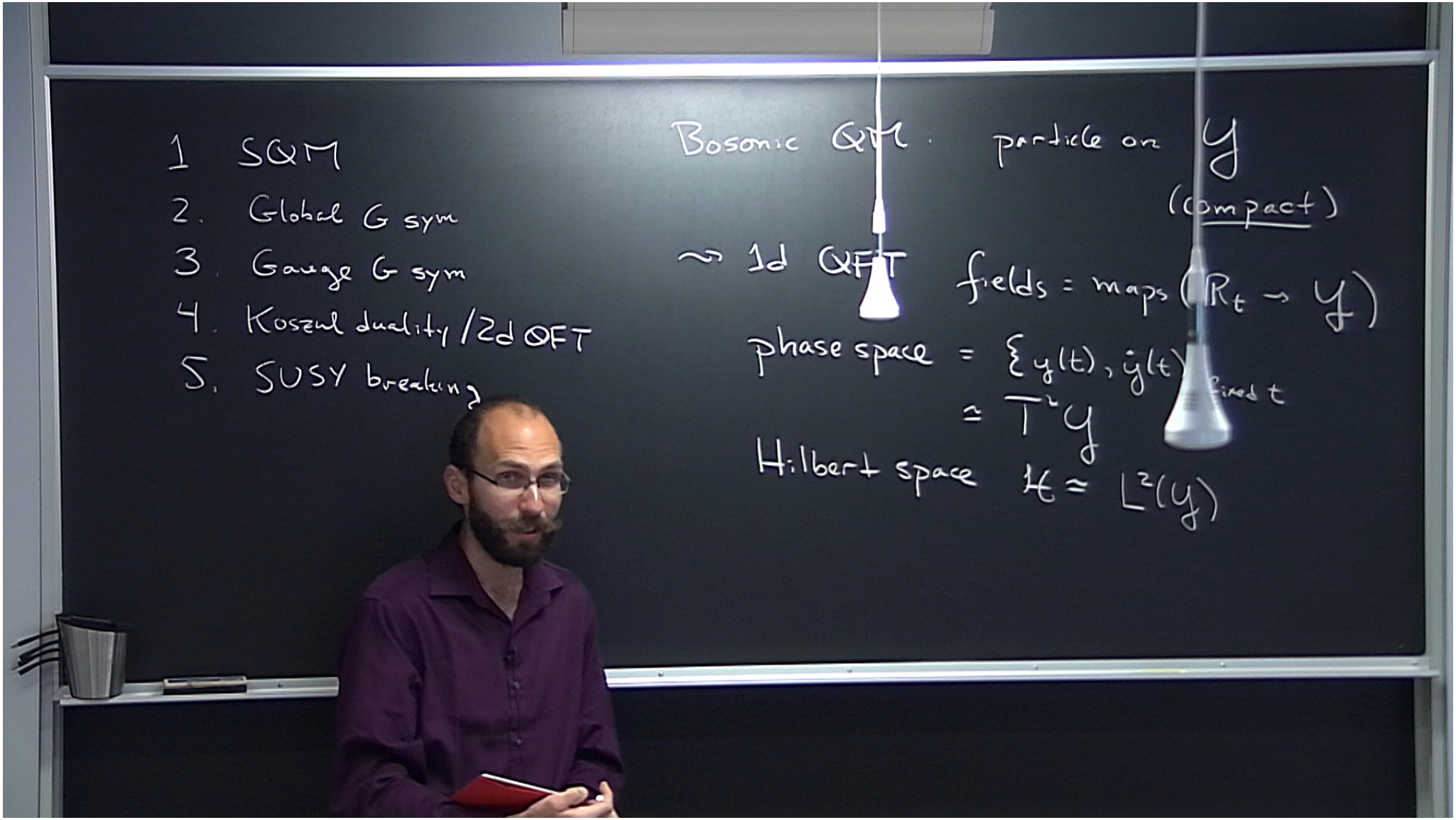
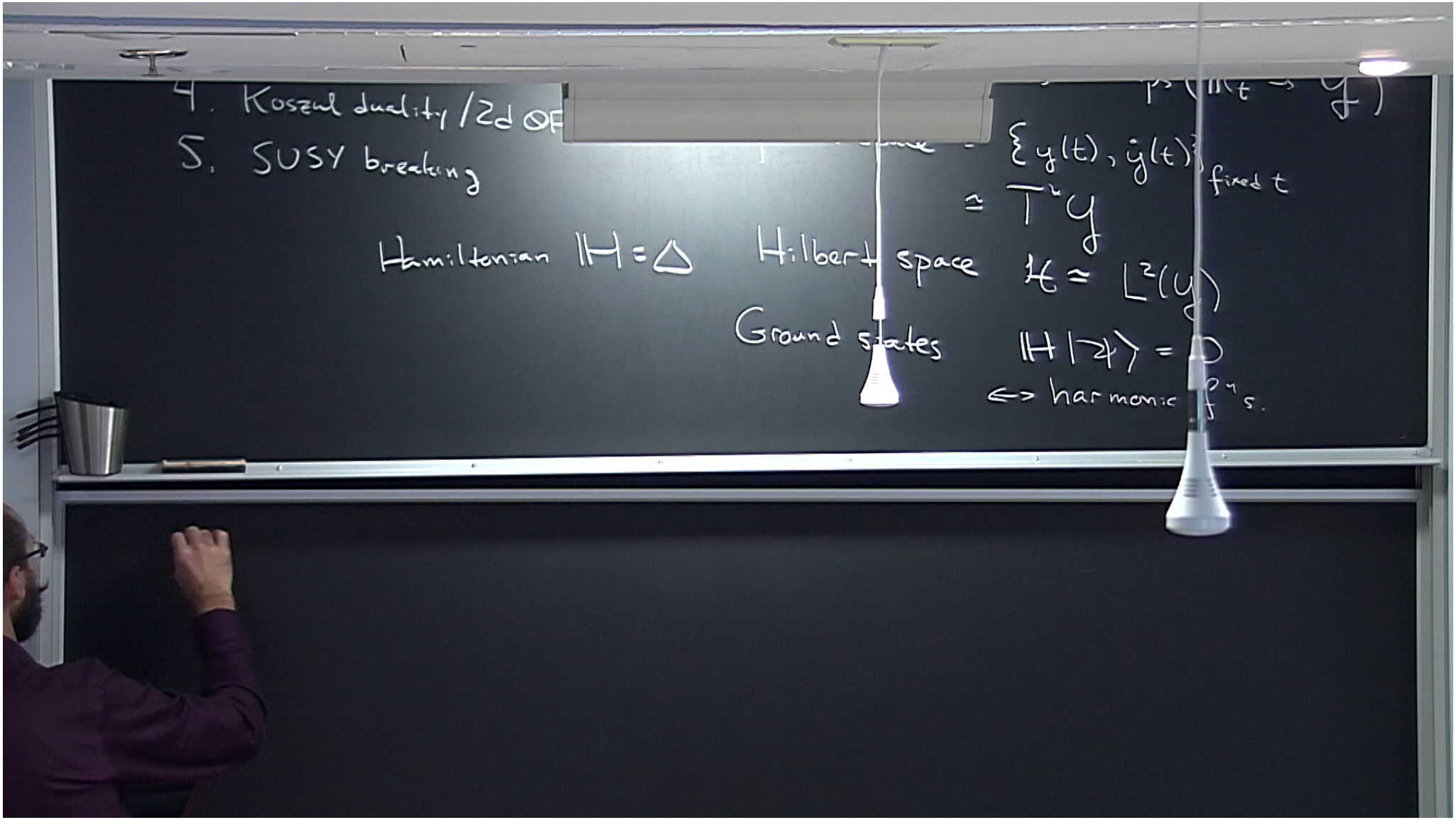
Title: G-actions in quantum mechanics (and spectral sequences and the cosmological constant)

