Title: G-actions in quantum mechanics (and spectral sequences and the cosmological constant)

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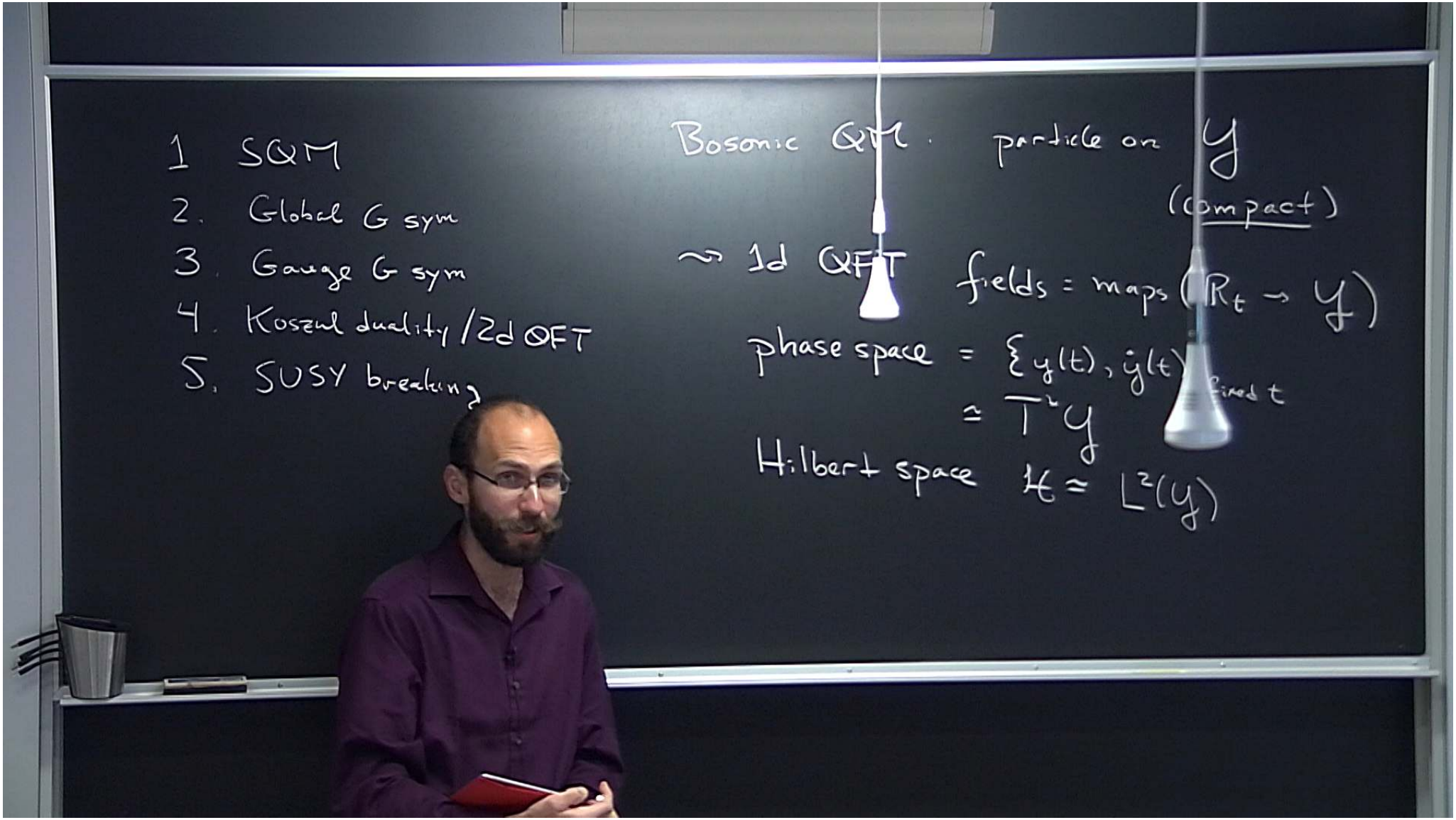
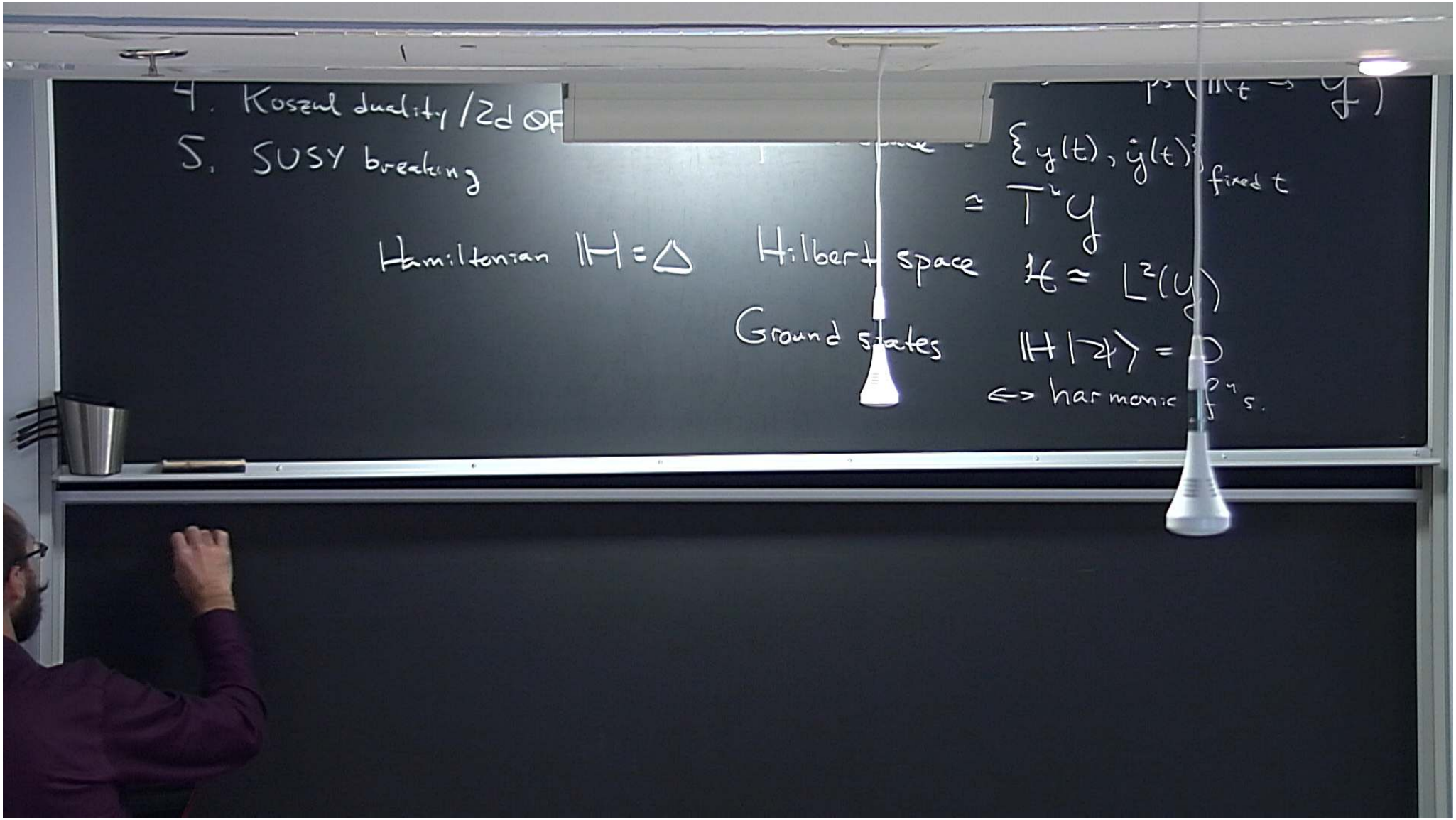
