Title: String Theory for Mathematicians - Lecture 2

Date: Apr 25, 2017 02:00 PM

URL: http://pirsa.org/17040080

Abstract:

A-model

X sympl manifold

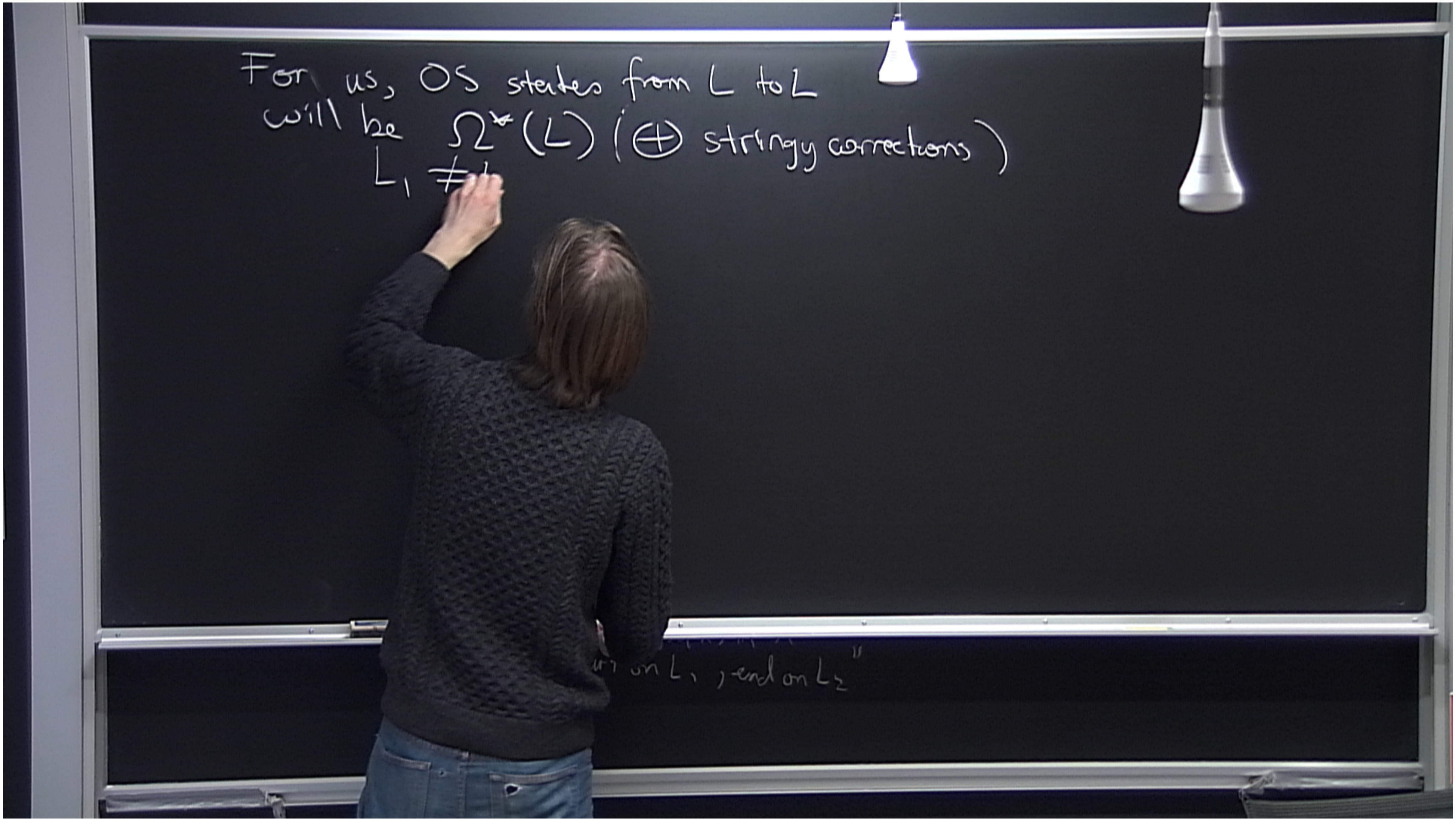
Branes $L \subseteq X$ Lagrangian

Open-String States

$=$ Hom's in Fukaya cat.

$HF^*(L_1, L_2)$ Floer cohomology

$HF^*(L_1, L_2) =$ "semi-infinite coho. of space of paths in X start on L_1, end on L_2"

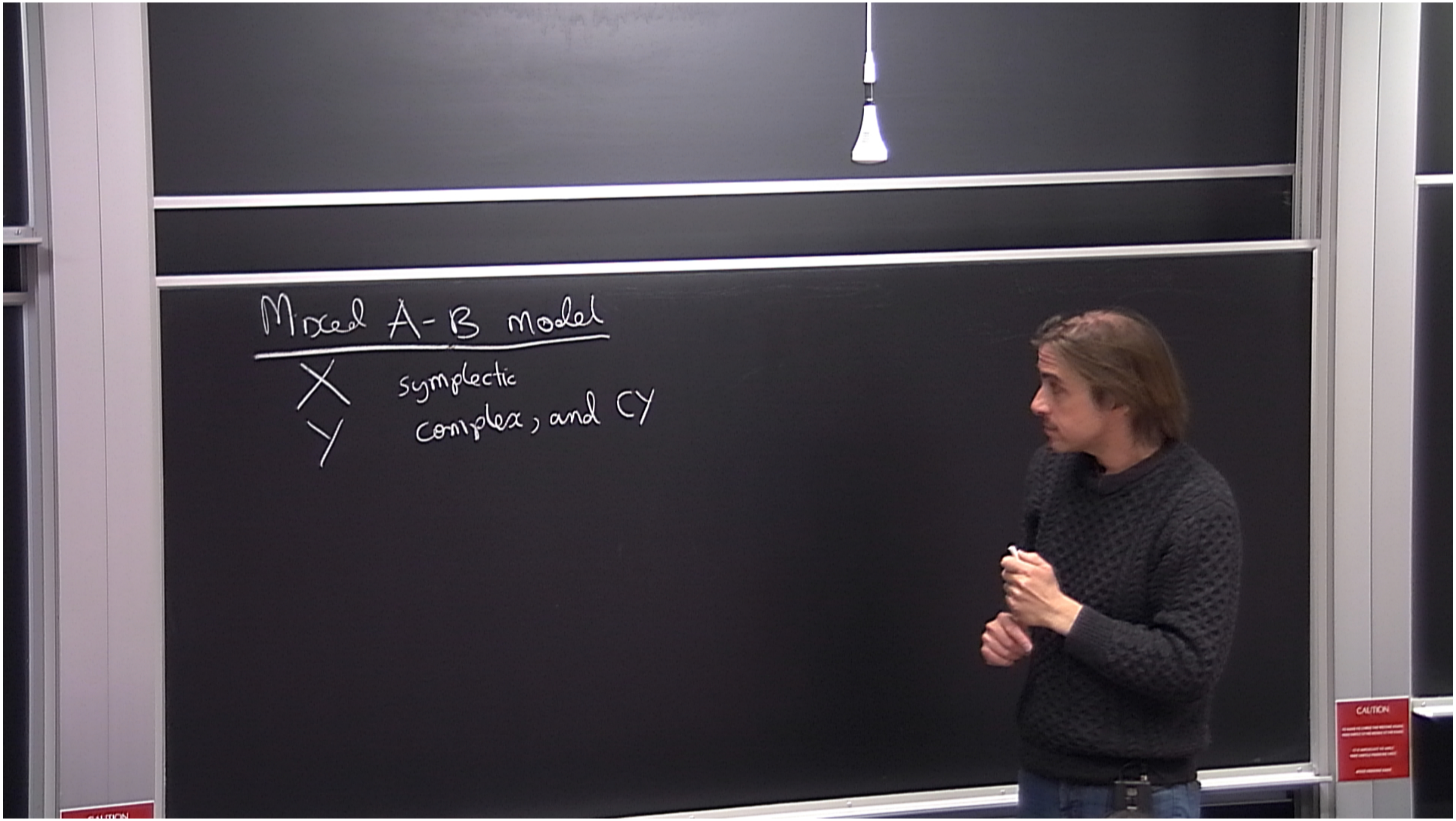

For us, OS states from L to L
will be Ω*(L) (⊕ stringy corrections)
on L₁, end on L₂"

For us, OS states from L to L
will be $\Omega^*(L)$ ($\oplus$ stringy corrections)

$L_1 \neq L_2$, it's

$\Omega^*(L_1 \cap L_2)$($\oplus$ . . . . .)

space of paths in X
start on L_1, end on L_2"


Mixed A-B model
X symplectic
Y complex, and CY

Mixed A-B model

X symplectic

Y complex, and CY

Mixed string theory on $X \times Y$

A-model on X

B-model on Y

Branes are a product

& $L \subseteq X$ Lagrangian

$Z \subseteq Y$ holomorphic submanifold

Mixed A-B model

X symplectic

Y complex, and CY

Mixed string theory on $X \times Y$

A-model on X

B-model on Y

Branes are a product

& $L \subseteq X$ Lagrangian

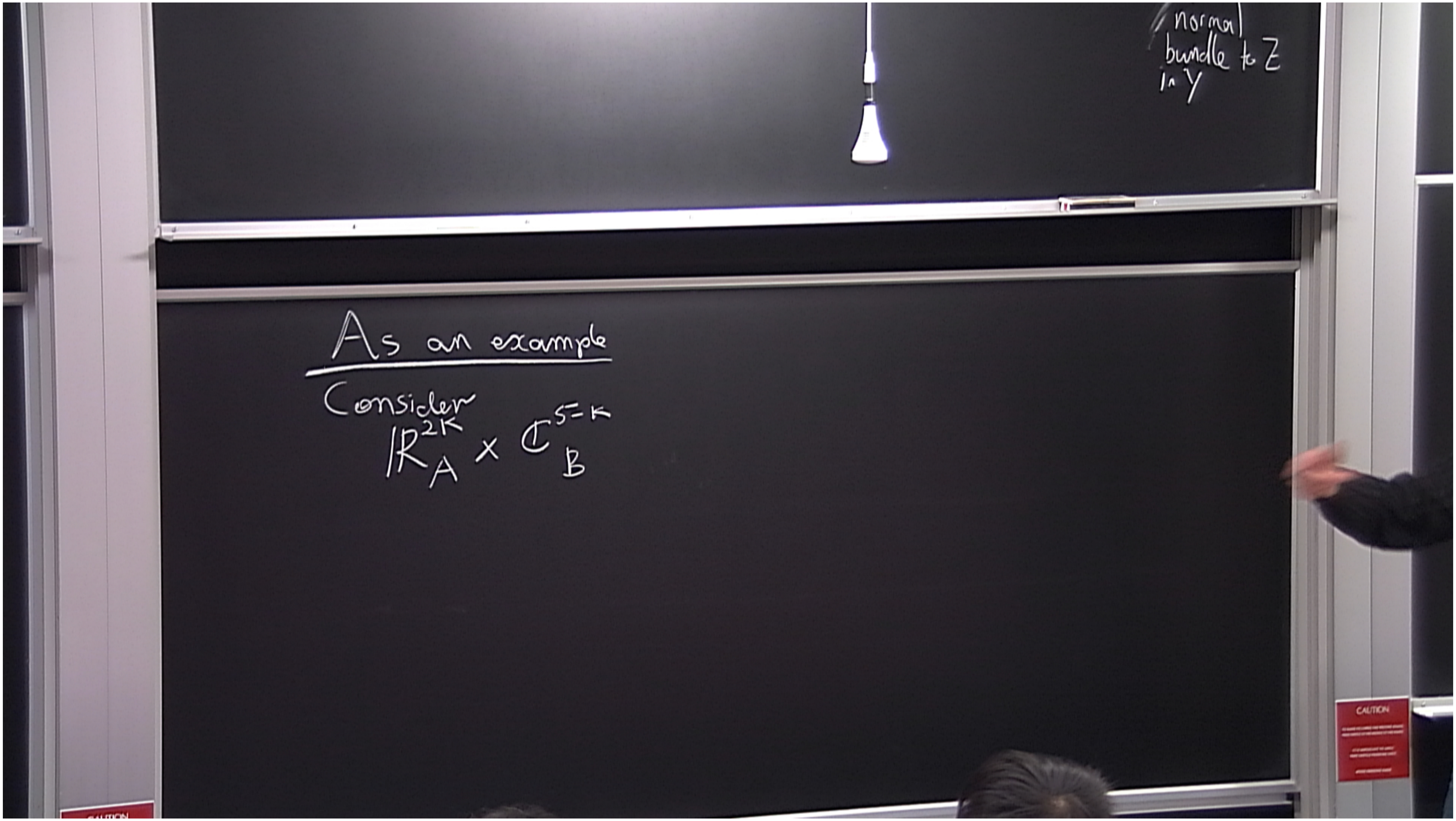
$Z \subseteq Y$ holomorphic submanifold

Open-string states (w/o α' corrections)

are

$$\Omega^*(L) \otimes \Omega^{0,*}(Z, \Lambda^* N_{Y/Z})$$

normal bundle to Z in Y


normal bundle to Z in Y
As an example
Consider
$\mathbb{R}^{2K}_{A} \times \mathbb{C}^{5-K}_{B}$

normal bundle to Z in Y

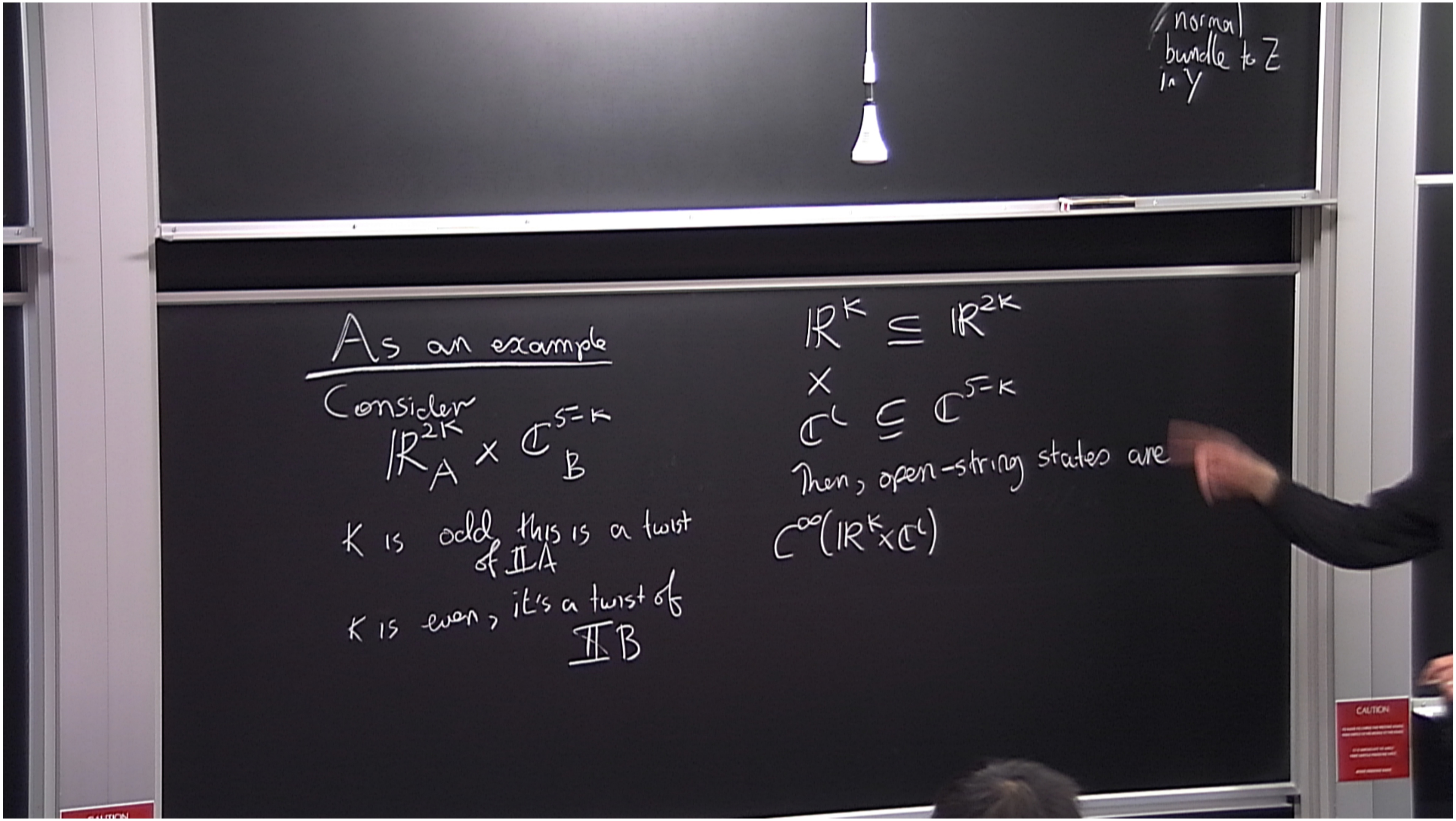
As an example

Consider

$$\mathbb{R}^{2K}_A \times \mathbb{C}^{5-K}_B$$

K is odd, this is a twist of IIA

K is even, it's a twist of IIB


normal bundle to Z in Y
As an example
Consider
$\mathbb{R}^{2K}_A \times \mathbb{C}^{5-K}_B$
K is odd, this is a twist of IIA
K is even, it's a twist of IIB
$\mathbb{R}^K \subseteq \mathbb{R}^{2K}$
$\times$
$\mathbb{C}^L \subseteq \mathbb{C}^{5-K}$
Then, open-string states are
$C^\infty(\mathbb{R}^K \times \mathbb{C}^L)$

