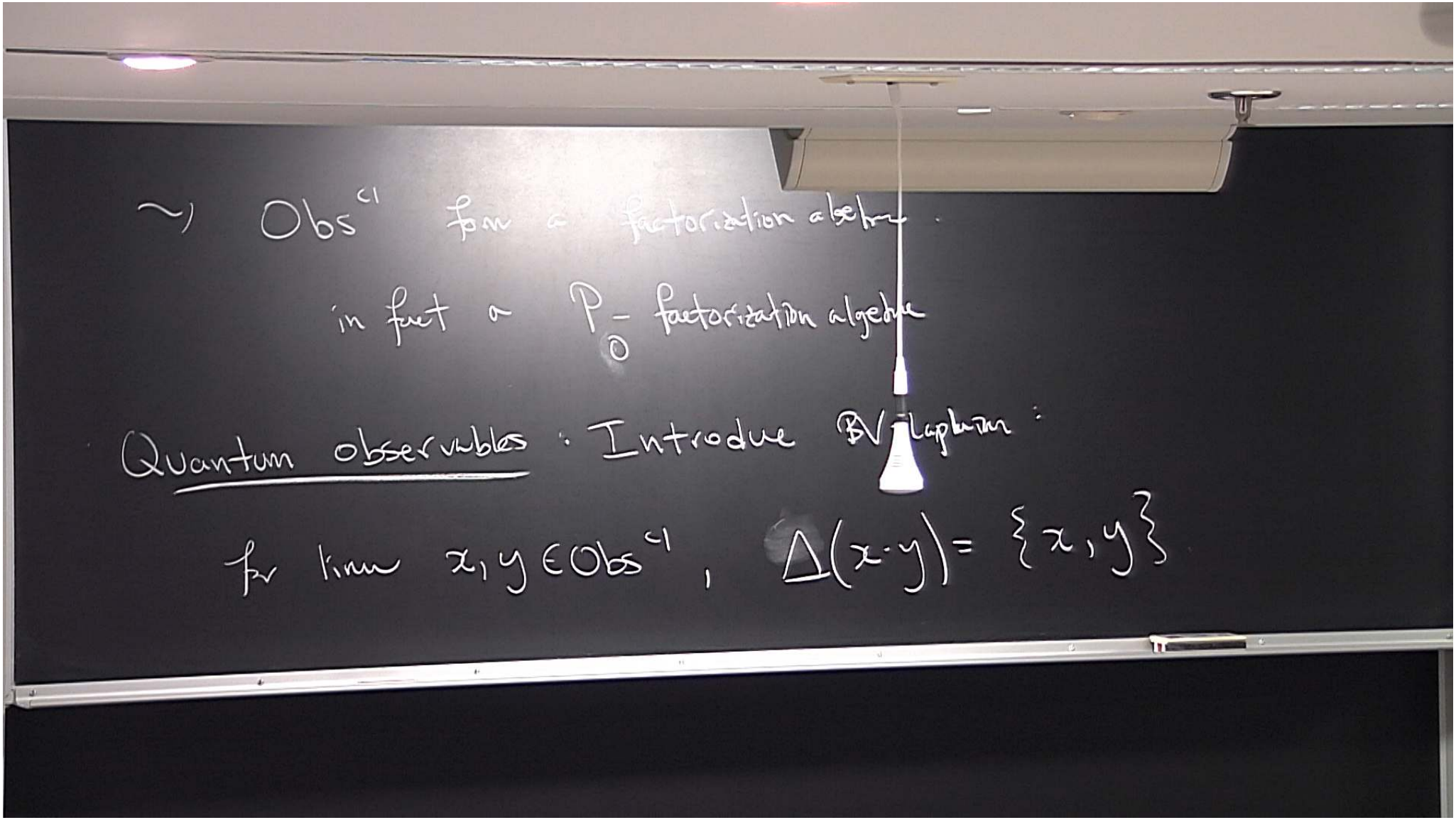
URL: http://pirsa.org/17050020

Abstract:


Boundary observables for a simple 2-1 theory
Review of observables:
fields
(-1) symplectic
Classical BV-thy = ($\mathcal{E}$, ω , $S = \omega(Q^-, -) + I$) on M.
$\mathcal{O}_{loc}(\mathcal{E})$
int. action
CME $\{S, S\} = 0 \Leftrightarrow QI + \frac{1}{2}\{I, I\} = 0$
$Obs^{cl} = (\mathcal{O}(\mathcal{E}) , Q + \{I, -\})$
$\{S, -\}$
chain complex


$\rightsquigarrow$ Obs^{cl} for a factorization algebra.
in fact a P_0- factorization algebra
Quantum observables: Introduce BV-Laplacian:
for time $x, y \in Obs^{cl}$, $\Delta(x \cdot y) = \{x, y\}$

Quantum observables : Introduce BV-Laplacian :

Problem

cannot define on Obs^{cl}

for linear $x, y \in Obs^{cl} = \Gamma(\mathcal{E}^\vee)$, $\Delta(x \cdot y) = \{x, y\}$

$\mathcal{E}^\vee \hookrightarrow$ distributional sections

Extends to observables by the rule $\Delta(fg) = \Delta(f)g \pm f\Delta g \pm \{f, g\}$

Equivalently: $id \in \mathcal{E}^\vee \otimes \mathcal{E}$, via symplectic pairing ω

$\hookrightarrow K_0 \in \bar{\mathcal{E}} \otimes \bar{\mathcal{E}}$

$\Delta =$ Contraction with K_0

Soln's: "Smeared observables"

$\tilde{\mathcal{O}}(\mathcal{E}) \tilde{\simeq} \mathcal{O}(\mathcal{E})$

think: $(\Omega^\cdot_c \hookrightarrow \bar{\Omega}^\cdot_c)$

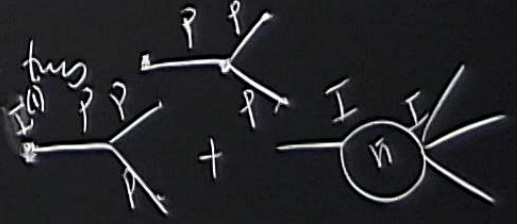
$\int \simeq$ ⋮ $\rightsquigarrow [Q, Q^{GF}] = D$ gauged Laplacian

$\mathcal{E} \otimes \mathcal{E} \Rightarrow K_L$ $\quad DK_t - \frac{\partial K_t}{\partial t} = 0.$

