Title: Random variables, entanglement and nonlocality in infinite > translation-invariant systems

Date: Apr 11, 2017 03:30 PM

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Abstract: <p>We consider the problem of certifying entanglement and nonlocality in one-dimensional translation-invariant (TI) infinite systems when just averaged near-neighbor correlators are available. Exploiting the triviality of the marginal problem for 1D TI distributions, we arrive at a practical characterization of the near-neighbor density matrices of multi-separable TI quantum states. This allows us, e.g., to identify a family of separable two-qubit states which only admit entangled TI extensions. For nonlocality detection, we show that, when viewed as a vector in R^n, the set of boxes admitting an infinite TI classical extension forms a polytope, i.e., a convex set defined by a finite number of linear inequalities. Using DMRG, we prove that some of these inequalities can be violated by distant parties conducting identical measurements on an infinite TI quantum state. Both our entanglement witnesses and our Bell inequalities can be used to certify entanglement and nonlocality in large spin chains (namely, finite, and not TI chains) via neutron scattering.</p>

<p> </p>

<p>Our attempts at generalizing our results to TI systems in 2D and 3D lead us to the virtually unexplored problem of characterizing the marginal distributions of infinite TI systems in higher dimensions. In this regard, we show that, for random variables which can only take a small number of possible values (namely, bits and trits), the set of nearest (and next-to-nearest) neighbor distributions admitting a 2D TI infinite extension forms a polytope. This allows us to compute exactly the ground state energy per site of any classical nearest-neighbor Ising-type TI Hamiltonian in the infinite square or triangular lattice. Remarkably, some of these results also hold in 3D.</p>

<p>In contrast, when the cardinality of the set of possible values grows (but remaining finite), we show that the marginal nearest-neighbor distributions of 2D TI systems are not described by a polytope or even a semi-algebraic set. Moreover, the problem of computing the exact ground state energy per site of arbitrary 2D TI Hamiltonians is undecidable.</p>

Random variables, entanglement and nonlocality in infinite TI systems

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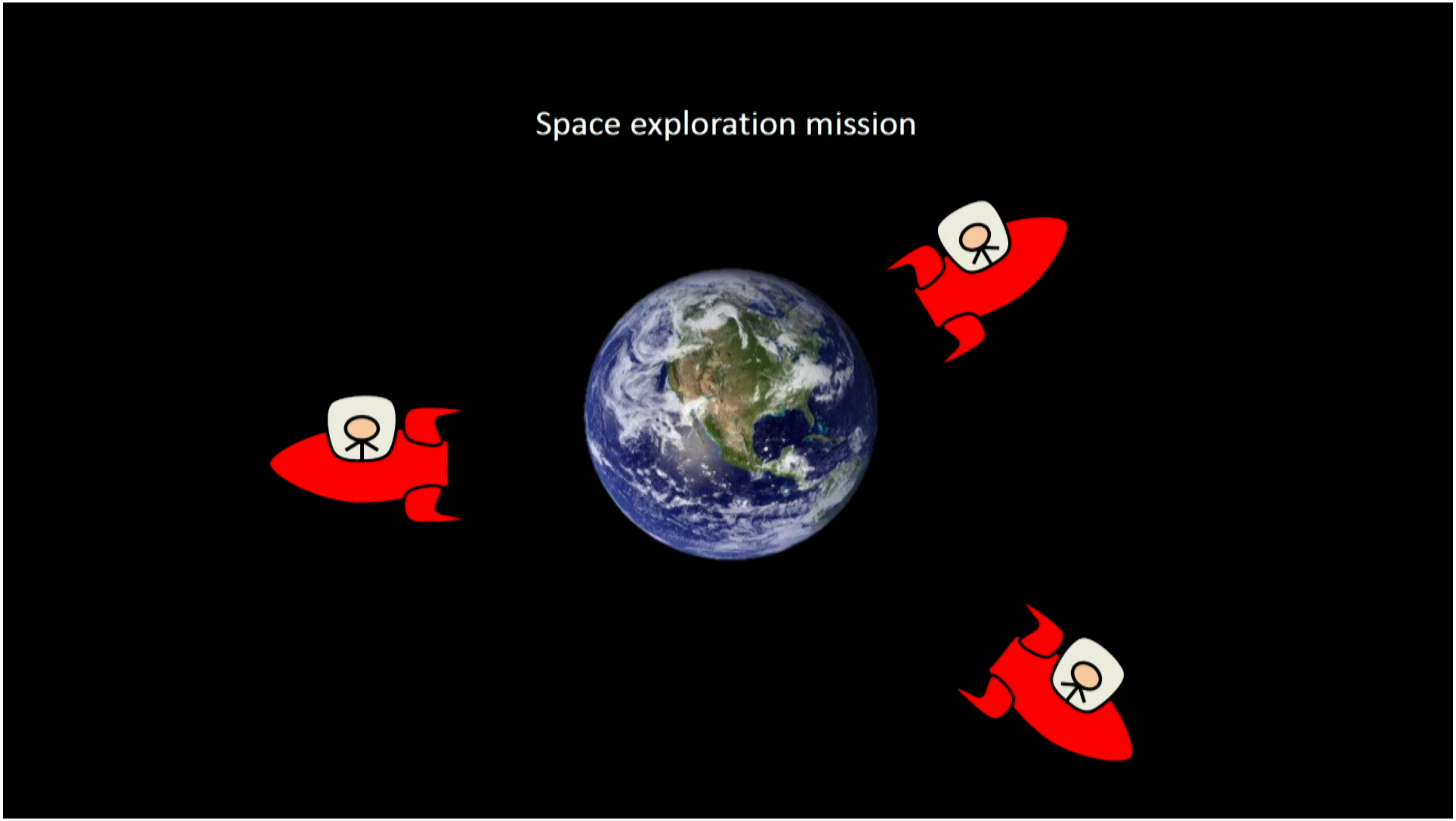
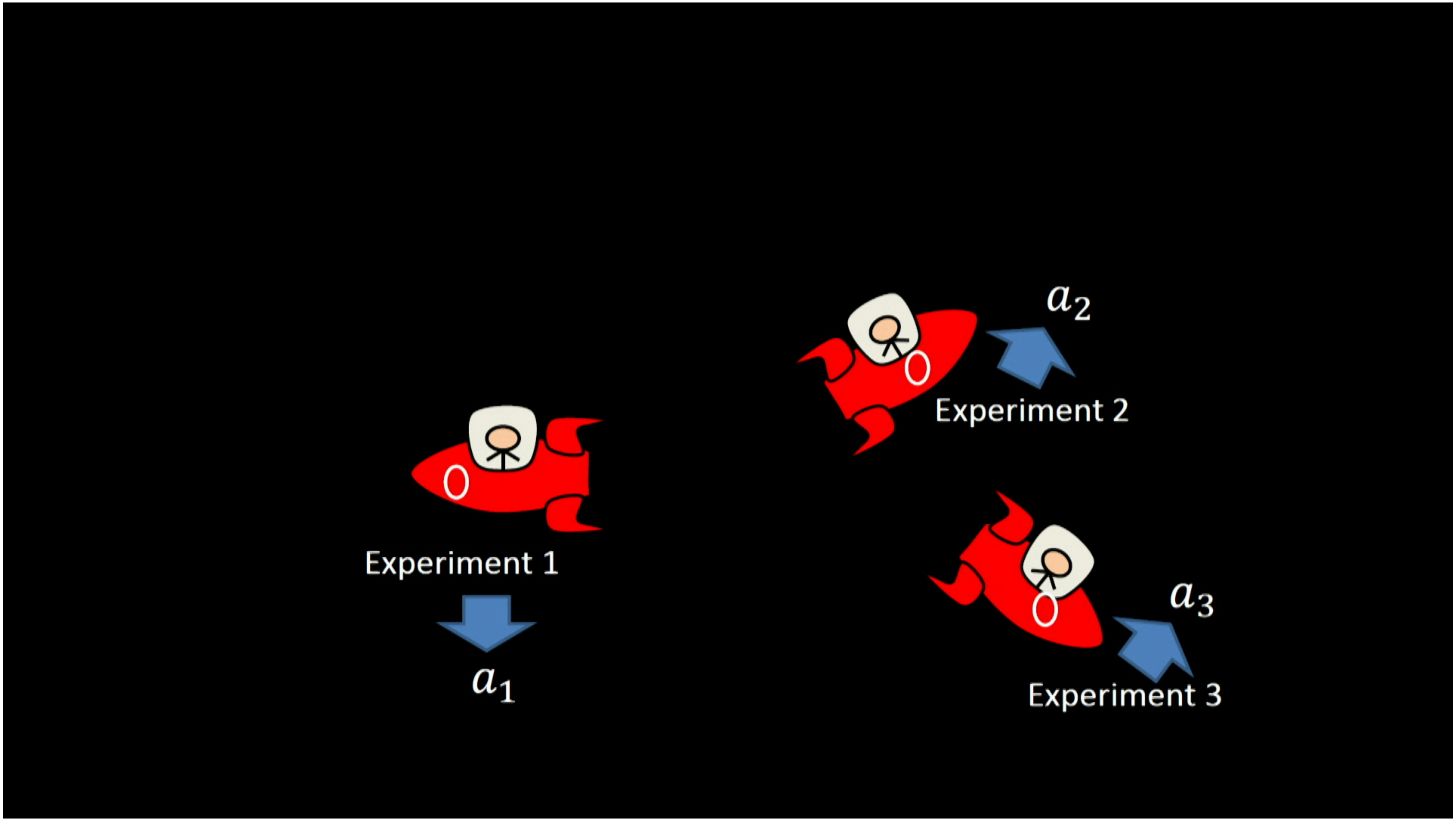
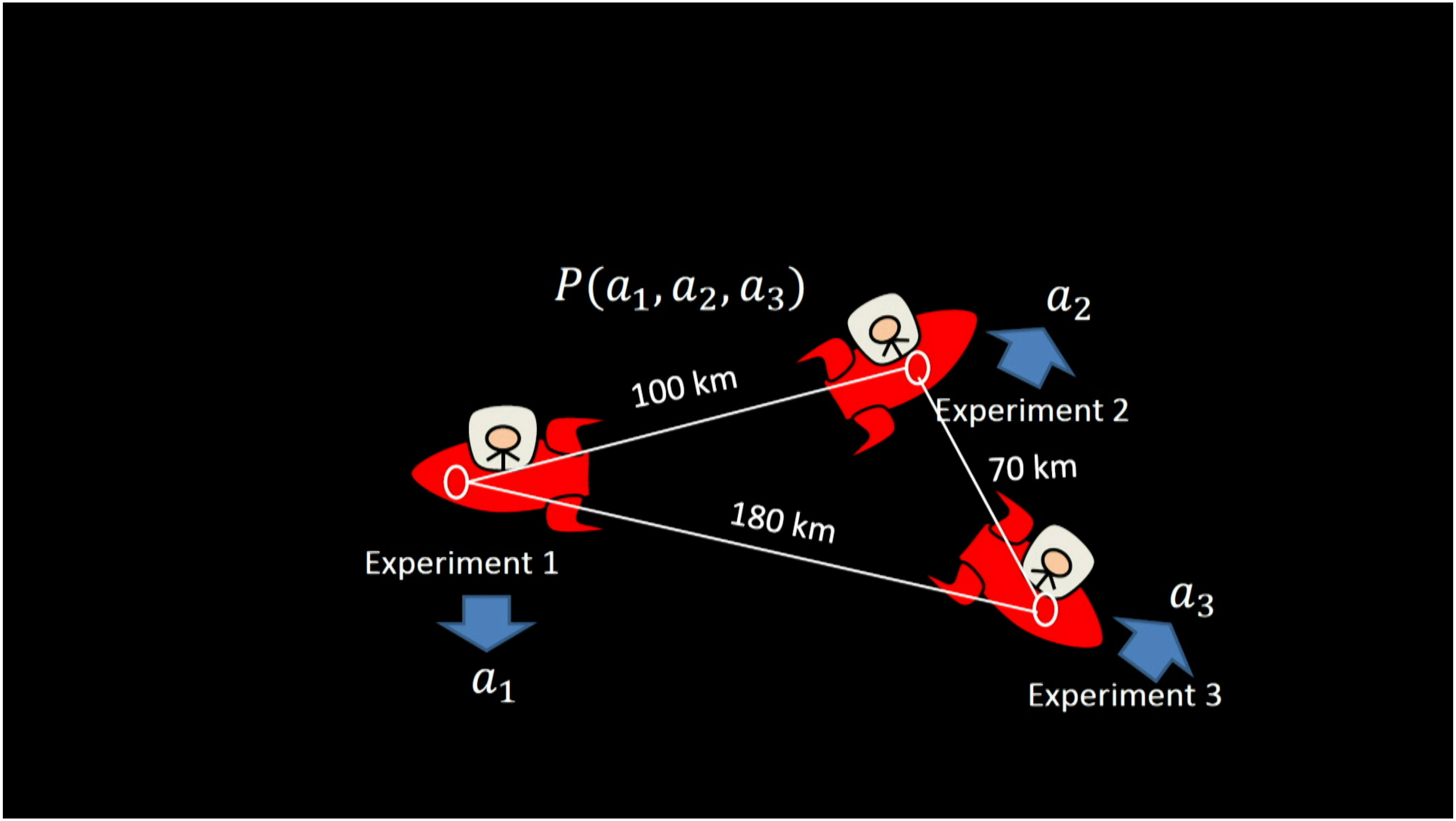
Institute for Quantum Optics and Quantum Information (IQOQI), Vienna