normal bundle to Z in Y

As an example

Consider

$$\mathbb{R}^{2K}_{A} \times \mathbb{C}^{5-K}_{B}$$

K is odd, this is a twist of IIA

K is even, it's a twist of IIB

$$\mathbb{R}^{K} \subseteq \mathbb{R}^{2K}$$

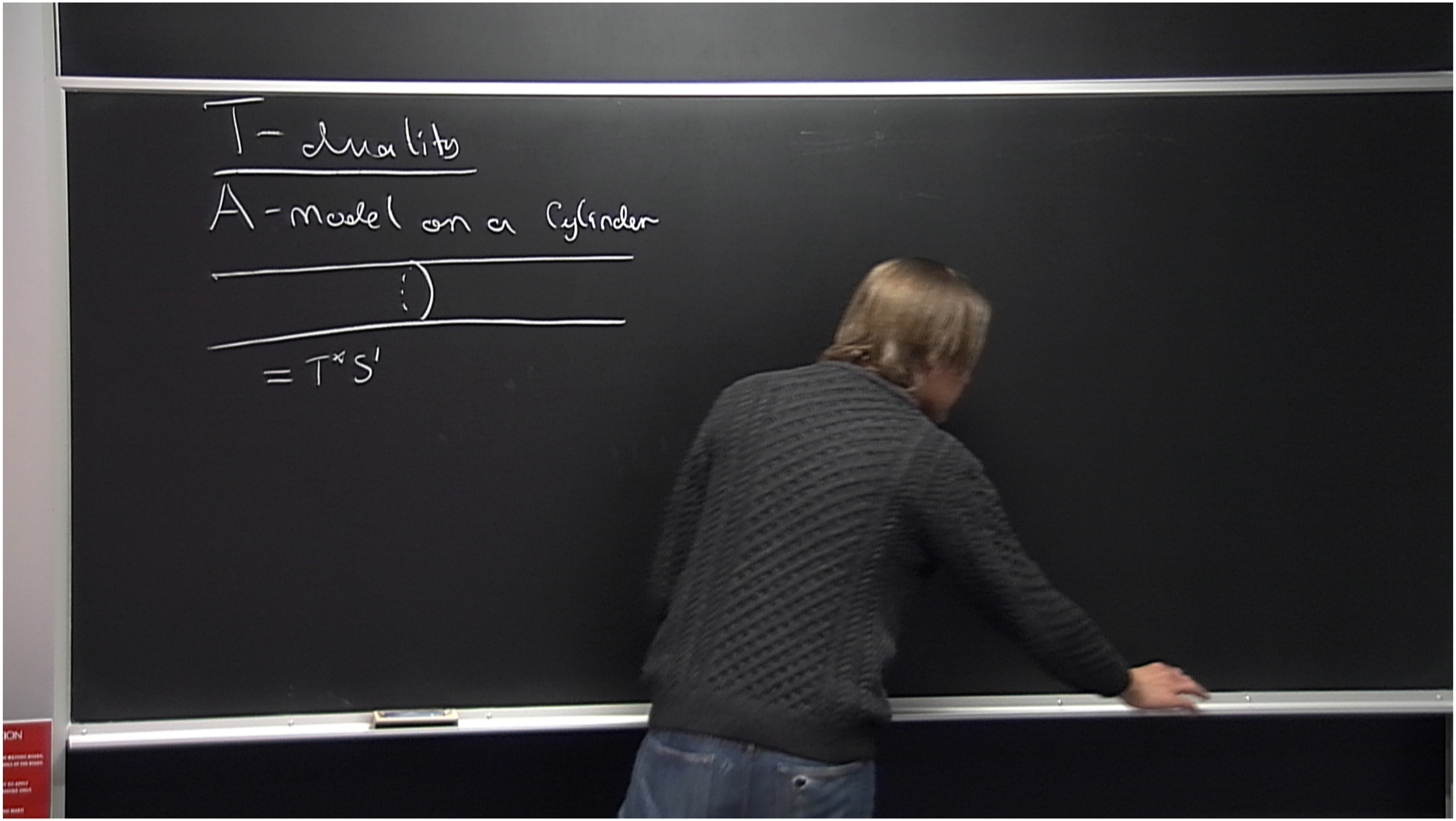
$$\times$$

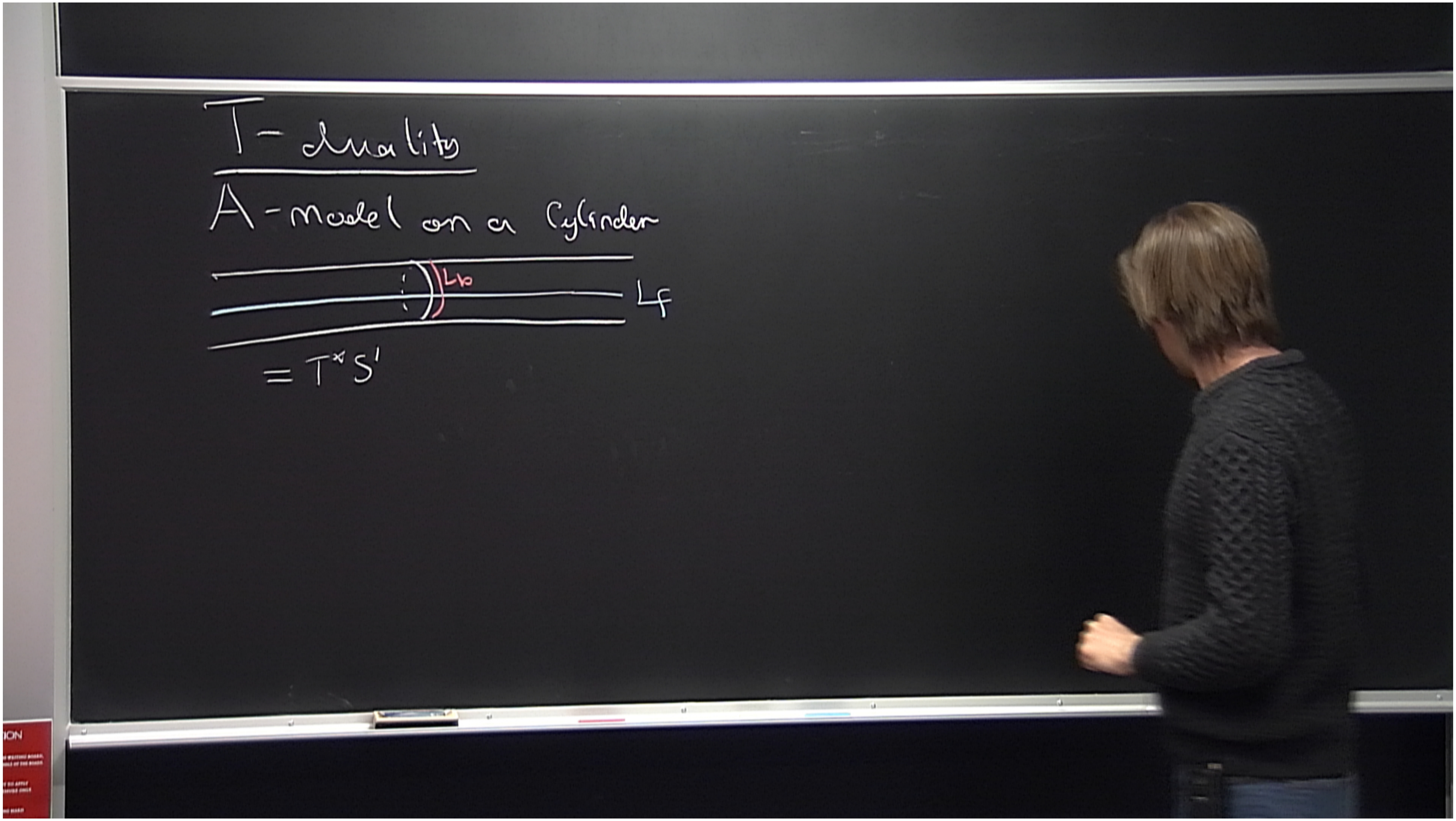
$$\mathbb{C}^{L} \subseteq \mathbb{C}^{5-K}$$

Then, open-string states are

$$C^{\infty}(\mathbb{R}^{K} \times \mathbb{C}^{L})[dx_1 \dots dx_K, d\bar{z}_1, \dots, d\bar{z}_L, \varepsilon_1, \dots, \varepsilon_{5-K-L}]$$

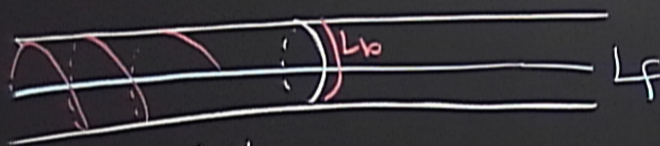
$$d = \sum dx_i \frac{\partial}{\partial x_i} + \sum d\bar{z}_i \frac{\partial}{\partial \bar{z}_i}$$


T-duality
A-model on a Cylinder
$= T^*S^1$


T-duality
A-model on a Cylinder
Lb
Lf
= T*S'

T-duality

A-model on a Cylinder

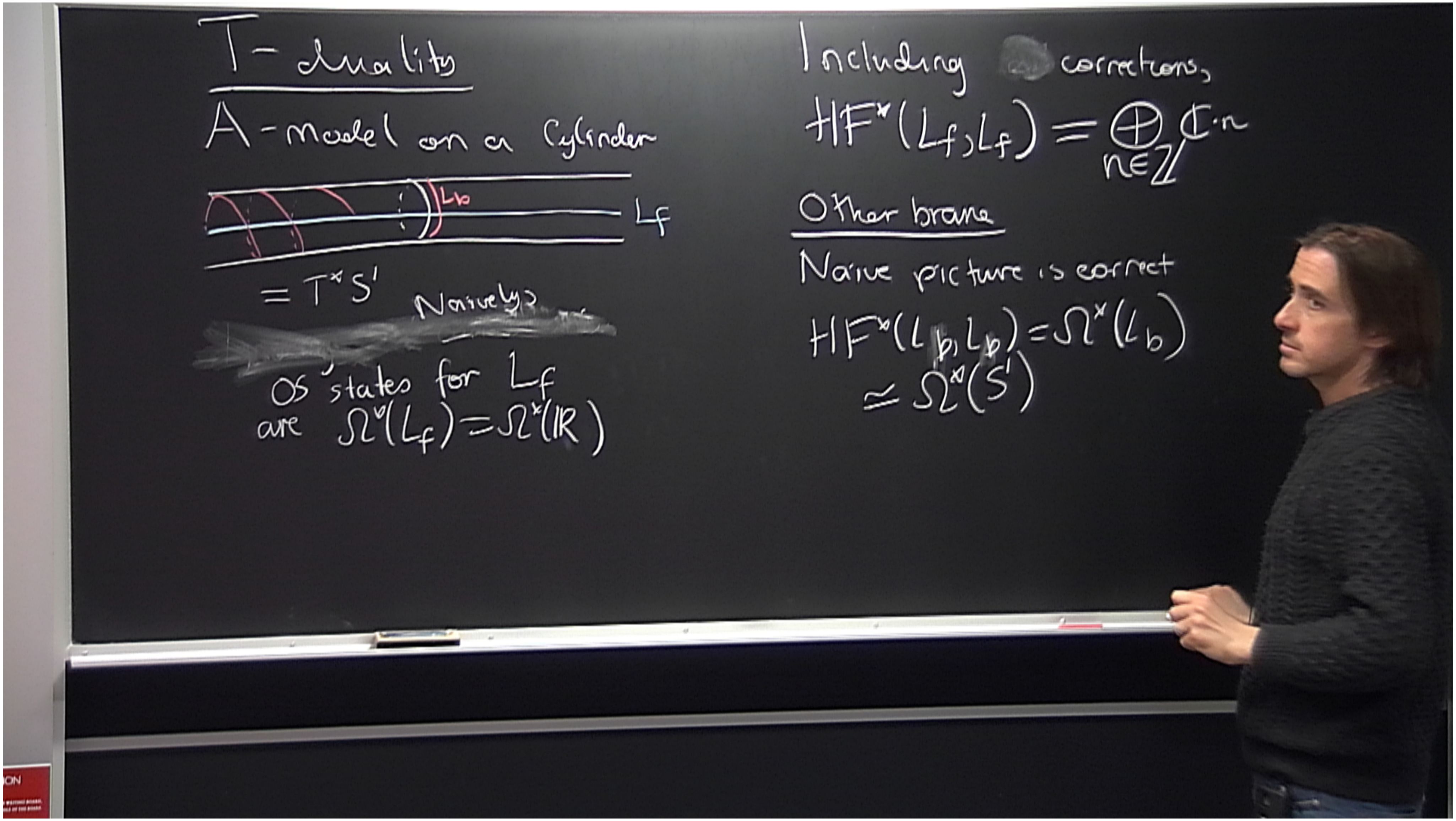


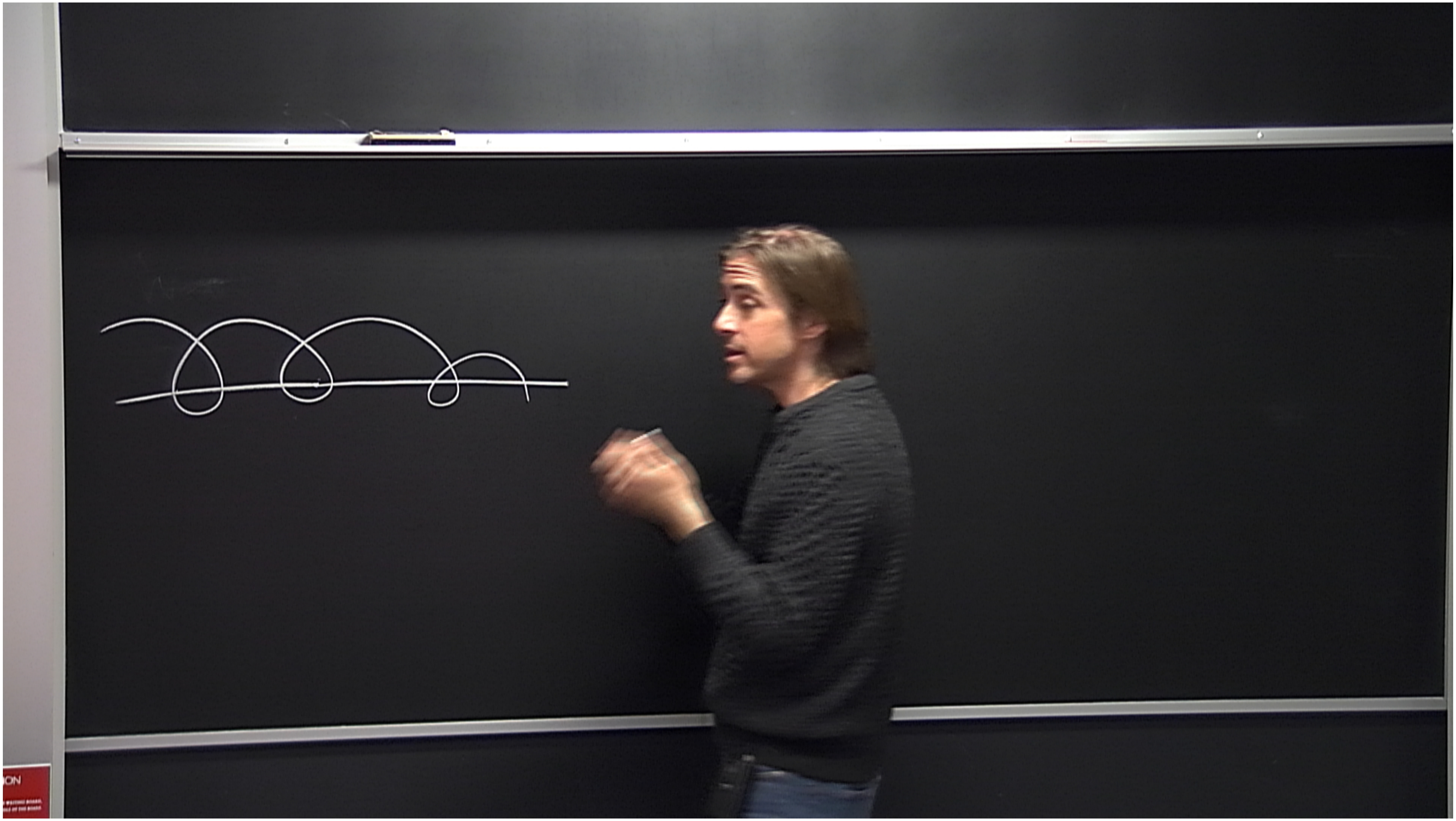
$= T^*S^1$

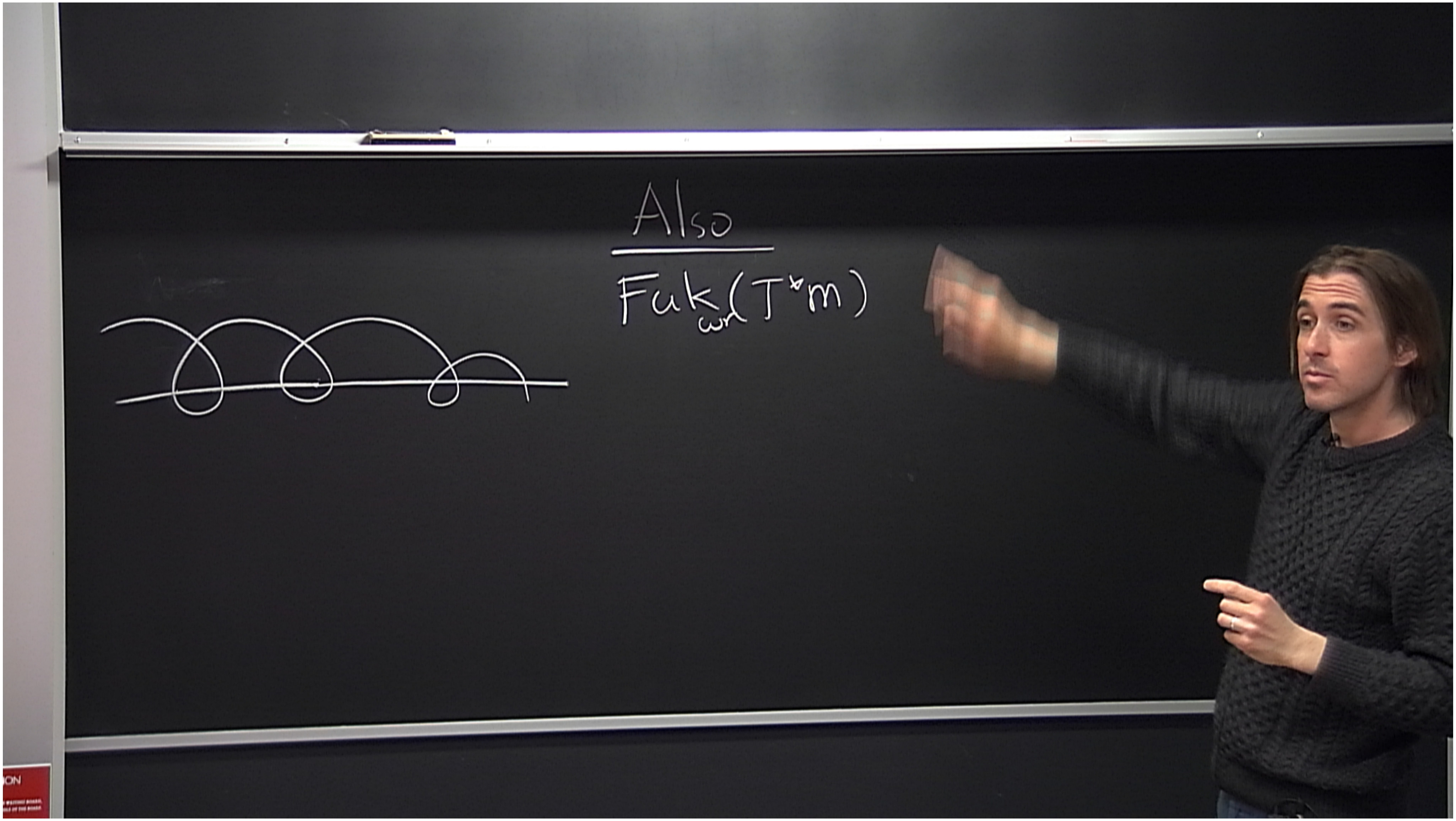
Ignoring α' corrections,
OS states for L_f
are $\Omega^\bullet(L_f) = \Omega^*(\mathbb{R})$