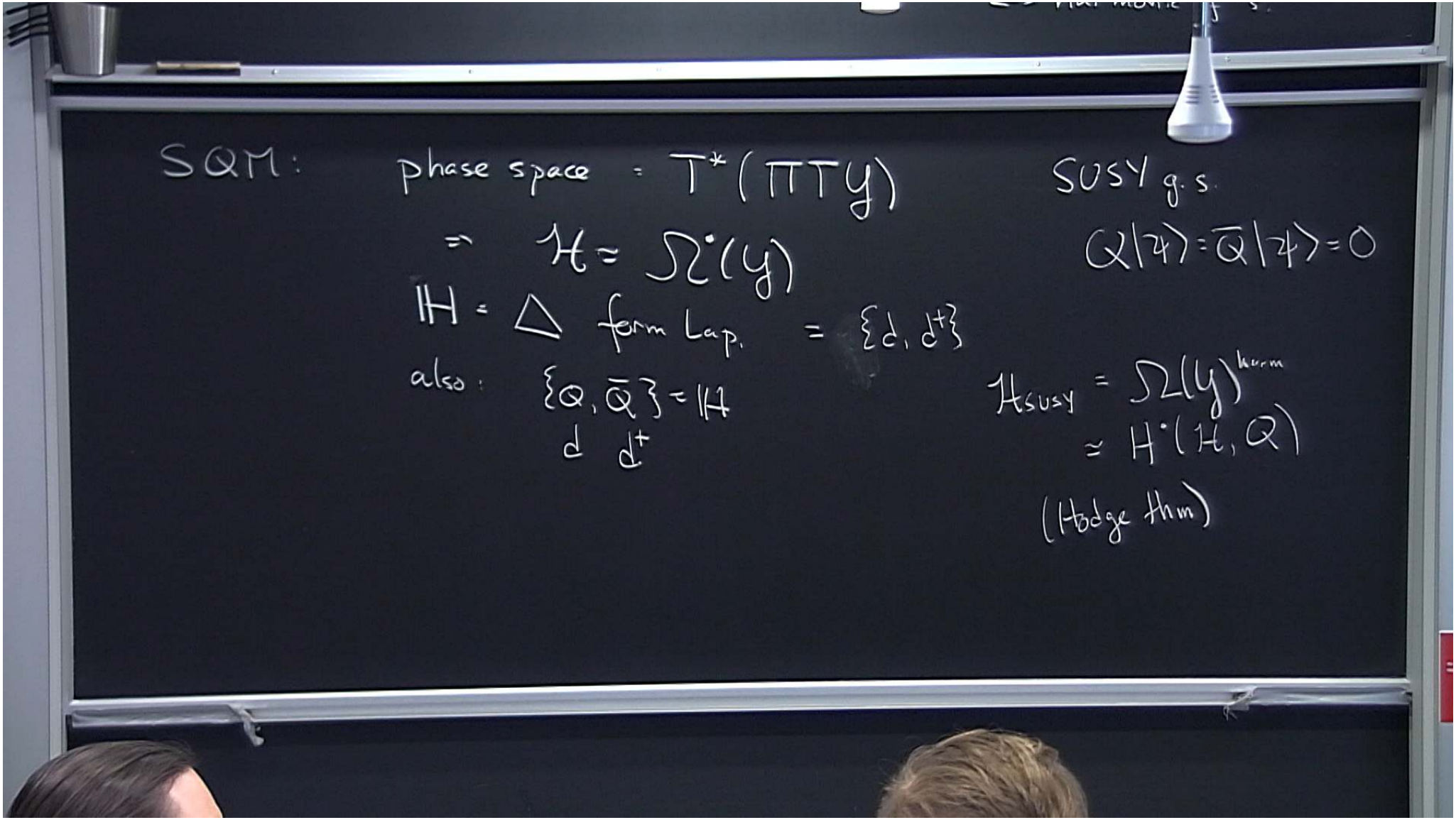
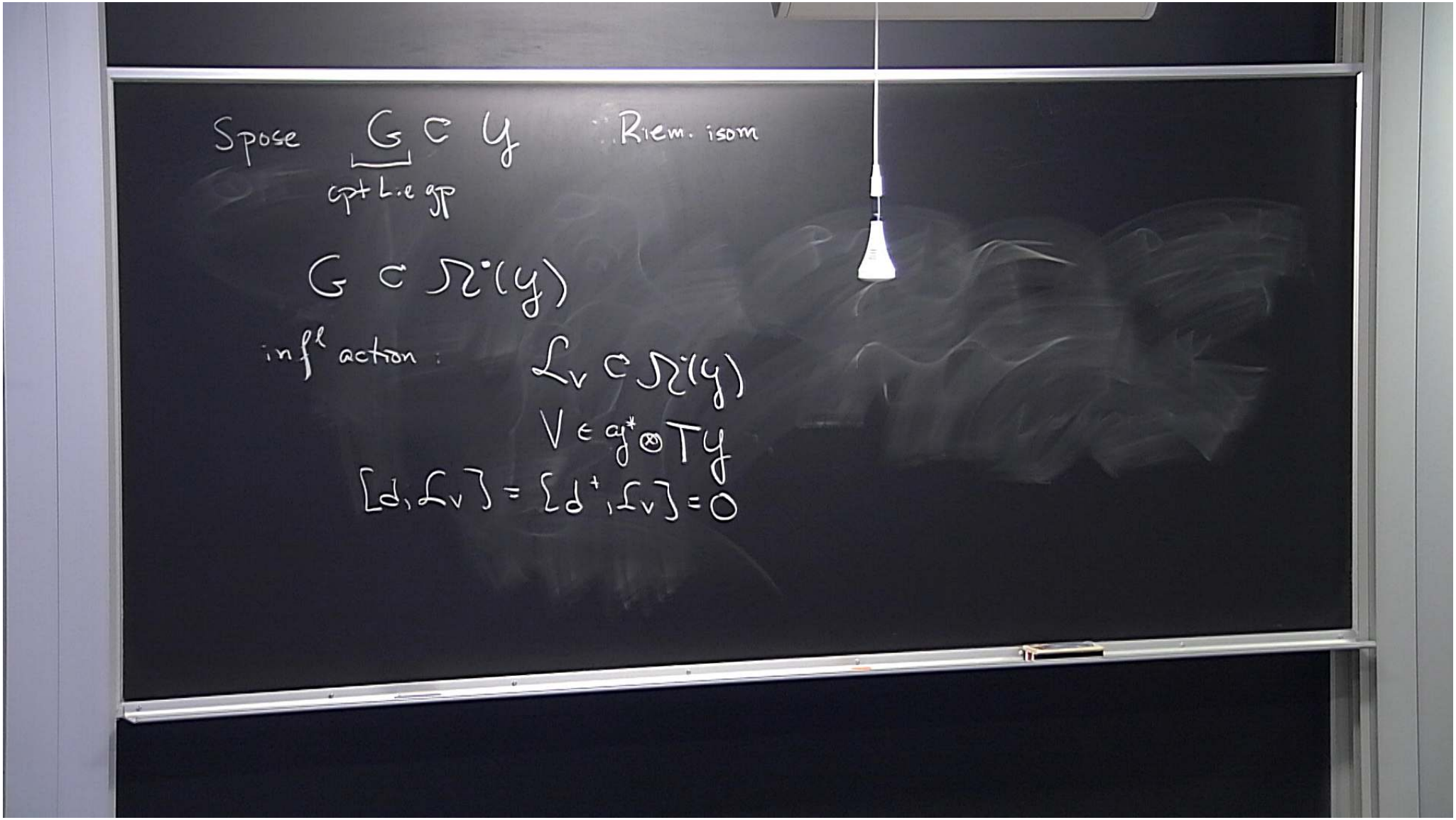
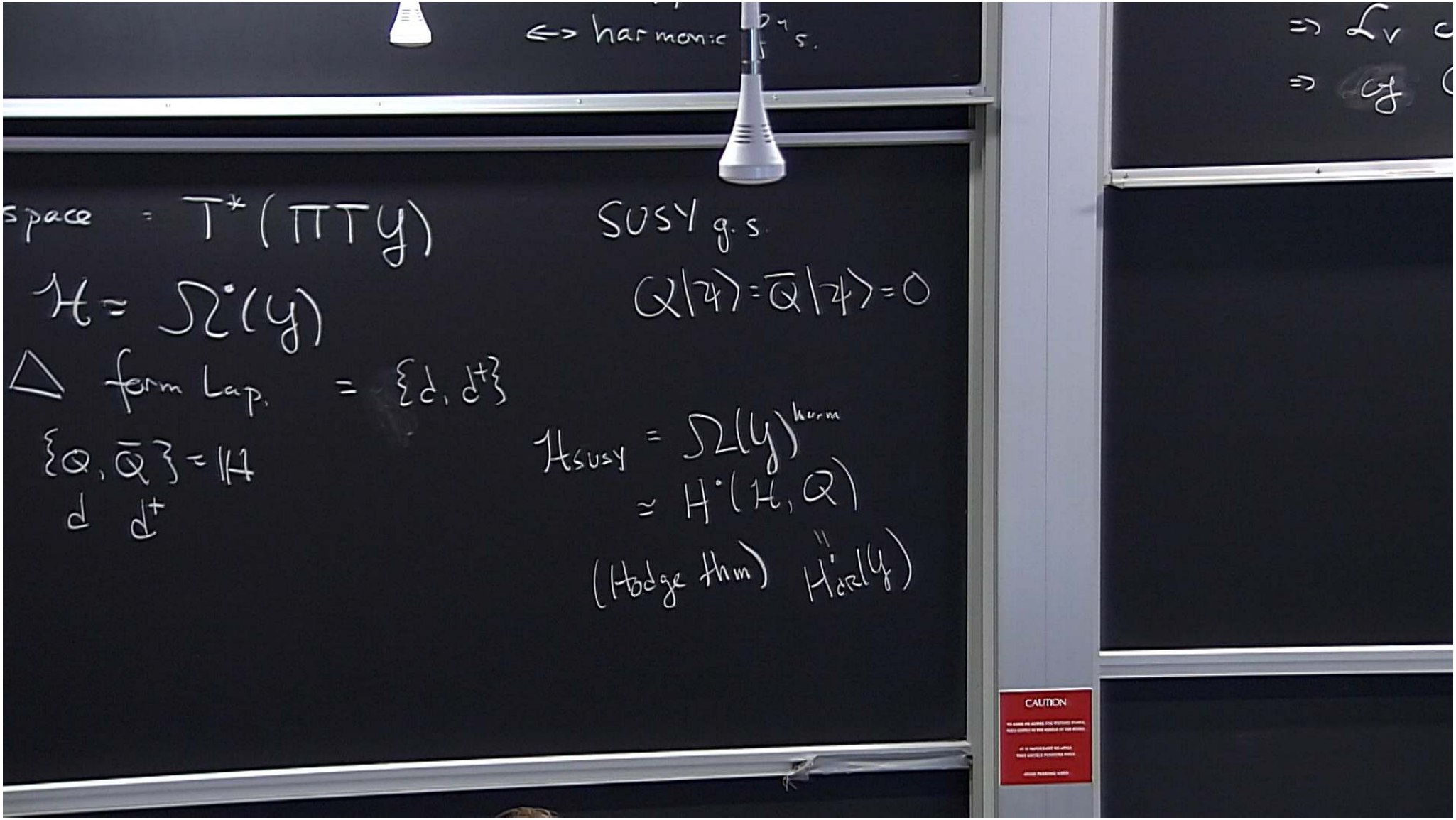
Abstract:

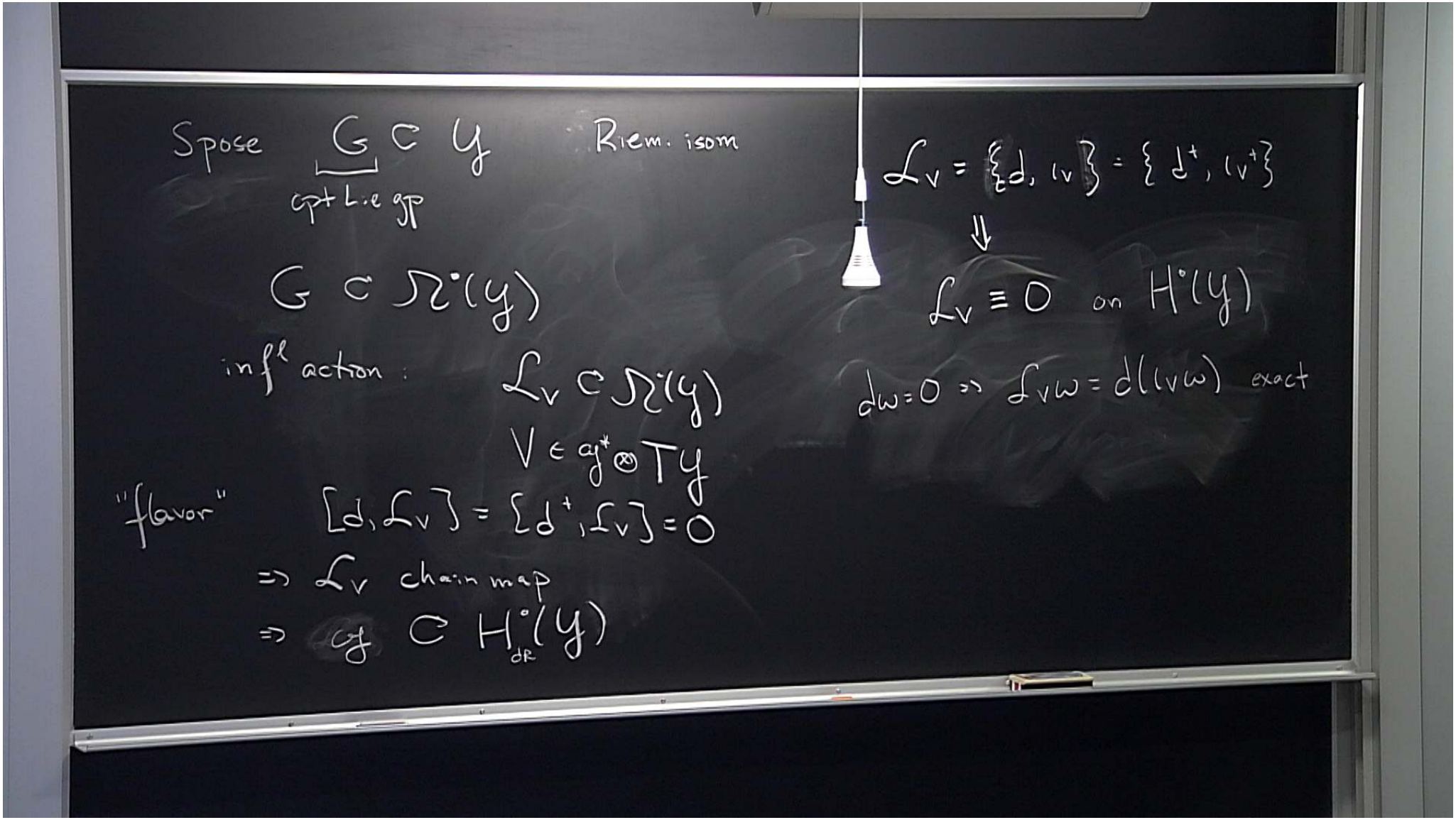

1 SQM
2. Global G sym
3. Gauge G sym
4. Koszul duality / 2d QFT
5. SUSY breaking
Bosonic QM: particle on Y (compact)
~ 1d QFT
fields = maps($\mathbb{R}_t \to Y$)
phase space = $\{y(t), \dot{y}(t)$ fixed t
$\cong T^*Y$
Hilbert space $\mathcal{H} \cong L^2(Y)$

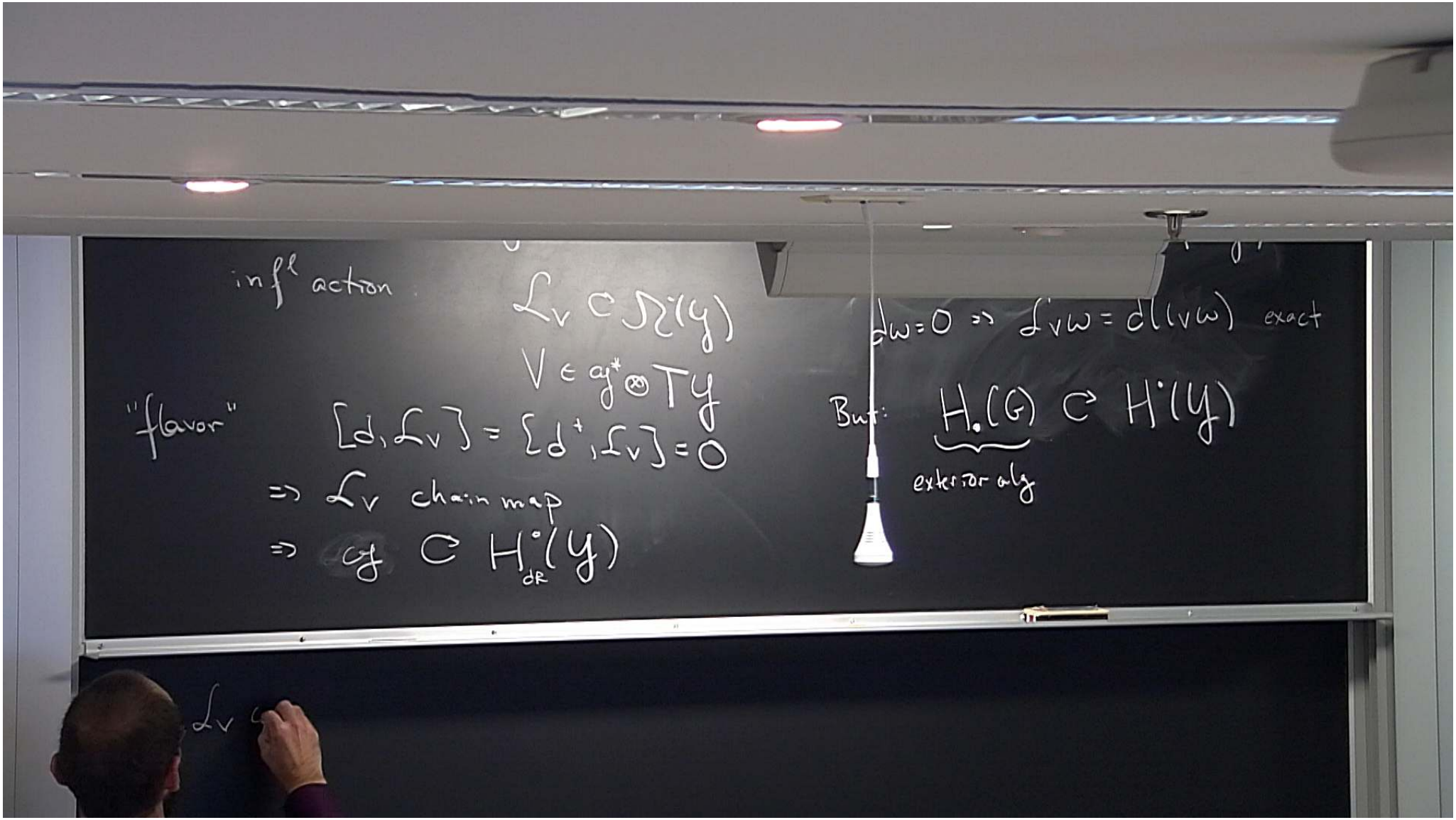

4. Koszul duality / 2d
5. SUSY breaking
Hamiltonian
Hilbert space
Ground states
fixed t
harmonic


SQM:
phase space = $T^*(\Pi TY)$
$\mathcal{H} = \Omega^\cdot(Y)$
$H = \Delta$ form Lap. $= \{d, d^\dagger\}$
also: $\{Q, \bar{Q}\} = H$
d $d^\dagger$
SUSY g.s.
$Q|\psi\rangle = \bar{Q}|\psi\rangle = 0$
$\mathcal{H}_{SUSY} = \Omega(Y)^{harm}$
$\simeq H^\cdot(\mathcal{H}, Q)$
(Hodge thm)


Spose G ⊂ Y
Riem. isom
cpt Lie gp
G ⊂ Ω⁰(Y)
infl action :
L_V ⊂ Ω⁰(Y)
V ∈ g* ⊗ TY
[d, L_V] = [d*, L_V] = 0


⇔ harmonic
space = $T^*(\Pi TY)$
$\mathcal{H} = \Omega^\bullet(Y)$
Δ form Lap. = $\{d, d^\dagger\}$
$\{Q, \bar{Q}\} = H$
d $d^\dagger$
susy g.s.
$Q|\psi\rangle = \bar{Q}|\psi\rangle = 0$
$\mathcal{H}_{susy} = \Omega(Y)^{harm}$
$\simeq H^\bullet(\mathcal{H}, Q)$
(Hodge thm) $H^\bullet_{dR}(Y)$
$\Rightarrow \mathcal{L}_V$
$\Rightarrow g$
CAUTION


