Title: String Theory for Mathematicians - Lecture 1

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Abstract:

Topological Strings

A and B models

B-model. Defined on a CY manifold X, $\dim_{\mathbb{C}} X$ is odd

Branes $Y \subseteq X$

complex submanifold of $\dim_{\mathbb{C}}^{n} k$

This has a B-brane.

$\mathbb{C}^k \subseteq \mathbb{C}^d$

Open-String States

Connecting $\mathbb{C}^k$ with itself $\quad \{\xi_i, d\bar{z}_j\} = 0$

are states

$$\Omega^{0,*}(\mathbb{C}^k)[\xi_1, \ldots, \xi_{d-k}]$$

$$\Omega^{0,*}(\mathbb{C}^k) = C^\infty(\mathbb{C}^k)[d\bar{z}_1 \ldots d\bar{z}_k]$$

$$\bar{\partial} = \sum d\bar{z}_i \frac{\partial}{\partial \bar{z}_i}$$

If we have N branes wrapping $\mathbb{C}^k$,

open string states are

$$\Omega^{0,\nu}(\mathbb{C}^k)[\varepsilon_1 \ldots \varepsilon_{d-k}] \otimes gl_N$$

This is a differential graded algebra

Product is obvious

Differential is $\bar{\partial}$.

$$\mathrm{Tr}: \Omega^{0,\nu}(\mathbb{C}^k)[\varepsilon_1] \otimes gl_N$$

$$A \longrightarrow \int_{\mathbb{C}^{k|d-k}} dz_1 \ldots dz_k \, d\varepsilon_1 \ldots d\varepsilon_{d-k} \, \mathrm{Tr} A$$

Open String Field Theory

Fields to be $T\mathcal{A}$

$$\mathcal{A} = \Omega^{0*}(\mathbb{T}^k)[\varepsilon_i] \otimes \mathfrak{gl}_N$$

If $\alpha \in T_\cdot \mathcal{A}$

$$S(\alpha) = \frac{1}{2}\mathrm{Tr}(\alpha \bar{\partial} \alpha) + \frac{1}{3}\mathrm{Tr}\,\alpha^3$$

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Example $\xi_{1 \cdots} \xi_{d-k}$ are $d-k$ bosonic scalars in open string field theory

Relation to physical String

Consider type IIB string theory on $\mathbb{R}^{10}$

If $S_+ = 16d$ chiral spin rep of $Spin(10)$

SUSY of type IIB are

$$S_+ \oplus S_+$$

Consider $SU(5) \subseteq Spin(10)$

There are 2 supercharges Q_1, Q_2

invariant under $SU(5)$

Pick $Q = Q_1$

Then we will "twist" using this.

- Brane is preserved by Q if it lives on a complex subspace $\mathbb{C}^* \subseteq \mathbb{C}^5 = \mathbb{R}^{10}$

- Twisting For branes which preserved by Q, add Q to BRST operator of open-string fields.

Closed string fields

Fancy approach

$Q \in$ local SUSYs which give ghosts in type IIB SUGRA

We're working in type IIB where super-ghost has expectation value given by Q.

$Q \in$ local SUSYs

which give ghosts in type IIB SUGRA

We're working in type IIB where super-ghost has expectation value given by Q.

Concretely

Add Q to BRST operator closed string states.

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working in type IIB
super-ghost has
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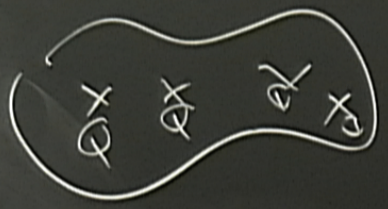
to BRST operator
string states.