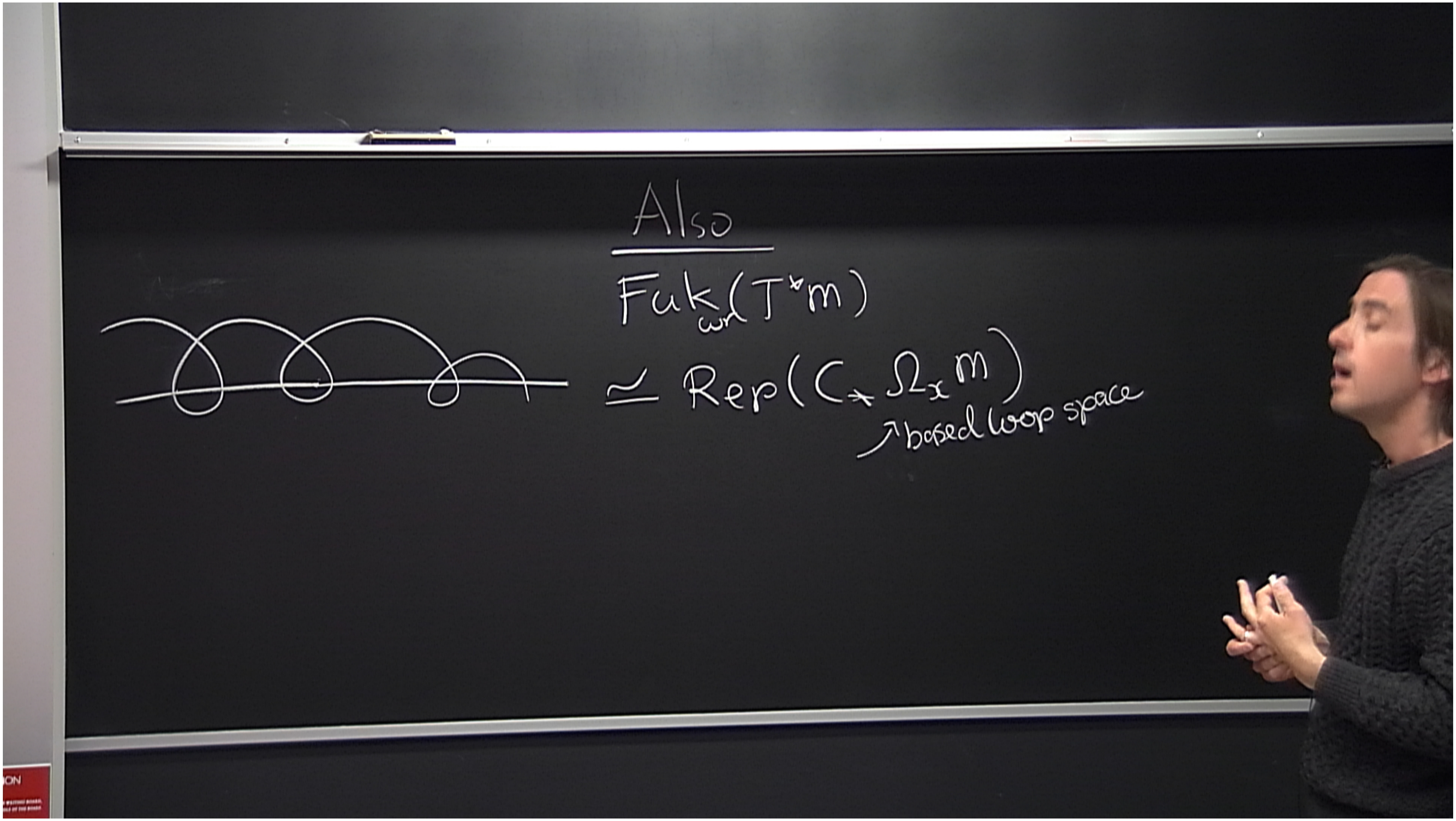
Including α' corrections,

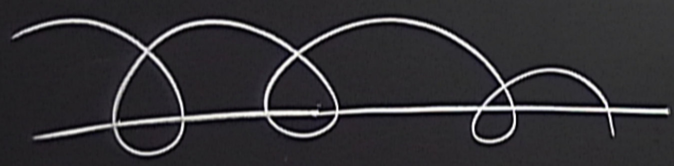
$$HF^*(L_f, L_f) = \bigoplus_{n\in\mathbb{Z}} \mathbb{C}\cdot n$$


T-duality
A-model on a Cylinder
L_b
L_f
$= T^*S^1$
Naively,
0s states for L_f
are $\Omega^*(L_f) = \Omega^*(\mathbb{R})$
Including corrections,
$HF^*(L_f, L_f) = \bigoplus_{n \in \mathbb{Z}} \mathbb{C} \cdot n$
Other brane
Naive picture is correct
$HF^*(L_b, L_b) = \Omega^*(L_b)$
$\simeq \Omega^*(S^1)$




Also
$\mathrm{Fuk}_{wr}(T^*m)$


Also
Fuk_wr(T*M)
≃ Rep(C_* Ω_x M)
based loop space



Also

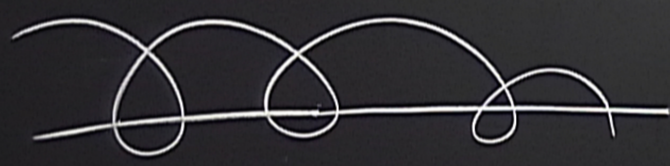
$\mathrm{Fuk}_{wr}(T^*M)$

$\simeq \mathrm{Rep}(C_* \Omega_x M)$

based loop space

Fibre at x
$\implies C_* \Omega_x M$, regular rep

Base $\implies \mathbb{C}$, augmentation rep



Also

$\mathrm{Fuk}_{wr}(T^*M)$ ⋯→ $\Omega_x S^1 \simeq \mathbb{Z}$

$C_*\Omega_x S^1 \simeq \mathbb{C}[z, z^{-1}]$

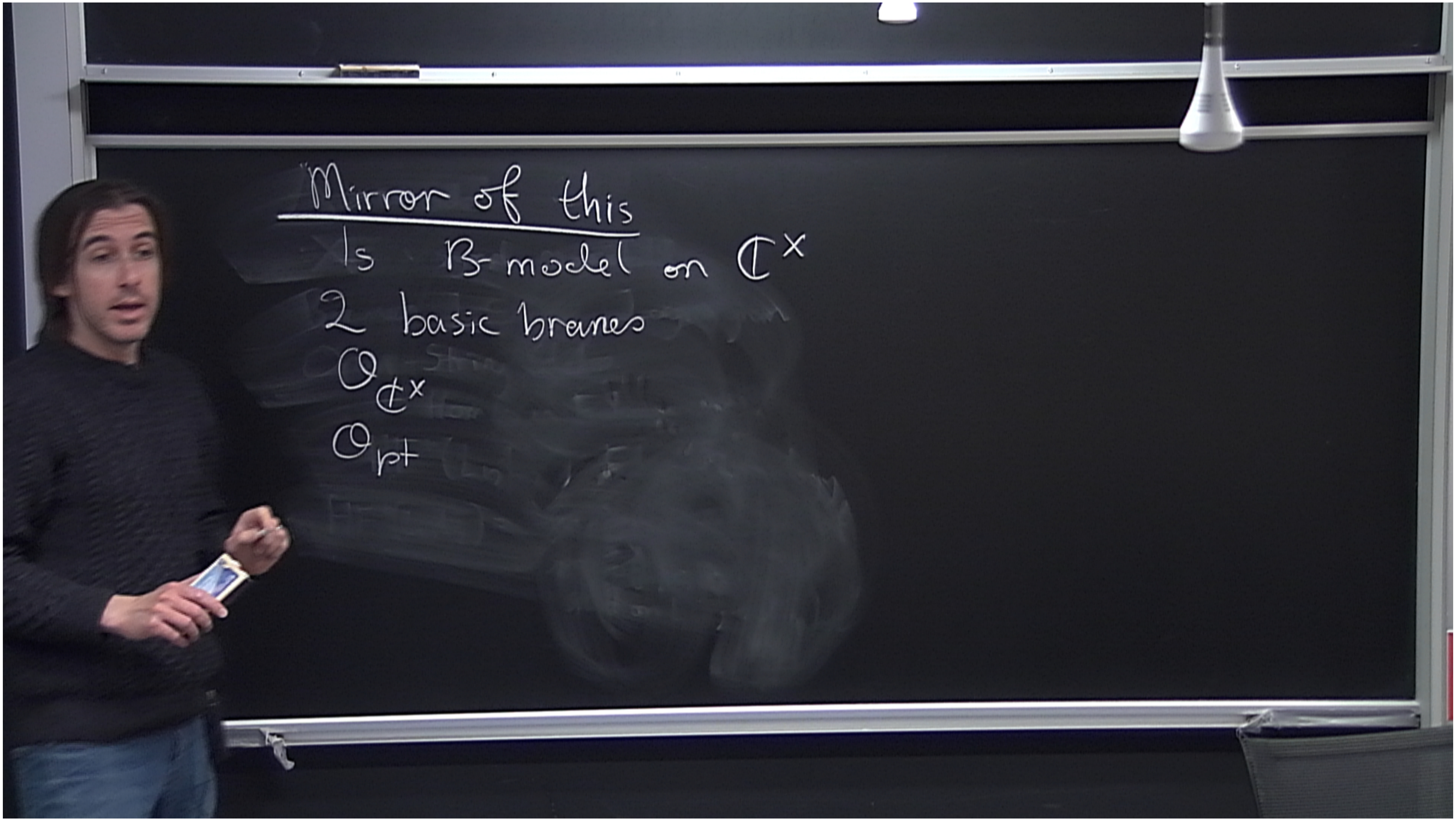
$\simeq \mathrm{Rep}(C_*\Omega_x M)$

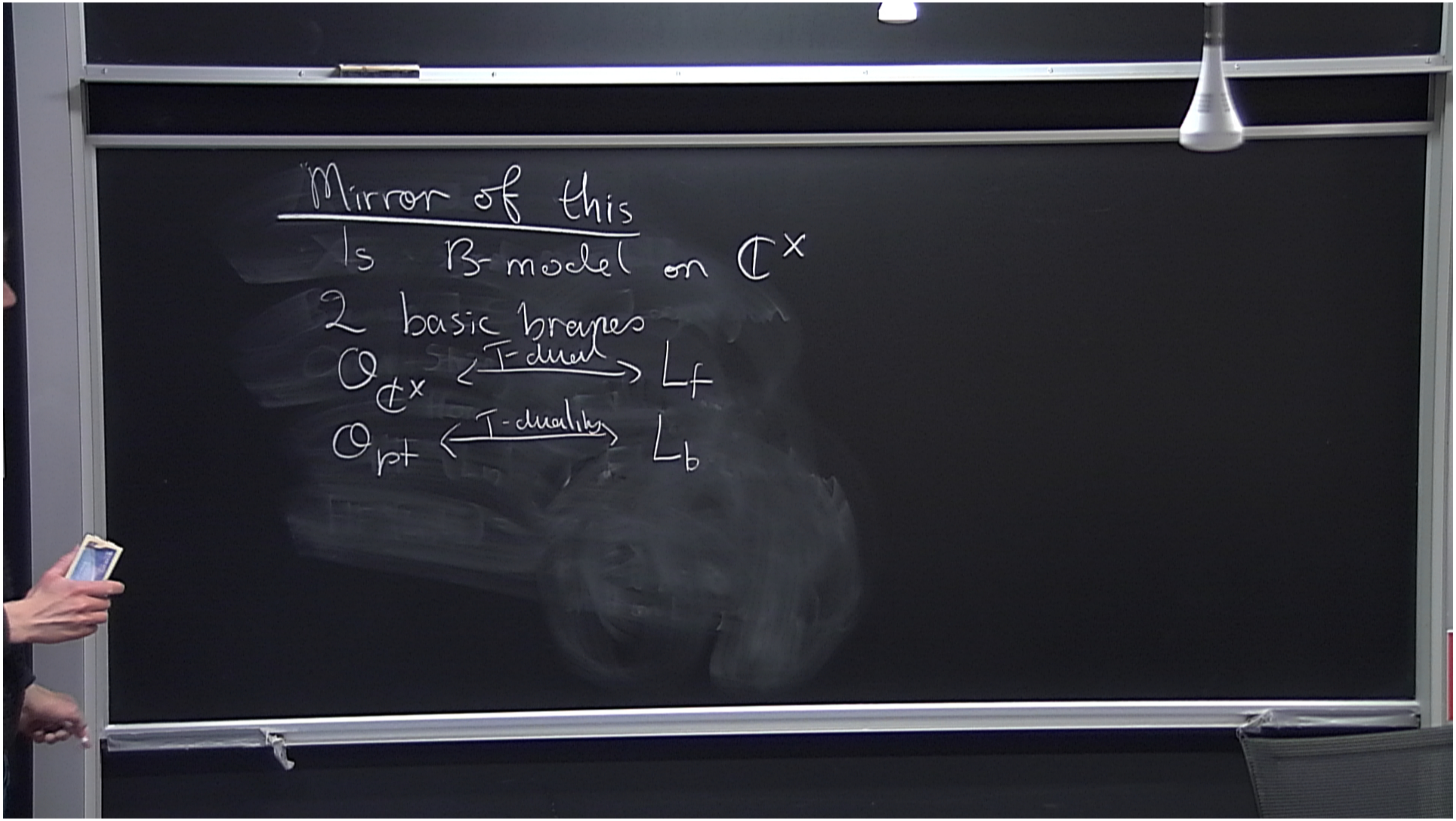
↗ based loop space

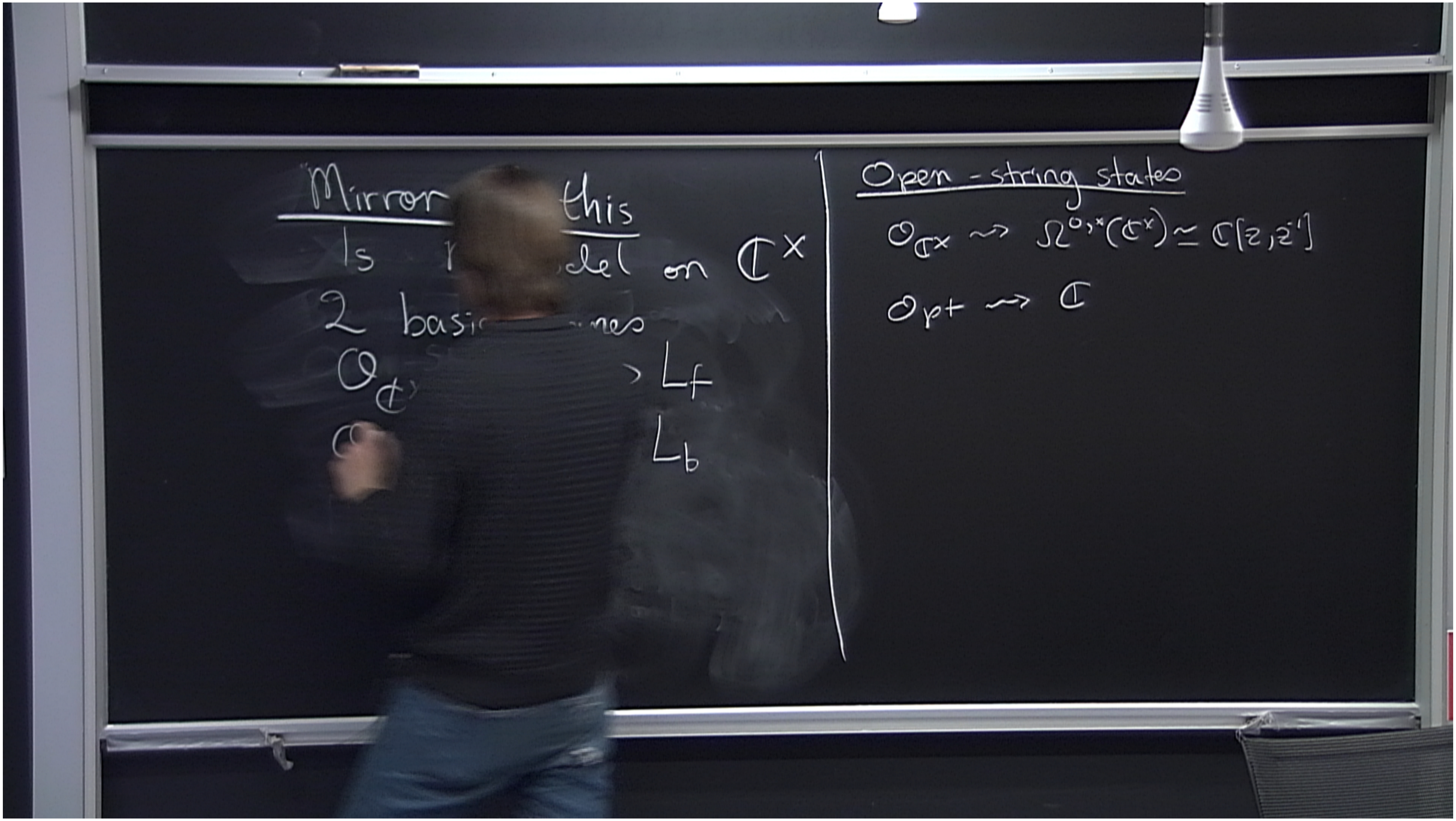
Fibre at x

$\Longrightarrow C_*\Omega_x M$, regular rep

Base $\Longrightarrow \mathbb{C}$, augmentation rep


Mirror of this
Is B-model on $\mathbb{C}^\times$
2 basic branes
$\mathcal{O}_{\mathbb{C}^\times}$
$\mathcal{O}_{pt}$


