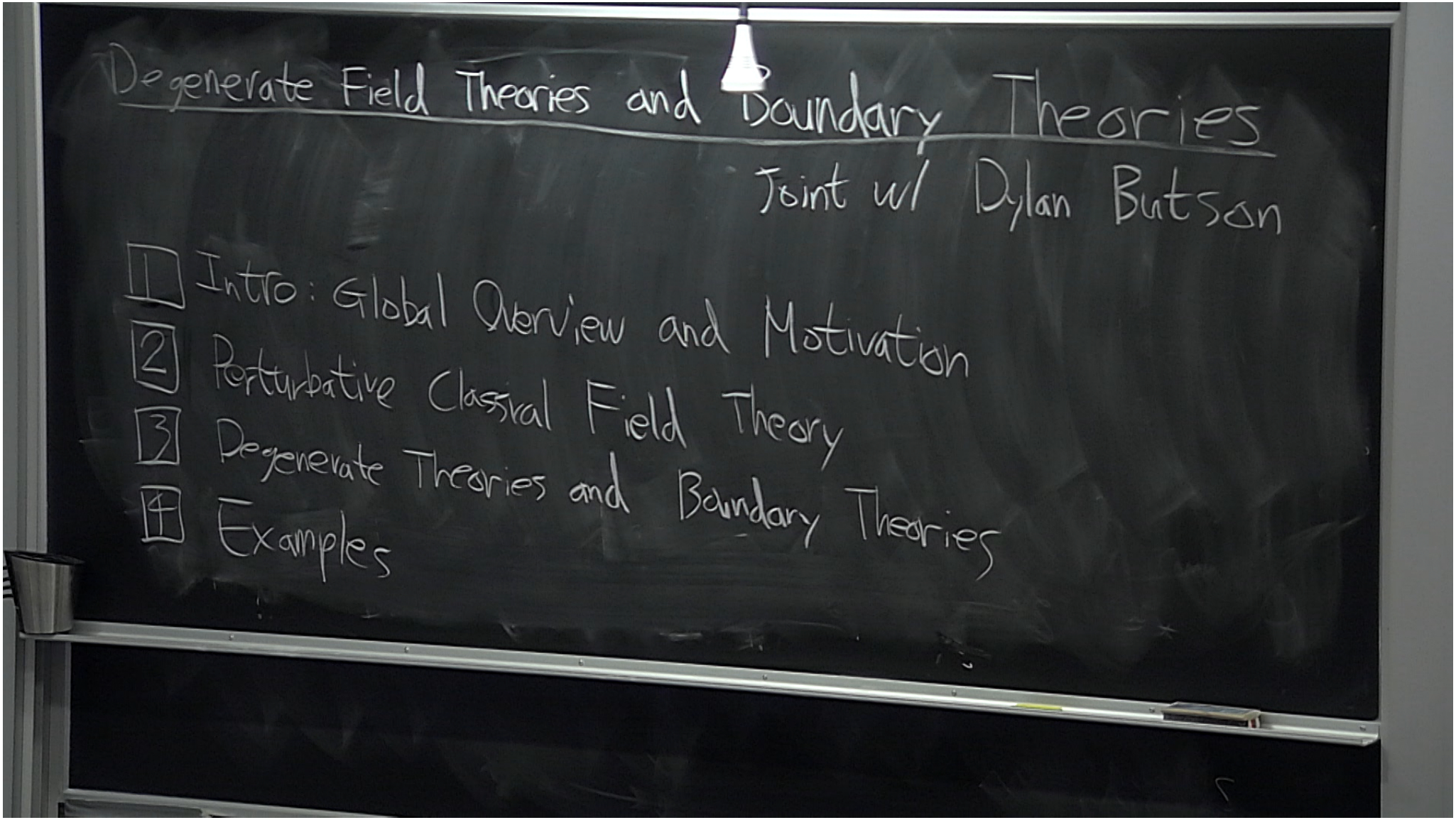
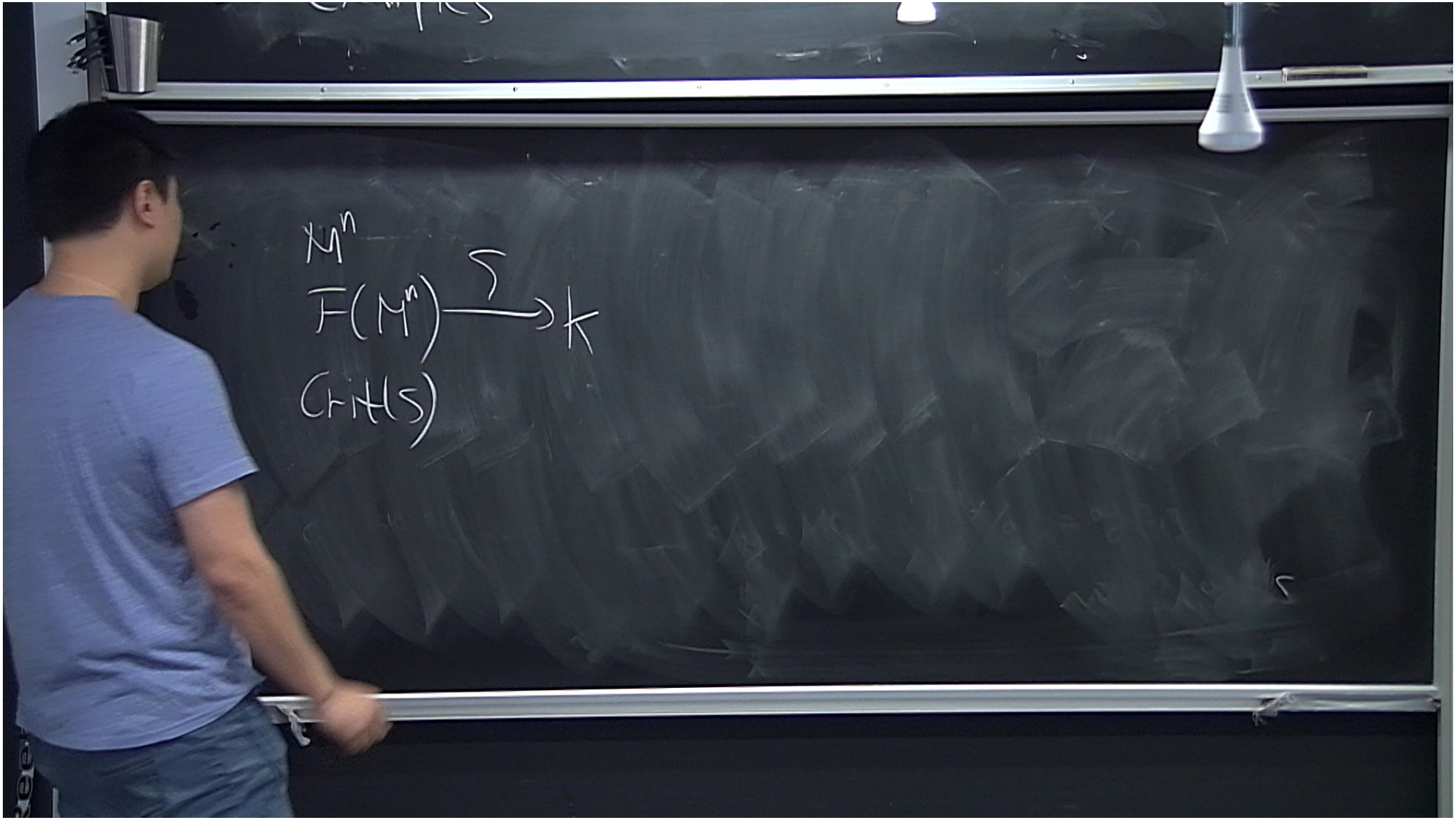
Title: Degenerate Field Theories and Boundary Theories

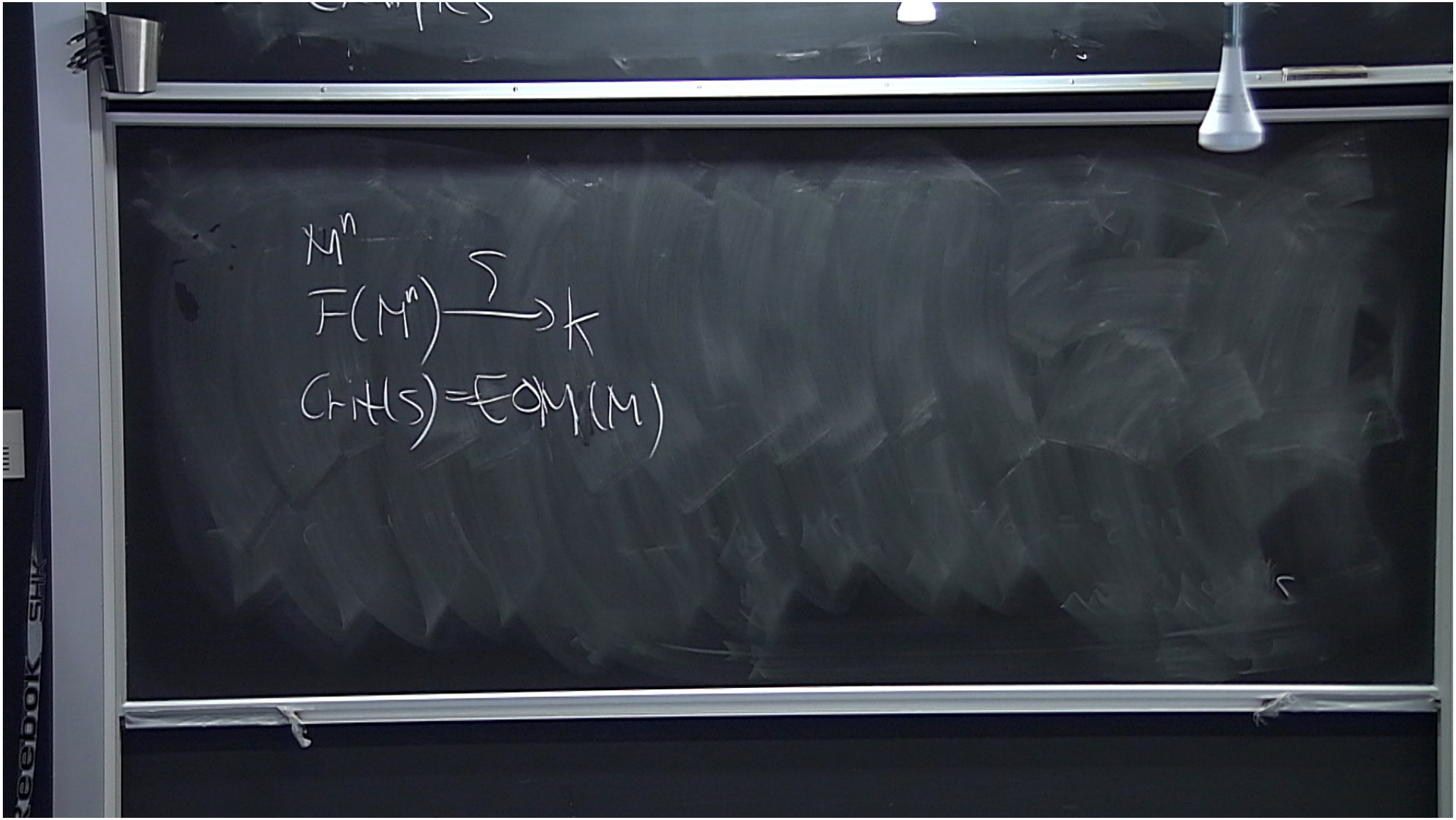
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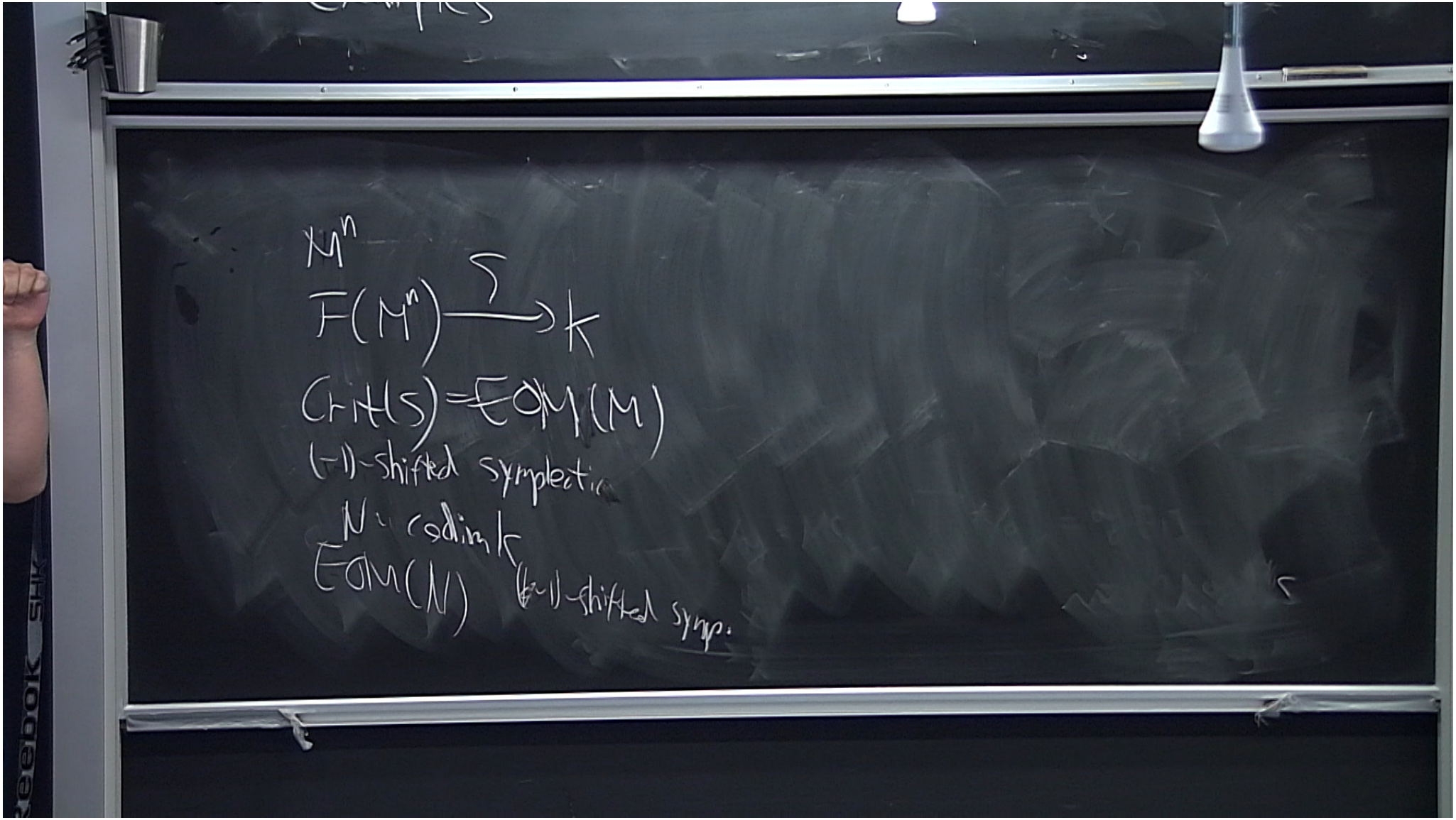
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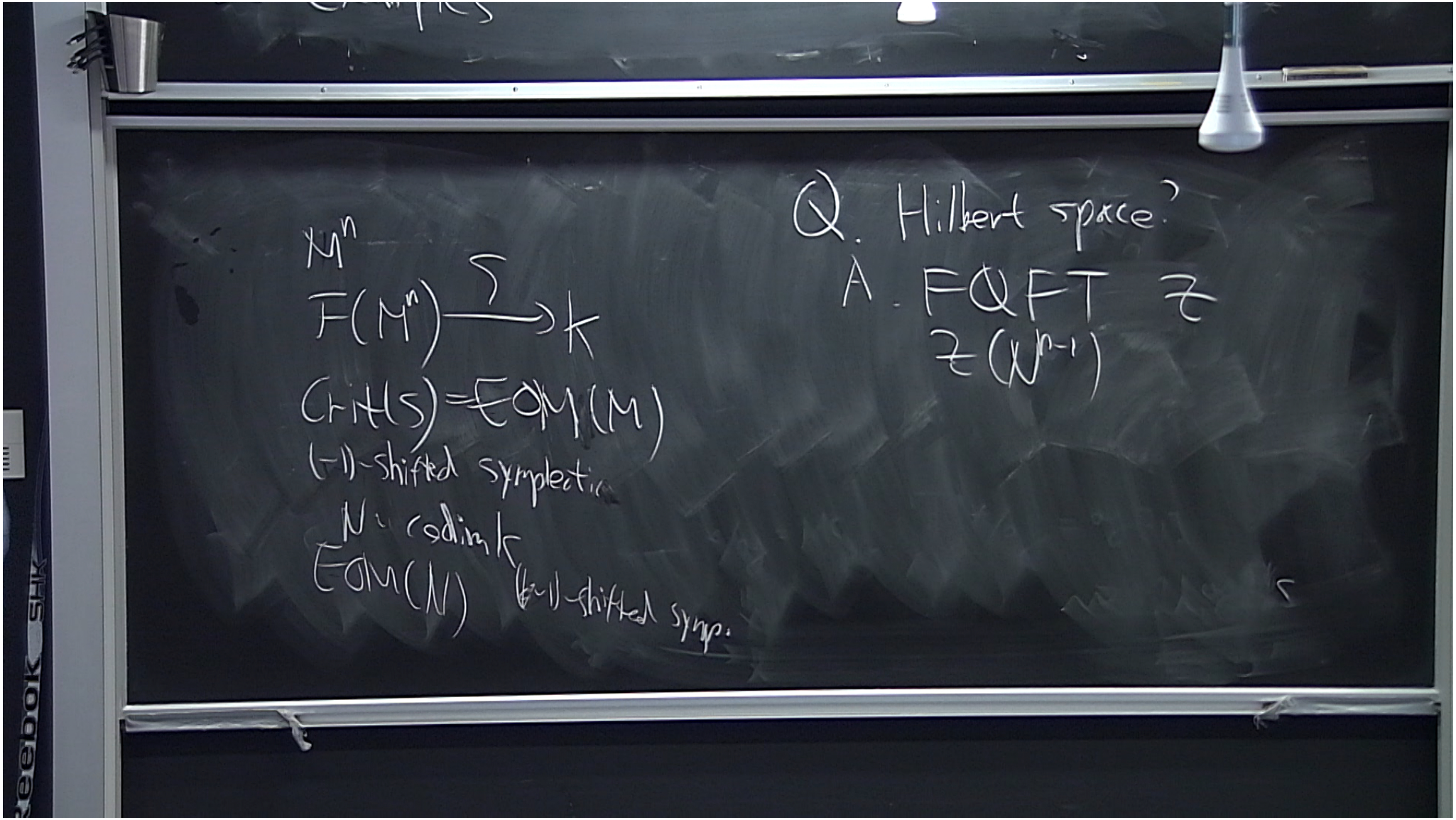
Abstract:


Degenerate Field Theories and Boundary Theories
Joint w/ Dylan Butson
1 Intro: Global Overview and Motivation
2 Perturbative Classical Field Theory
3 Degenerate Theories and Boundary Theories
4 Examples


M^n
$F(M^n) \xrightarrow{S} k$
$Crit(S)$

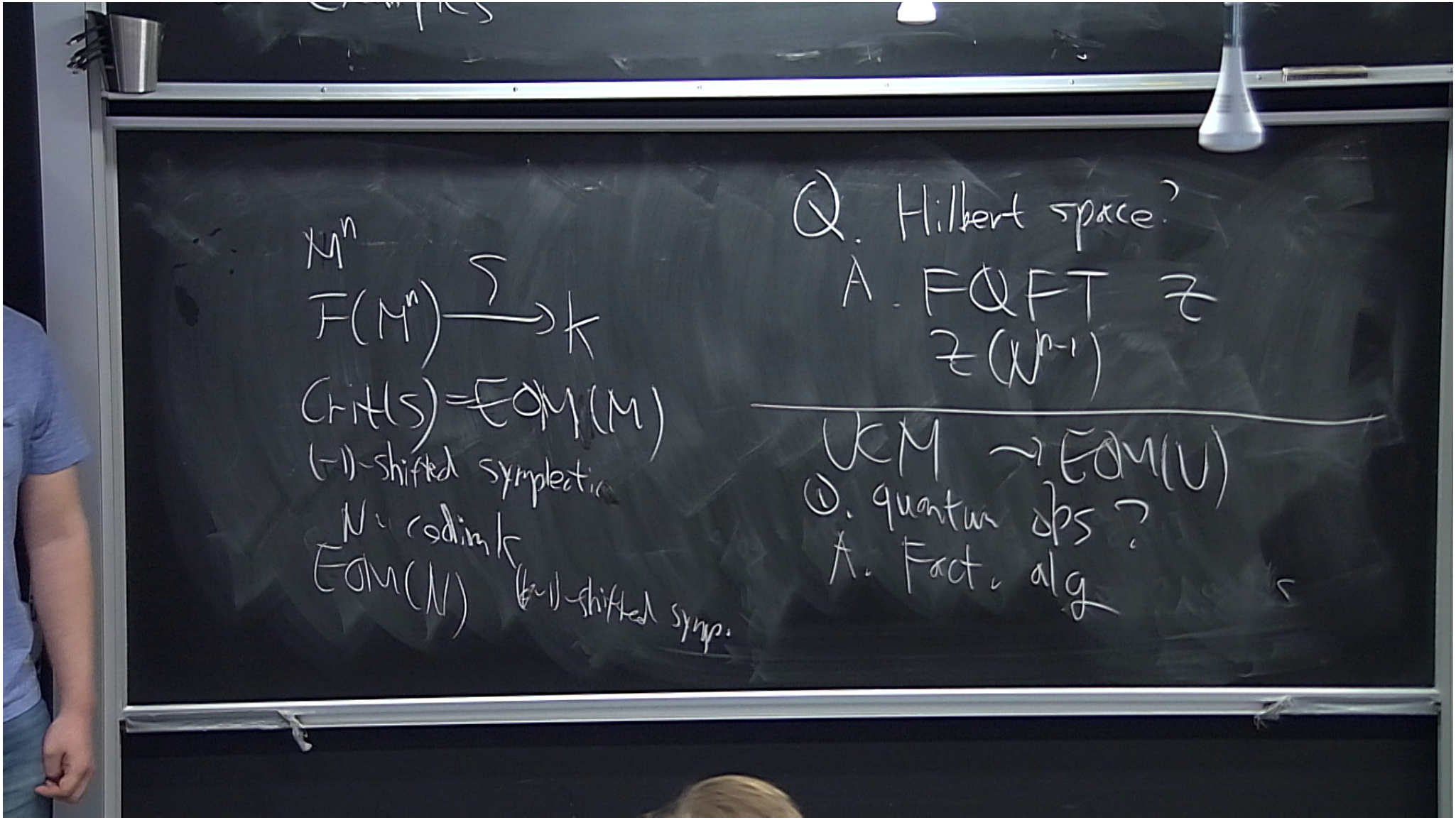

M^n
$F(M^n) \xrightarrow{S} k$
$C_r H(S) = \sum_M (M)$

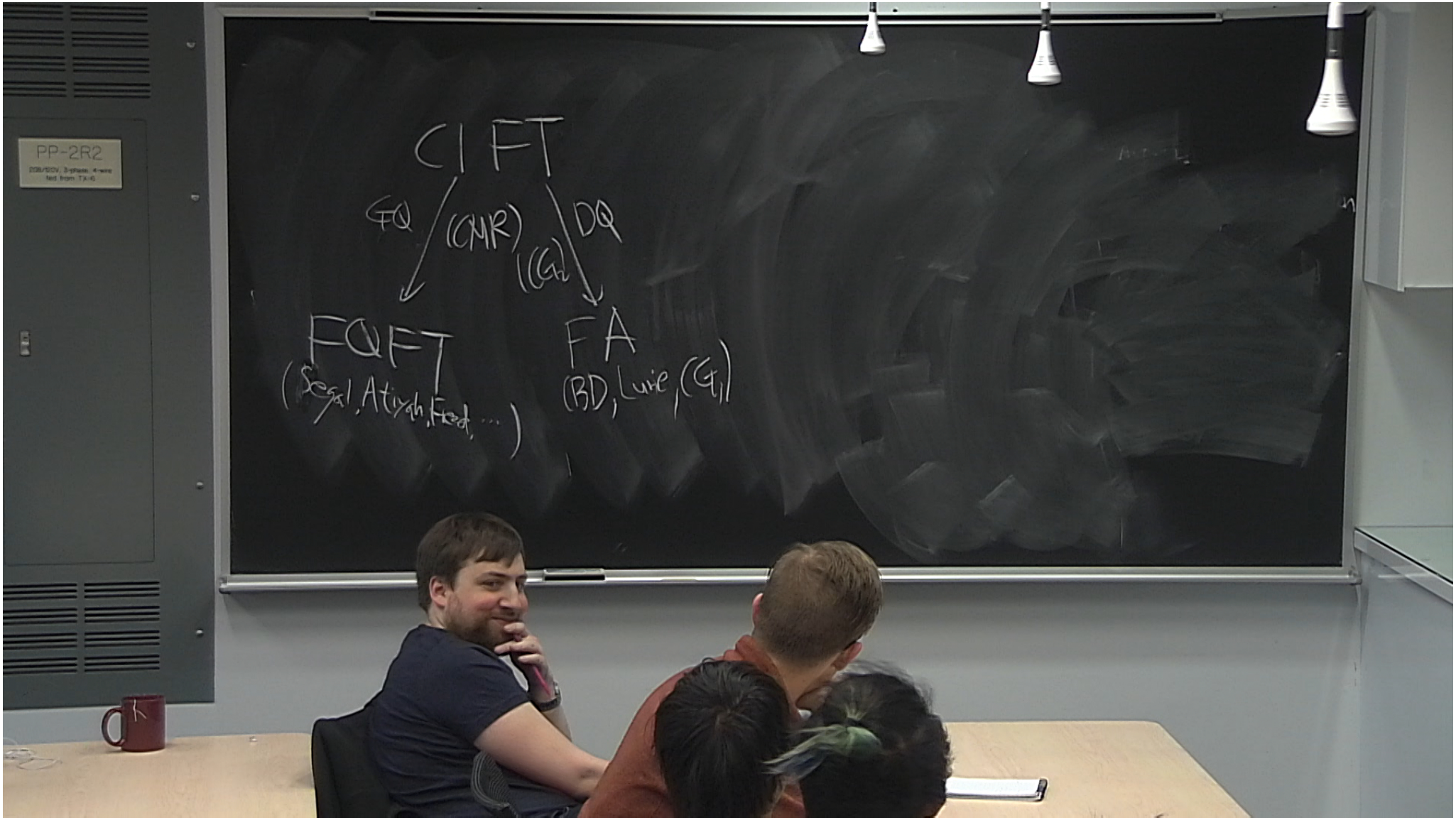

M^n
$F(M^n) \xrightarrow{S} k$
$Crit(S) = \mathbb{E}QM(M)$
(-1)-shifted symplectic
N : codim k
$\mathbb{E}QM(N)$ (-1)-shifted symp.


M^n
$F(M^n) \xrightarrow{\Sigma} k$
$Crit(S) = EOM(M)$
(-1)-shifted symplectic
N: codim k
$EOM(N)$ (-1)-shifted symp.
Q. Hilbert space?
A. FQFT Z
$Z(N^{n-1})$

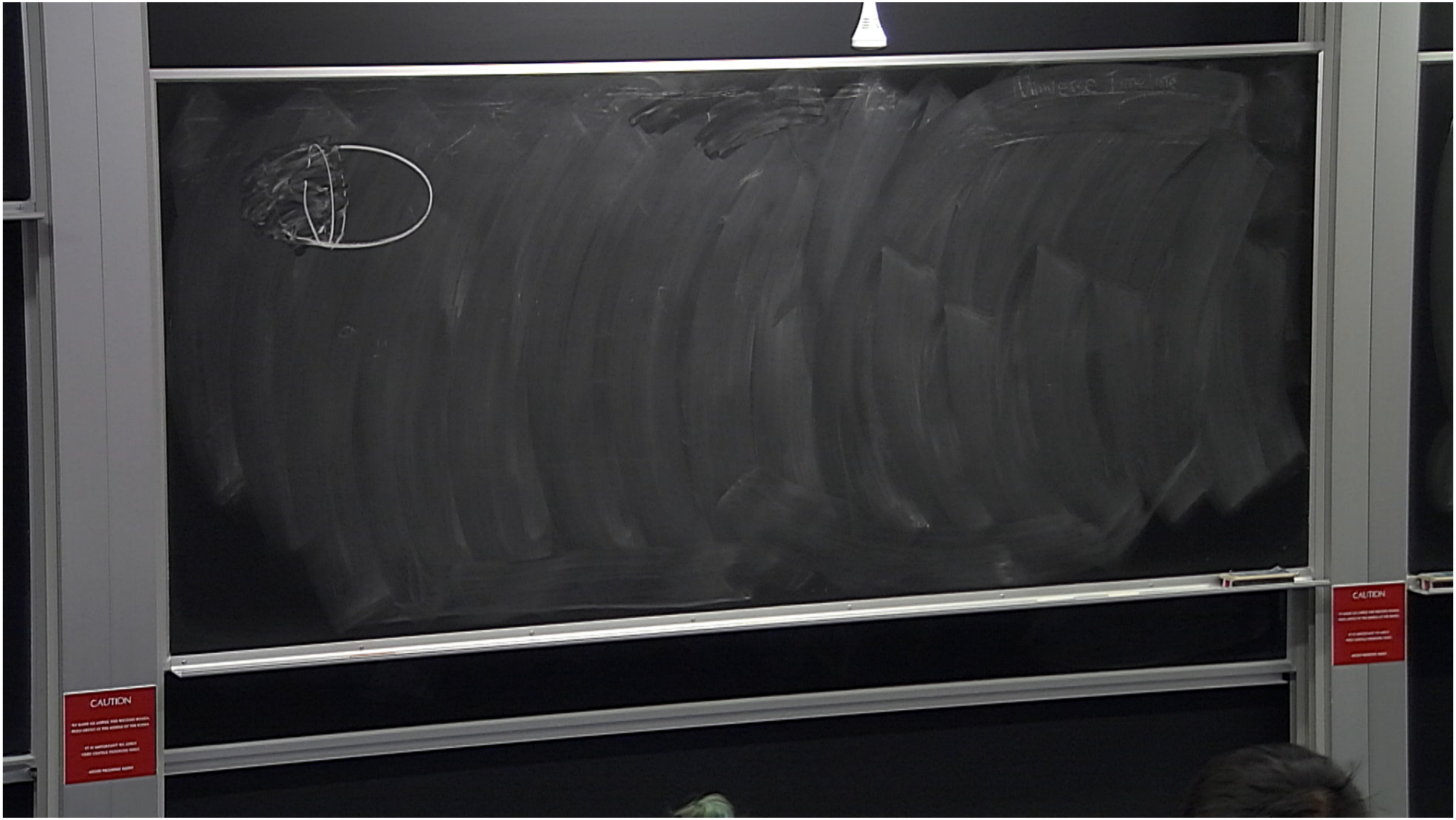

CIFT
FQ
FQFT
(Segal, Atiyah, Freed, ...)
PP-2R2


PP-2R2
CIFT
FQ
(CMR)
FQFT

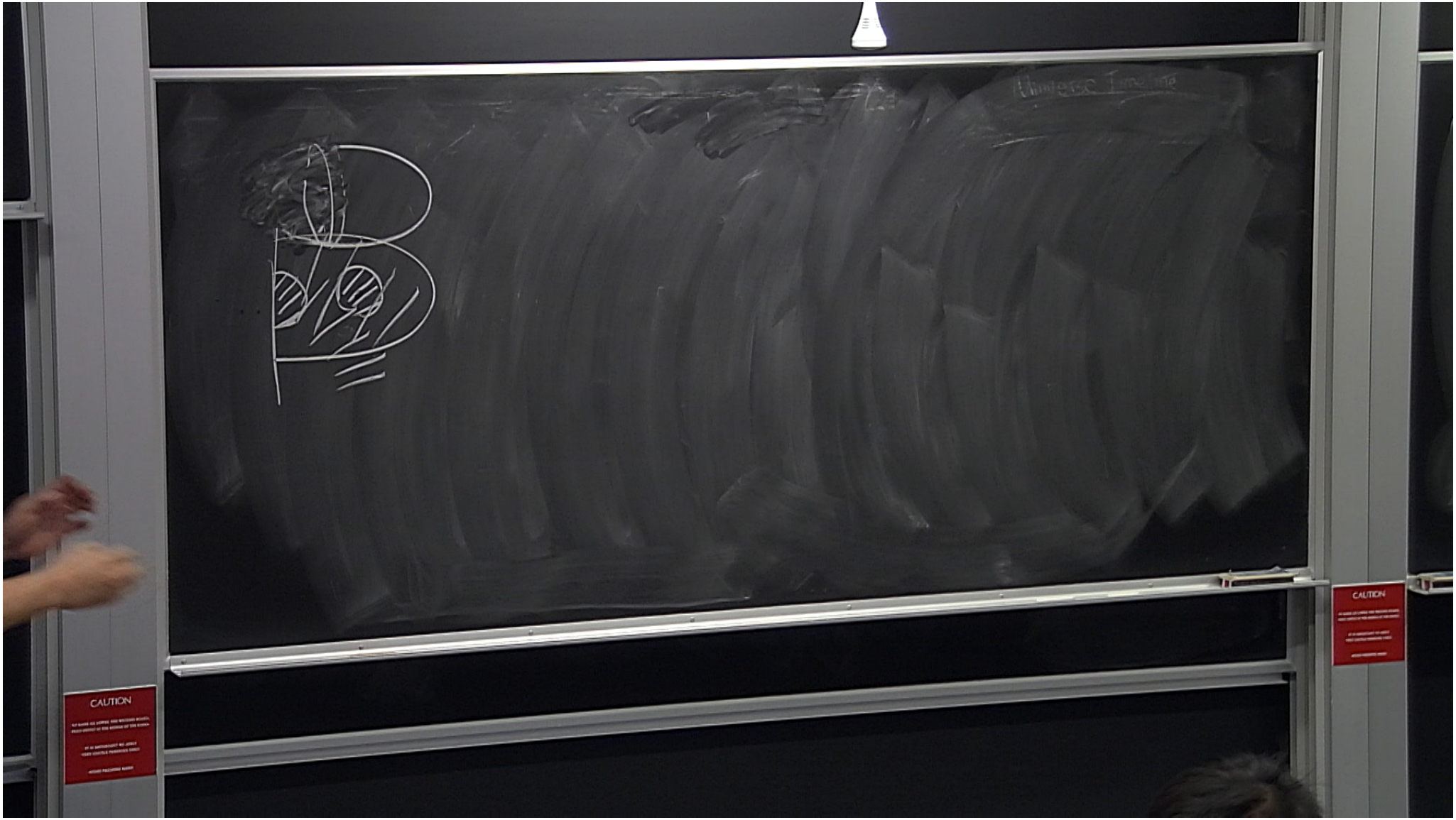

M^n
$F(M^n) \xrightarrow{S} k$
$Crit(S) = EOM(M)$
(-1)-shifted symplectic
N: codim k
$EOM(N)$ (-1)-shifted symp.
Q. Hilbert space?
A. FQFT Z
$Z(N^{n-1})$
$U \subset M \rightsquigarrow EOM(U)$
①. quantum obs?
A. Fact. alg

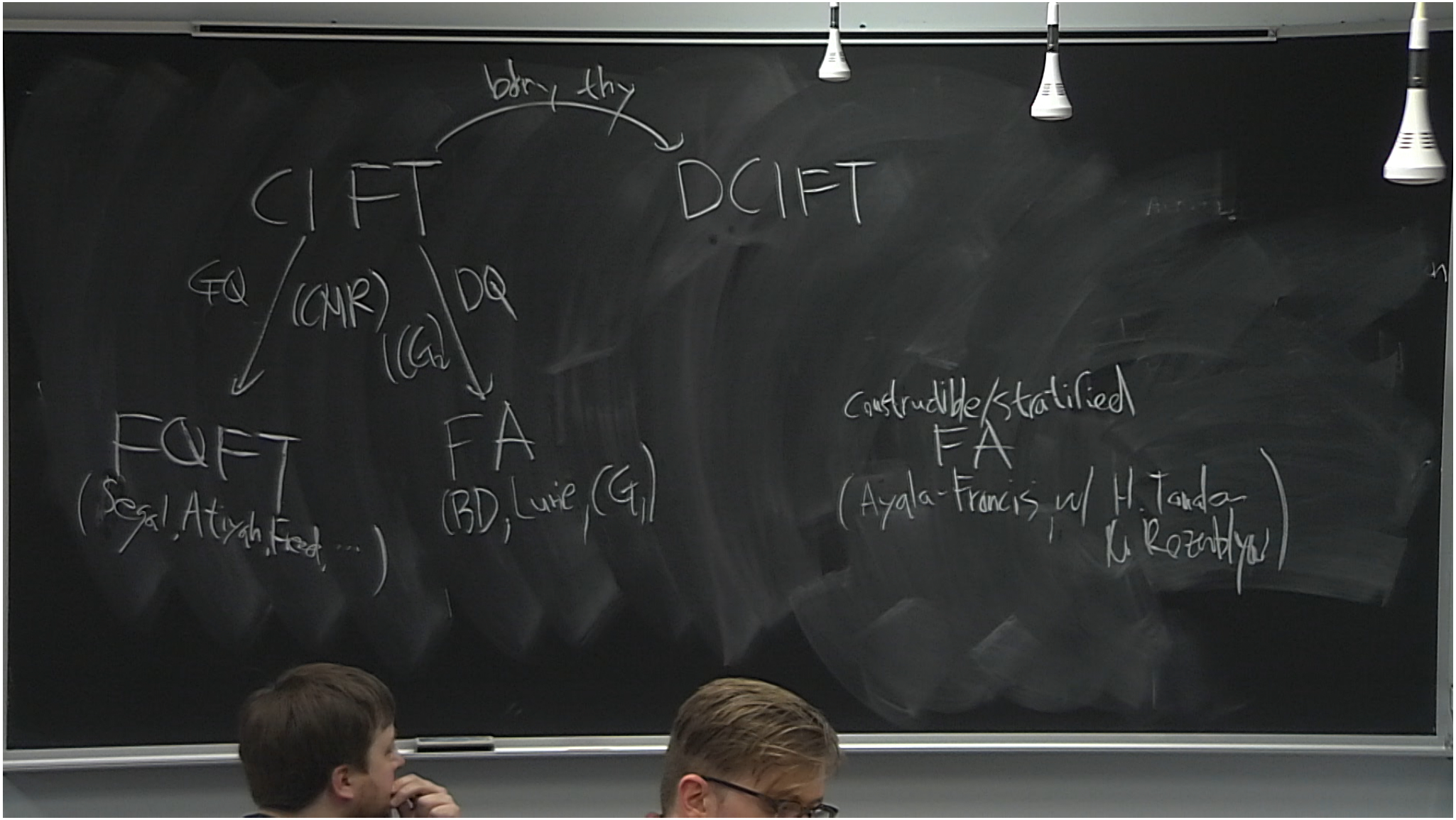

CIFT
FQ
(CMR)
(G2)
DQ
FQFT
FA
(BD, Lurie, (G1)