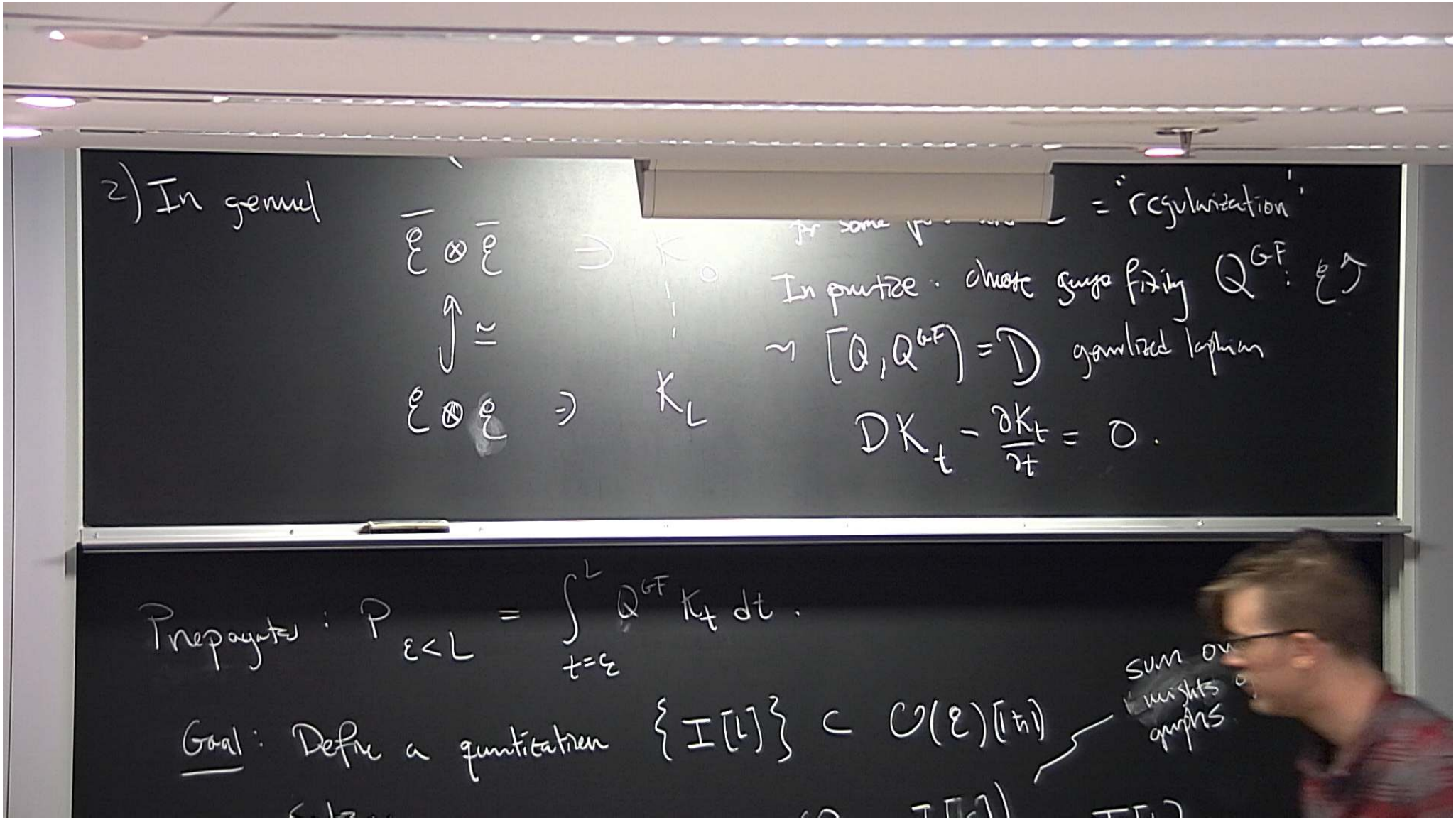
Propagator: $P_{\varepsilon<L} = \int_{t=\varepsilon}^{L} Q^{GF} K_t \, dt.$

Goal: Define a quantization $\{I[L]\} \subset \mathcal{O}(\mathcal{E})[[\hbar]]$ — sum over weights of graphs

Satisfying homotopy Rb flow $W(P_{\varepsilon<L}, I[\varepsilon]) = I[L]$.

order $\hbar^0$: 

order $\hbar$:


2) In general
$\bar{\mathcal{E}} \otimes \bar{\mathcal{E}}$
$\cong$
$\mathcal{E} \otimes \mathcal{E} \supset K_L$
= "regularization"
In practice: choose gauge fixing Q^{GF}: $\mathcal{E}$
$\rightsquigarrow [Q, Q^{GF}] = D$ generalized Laplacian
$DK_t - \frac{\partial K_t}{\partial t} = 0$.
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$\xi \otimes \xi \Rightarrow K_L$

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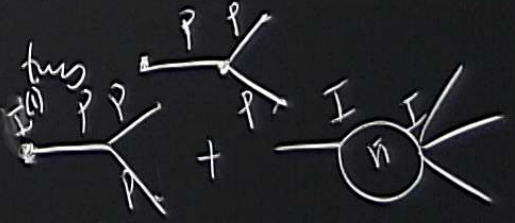
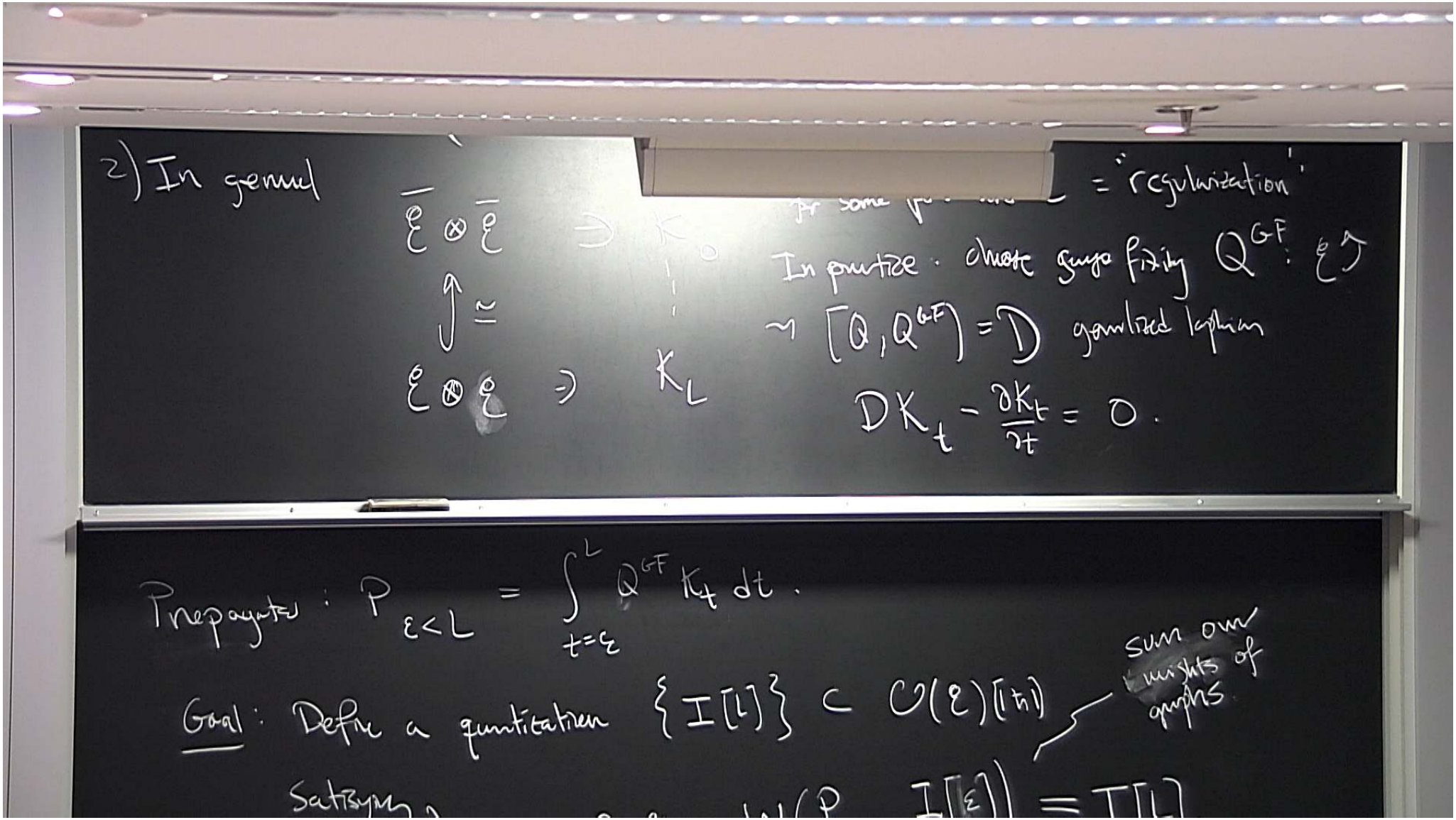
Satisfying 1) homotopy Rieflow $W(P_{\varepsilon<L}, I[\varepsilon]) = I[L]$.