Mirror of this
is B-model on $\mathbb{C}^\times$
2 basic branes
$\mathcal{O}_{\mathbb{C}^\times} \xleftrightarrow{\text{T-dual}} L_f$
$\mathcal{O}_{pt} \xleftrightarrow{\text{T-duality}} L_b$


Mirror
this
on $\mathbb{C}^\times$
2 basi
L_f
L_b
Open - string states
$\mathcal{O}_{\mathbb{C}^\times} \rightsquigarrow \Omega^{0,*}(\mathbb{C}^\times) \simeq \mathbb{C}[z, z^{-1}]$
$\mathcal{O}_{pt} \rightsquigarrow \mathbb{C}$

Mirror of this

is B-model on $\mathbb{C}^\times$

2 basic branes

$$\mathcal{O}_{\mathbb{C}^\times} \xleftrightarrow{\text{T-dual}} L_f$$

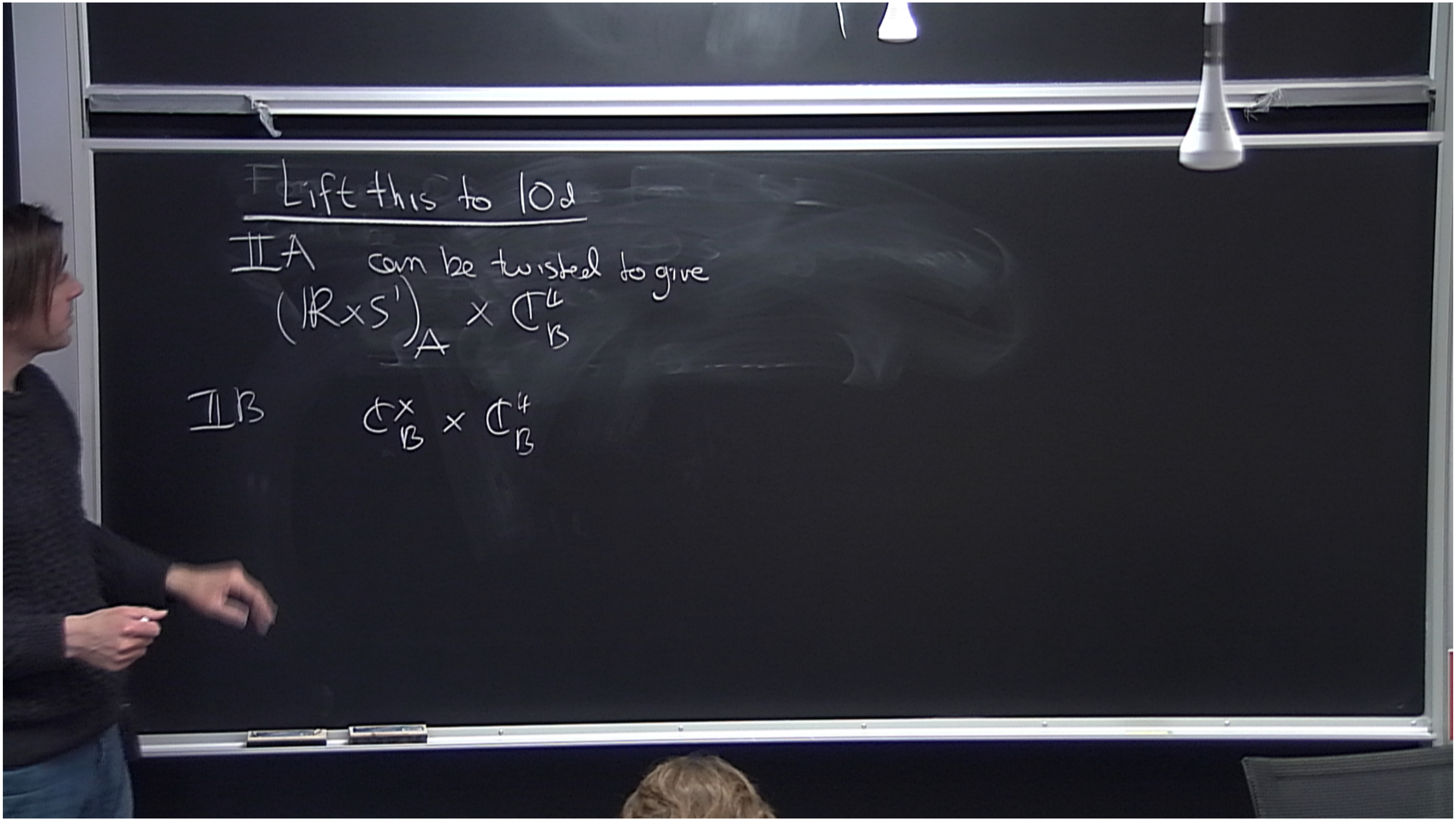
$$\mathcal{O}_{pt} \xleftrightarrow{\text{T-duality}} L_b$$

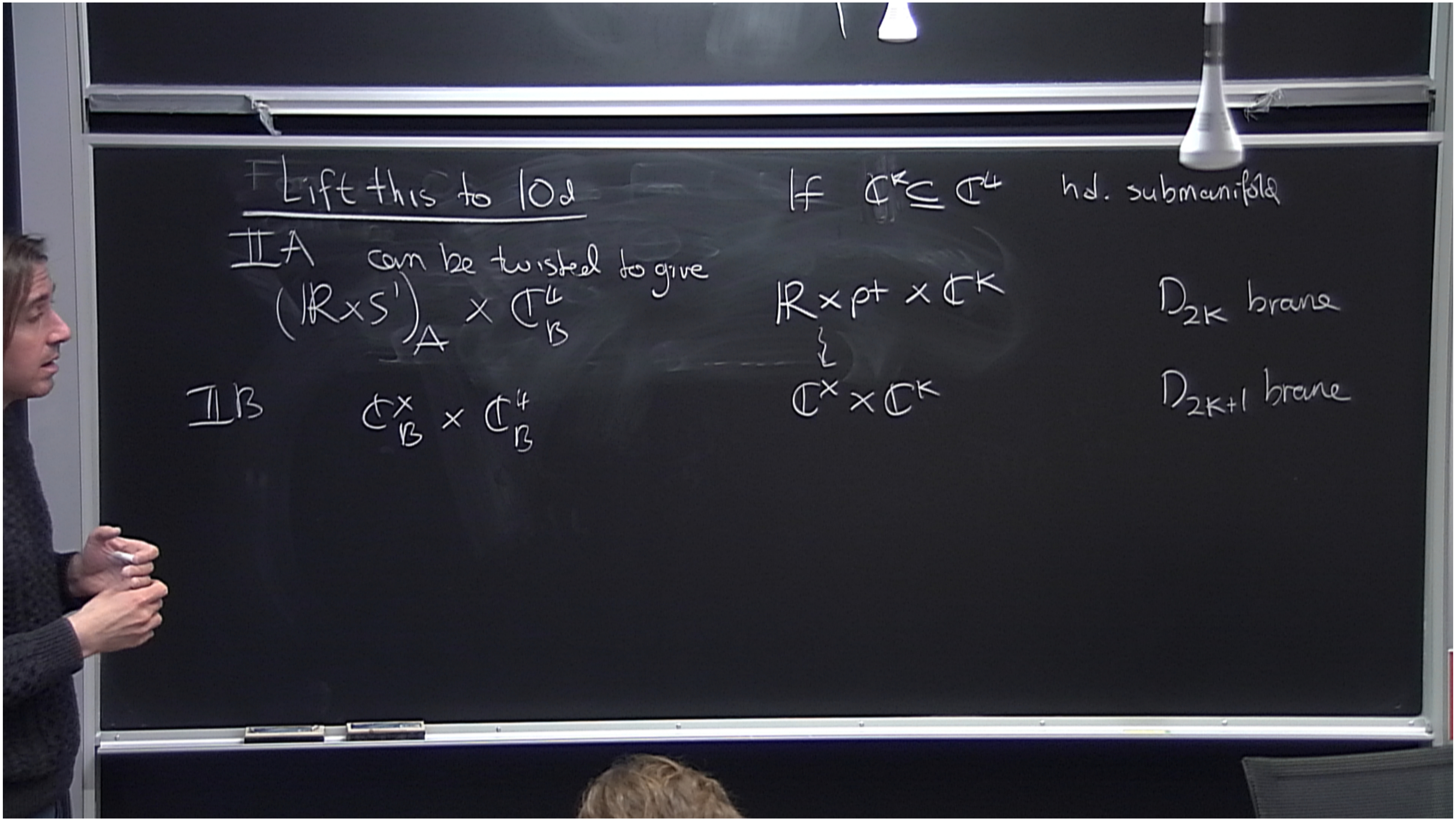
Open-string states

$$\mathcal{O}_{\mathbb{C}^\times} \rightsquigarrow \Omega^{0,*}(\mathbb{C}^\times) \simeq \mathbb{C}[z, z^{-1}]$$

$$\mathcal{O}_{pt} \rightsquigarrow \mathbb{C}[\varepsilon] = H^* S^1$$

these match.