Spose G ⊂ Y Riem. isom
cpt Lie gp
G ⊂ Ω°(Y)
inf^l action: L_V ⊂ Ω°(Y)
V ∈ g* ⊗ TY
"flavor" [d, L_V] = [d†, L_V] = 0
⇒ L_V chain map
⇒ g ⊂ H°_dR(Y)
L_V = {d, ι_V} = {d†, ι_V†}
⇓
L_V ≡ 0 on H°(Y)
dω = 0 ⇒ L_V ω = d(ι_V ω) exact


infl action
$\mathcal{L}_V \subset \mathfrak{X}(Y)$
$V \in \mathfrak{g}^* \otimes TY$
"flavor"
$[d, \mathcal{L}_V] = [d^\dagger, \mathcal{L}_V] = 0$
$\Rightarrow \mathcal{L}_V$ chain map
$\Rightarrow \mathfrak{g} \subset H^\bullet_{dR}(Y)$
$d\omega = 0 \Rightarrow \mathcal{L}_V\omega = d(V\omega)$ exact
But: $H_\bullet(G) \subset H^\bullet(Y)$
exterior alg

$\mathcal{L}_v \omega = d(\underbrace{\iota_v \omega}_{1\text{-form on } G})$

given $\gamma \in H_1(G)$ $\quad \gamma \cdot \omega = \underbrace{\oint_\gamma}_{\text{avg over } \gamma} \iota_v \omega$

$G \times Y \xrightarrow{\varphi} Y$

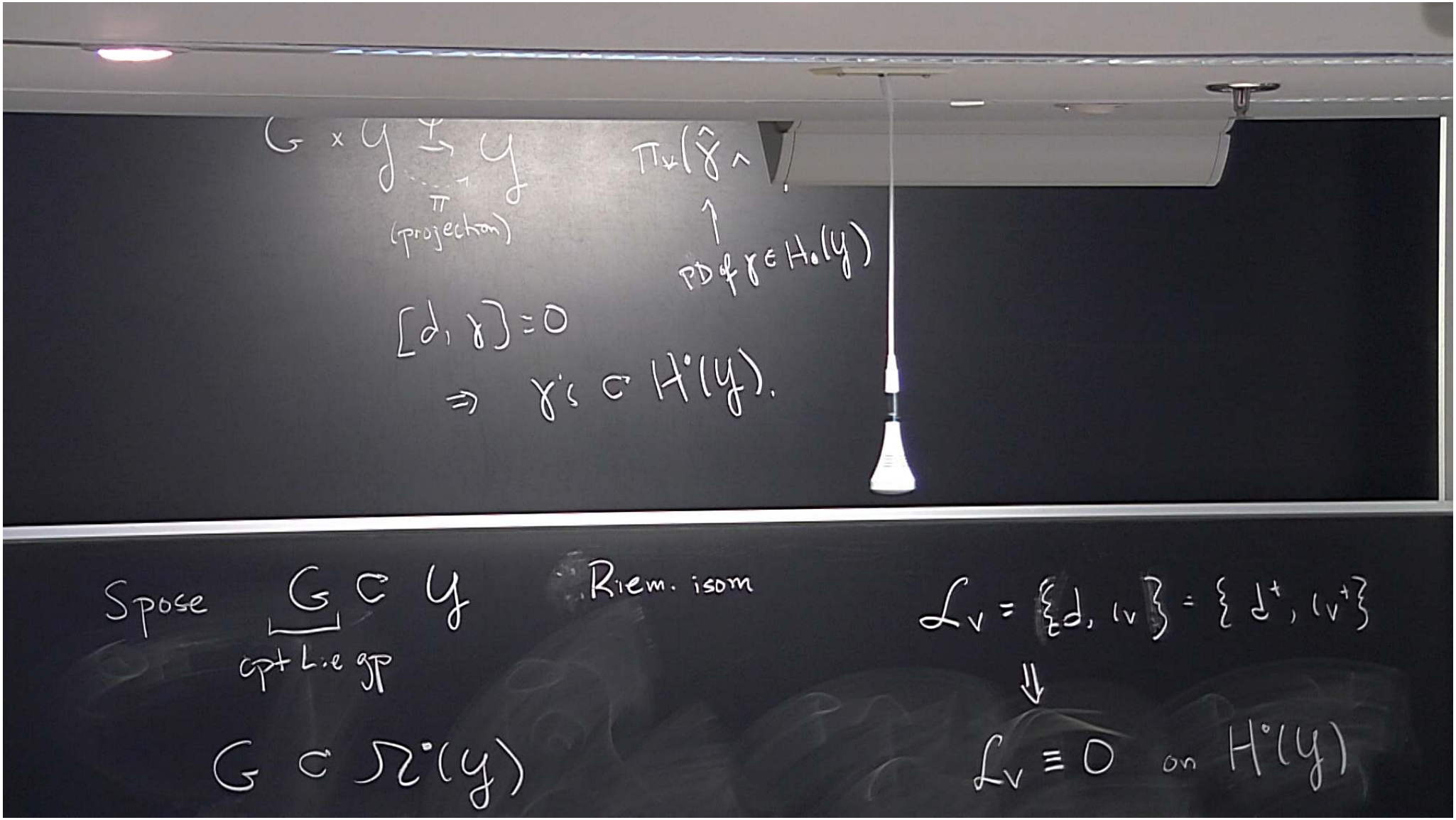
π (projection)

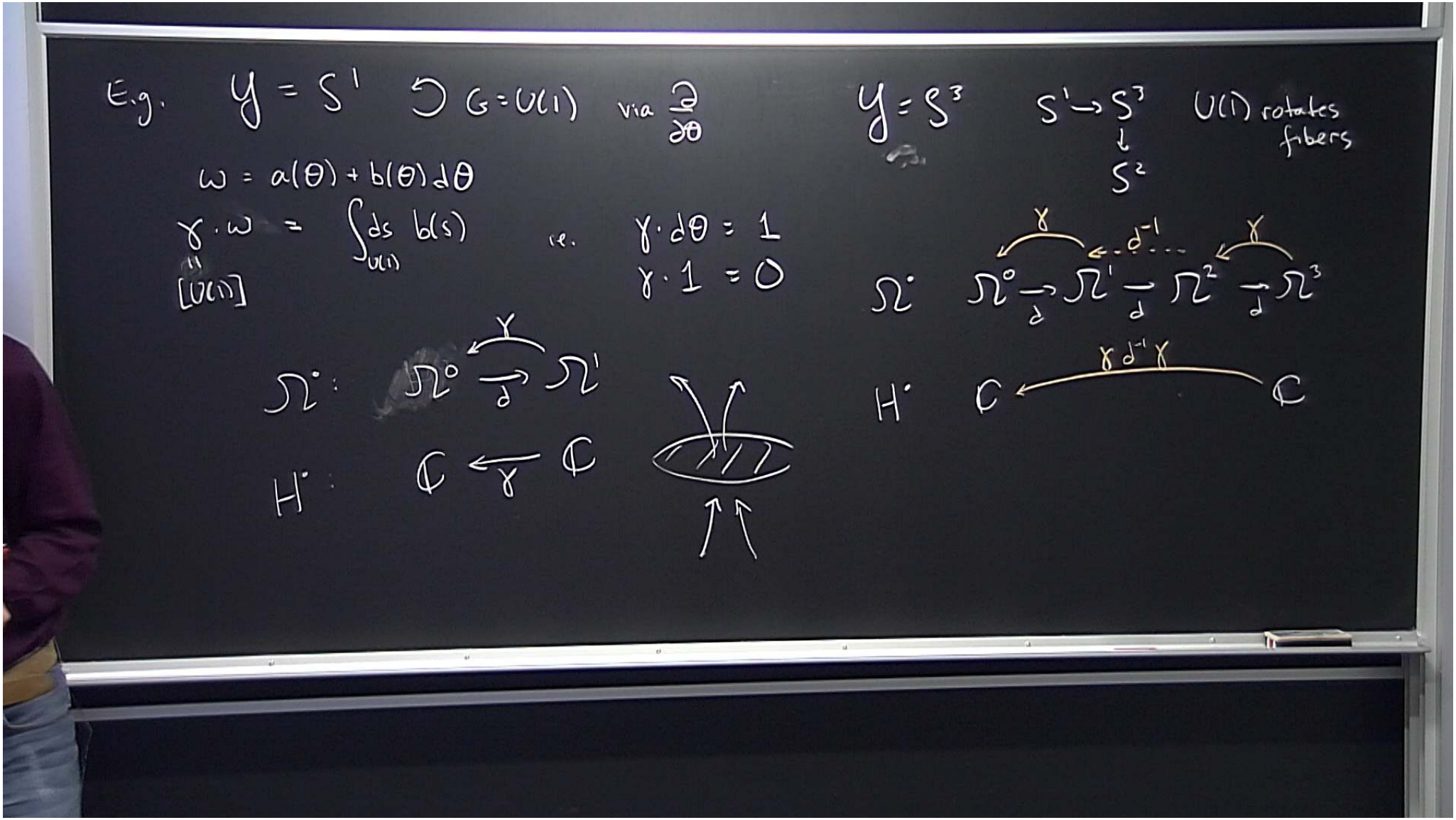
$\pi_*(\hat{\gamma} \wedge \varphi^* \omega) =: \gamma \cdot \omega$