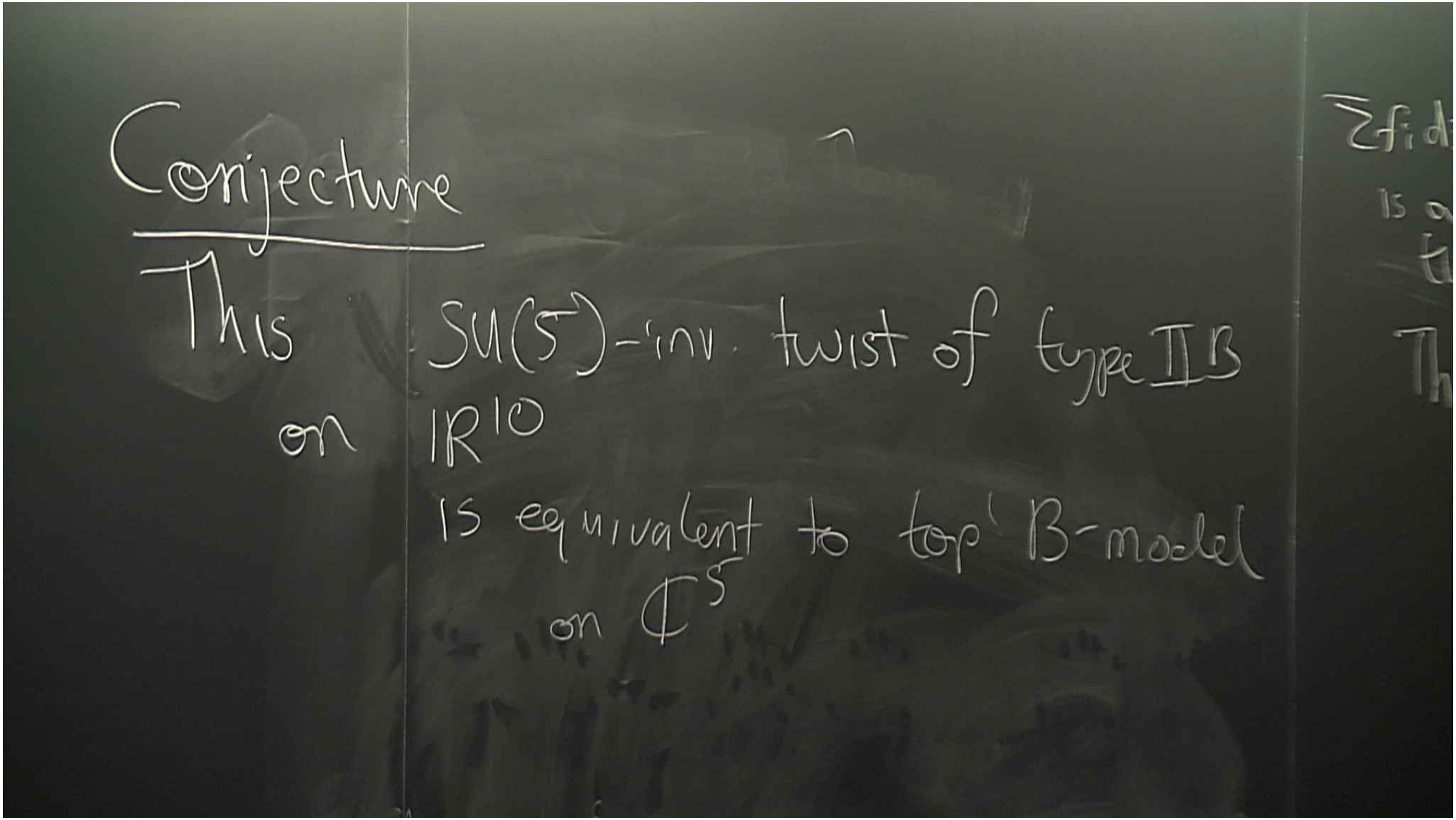


Fields to be

A

If α∈
S(α) =

Example ε1
scalar


Conjecture
This SU(5)-inv. twist of type IIB
on $\mathbb{R}^{10}$
is equivalent to top' B-model
on $\mathbb{C}^5$