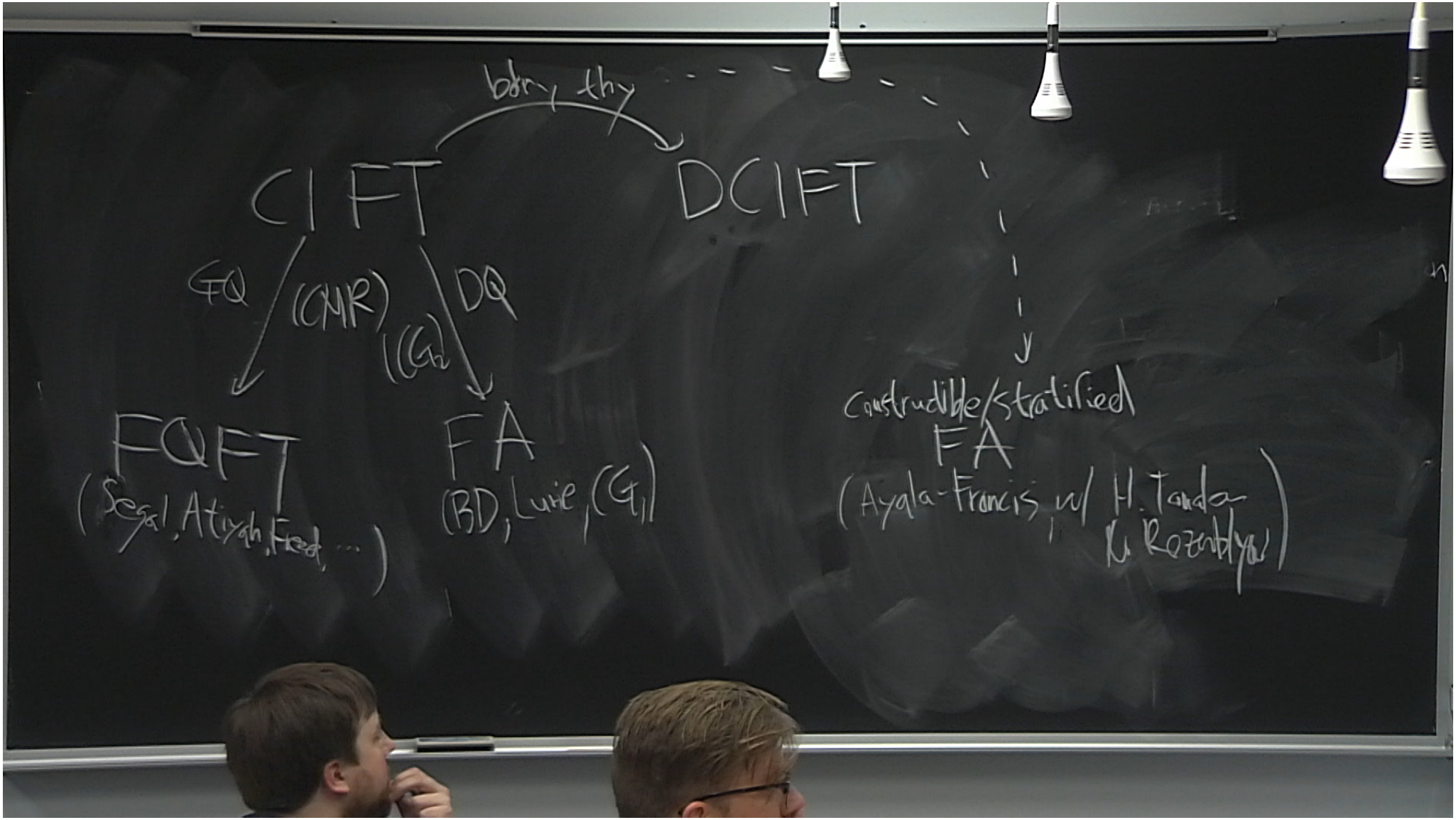

PP-2R2
CIFT
DCIFT
FQFT
FA

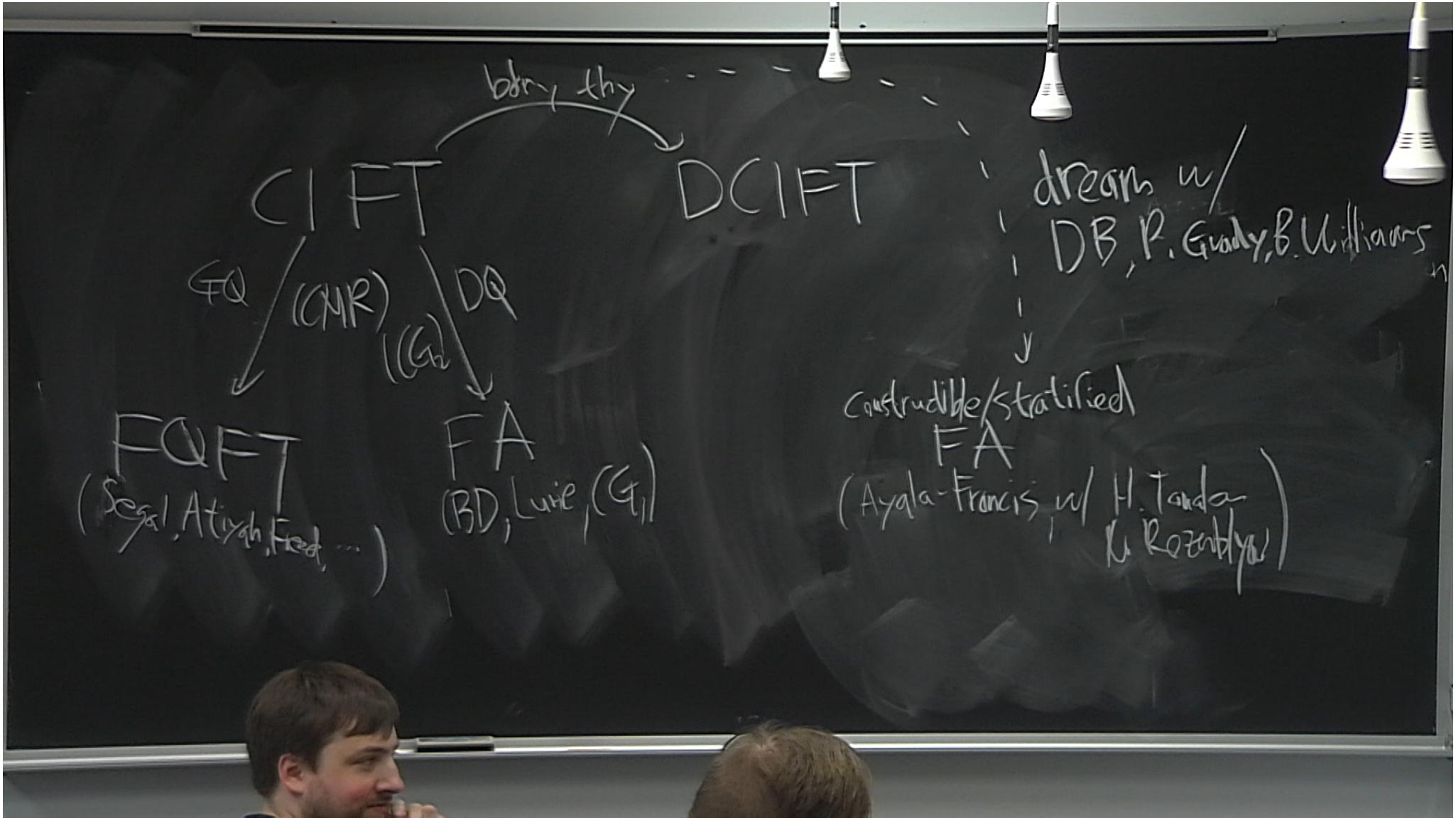

CAUTION
CAUTION


Universe Timeline
CAUTION
CAUTION

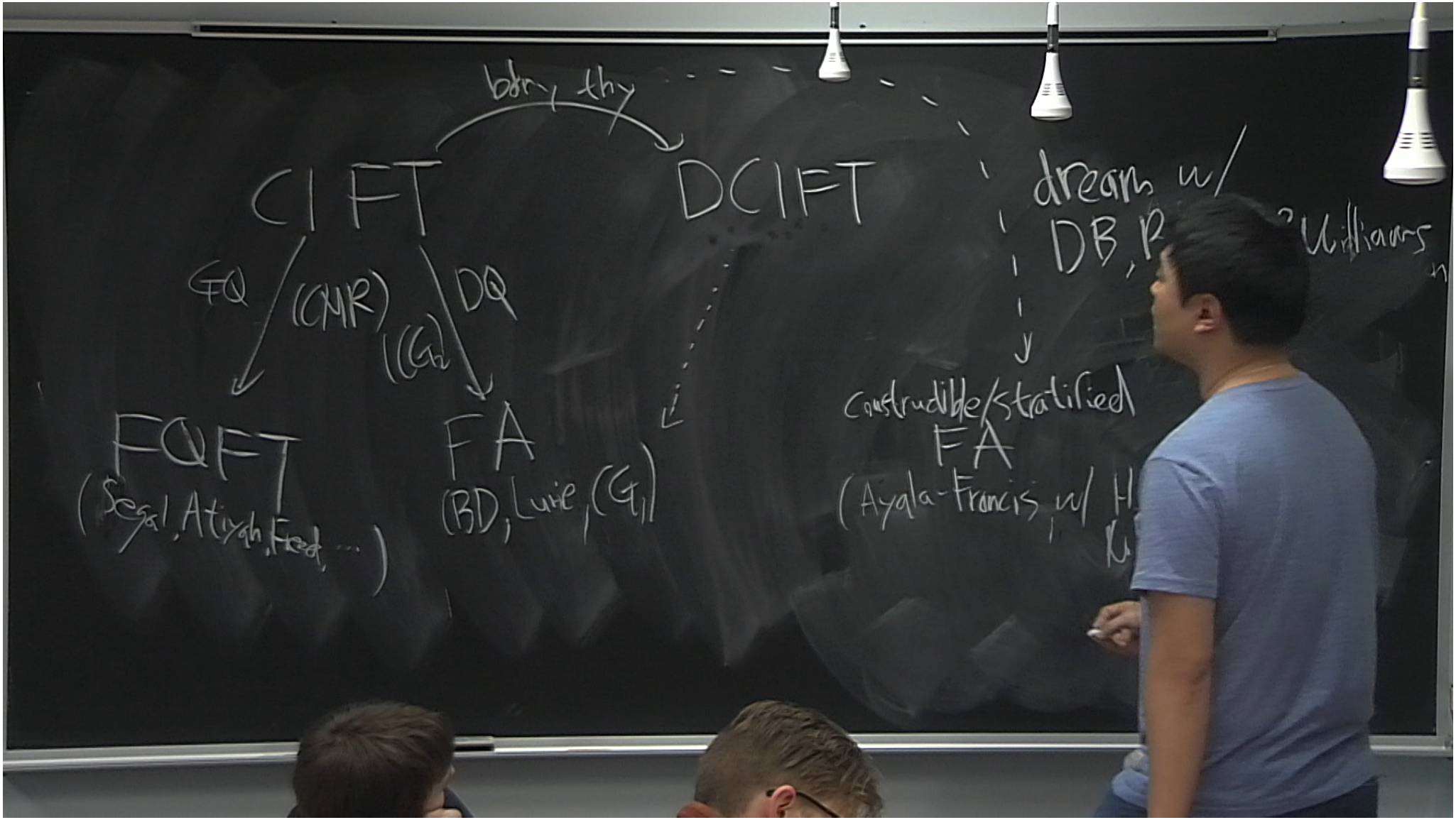

CAUTION
CAUTION

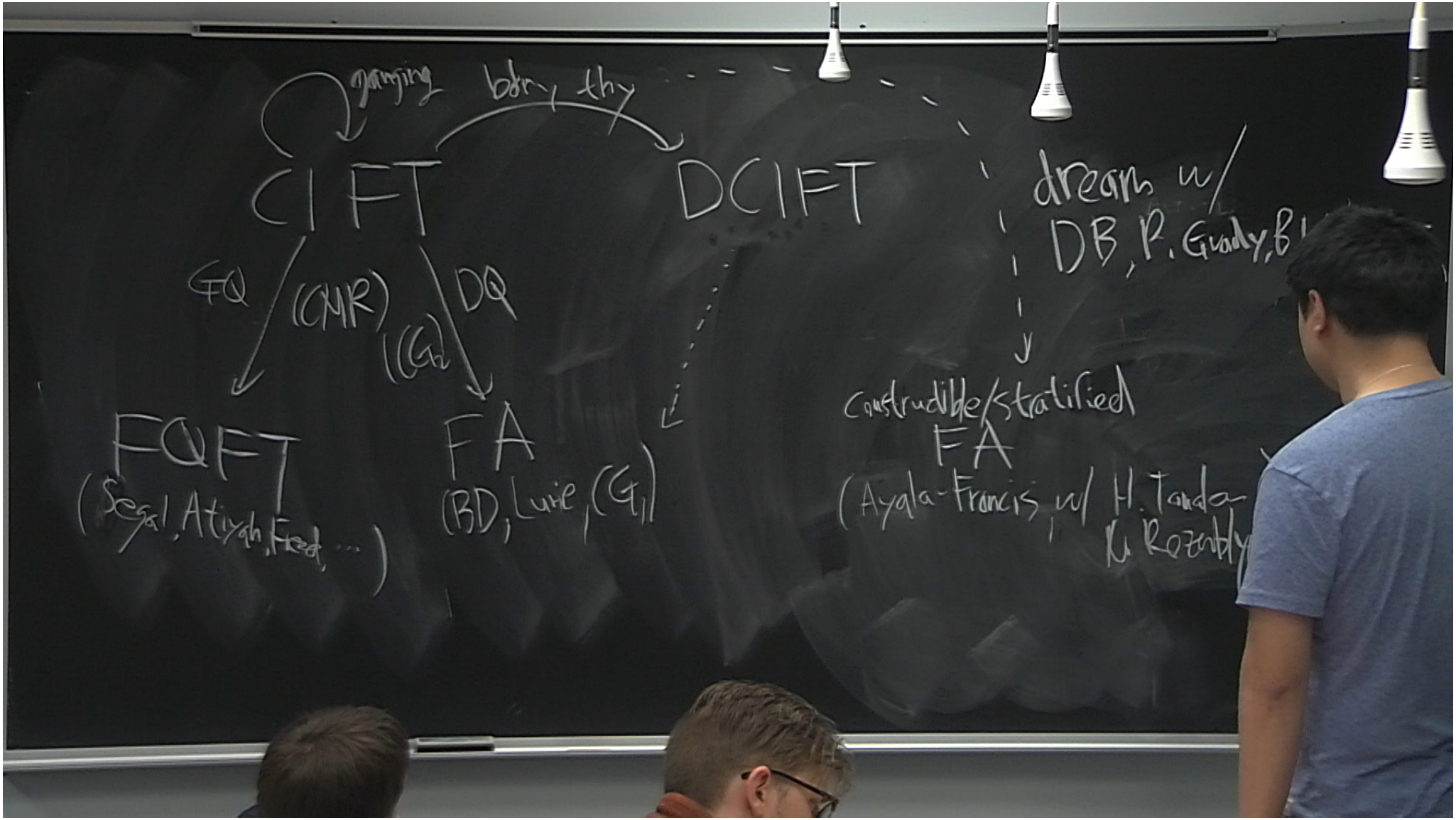

bdry thy
CIFT
DCIFT
FQ
(CMR)
(G2)
DQ
FQFT
(Segal, Atiyah, Freed, ...)
FA
(BD, Lurie, (G1)
Constructible/stratified
FA
(Ayala-Francis, w/ H. Tanda
N. Rozenblyum)

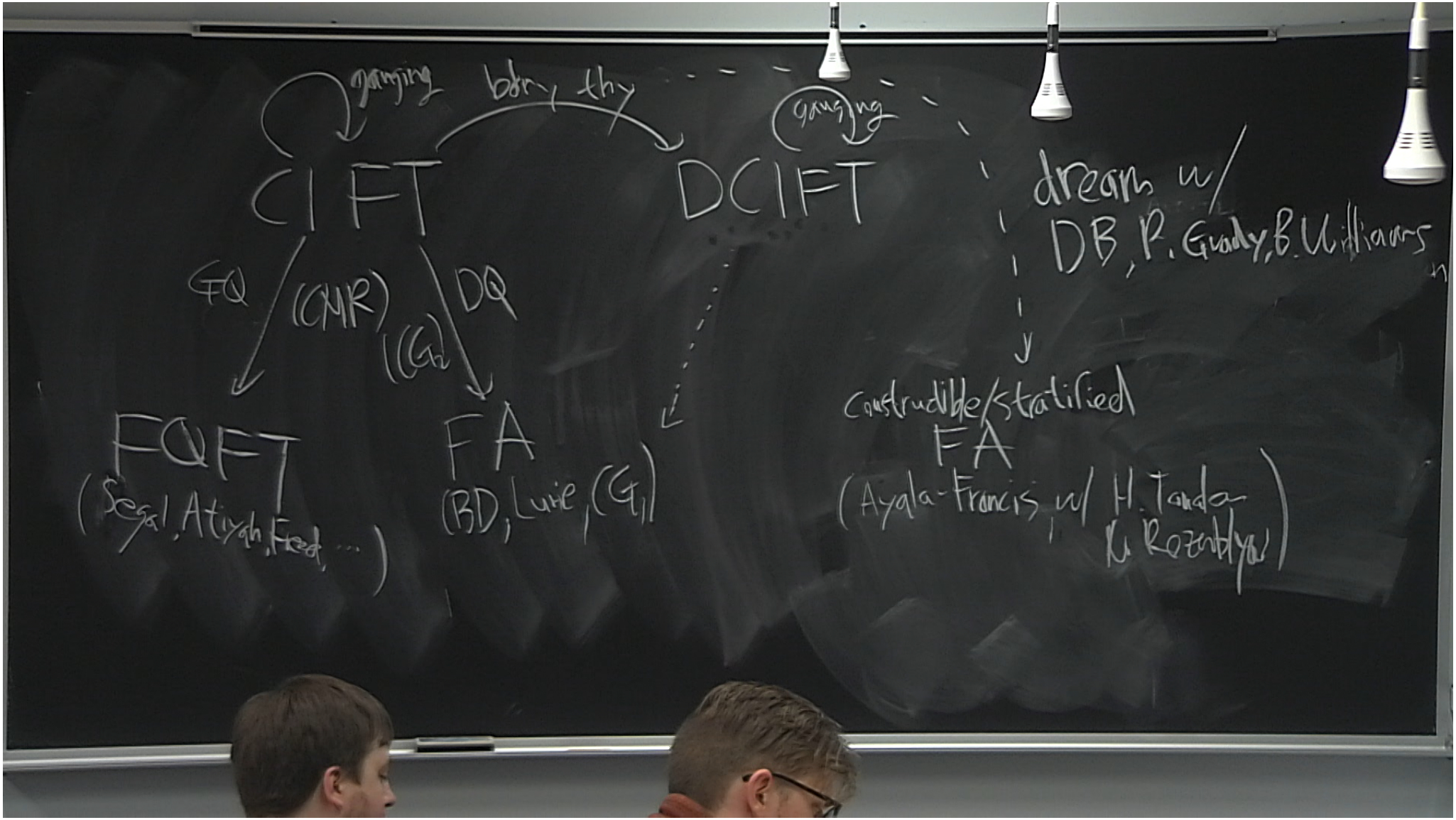

bdry thy
CIFT
DCIFT
FQ
(CMR)
(G2)
DQ
FQFT
(Segal, Atiyah, Freed, ...)
FA
(BD, Lurie, (G1)
Constructible/stratified
FA
(Ayala-Francis, w/ H. Tanda
N. Rozenblyum)

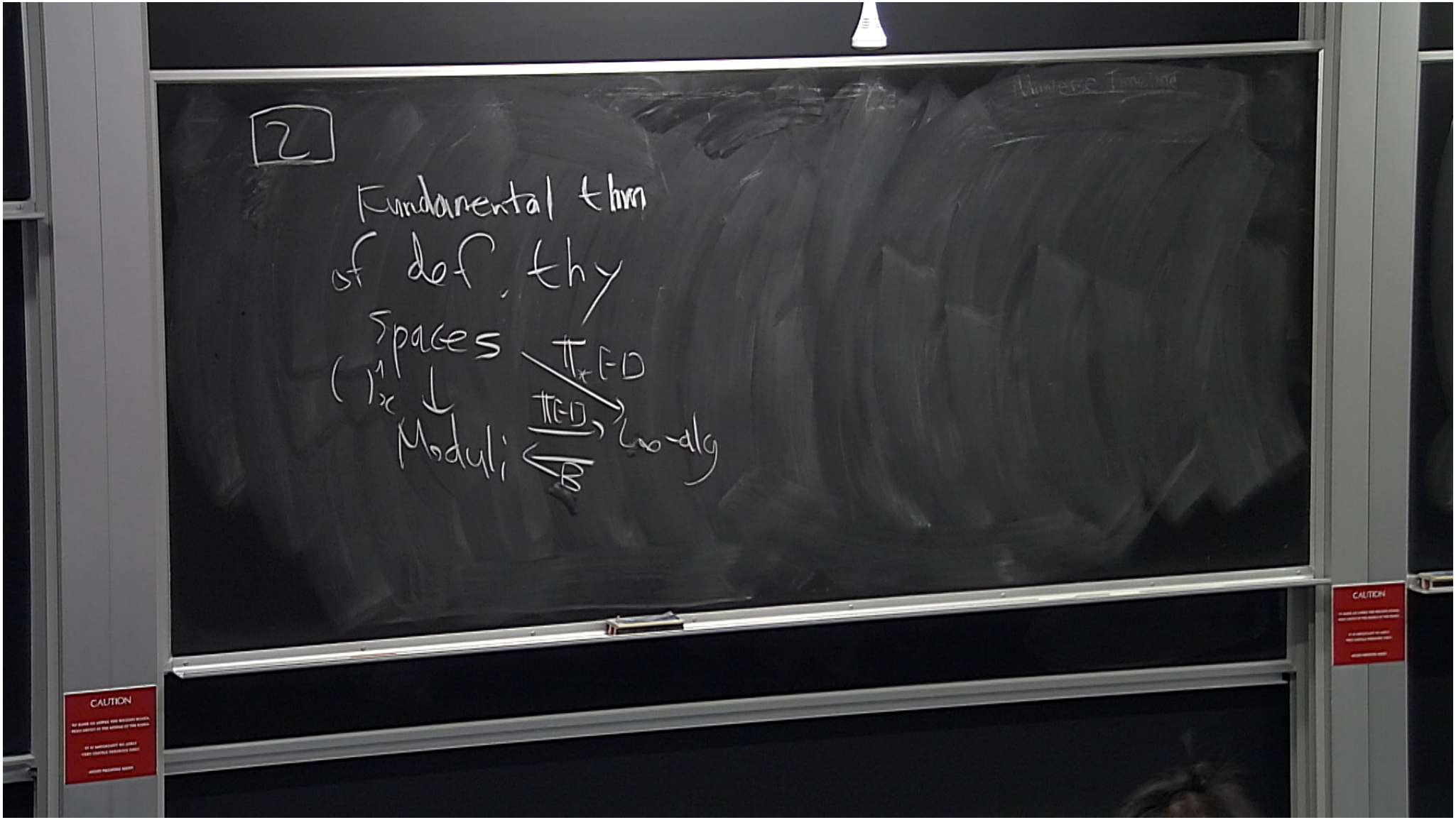

bdry thy
CIFT
DCIFT
dream w/
DB, P. Grady, B. Williams
FQ
(CMR)
(G2)
DQ
FQFT
(Segal, Atiyah, Freed, ...)
FA
(BD, Lurie, (G1)
constructible/stratified
FA
(Ayala-Francis, w/ H. Tanda
N. Rozenblyum)

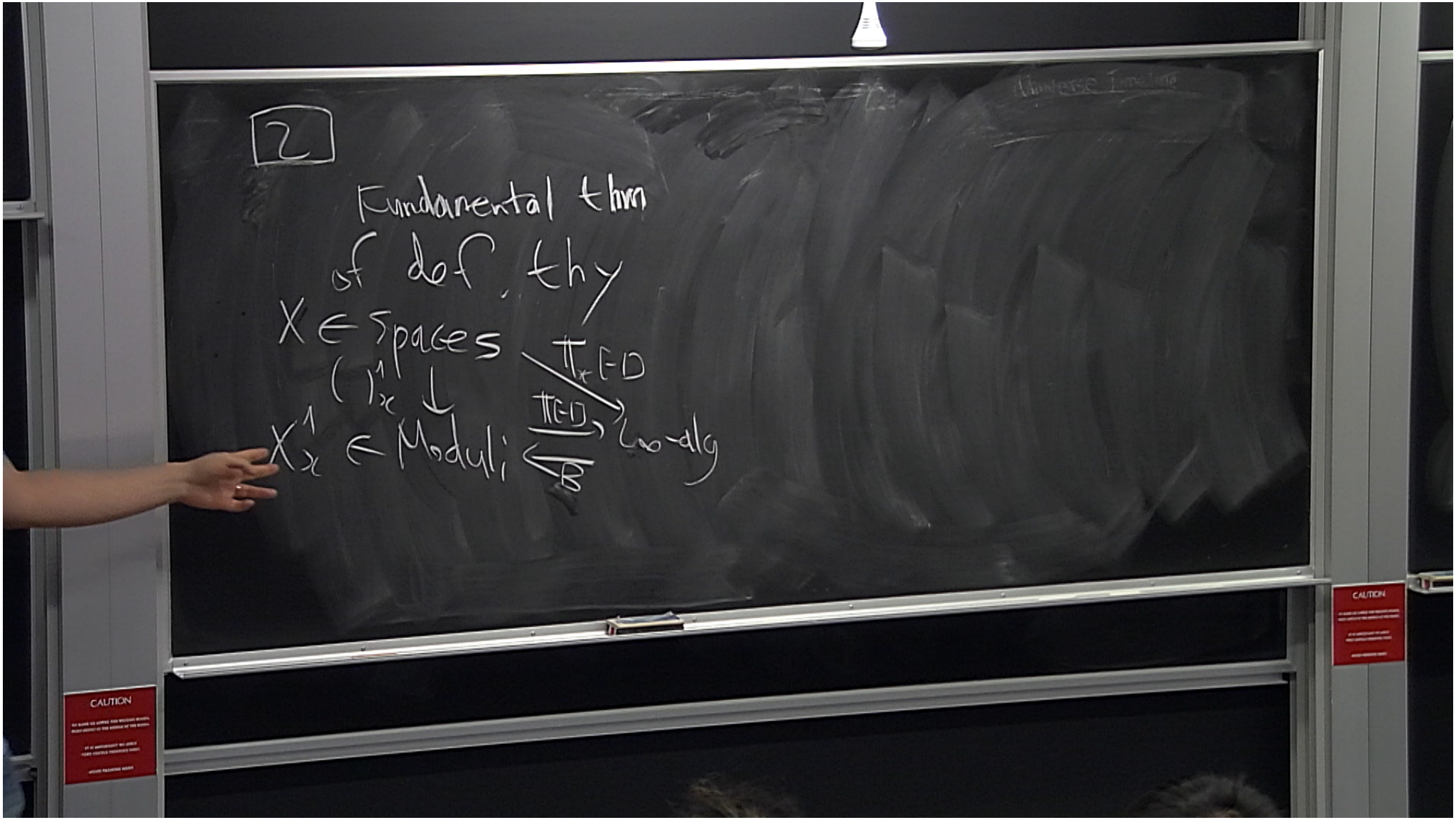

tail
CAUTION
CAUTION

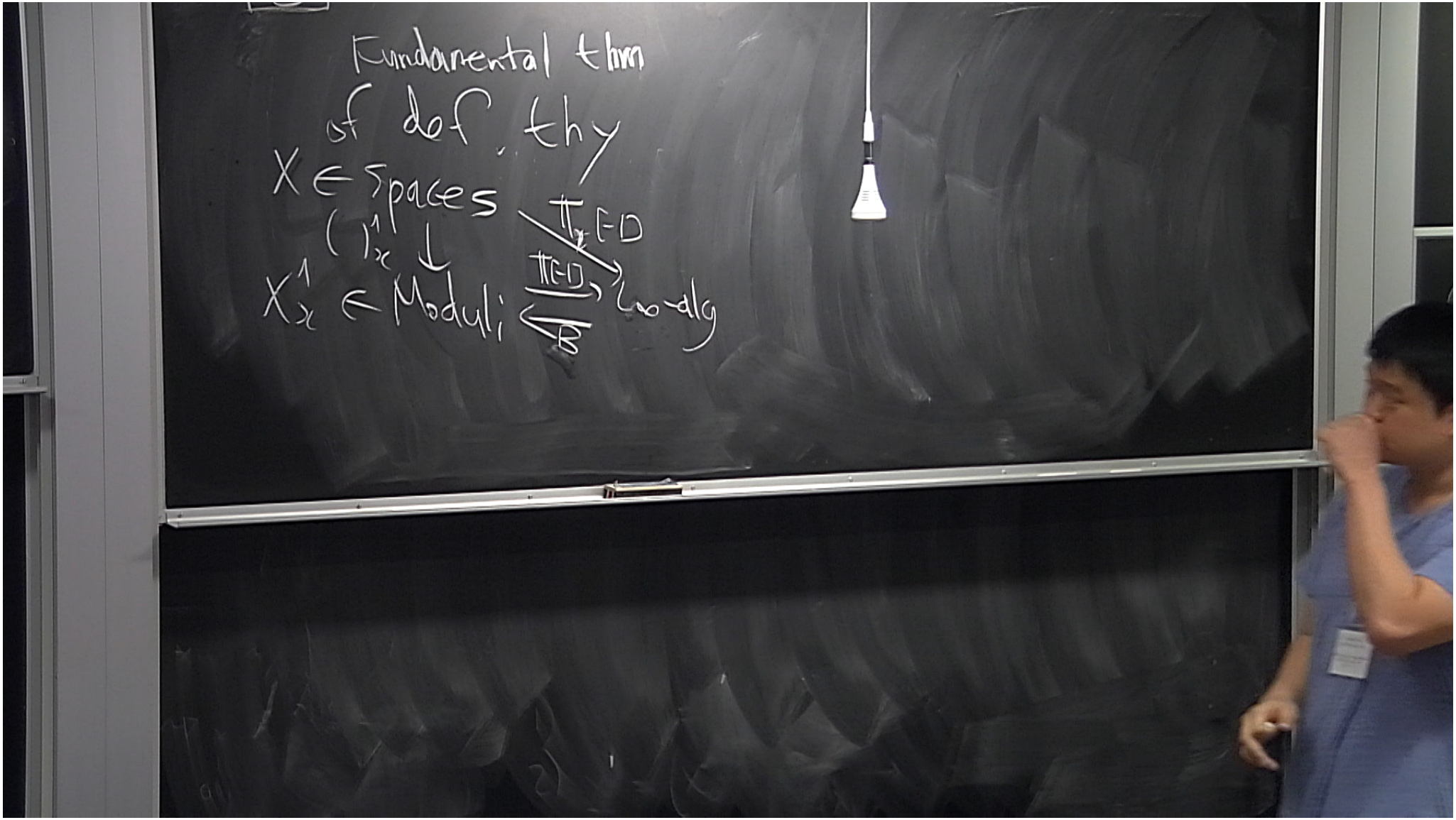

bdry thy
CIFT
DCIFT
dream w/
DB, R
Williams
FQ
(CMR)
(G2)
DQ
FQFT
(Segal, Atiyah, Freed, ...)
FA
(BD, Lurie, (G1)
Constructible/stratified
FA
(Ayala-Francis, w/ H
K

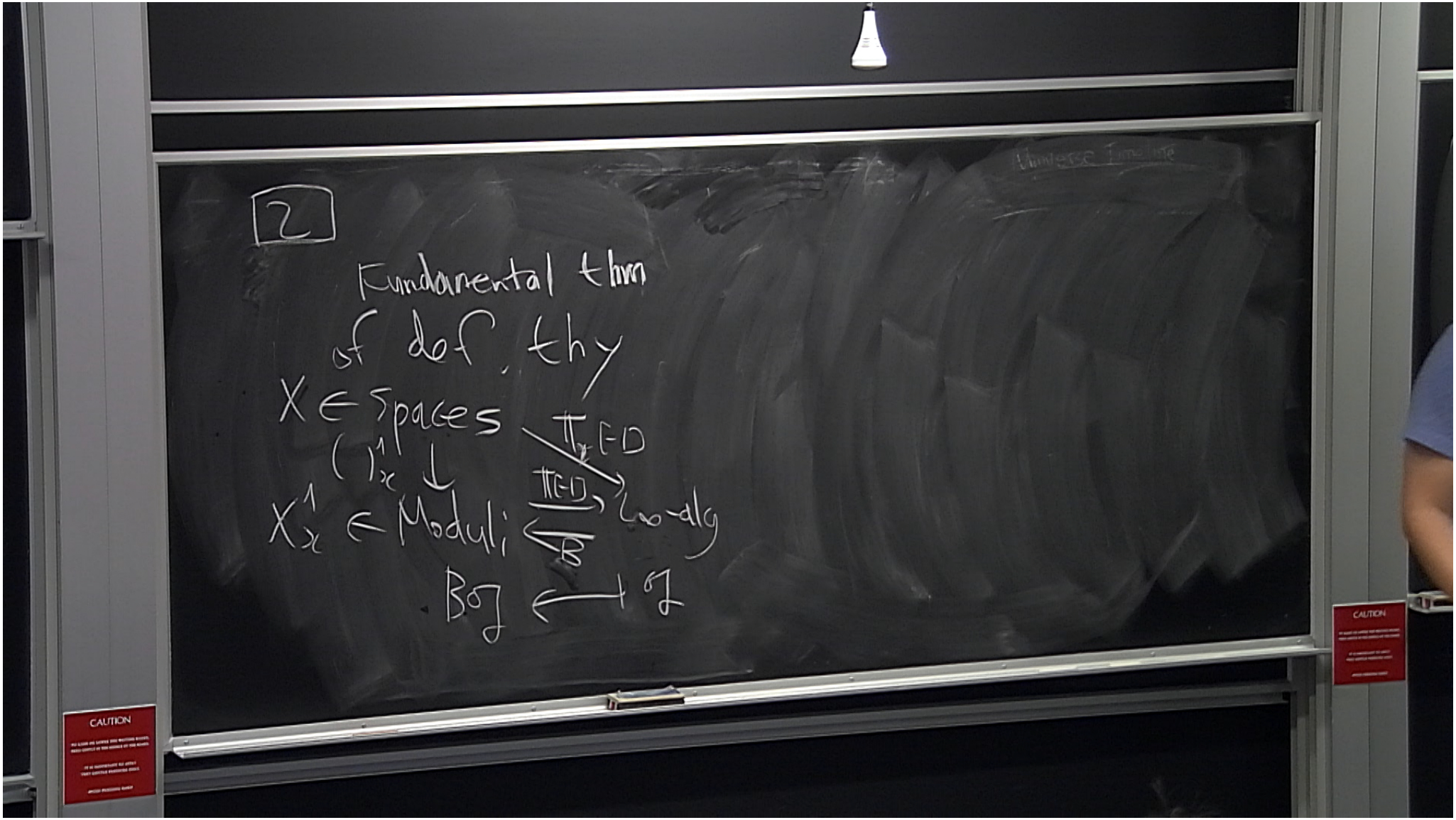

CIFT
DCIFT
dream w/
FQFT
FA
FA

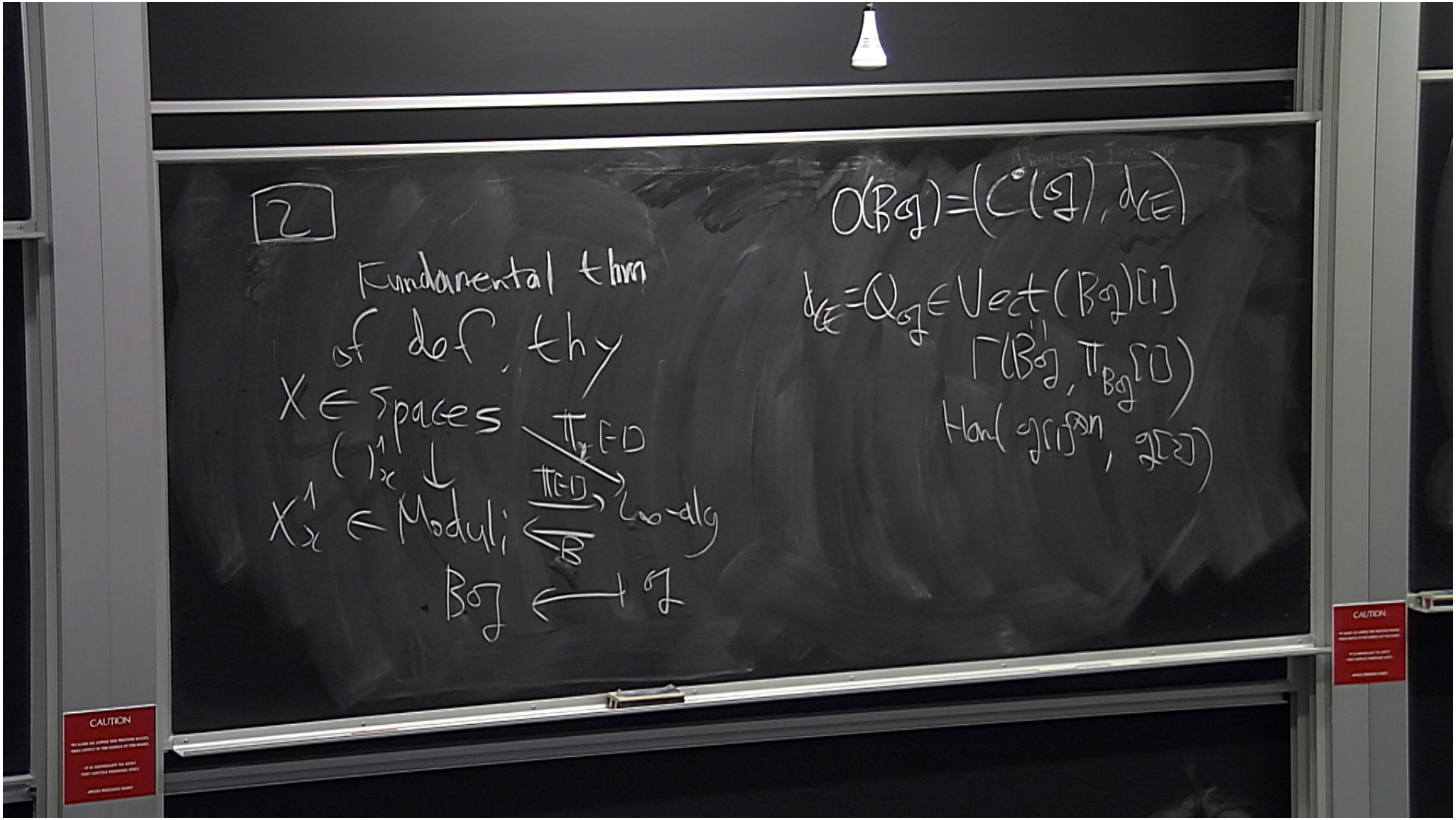

ranging
bdry thy
CIFT
DCIFT
ranging
dream w/
DB, P. Grady, B. Williams
FQ
(CMR)
(G2)
DQ
FQFT
(Segal, Atiyah, Freed, ...)
FA
(BD, Lurie, (G1)
constructible/stratified
FA
(Ayala-Francis, w/ H. Tanda
& Rozenblyum)