Lift this to 10d
IIA can be twisted to give
$(\mathbb{R}\times S^1)_A \times \mathbb{C}^4_B$
IIB $\mathbb{C}^\times_B \times \mathbb{C}^4_B$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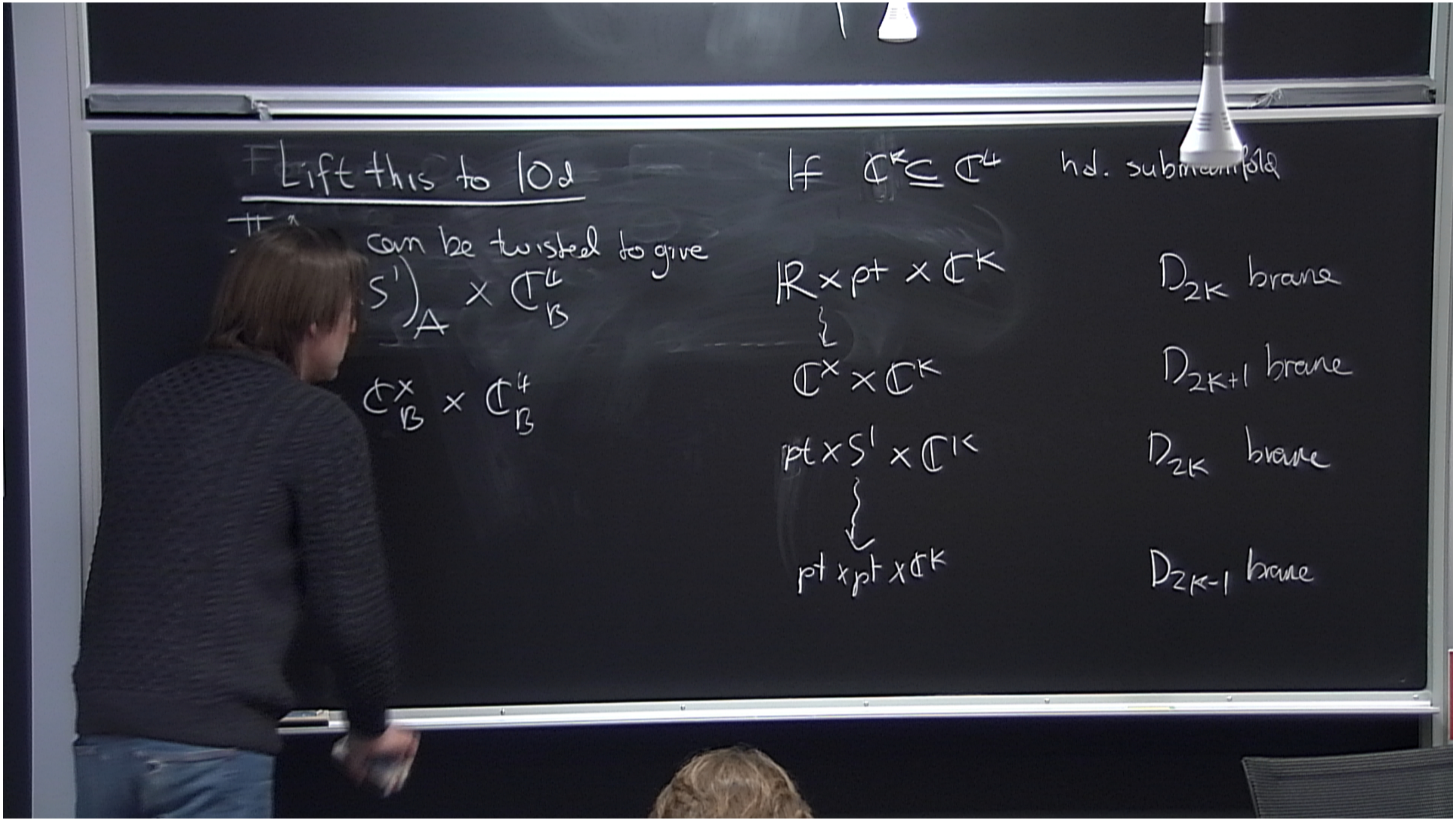
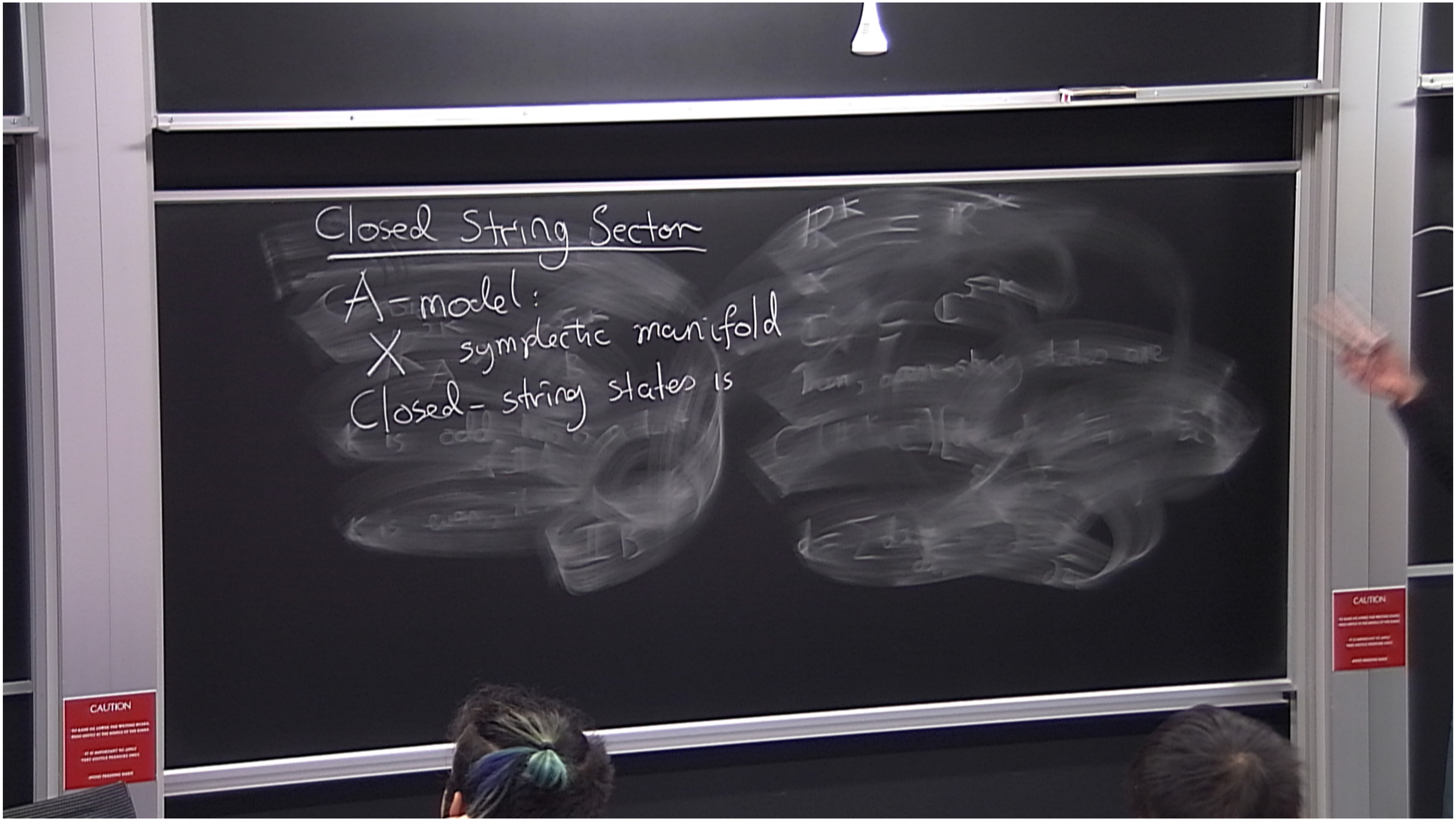
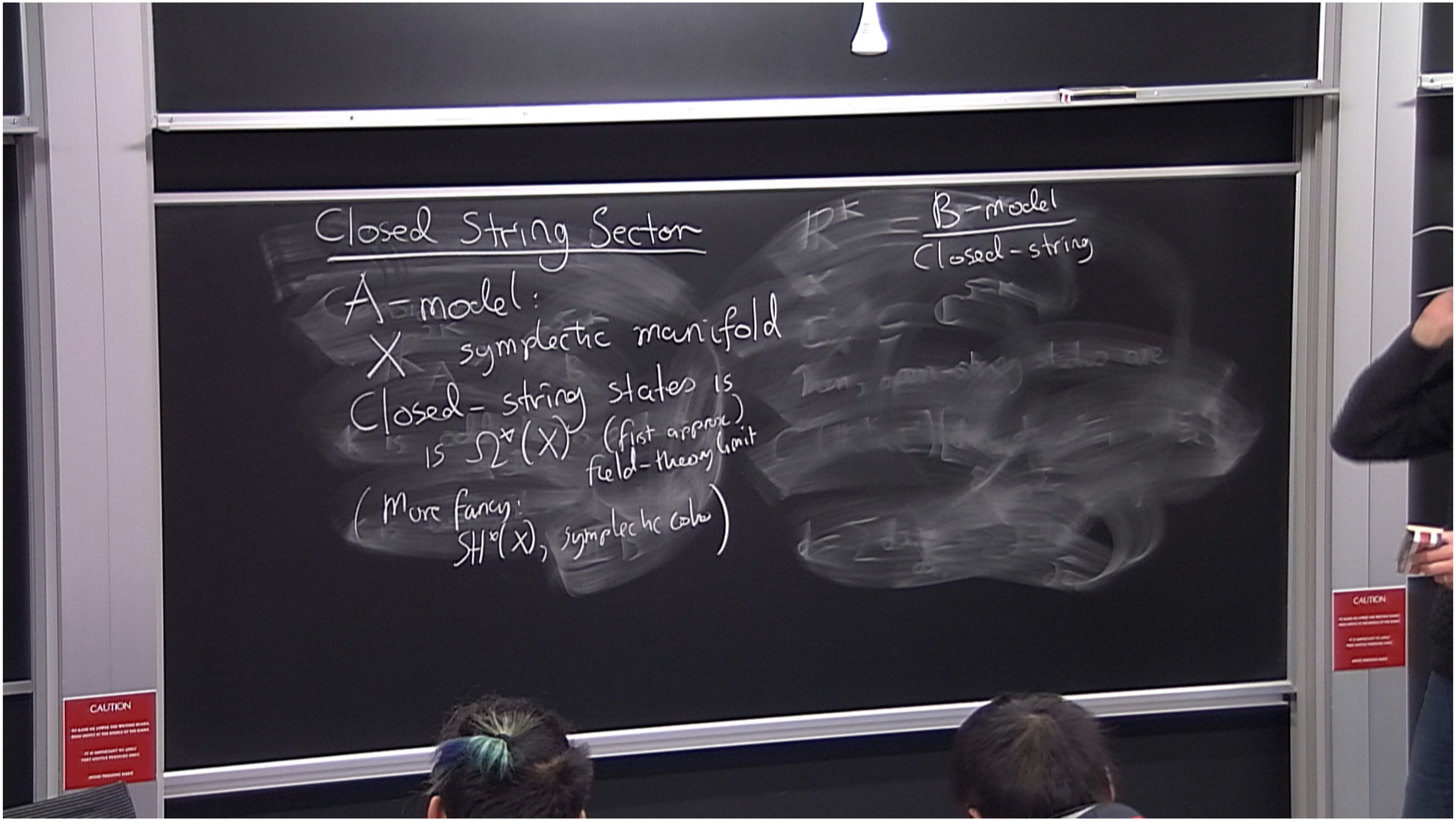
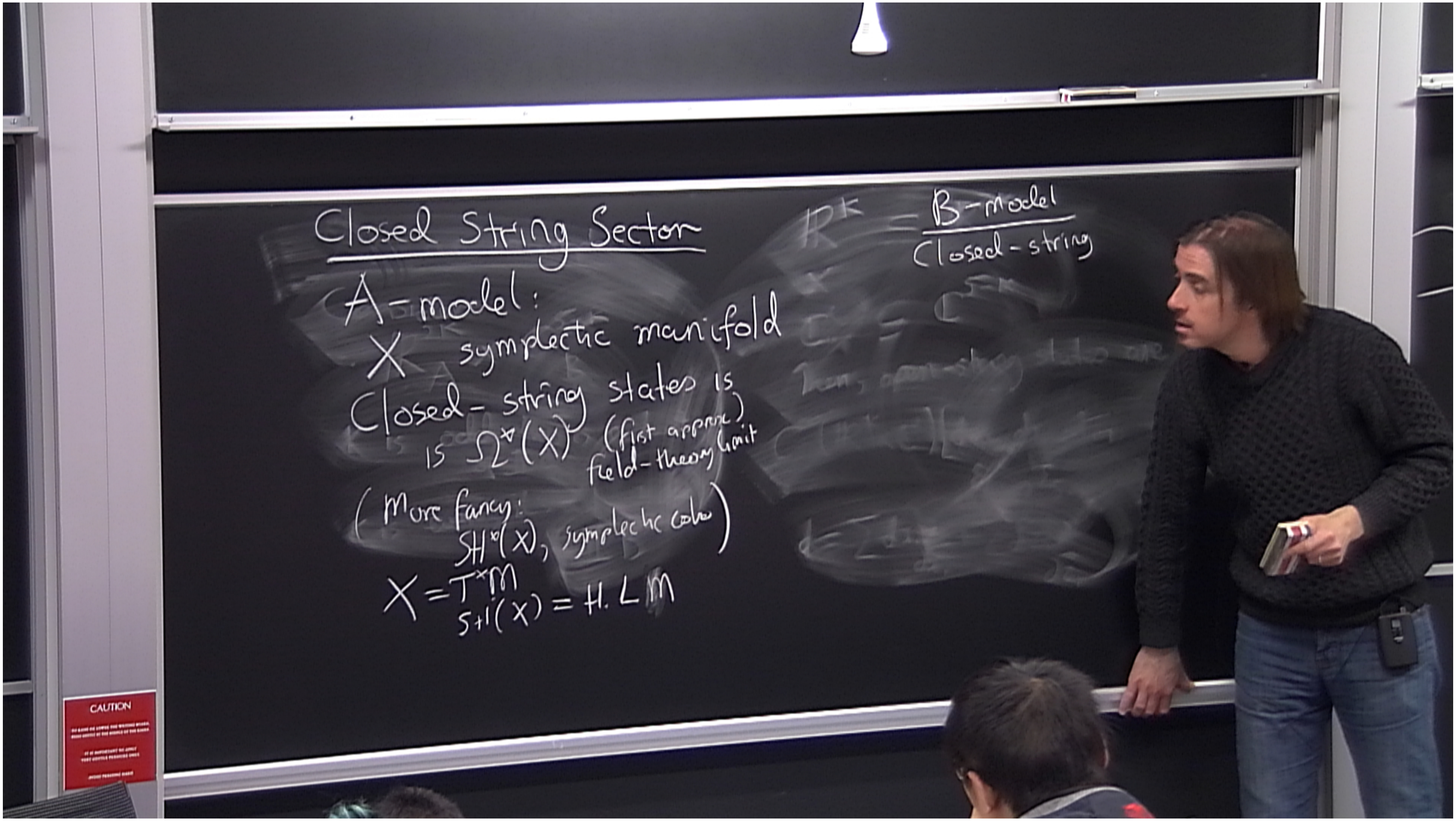
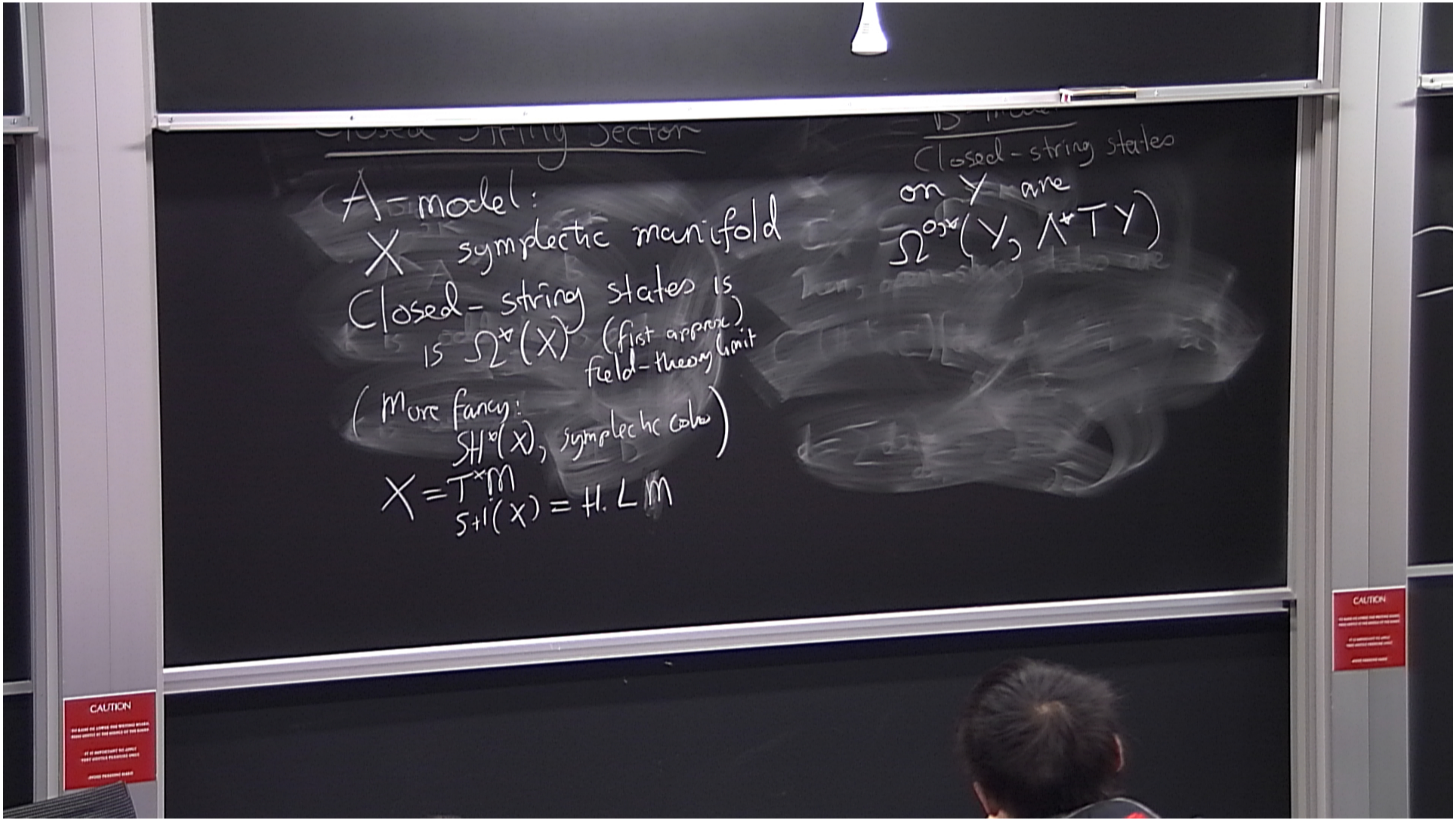
IIA can be twisted to give
$(\mathbb{R}\times S^1)_A \times \mathbb{C}^4_B$
IIB $\mathbb{C}^\times_B \times \mathbb{C}^4_B$
If $\mathbb{C}^k \subseteq \mathbb{C}^4$ hol. submanifold
$\mathbb{R} \times pt \times \mathbb{C}^k$
$\mathbb{C}^\times \times \mathbb{C}^k$
D_{2k} brane
D_{2k+1} brane


Lift this to 10d
can be twisted to give
$S^1)_A \times \mathbb{C}^4_B$
$\mathbb{C}^\times_B \times \mathbb{C}^4_B$
If $\mathbb{C}^K \subseteq \mathbb{C}^4$ hol. submanifold
$\mathbb{R} \times pt \times \mathbb{C}^K$
D_{2K} brane
$\mathbb{C}^\times \times \mathbb{C}^K$
D_{2K+1} brane
$pt \times S^1 \times \mathbb{C}^K$
D_{2K} brane
$pt \times pt \times \mathbb{C}^K$
D_{2K-1} brane


Closed String Sector
A-model:
X symplectic manifold
Closed-string states is
CAUTION


Closed String Sector
A-model:
X symplectic manifold
Closed-string states is
is $\Omega^*(X)$ (first approx)
field-theory limit
(More fancy: $SH^*(X)$, symplectic coho)
B-model
Closed-string
CAUTION
CAUTION


Closed String Sector
A-model:
X symplectic manifold
Closed-string states is
is Ω*(X) (first apprx)
field-theory limit
(More fancy:
SH*(X), symplectic coho)
X = T*M
SH(X) = H. L M
B-model
Closed-string



Closed String Sector
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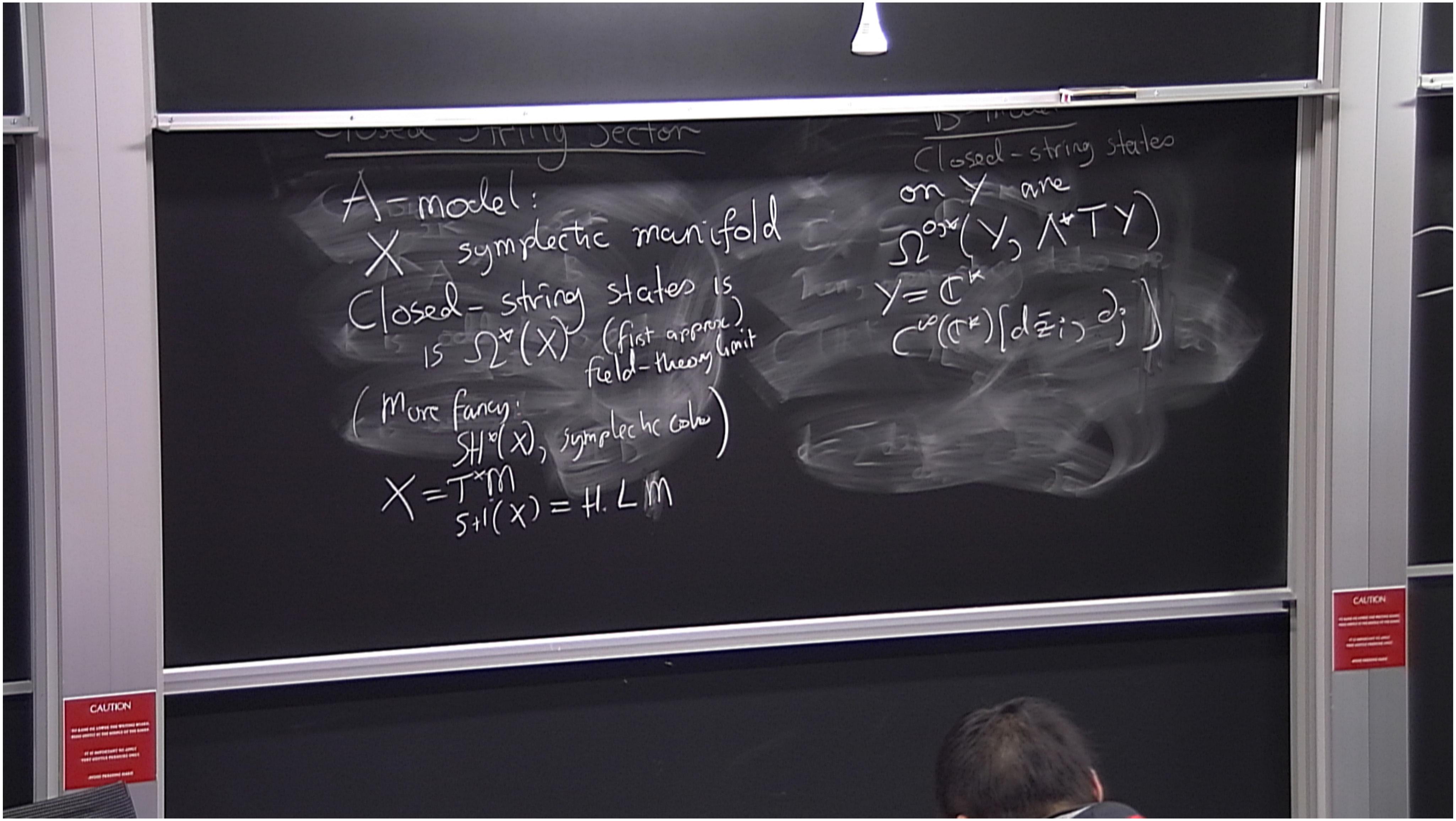
(More fancy:
SH*(X), symplectic coho)
X = T*M
SH(X) = H. L M
Closed-string states
on Y are
Ω^{0,*}(Y, Λ*TY)