2) QME

$Q I[L] + \hbar \Delta_L I[L] + \frac{1}{2}\{I[L], I[L]\}_L = 0$ — order $\hbar^0$: tree; order $\hbar^1$: +

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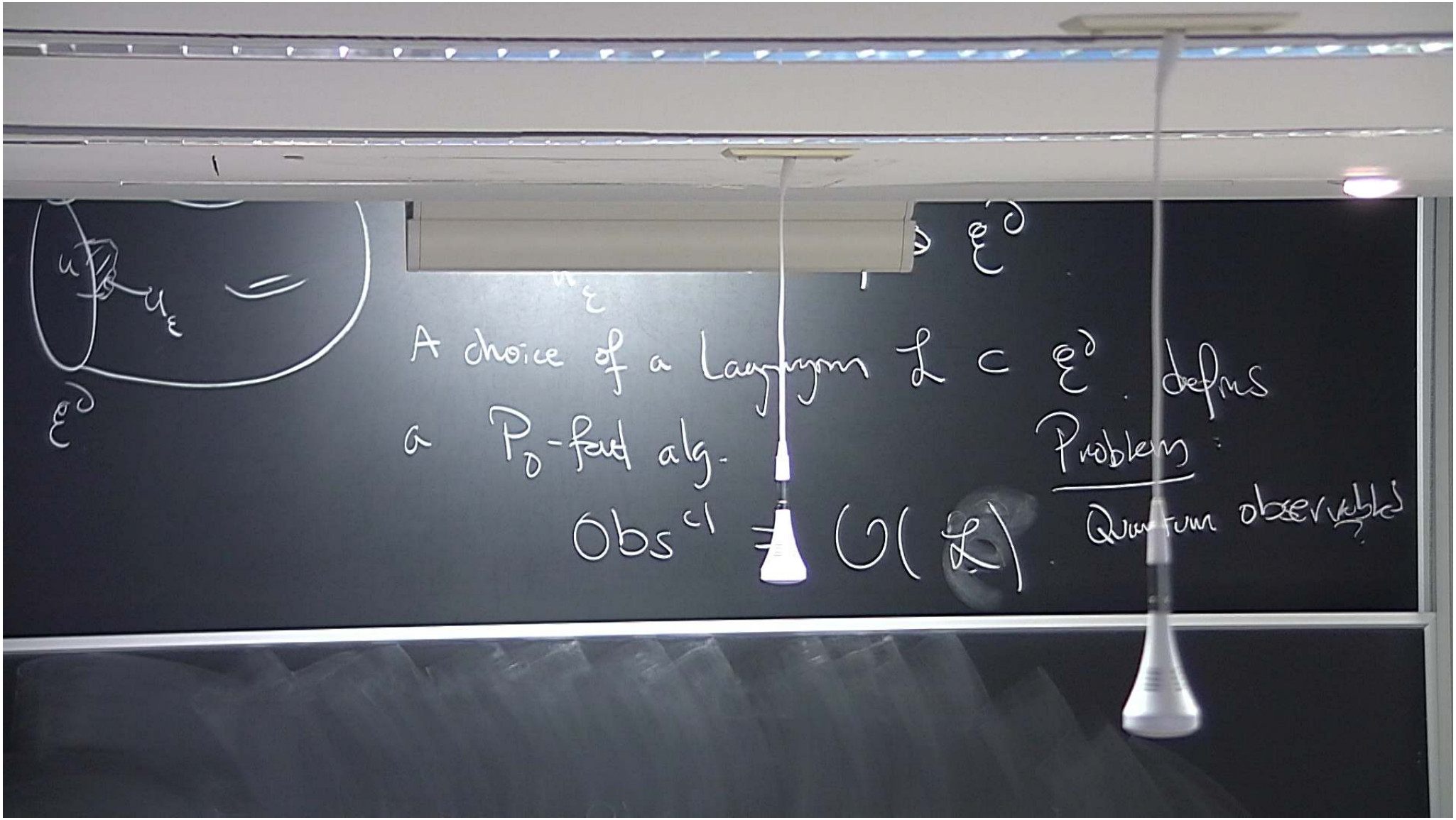