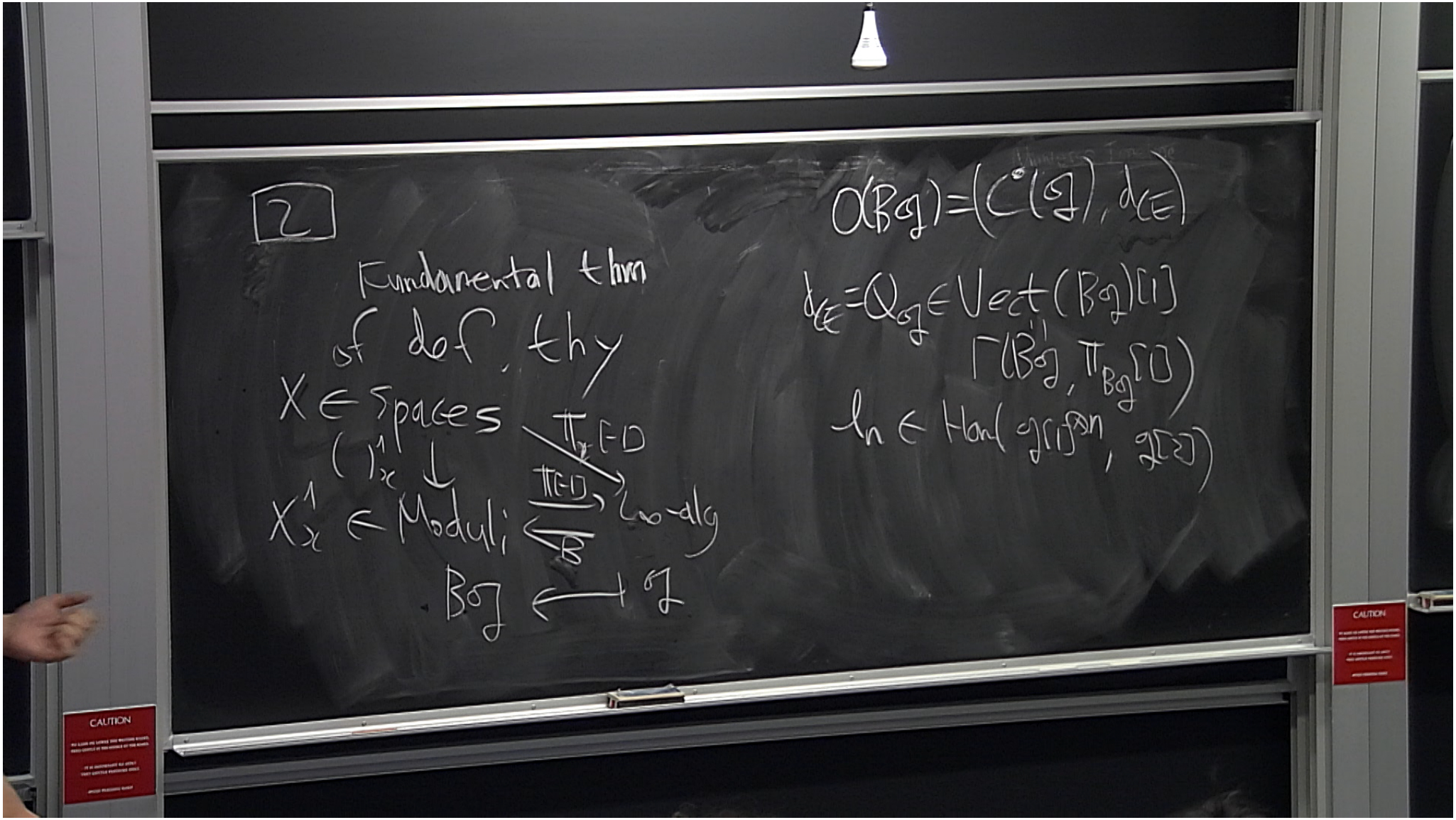

2
Fundamental thm
of def. thy
Spaces
Moduli
TFD
TFD
TFD
Loc-alg

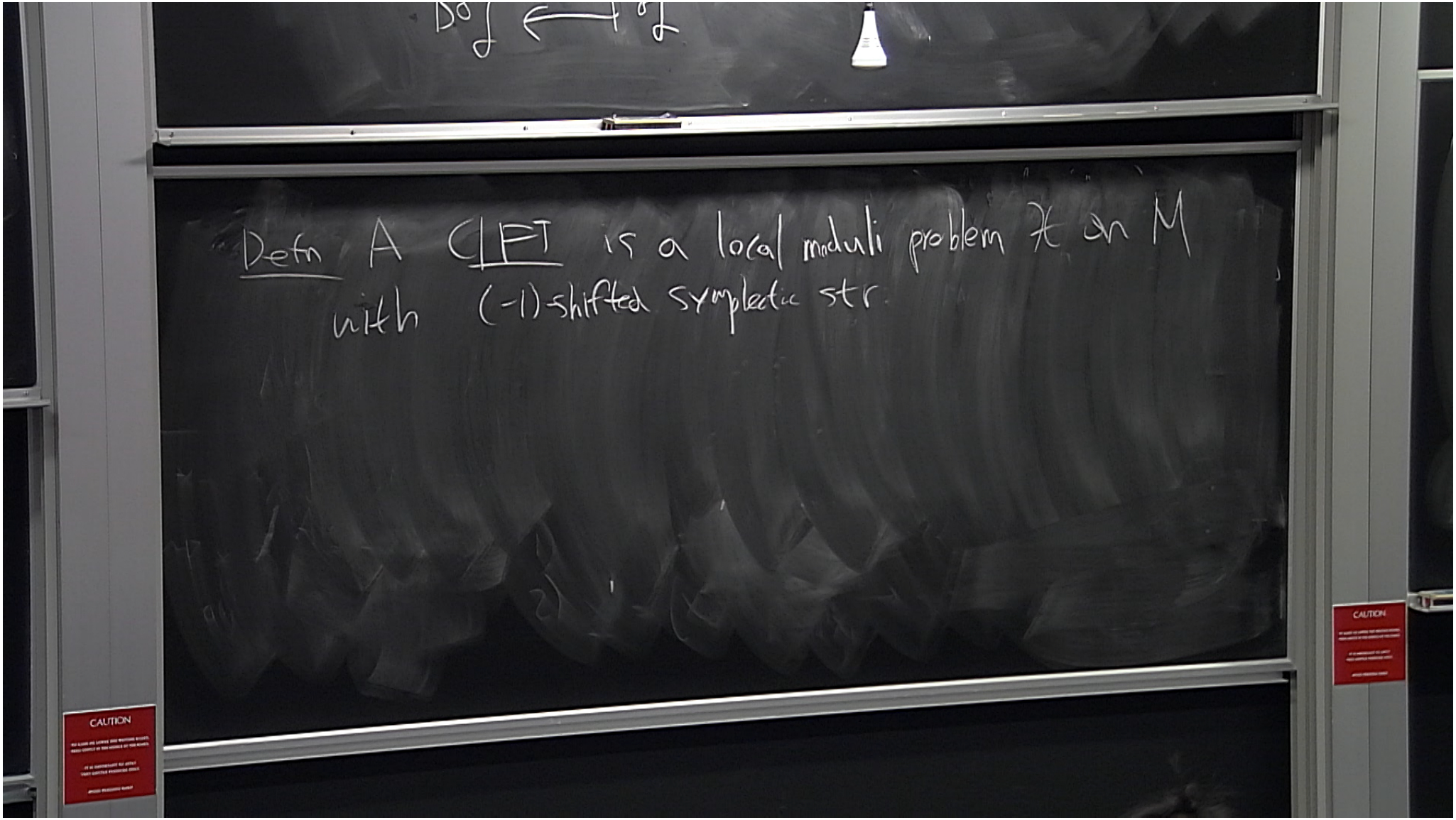

2
Fundamental thm
of def. thy
$X \in$ Spaces
$\Pi_{\leq 1}$-ED
Π-ED
∞-alg
$X^1_x \in$ Moduli
B


Fundamental thm
of def. thy
X ∈ Spaces
π_1 f-d
X^1_x ∈ Moduli
π f-d
R
∞-alg

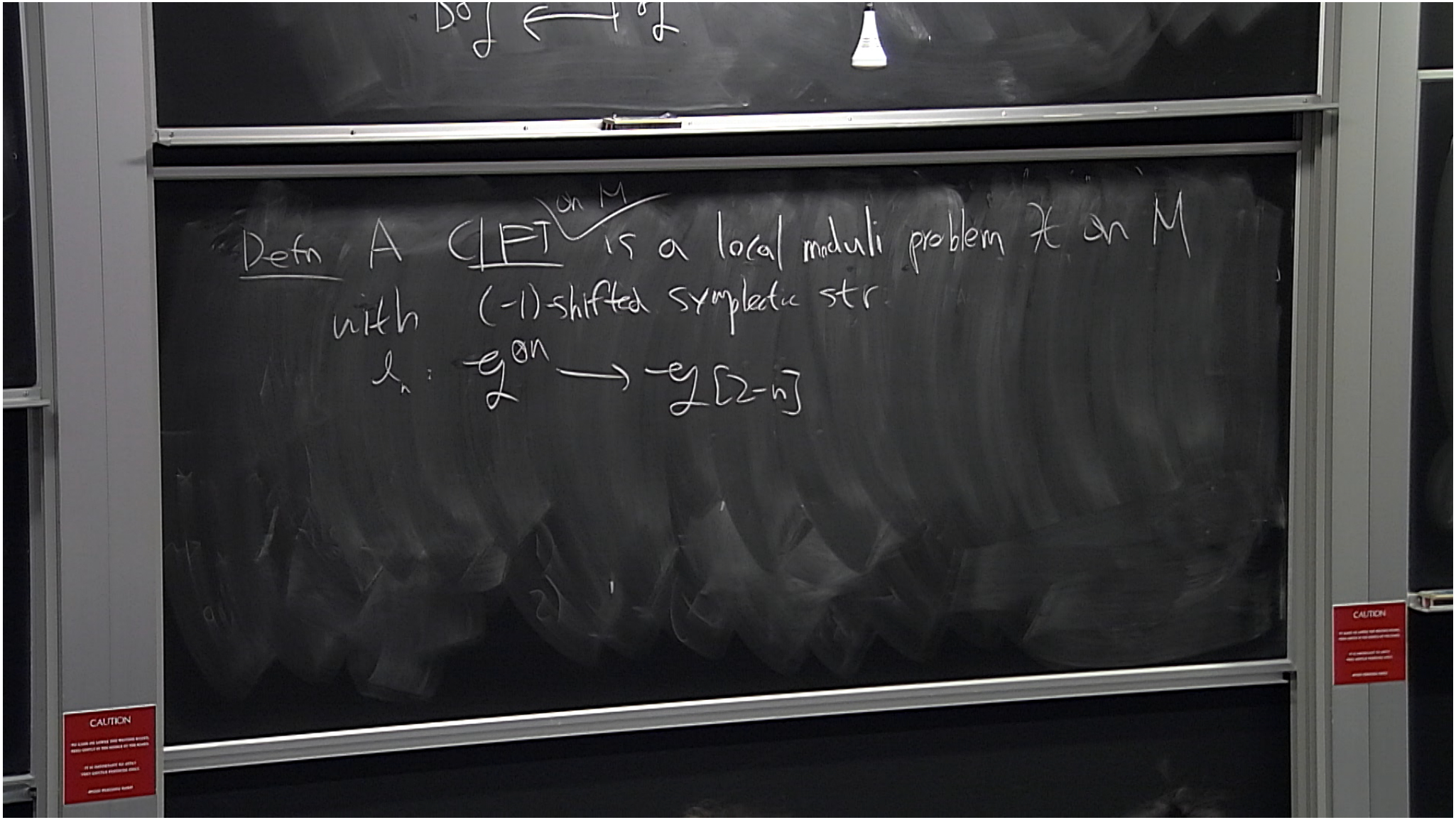

2
Fundamental thm
of def. thy
X ∈ Spaces
Moduli
Bσ
σ


2
Fundamental thm
of def. thy
$X \in$ Spaces
$X^1_x \in$ Moduli
$\pi_{\leq 0}$
$\pi_{\leq 0}$
$\mathbb{B}$
∞-alg
$B\mathfrak{g} \longleftarrow \mathfrak{g}$
$\mathcal{O}(B\mathfrak{g}) = (C^\bullet(\mathfrak{g}), d_{CE})$
$d_{CE} = Q_{\mathfrak{g}} \in Vect(B\mathfrak{g})[1]$
$\Gamma(B\mathfrak{g}, \pi_{B\mathfrak{g}}[1])$
Hom(grpd, $\mathbb{Z}[2]$)


2
Fundamental thm
of def. thy
$X \in$ Spaces
$X^1_x \in$ Moduli
Π_{∞}-D
Π-D
$\mathcal{B}$
∞-alg
$B\sigma_G \longleftarrow \sigma_G$
$\mathcal{O}(B\sigma_G) = (C^*(\sigma_G), d_{CE})$
$d_{CE} = Q_{\sigma_G} \in Vect(B\sigma_G)[1]$
$\Gamma(B\sigma_G, \Pi_{B\sigma_G}[1])$
$\ln \in Hom($ grips$^{\otimes n}, \mathbb{R}[2])$
CAUTION
CAUTION


Defn A CLFT is a local moduli problem $\mathcal{X}$ on M
with (-1)-shifted symplectic str.
CAUTION
CAUTION

Defn A CLFT on M is a local moduli problem $\mathcal{X}$ on M with (-1)-shifted symplectic str.


Defn A CLFT on M is a local moduli problem $\mathcal{X}$ on M
with (-1)-shifted symplectic str
$\omega_n : \mathcal{X}^{\partial M} \longrightarrow \mathcal{X}[2-n]$
CAUTION
CAUTION

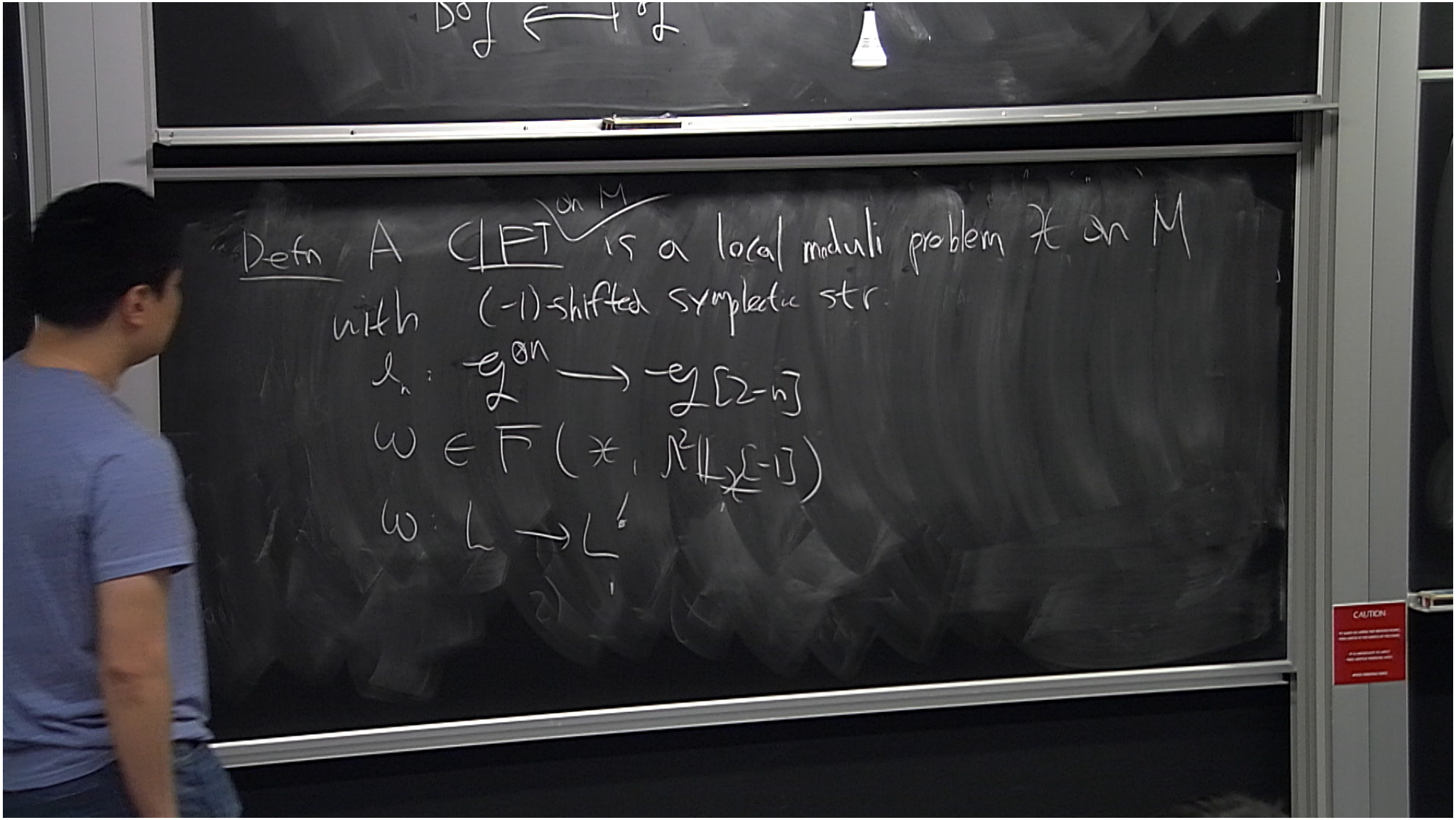
$\mathrm{Def}^0 \longleftarrow \mathfrak{g}$

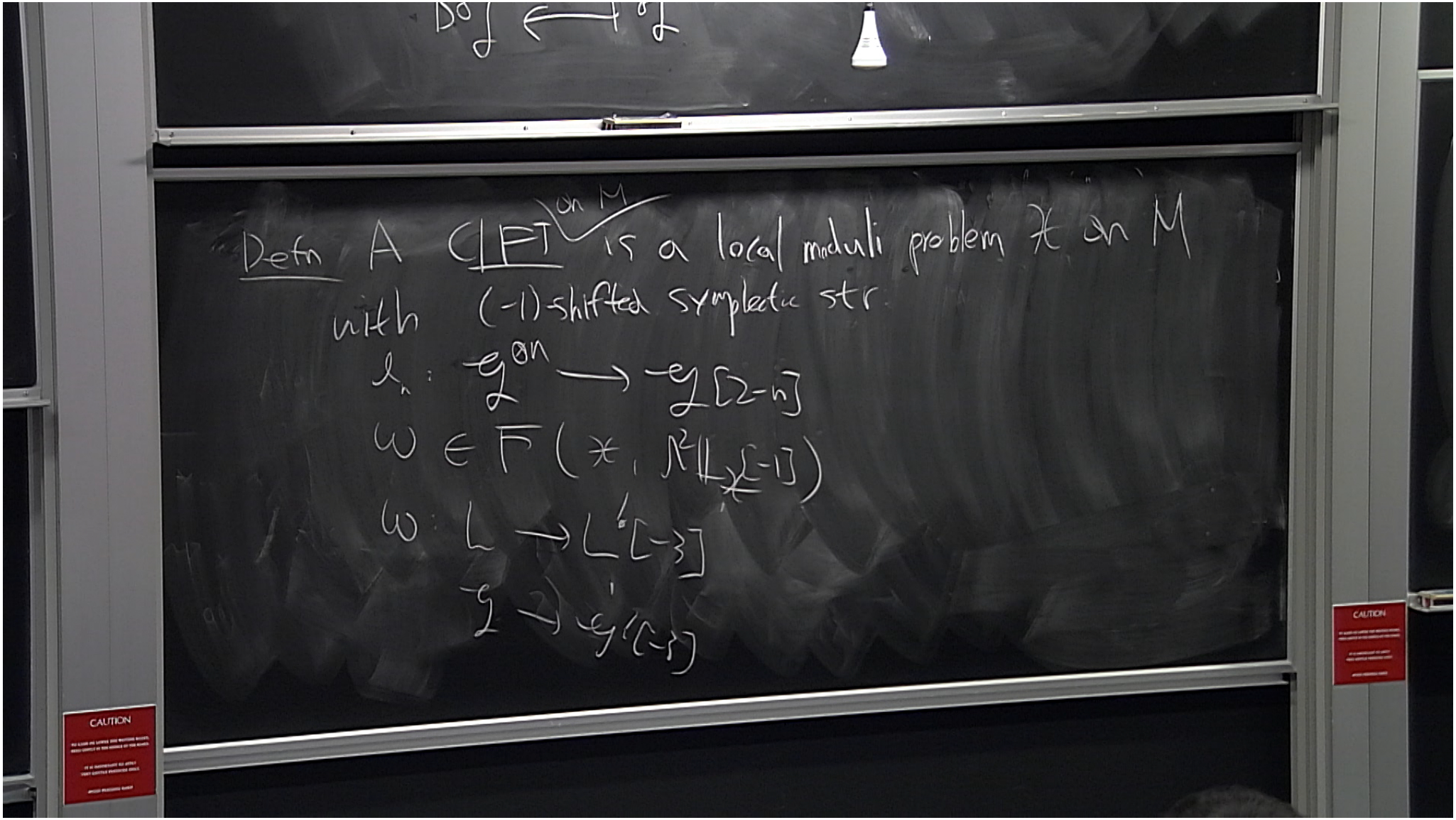
Defn A CLFT on M is a local moduli problem $\mathcal{X}$ on M with (-1)-shifted symplectic str.

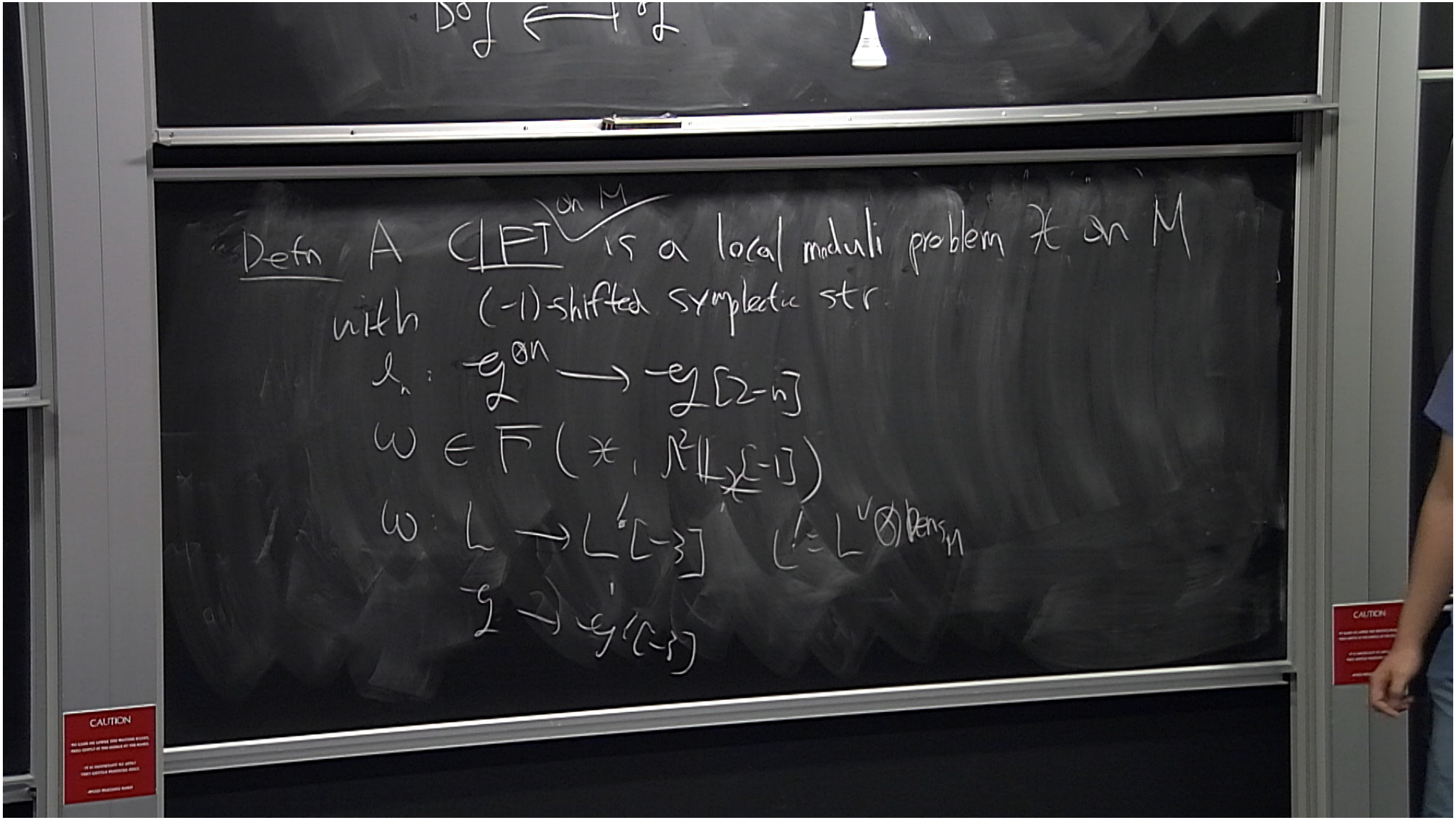
$\ell_n : \mathfrak{g}^{\otimes n} \longrightarrow \mathfrak{g}[2-n]$

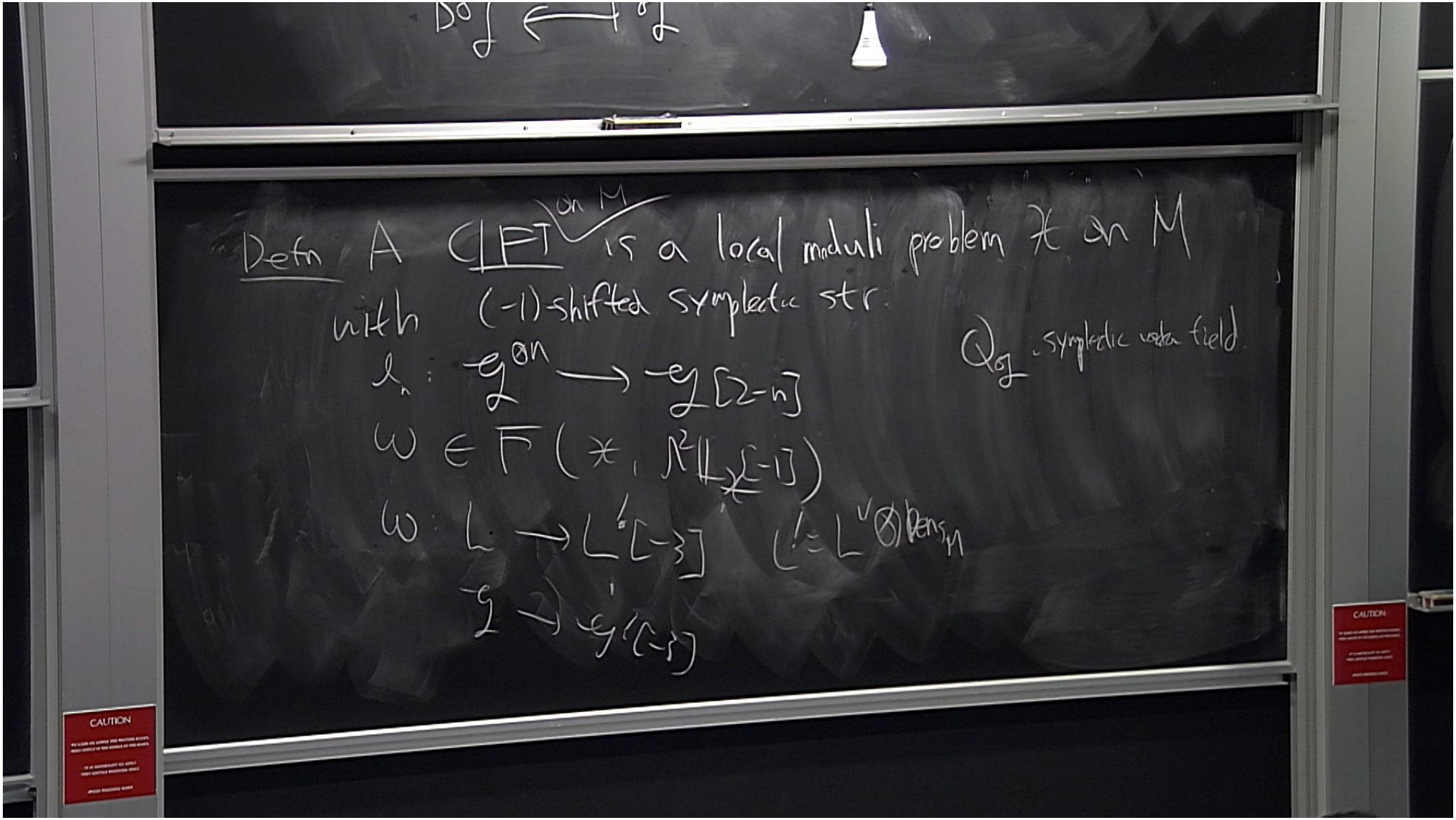
$\omega \in \mathbb{F}(*, \wedge^2 L_{\mathcal{X}}[-1])$

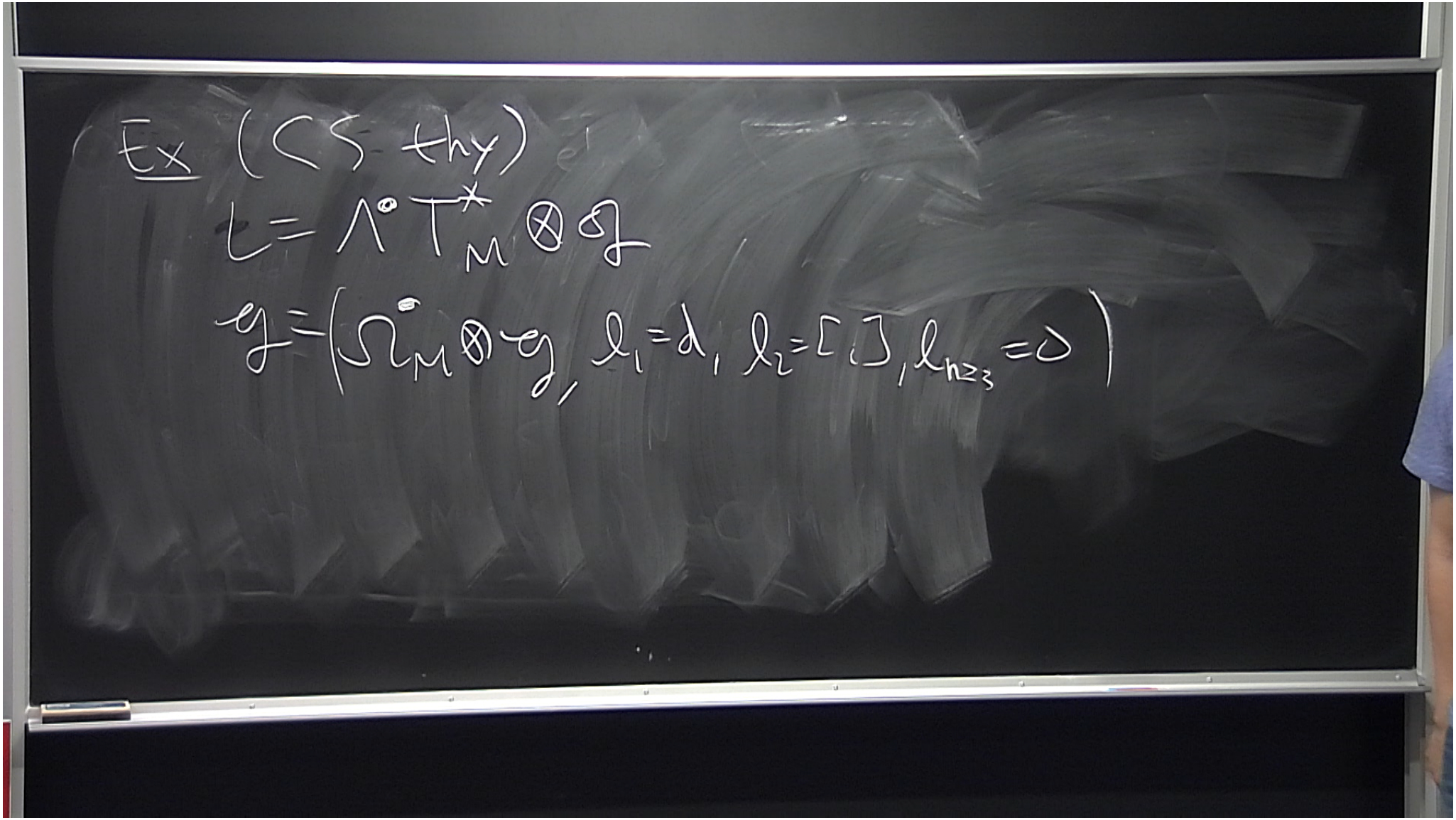
CAUTION

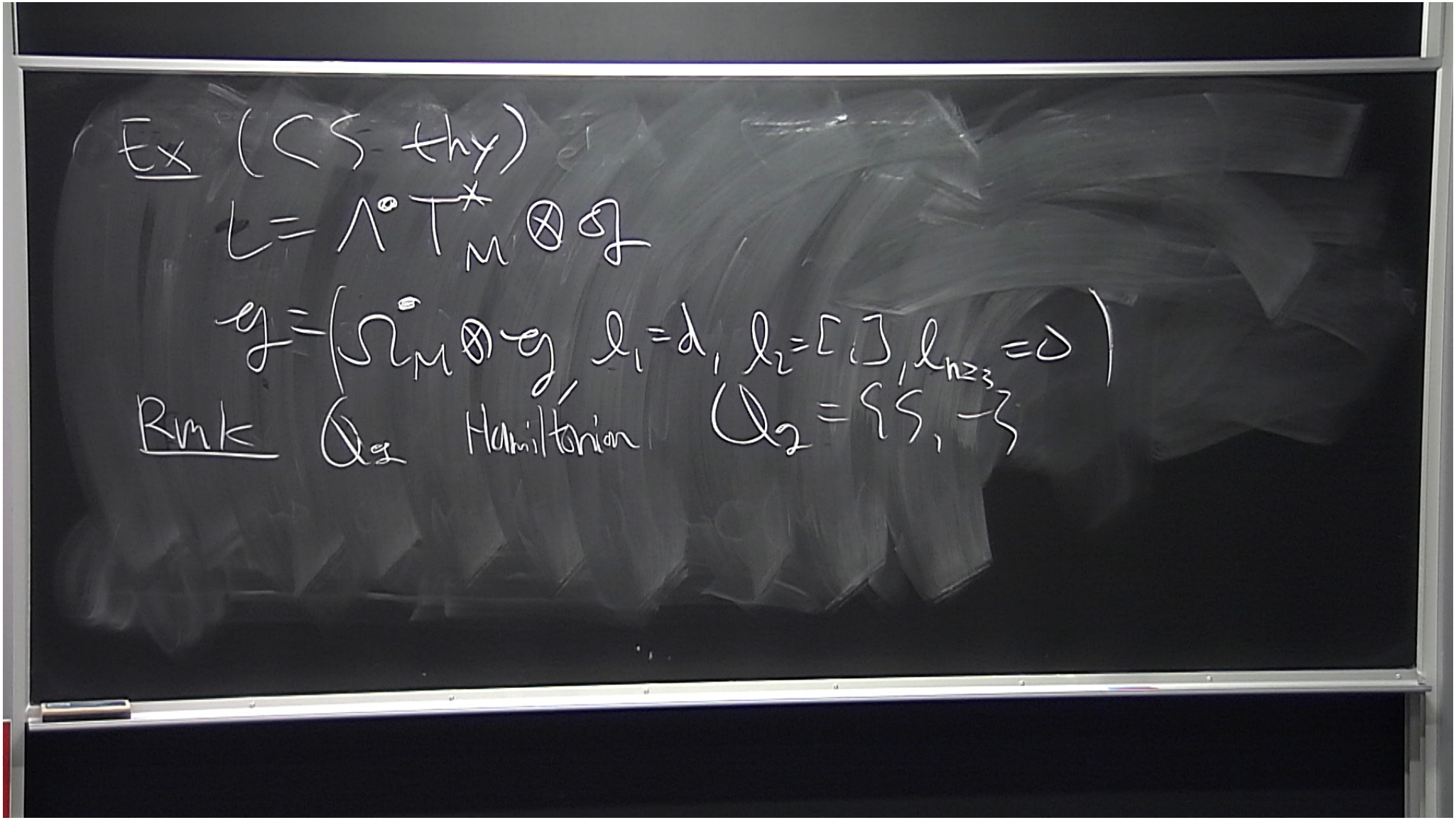

Defn A CLFT on M is a local moduli problem $\mathcal{X}$ on M
with (-1)-shifted symplectic str.
$\rho_n : \mathfrak{g}^{\otimes n} \to \mathfrak{g}[2-n]$
$\omega \in F(*, \wedge^2 L_{\mathcal{X}}[-1])$
$\omega : L \to L^!$
CAUTION