Dictionary

Physical IIB	Topological
D_{2k-1} brane on $\mathbb{C}^k \subseteq \mathbb{R}^{10} = \mathbb{C}^5$	$\Rightarrow$ B-brane on $\mathbb{C}^k \subseteq \mathbb{C}^5$
(Open-string states, $Q_{BRST} + Q$)	$\xRightarrow{\sim}$ (Open string states, $\bar{\partial}$)
H_0(twist of low-energy theory on the brane	$\Rightarrow$ Open-string field theory we discussed, $\prod \Omega^{0,*}(\mathbb{C}^k)[\varepsilon_i]\otimes \mathfrak{gl}_N$

Notation

Denote the theory whose fields are

$$\prod \Omega^{r}(\mathbb{C}^k)[\varepsilon_1 \cdots \varepsilon_{5-k}] \otimes \mathfrak{g}l_N$$

as holomorphic CS on $\mathbb{C}^{k|5-k}$

$$S(\alpha) = \int_{\mathbb{C}^{k|5-k}} dVol \, \frac{1}{2} Tr(\alpha \bar{\partial} \alpha) + \frac{1}{3} Tr(\alpha^3)$$

Theorem (

Consider the

Theorem (L Baulieu)

Consider the SU(5) inv. twist of max. SUSY gauge theory on $\mathbb{R}^{10}$.

This is equivalent to hol. CS on $\mathbb{C}^5$.

D_q brane (physical) $\longleftrightarrow$ D_q brane (top^{l})

Dimensional Reduction

If we reduce 10d susy ym
to 2k dimensions ($k \leq 5$)
we find theory living on a D_{2k-1}-brane.

Reduce hCS on $\mathbb{C}^5$ to $\mathbb{C}^k$:

$\prod C^\infty(\mathbb{C}^5)[d\bar{z}_1 \ldots d\bar{z}_{5-k}] \otimes gl_N$. Make fields

constant in $5-k$ dim^{n}s.

$d\bar{z}_{k+1} \ldots d\bar{z}_5$ identify w. $\varepsilon_1 \ldots \varepsilon_{5-k}$

$\sum f_i d\bar{z}_i \epsilon$
is a bo
the s

This is

Open-string states are

$\Omega^*(\mathbb{R}^n)$

Open-string fields (on N branes)

$T\Omega^*(\mathbb{R}^n) \otimes g_N$

Action is

$$\int_{\mathbb{R}^n} \frac{1}{2} Tr(\alpha d\alpha) + \frac{1}{3} Tr\,\alpha^3$$