$\uparrow$ PD of $\gamma \in H_0(Y)$

$[d, \gamma] = 0$

$\Rightarrow$


$G \times Y \xrightarrow{\Psi} Y$
π
(projection)
$\pi_*(\hat{\gamma} \wedge$
PD of $\gamma \in H_*(Y)$
$[d, \gamma] = 0$
$\Rightarrow \gamma$'s $\subset H^*(Y)$.
Spose $G \subset Y$
Riem. isom
cpt Lie gp
$G \subset \Omega^*(Y)$
$\mathcal{L}_V = \{d, \iota_V\} = \{d^\dagger, \iota_{V^\dagger}\}$
$\Downarrow$
$\mathcal{L}_V \equiv 0$ on $H^*(Y)$


E.g. $Y = S^1$ $\circlearrowright$ $G = U(1)$ via $\frac{\partial}{\partial\theta}$
$\omega = a(\theta) + b(\theta)\,d\theta$
$\gamma \cdot \omega = \int_{U(1)} ds\, b(s)$
$\gamma \in [U(1)]$
i.e. $\gamma \cdot d\theta = 1$
$\gamma \cdot 1 = 0$
$\Omega^\cdot$: $\Omega^0 \xrightarrow{d} \Omega^1$
$H^\cdot$: $\mathbb{C} \xleftarrow{\gamma} \mathbb{C}$
$Y = S^3$
$S^1 \to S^3$
$\downarrow$
S^2
$U(1)$ rotates fibers
$\Omega^\cdot$: $\Omega^0 \xrightarrow{d} \Omega^1 \xrightarrow{d} \Omega^2 \xrightarrow{d} \Omega^3$
γ, d^{-1}, γ
$H^\cdot$: $\mathbb{C} \xleftarrow{\gamma d^{-1} \gamma} \mathbb{C}$

$\Omega^\bullet(y) \in H_\bullet(G)\text{-dg-mod}$

$\mathcal{H} = (\Omega^\bullet(y) \otimes \mathbb{C}[\sigma])^G$

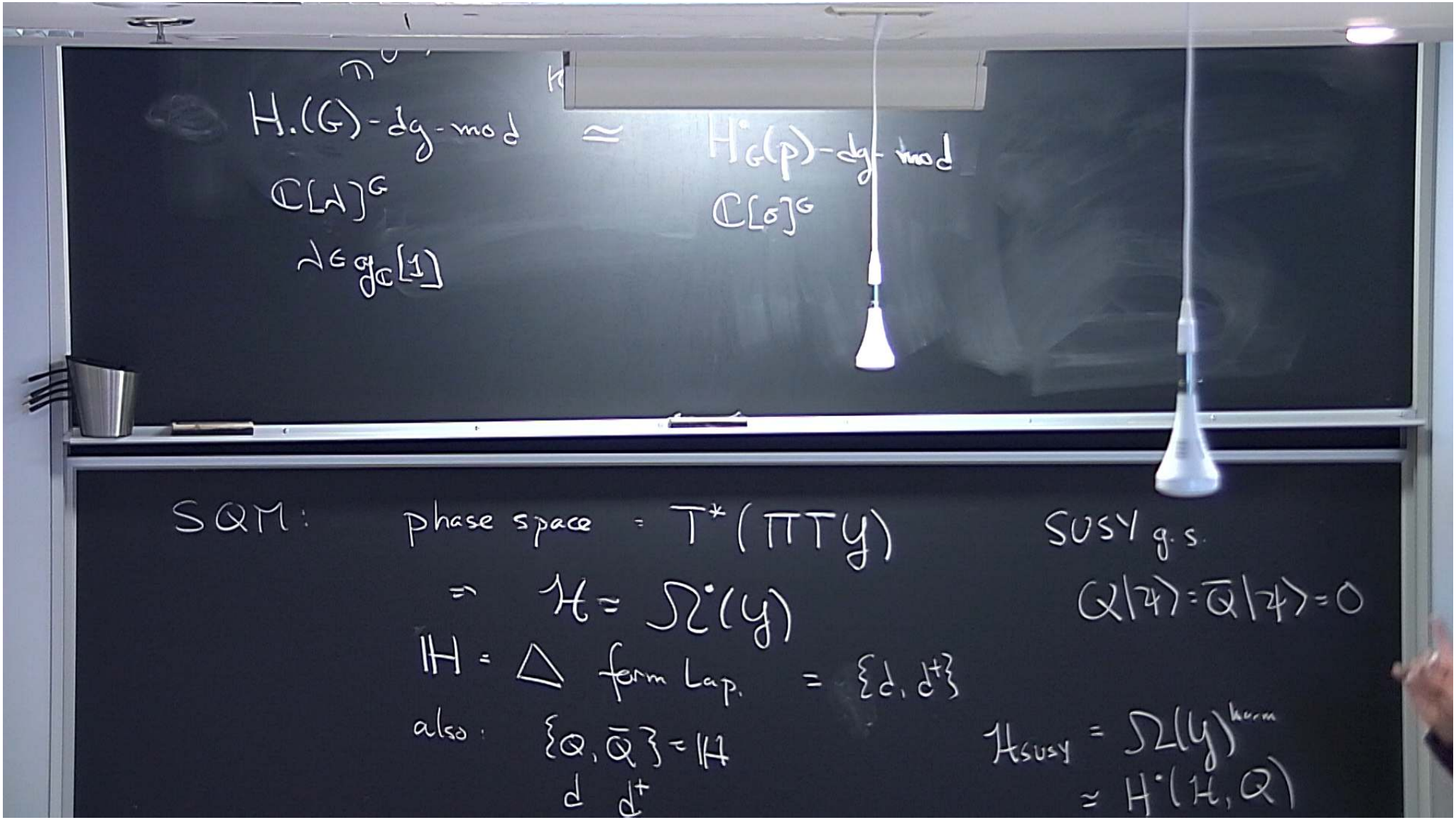
vs <u>gauge sym</u>: introduce principal $\overset{\text{super}}{G}$-bundle, integrate over $\overset{\text{super}}{\text{connections}}$