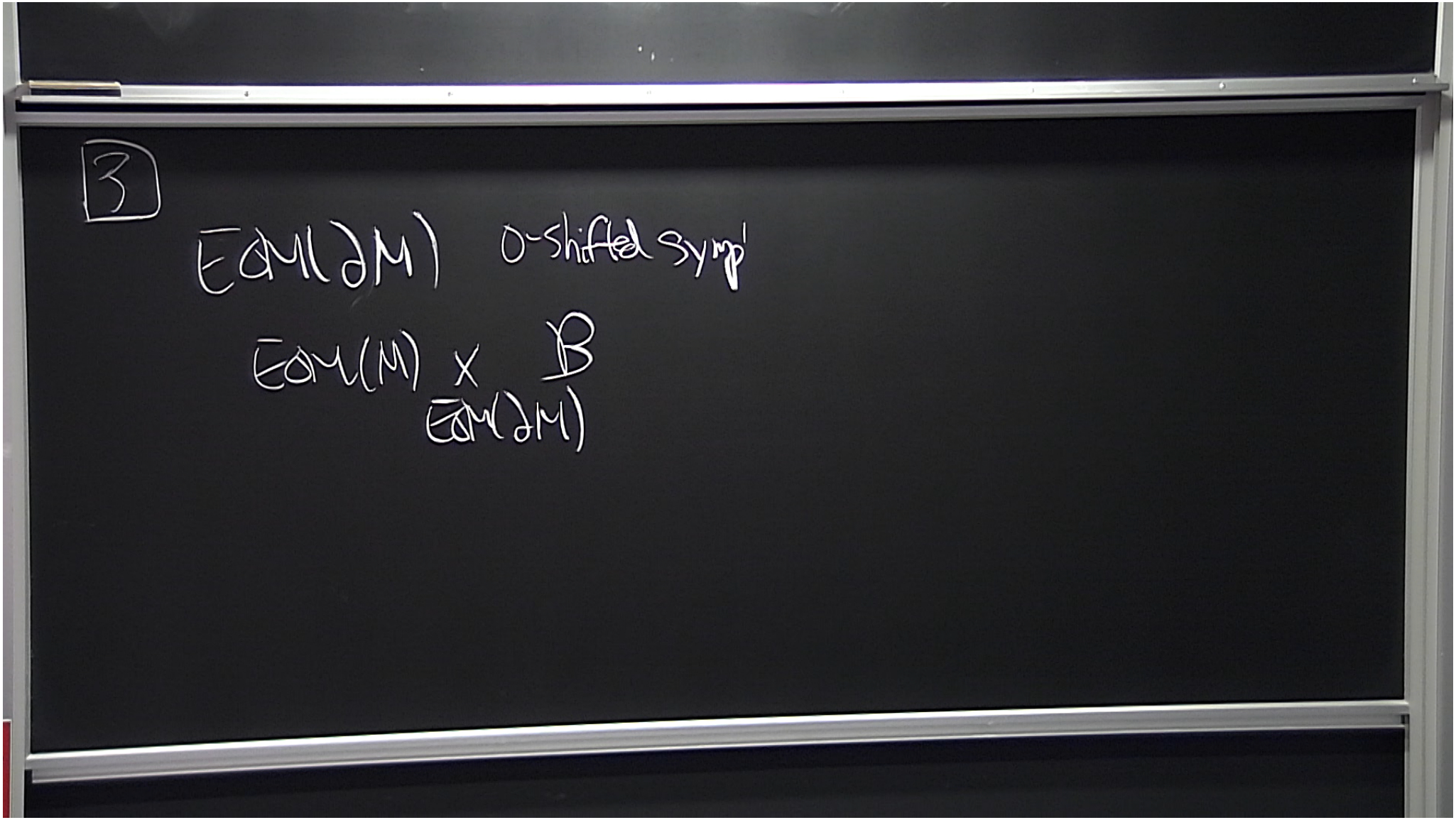

Defn A CLFT on M is a local moduli problem $\mathcal{X}$ on M
with (-1)-shifted symplectic str.
$\rho_n : \mathfrak{g}^{\otimes n} \longrightarrow \mathfrak{g}[2-n]$
$\omega \in F(*, \wedge^2 L_{\mathcal{X}}[-1])$
$\omega : L \to L^\vee[-3]$
$\mathfrak{g} \to \mathfrak{g}^\vee[-3]$

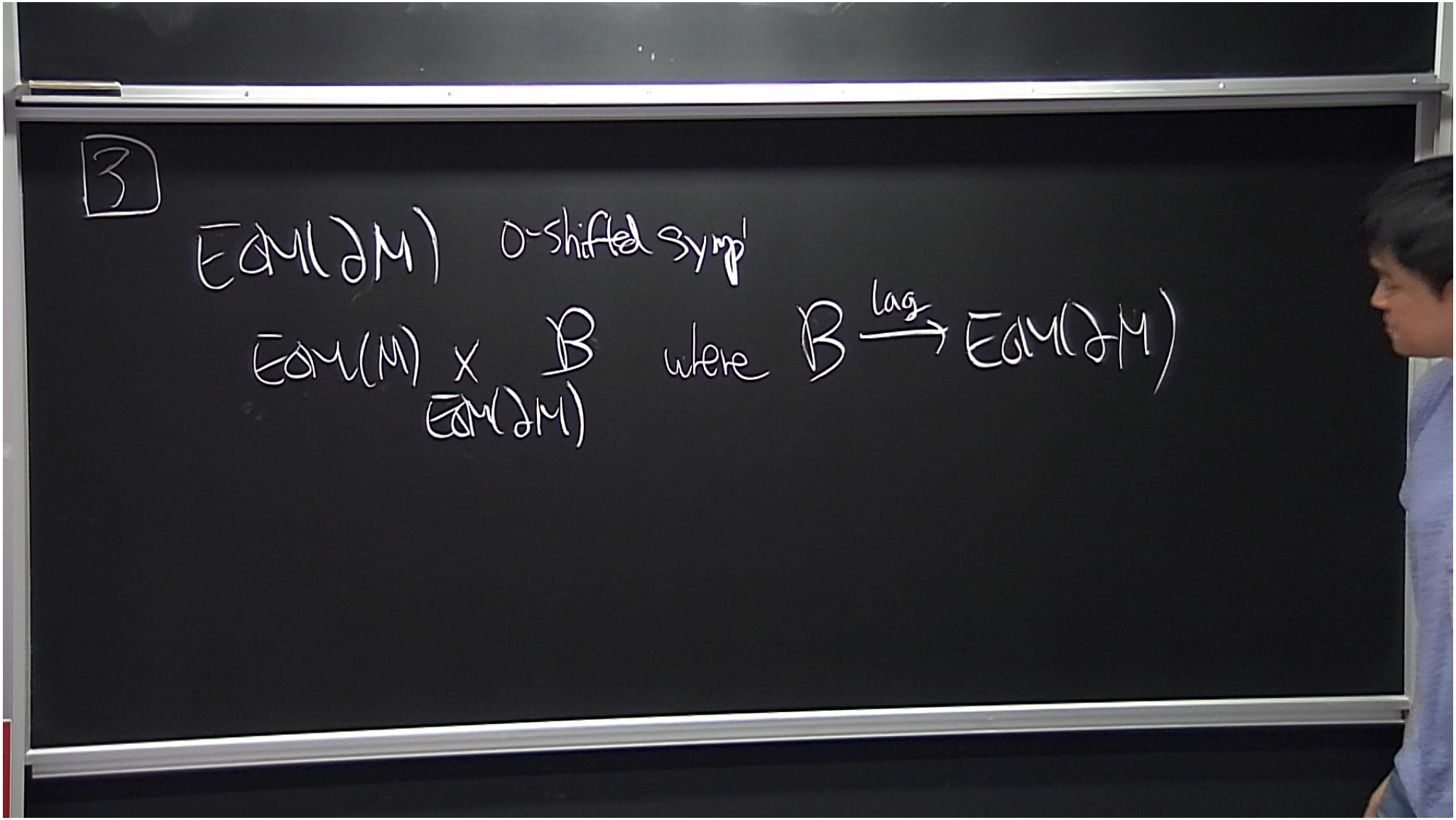

Defn A CLFT on M is a local moduli problem $\mathcal{X}$ on M
with (-1)-shifted symplectic str
$\omega \in F(*, \wedge^2 L_{\mathcal{X}}[-1])$
$\omega: L \to L^\vee[-3]$ $L^\vee = L^\vee \otimes \mathrm{Dens}_M$
CAUTION

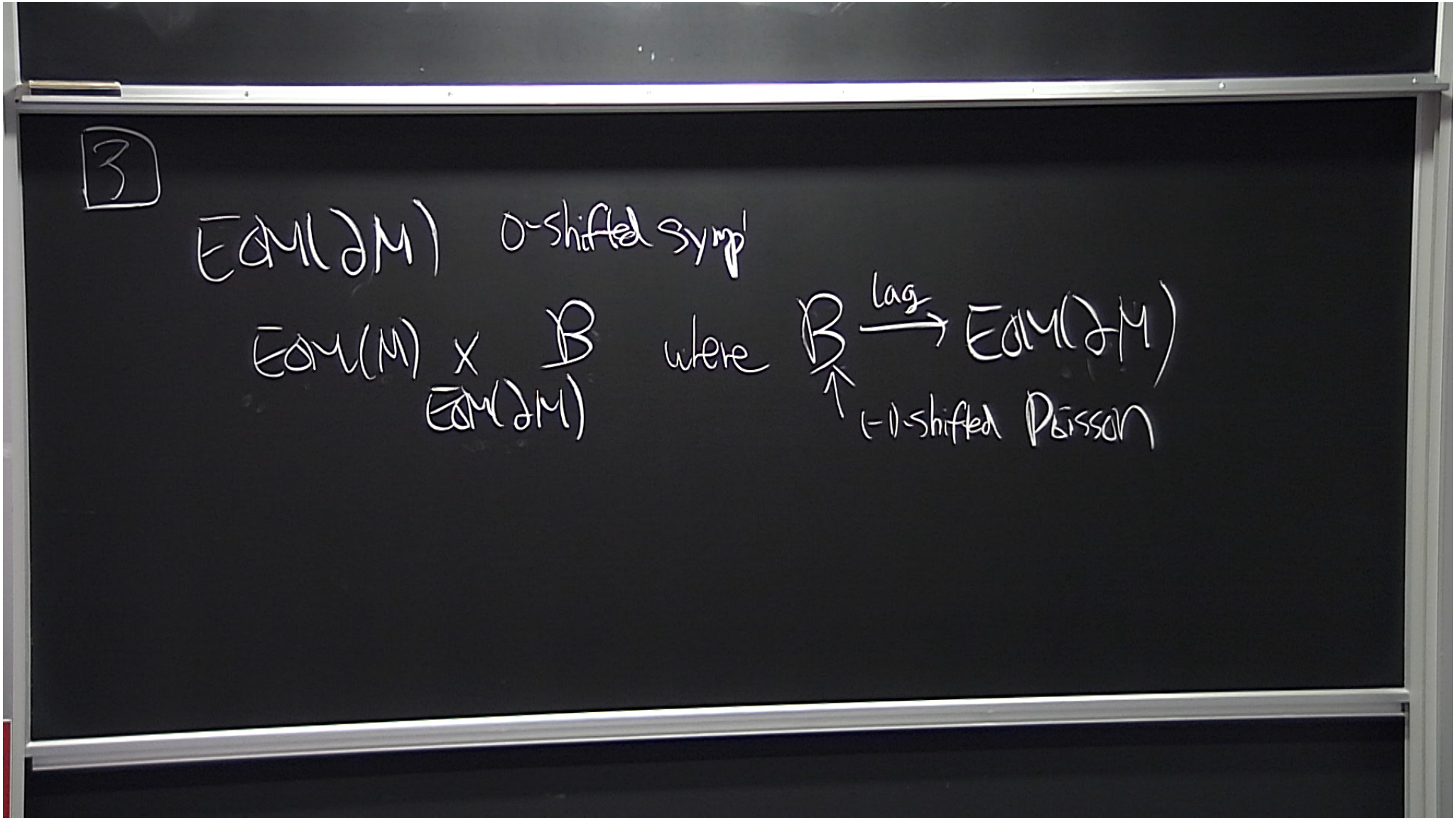

Defn A CLFT on M is a local moduli problem $\mathcal{X}$ on M
with (-1)-shifted symplectic str.
$\ell_n : \mathfrak{g}^{\otimes n} \to \mathfrak{g}[2-n]$
$Q_{\mathfrak{g}}$: symplectic vector field
$\omega \in F(*, \wedge^2 L_{\mathcal{X}}[-1])$
$\omega : L \to L^!\,[-3]$ $\quad L^! = L^\vee \otimes \mathrm{Dens}_M$
$\mathfrak{g} \to \mathfrak{g}^!\,[-3]$
CAUTION
CAUTION

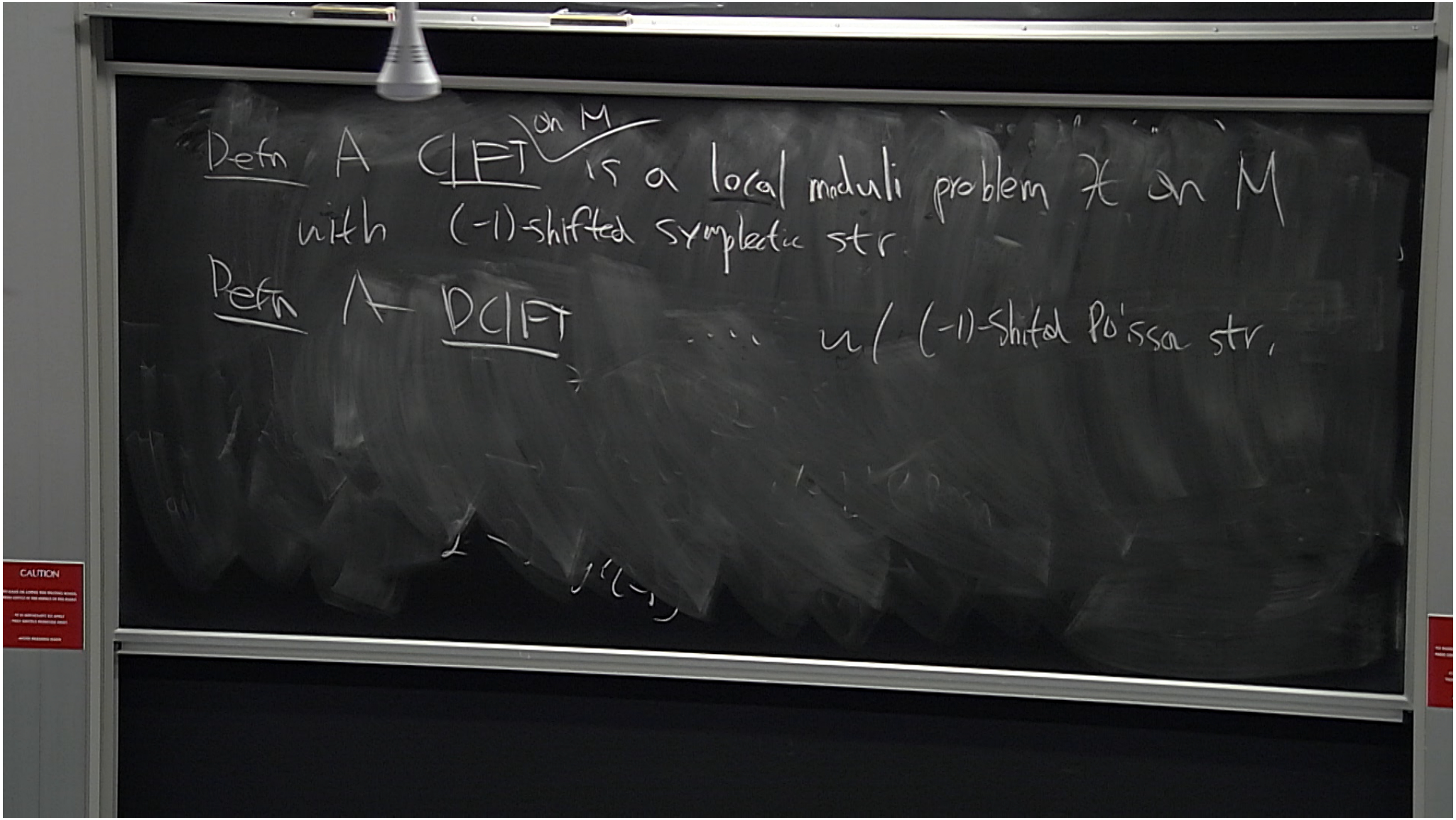

Ex (CS thy)
$L = \Lambda^\bullet T^*_M \otimes \mathfrak{g}$
$\mathfrak{g} = (\Omega^\bullet_M \otimes \mathfrak{g},\ l_1 = d,\ l_2 = [\,,\,],\ l_{n\geq 3} = 0)$

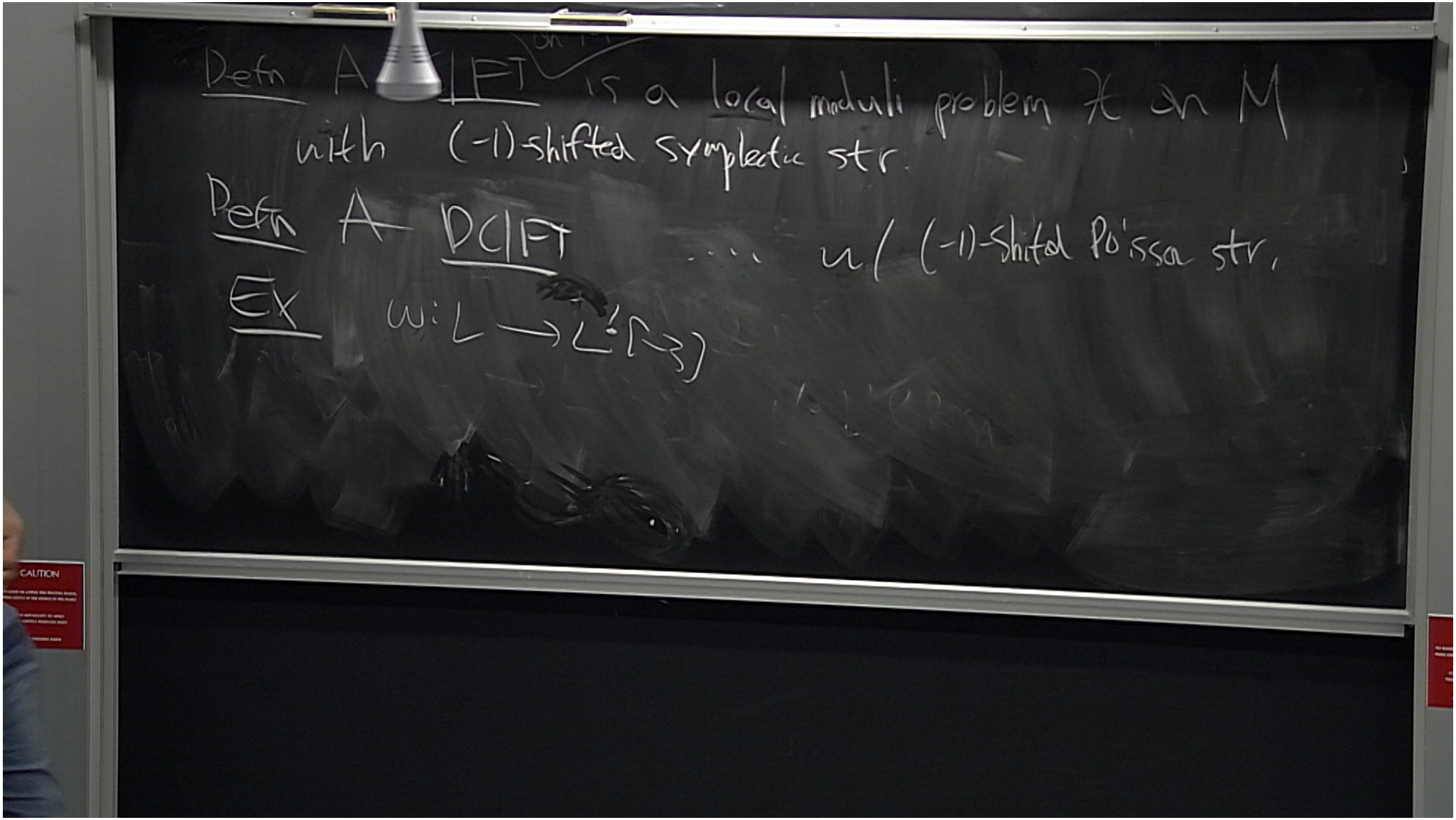

Ex (CS thy)
$L = \Lambda^\bullet T^*_M \otimes \mathfrak{g}$
$g = (\Omega^\bullet_M \otimes \mathfrak{g}, l_1 = d, l_2 = [,], l_{n\geq 3} = 0)$
Rmk Q_1 Hamiltonian $Q_2 = \{S, -\}$

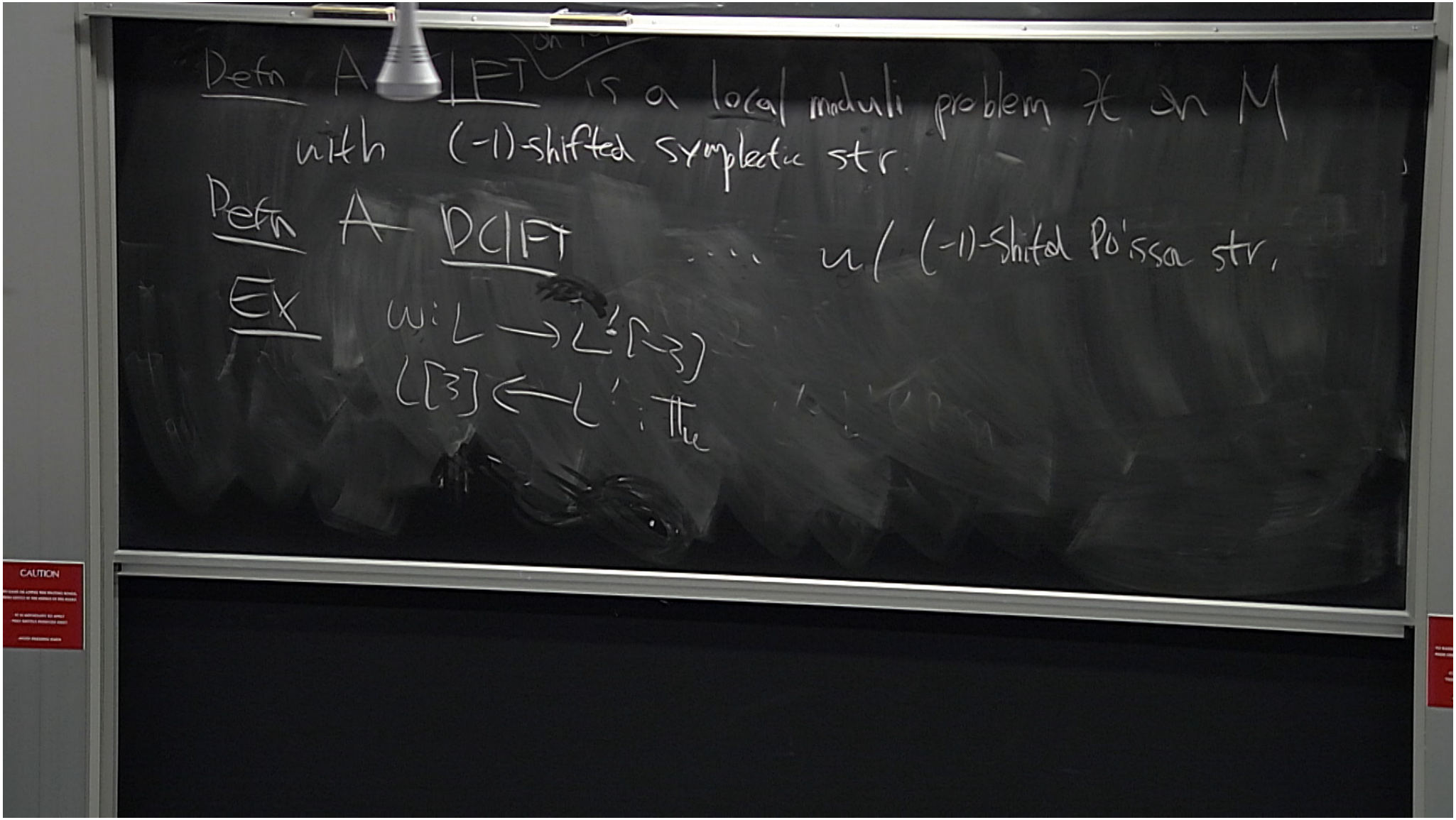

3
EQM(∂M) 0-shifted symp'
EQM(M) ×_{EQM(∂M)} B

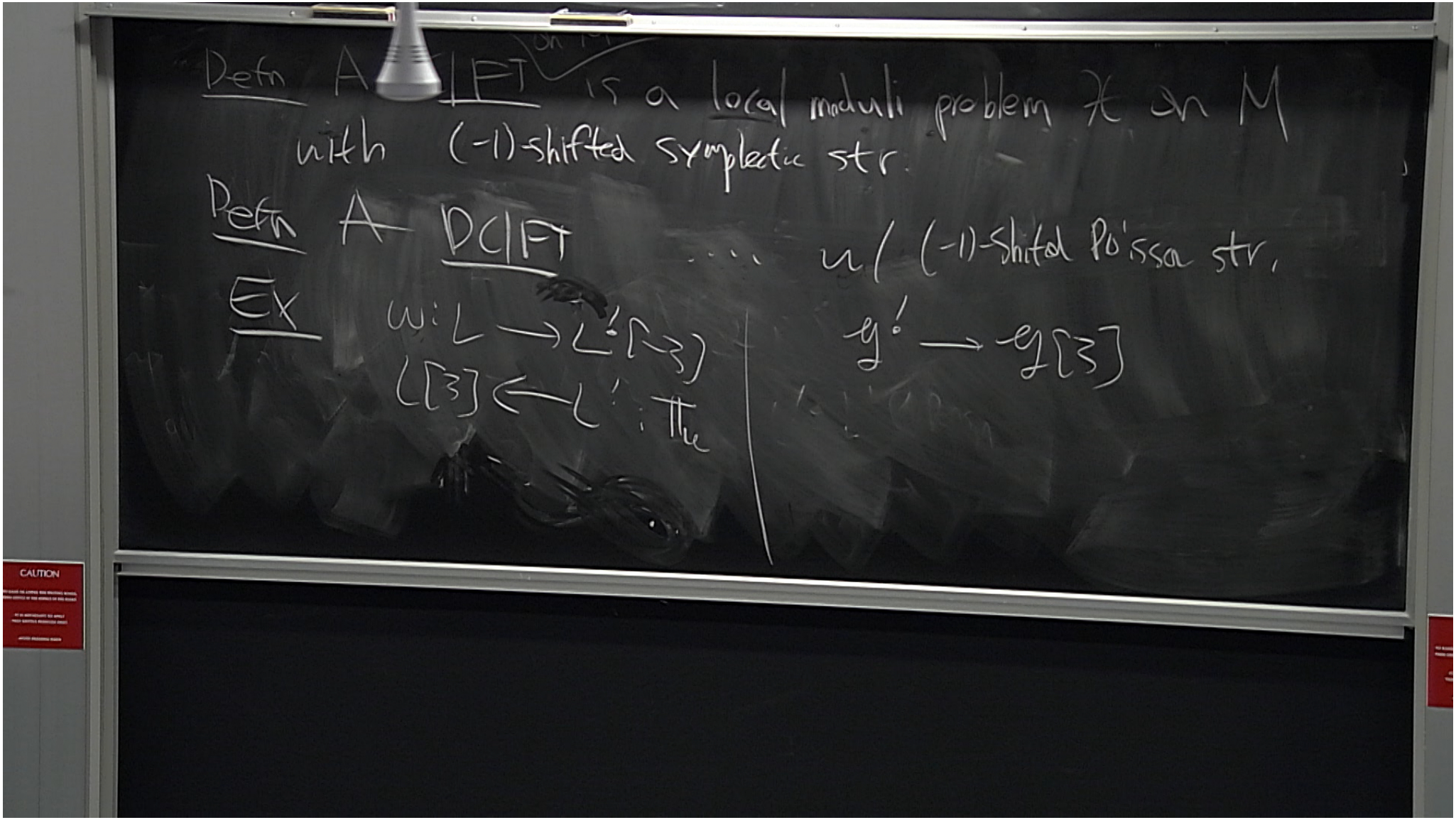

3
$\mathrm{Fact}(\partial M)$ 0-shifted Sympl
$\mathrm{Fact}(M) \times_{\mathrm{Fact}(\partial M)} B$ where $B \xrightarrow{\text{lag}} \mathrm{Fact}(\partial M)$

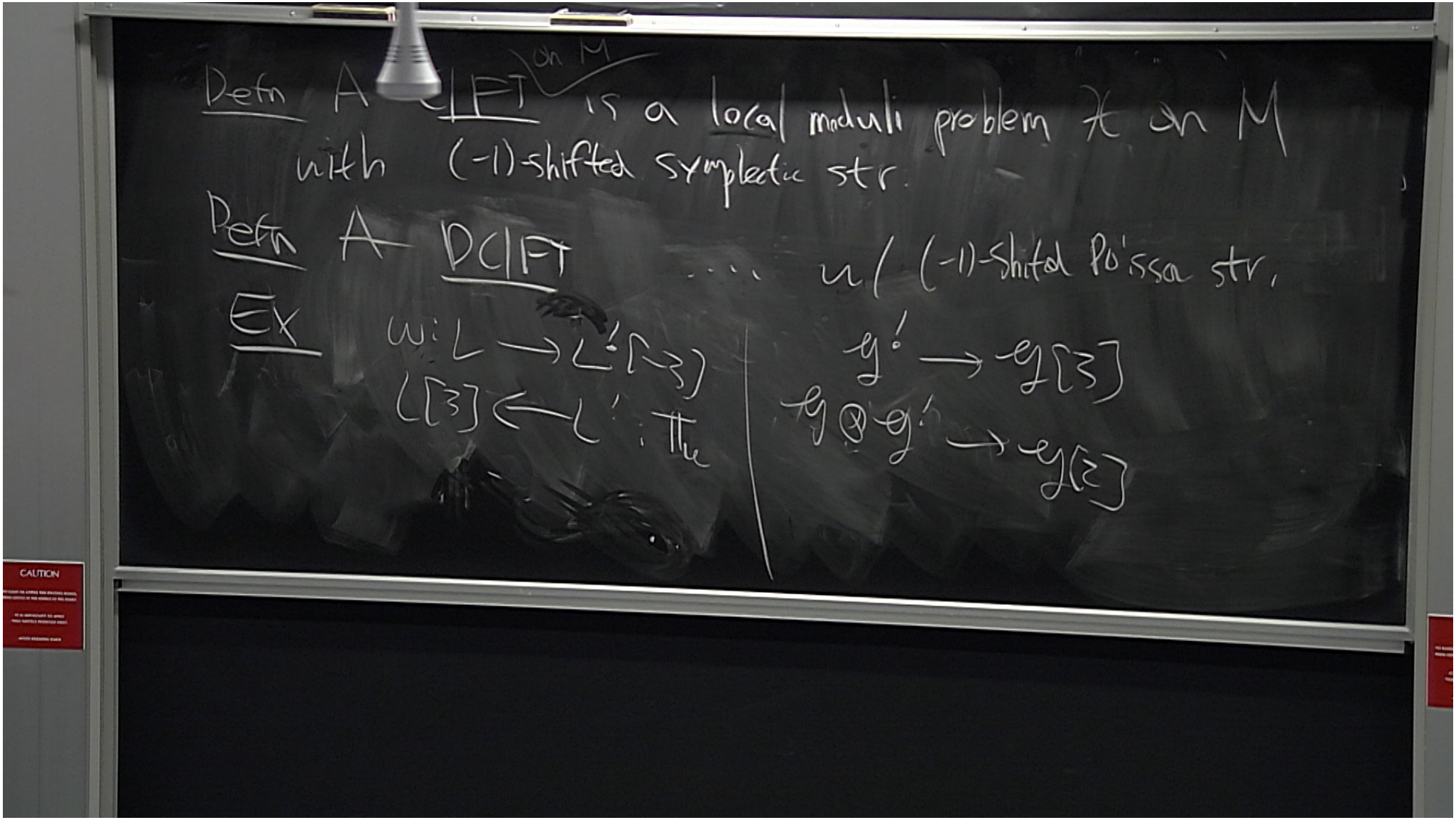

3
$Fact(\partial M)$ 0-shifted symp'
$Fact(M) \times_{Fact(\partial M)} B$ where $B \xrightarrow{\text{lag}} Fact(\partial M)$
B (-1)-shifted Poisson

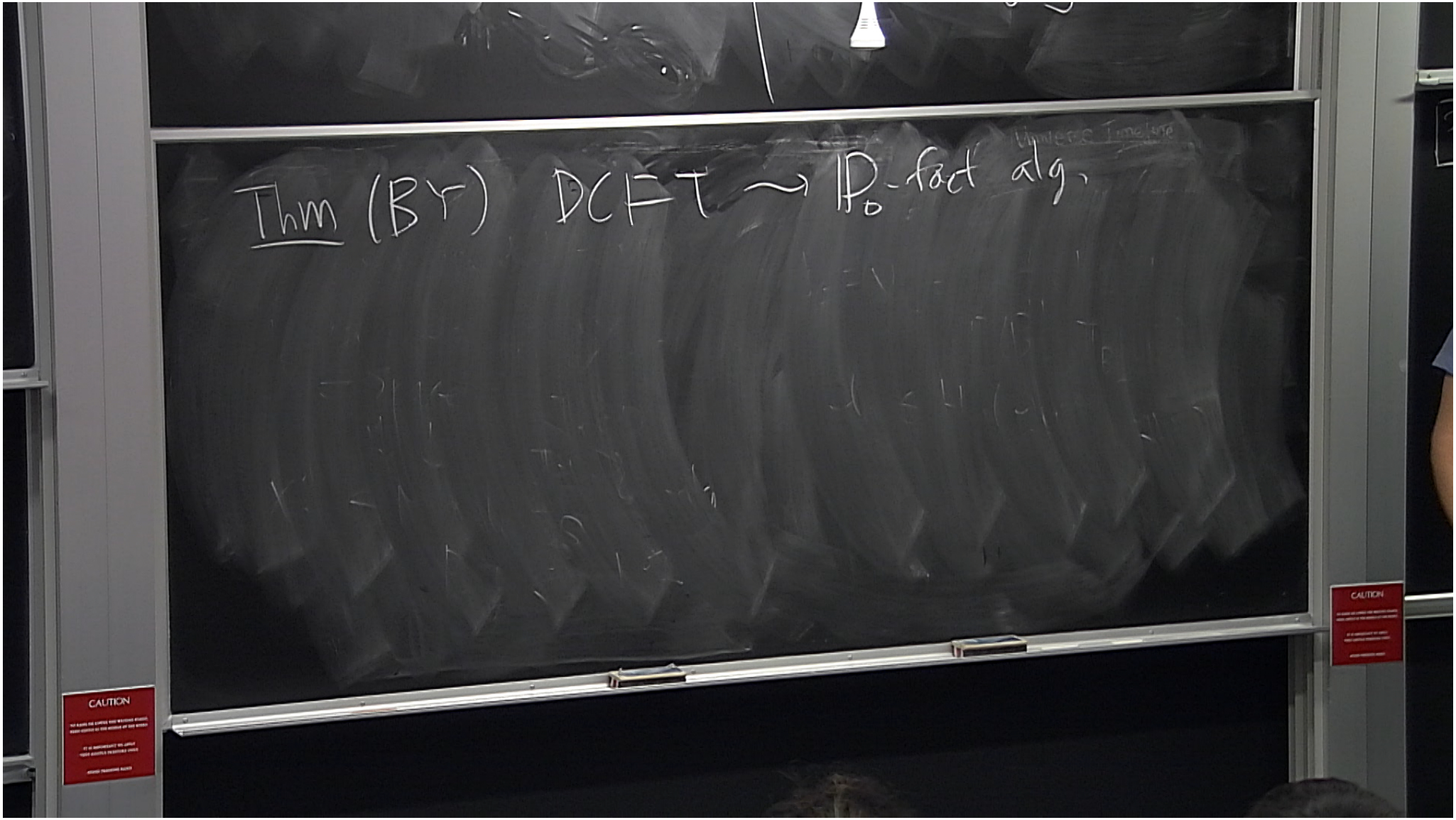

Defn A CLFT on M is a local moduli problem $\mathcal{X}$ on M
with (-1)-shifted symplectic str.
Defn A DCLFT ... w/ (-1)-shifted Poisson str.