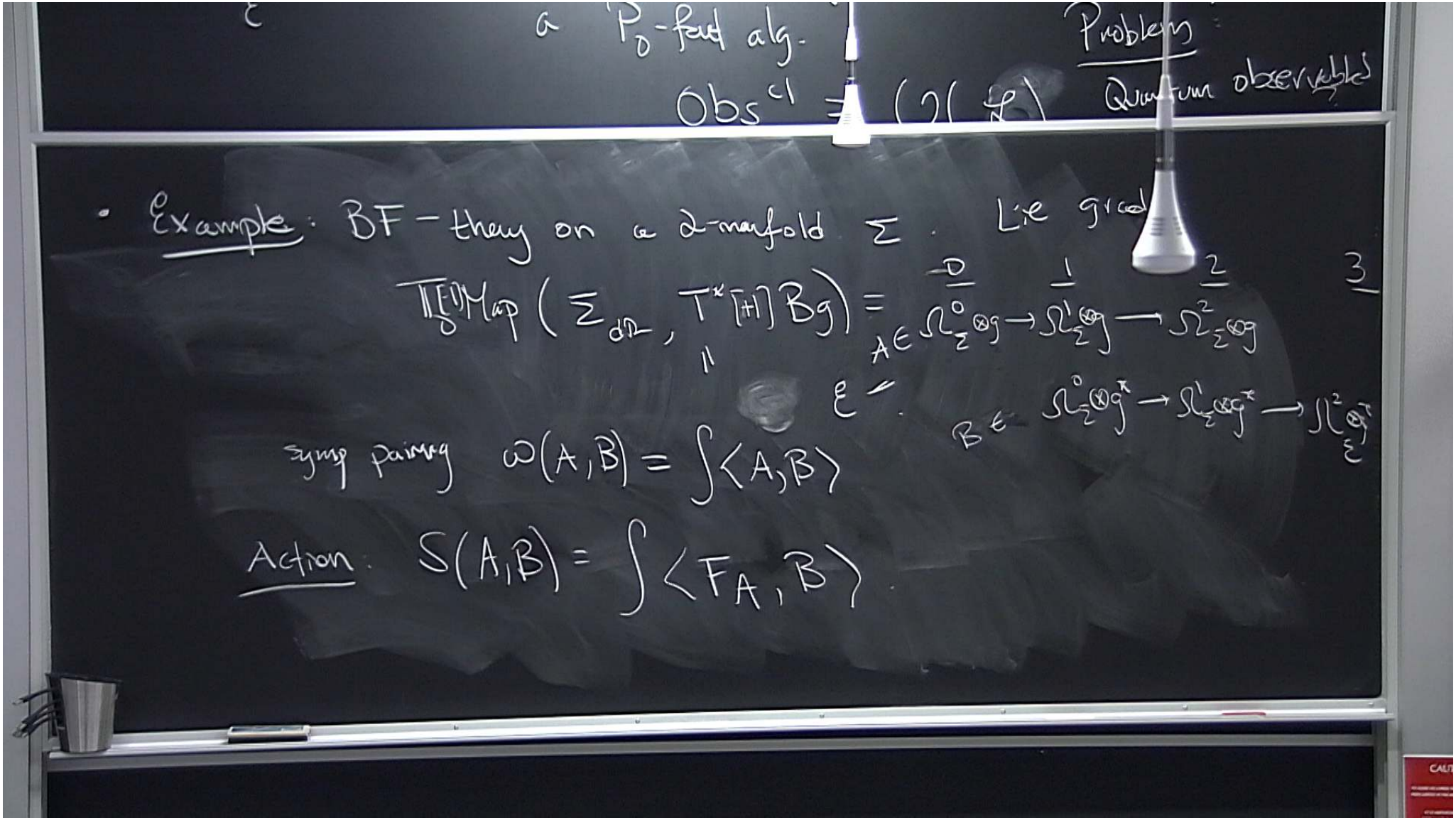
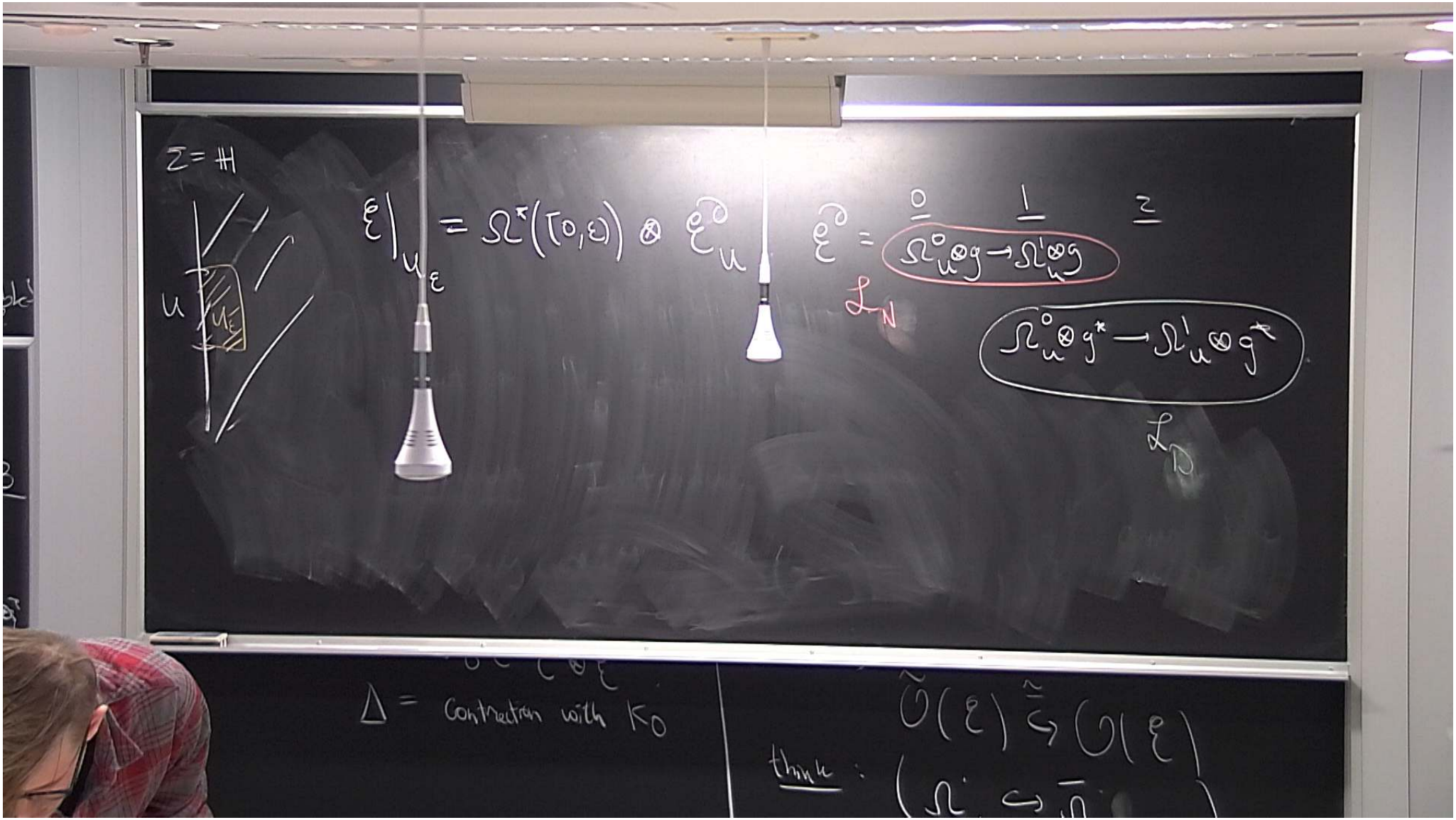
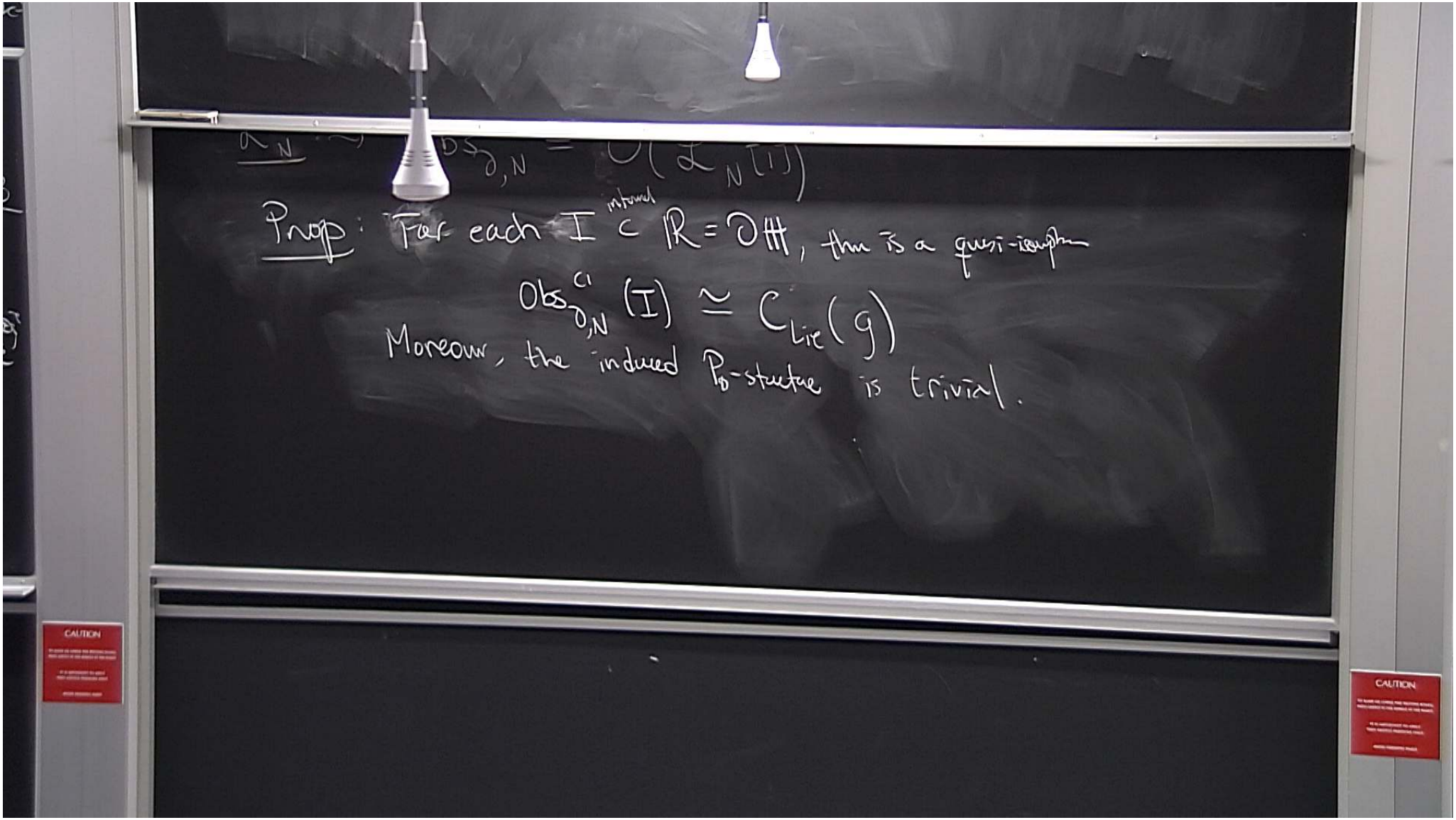
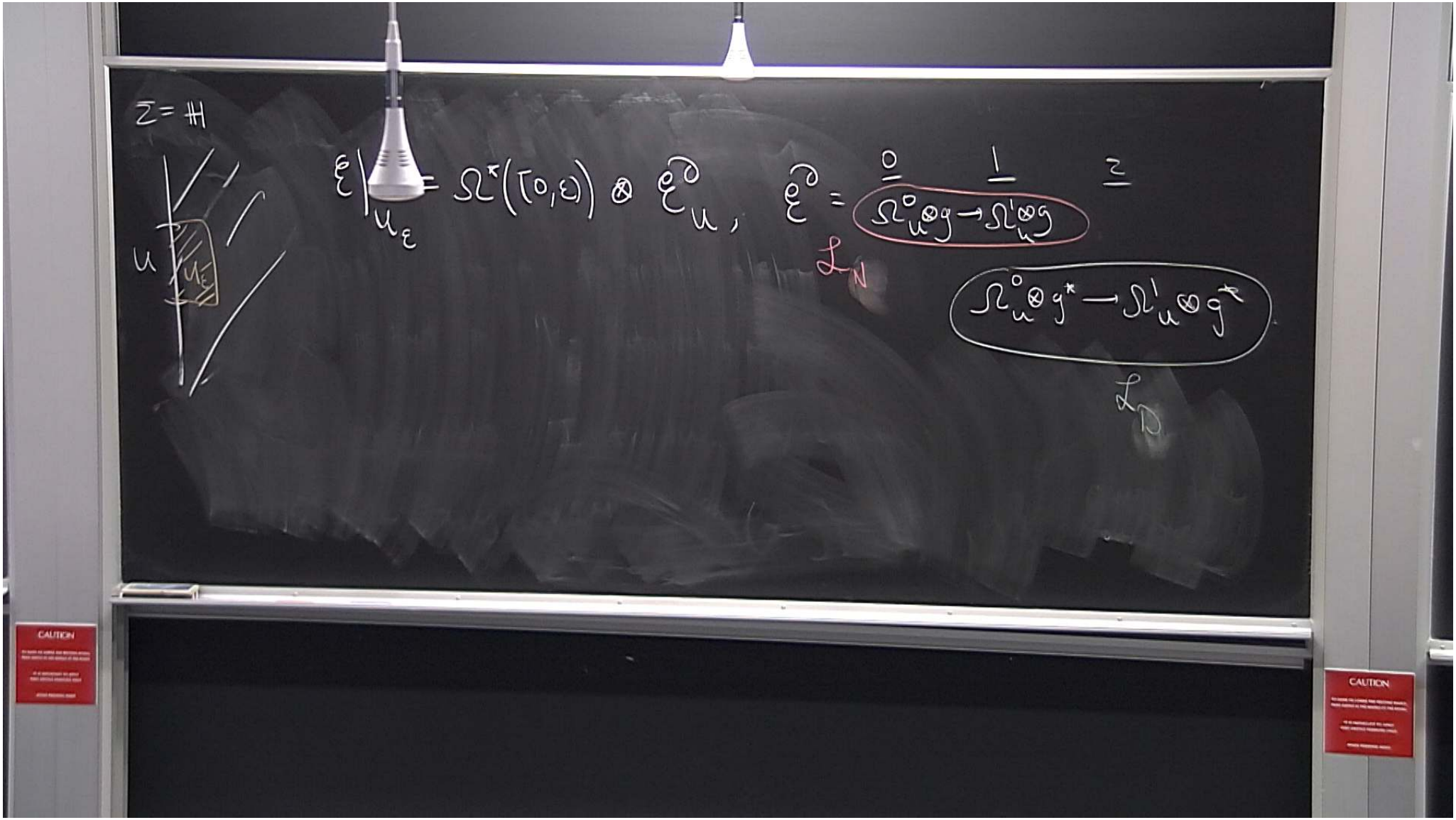
Satisfying