Notat

Denote

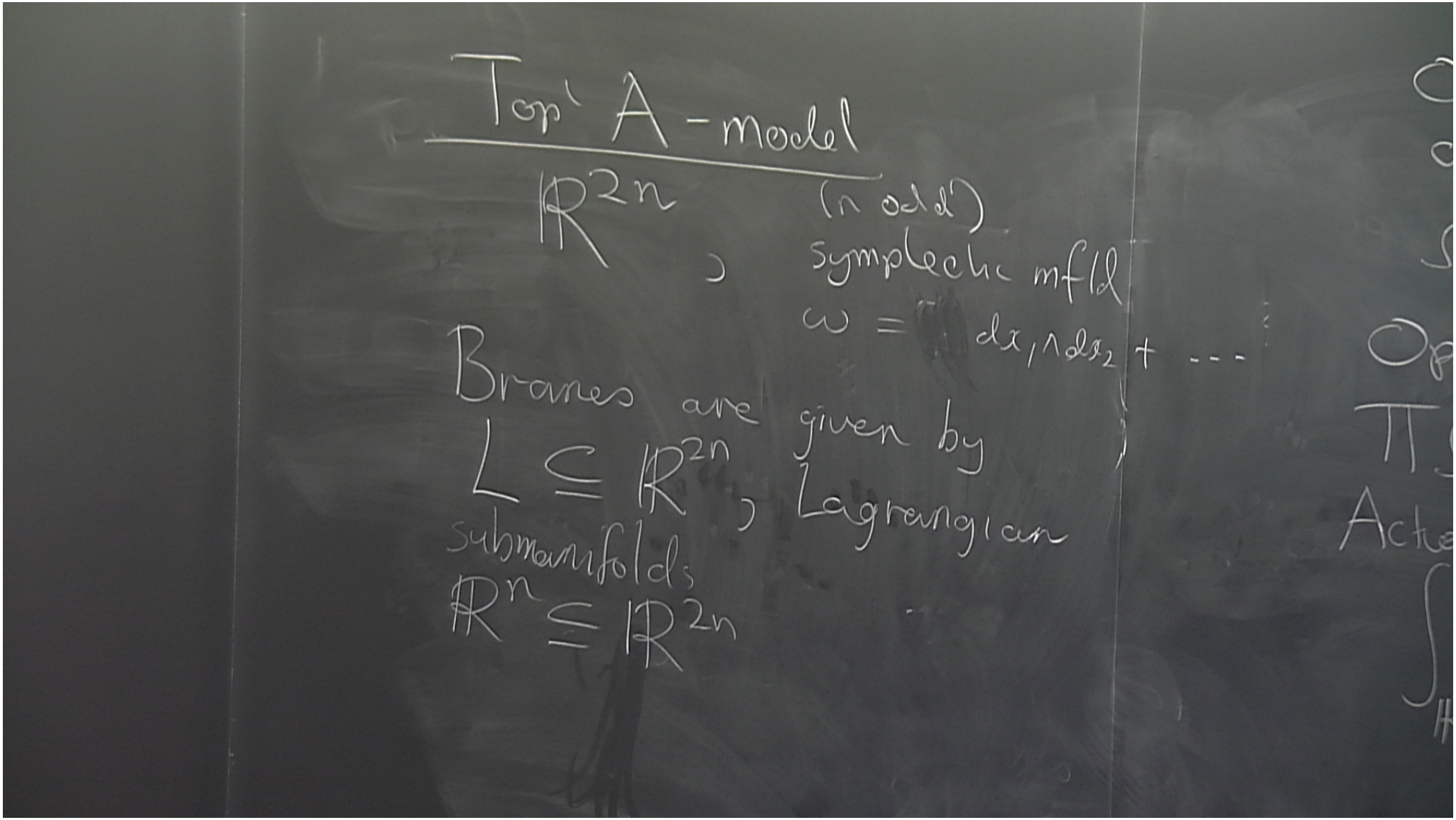
fields

$T\Omega^*$

as hol

$\mathbb{C}^{*/5}$

$S(\alpha) =$


Top' A-model
$\mathbb{R}^{2n}$ (n odd)
, symplectic mfld
$\omega = dx_1 \wedge dx_2 + \cdots$
Branes are given by
$L \subseteq \mathbb{R}^{2n}$, Lagrangian
submanifolds
$\mathbb{R}^n \subseteq \mathbb{R}^{2n}$