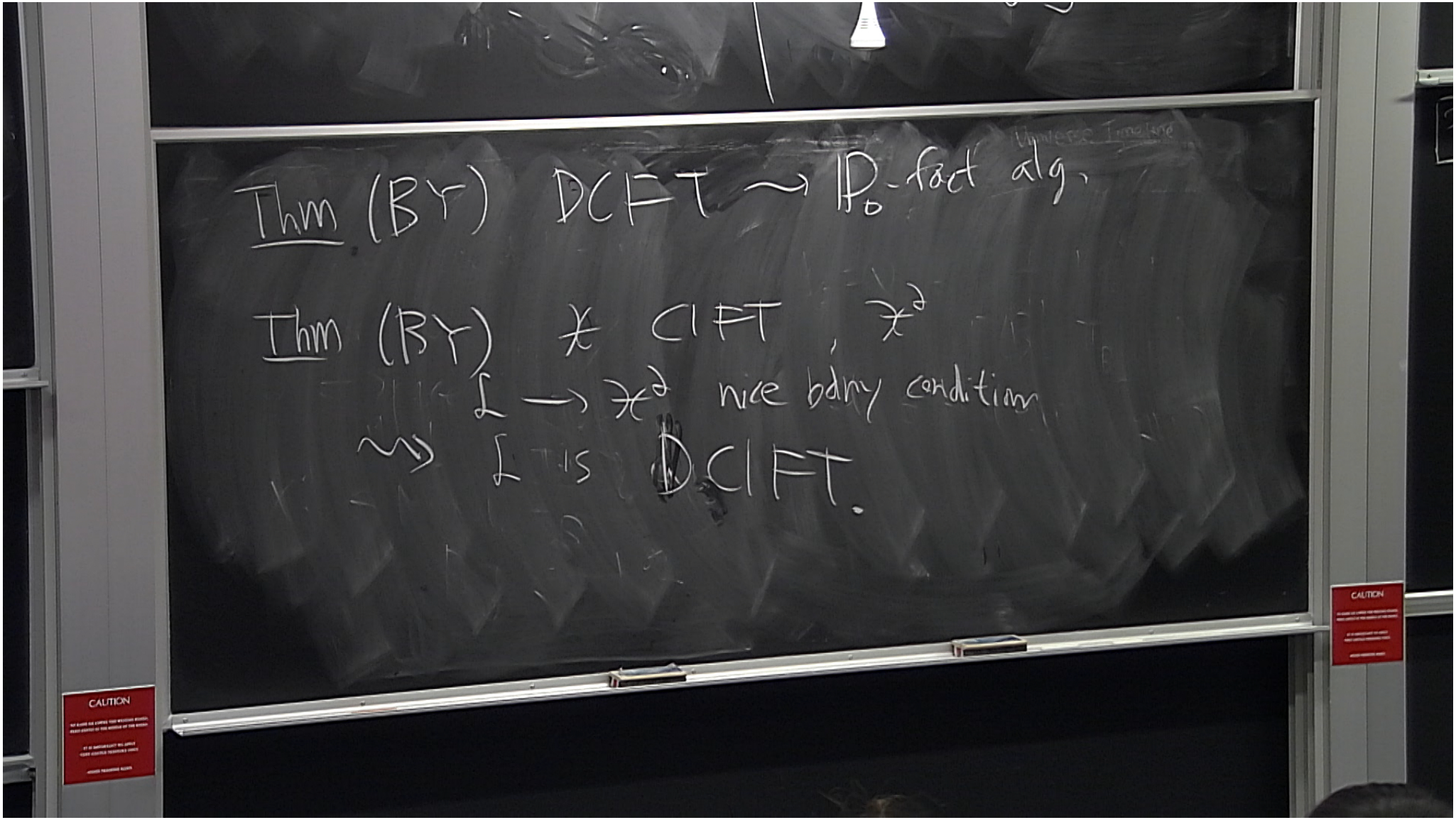

Defn A IFT is a local moduli problem $\mathcal{X}$ on M
with (-1)-shifted symplectic str.
Defn A DCIFT ... w/ (-1)-shifted Poisson str.
Ex $\omega: L \to L^![-3]$
CAUTION

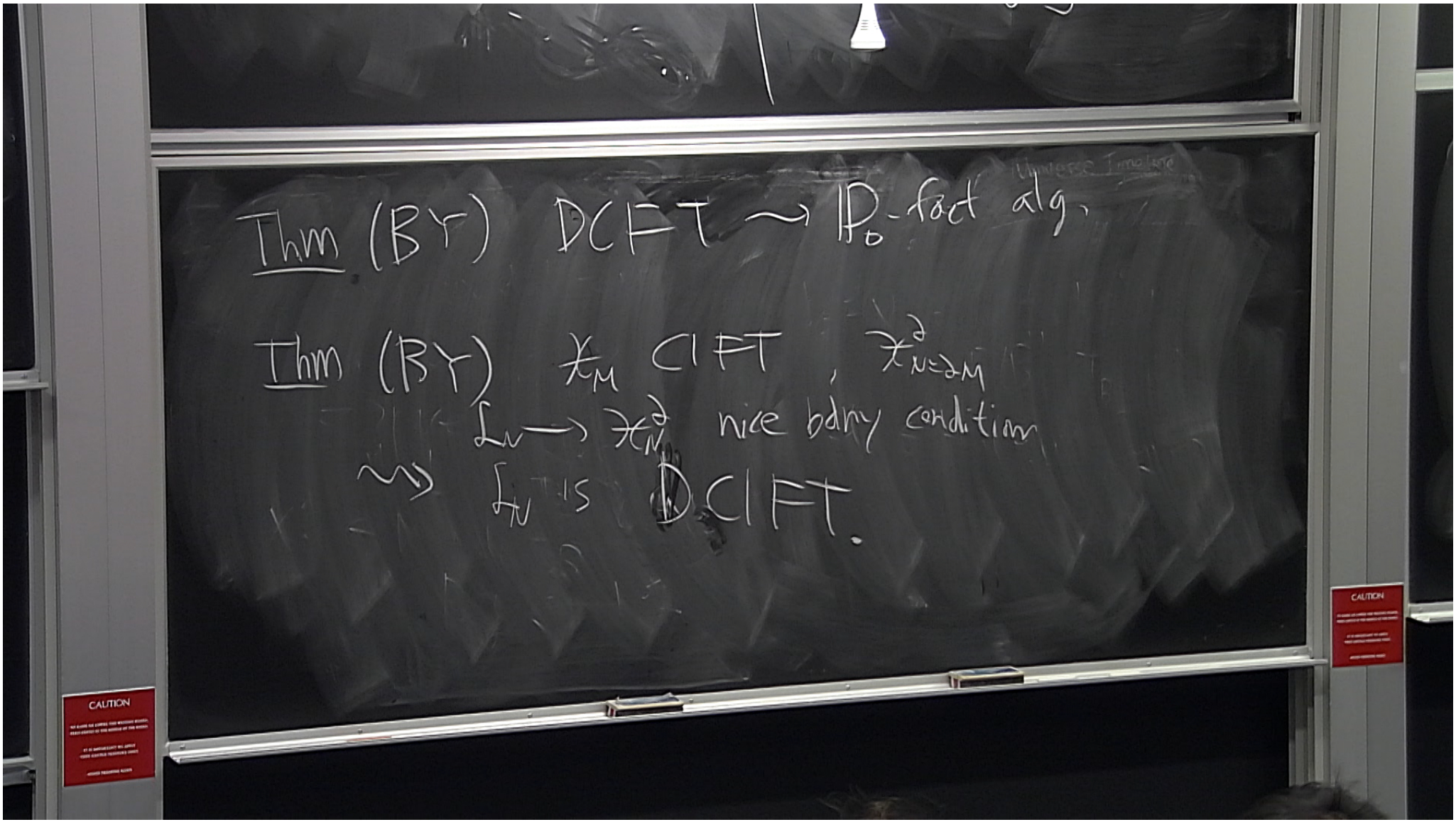

Defn A LFT is a local moduli problem $\mathcal{X}$ on M
with (-1)-shifted symplectic str.
Defn A DCLFT ... w/ (-1)-shifted Poisson str.
Ex $w: L \to L^![-3]$
$L[3] \leftarrow L'$: the
CAUTION

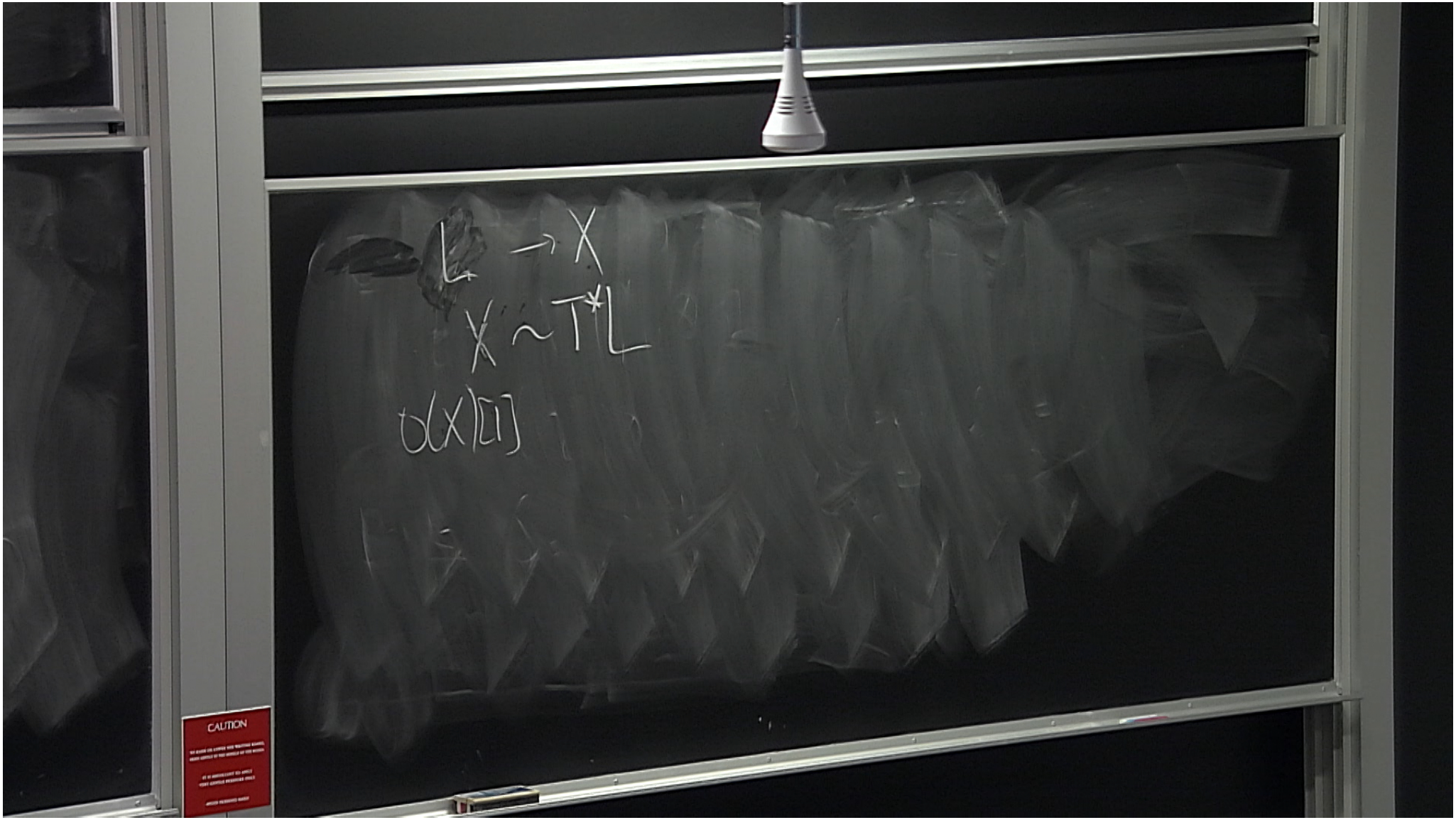

Defn A LFT is a local moduli problem $\mathcal{X}$ on M
with (-1)-shifted symplectic str.
Defn A DCLFT ... w/ (-1)-shifted Poisson str.
Ex $w: L \to L^![-3]$
$L[3] \leftarrow L^!$: the
$\gamma^! \to \gamma[3]$
CAUTION

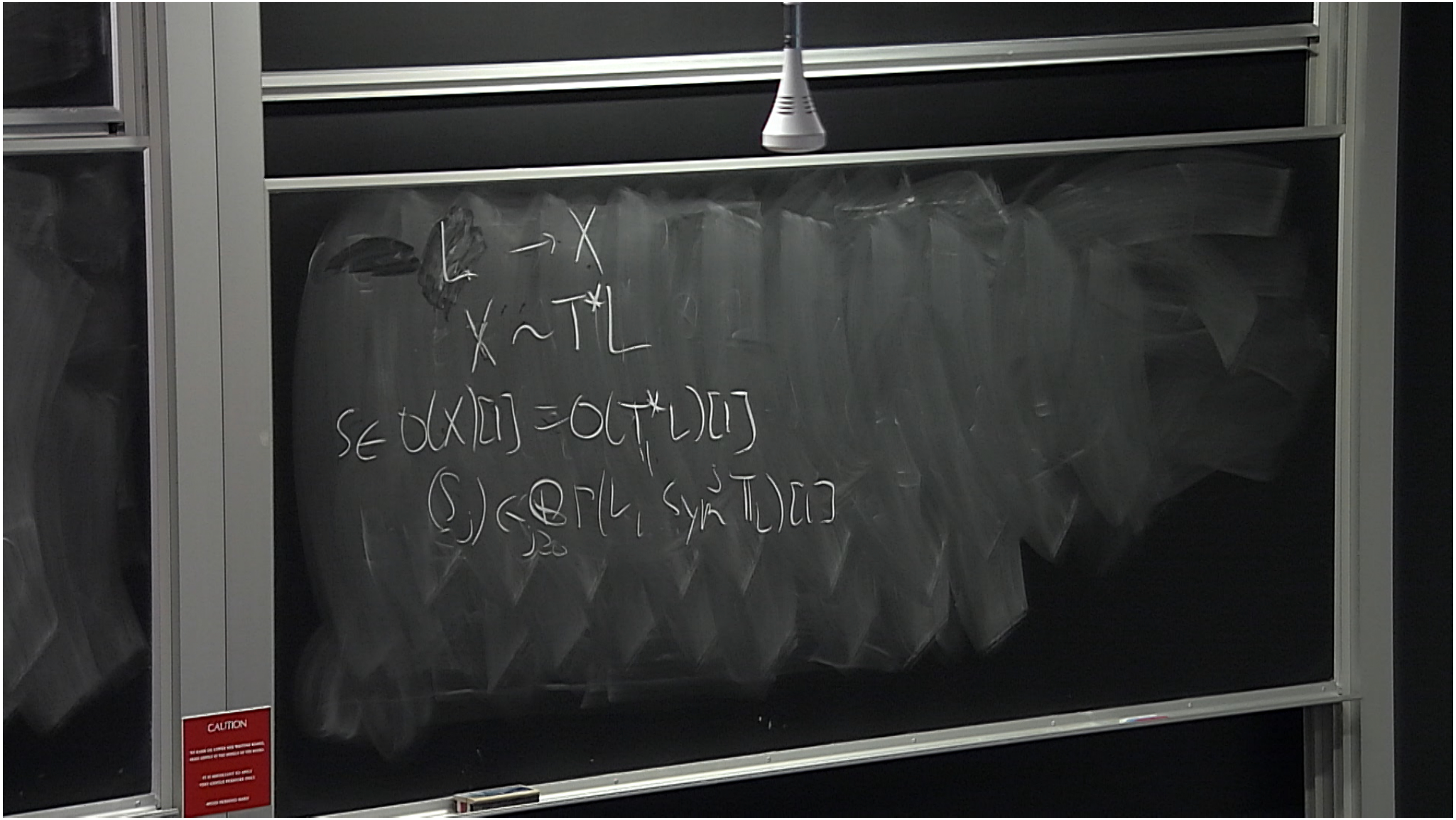

Defn A CIFT on M is a local moduli problem X on M
with (-1)-shifted symplectic str.
Defn A DCIFT ... w/ (-1)-shifted Poisson str.
Ex w: L → L^![-3]
L[3] ← L^!: the
y^! → y[3]
y ⊗ y^! → y[2]
CAUTION

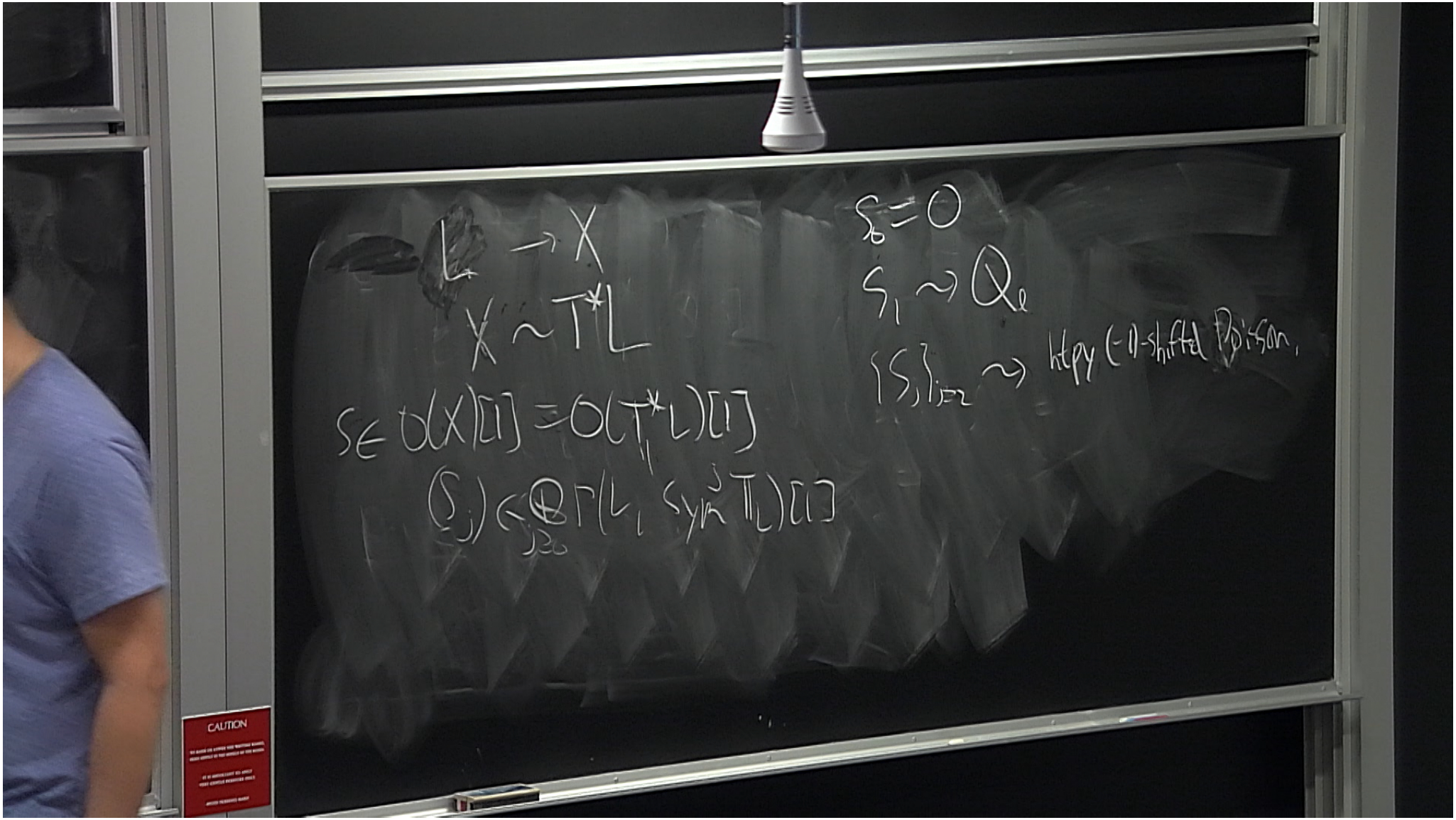

Thm (Br) DCFT ~ $\mathbb{P}_0$-fact alg.
CAUTION
CAUTION

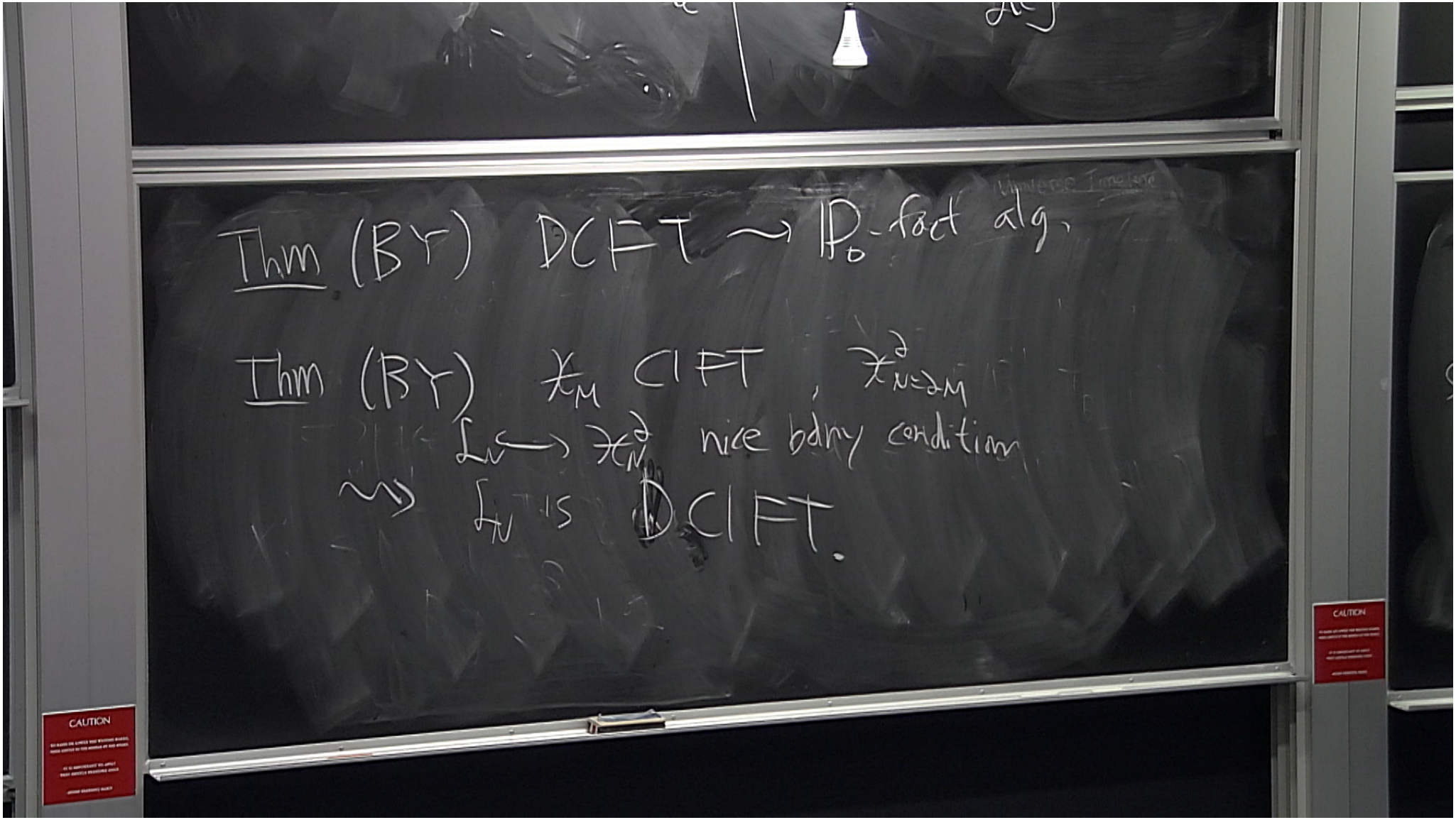

Thm (BY) DCFT ⇝ $\mathbb{P}_0$-fact alg.
Thm (BY) $\mathcal{I}$ CIFT, $\mathcal{I}^d$
$\mathcal{I} \to \mathcal{I}^d$ nice bdry condition
⇝ $\mathcal{I}$ is DCIFT.

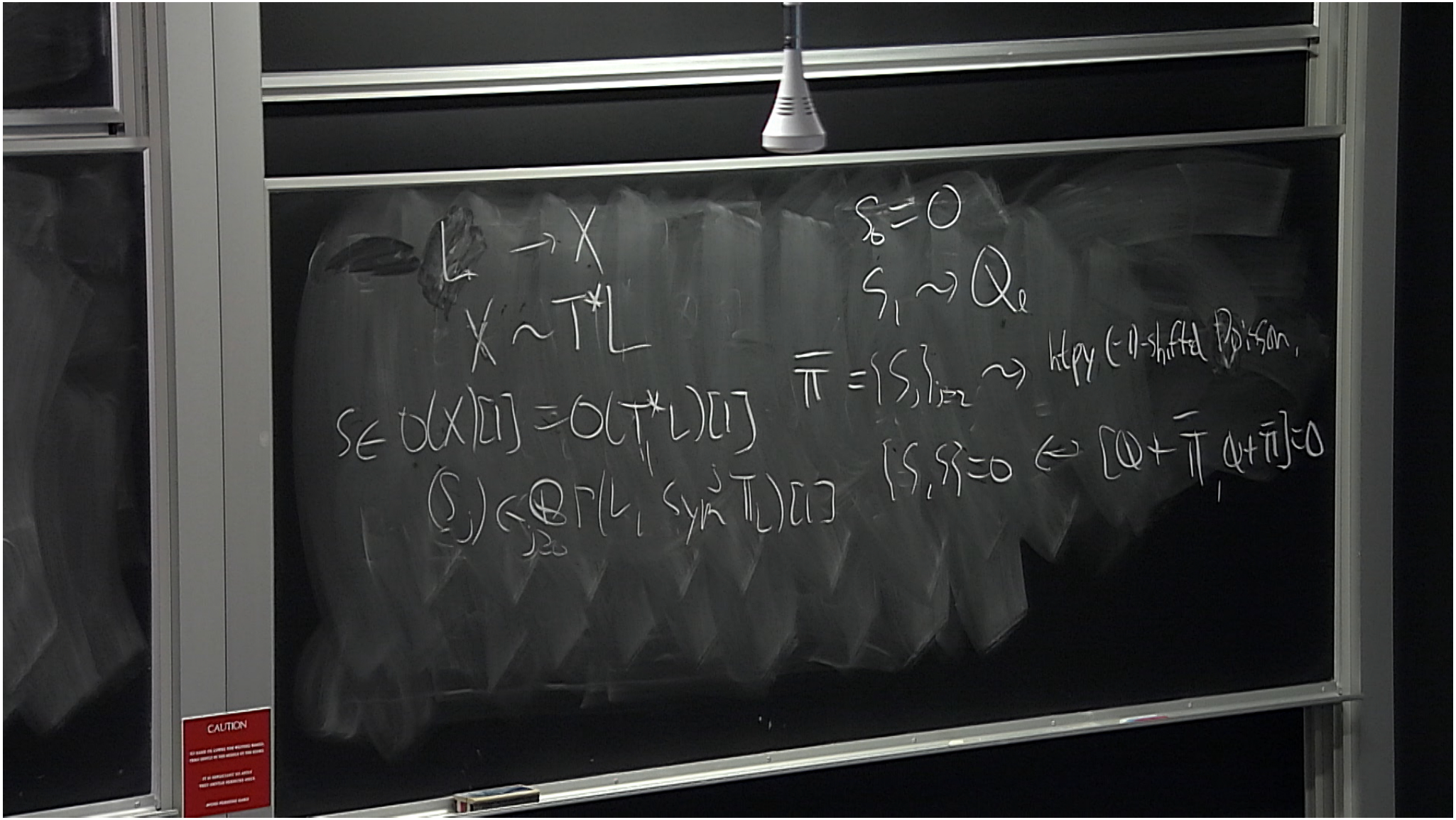

Thm (BY) DCFT ⇝ $\mathbb{P}_0$-fact alg.
Thm (BY) $\mathcal{X}_M$ CFT, $\mathcal{X}^2_{N=\partial M}$
$I_N \longrightarrow \mathcal{X}^2_N$ nice bdry condition
⇝ I_N is DCFT.
CAUTION
CAUTION

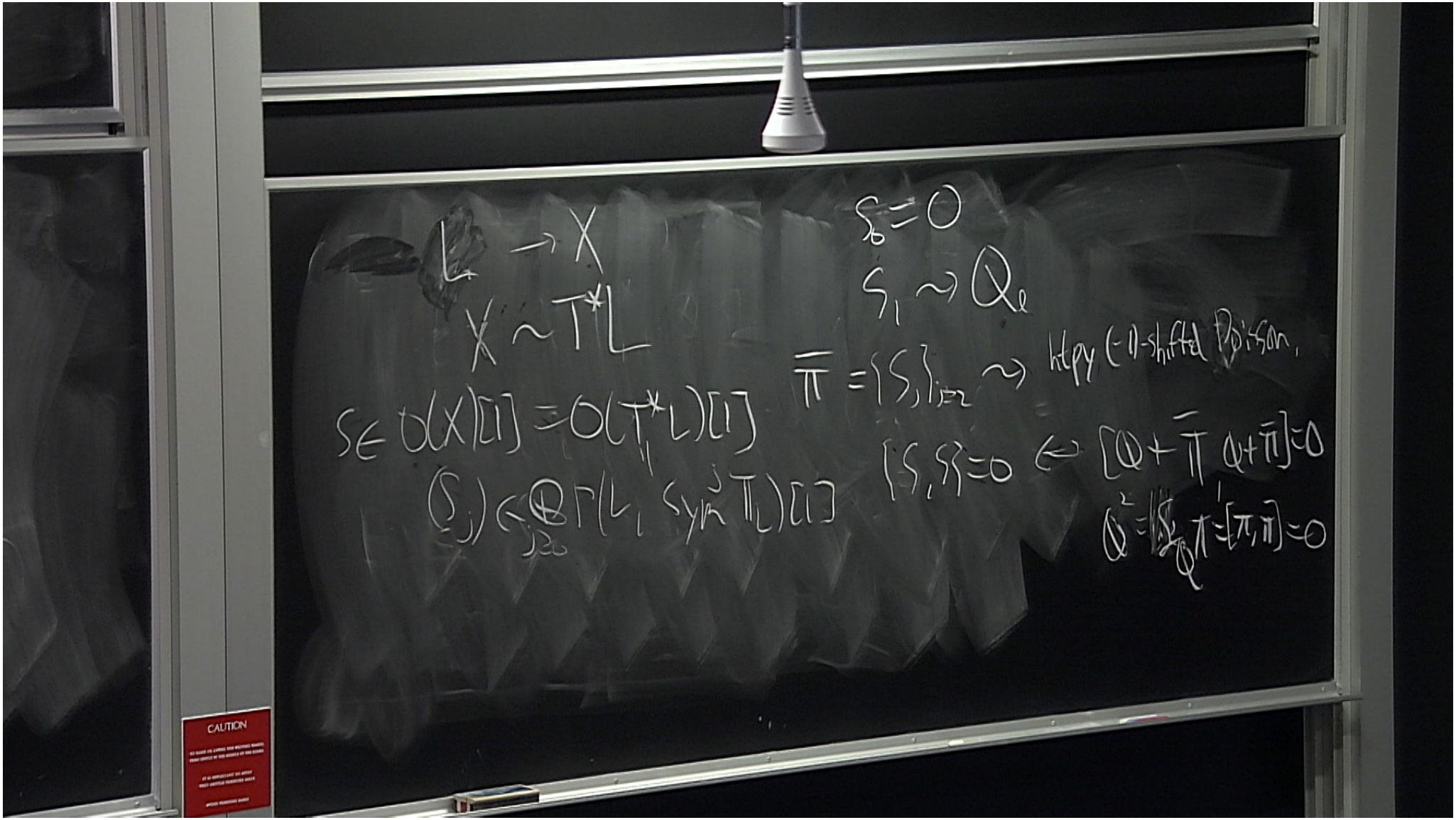

L → X
X ~ T*L
O(X)[1]
CAUTION

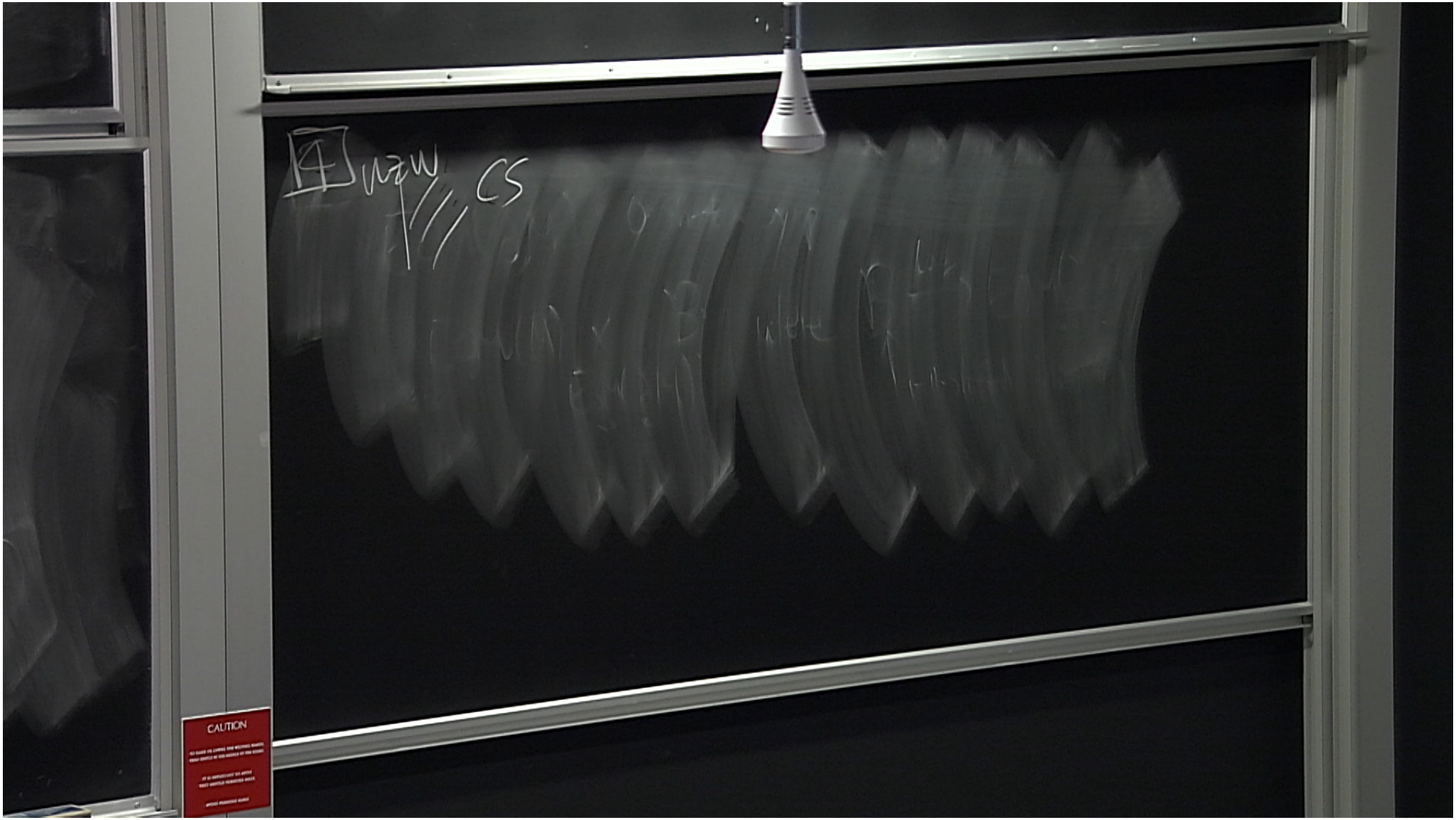

L → X
X ~ T*L
S ∈ O(X)[1] = O(T*L)[1]
CAUTION

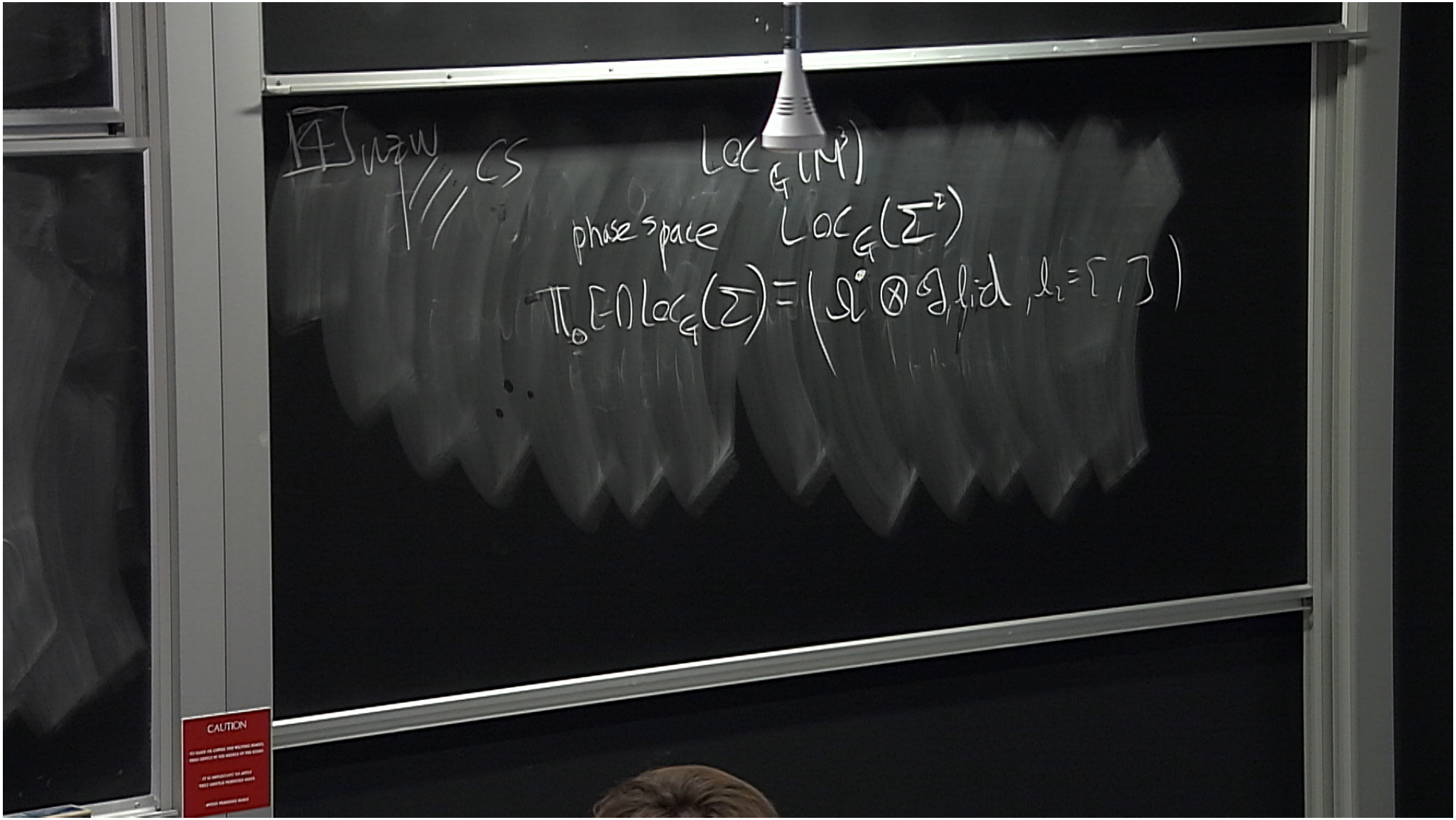

L → X
X ~ T*L
S ∈ O(X)[0] = O(T*L)[0]
S_0 = 0
S_1 ~ Q_ℓ
{S_j}_{j≥2} ~ htpy (-1-shifted Poisson,
CAUTION

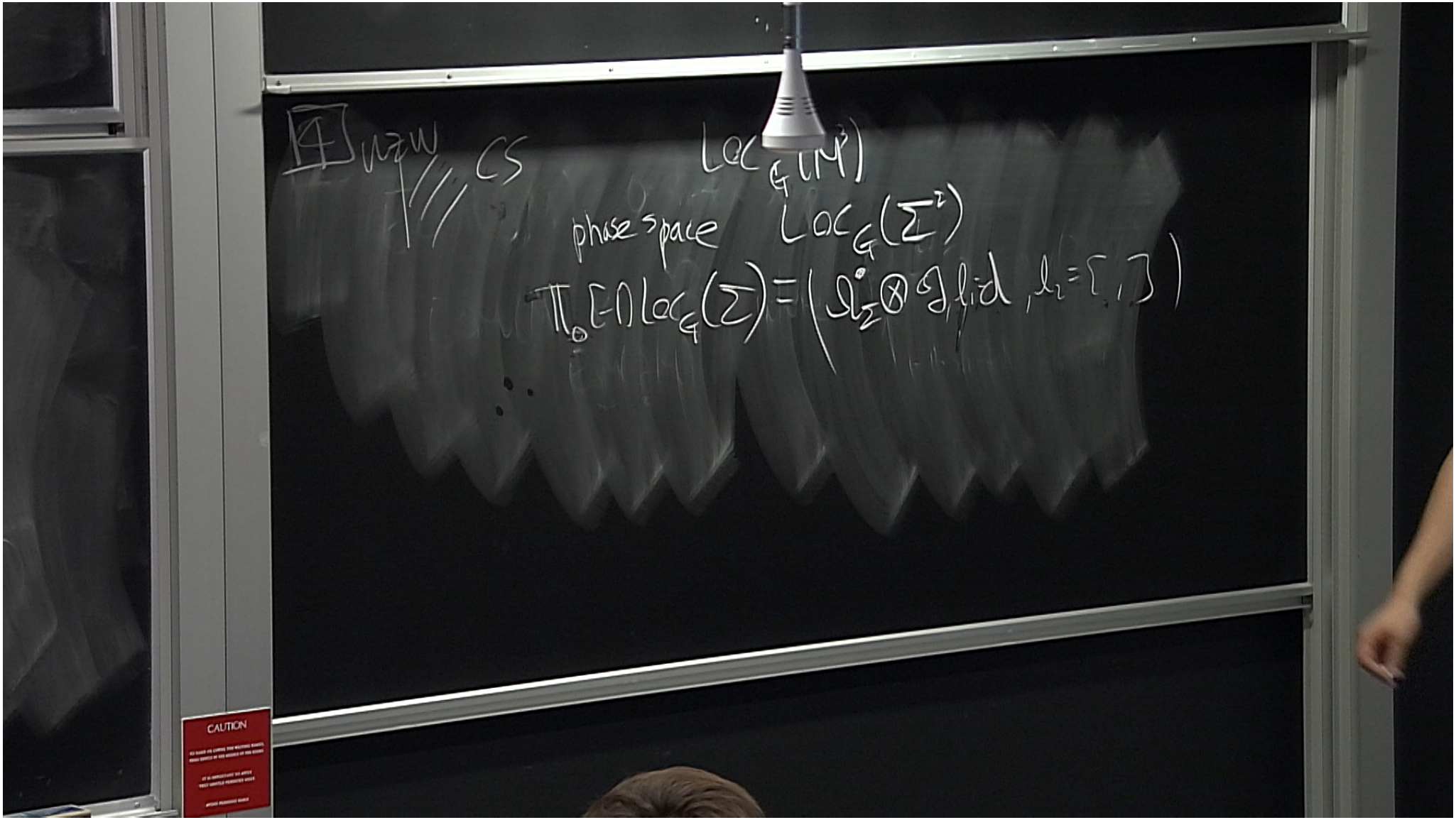

Thm (BY) DCFT ⇝ $\mathbb{P}_0$-fact alg.
Thm (BY) $\mathcal{X}_M$ CIFT, $\mathcal{Z}^2_{N=\partial M}$
$L_N \longrightarrow \mathcal{Z}^2_N$ nice bdry condition
⇝ L_N is DCIFT.
CAUTION
CAUTION