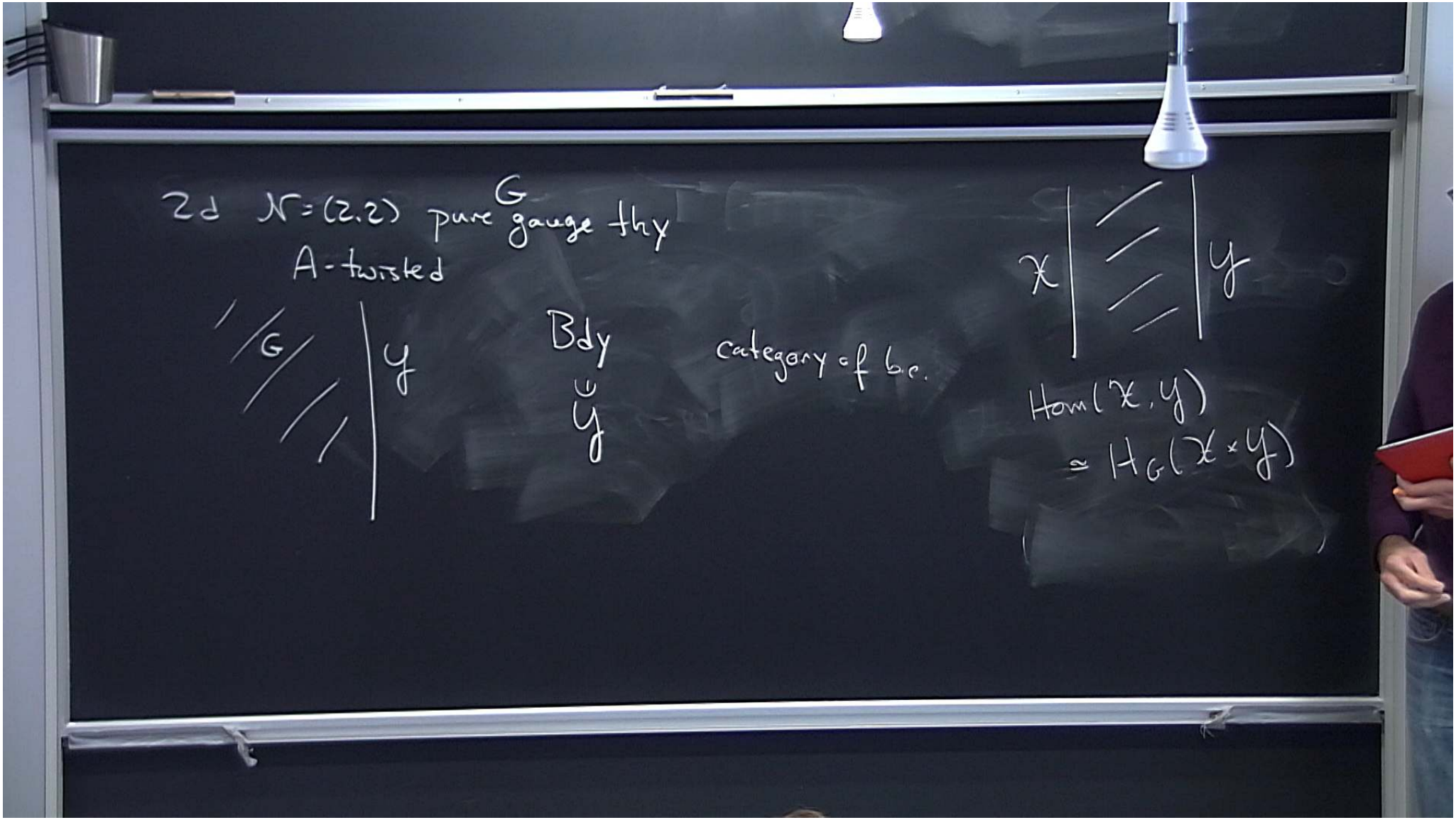
$\sigma \in \mathfrak{g}_0$

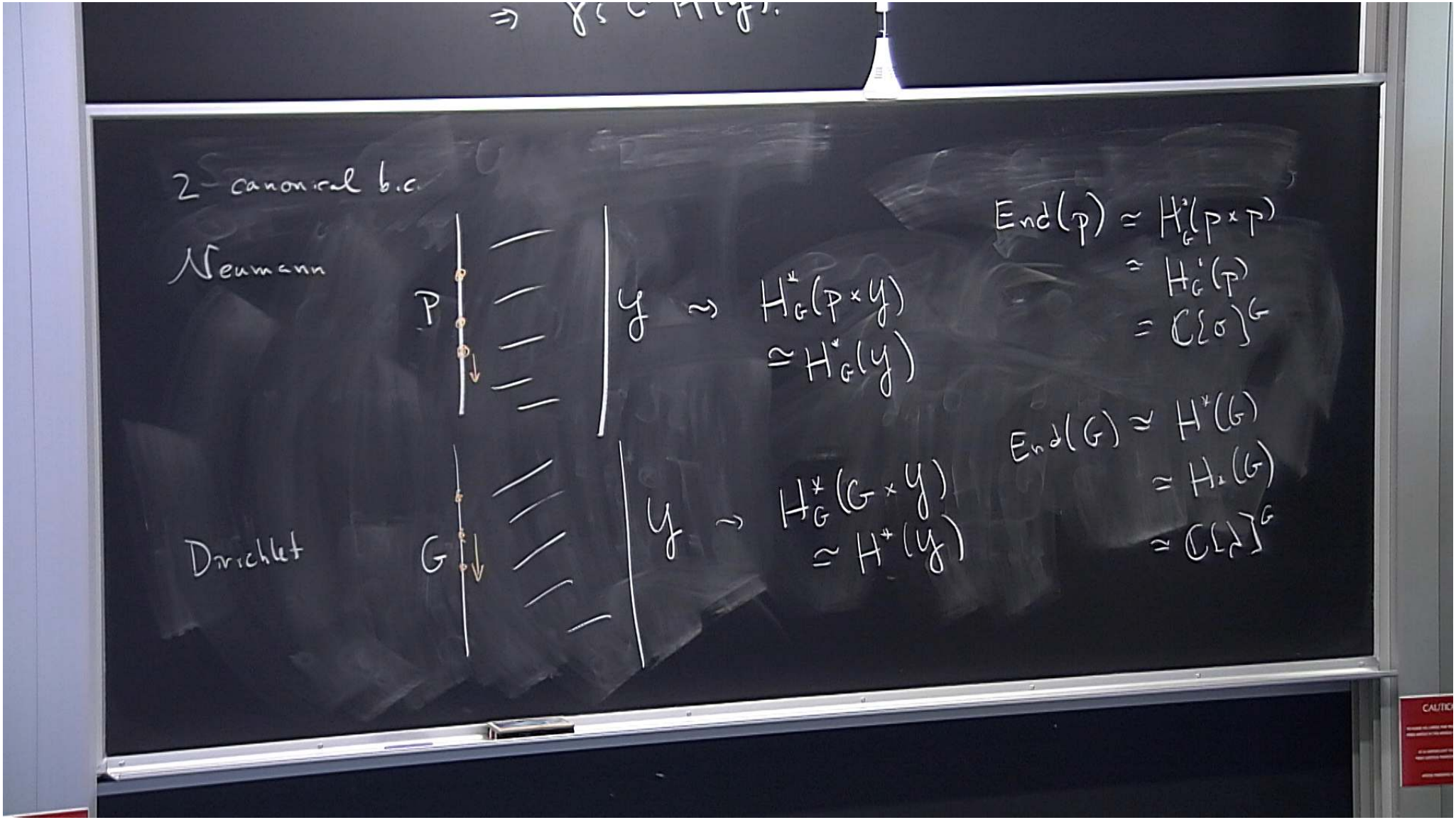
$\mathcal{H}_{\text{susy}} = H^\bullet(\mathcal{H}, d + \sigma(v))$