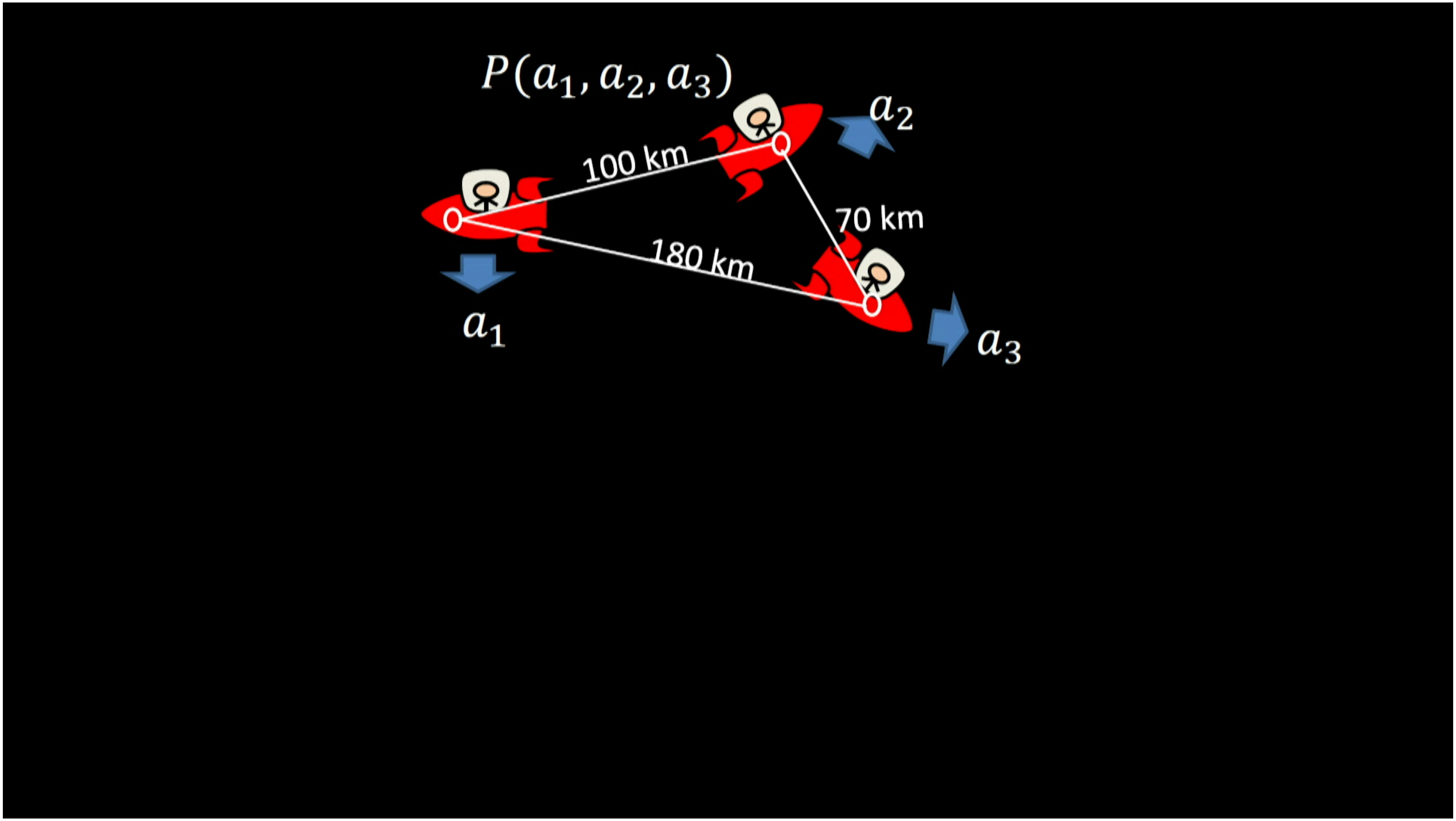
Z. Wang, S. Singh and MN, arXiv:1608.03485.
Z. Wang and MN, arXiv:1703.05640.