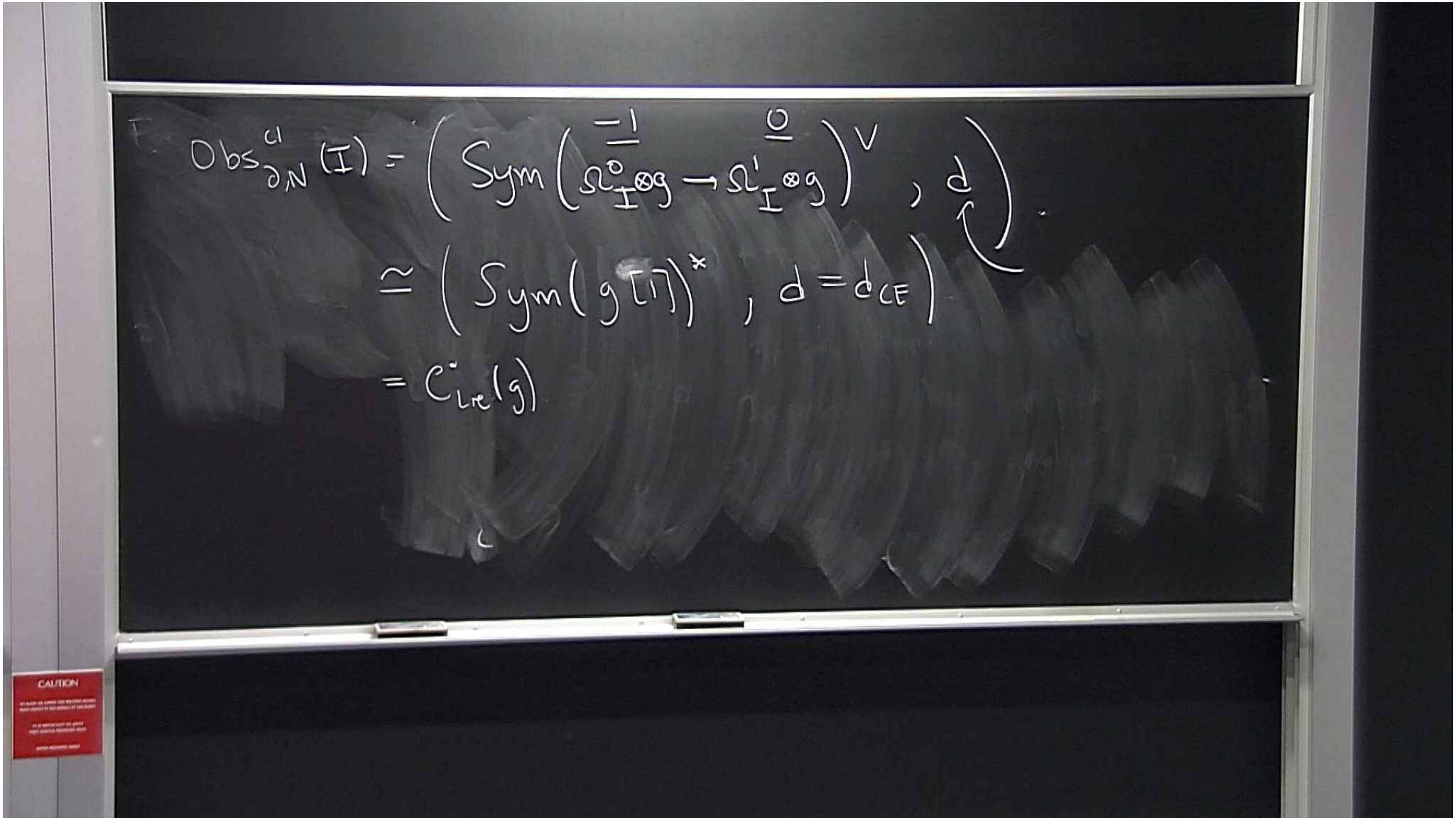

A choice of a Lagrangian $\mathcal{L} \subset \mathcal{E}^0$ defines
a P_0-fact alg.
$Obs^{cl} \cong \mathcal{O}(\mathcal{L})$
Problem:
Quantum observables