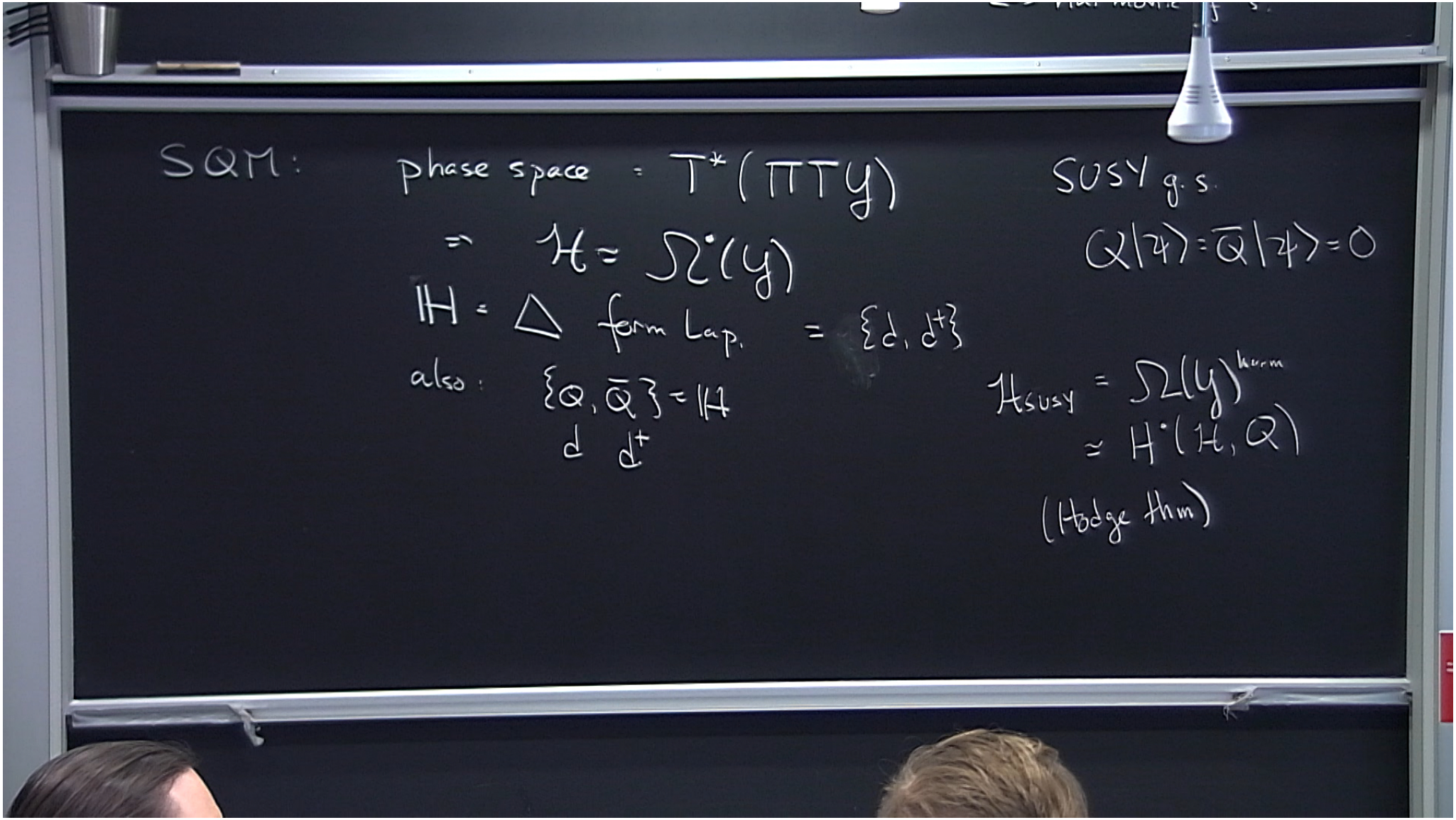
Date: May 08, 2017 11:00 AM

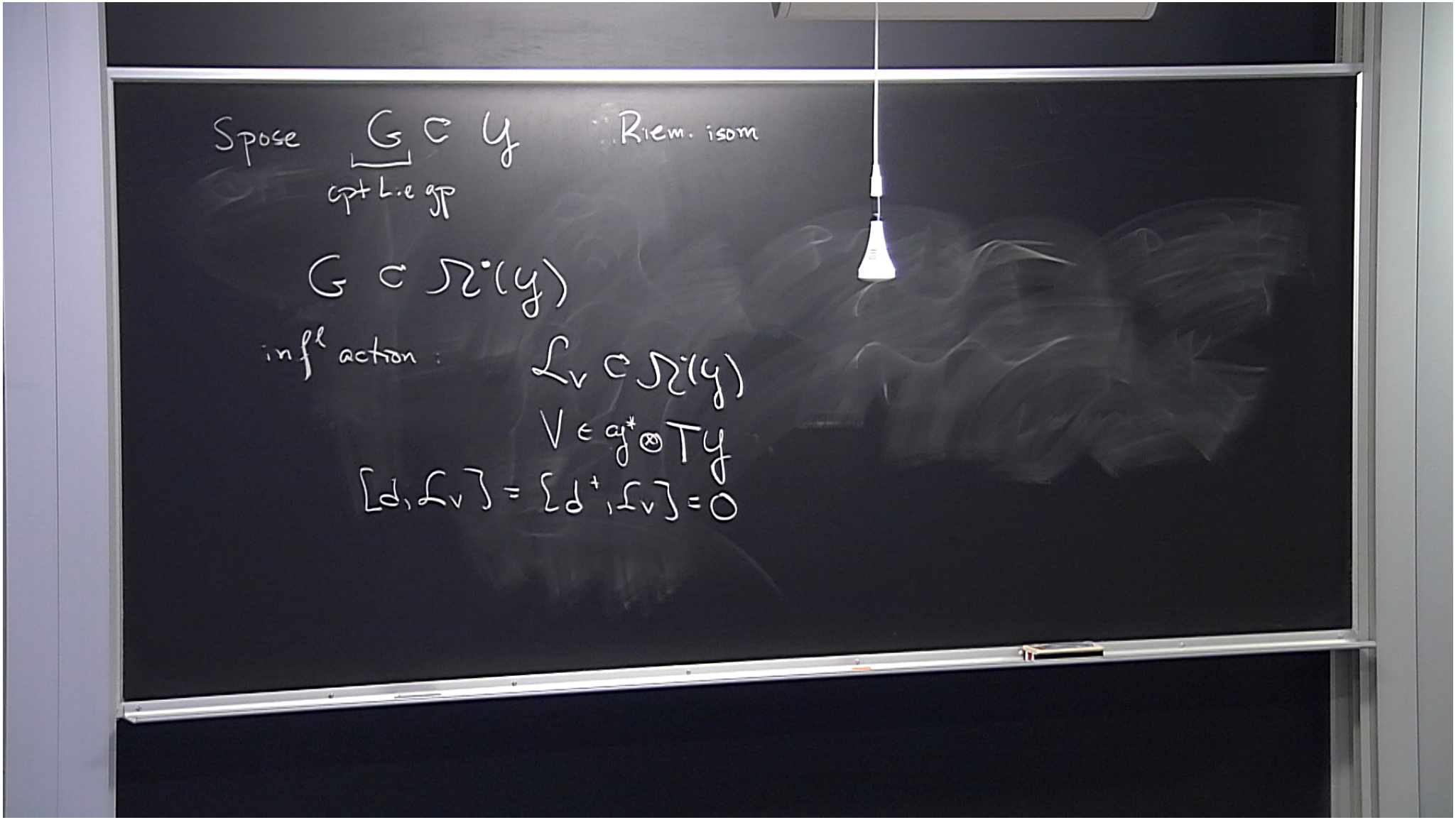