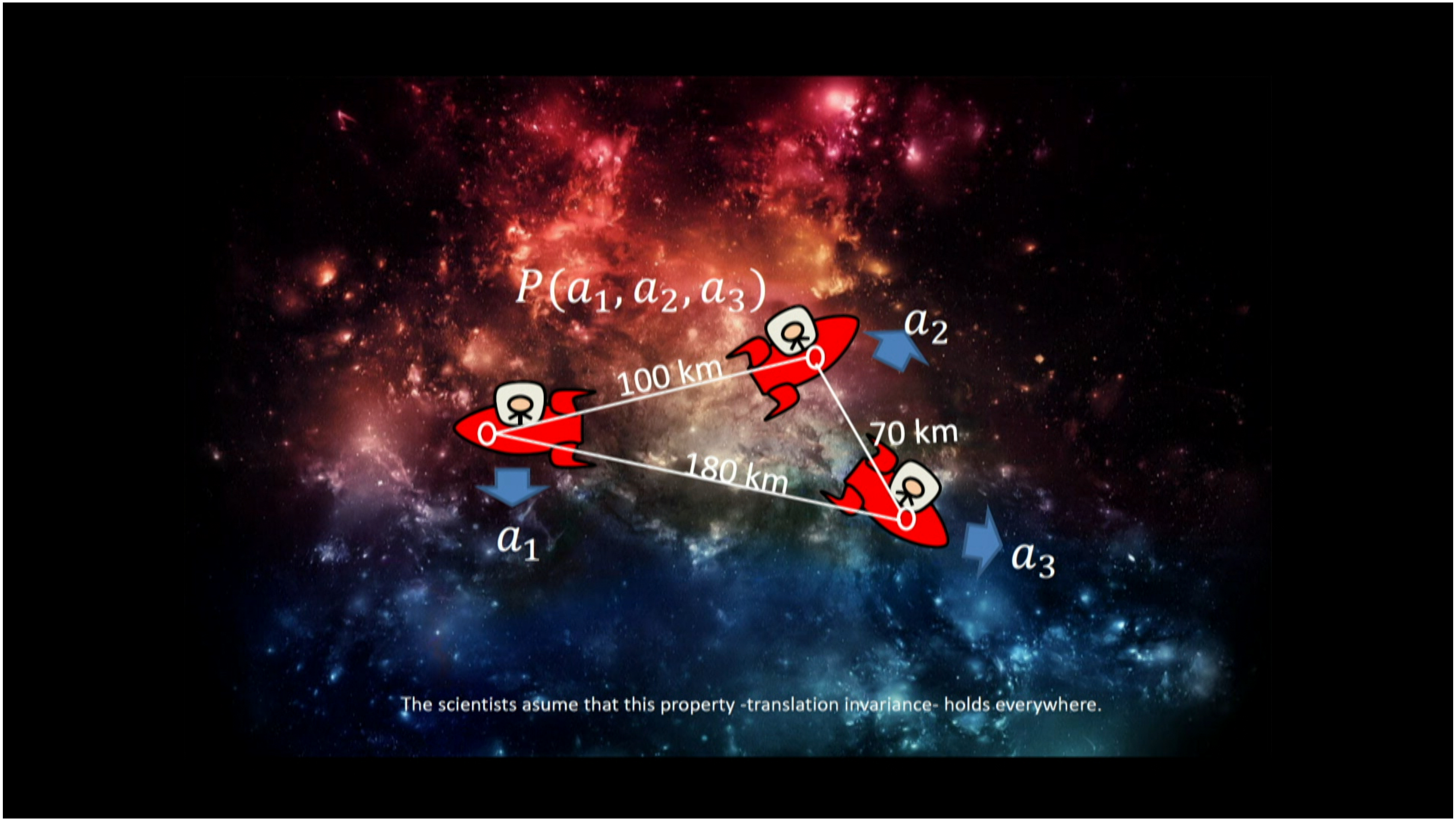

Space exploration mission

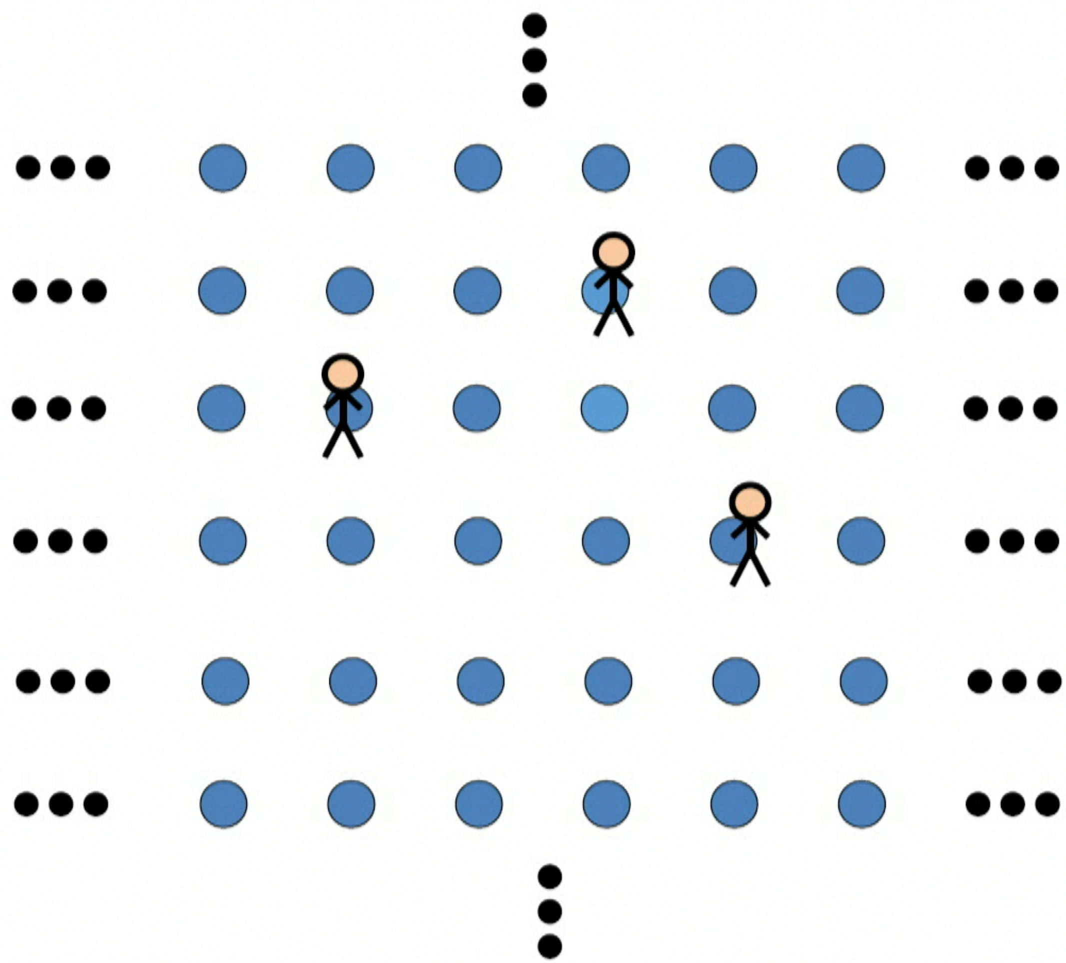

a_2
Experiment 2
Experiment 1
a_3
a_1
Experiment 3



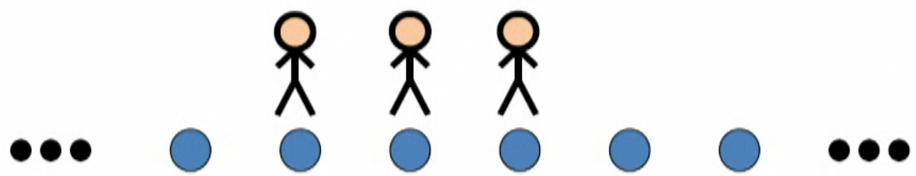
$P(a_1, a_2, a_3)$
a_2
100 km
Experiment 2
70 km
180 km
Experiment 1
a_3
a_1
Experiment 3


$P(a_1, a_2, a_3)$
100 km
70 km
180 km
a_1
a_2
a_3

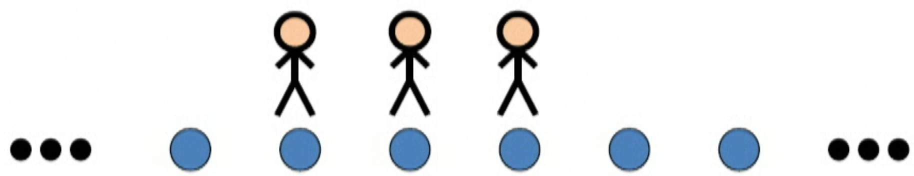

$P(a_1, a_2, a_3)$
a_2
100 km
70 km
180 km
a_1
a_3
The scientists asume that this property -translation invariance- holds everywhere.



They are tapping the sites of a TI infinite system

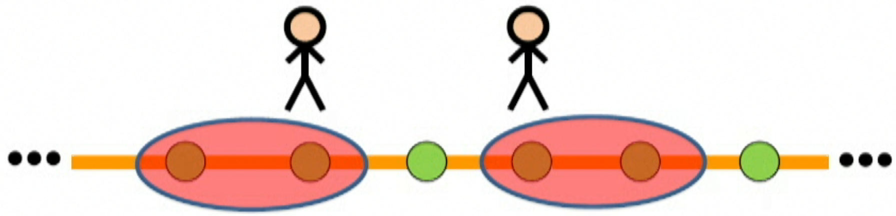


For simplicity, let us asume 1D



From this local information, what are the global properties of ω?

Is ω entangled?



Prior literature focused on bipartite entanglement

A. Osterloh, L. Amico, G. Falci and R. Fazio, Nature, **416** (6881), 608–610 (2002).

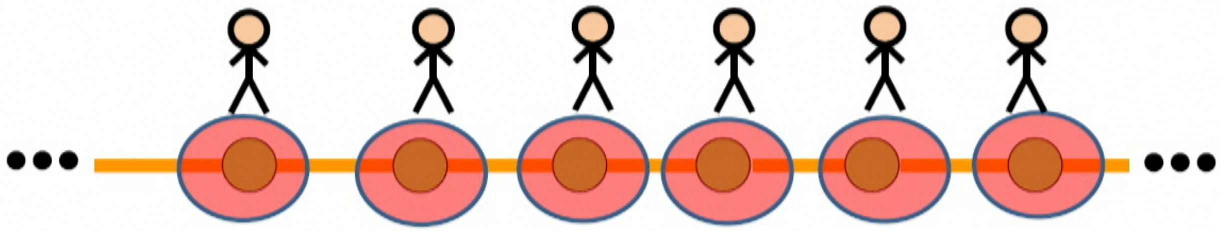
W. K. Wootters, Cont. Math., **305**, 299 (2002).

M. Wolf, F. Verstraete and J. I. Cirac, Int. J. Quant. Inf., **01** (04), 465–477 (2003).

J. Eisert, M. Cramer and M. B. Plenio, Rev. Mod. Phys., **82**, 277–306 (2010).

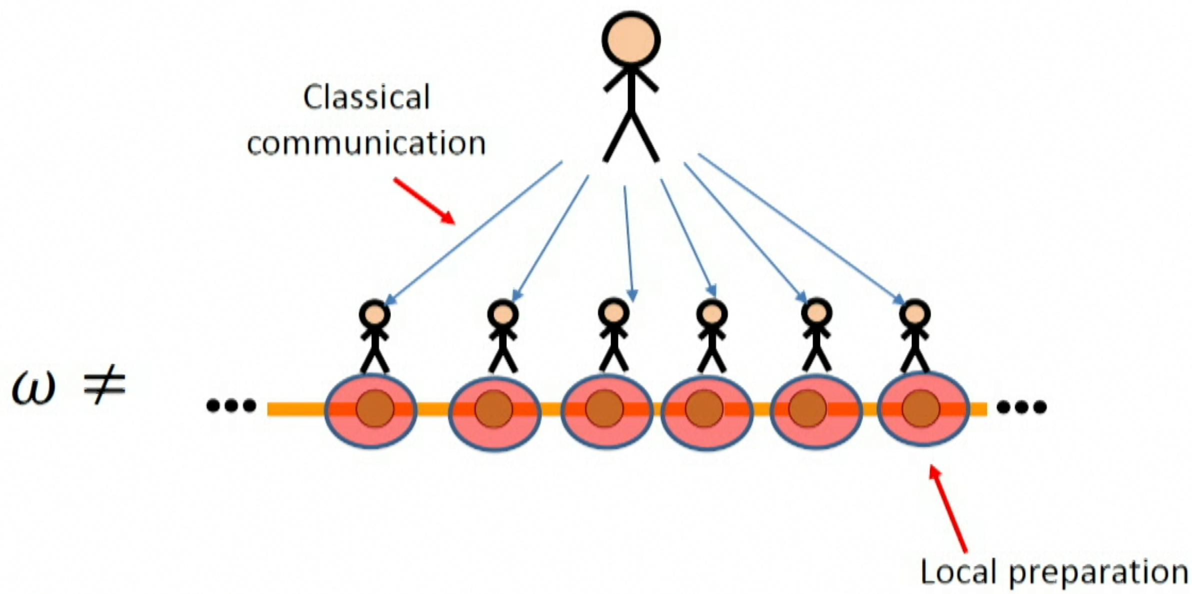
M. Wiesniak, V. Vedral and C. Brukner, New J. Phys. 7, 258 (2005).

Is ω entangled?

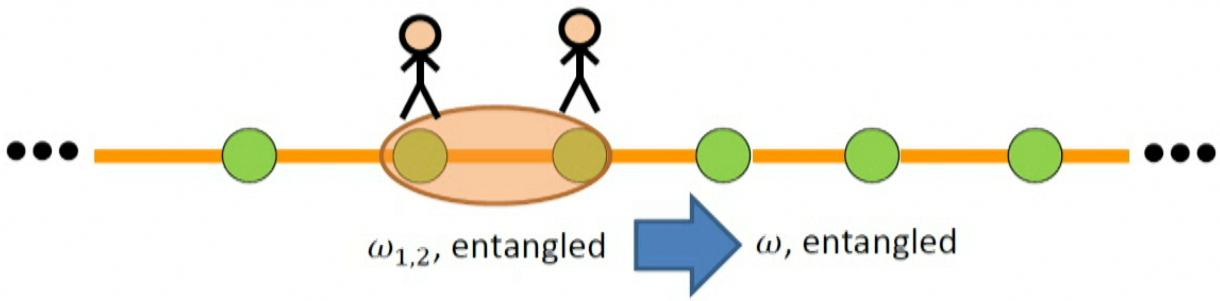


ω, entangled $\Leftrightarrow$ $\omega \neq \sum_i p_i \ldots \otimes \omega_{-1}^{(i)} \otimes \omega_0^{(i)} \otimes \omega_1^{(i)} \otimes \ldots$

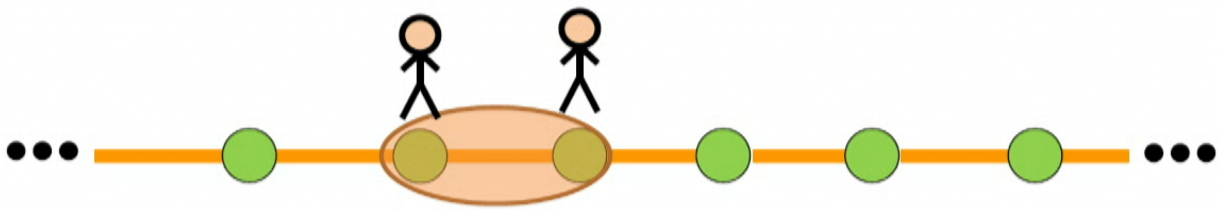
Is ω entangled?



Is ω entangled?