a P_0-fact alg.
Problems
Quantum observables
$Obs^{cl} \cong H^0(\mathcal{E})$
Example: BF-theory on a 2-manifold Σ. Lie grd
$T^*[1]B\mathfrak{g}$
$\mathcal{E}$
sympl pairing $\omega(A,B) = \int \langle A, B \rangle$
Action: $S(A,B) = \int \langle F_A, B \rangle$


Z = H
$\mathcal{L}_N$
$\mathcal{L}_D$
Δ = Contraction with K0
think:


$\mathcal{L}_N$
$Obs_{\partial,N} = \mathcal{O}(\mathcal{L}_N[1])$
Prop: For each $I \overset{\text{intvl}}{\subset} \mathbb{R} = \partial\mathbb{H}$, there is a quasi-isomorphism
$Obs^{cl}_{\partial,N}(I) \simeq C_{Lie}(\mathfrak{g})$
Moreover, the induced P_0-structure is trivial.
CAUTION
CAUTION


$\mathcal{Z} = \#$
$\mathcal{E}|_{U_\varepsilon} = \Omega^*((0,\varepsilon)) \otimes \mathcal{E}^{\mathcal{O}}_U$, $\mathcal{E}^{\mathcal{O}} =$
$\Omega^0_U \otimes g \to \Omega^1_U \otimes g$
$\mathcal{L}_N$
$\Omega^0_U \otimes g^* \to \Omega^1_U \otimes g^*$
$\mathcal{L}_D$
U
U_ε
CAUTION
CAUTION


$\mathrm{Obs}^{cl}_{\partial,N}(I) = \left(\mathrm{Sym}\left(\Omega^{0}_{I}\otimes g \xrightarrow{-1} \Omega^{1}_{I}\otimes g \right)^{\vee} , \overset{0}{d} \right)$.
$\simeq \left(\mathrm{Sym}(g[1])^{*} , d = d_{CE} \right)$
$= C^{\cdot}_{Lie}(g)$

$Obs_{D,N}(1) = C_{lc}(g)[[\hbar]]$.

- $\mathcal{L}_D = \Omega^0 \otimes g^{\overset{1}{\alpha}} \to \Omega^1 \otimes g^{\overset{2}{\alpha}}$

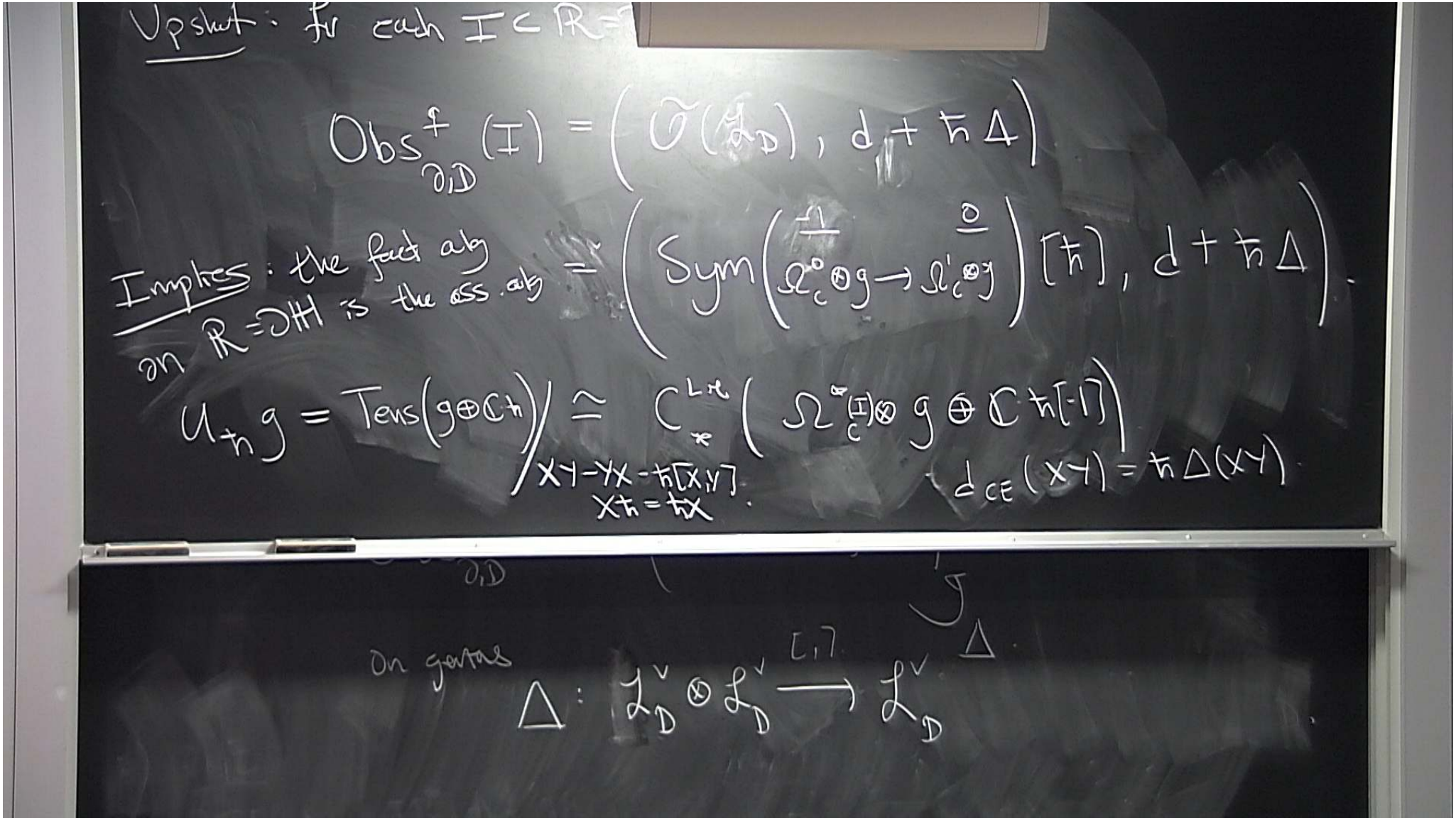
Trivial Lie structure.