URL: http://pirsa.org/17050015

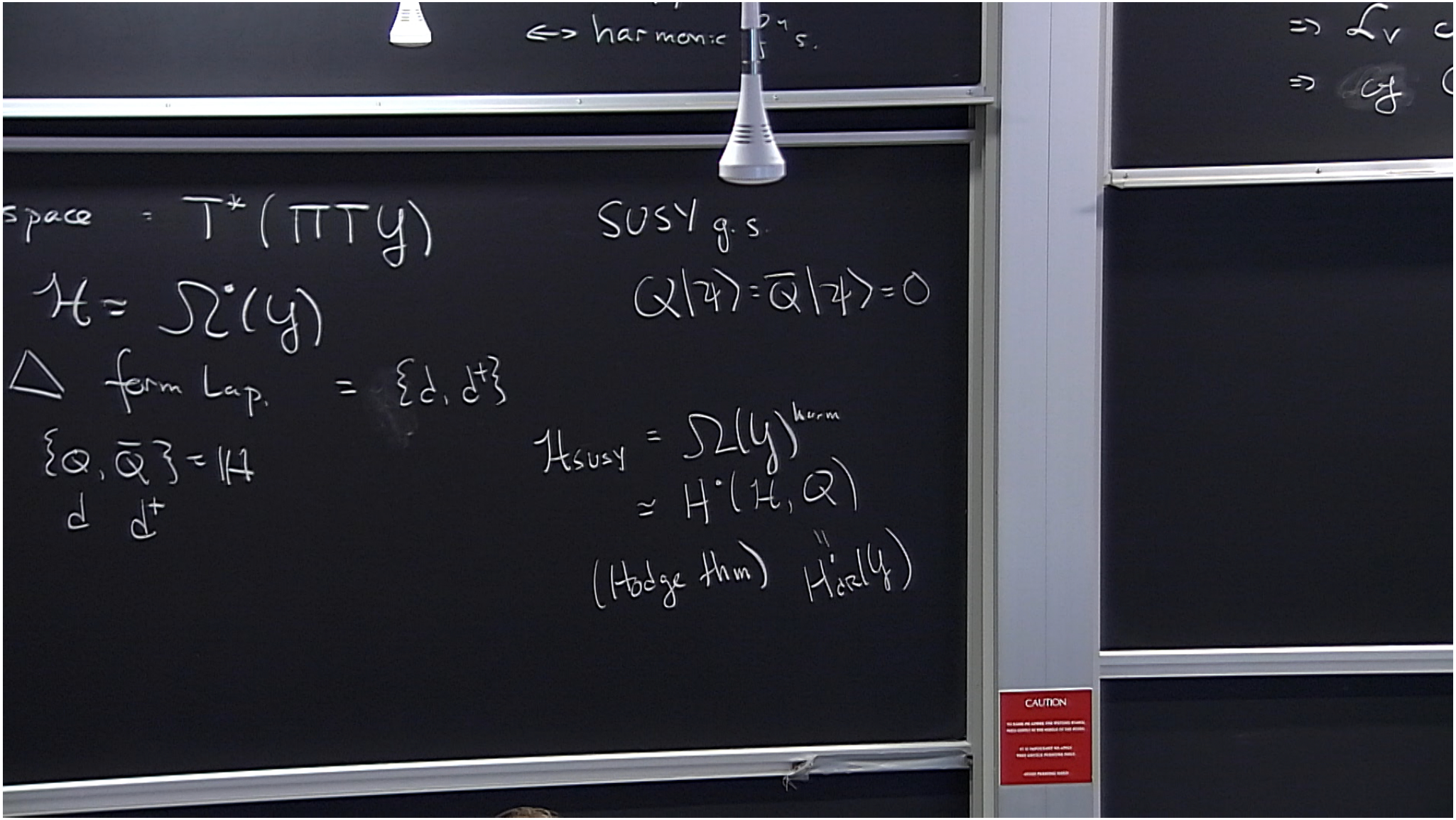
Abstract:

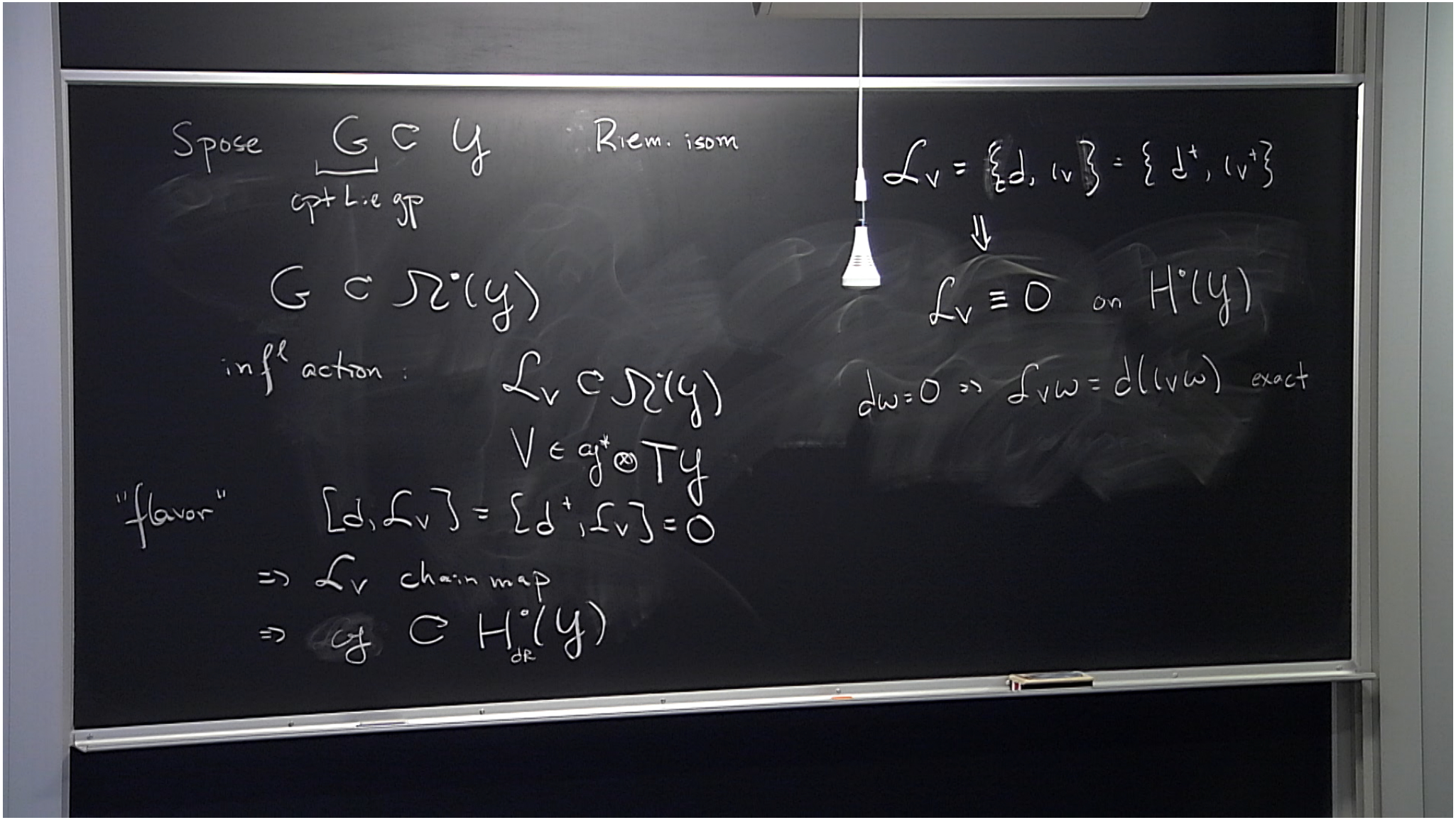

1 SQM
2. Global G sym
3. Gauge G sym
4. Koszul duality / 2d QFT
5. SUSY breaking
Bosonic QM: particle on Y (compact)
~ 1d QFT
fields = maps($\mathbb{R}_t \to Y$)
phase space = $\{y(t), \dot{y}(t)$ fixed t
$\simeq T^*Y$
Hilbert space $\mathcal{H} \simeq L^2(Y)$