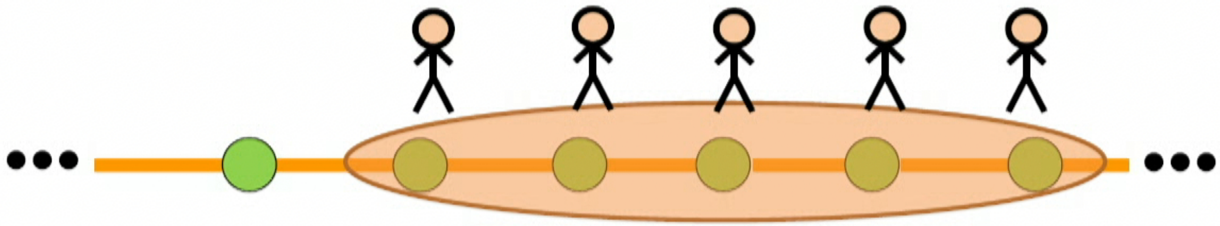


Is ω entangled?



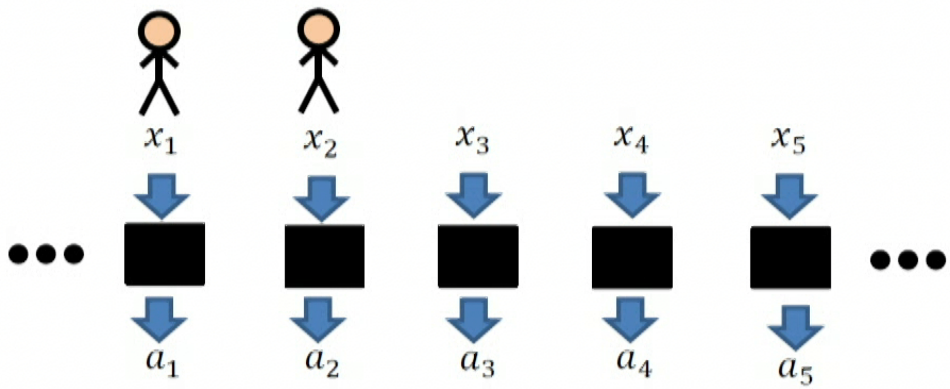
$\omega_{1,2}$, separable, but $\nexists\Omega$, separable and TI such that
$\Omega_{1,2} = \omega_{1,2}$

Is ω entangled?



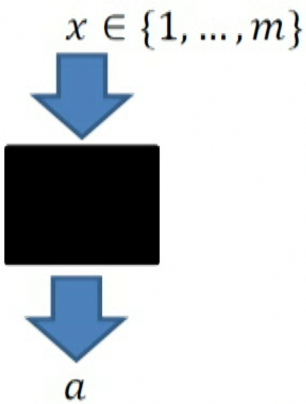
Is ω local?

(Can we simulate the statistics of the box with a classical device?)



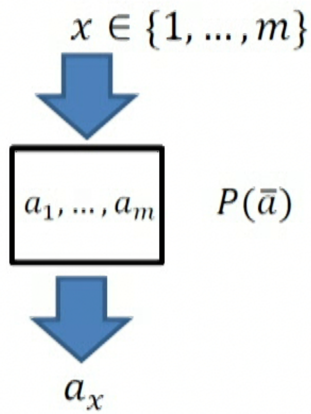
Is ω local?

A single-site black box



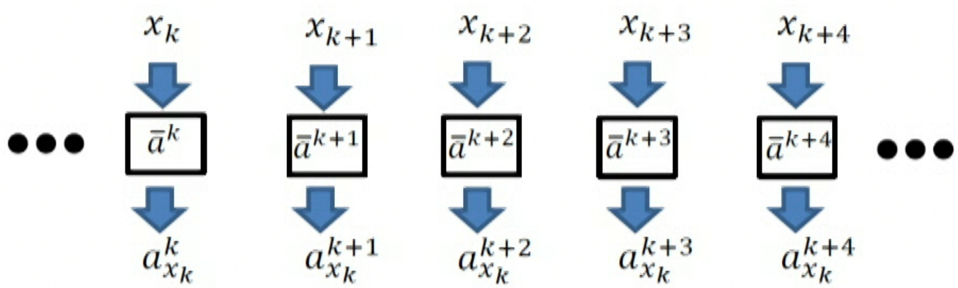
Is ω local?

A single-site *classical* black box



Is ω local?

A *classical* multipartite black box

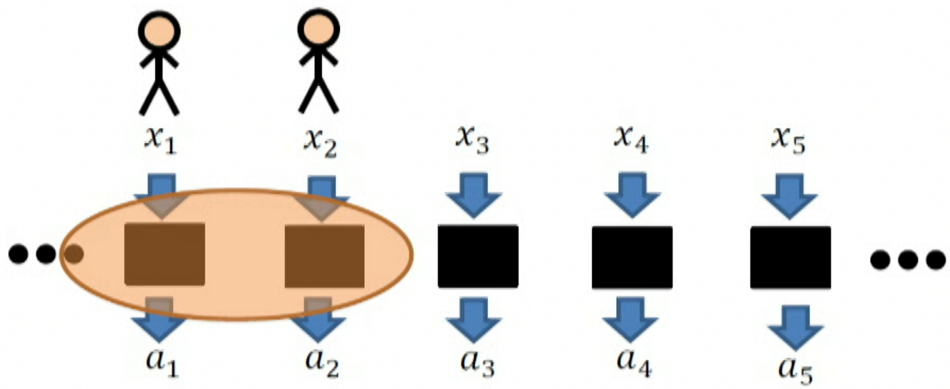


$$P(\bar{a}^{-\infty}, \dots, \bar{a}^{\infty})$$

(local hidden variable model)

Is ω local?

(Can we simulate the statistics of the box with a classical device?)

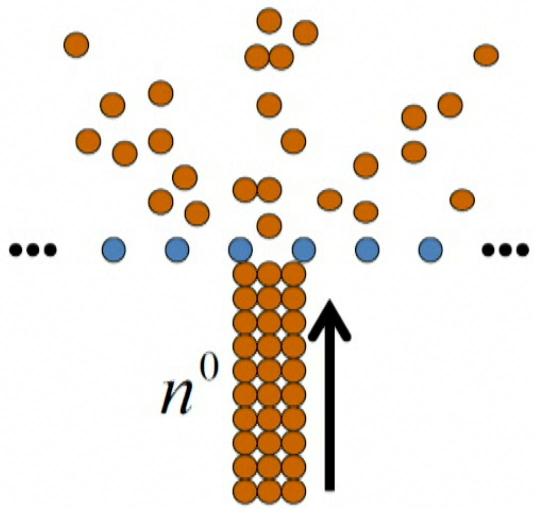


$Q(a_1, a_2|x_1, x_2)$, local? $\neq$ $\exists Q(\ldots, a_1, a_2 \ldots | \ldots, x_1, x_2, \ldots)$, local, TI such that $Q(a_1, a_2|x_1, x_2) = \omega_{1,2}$

Motivation (alternative)
Entanglement and nonlocality in condensed matter systems

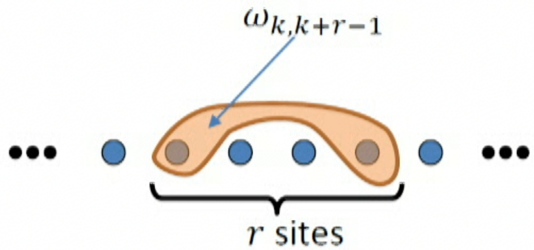
structure factors

$$\sum_k \langle S_a^k S_b^{k+r} \rangle \equiv \Theta_{ab}^r$$



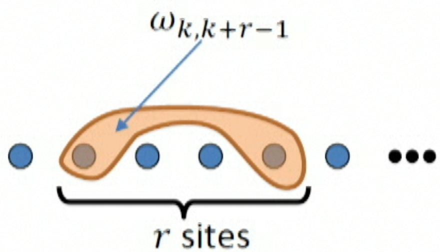
structure factors

$$\tilde{\omega}^{(N)}_{[r]} \equiv \frac{1}{N-r+1} \sum_{k=1}^{N-r+1} \omega_{k,k+r-1}$$



structure factors

$$\tilde{\omega}^{(N)}_{[r]} \equiv \frac{1}{N-r+1} \sum_{k=1}^{N-r+1} \omega_{k,k+r-1}$$

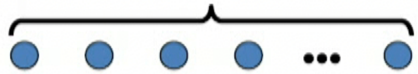


Is the chain entangled?

Is the chain non-local?

Finite system, no symmetries

$N \gg 1$ sites

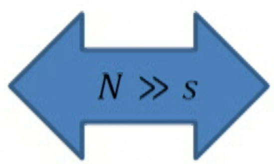


Entanglement? Nonlocality?

Entanglement witness/
Bell inequality for N-partite states

$$W\left(\left\{\omega_{[r]}^{(N)}\right\}_{r=2,\ldots,s}\right) \geq R$$

Maximally violated by ground states of local TI Hamiltonians



Infinite, TI system

Entanglement? Nonlocality?

Entanglement witness/
Bell inequality for TI states

$$W\left(\left\{\Omega_{1,r}\right\}_{r=2,\ldots,s}\right) \geq R$$

Characterizing entanglement/locality
in TI systems, but how?