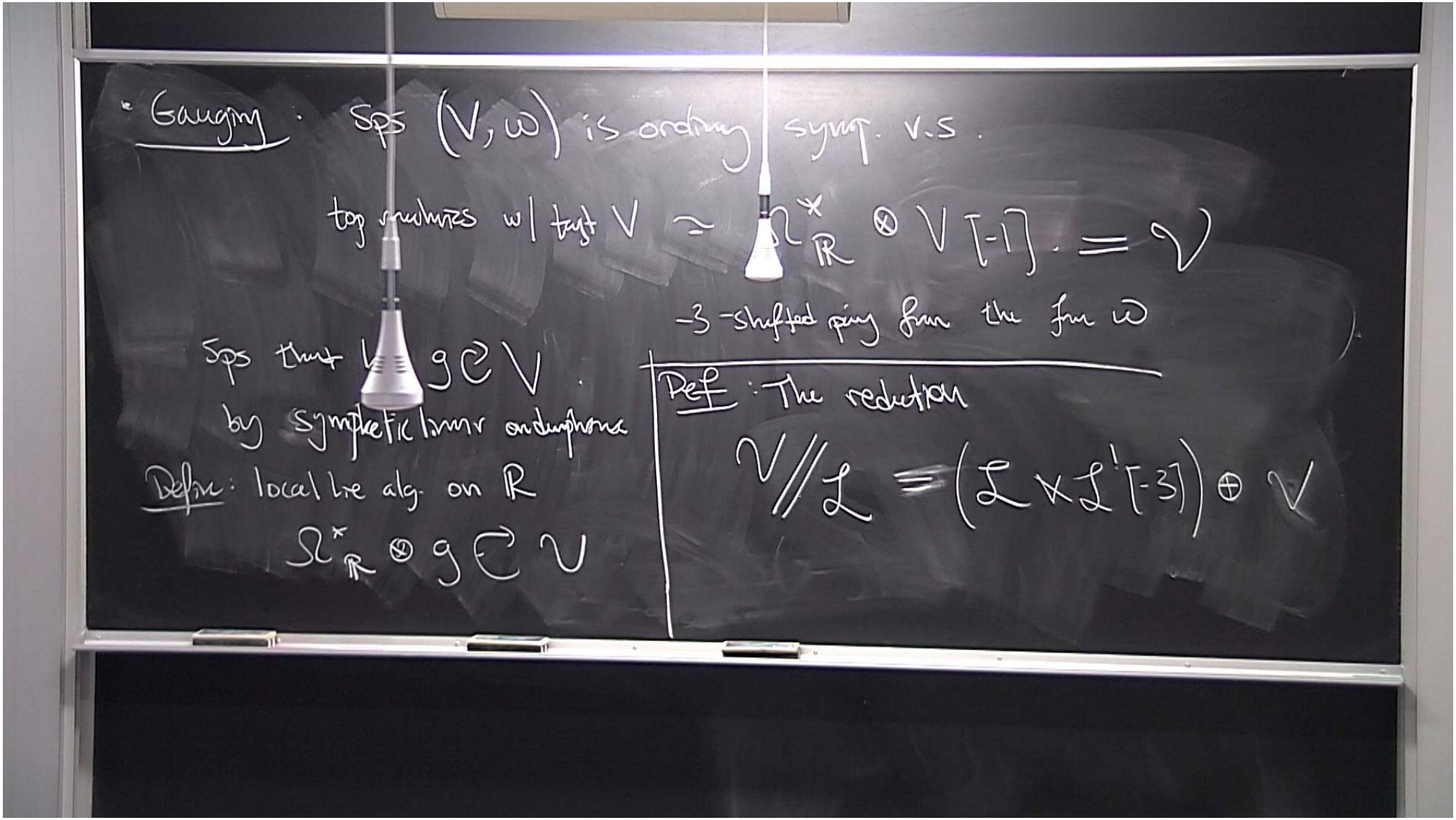
- P_0 structure is identified w/ $\wedge \otimes [\,,\,]_g$

$$\Gamma(\mathcal{L}_D ; \mathrm{Sym}^2 \mathcal{L}_D)$$

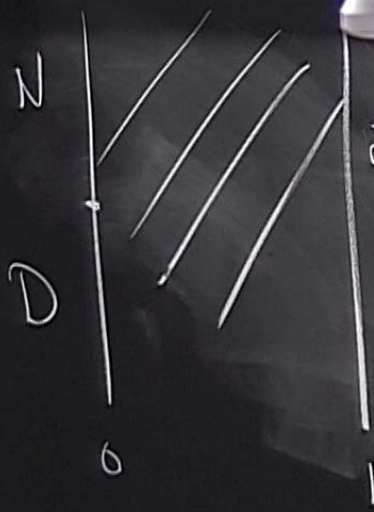
linear P_0-structure

$$\mathcal{L}_D \longrightarrow \mathrm{Sym}^2 \mathcal{L}_D \Leftrightarrow \mathcal{L}_D \otimes \mathcal{L}_N \xrightarrow{\wedge \otimes [\,,\,]} \mathcal{L}_D .$$


Upshot: for each I ⊂ R
$Obs^{\hbar}_{\partial,D}(I) = \left(\mathcal{O}(\mathcal{L}_D), d_{\mathcal{L}} + \hbar\Delta\right)$
Implies: the fact alg on $R = \partial H$ is the ass. alg
$\simeq \left(Sym\left(\mathcal{L}^{-1}_c \oplus g \to \mathcal{L}^{0}_c \oplus g\right)[\hbar], d + \hbar\Delta\right)$.
$U_\hbar g = Tens(g \oplus \mathbb{C}\hbar)/(xy - yx - \hbar[x,y],\ x\hbar = \hbar x) \simeq C^{Lie}_*\left(\Omega^{\bullet}_c(I) \otimes g \oplus \mathbb{C}\hbar[-1]\right)$
$d_{CE}(xy) = \hbar\Delta(x,y)$
on germs
$\Delta : \mathcal{L}^{\vee}_D \otimes \mathcal{L}^{\vee}_D \xrightarrow{[,]} \mathcal{L}^{\vee}_D \xrightarrow{\Delta} \mathbb{C}$


Gauging: Sps (V,ω) is ordinary symp. v.s.
tog resolves w/ tgt $V \simeq \Omega^*_{\mathbb{R}} \otimes V[-1] =: \mathcal{V}$
-3-shifted pairing from the form ω
Sps that $\mathfrak{g} \subset V$.
by symplectic linear endomorphisms
Defn: local Lie alg. on $\mathbb{R}$
$\Omega^*_{\mathbb{R}} \otimes \mathfrak{g} \subset \mathcal{V}$
Def: The reduction
$\mathcal{V}/\!/\mathcal{L} = (\mathcal{L} \times \mathcal{L}^1[-3]) \oplus \mathcal{V}$

$\mathcal{L} = \Omega^*_{\mathbb{R}} \otimes \mathfrak{g} \subset V$



Consider bulk thy

$\mathcal{L} \times V$

$\Omega^*_{[0,1]} \otimes \left(\mathcal{L} \times \mathcal{L}^![-2] \right) \simeq$ BF thy.