Closed String Sector
A-model:
X symplectic manifold
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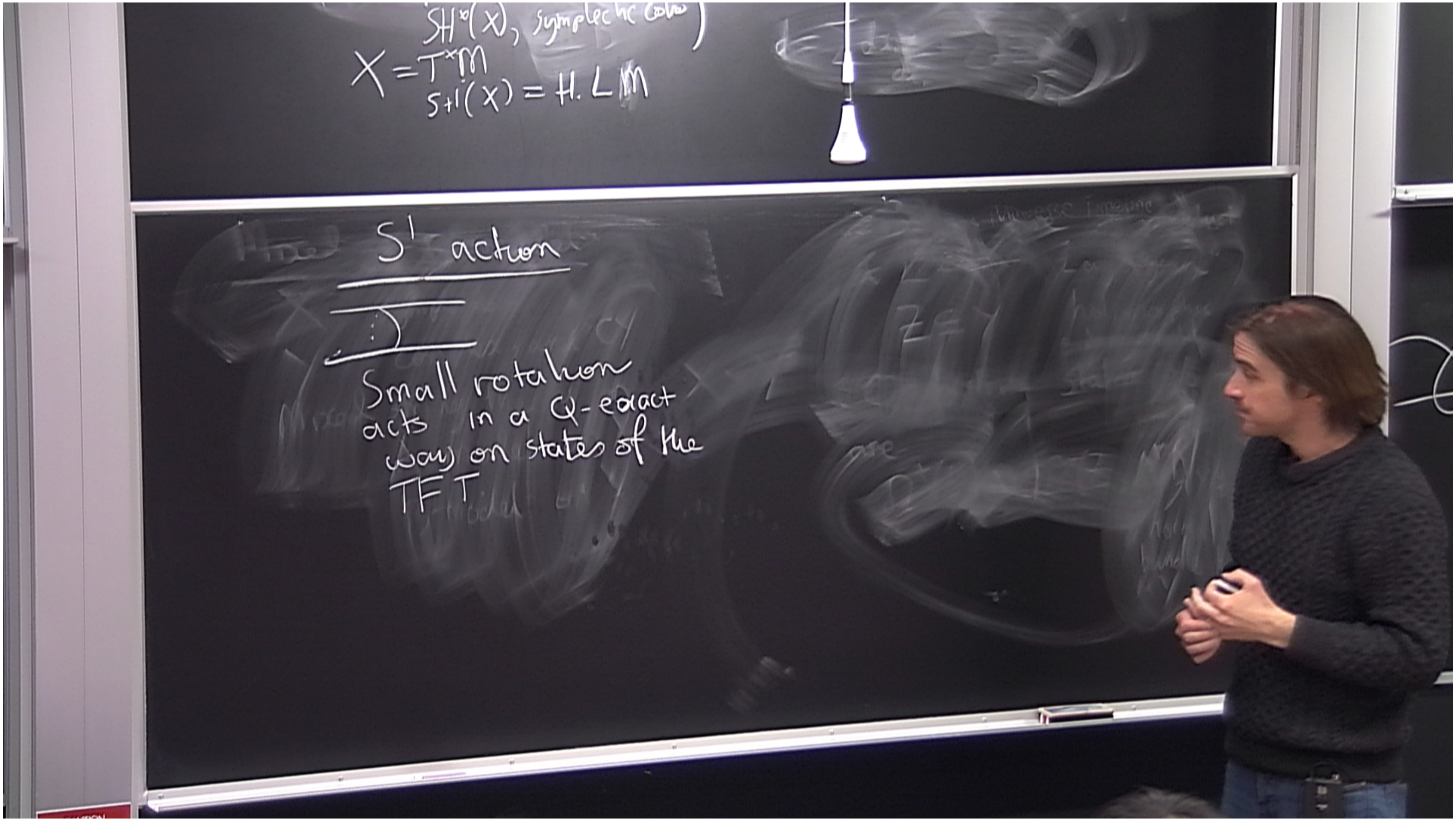
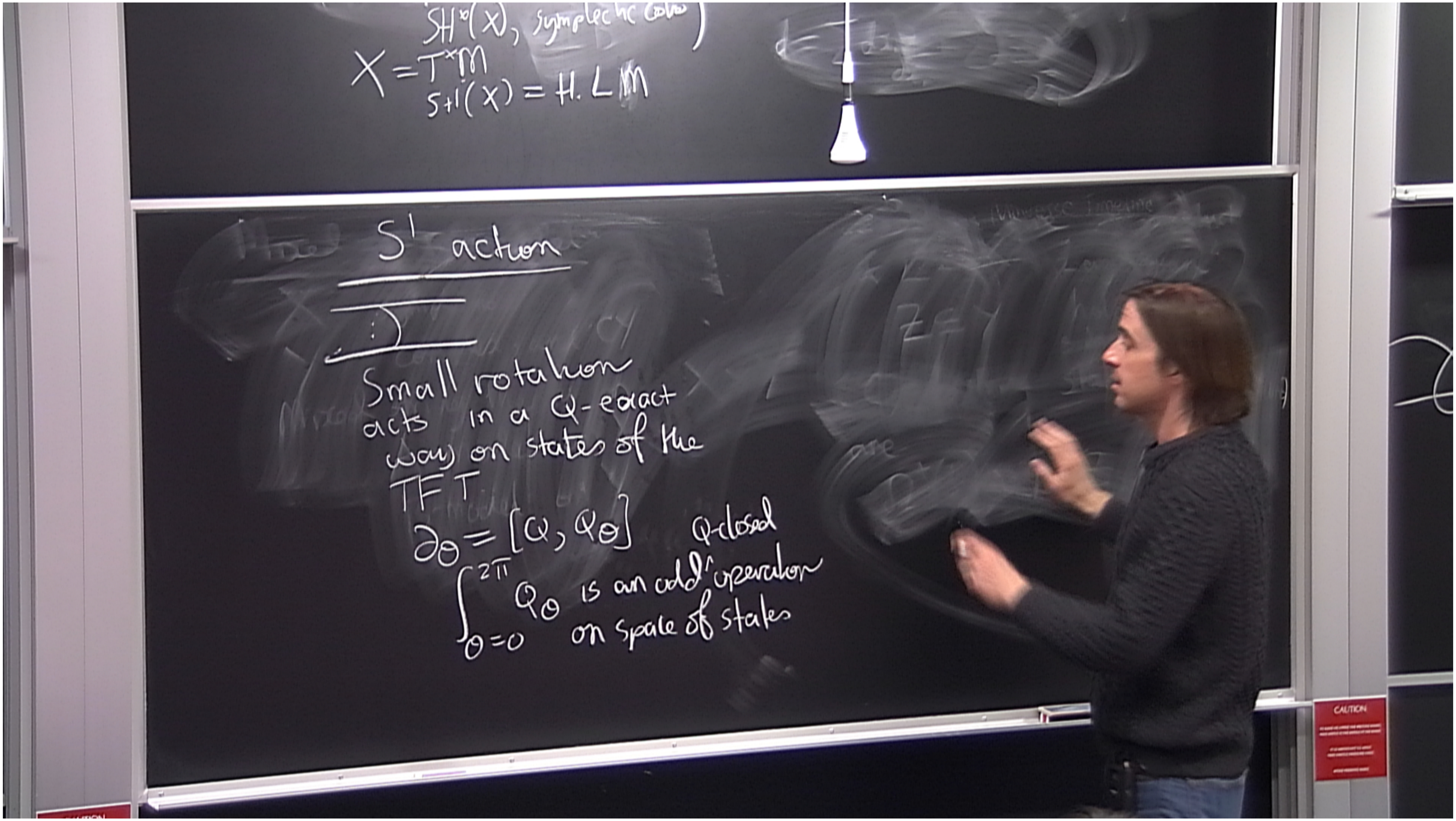
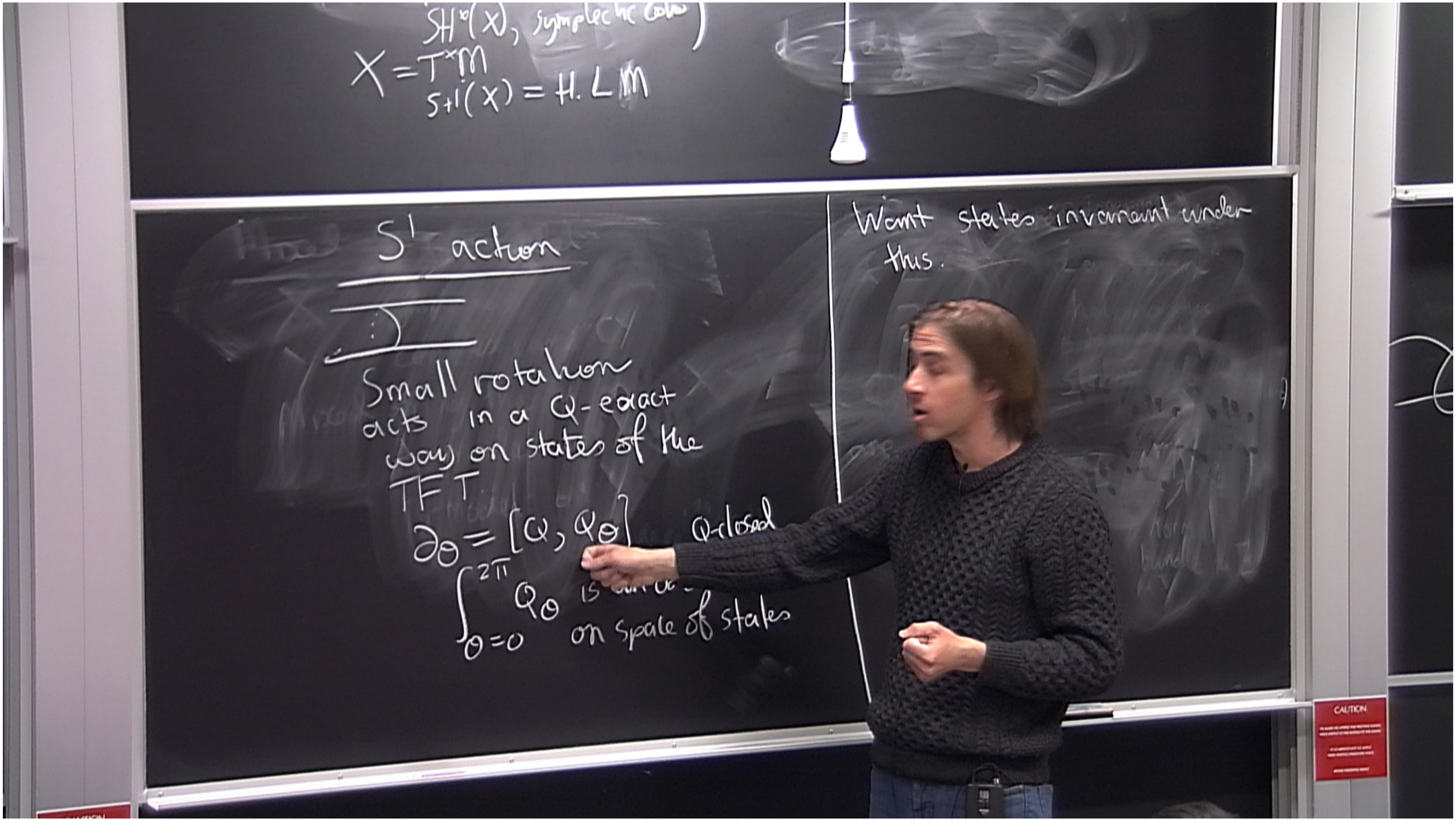
(More fancy: SH*(X), symplectic coho)
X = T*M
SH(X) = H. L M
Closed-string states
on Y are
Ω^{0,*}(Y, Λ*TY)
Y = C^k
C^∞(C^k)[dz̄_i, ∂_j]


X=T*M
S¹ action
Small rotation acts in a Q-exact way on states of the TFT


SH*(X), symplectic cohom)
X = T*M
SH(X) = H. LM
S¹ action
Small rotation
acts in a Q-exact
way on states of the
TFT
∂θ = [Q, Qθ]   Q-closed
∫ from θ=0 to 2π Qθ is an odd operator
on space of states
CAUTION



X = T*M
S¹ action
Small rotation
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TFT
∂θ = [Q, Qθ]
Q-closed
∫ θ=0 to 2π Qθ
on space of states
Want states invariant under
this.

$SH^*(X)$, symplectic coho)

$X = T^*M$

$SH^*(X) = H_\cdot LM$

S^1 action

Small rotation acts in a Q-exact way on states of the TFT

$\partial_\theta = [Q, Q_\theta]$ Q-closed

$\int_{\theta=0}^{2\pi} Q_\theta$ is an odd operator on space of states

Want states invariant under this.

A-model

$X = T^*M$

$SH^* X = H_\cdot LM$

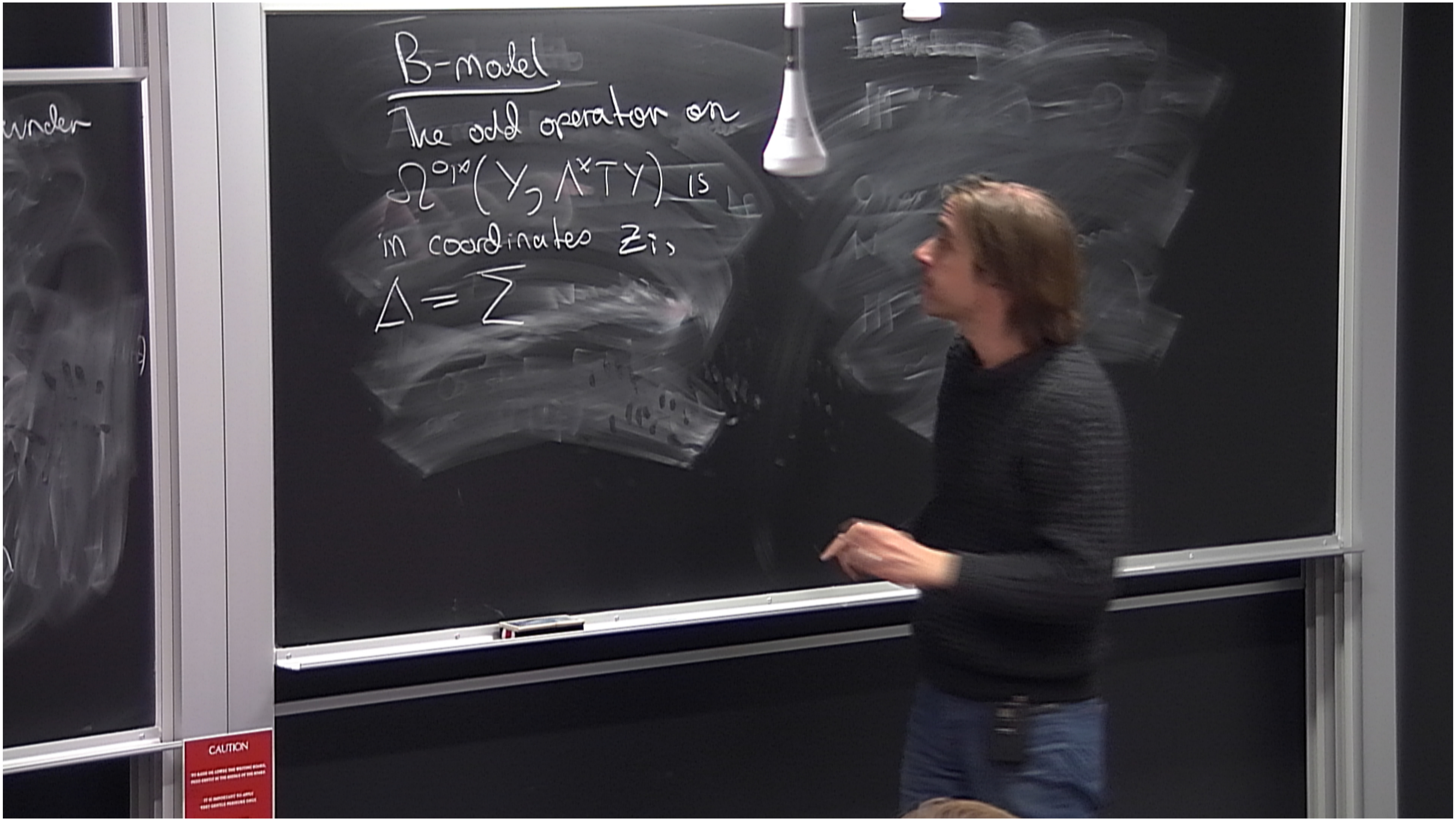
$S^1 \times LM \longrightarrow LM$

$H_\cdot S^1 \otimes H_\cdot LM \to H_\cdot LM$

$[S^1] \in H_1 S^1$ gives a map

$H_k LM \longrightarrow H_{k+1} LM$

which is

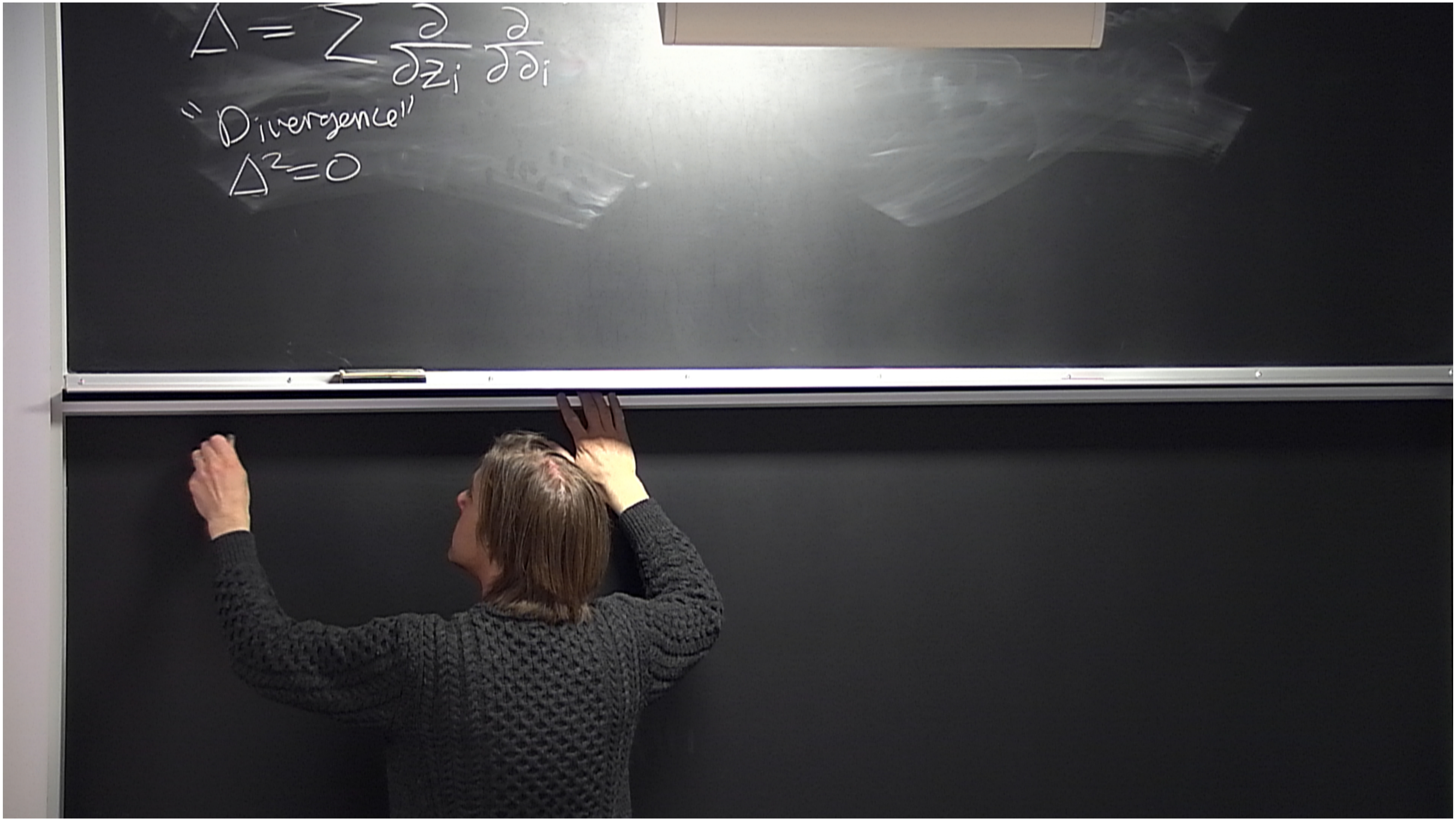

B-model
The odd operator on
$\Omega^{0,*}(Y, \Lambda^* TY)$ is
in coordinates z_i,
$\Delta = \sum$
CAUTION