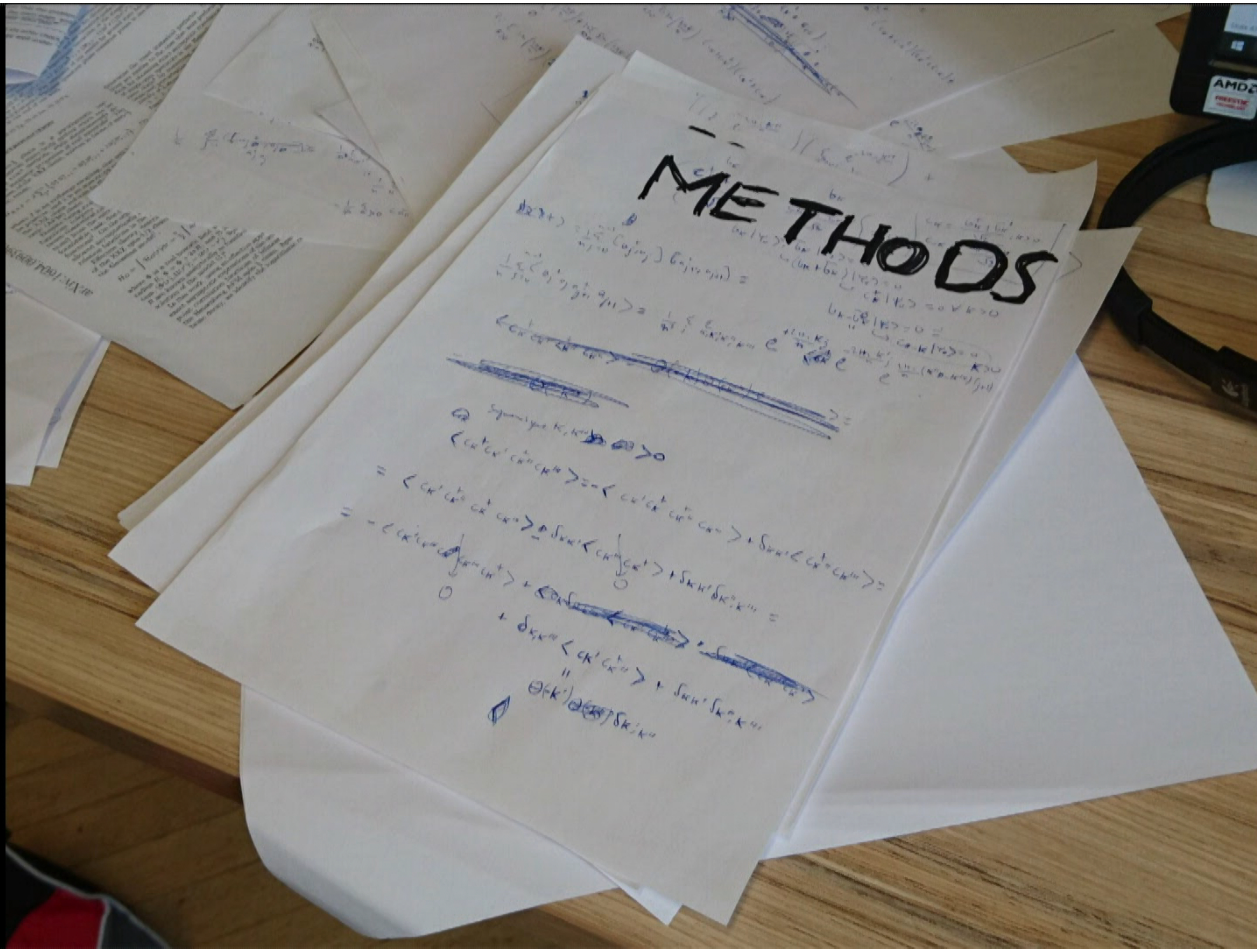
European Research Council

Established by the European Commission

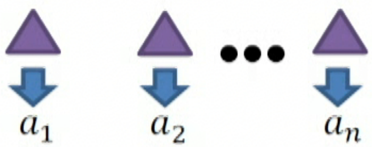
b. Methodology

Describe the proposed methodology in detail including, as appropriate, key intermediate goals. Explain and justify the methodology in relation to the state of the art, including any particularly novel or unconventional aspects addressing 'high-risk/high-gain' balance. Highlight any intermediate stages where results may require adjustments to the project planning. In case it is proposed that team members engaged by another host institution participate in the project, their participation has to be fully justified. This should be done emphasising the scientific added value they bring to the project.

ERC Starting and Consolidator grants--applicant's guide

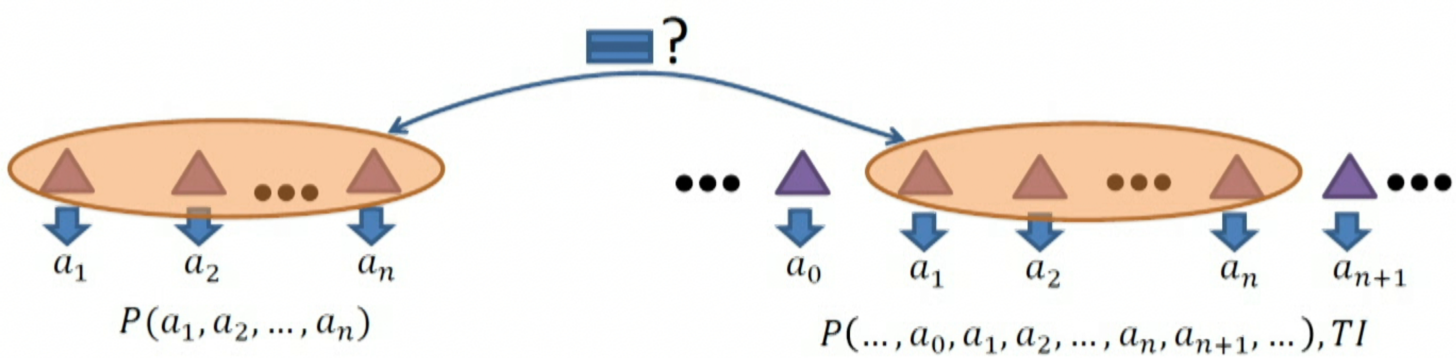

METHODS

Marginal problem for 1-D classical TI distributions



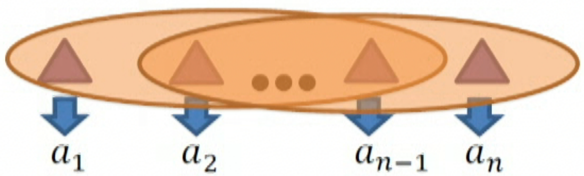
$P(a_1, a_2, \ldots, a_n)$

Marginal problem for 1-D classical TI distributions



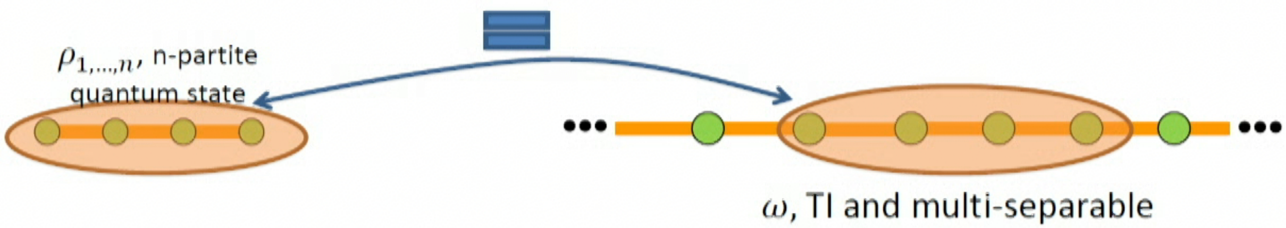
Marginal problem for 1-D classical TI distributions

Solution:



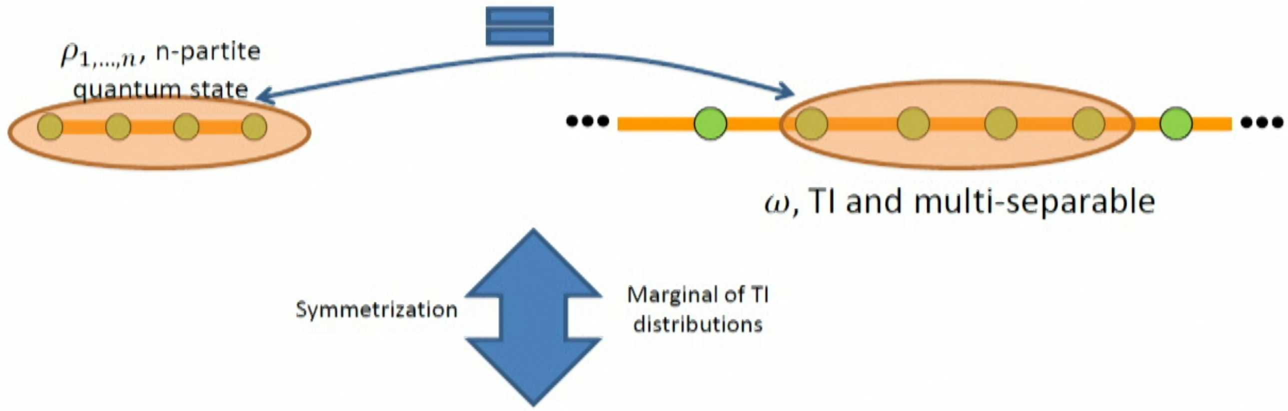
$$P_{1,\dots,n-1}(a_1, a_2, \dots, a_{n-1}) = P_{2,\dots,n}(a_1, a_2, \dots, a_{n-1})$$

Z. Wang, S. Singh and MN, arXiv:1608.03485.



$\rho_{1,\ldots,n}$ admits a Translation-Invariant Separable (TIS) extension

Characterization of states admitting a TIS extension



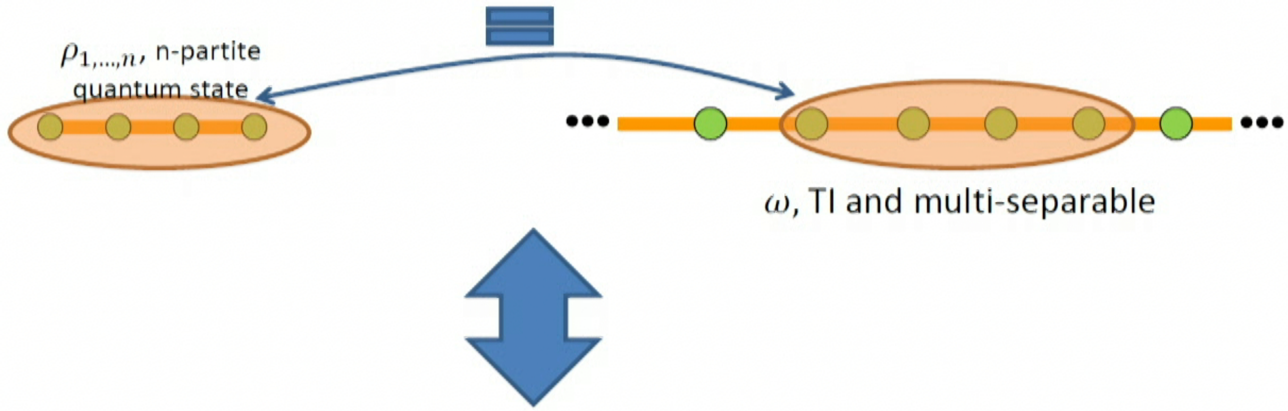
$$\rho_{1,\ldots,n} = \int d\vec{\psi} P(\psi_1,\ldots,\psi_n)|\psi_1\rangle\langle\psi_1| \otimes \ldots \otimes |\psi_n\rangle\langle\psi_n|,$$

$$P(\psi_1,\ldots,\psi_n) \geq 0,$$

$$P_{1,\ldots,n-1}(\psi_1,\ldots,\psi_{n-1}) = P_{2,\ldots,n}(\psi_1,\ldots,\psi_{n-1})$$

Z. Wang, S. Singh and MN, arXiv:1608.03485.

Characterization of states admitting a TIS extension



$$\rho_{1,\ldots,n} = \int d\vec{\psi} P(\psi_1,\ldots,\psi_n)|\psi_1\rangle\langle\psi_1| \otimes \ldots \otimes |\psi_n\rangle\langle\psi_n|,$$

$$P(\psi_1,\ldots,\psi_n) \geq 0,$$

→ Separability

$$P_{1,\ldots,n-1}(\psi_1,\ldots,\psi_{n-1}) = P_{2,\ldots,n}(\psi_1,\ldots,\psi_{n-1})$$