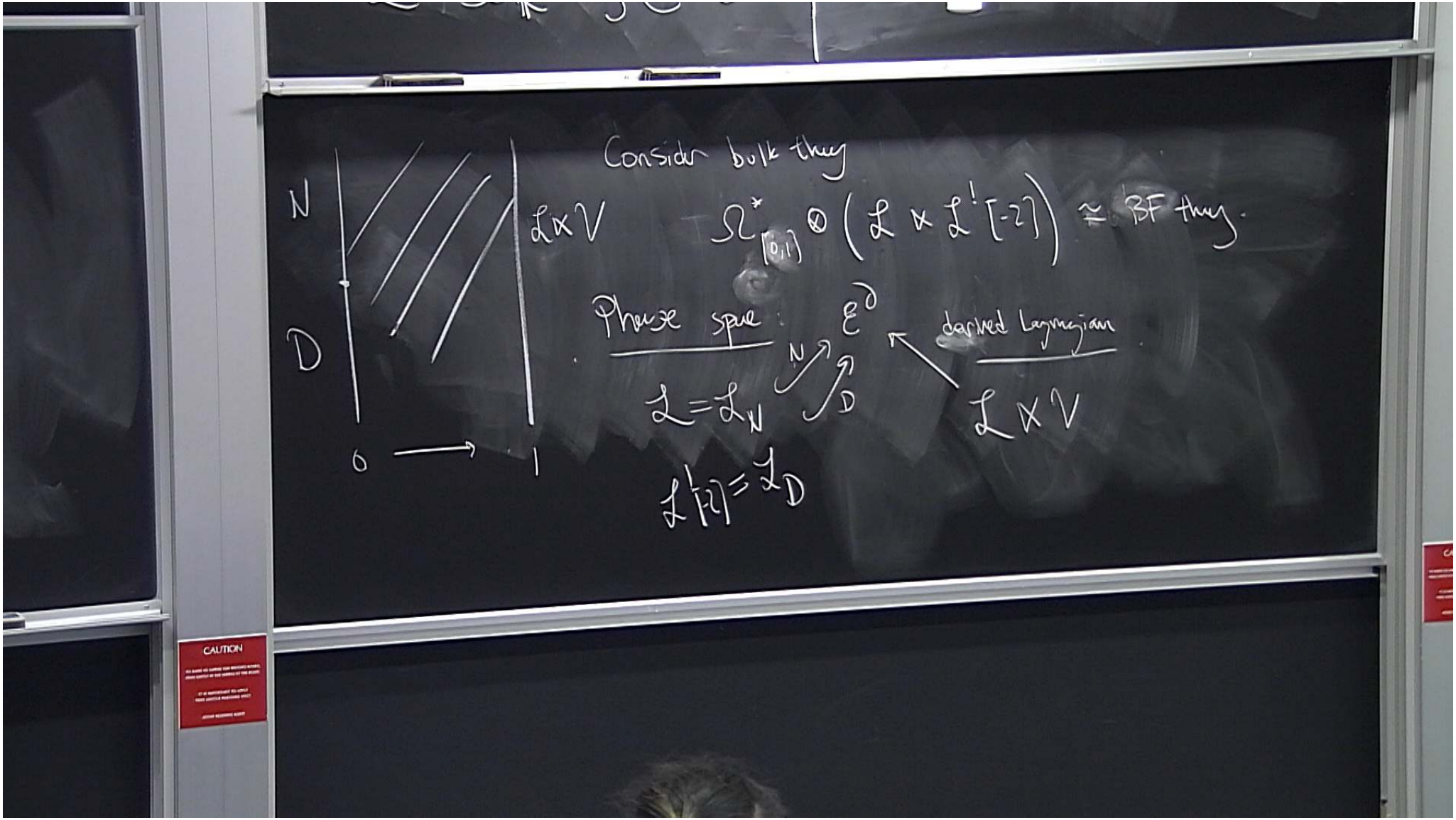
Phase space:

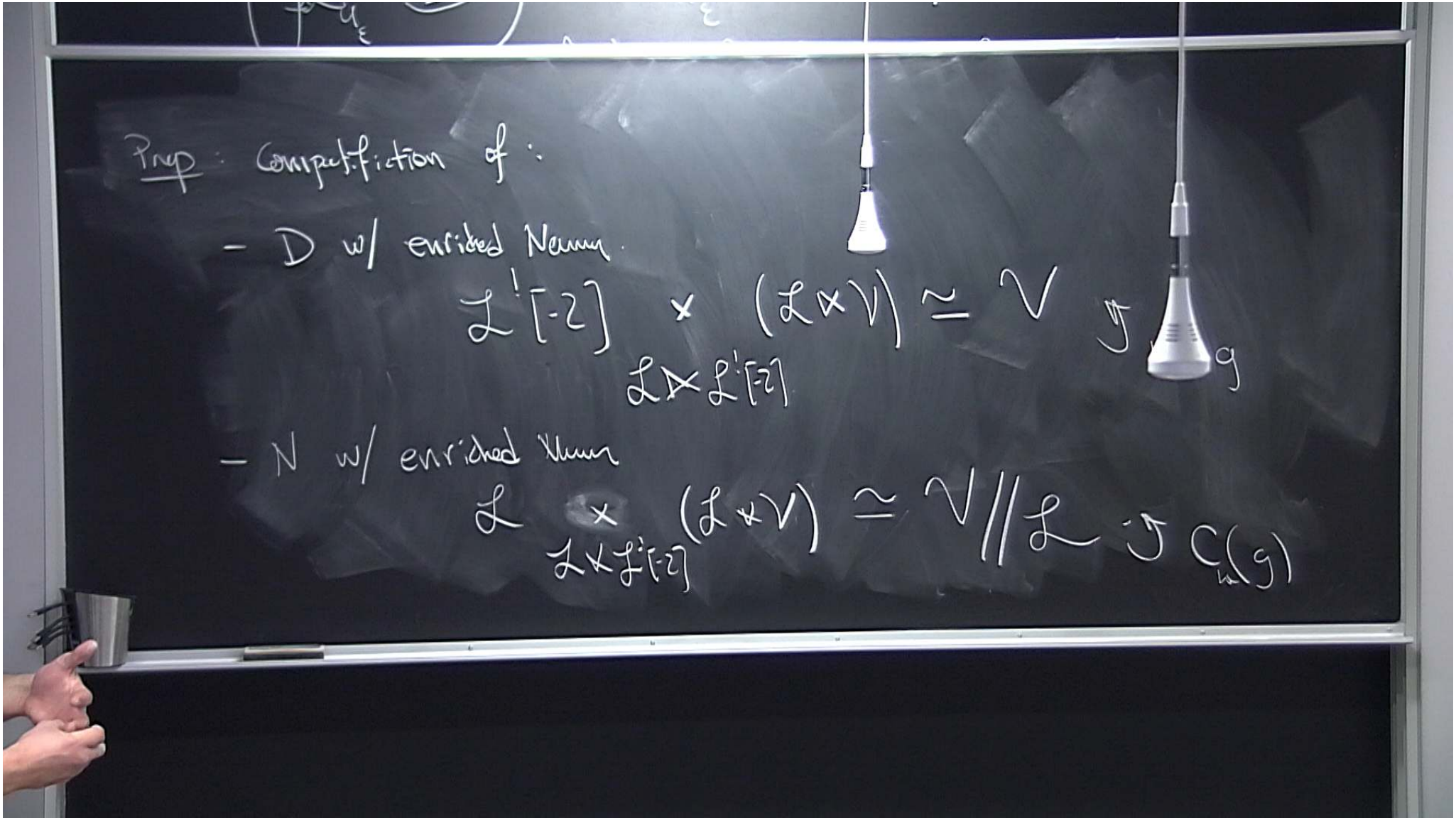
$\mathcal{L} = \mathcal{L}_N$ (N) $\mathcal{L}^\partial$ (D)

$\mathcal{L}^![-1] = \mathcal{L}_D$

derived Lagrangian

$\mathcal{L} \times V$


N
D
0
1
Consider bulk thy
$\mathcal{L} \times V$
$\Omega^*_{[0,1]} \otimes (\mathcal{L} \times \mathcal{L}'[-2]) \approx$ BF thy.
Phase space:
$\mathcal{L} = \mathcal{L}_N$
$\mathcal{L}'[-1] = \mathcal{L}_D$
$\mathcal{E}^0$
N
D
darboux Lagrangian
$\mathcal{L} \times V$


Prop: Compactification of:
- D w/ enriched Neumann
$\mathcal{L}^![-2] \underset{\mathcal{L}\times\mathcal{L}^![-2]}{\times} (\mathcal{L}\times\mathcal{V}) \simeq \mathcal{V}$
- N w/ enriched Klumm
$\mathcal{L} \underset{\mathcal{L}\times\mathcal{L}^![-2]}{\times} (\mathcal{L}\times\mathcal{V}) \simeq \mathcal{V}//\mathcal{L}$