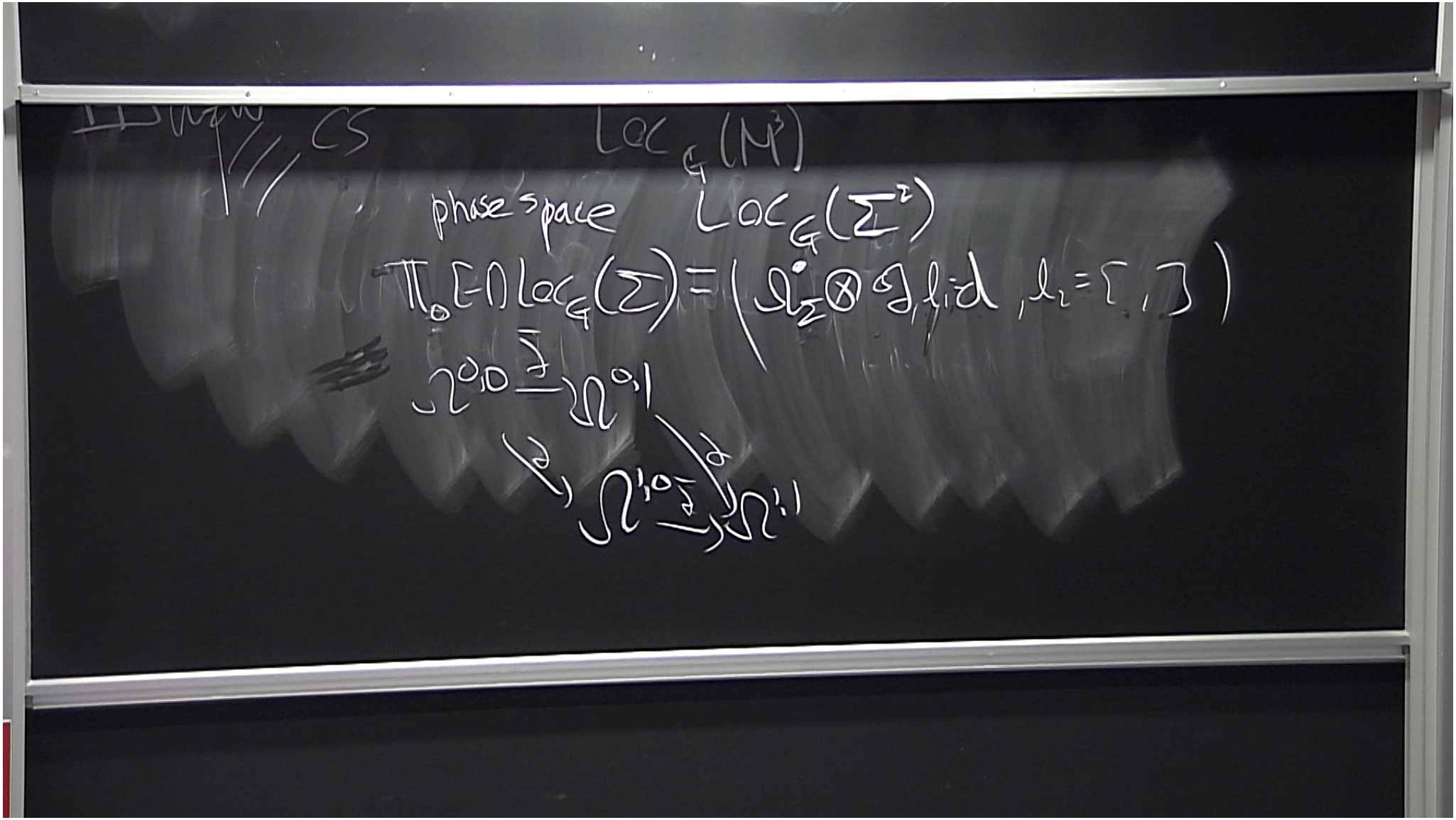

$L \to X$
$X \sim T^*L$
$S_0 = 0$
$S_1 \sim Q_\ell$
$\bar{\pi} = \{S_j\}_{j \geq 2} \sim$ Holpy (-1)-shifted Poisson.
$S \in \mathcal{O}(X)[1] = \mathcal{O}(T^*L)[1]$
$(S_j) \in \bigoplus_{j \geq 0} \Gamma(L, \mathrm{Sym}^j \mathbb{T}_L)[1]$
$\{S, S\} = 0 \Leftrightarrow [Q + \bar{\pi}, Q + \bar{\pi}] = 0$
CAUTION

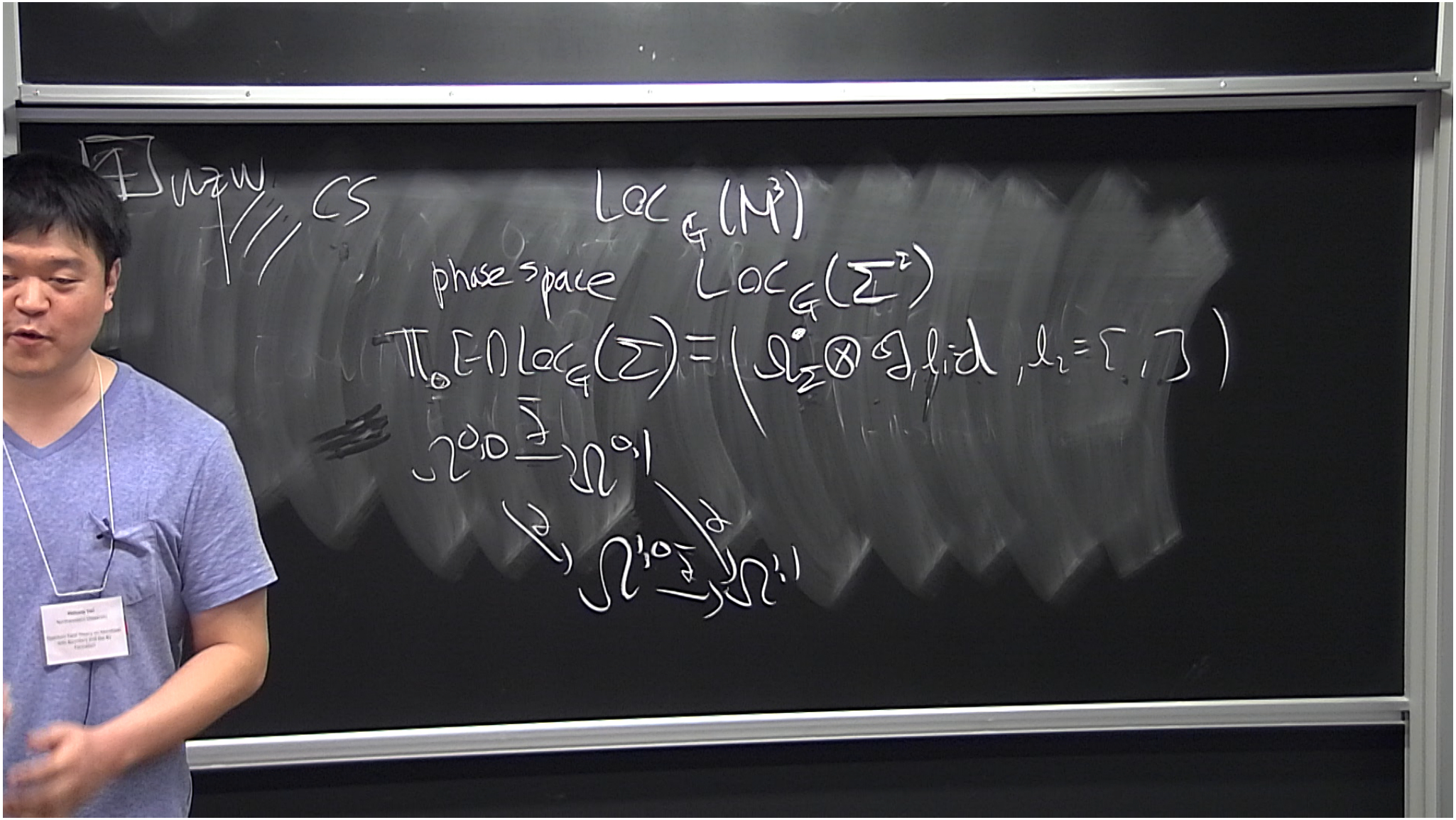

$L \to X$
$X \sim T^*L$
$S_0 = 0$
$S_1 \sim Q_\ell$
$\pi = \{S_j\}_{j \geq 2} \sim$ Hopf (-1)-shifted Poisson.
$S \in \mathcal{O}(X)[1] = \mathcal{O}(T^*L)[1]$
$(S_j) \in \bigoplus_{j \geq 0} \Gamma(L, \mathrm{Sym}^j T_L)[1]$
$\{S, S\} = 0 \Leftrightarrow [Q + \pi, Q + \pi] = 0$
$Q^2 = 0, \quad [Q, \pi] = 0, \quad [\pi, \pi] = 0$
CAUTION

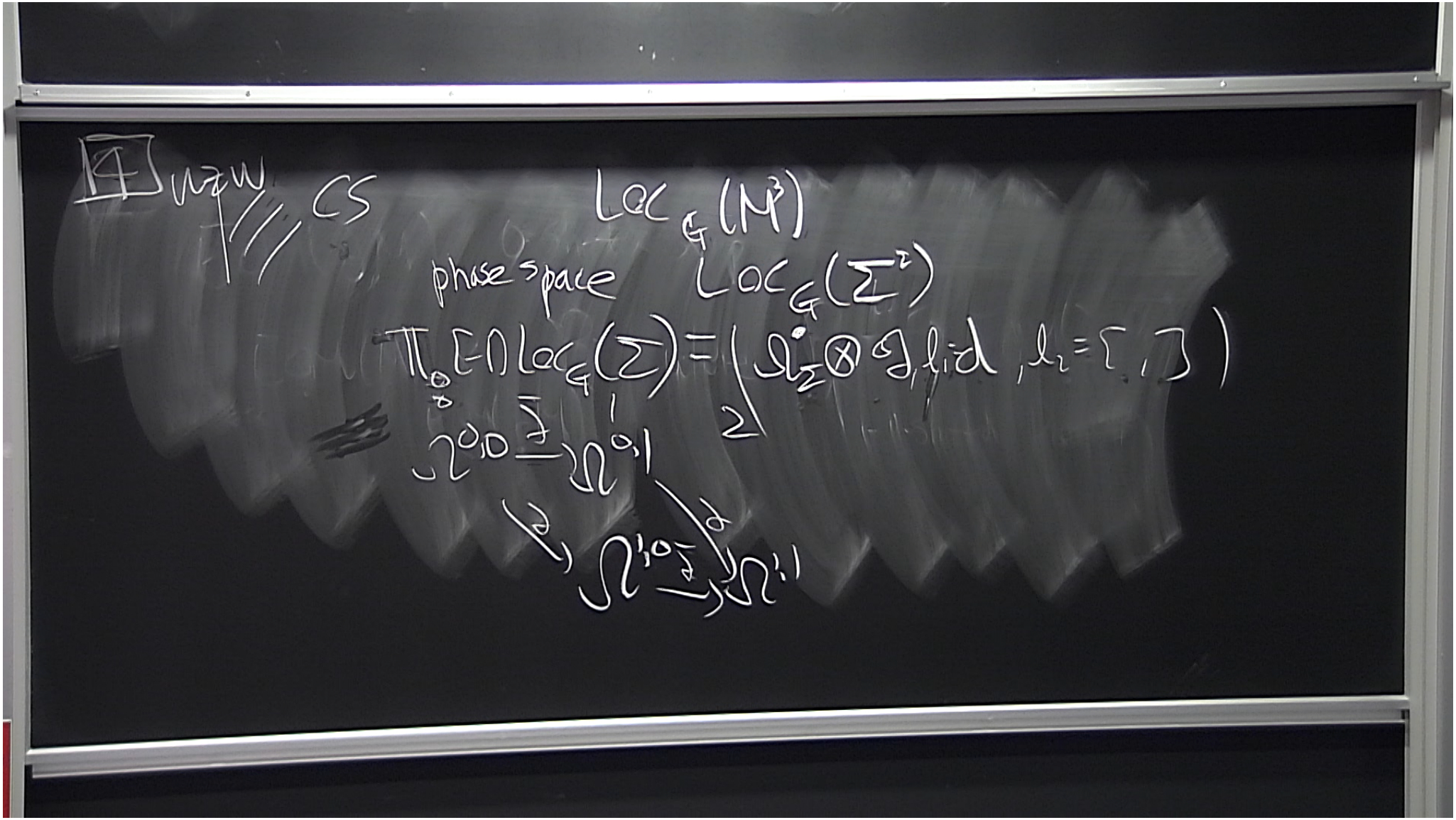

CAUTION

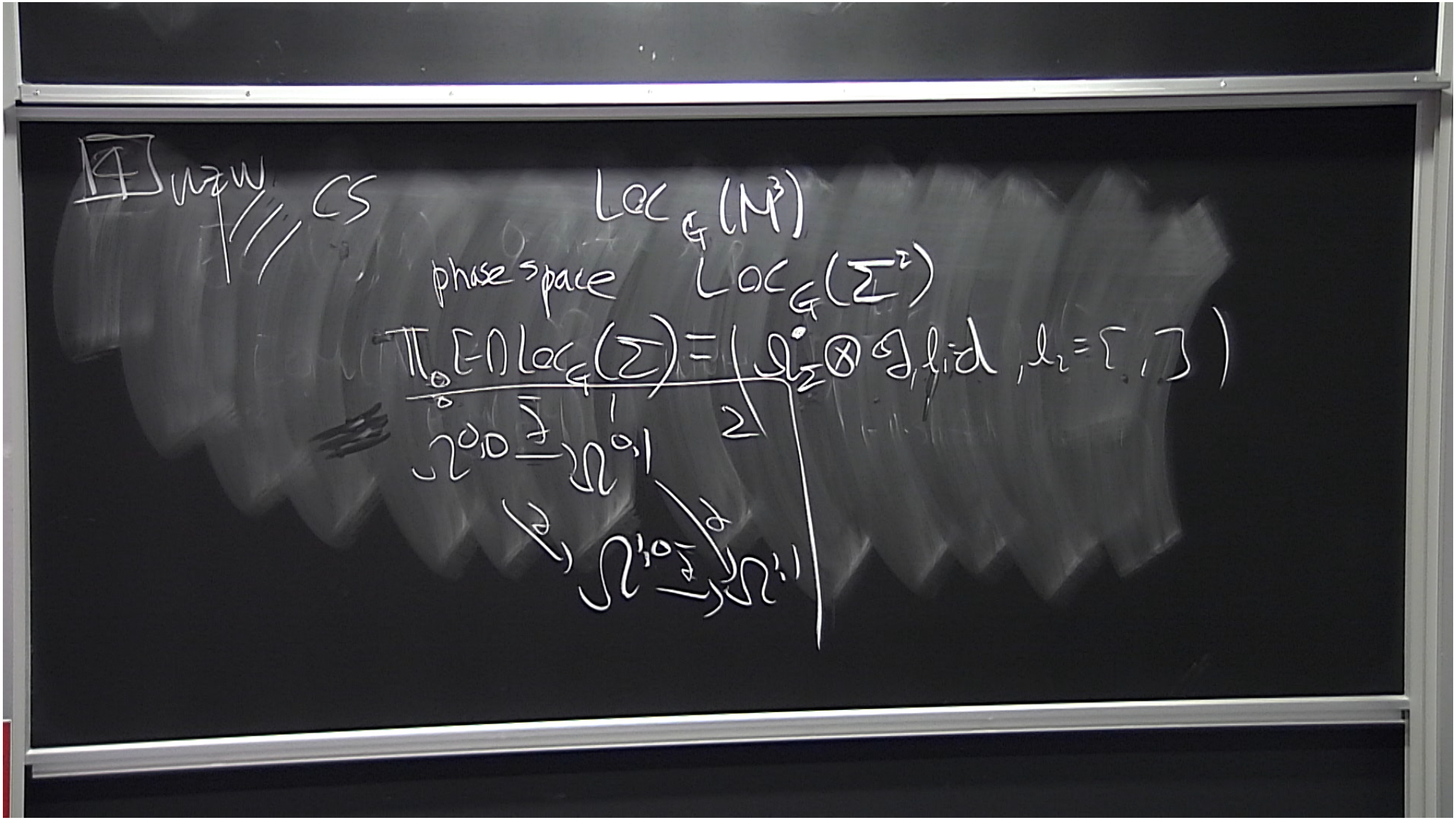

CS
phase space
CAUTION

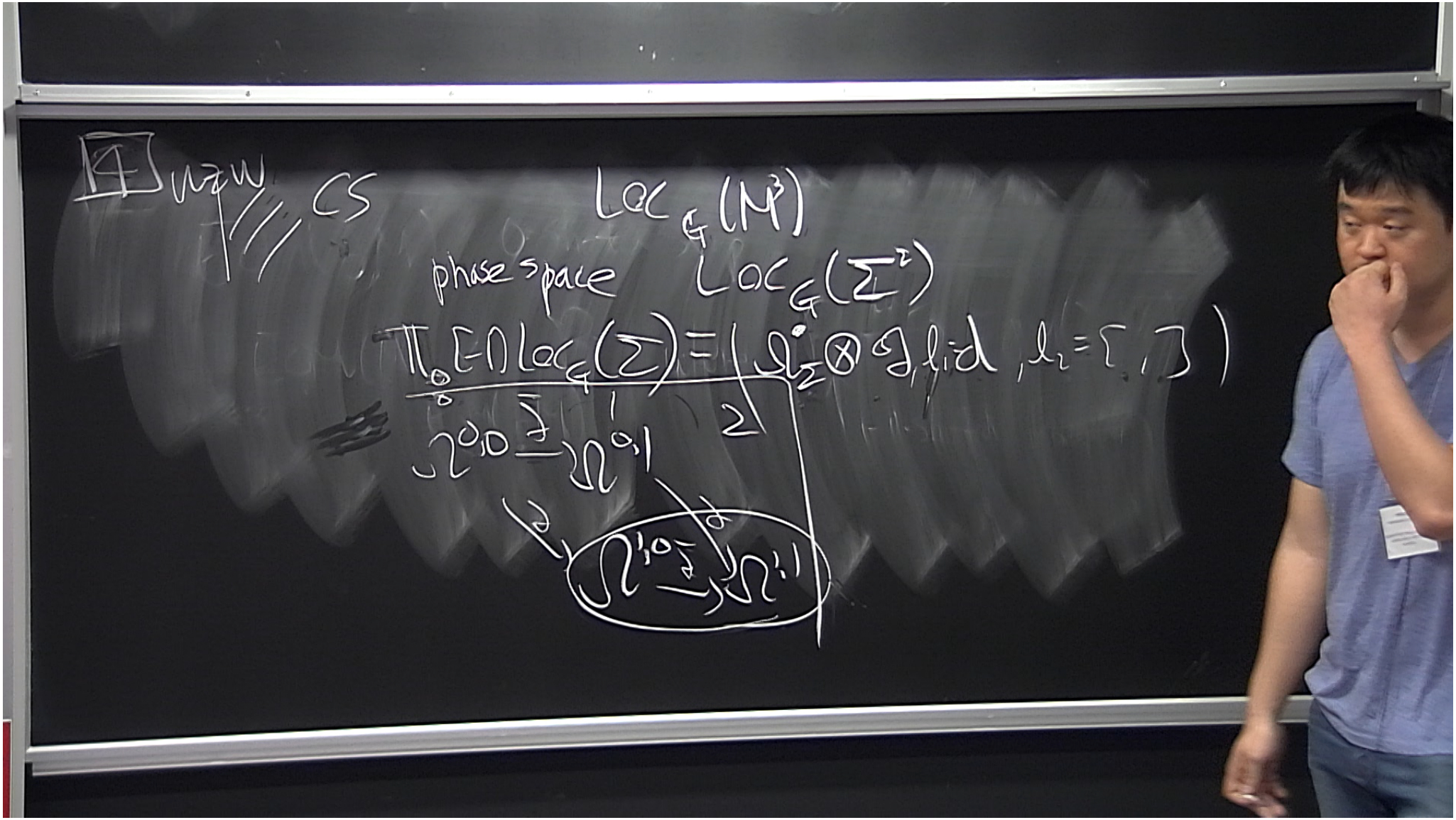

CS
phase space
CAUTION

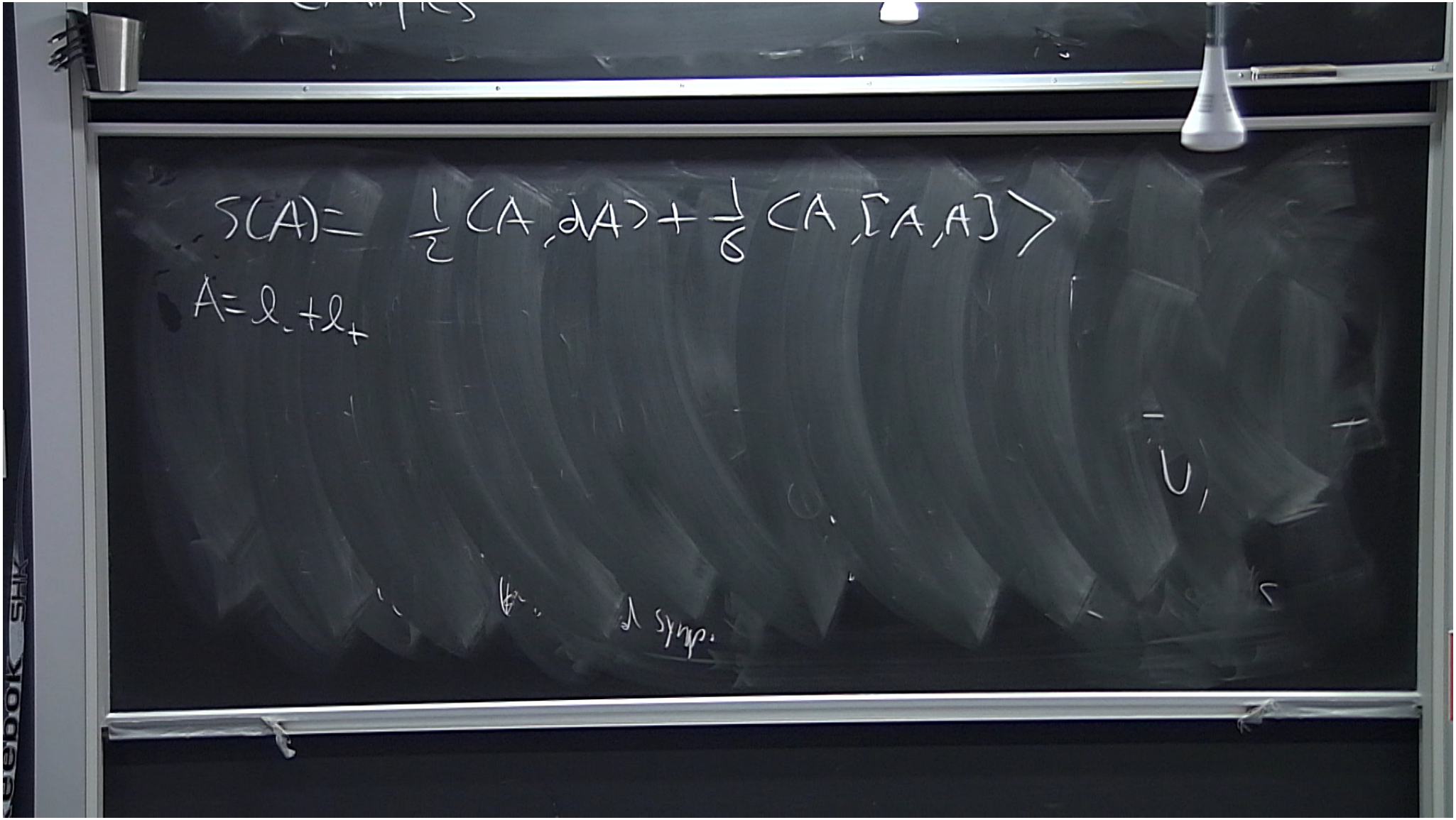

CS
$Loc_G(M^3)$
phase space
$Loc_G(\Sigma^2)$
$\pi_0 \Gamma(\Omega^{0,1} Loc_G(\Sigma)) = (\mathfrak{sl}_2 \otimes \mathcal{O}_{fld}, d_1 = [\ ,\ ])$
$\Omega^{0,0}$
$\Omega^{0,1}$
$\Omega^{1,0}$
$\Omega^{1,1}$

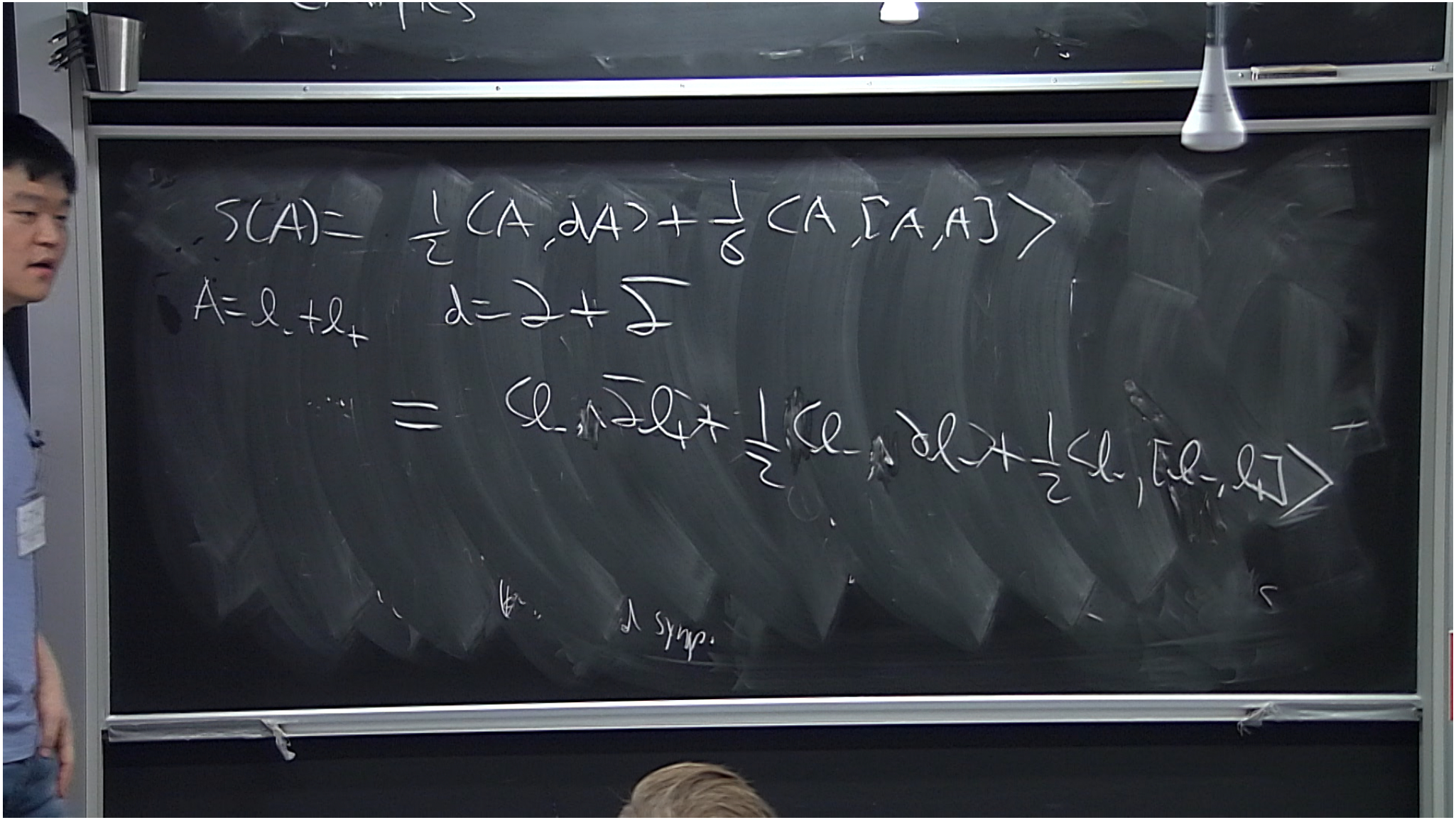

CS
$Loc_G(M^3)$
phase space
$Loc_G(\Sigma^2)$
$\pi_0 \Gamma(\mathcal{L}oc_G(\Sigma)) = (\mathfrak{sl}_2 \otimes g, \text{fid}, k_i = [\ , 3])$

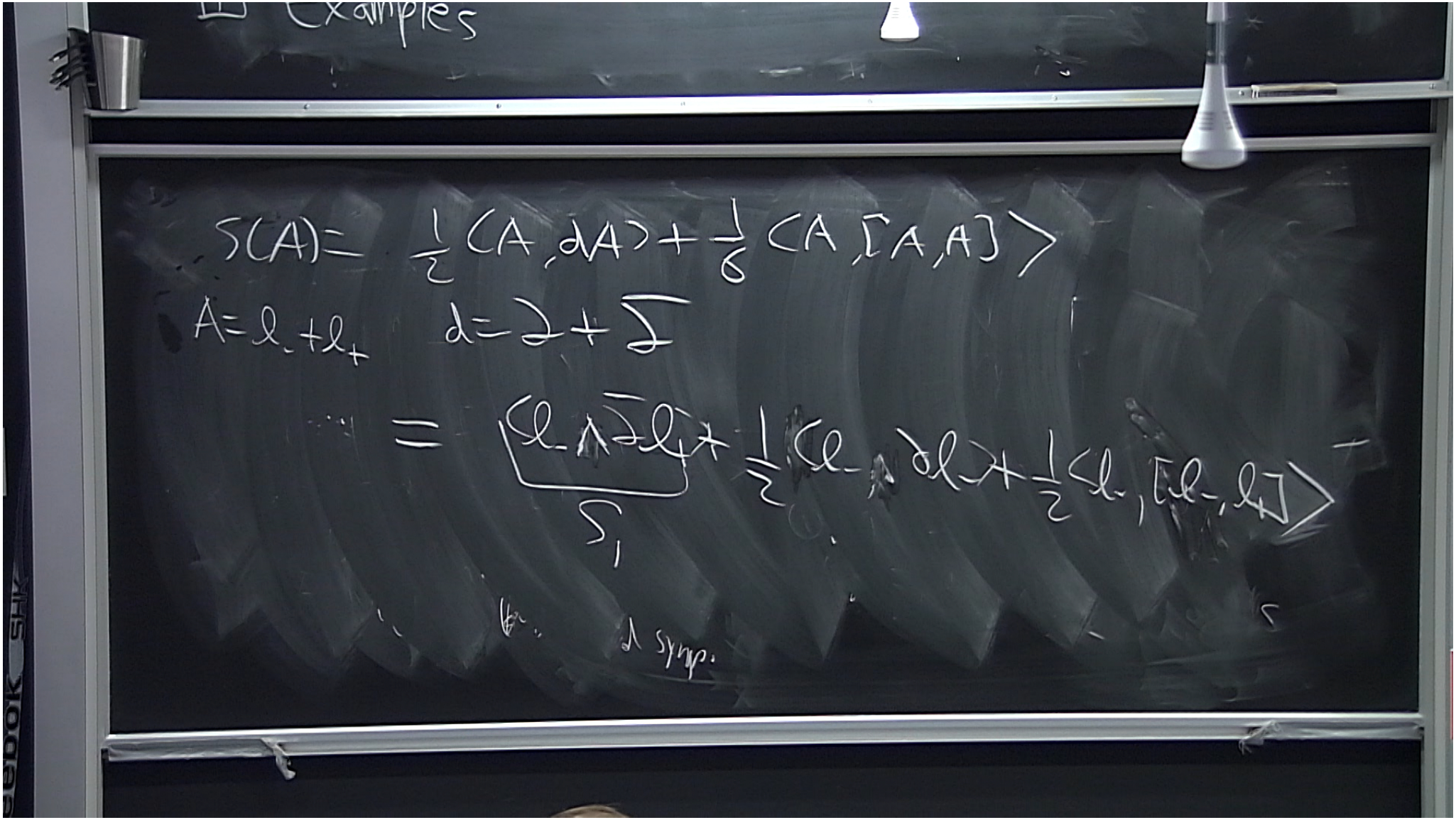

CS
$Loc_G(M^3)$
phase space
$Loc_G(\Sigma^2)$
$\pi_0 \mathcal{E}\text{-}1 Loc_G(\Sigma) = (\Omega^2_\Sigma \otimes g, f.d., d_i = 1, 3)$
$\Omega^{2,0}$
$\Omega^{0,1}$
$\Omega^{1,0}$
$\Omega^{0,1}$

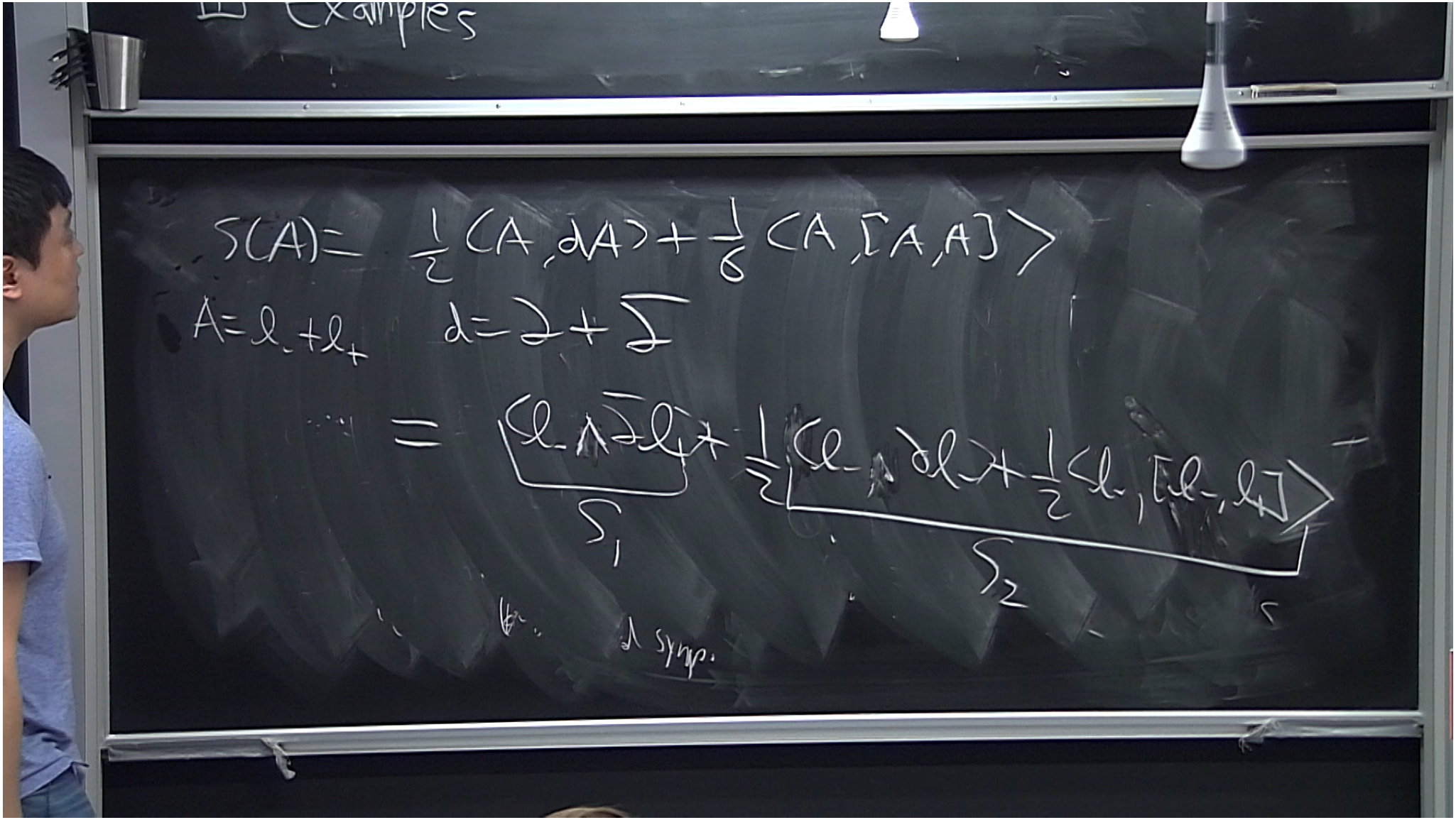

CS
$Loc_G(M^3)$
phase space
$Loc_G(\Sigma^2)$
$\pi_0 \Gamma \cap Loc_G(\Sigma) = (\Omega^\bullet_\Sigma \otimes g, \mathrm{fid}, d, d_i = \Gamma, \mathcal{J})$
$\Omega^{2,0}$
$\Omega^{0,1}$
$\Omega^{1,0}$
$\Omega^{1,1}$

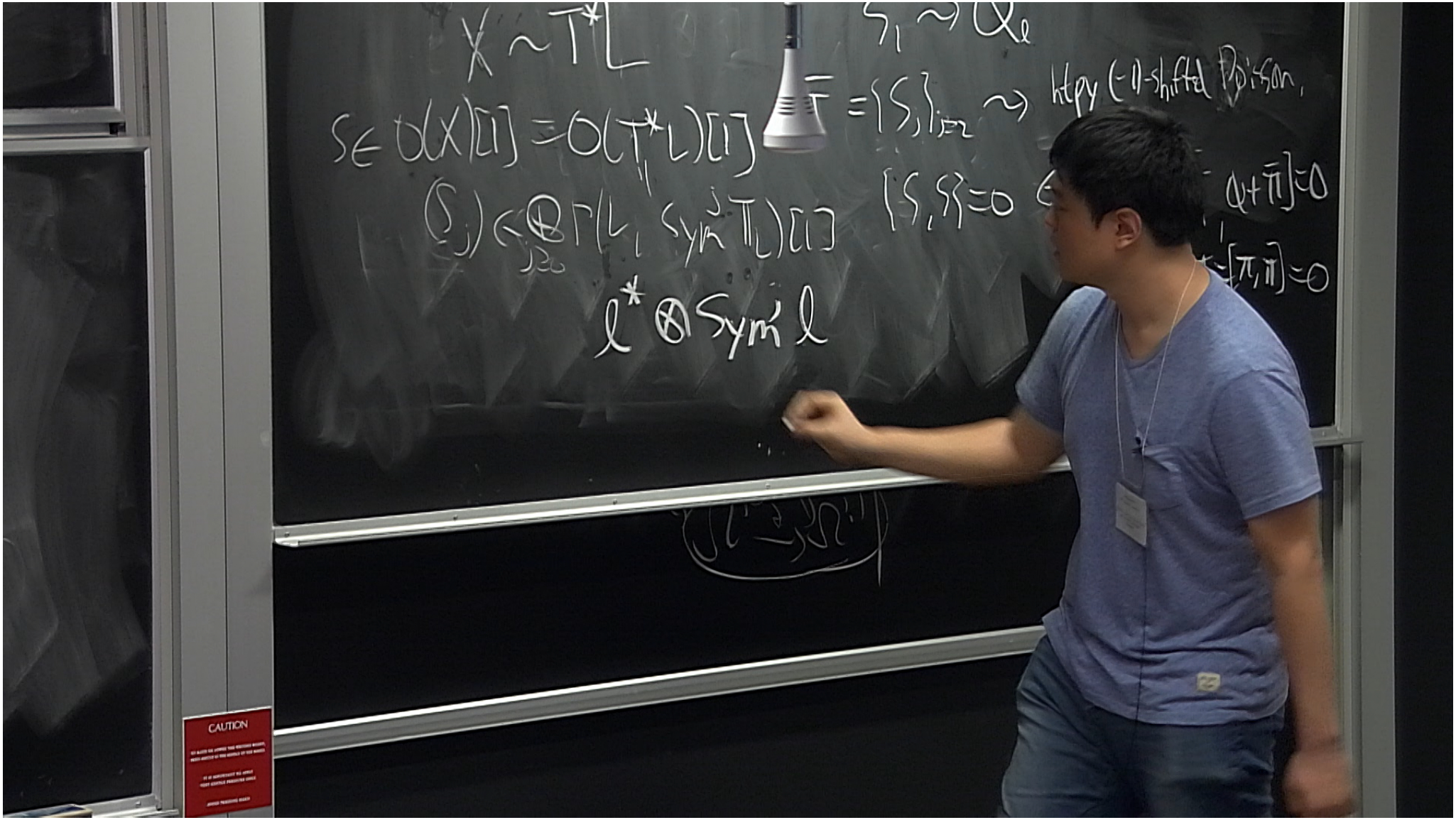

CS
$Loc_G(M^3)$
phase space
$Loc_G(\Sigma^2)$
$\pi_0 T^*[1] Loc_G(\Sigma) = (\Omega^*_{\Sigma} \otimes g, [\cdot,\cdot], d, \deg = [\ ,3])$
$\Omega^{2,0}$
$\Omega^{0,1}$
$\Omega^{1,0}$
$\Omega^{1,1}$

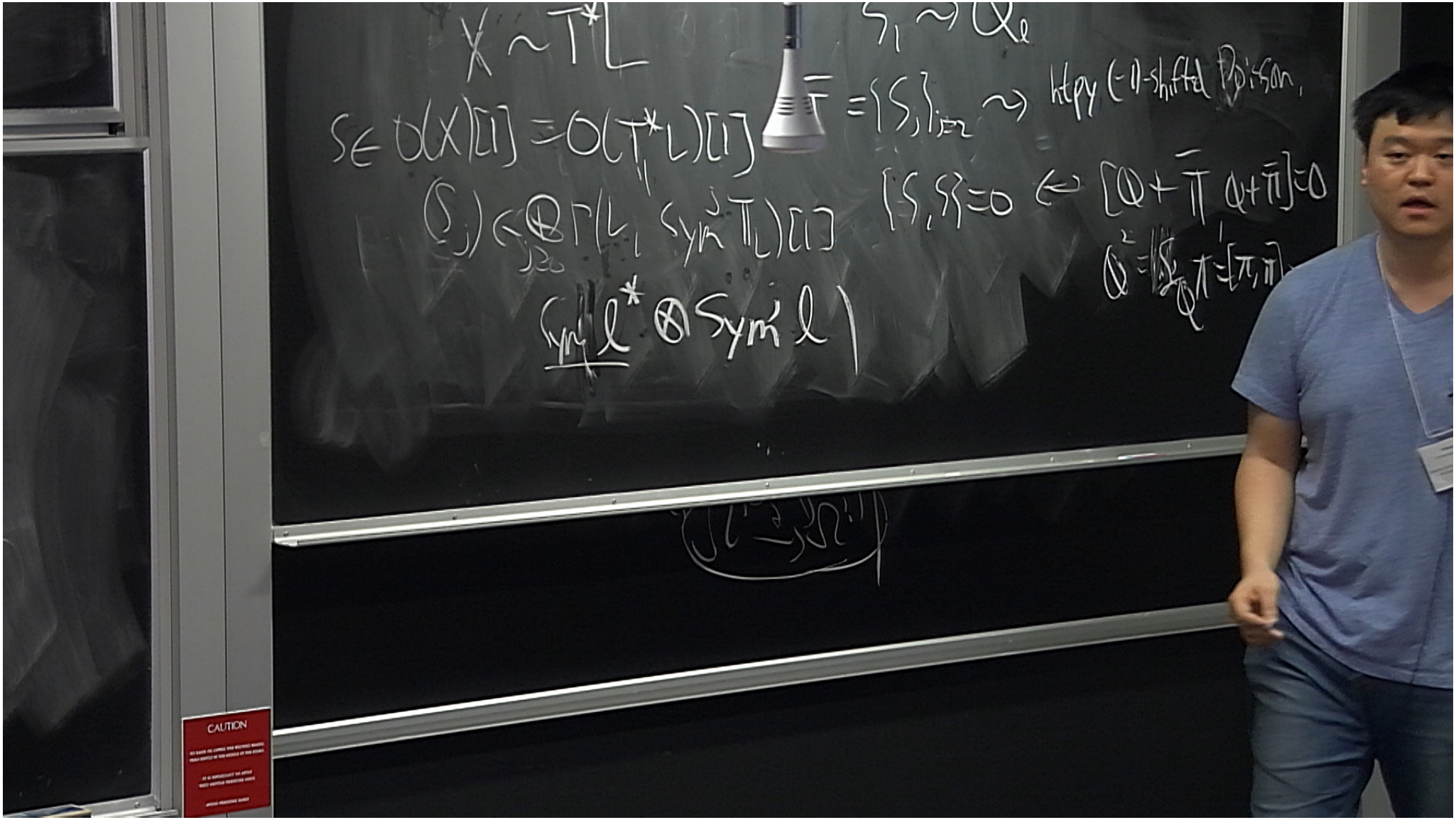

$S(A) = \frac{1}{2}\langle A, dA\rangle + \frac{1}{6}\langle A, [A, A]\rangle$
$A = \ell_- + \ell_+$
symp.