→ Existence of a TI extension of $P(\psi_1,\ldots,\psi_n)$

Z. Wang, S. Singh and MN, arXiv:1608.03485.

Characterization of states admitting a TIS extension

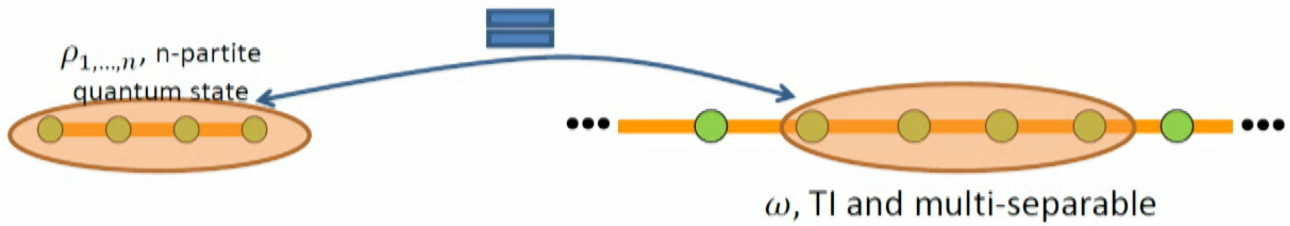
$$\rho_{1,\ldots,n} = \int d\vec{\psi} P(\psi_1, \ldots, \psi_n) |\psi_1\rangle\langle\psi_1| \otimes \ldots \otimes |\psi_n\rangle\langle\psi_n|,$$

$$P(\psi_1, \ldots, \psi_n) \geq 0,$$

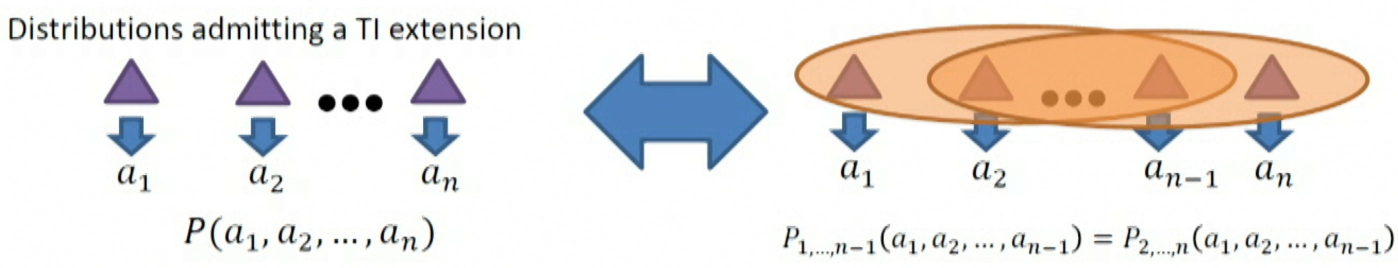
$$P_{1,\ldots,n-1}(\psi_1, \ldots, \psi_{n-1}) = P_{2,\ldots,n}(\psi_1, \ldots, \psi_{n-1})$$

This characterization is as useful for entanglement detection as the definition of LOCC for entanglement distillation :-P

Simpler characterization?



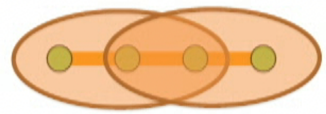
Simpler characterization?



Simpler characterization?

States admitting a TI extension

$\rho_{1,\ldots,n}$



$\rho_{1,\ldots,n-1} = \rho_{2,\ldots,n}$

$$\rho_{1,2} = |\phi\rangle\langle\phi|,$$

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|0,0\rangle + |1,1\rangle)$$

$\rho_1 = \rho_2 = \frac{\mathbb{I}}{2}$

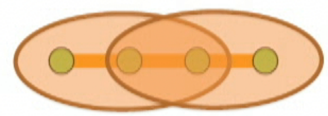
$\rho_{1,2}$, not extendible

Z. Wang, S. Singh and MN, arXiv:1608.03485.

Simpler characterization?

States admitting a TIS extension

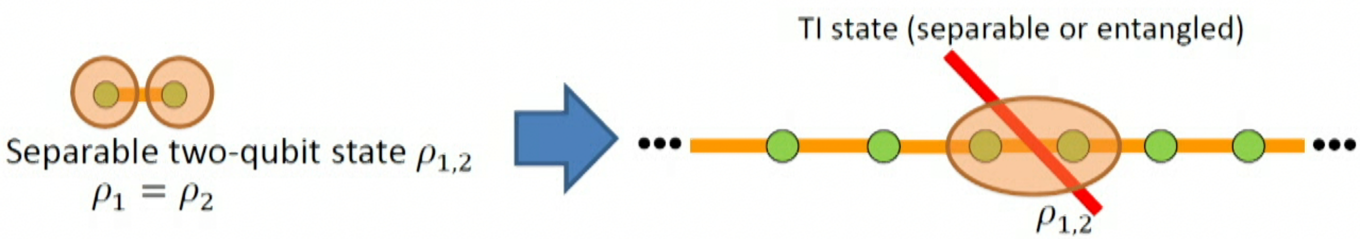
$\rho_{1,\ldots,n}$



$\rho_{1,\ldots,n-1} = \rho_{2,\ldots,n}$

$\rho_{1,\ldots,n}$, separable

Simpler characterization?

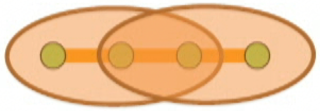


Z. Wang, S. Singh and MN, arXiv:1608.03485.

Simpler characterization?

States admitting a TIS extension

$\rho_{1,\ldots,n}$

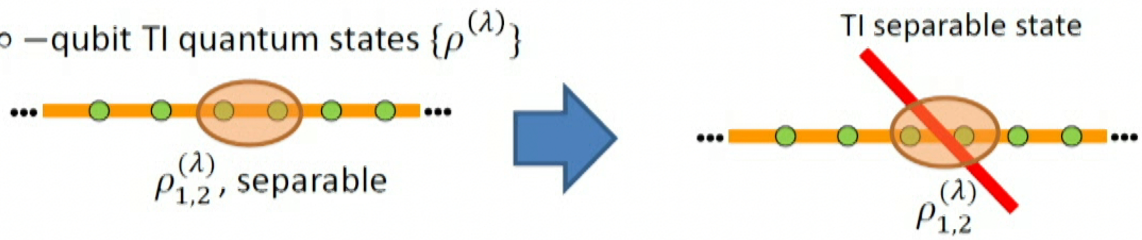


$\rho_{1,\ldots,n-1} = \rho_{2,\ldots,n}$

$\rho_{1,\ldots,n}$, separable, admits a TI extension

Simpler characterization?

∞ −qubit TI quantum states $\{\rho^{(\lambda)}\}$



Z. Wang, S. Singh and MN, arXiv:1608.03485.

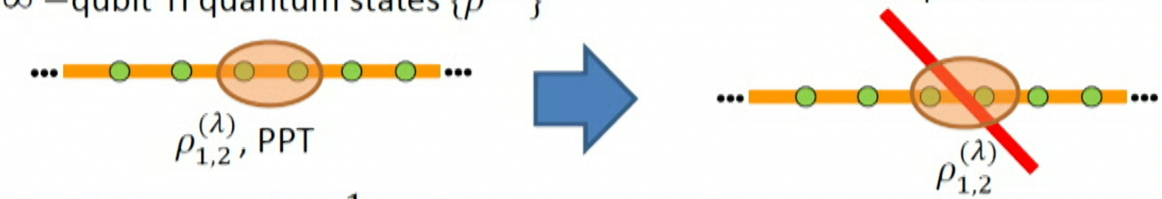
Simpler characterization?

Method?

Jordan-Wigner transformation

∞ −qubit TI quantum states $\{\rho^{(\lambda)}\}$

TI separable state



$$tr\left(\rho_{1,2}^{(\lambda)}\sigma_x \otimes \sigma_y\right) = \frac{1}{2} + O(\lambda)$$

$\rho_{1,2}$ admits a TIS extension $\Rightarrow$ $tr(\rho_{1,2}\sigma_x \otimes \sigma_y) \leq \frac{1}{2}$

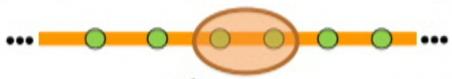
Z. Wang, S. Singh and MN, arXiv:1608.03485.

Simpler characterization?

Method?

Jordan-Wigner transformation

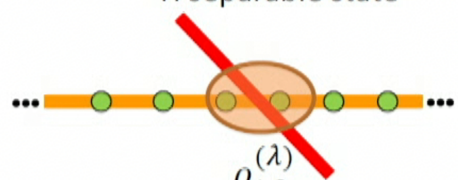
∞ −qubit TI quantum states $\{\rho^{(\lambda)}\}$



$\rho_{1,2}^{(\lambda)}$, PPT

$$tr\left(\rho_{1,2}^{(\lambda)}\sigma_x\otimes\sigma_y\right)=\frac{1}{2}+O(\lambda)$$

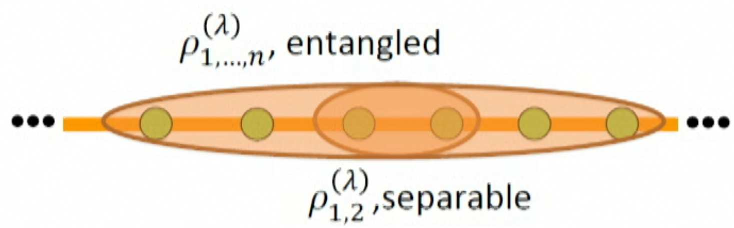
TI separable state



$\rho_{1,2}^{(\lambda)}$

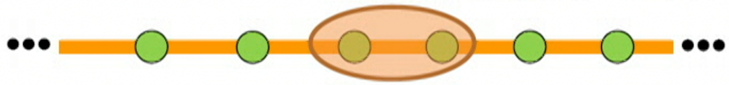
$\rho_{1,2}$ admits a TIS extension ⇒ $tr(\rho_{1,2}\sigma_x\otimes\sigma_y)\leq\frac{1}{2}$

Z. Wang, S. Singh and MN, arXiv:1608.03485.



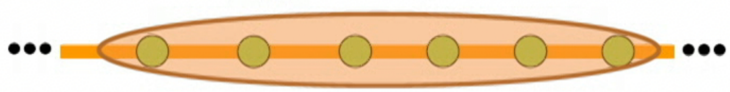
How large can n be?