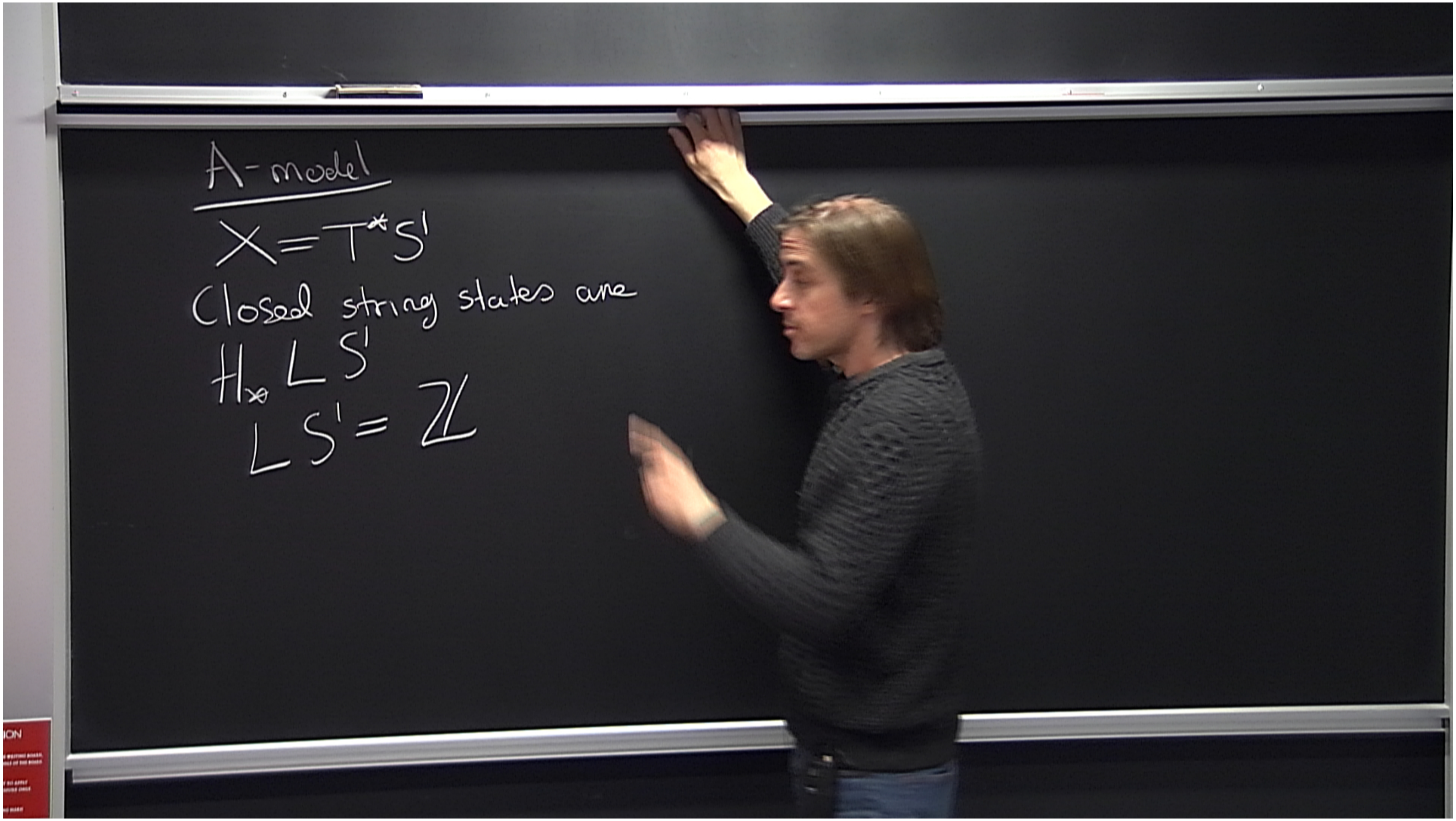
B-model

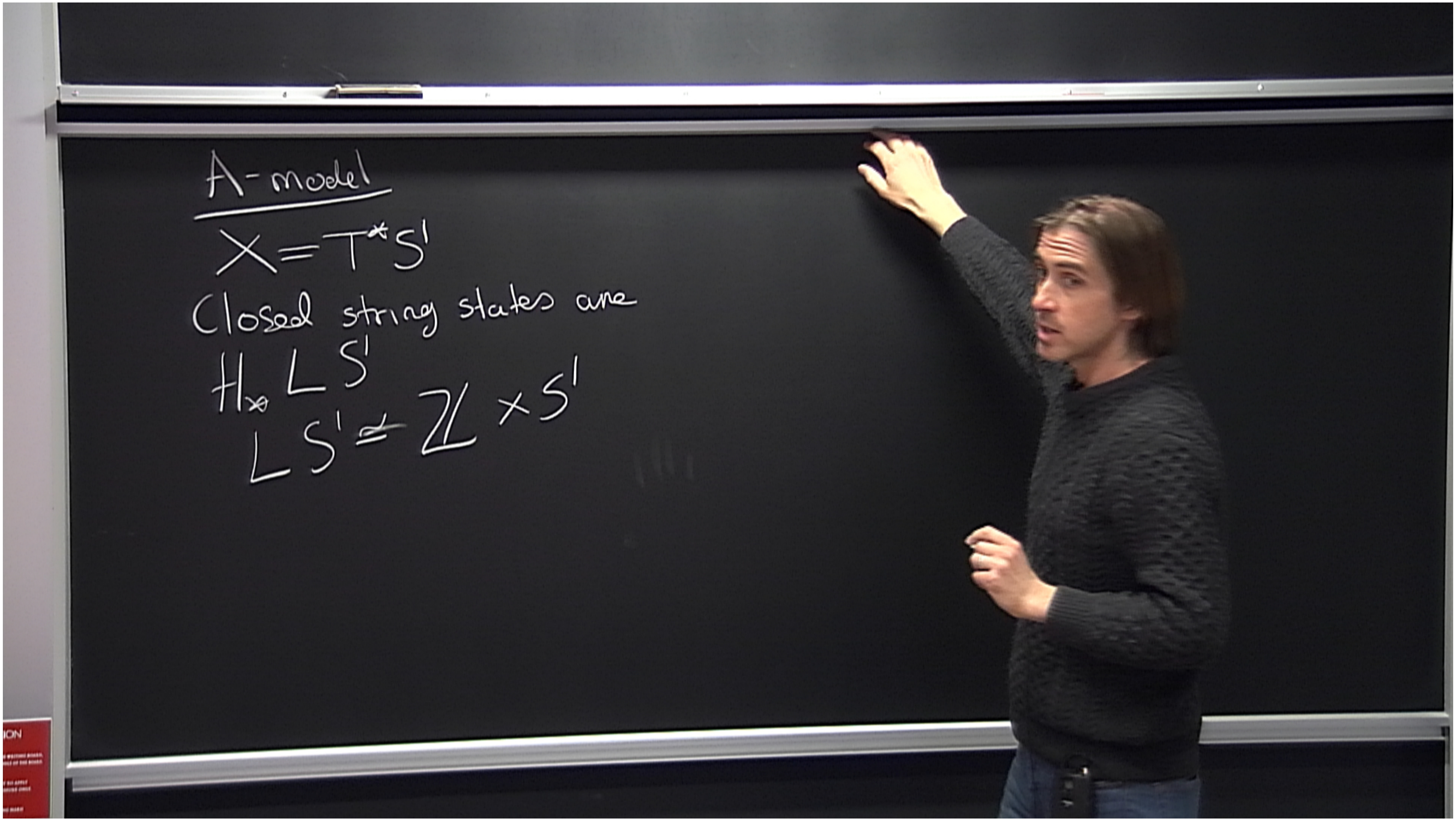
The odd operator on $\Omega^{0,*}(Y, \Lambda^* TY)$ is

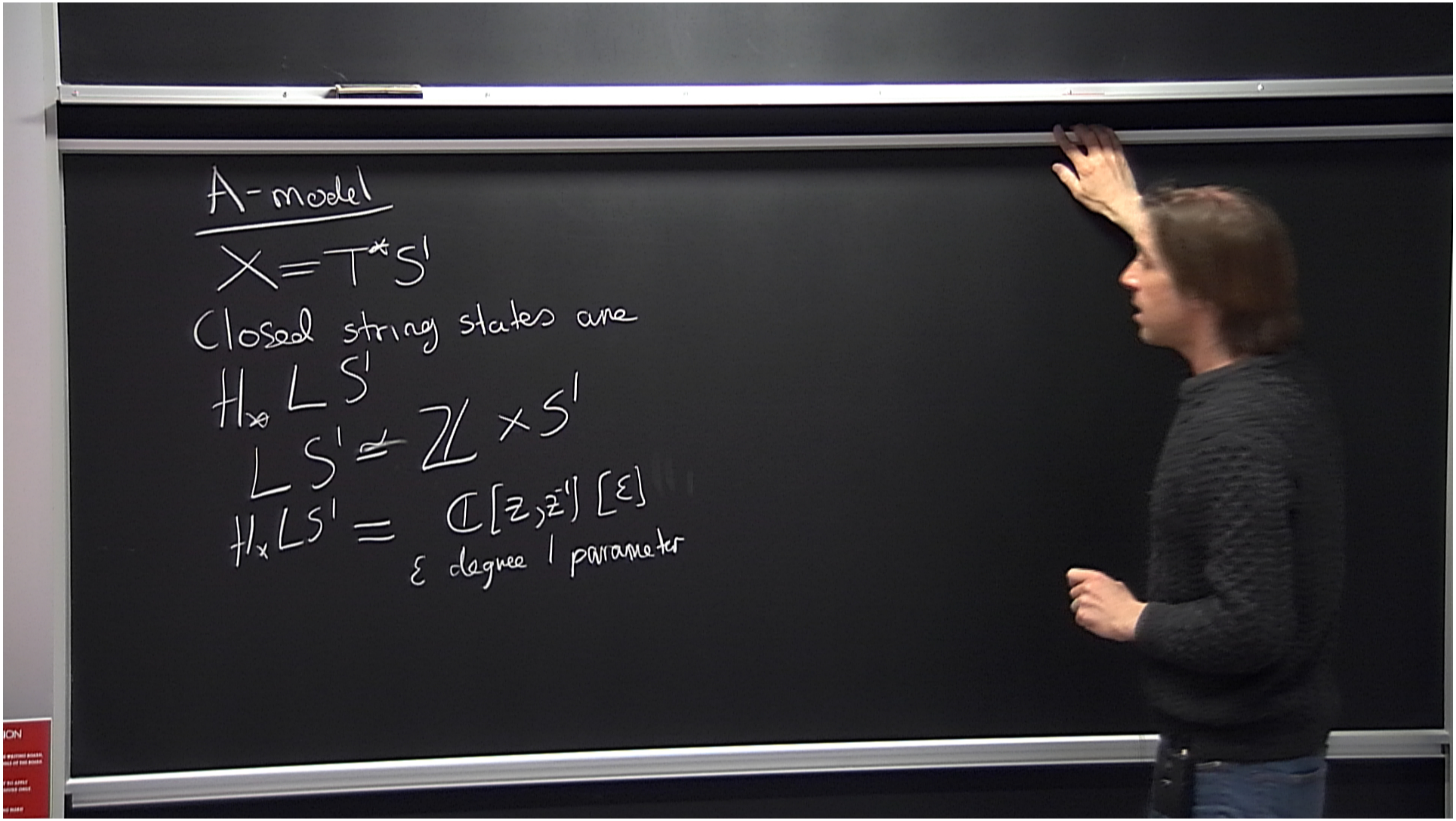
in coordinates z_i,

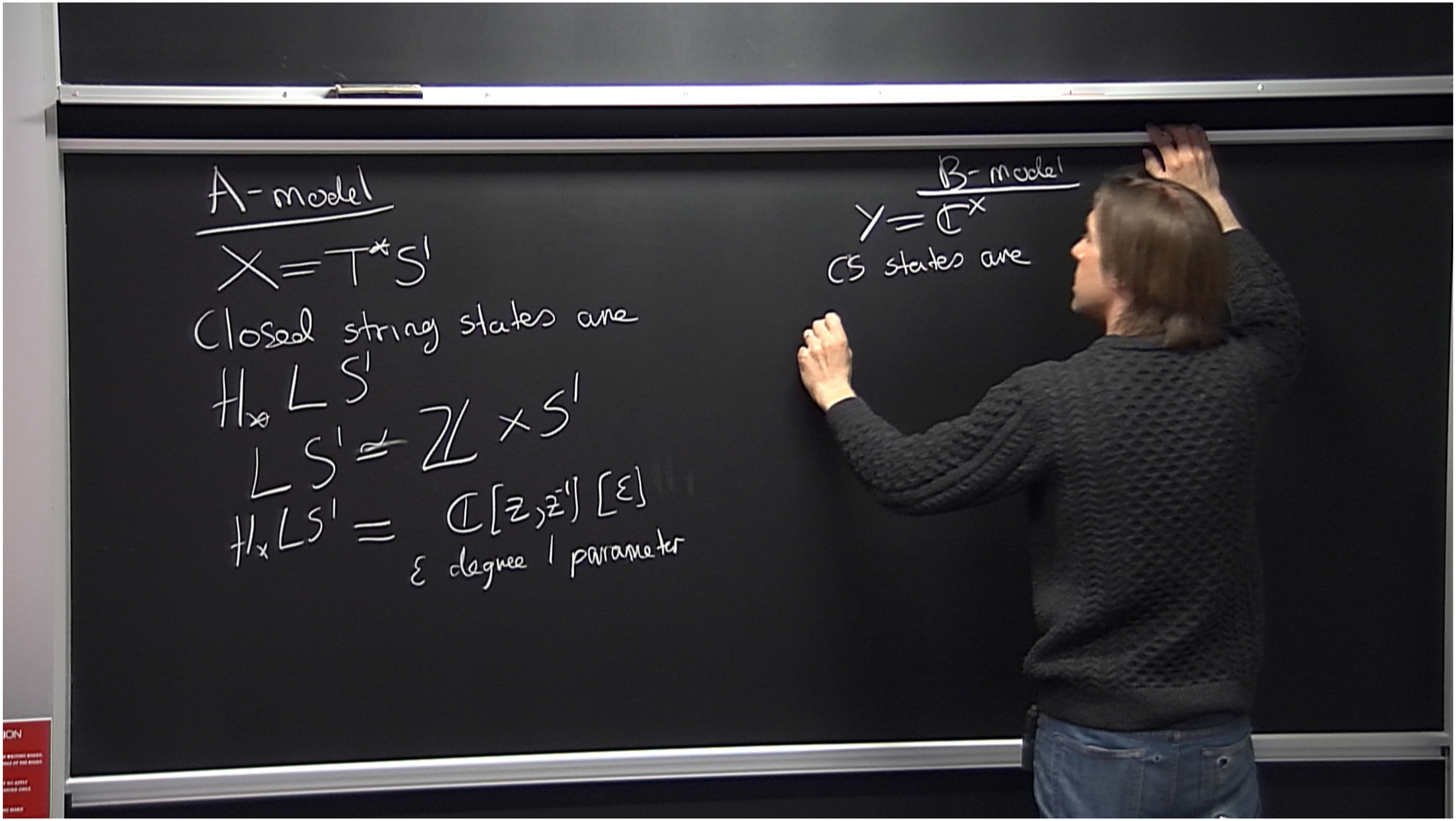
$$\Delta = \sum \frac{\partial}{\partial z_i} \frac{\partial}{\partial \theta_i}$$


$\Delta = \sum \frac{\partial}{\partial z_i} \frac{\partial}{\partial \theta_i}$
"Divergence"
$\Delta^2 = 0$


A-model
$X = T^*S^1$
Closed string states are
$H_* LS^1$
$LS^1 = \mathbb{Z}$


A-model
X = T*S'
Closed string states are
H* LS'
LS' ≃ Z × S'


A-model
$X = T^*S^1$
Closed string states are
$H_* L S^1$
$L S^1 \simeq \mathbb{Z} \times S^1$
$H_* L S^1 = \mathbb{C}[z, z^{-1}][\varepsilon]$
ε degree 1 parameter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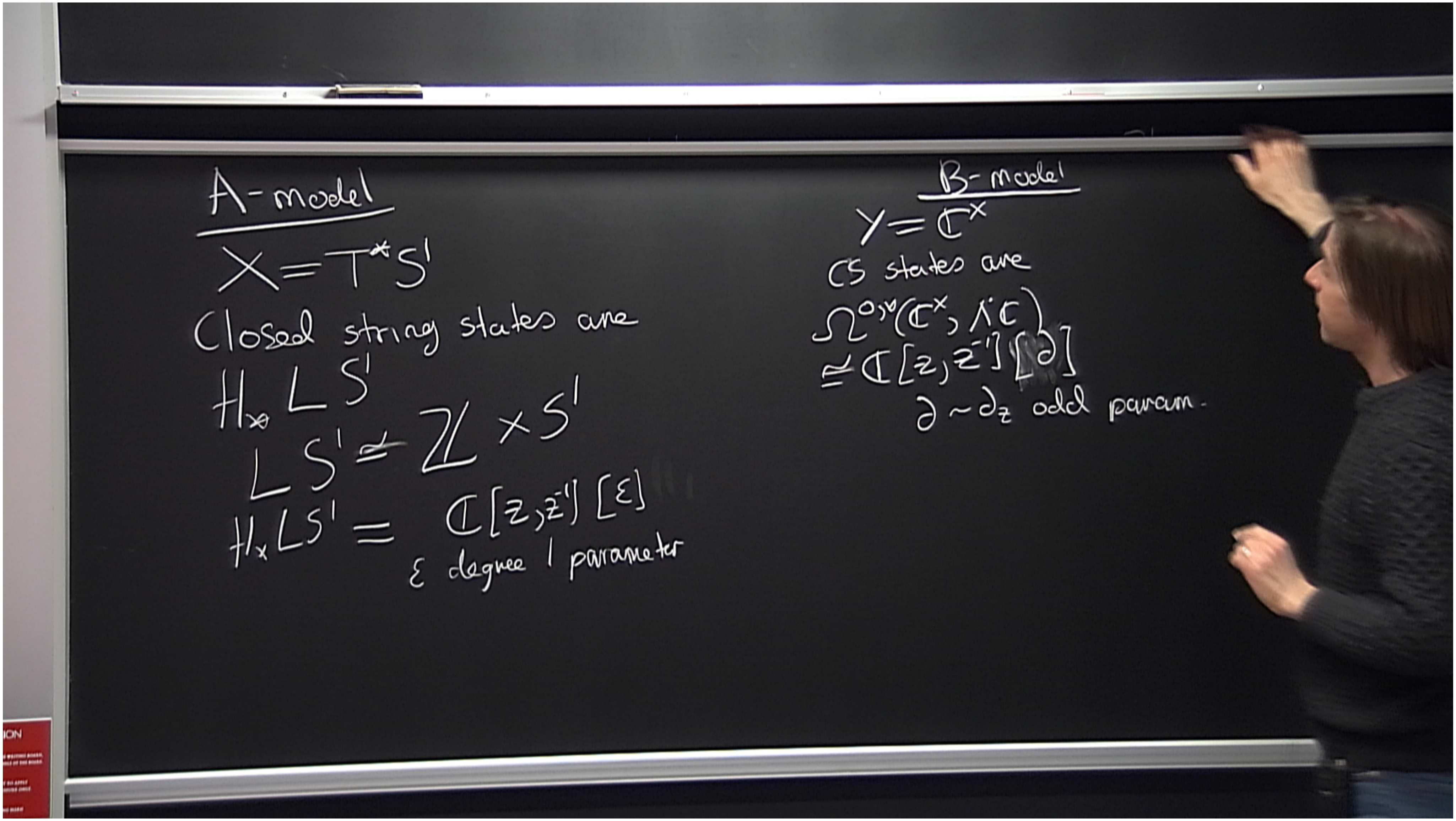
ε degree 1 parameter
B-model
$Y = \mathbb{C}^\times$
CS states are


A-model
$X = T^*S^1$
Closed string states are
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$LS^1 \simeq \mathbb{Z} \times S^1$
$H_* LS^1 = \mathbb{C}[z, z^{-1}][\varepsilon]$
ε degree 1 parameter
B-model
$Y = \mathbb{C}^\times$
CS states are
$\Omega^{0,*}(\mathbb{C}^\times, \wedge^* \mathbb{C})$
$\simeq \mathbb{C}[z, z^{-1}][\partial]$
$\partial \sim \partial_z$ odd param.