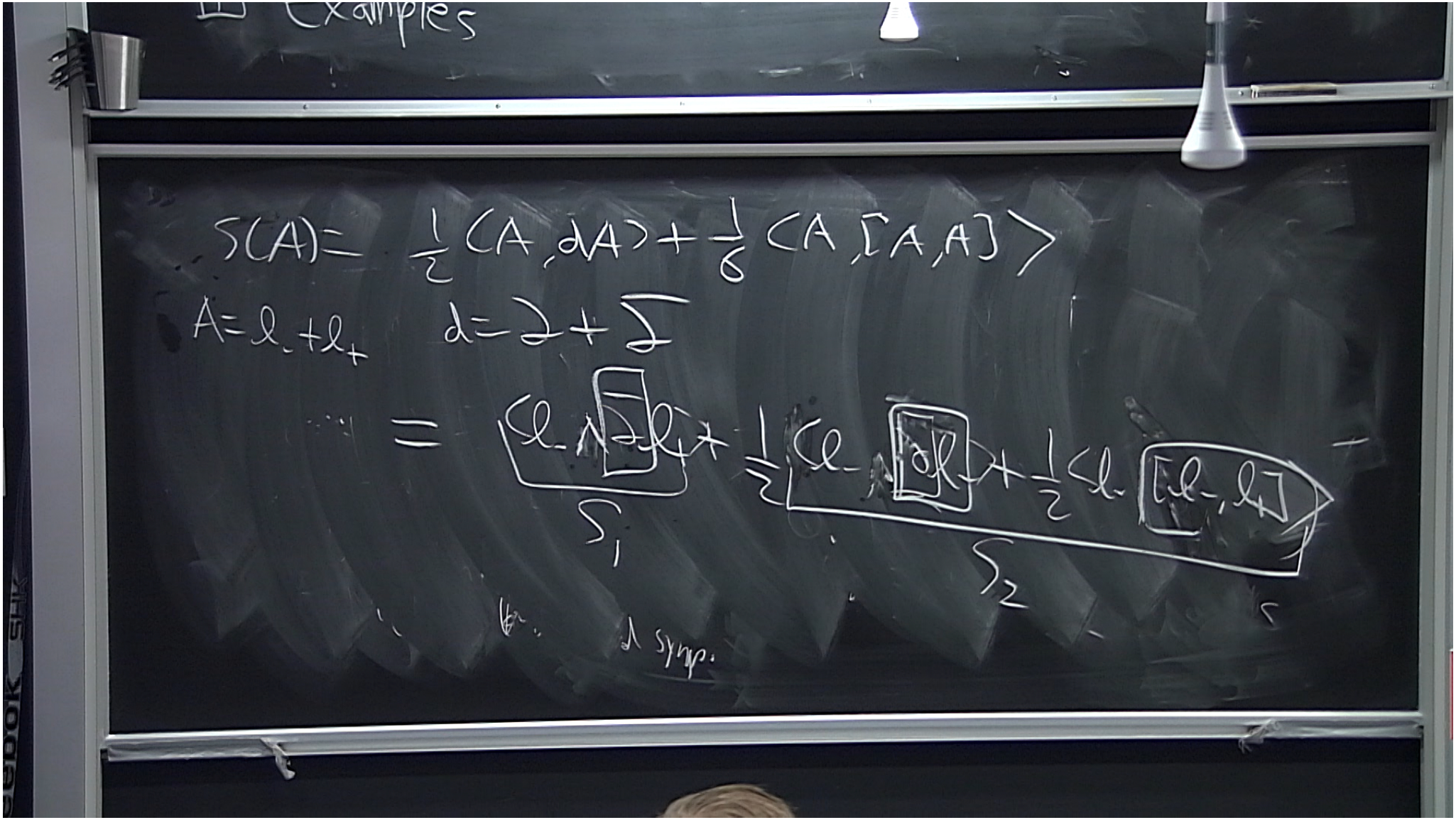

$S(A) = \frac{1}{2}\langle A, dA\rangle + \frac{1}{6}\langle A, [A,A]\rangle$
$A = \ell_- + \ell_+$
$d = \partial + \bar{\partial}$
$= \langle \ell_-, \bar{\partial}\ell_+ \rangle + \frac{1}{2}\langle \ell_-, \partial \ell_- \rangle + \frac{1}{2}\langle \ell_-, [\ell_-, \ell_+] \rangle$
symp.

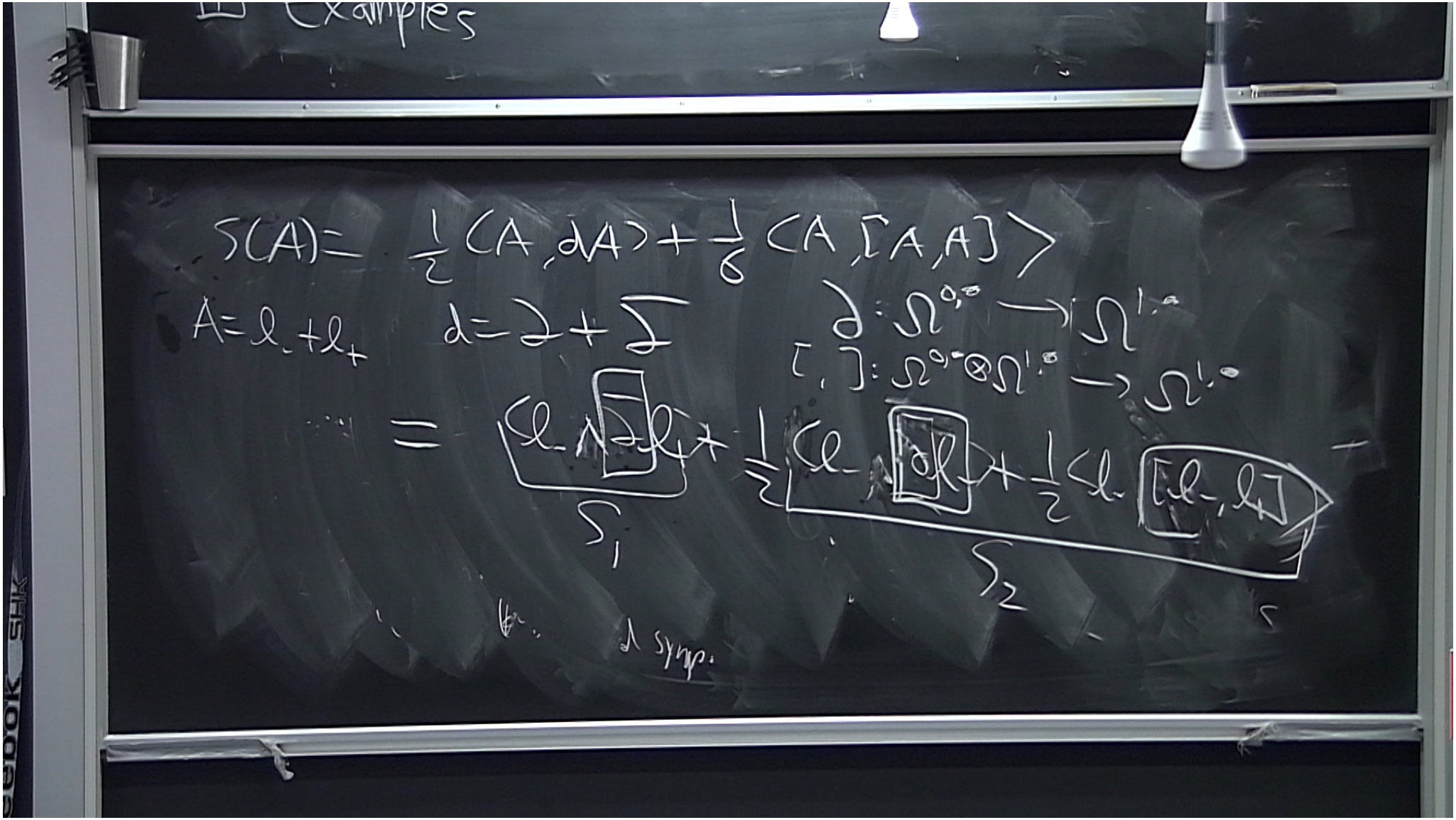

Examples
$S(A) = \frac{1}{2}\langle A, dA\rangle + \frac{1}{6}\langle A, [A,A]\rangle$
$A = \ell_- + \ell_+$
$d = \partial + \bar{\partial}$
$= \underbrace{\langle \ell_- \wedge \bar{\partial} \ell_+ \rangle}_{S_1} + \frac{1}{2}\langle \ell_- \wedge \partial \ell_- \rangle + \frac{1}{2}\langle \ell_-, [\ell_-, \ell_+] \rangle$
ω symp.

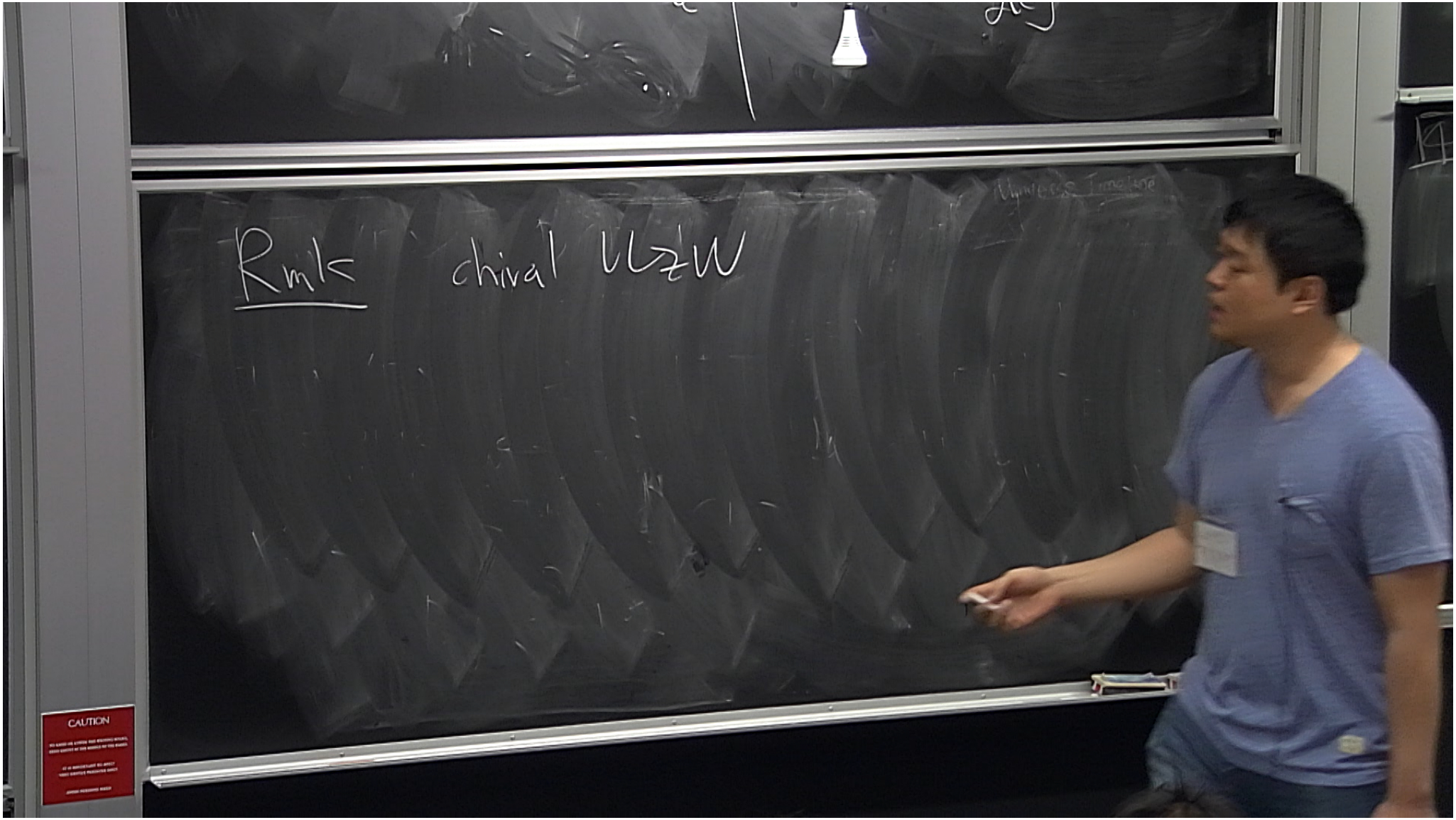

Examples
$S(A) = \frac{1}{2}\langle A, dA\rangle + \frac{1}{6}\langle A, [A,A]\rangle$
$A = \ell_- + \ell_+$
$d = \partial + \bar{\partial}$
$= \underbrace{\langle \ell_-, \bar{\partial}\ell_+\rangle}_{S_1} + \frac{1}{2}\underbrace{\langle \ell_-, \partial\ell_+ + \frac{1}{2}\langle \ell_-, [\ell_-, \ell_+]\rangle}_{S_2}$
∂ symp.

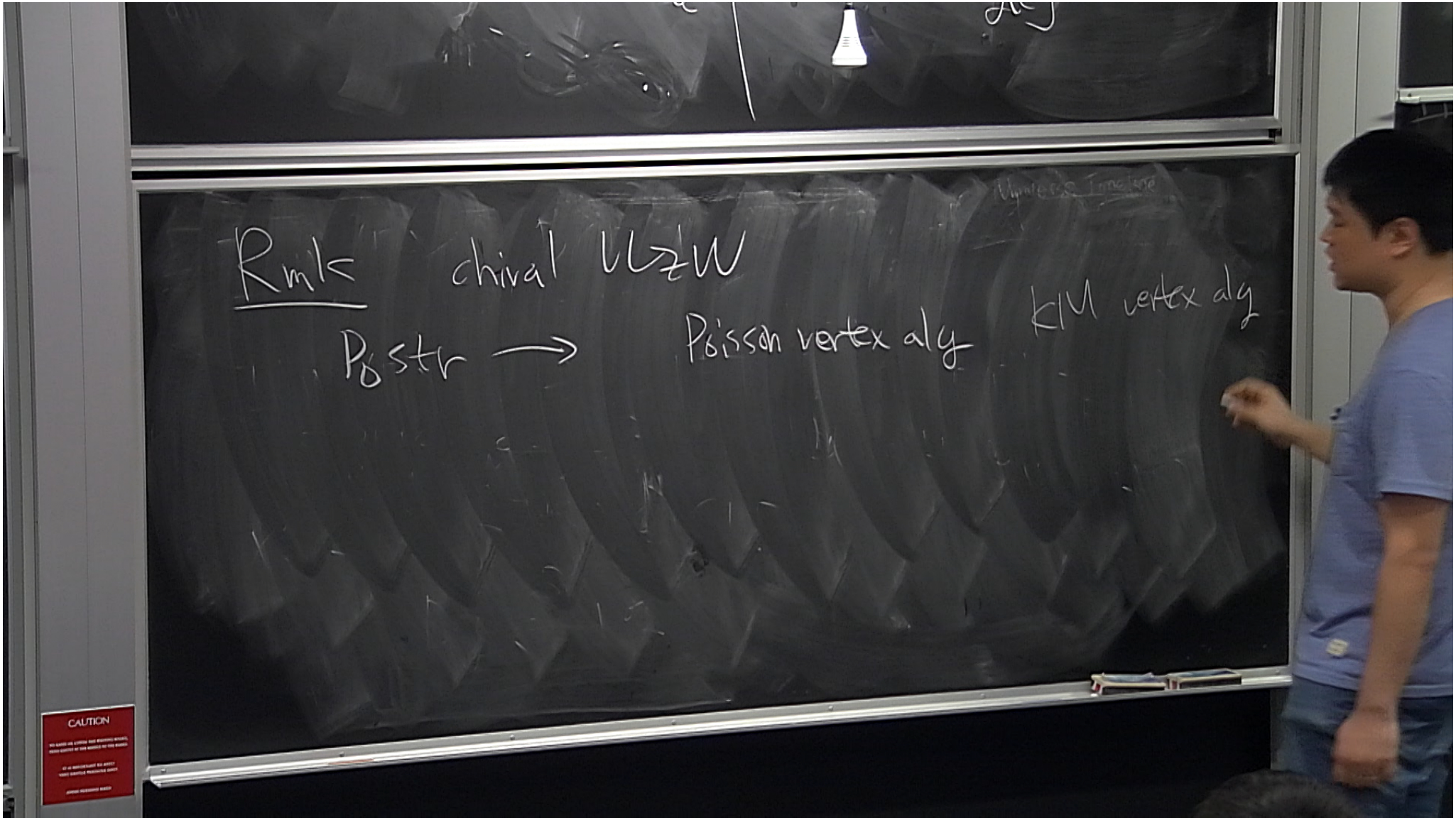

$X \sim T^*L$
$S \in \mathcal{O}(X)[1] = \mathcal{O}(T^*L)[1]$
$(S_j) \in \bigoplus_{j \geq 0} \Gamma(L, \mathrm{Sym}^j T_L)[1]$
$\ell^* \otimes \mathrm{Sym}^j \ell$
$S_1 \sim Q_\ell$
$\{S_i\}_{i=1} \sim$ Htpy (-1)-shifted Poisson,
$\{S, S\} = 0 \in$
$Q + \bar{\pi}] = 0$
$[\pi, \pi] = 0$
CAUTION

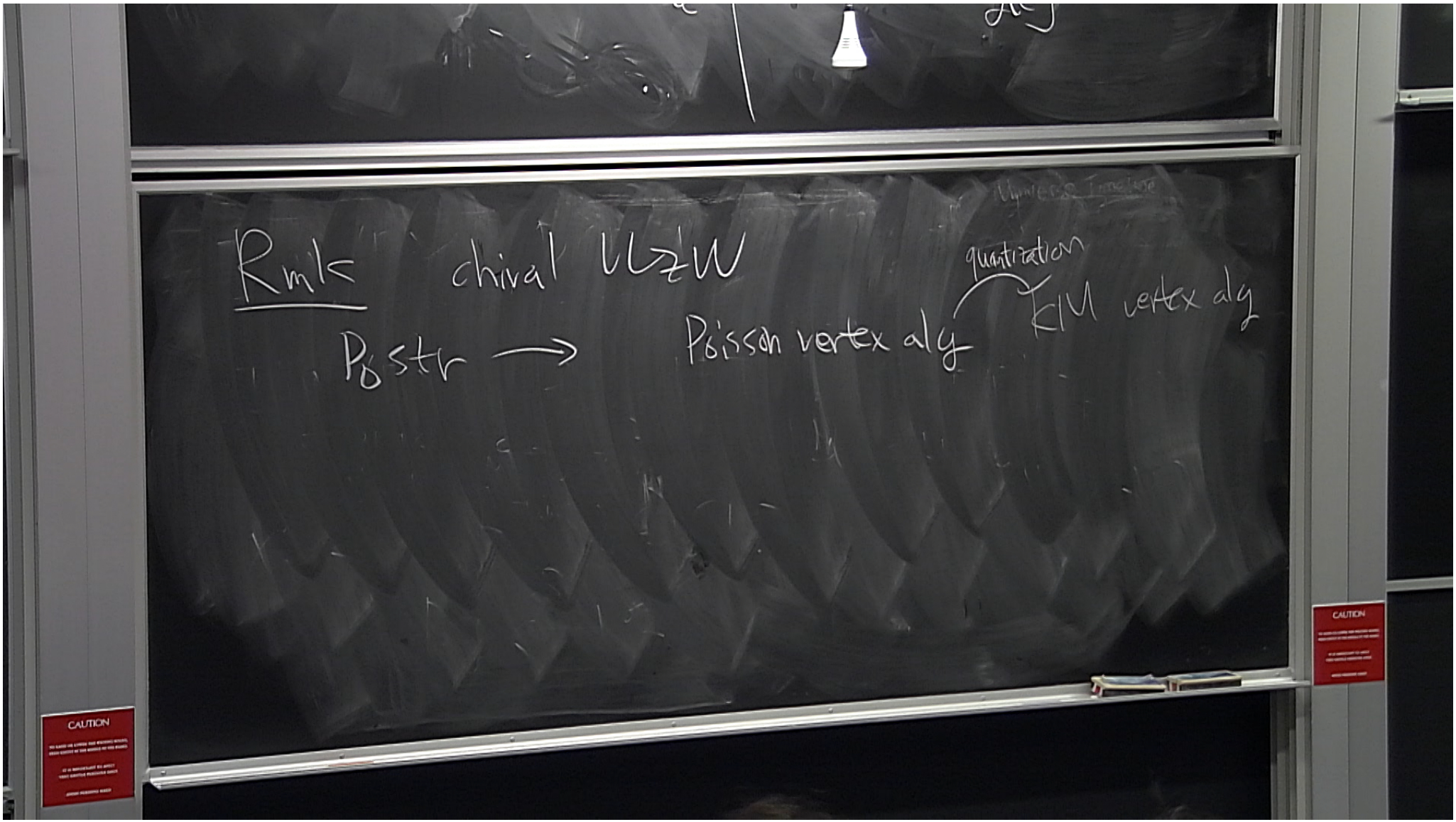

X ~ T*L
S ∈ O(X)[1] = O(T*L)[1]
Sym^k l* ⊗ Sym^k l


Examples
$S(A) = \frac{1}{2}\langle A, dA\rangle + \frac{1}{6}\langle A, [A,A]\rangle$
$A = l_- + l_+$
$d = \partial + \bar{\partial}$
$= \langle l_- \wedge \partial l_+ + \frac{1}{2}\langle l_-, \partial l_-\rangle + \frac{1}{2}\langle l_-, [l_-, l_+]$
S_1
S_2
ω symp.

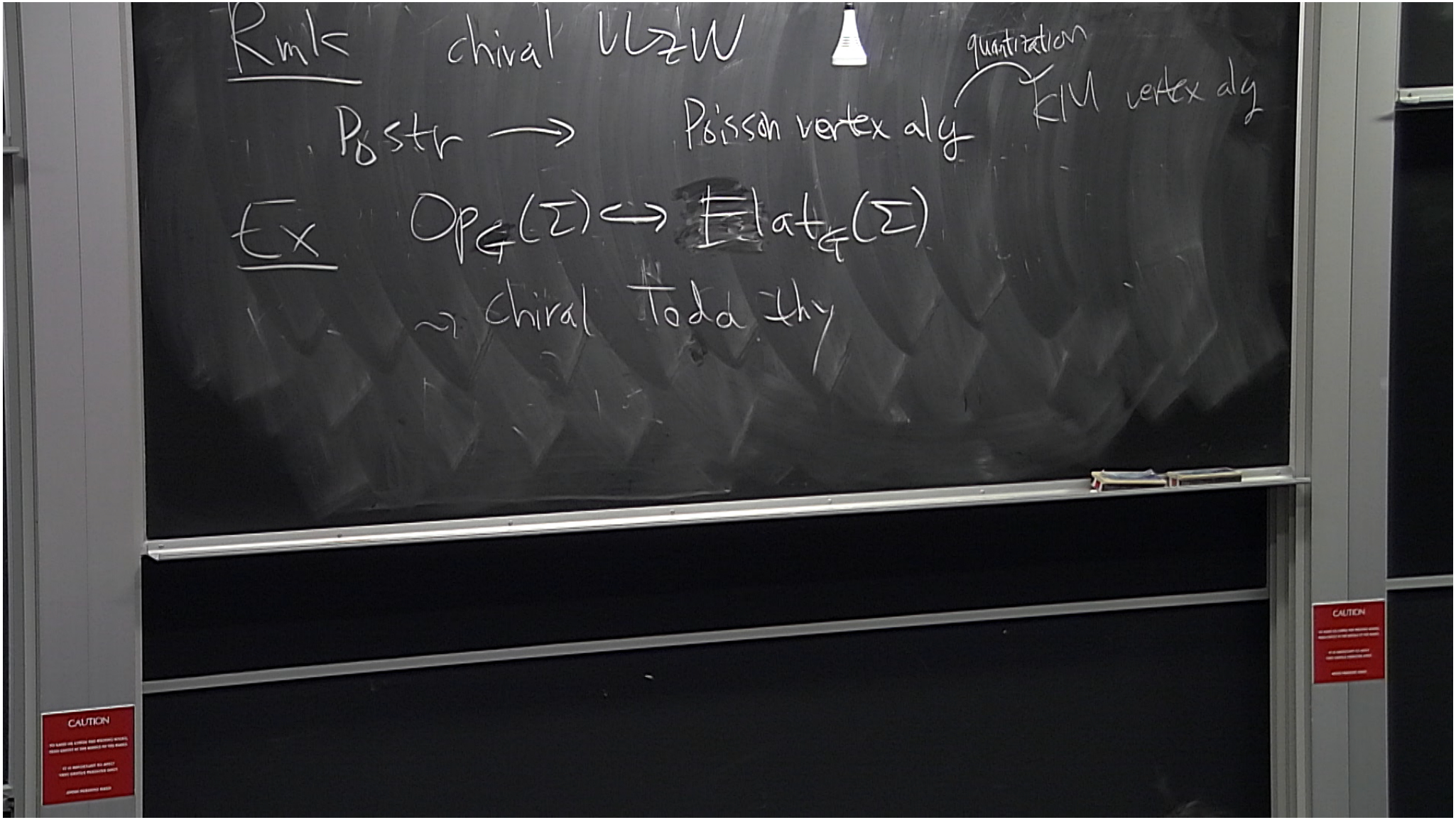

Examples
$S(A) = \frac{1}{2}\langle A, dA\rangle + \frac{1}{6}\langle A, [A,A]\rangle$
$A = l_- + l_+$
$d = \partial + \bar{\partial}$
$\partial : \Omega^{0,\bullet} \to \Omega^{1,\bullet}$
$[\,,\,] : \Omega^{0,\bullet} \otimes \Omega^{1,\bullet} \to \Omega^{1,\bullet}$
$= \langle l_- \wedge \bar{\partial} l_+ \rangle + \frac{1}{2}\langle l_- [l_-, l_+] \rangle$
S_1
S_2
symp.