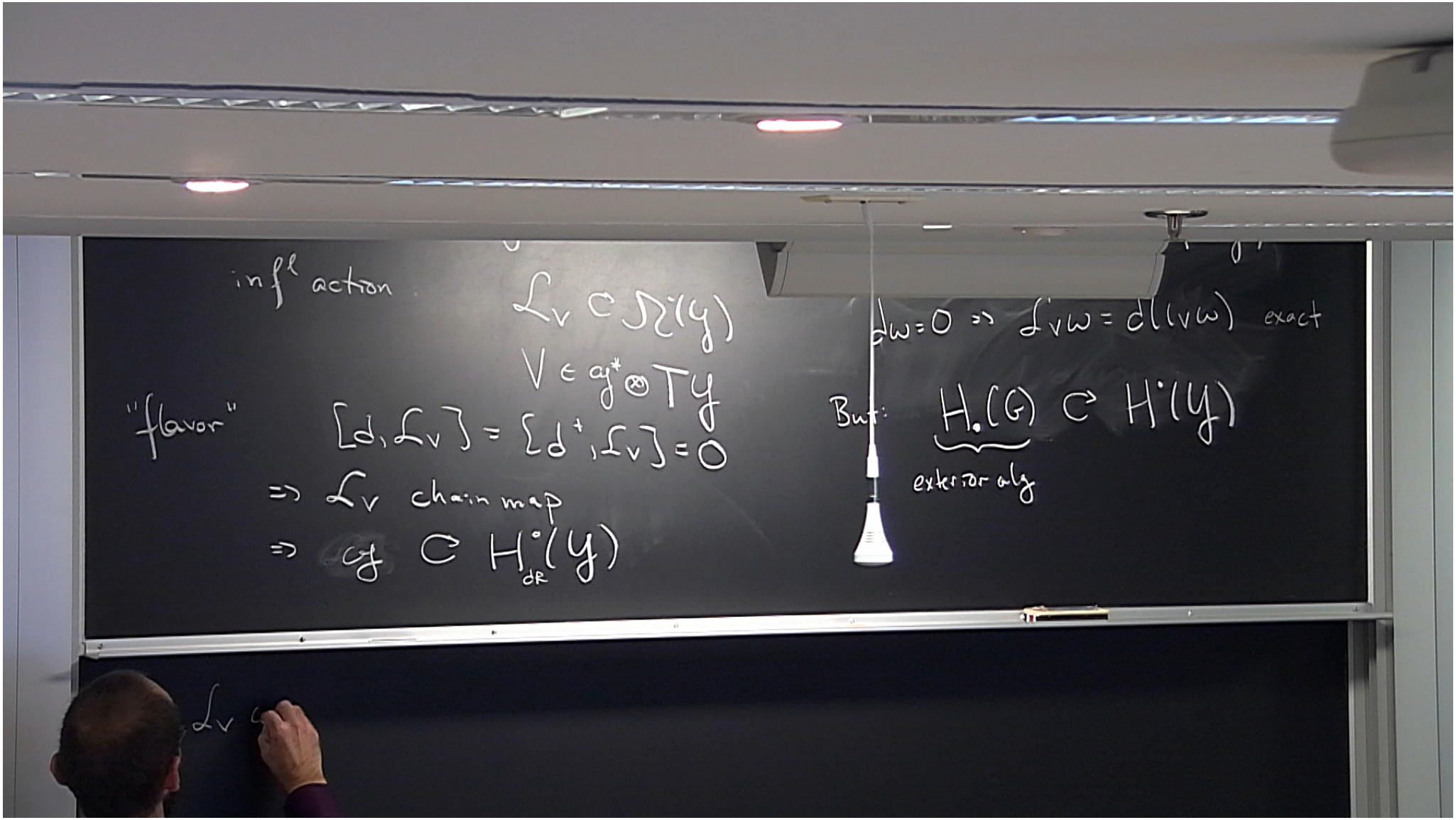

4. Koszul duality / 2d
5. SUSY breaking
Hamiltonian
Hilbert space
Ground states
fixed t
harmonic

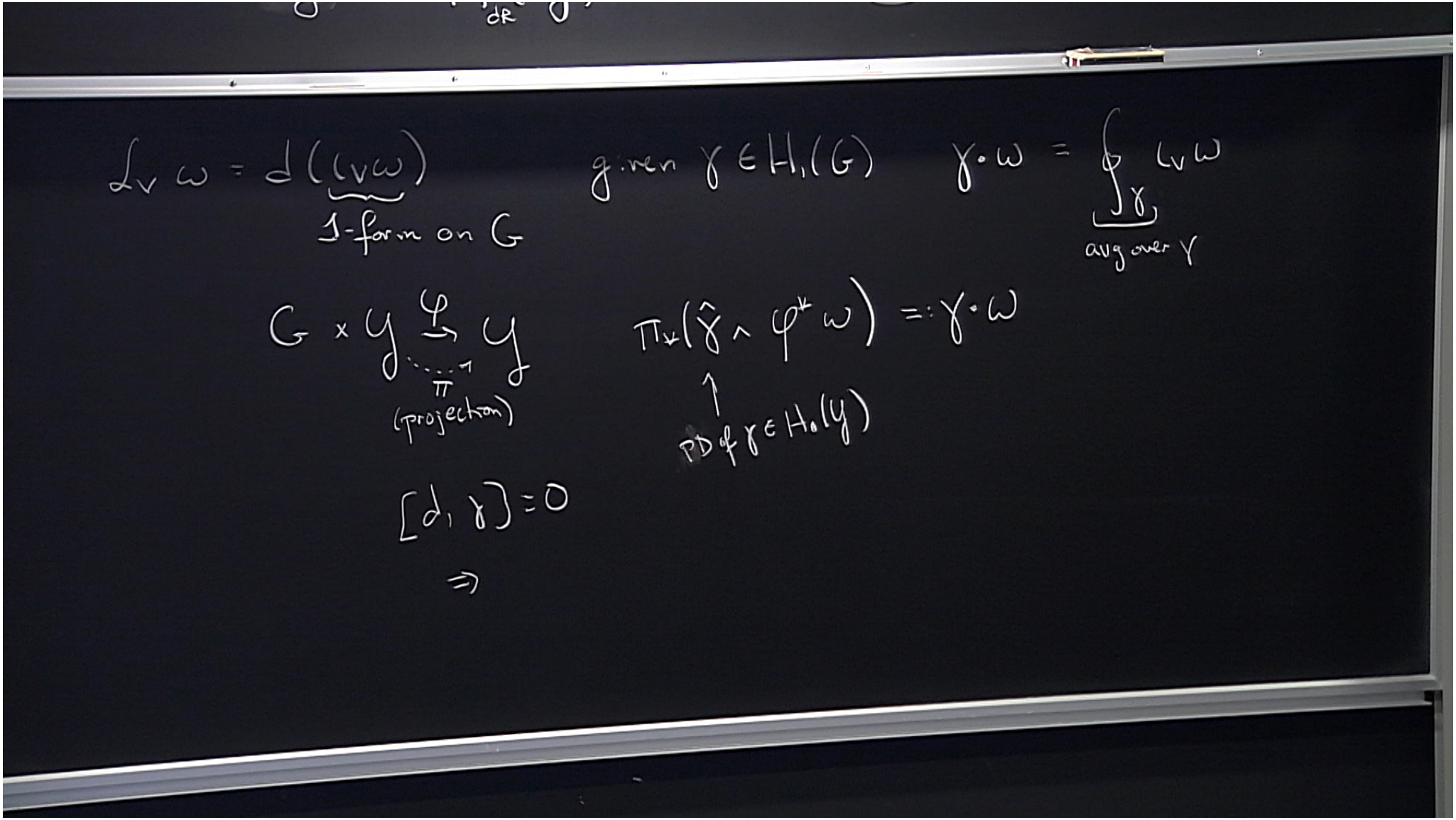

SQM:
phase space = $T^*(\Pi TY)$
$\mathcal{H} = \Omega^\cdot(Y)$
$H = \Delta$ form Lap. $= \{d, d^\dagger\}$
also: $\{Q, \bar{Q}\} = H$
$d \quad d^\dagger$
SUSY g.s.
$Q|\psi\rangle = \bar{Q}|\psi\rangle = 0$
$\mathcal{H}_{SUSY} = \Omega(Y)^{harm}$
$\simeq H^\cdot(\mathcal{H}, Q)$
(Hodge thm)