A-model

$X = T^*S^1$

Closed string states are

$H_* LS^1$

$LS^1 \simeq \mathbb{Z} \times S^1$

$H_* LS^1 = \mathbb{C}[z, z^{-1}][\varepsilon]$

ε degree 1 parameter

B-model

$Y = \mathbb{C}^\times$

CS states are

$\Omega^{0,*}(\mathbb{C}^\times, \wedge^* T)$

$\cong \mathbb{C}[z, z^{-1}][\partial]$

$\partial \sim \partial_z$ odd param.

Δ measures failure of $f(z)\partial_z$ to preserve dz/z

A-model

$X = T^* S^1$

Closed string states are

$H_* LS^1$

$LS^1 \simeq \mathbb{Z} \times S^1$

$H_* LS^1 = \mathbb{C}[z, z^{-1}][\varepsilon]$

ε degree 1 parameter

B-model

$Y = \mathbb{C}^\times$

CS states are

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$\partial \sim \partial_z$ odd param.

Δ measures failure of $f z \partial_z$ to preserve dz/z

A-model

$X = T^*S^1$

Closed string states are

$H_* LS^1$

$LS^1 \simeq \mathbb{Z} \times S^1$

$H_* LS^1 = \mathbb{C}[z, z^{-1}][\varepsilon]$

ε degree 1 parameter

B-model

$Y = \mathbb{C}^\times$

CS states are

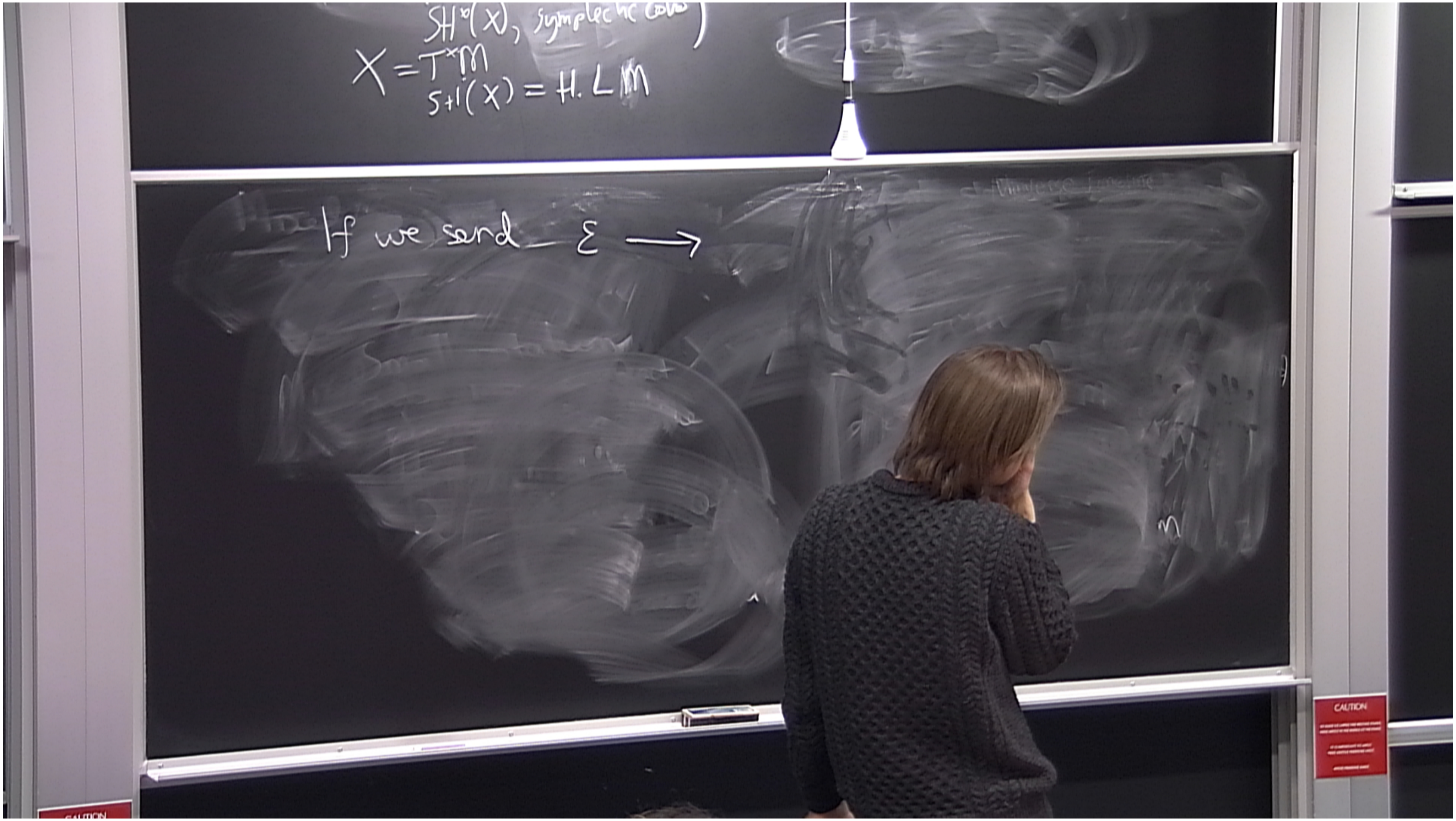
$\Omega^{0,*}(\mathbb{C}^\times, \wedge^* \mathbb{C})$

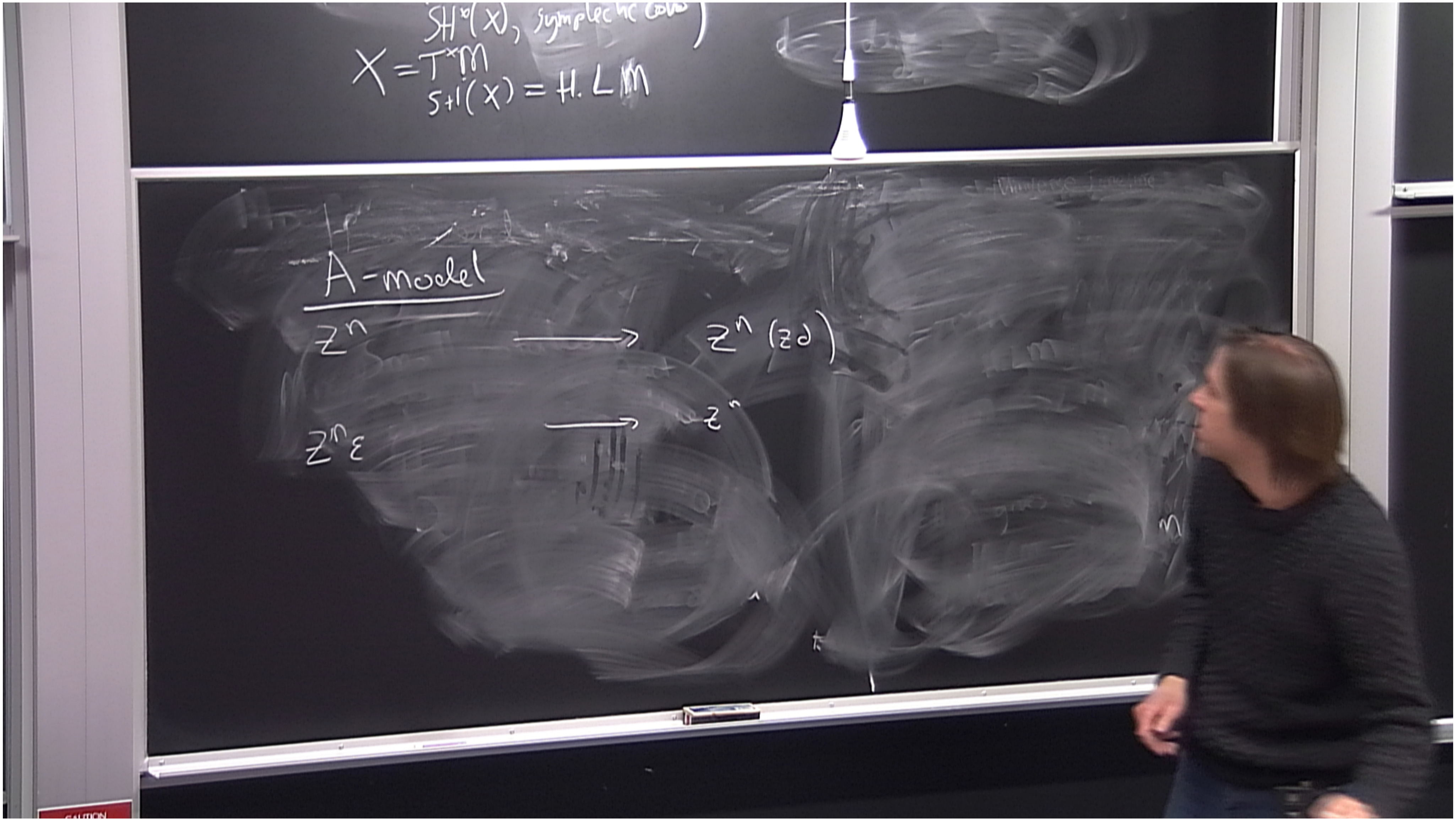
$\simeq \mathbb{C}[z, z^{-1}][z\partial]$

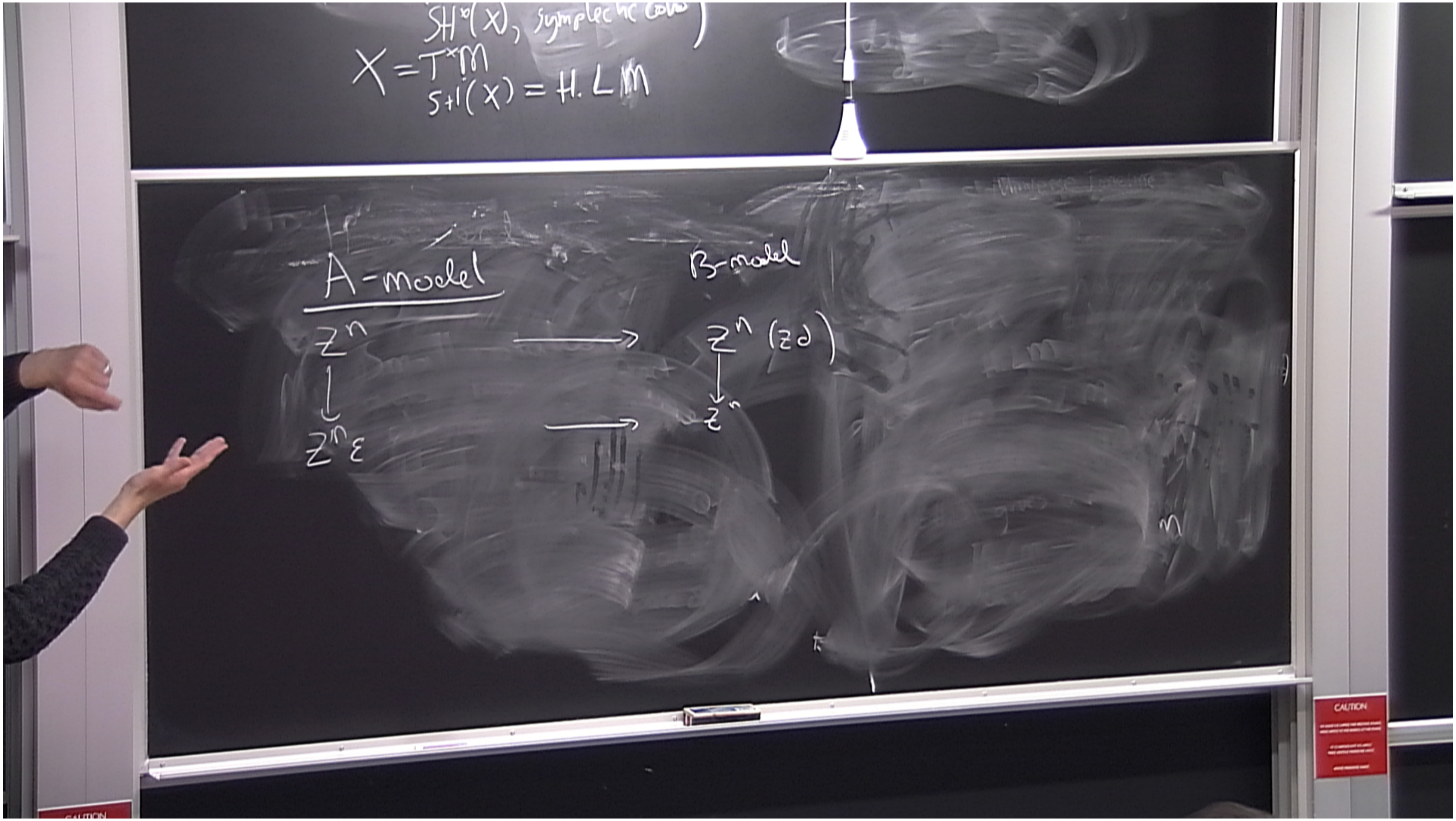
$\partial \sim \partial_z$ odd param.

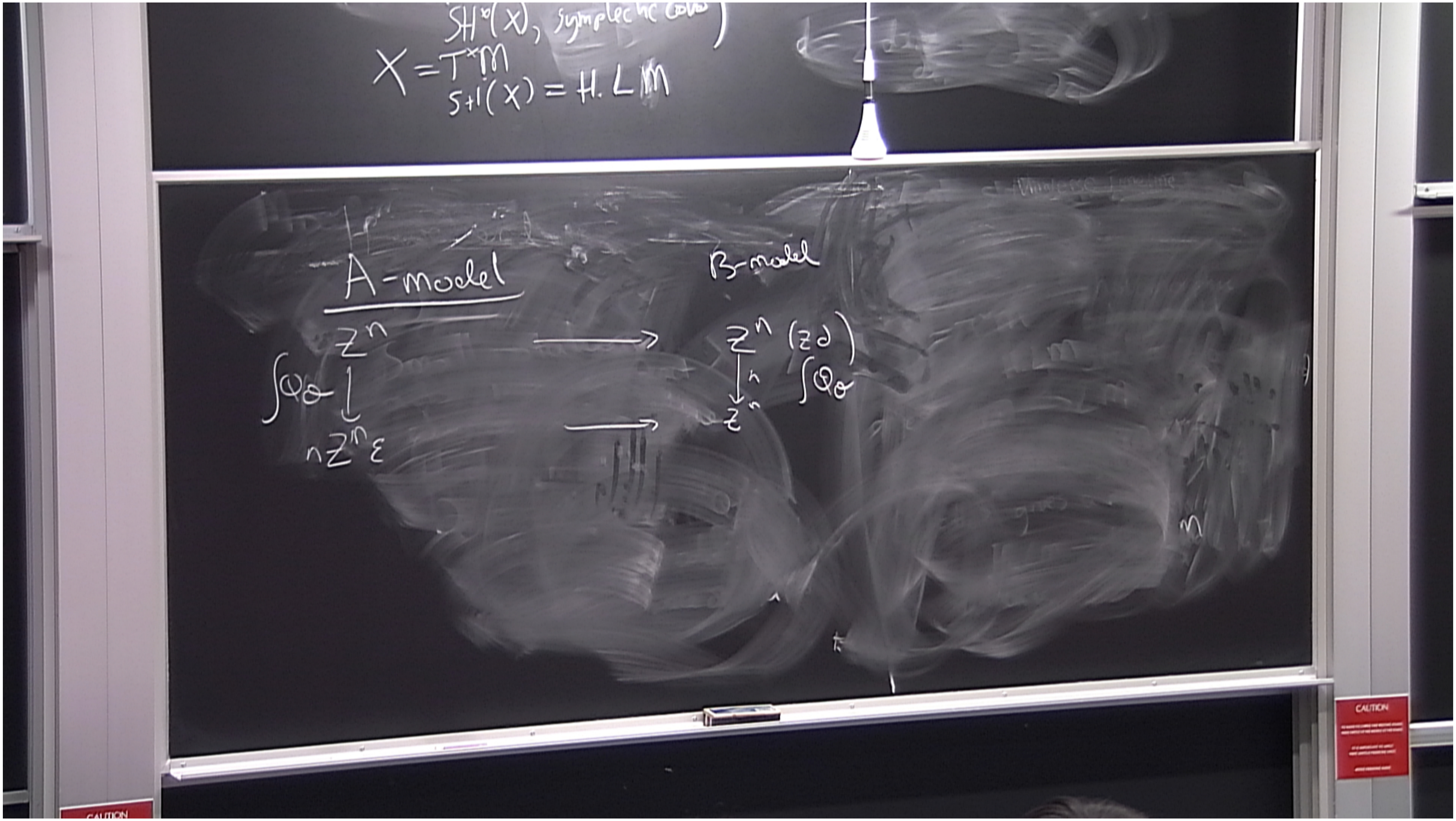
Δ measures failure of $f z \partial_z$ to preserve dz/z

$\Delta(f z \partial_z) = z f'$


SH*(X), symplectic cohom)
X = T*M
SH(X) = H. LM
If we send ε →


SH*(X), symplectic coho)
X = T*M
SH(X) = H. LM
A-model
Zⁿ → Zⁿ (z∂)
Zⁿ ε → Zⁿ


$SH^*(X)$, symplectic coho
$X = T^*M$
$SH(X) = H_\cdot LM$
A-model
B-model
Z^n
$Z^n\,(z\partial)$
$Z^n \varepsilon$
Z^n


SH*(X), symplectic coho)
X = T*M
SH(X) = H. LM
A-model
B-model
Z^n
Z^n (zd)
∫Q_θ
nZ^n ε
z^n