$\rho_{1,2}$, $tr(\rho_{1,2}W) = S + \Delta$



$$n = \left\lfloor \frac{S - tr(\rho_1^{\otimes 2}W)}{\Delta} \right\rfloor + 2$$

$\rho_{1,\dots,n}$, entangled



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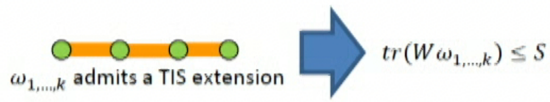
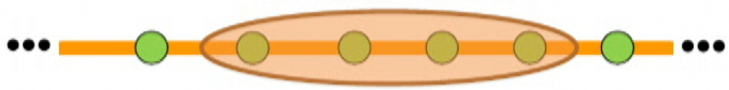
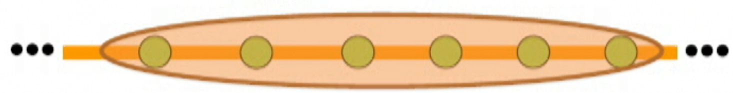


$\rho_{1,\dots,k}$, $tr(\rho_{1,\dots,k}W) = S + \Delta$



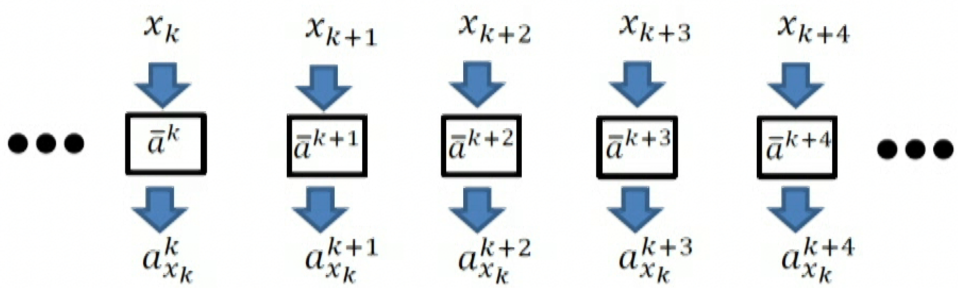
$$n = O\left(\frac{1}{\Delta}\right)$$

$\rho_{1,\dots,n}$, entangled

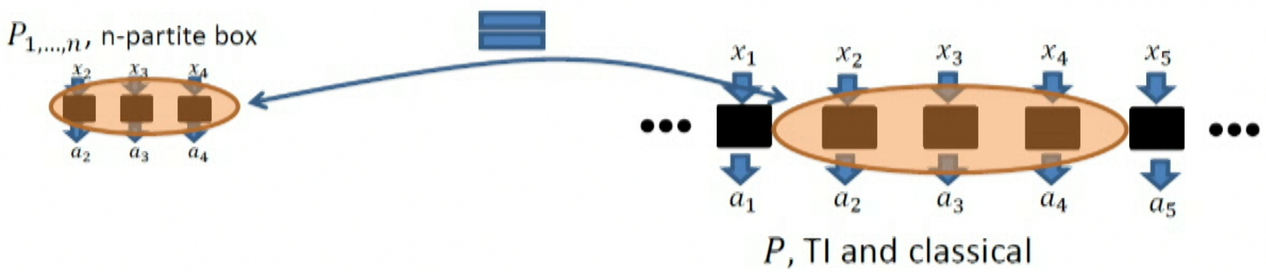


Z. Wang, S. Singh and MN, arXiv:1608.03485.

A *classical* TI black box



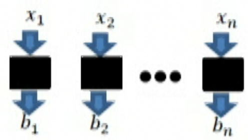
$P(\bar{a}^{-\infty}, \ldots, \bar{a}^{\infty})$, TI

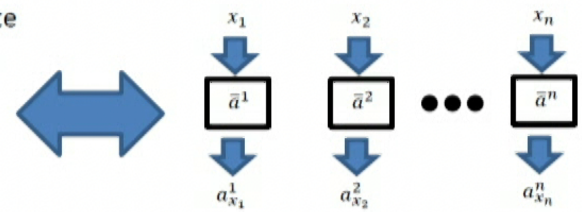


$P_{1,\ldots,n}$ admits a Translation-Invariant Local (TIL) extension

Characterization of boxes admitting a TIL extension

$P(b_1, \dots, b_n | x_1, \dots, x_n)$, n-partite box admitting a TIL extension



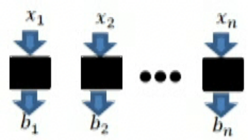


$$P(b_1, \dots, b_n | x_1, \dots, x_n) = P(a^1_{x_1} = b_1, \dots, a^n_{x_n} = b_n)$$

$P(\bar{a}^1, \dots, \bar{a}^n)$, admits a TI extension

Characterization of boxes admitting a TIL extension

$P(b_1, \dots, b_n | x_1, \dots, x_n)$, n-partite box admitting a TIL extension



$$P(b_1, \dots, b_n | x_1, \dots, x_n) = P\left(a^1_{x_1} = b_1, \dots, a^n_{x_n} = b_n\right),$$

s.t.

$$P_{1,\dots,n}(\bar{a}_1, \dots, \bar{a}_n) \geq 0, \sum_{\bar{a}_1,\dots,\bar{a}_n} P_{1,\dots,n}(\bar{a}_1, \dots, \bar{a}_n) = 1$$

$$P_{1,\dots,n-1}(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_{n-1}) = P_{2,\dots,n}(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_{n-1})$$

The set of local TI boxes can be characterized via linear programming!!

Z. Wang, S. Singh and MN, arXiv:1608.03485.

Linear programming

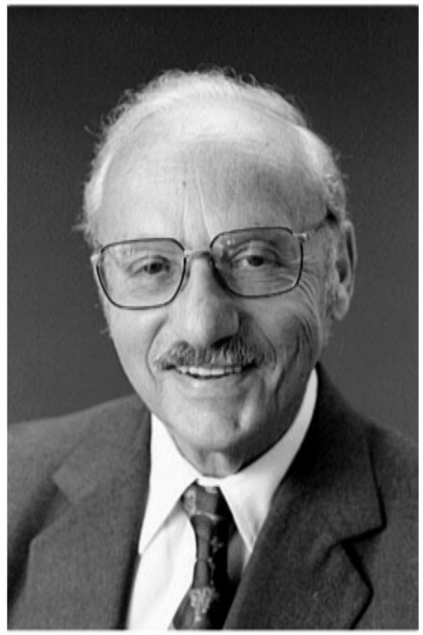
$\vec{c}, \vec{v}_0, \{\vec{v}_i : i\}$ ⇒

$$p^* = \min \vec{c} \cdot \vec{x}$$

$$\vec{v}_0 + \sum_i x_i \vec{v}_i \geq 0$$

Evar D. Nering and Albert W. Tucker, 1993, Linear Programs and Related Problems, Academic Press.

Simplex method



George Dantzig

1947

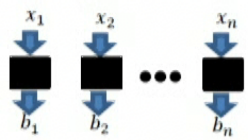

FINANCIAL CRASH DETECTOR
Early warning system wards off disaster
NewScientist
THE ALGORITHM THAT RUNS THE WORLD
HOT ON THE MARTIAN TRAIL
Curiosity begins its epic search for signs of life
ME, MYSELF AND I
AUGMENTED EYEBALL

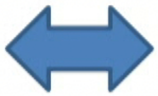
"Probably tens or hundreds of thousands of calls of the simplex method are made every minute“.

Jacek Gondzio, University of Edinborough

Characterization of boxes admitting a TIL extension

$P(b_1, \ldots, b_n | x_1, \ldots, x_n)$, n-partite box admitting a TIL extension





$$P(b_1, \ldots, b_n | x_1, \ldots, x_n) = P\left(a^1_{x_1} = b_1, \ldots, a^n_{x_n} = b_n\right),$$

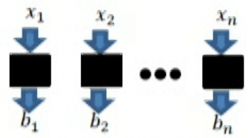
s.t.

$$P_{1,\ldots,n}(\bar{a}_1, \ldots, \bar{a}_n) \geq 0, \sum_{\bar{a}_1,\ldots,\bar{a}_n} P_{1,\ldots,n}(\bar{a}_1, \ldots, \bar{a}_n) = 1$$

$$P_{1,\ldots,n-1}(\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_{n-1}) = P_{2,\ldots,n}(\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_{n-1})$$

Characterization of boxes admitting a TIL extension

$P(b_1, \dots, b_n | x_1, \dots, x_n)$, n-partite box admitting a TIL extension



$\Leftrightarrow$

$$\left[\min 0\right]$$

$$P(b_1, \dots, b_n | x_1, \dots, x_n) = P\left(a^1_{x_1} = b_1, \dots, a^n_{x_n} = b_n\right),$$

$$\text{s.t.} \left| \begin{array}{l} P_{1,\dots,n}(\bar{a}_1, \dots, \bar{a}_n) \geq 0, \sum_{\bar{a}_1,\dots,\bar{a}_n} P_{1,\dots,n}(\bar{a}_1, \dots, \bar{a}_n) = 1 \\ P_{1,\dots,n-1}(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_{n-1}) = P_{2,\dots,n}(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_{n-1}) \end{array} \right.$$

General linear program

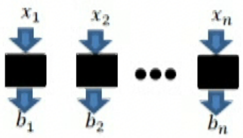
$$p^* = \min \vec{c} \cdot \vec{x}$$

$$\vec{v}_0 + \sum_i x_i \vec{v}_i \geq 0$$

Computation of minimum values of linear functionals

$$\min \sum_{b_1,\ldots,b_n,x_1,\ldots,x_n} c_{b_1,\ldots,b_n,x_1,\ldots,x_n} P(b_1,\ldots,b_n|x_1,\ldots,x_n)$$

$P(b_1,\ldots,b_n|x_1,\ldots,x_n)$, n-partite box admitting a TIL extension



↔

$$\min \sum_{b_1,\ldots,b_n,x_1,\ldots,x_n} c_{b_1,\ldots,b_n,x_1,\ldots,x_n} P\left(a^1_{x_1}=b_1,\ldots,a^n_{x_n}=b_n\right)$$

s.t. $P_{1,\ldots,n}(\bar{a}_1,\ldots,\bar{a}_n) \geq 0, \sum_{\bar{a}_1,\ldots,\bar{a}_n} P_{1,\ldots,n}(\bar{a}_1,\ldots,\bar{a}_n) = 1$

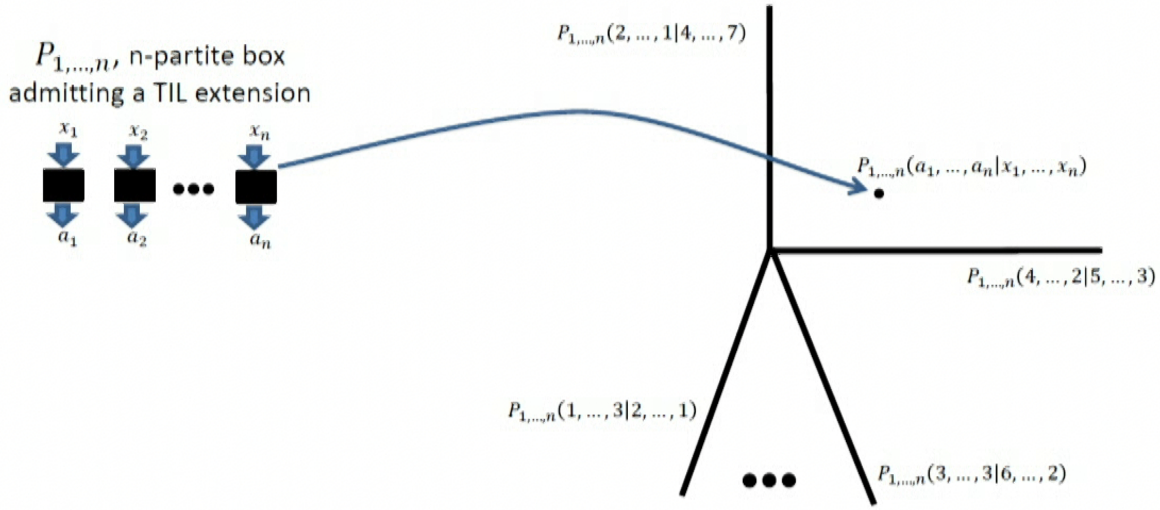
$P_{1,\ldots,n-1}(\bar{a}_1,\bar{a}_2,\ldots,\bar{a}_{n-1}) = P_{2,\ldots,n}(\bar{a}_1,\bar{a}_2,\ldots,\bar{a}_{n-1})$