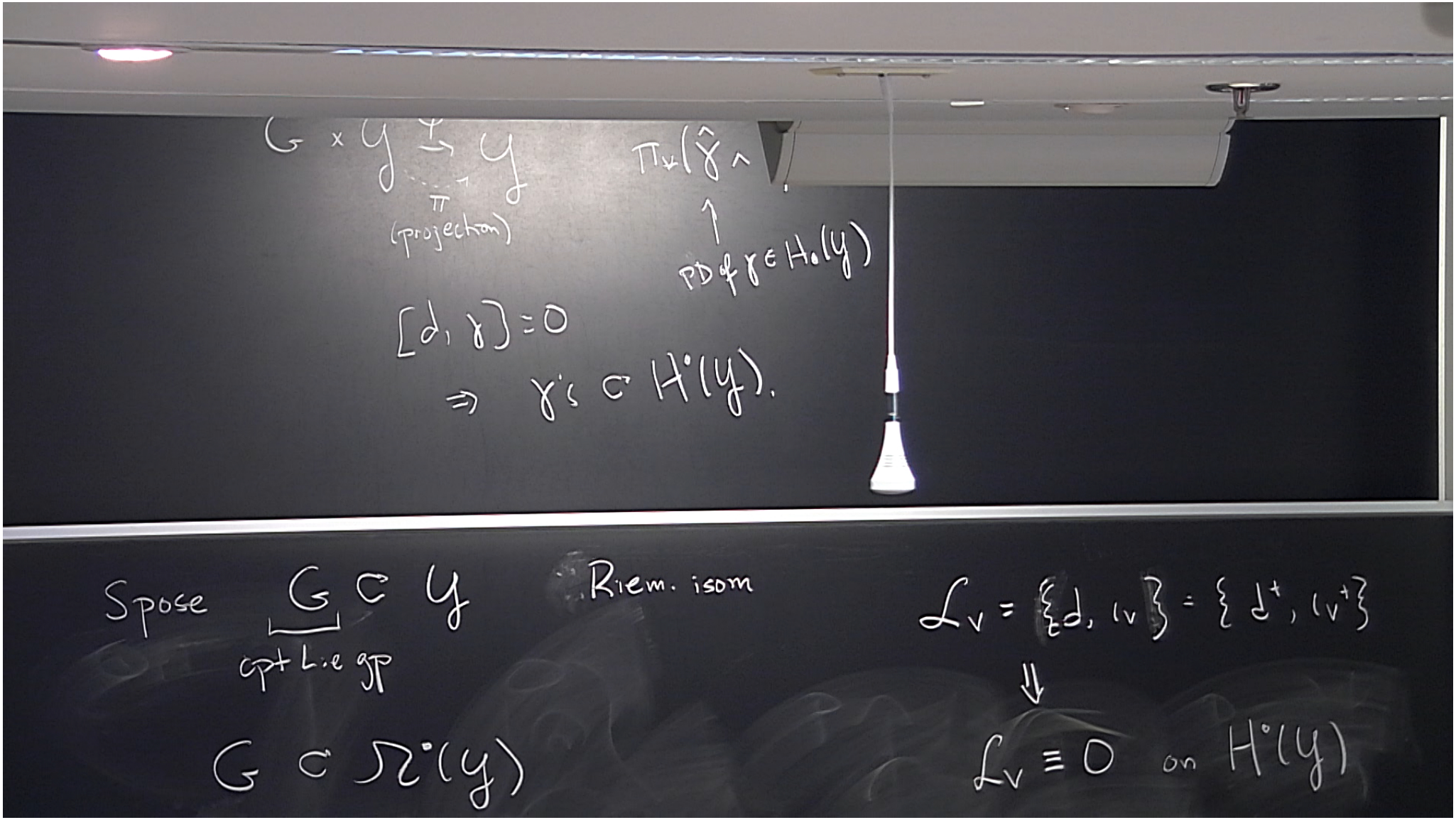

Spose G ⊂ Y
cpt Lie gp
Riem. isom
G ⊂ Ω⁰(Y)
infl action :
L_V ⊂ Ω⁰(Y)
V ∈ g* ⊗ TY
[d, L_V] = [d*, L_V] = 0

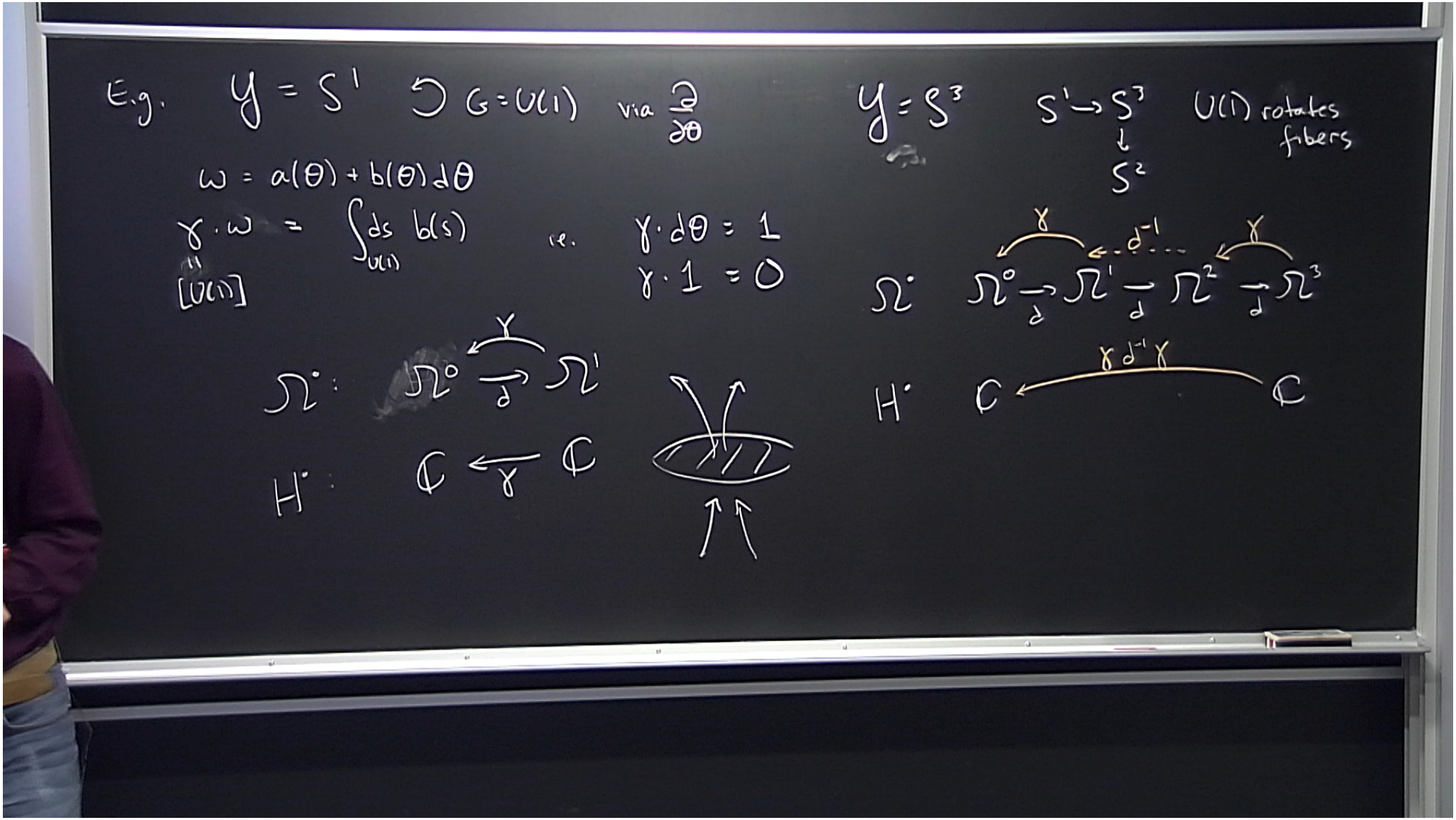

⇔ harmonic
space = $T^*(\Pi TY)$
$\mathcal{H} = \Omega^\bullet(Y)$
Δ form Lap. $= \{d, d^\dagger\}$
$\{Q, \bar{Q}\} = H$
d $d^\dagger$
susy g.s.
$Q|\psi\rangle = \bar{Q}|\psi\rangle = 0$
$\mathcal{H}_{susy} = \Omega(Y)^{harm}$
$\cong H^\bullet(\mathcal{H}, Q)$
(Hodge thm) $\cong H^\bullet_{dR}(Y)$
⇒ $\mathcal{L}_V$
⇒ g
CAUTION


Spose G ⊂ Y Riem. isom
cpt L.e gp
G ⊂ Ω°(Y)
inf'l action: $\mathcal{L}_V \subset \Omega^\circ(Y)$
$V \in \mathfrak{g}^* \otimes TY$
"flavor" $[d, \mathcal{L}_V] = [d^+, \mathcal{L}_V] = 0$
⇒ $\mathcal{L}_V$ chain map
⇒ $\mathfrak{g} \subset H^\circ_{dR}(Y)$
$\mathcal{L}_V = \{d, \iota_V\} = \{d^+, \iota_V^+\}$
⇓
$\mathcal{L}_V \equiv 0$ on $H^\circ(Y)$
$dw = 0 \Rightarrow \mathcal{L}_V w = d(\iota_V w)$ exact


infl action
$\mathcal{L}_V \subset \mathfrak{X}(y)$
$V \in \mathfrak{g}^* \otimes Ty$
"flavor"
$[d, \mathcal{L}_V] = [d^+, \mathcal{L}_V] = 0$
$\Rightarrow \mathcal{L}_V$ chain map
$\Rightarrow \mathfrak{g} \subset H^\bullet_{dR}(y)$
$dw = 0 \Rightarrow \mathcal{L}_V w = d(Vw)$ exact
But: $H_\bullet(G) \subset H^\bullet(y)$
exterior alg