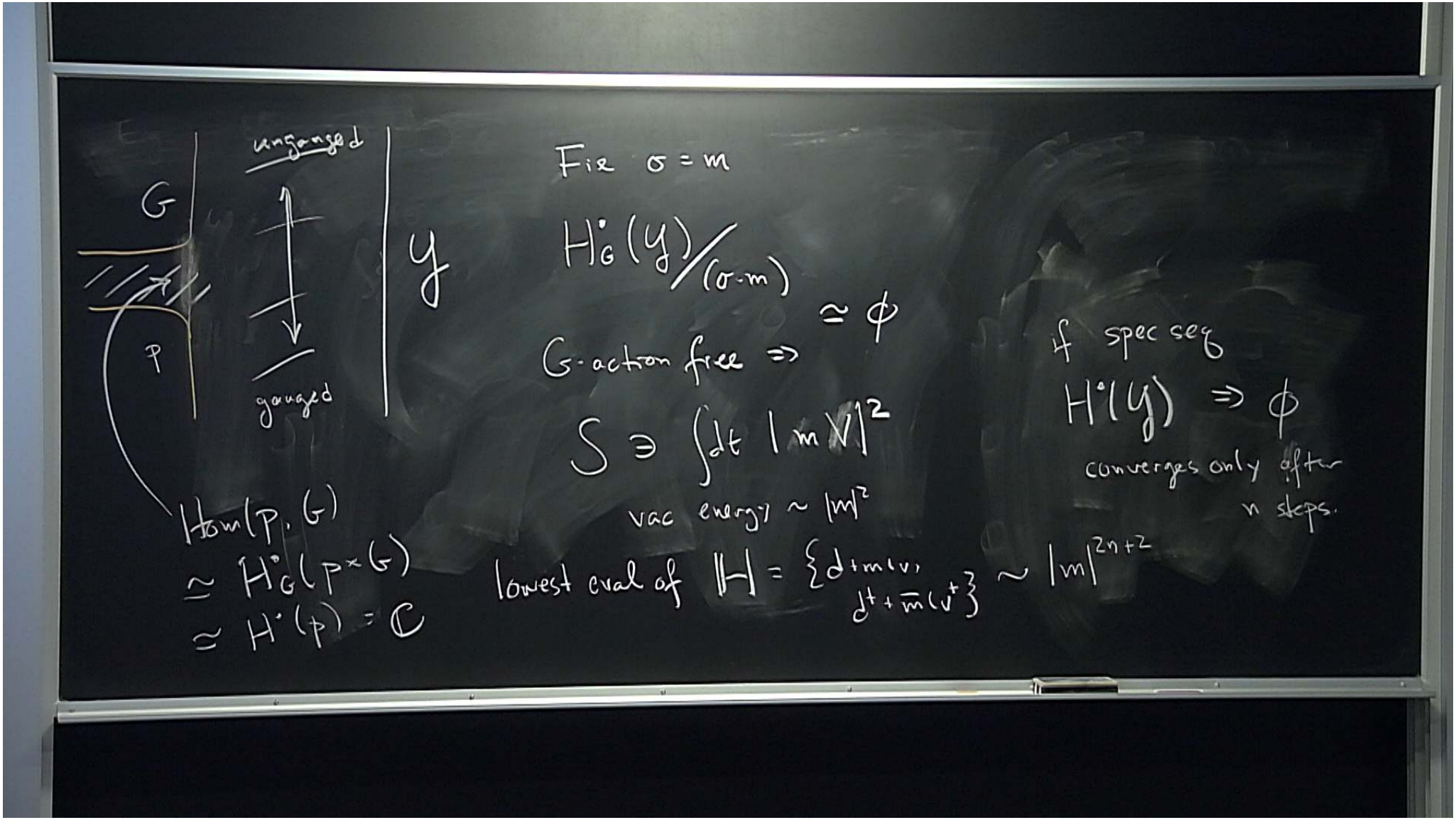
is

$H^\bullet_G(y)$


$H_\cdot(G)$-dg-mod $\simeq$ $H^\cdot_G(p)$-dg-mod
$\mathbb{C}[\mathfrak{h}]^G$
$\mathbb{C}[\sigma]^G$
$\wedge^\cdot \mathfrak{g}_\mathbb{C}[1]$
SQM: phase space $= T^*(\Pi TY)$
$\mathcal{H} = \Omega^\cdot(Y)$
$\mathbb{H} = \Delta$ form Lap. $= \{d, d^\dagger\}$
also: $\{Q, \bar{Q}\} = \mathbb{H}$
d $d^\dagger$
SUSY g.s.
$Q|\psi\rangle = \bar{Q}|\psi\rangle = 0$
$\mathcal{H}_{SUSY} = \Omega(Y)^{horiz}$
$\simeq H^\cdot(\mathcal{H}, Q)$


2d $\mathcal{N}=(2,2)$ pure G gauge thy
A-twisted
G
y
Bdy
y
category of b.c.
x
y
Hom(x, y)
$\simeq H_G(X \times Y)$


2 canonical b.c.
Neumann
P
$y \rightsquigarrow H^*_G(P \times y) \simeq H^*_G(y)$
$End(P) \simeq H^*_G(P \times P) \simeq H^*_G(P) \simeq \mathbb{C}[\sigma]^G$
Dirichlet
G
$y \rightsquigarrow H^*_G(G \times y) \simeq H^*(y)$
$End(G) \simeq H^*(G) \simeq H_*(G) \simeq \mathbb{C}[G]^G$


ungauged
G
P
gauged
y
Fix σ = m
$H^\bullet_G(y)/_{(\sigma-m)} \simeq \phi$
G-action free ⇒
$S \ni \int dt\, |mV|^2$
vac energy ~ $|m|^2$
lowest eval of $\|H\| = \{d+m|v, d^\dagger+\bar{m}|v^\dagger\} \sim |m|^{2n+2}$
if spec seq
$H^\bullet(y) \Rightarrow \phi$
converges only after
n steps.
$Hom(P, G)$
$\simeq H^\bullet_G(P \times G)$
$\simeq H^\bullet(P) = \mathbb{C}$