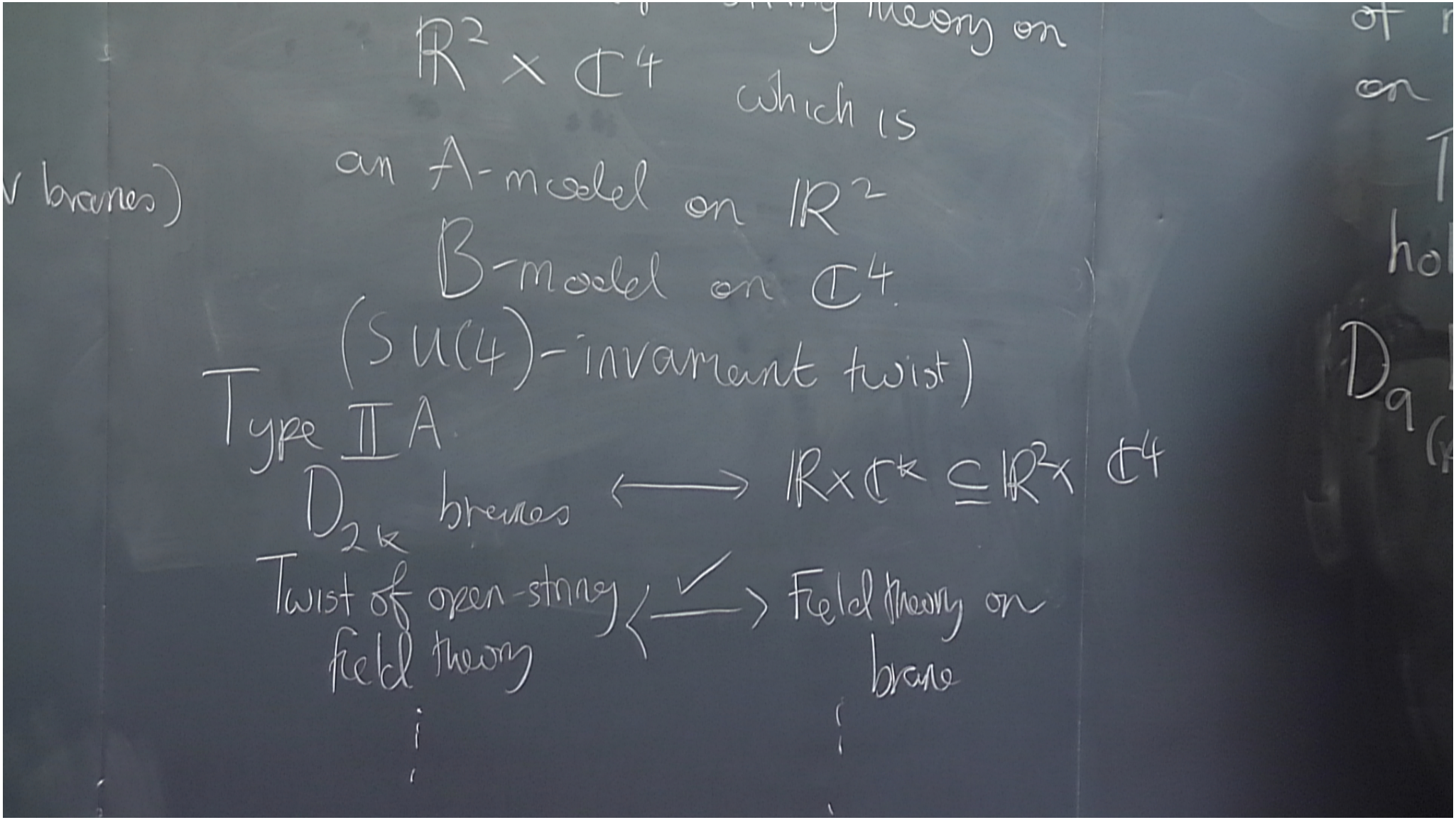
Claim

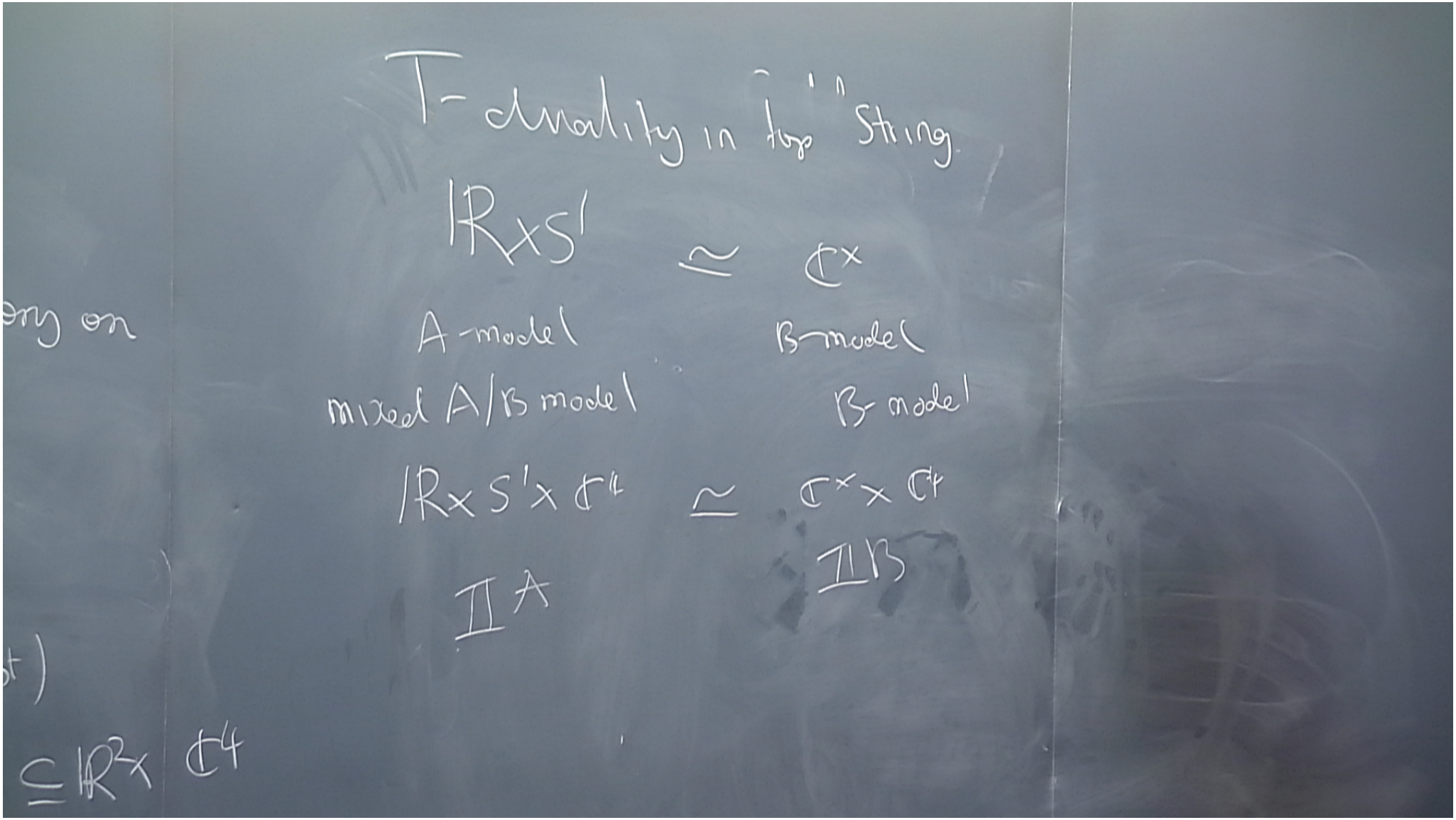
Twist of type IIA is the top' string theory on $\mathbb{R}^2 \times \mathbb{C}^4$ which is

an A-model on $\mathbb{R}^2$

B-model on $\mathbb{C}^4$.

(SU(4)-invariant twist)


$\mathbb{R}^2 \times \mathbb{C}^4$
which is
an A-model on $\mathbb{R}^2$
B-model on $\mathbb{C}^4$
(SU(4)-invariant twist)
Type II A
D_{2k} branes $\longleftrightarrow$ $\mathbb{R} \times \mathbb{C}^* \subseteq \mathbb{R}^2 \times \mathbb{C}^4$
Twist of open-string field theory
Field theory on brane


T-duality in the "" String
$\mathbb{R}\times S^1 \simeq \mathbb{C}^\times$
A-model
B-model
mixed A/B model
B-model
$\mathbb{R}\times S^1\times\mathbb{C}^4 \simeq \mathbb{C}^\times\times\mathbb{C}^4$
II A
II B
ory on
st)
$\subseteq \mathbb{R}^3\times\mathbb{C}^4$