$\mathcal{L}_V \omega = d(\underbrace{\iota_V \omega}_{1\text{-form on } G})$
given $\gamma \in H_1(G)$
$\gamma \cdot \omega = \underbrace{\oint_\gamma}_{\text{avg over } \gamma} \iota_V \omega$
$G \times Y \xrightarrow{\varphi} Y$
π (projection)
$\pi_*(\hat{\gamma} \wedge \varphi^* \omega) =: \gamma \cdot \omega$
$\uparrow$
PD of $\gamma \in H_0(Y)$
$[d, \gamma] = 0$
$\Rightarrow$


G × Y → Y
π
(projection)
[d, γ] = 0
⇒ γ's ∈ H°(Y).
π*(γ̂ ∩
↑
PD of γ ∈ H*(Y)
Spose G ⊂ Y
Riem. isom
gp + Lie gp
G ⊂ Ω°(Y)
ℒ_V = {d, ι_V} = {d†, ι_V†}
⇓
ℒ_V ≡ 0 on H°(Y)


E.g. $Y = S^1$ $\circlearrowright$ $G = U(1)$ via $\frac{\partial}{\partial\theta}$
$\omega = a(\theta) + b(\theta)\,d\theta$
$\gamma \cdot \omega = \int_{U(1)} ds\, b(s)$
$\gamma \in [U(1)]$
i.e. $\gamma \cdot d\theta = 1$
$\gamma \cdot 1 = 0$
$\Omega^\cdot$: $\Omega^0 \xrightarrow{d} \Omega^1$
$H^\cdot$: $\mathbb{C} \xleftarrow{\gamma} \mathbb{C}$
$Y = S^3$
$S^1 \to S^3 \downarrow S^2$
$U(1)$ rotates fibers
$\Omega^\cdot$: $\Omega^0 \xrightarrow{d} \Omega^1 \xrightarrow{d} \Omega^2 \xrightarrow{d} \Omega^3$
γ, d^{-1}, γ
$H^\cdot$: $\mathbb{C} \xleftarrow{\gamma d^{-1} \gamma} \mathbb{C}$

$\Omega^\bullet(\mathcal{Y}) \in H_\bullet(G)\text{-dg-mod}$

$\mathcal{H} = (\Omega^\bullet(\mathcal{Y}) \otimes \mathbb{C}[\sigma])^G$

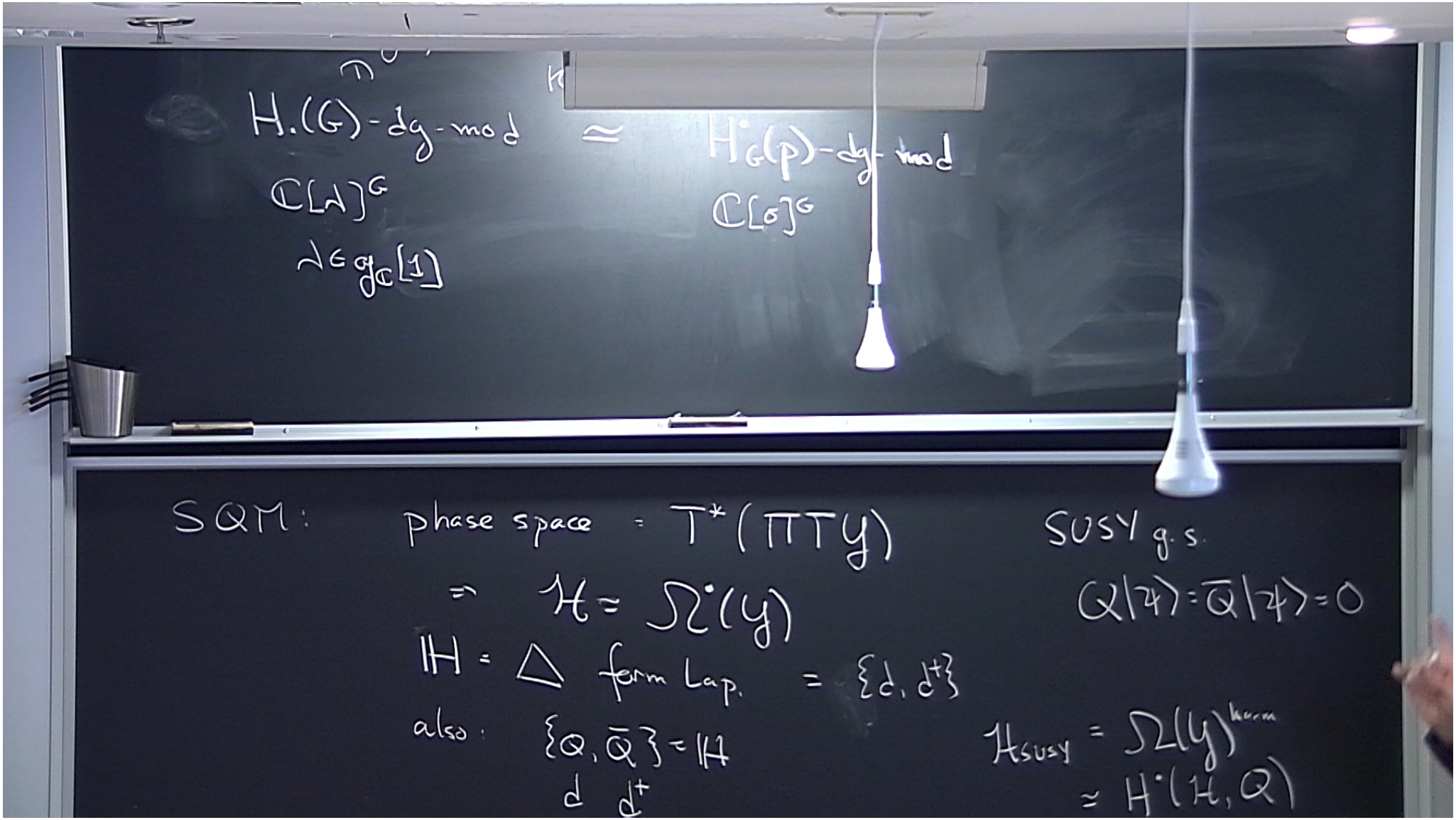
vs <u>gauge sym</u>: introduce principal $\overset{\text{super}}{G}$-bundle, integrate over $\overset{\text{super}}{\text{connections}}$