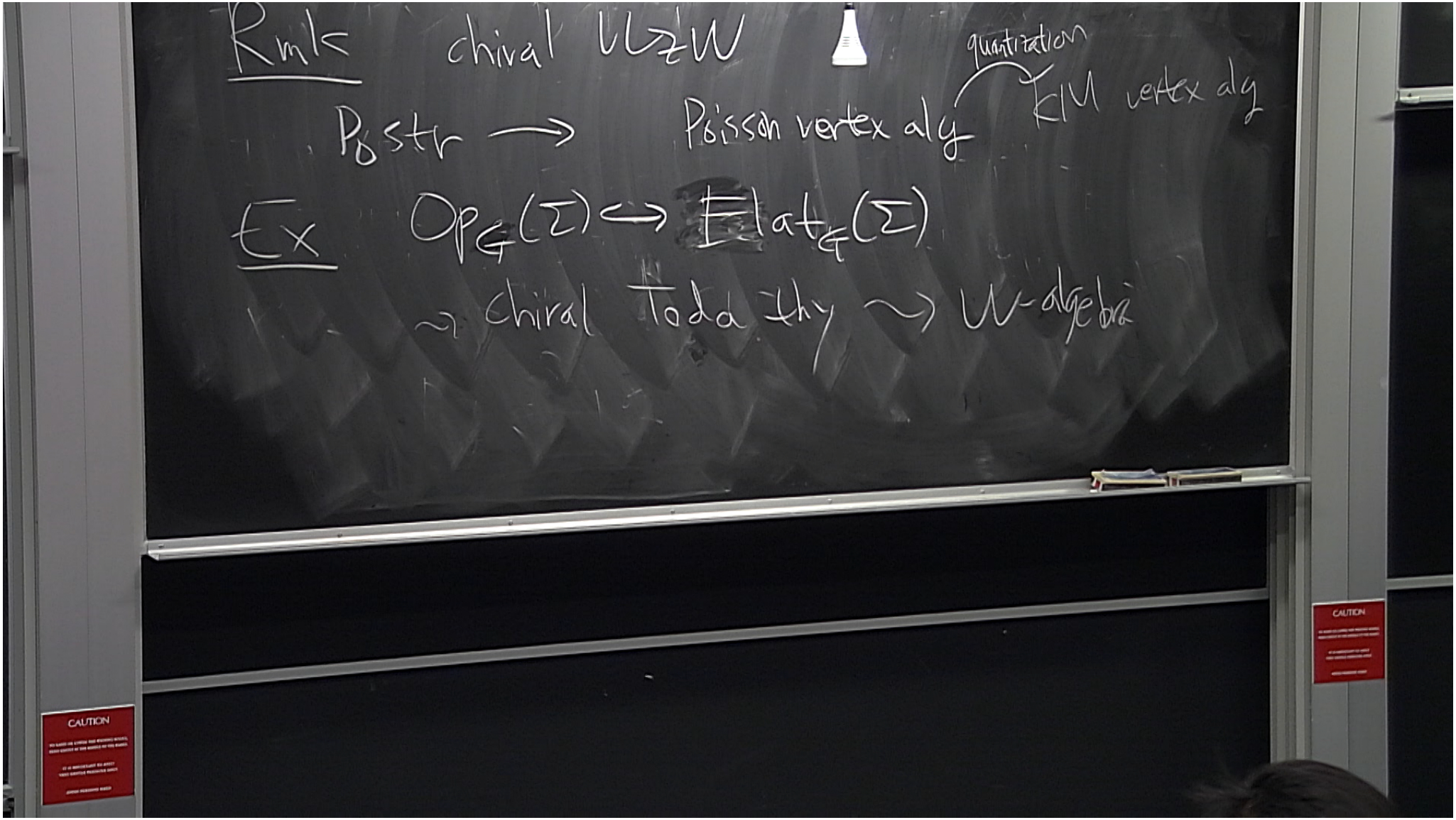

Rmk
chiral
CAUTION

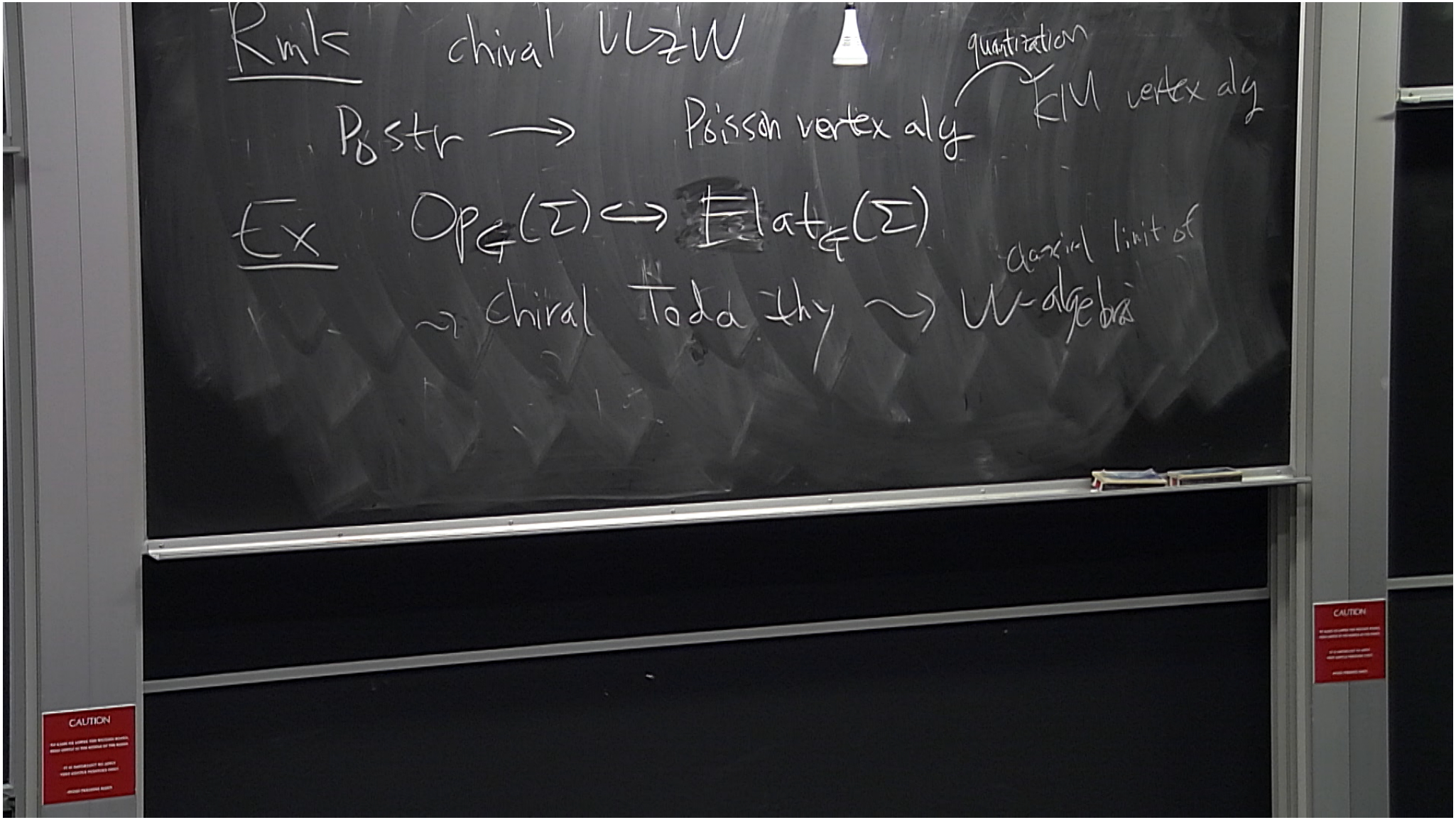

Rmk chiral WZW
Poisson vertex alg
KM vertex alg
CAUTION


Rmk chiral WZW
quantization
Poisson vertex alg
KM vertex alg
CAUTION

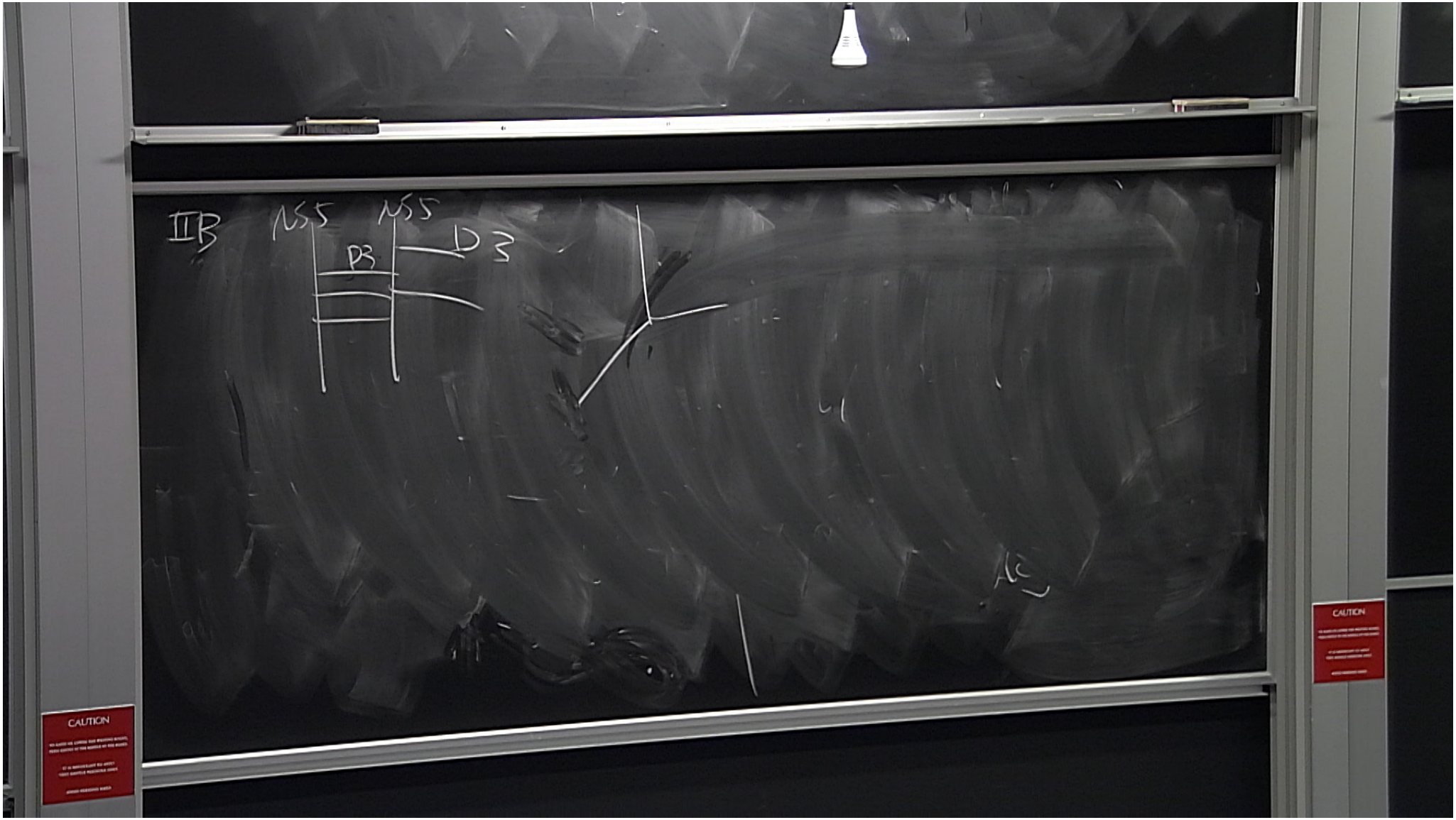

Rmk chiral WZW
quantization
Poisson vertex alg
KM vertex alg
Ex

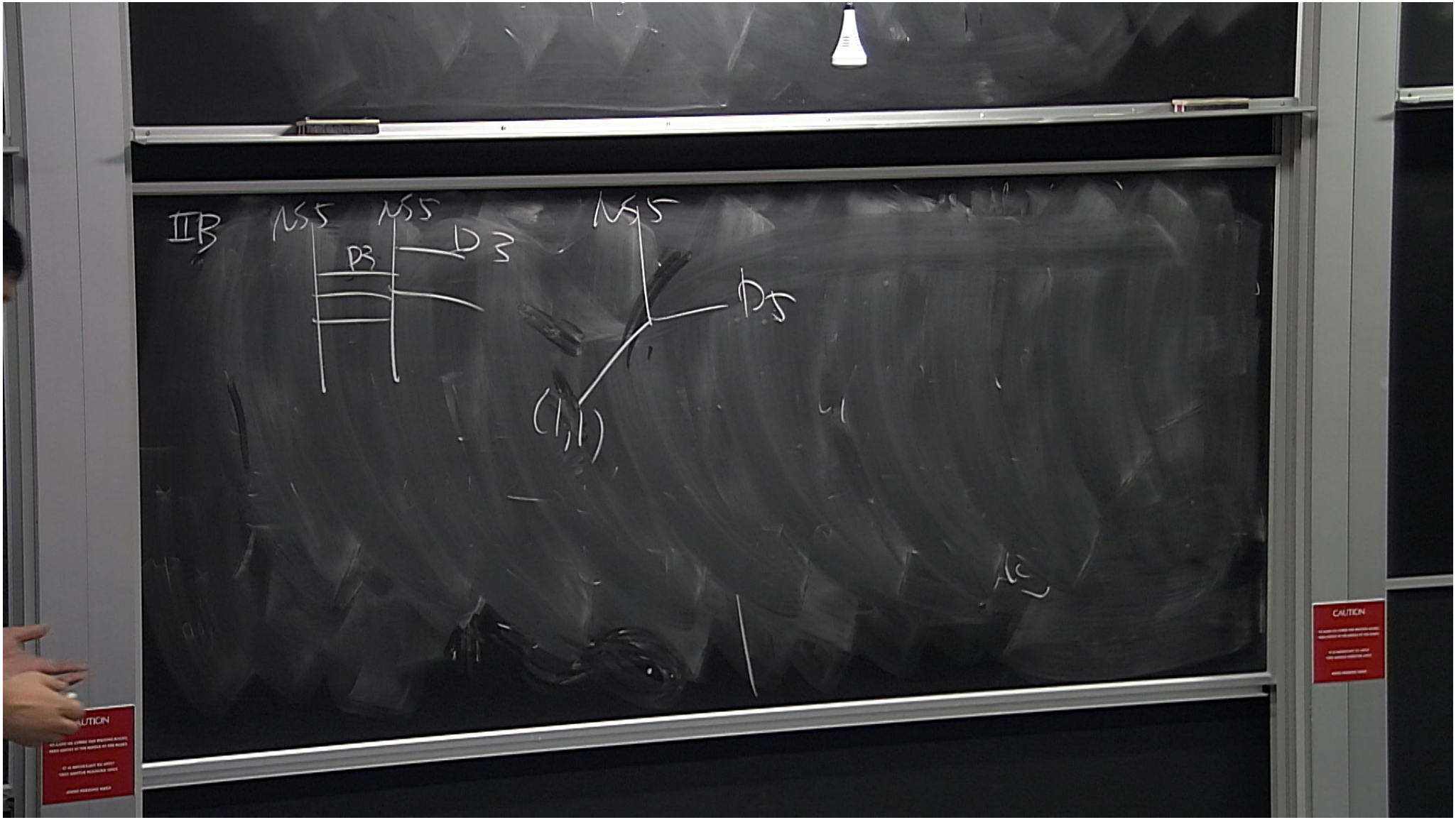

Rmk chiral WZW
quantization
Poisson vertex alg
KM vertex alg
Ex
chiral Toda thy

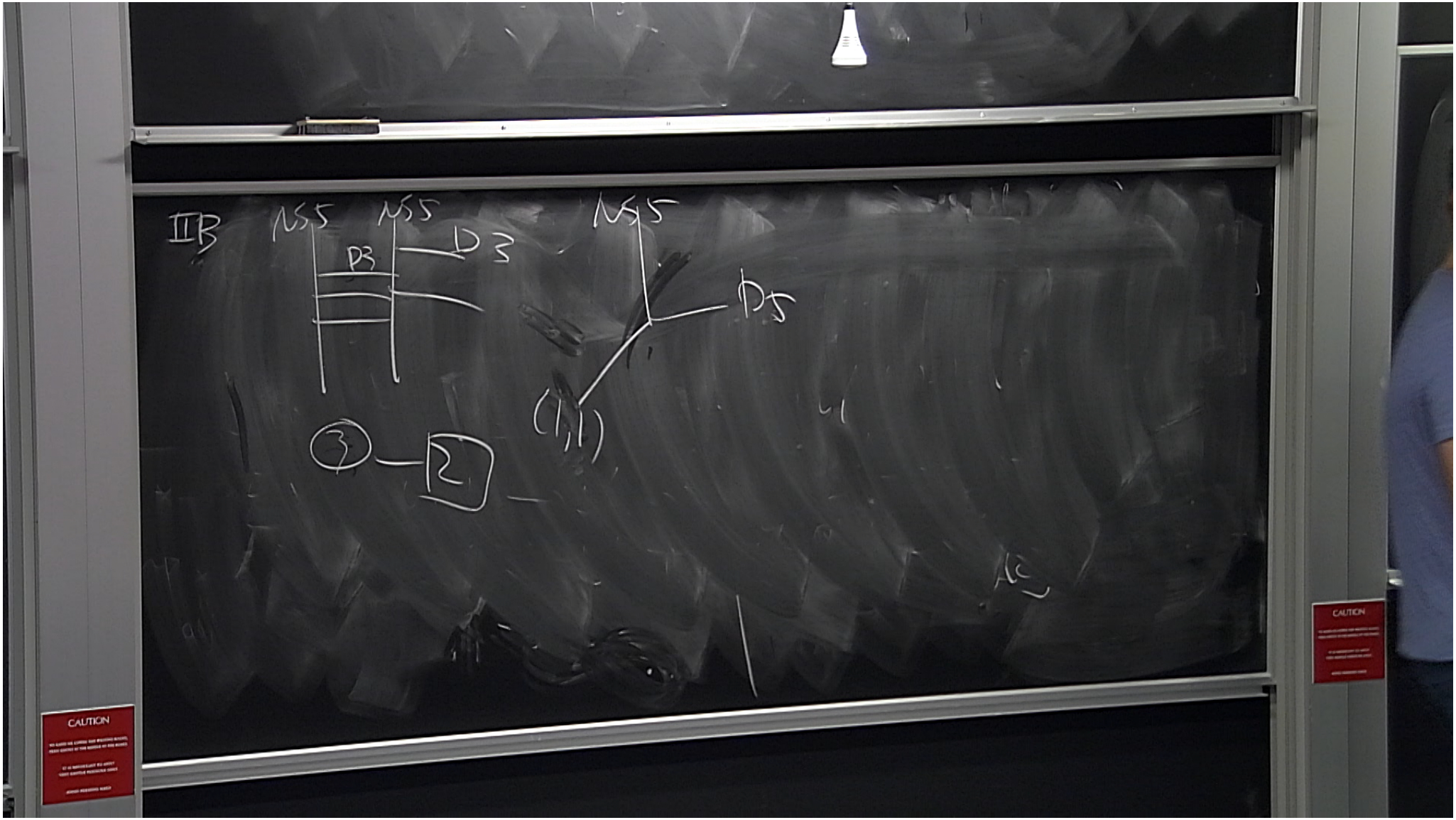

Rmk chiral WZW
quantization
Poisson vertex alg
KM vertex alg
Ex
chiral Toda thy
W-algebra

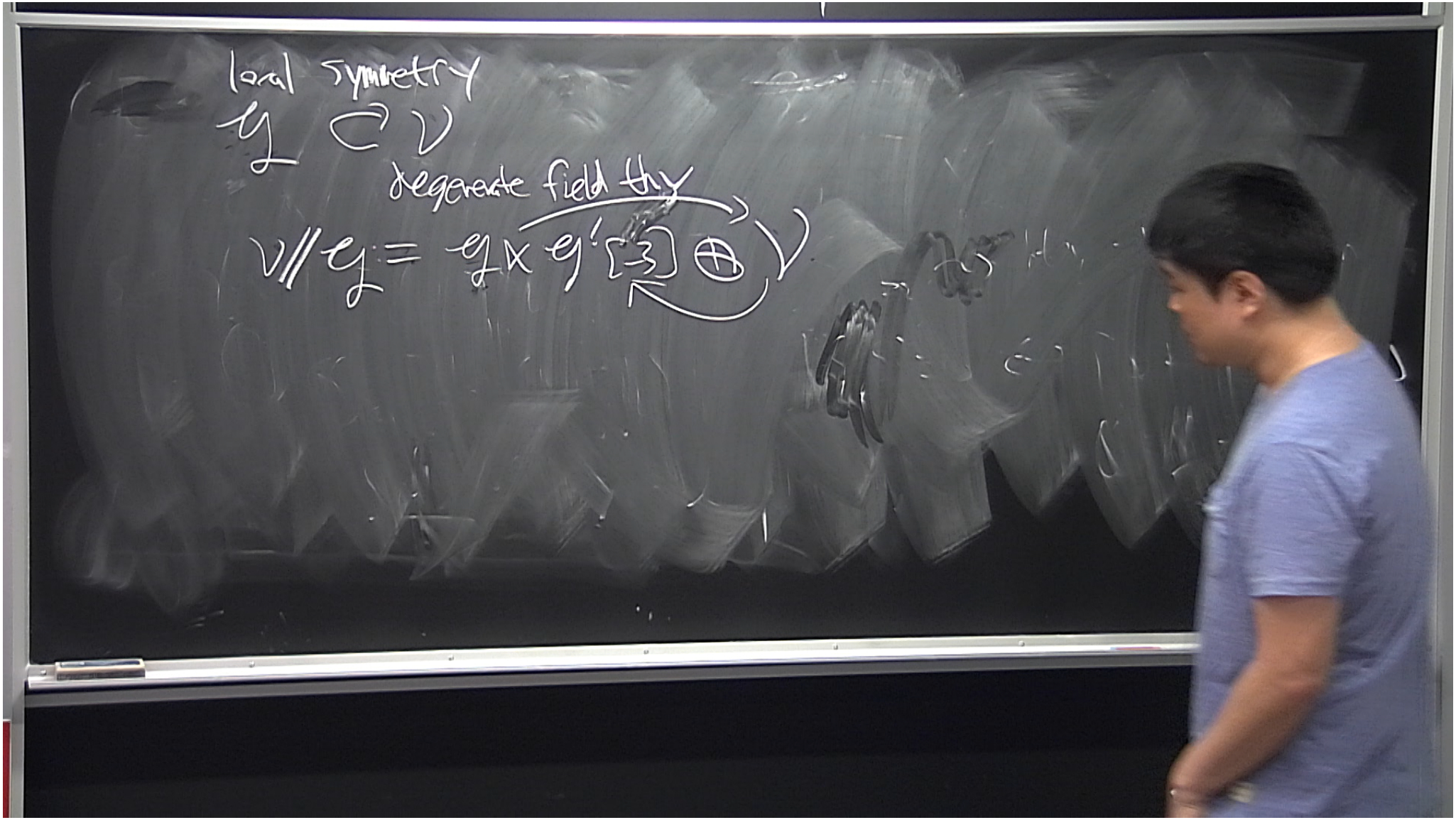

Rmk chiral WZW
quantization
Poisson vertex alg
KM vertex alg
Ex
chiral Todd thy
classical limit of
W-algebra

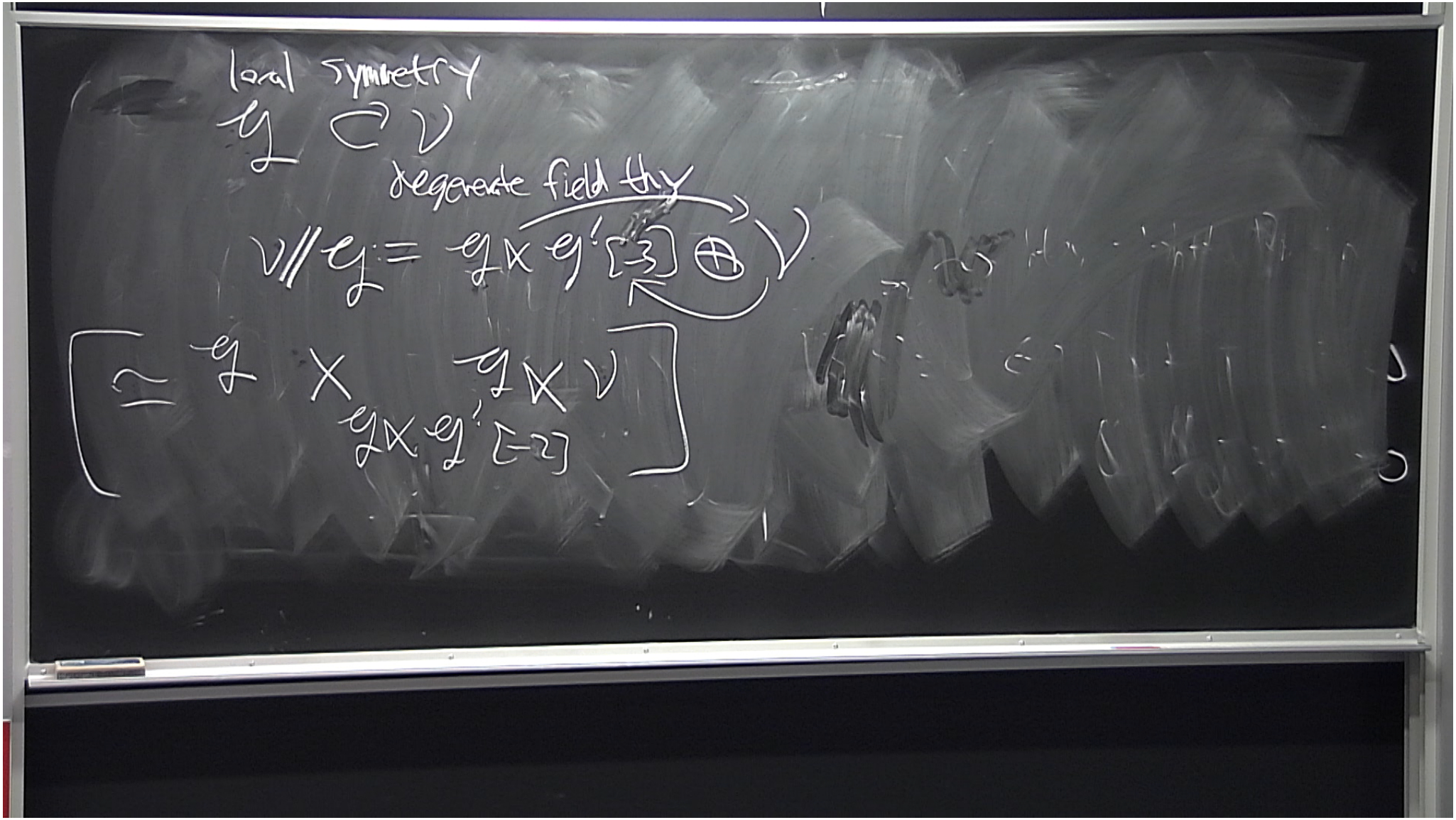

IIB
NS5
NS5
D3
D3
CAUTION
CAUTION

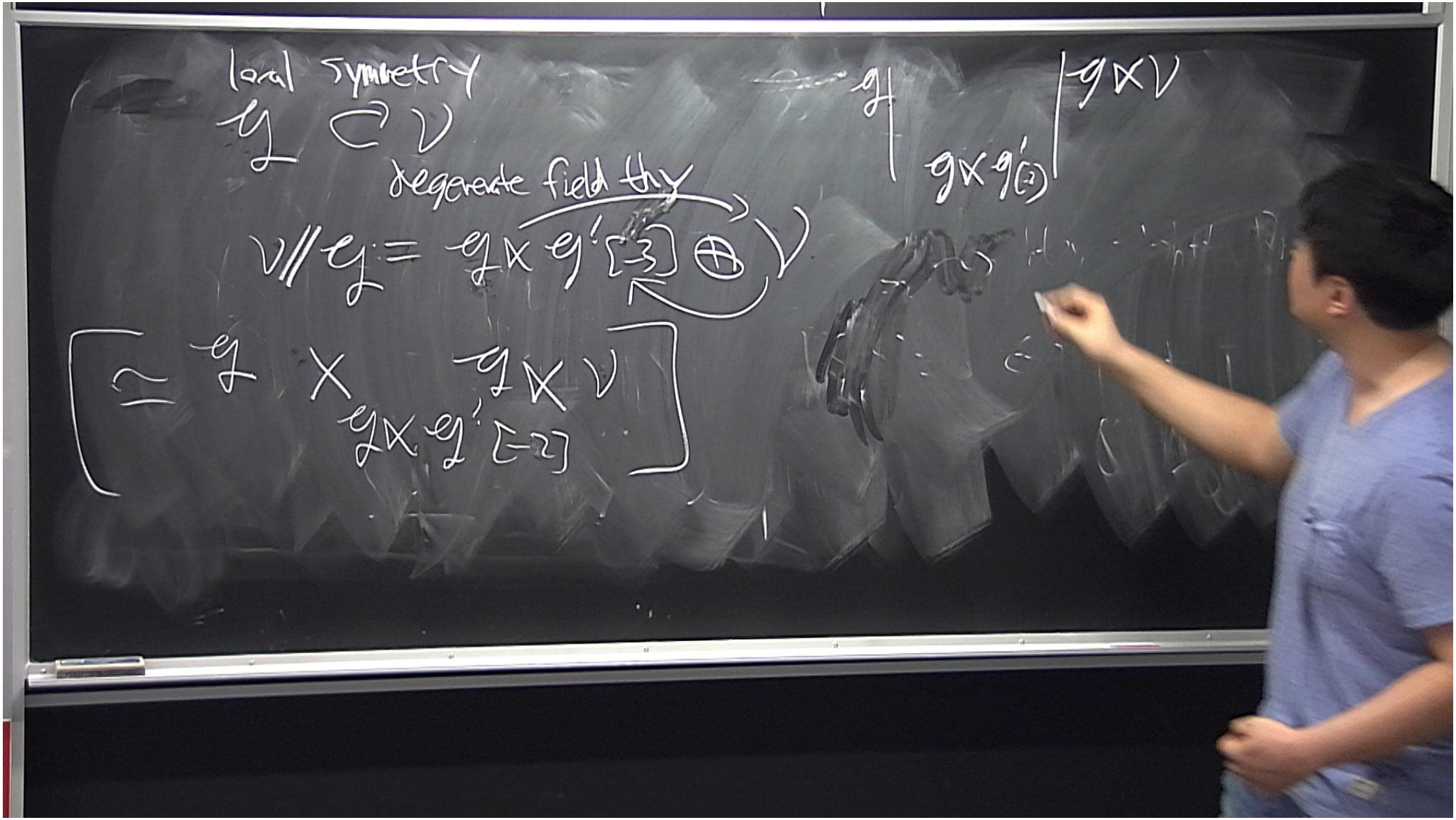

IIB
NS5
NS5
D3
D3

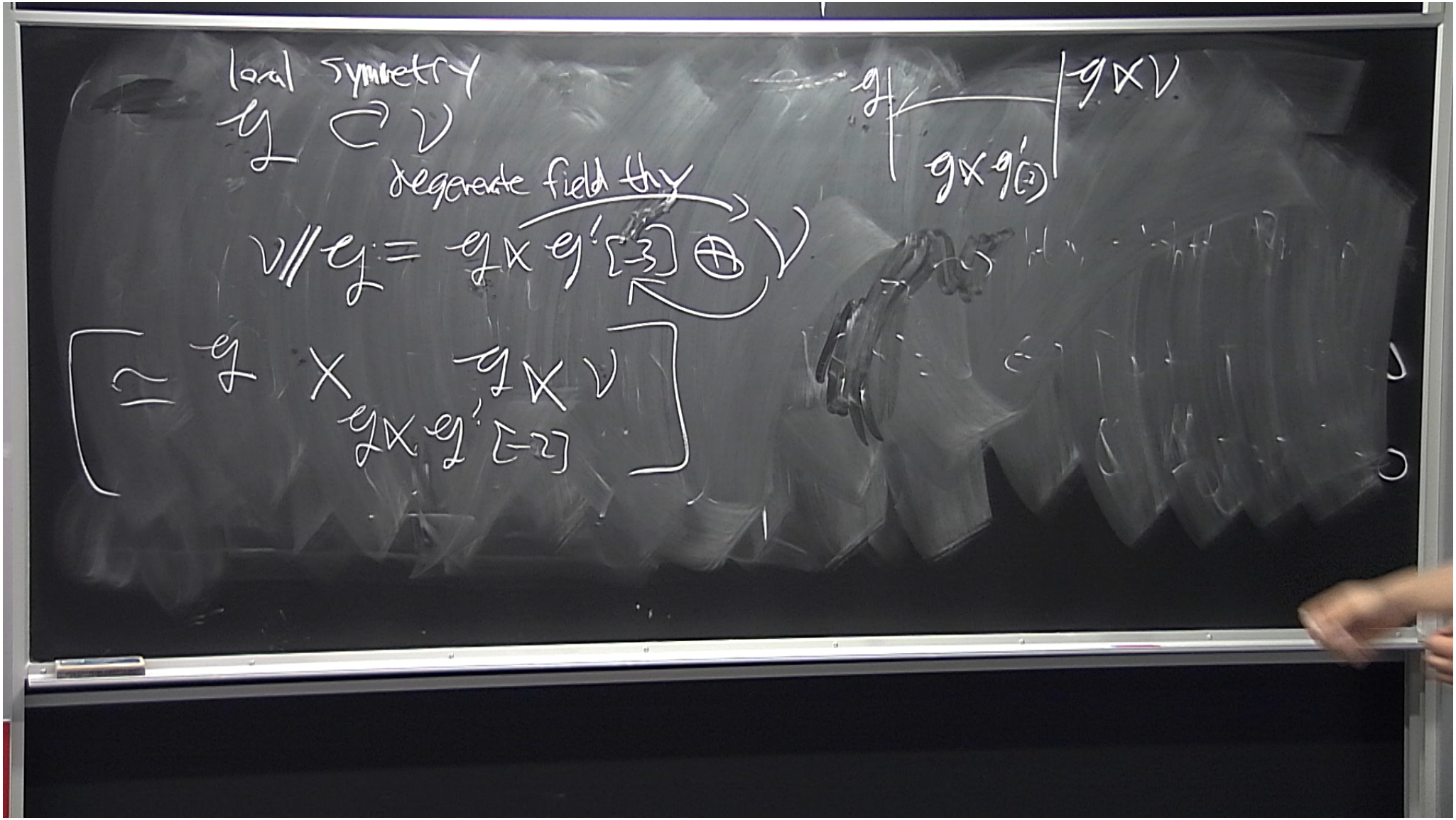

IIB
NS5
NS5
D3
D3
NS5
D5
(1,1)

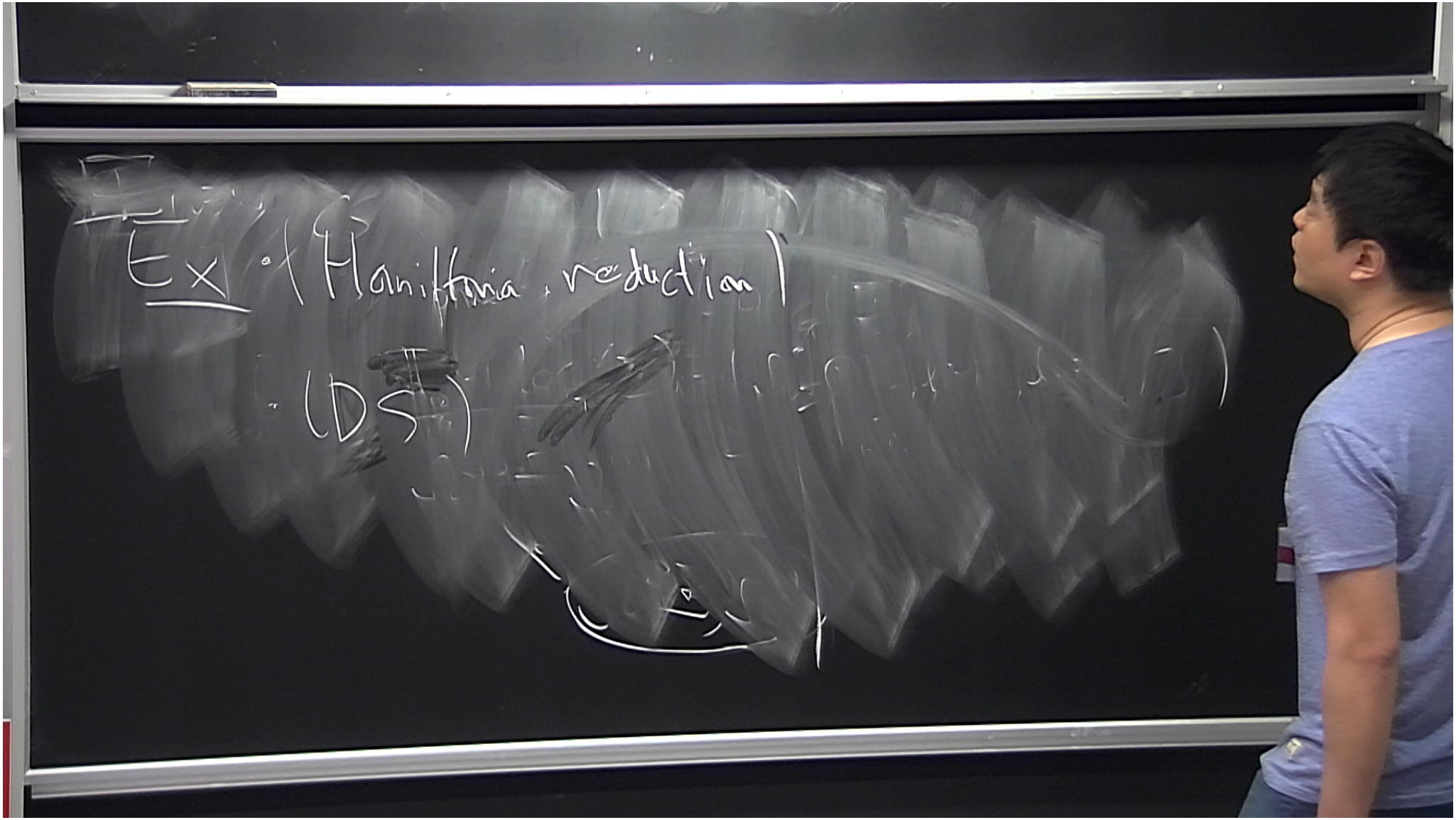

IIB
NS5
NS5
D3
NS5
D5
(1,1)
3
2

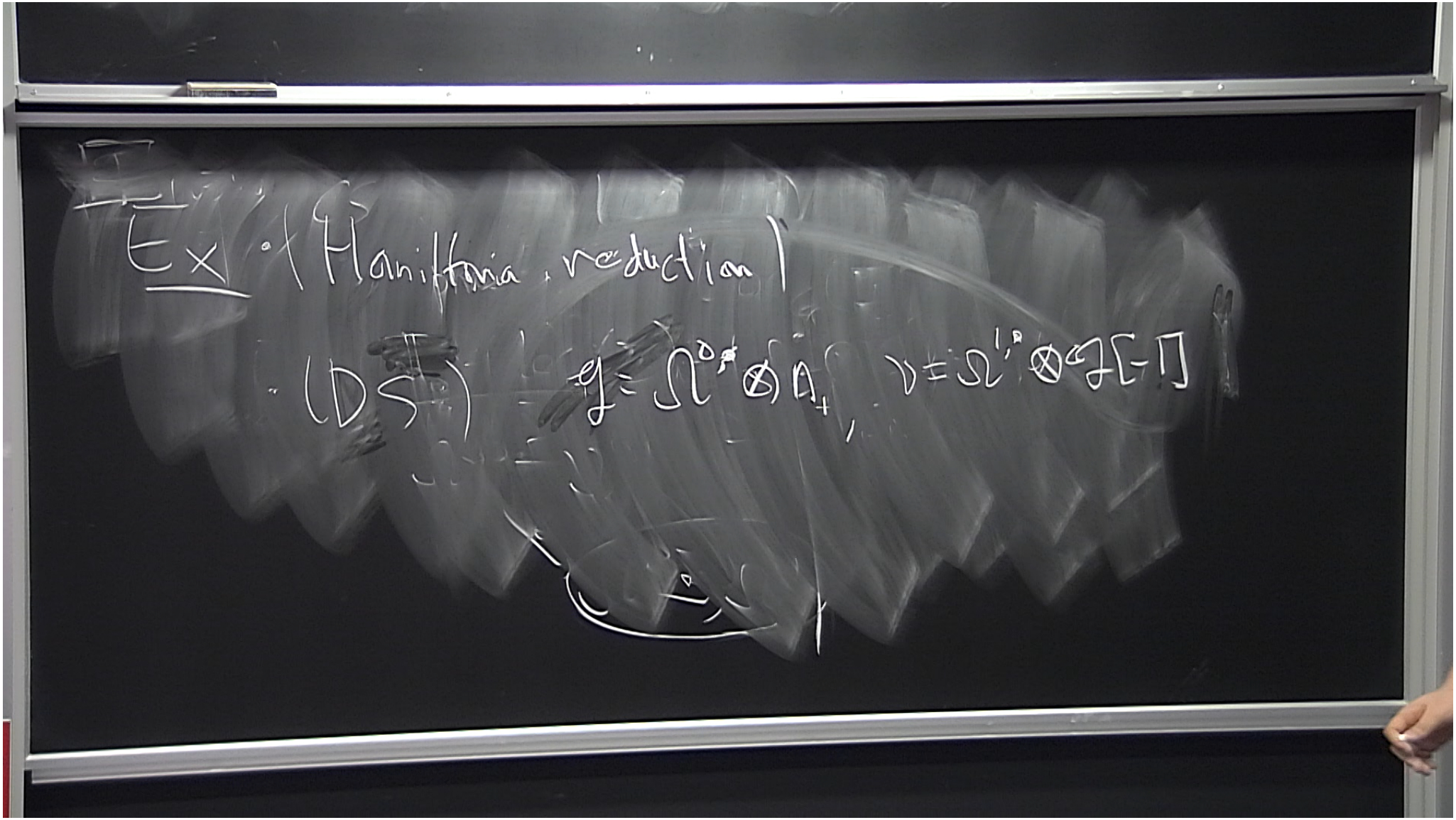

local symmetry
$\mathfrak{g} \subset V$
degenerate field thy
$V /\!\!/ \mathfrak{g} = \mathfrak{g} \times \mathfrak{g}'[-3] \oplus V$

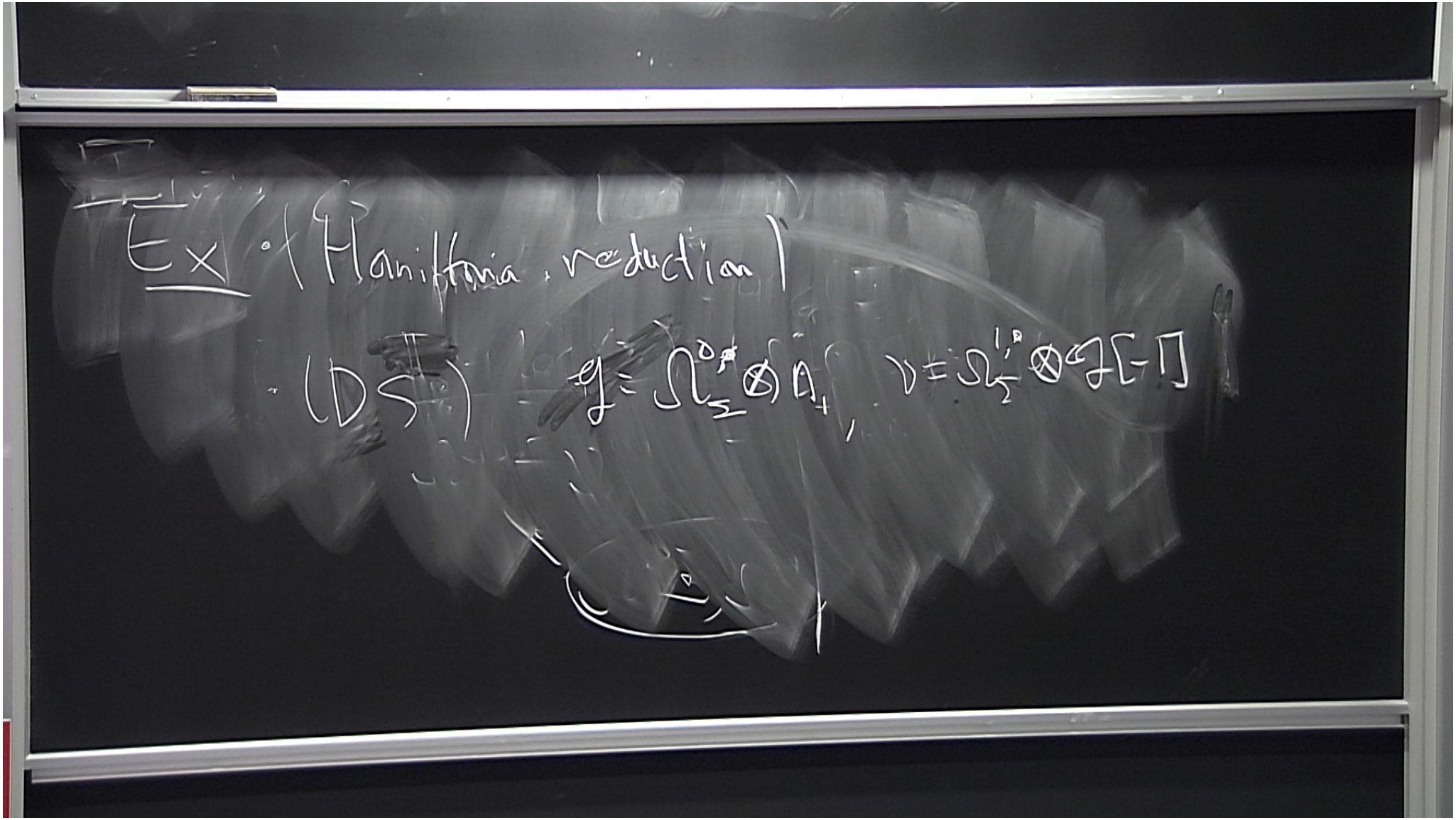

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Ex: (Hamiltonian reduction)
(DS)


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(DS)
$\mathcal{G} = \Omega^{0,*} \otimes \mathfrak{n}_+$, $\mathcal{V} = \Omega^{1,0} \otimes \mathfrak{g}[-1]$


Ex. (Hamiltonian reduction)
(DS)
$\mathcal{J} = \Omega^{0,*}_{\Sigma} \otimes \Pi_+$, $\mathcal{V} = \Omega^{1,0}_{\Sigma} \otimes \mathbb{C}[-1]$