$\sigma \in \mathfrak{g}_0$

$\mathcal{H}_{\text{susy}} = H^\bullet(\mathcal{H}, d + \sigma(v))$

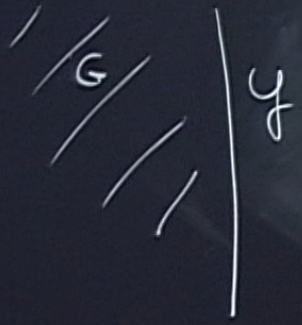
is

$H^\bullet_G(\mathcal{Y})$


$H_\cdot(G)$-dg-mod $\simeq$ $H^\cdot_G(p)$-dg-mod
$\mathbb{C}[\mathfrak{h}]^G$
$\mathbb{C}[\sigma]^G$
$\wedge^\cdot \mathfrak{g}_\mathbb{C}[1]$
SQM: phase space $= T^*(\Pi TY)$
$\mathcal{H} = \Omega^\cdot(Y)$
$\mathbb{H} = \Delta$ form Lap. $= \{d, d^+\}$
also: $\{Q, \bar{Q}\} = \mathbb{H}$
d d^+
susy g.s.
$Q|\psi\rangle = \bar{Q}|\psi\rangle = 0$
$\mathcal{H}_{susy} = \Omega(Y)^{harm}$
$\simeq H^\cdot(\mathcal{H}, Q)$

2d $\mathcal{N}=(2,2)$ pure G gauge thy

A-twisted

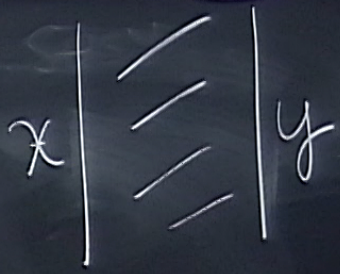


Bdy

$\cup$

y

category of b.c.



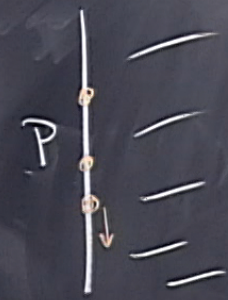
$\mathrm{Hom}(x,y)$

$\simeq H_G(x \times y)$

$\Rightarrow \gamma_i \in \mathrm{Hig}_i$

2 canonical b.c.

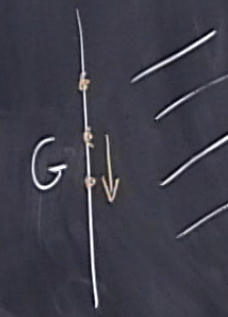
Neumann



$y \rightsquigarrow H^*_G(P \times y) \simeq H^*_G(y)$

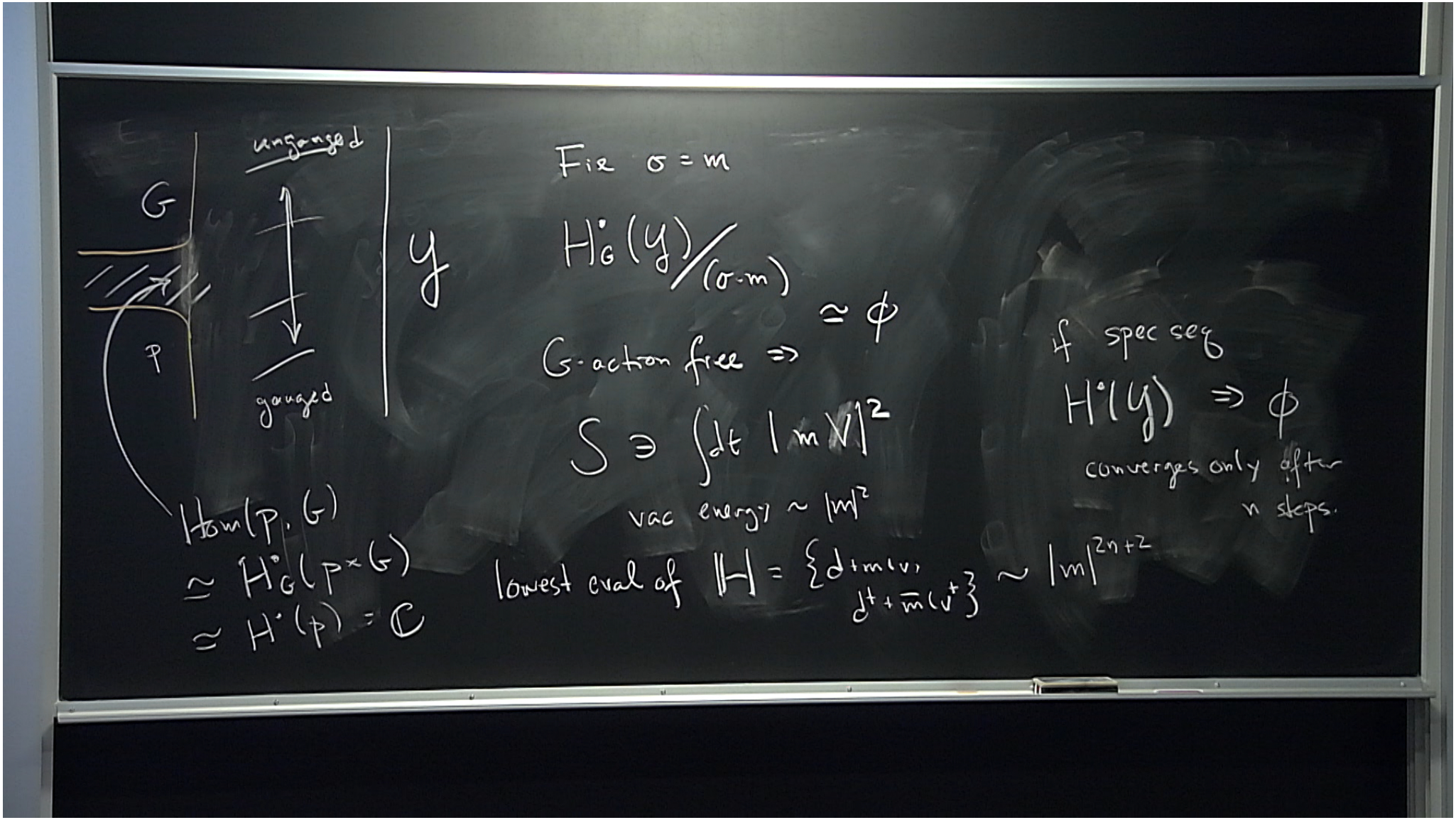
$\mathrm{End}(P) \simeq H^*_G(P \times P) \simeq H^*_G(P) = \mathbb{C}[\sigma]^G$

Dirichlet



$y \rightsquigarrow H^*_G(G \times y) \simeq H^*(y)$

$\mathrm{End}(G) \simeq H^*(G) \simeq H_*(G) \simeq \mathbb{C}[\lambda]^G$


ungauged
G
P
gauged
y
Fix σ = m
$H^\bullet_G(y)/(\sigma - m) \simeq \phi$
G-action free ⇒
$S \ni \int dt\, |mV|^2$
vac energy ~ $|m|^2$
lowest eval of $\|H\| = \{d + m\iota_v, d^\dagger + \bar{m}\iota_{v^\dagger}\} \sim |m|^{2n+2}$
if spec seq
$H^\bullet(y) \Rightarrow \phi$
converges only after
n steps.
$\mathrm{Hom}(P, G)$
$\simeq H^\bullet_G(P \times G)$
$\simeq H^\bullet(P) = \mathbb{C}$