General linear program

$$p^* = \min \vec{c}\cdot\vec{x}$$

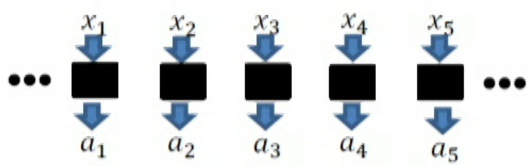
$$\vec{v}_0 + \sum_i x_i\,\vec{v}_i \geq 0$$

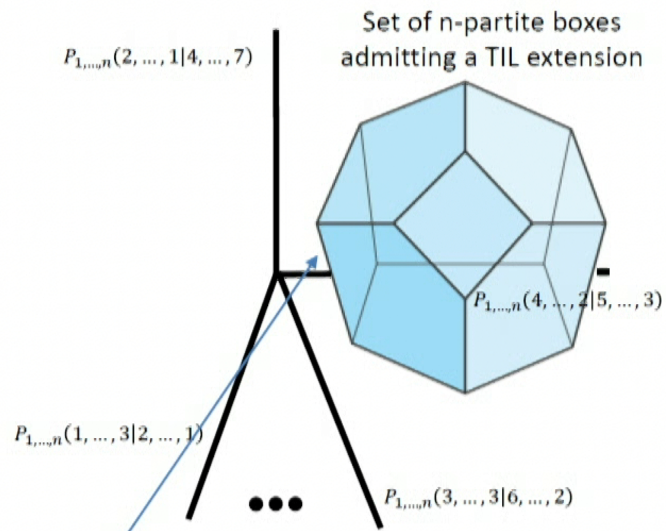
Characterization of boxes admitting a TIL extension



Characterization of boxes admitting a TIL extension

1-D TIL extension



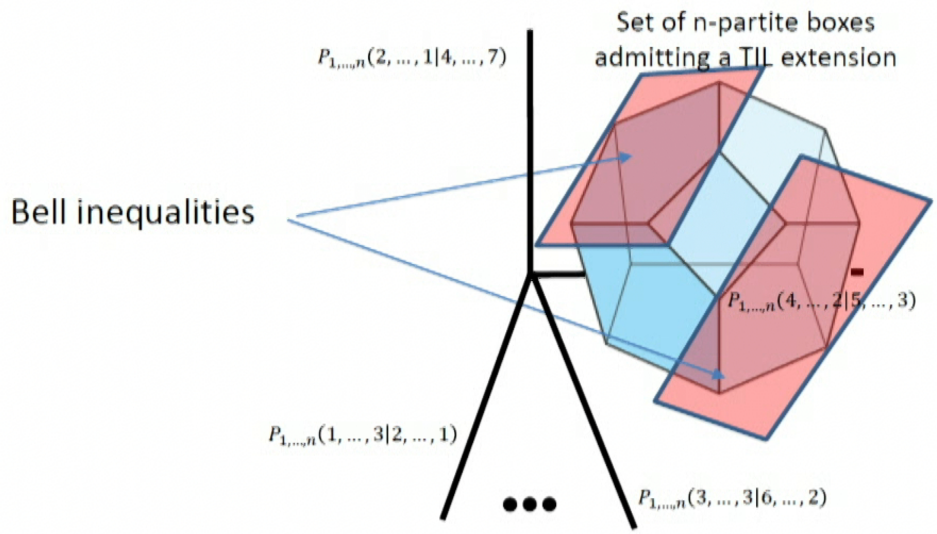


Convex polytope

Convex set defined by a finite number of linear inequalities

Z. Wang, S. Singh and MN, arXiv:1608.03485.

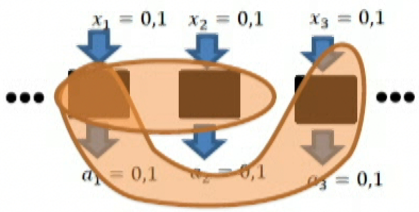
TI Bell inequalities



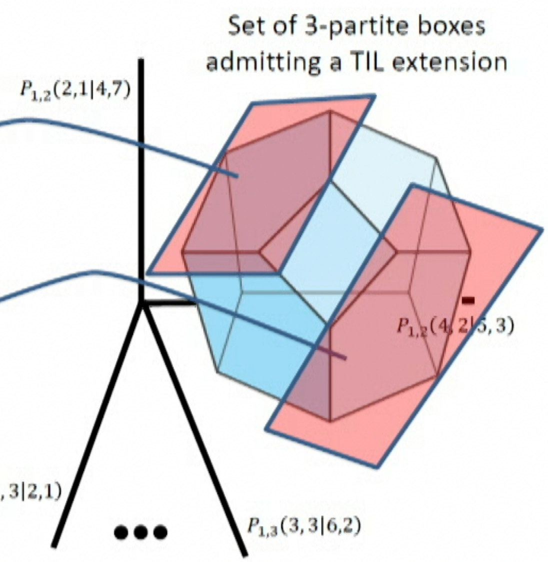
$$E_x \equiv \sum_{a=0,1} P_1(a|x)(-1)^a$$
$$E^{i,j}_{xy} \equiv \sum_{a,b=0,1} P_{i,j}(a,b|x,y)(-1)^a(-1)^b$$

Lörwald Stefan, and Gerhard Reinelt (2015), "PANDA: a software for polyhedral transformations," EURO Journal on Computational Optimization, 1–12.

TI Bell inequalities



$P_{1,2}(a,b|x,y)$, $P_{1,3}(a,b|x,y)$
Structure factors



$$I_{\mathrm{T}} \equiv -2E_0 - 4E_1 - 2E_{00}^{1,2} + 2E_{01}^{1,2} + 2E_{10}^{1,2} + 2E_{11}^{1,2} + E_{00}^{1,3} + E_{11}^{1,3} \geq -4,$$

$$I_{\mathrm{G}} \equiv -4E_0 - 6E_1 - 3E_{00}^{1,2} + 2E_{01}^{1,2} + 3E_{10}^{1,2} + 2E_{11}^{1,2} + 2E_{00}^{1,3} + E_{10}^{1,3} + E_{11}^{1,3} \geq -6,$$

$$E_x \equiv \sum_{a=0,1} P_1(a|x)(-1)^a$$
$$E_{xy}^{i,j} \equiv \sum_{a,b=0,1} P_{i,j}(a,b|x,y)(-1)^a(-1)^b$$

Lörwald Stefan, and Gerhard Reinelt (2015), "PANDA: a software for polyhedral transformations," EURO Journal on Computational Optimization, 1–12.

TI Bell inequalities

$$I_{\mathrm{T}} \equiv -2E_0 - 4E_1 - 2E_{00}^{1,2} + 2E_{01}^{1,2} + 2E_{10}^{1,2} + 2E_{11}^{1,2} + E_{00}^{1,3} + E_{11}^{1,3} \geq -4,$$

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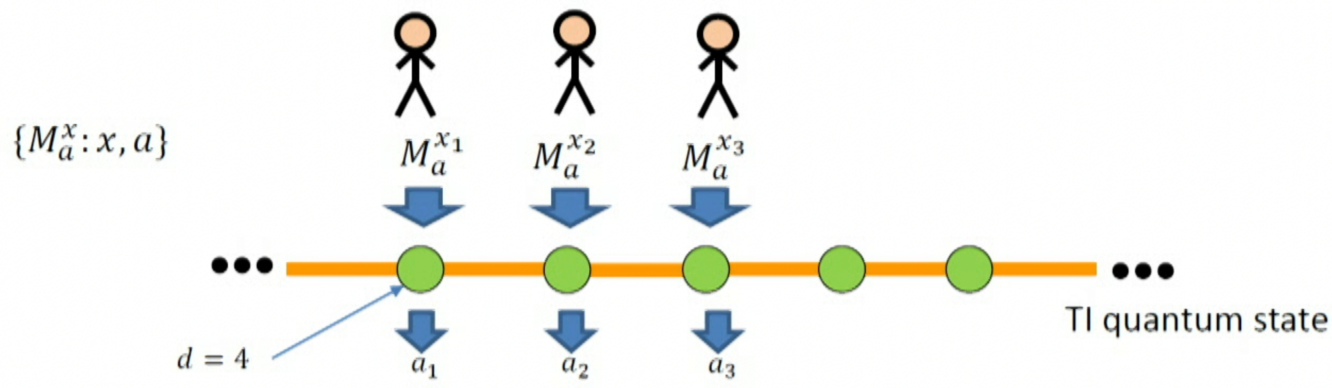
Can they be violated by TI quantum systems?

$$E_x \equiv \sum_{a=0,1} P_1(a|x)(-1)^a$$

$$E_{xy}^{i,j} \equiv \sum_{a,b=0,1} P_{i,j}(a,b|x,y)(-1)^a(-1)^b$$

Optimization over quantum TI boxes



3-local, 2-body operator

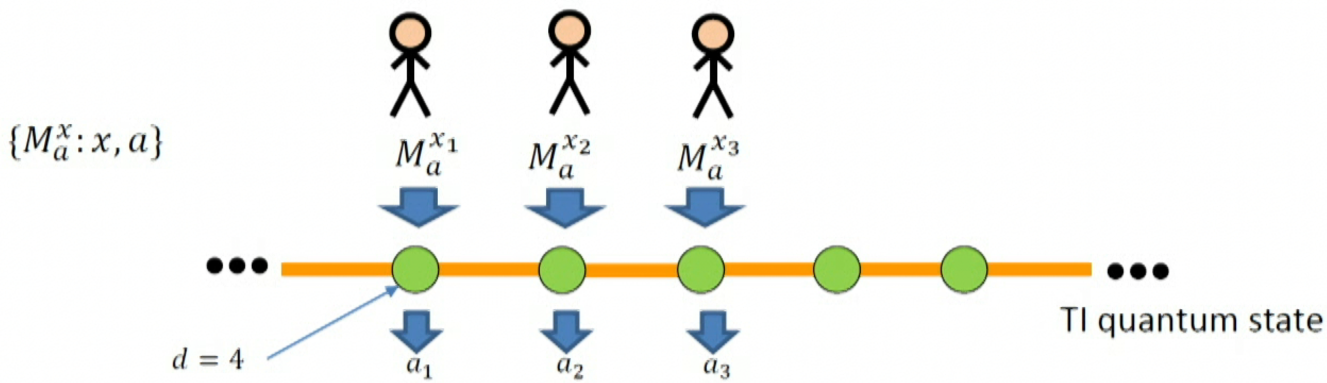
Bell functional

$$C_0E_0 + C_1E_1 + \sum_{x,y=0,1} C_{xy}^{1,2}E_{xy}^{1,2} + C_{xy}^{1,3}E_{xy}^{1,3} \Rightarrow h(\theta,\varphi) = C_0A_0^1 + C_1A_1^1 + \sum_{x,y=0,1} C_{xy}^{1,2}A_x^1 \otimes A_y^2 + C_{xy}^{1,3}A_x^1 \otimes A_y^3$$

$$M_0^0 - M_1^0 = \quad A_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

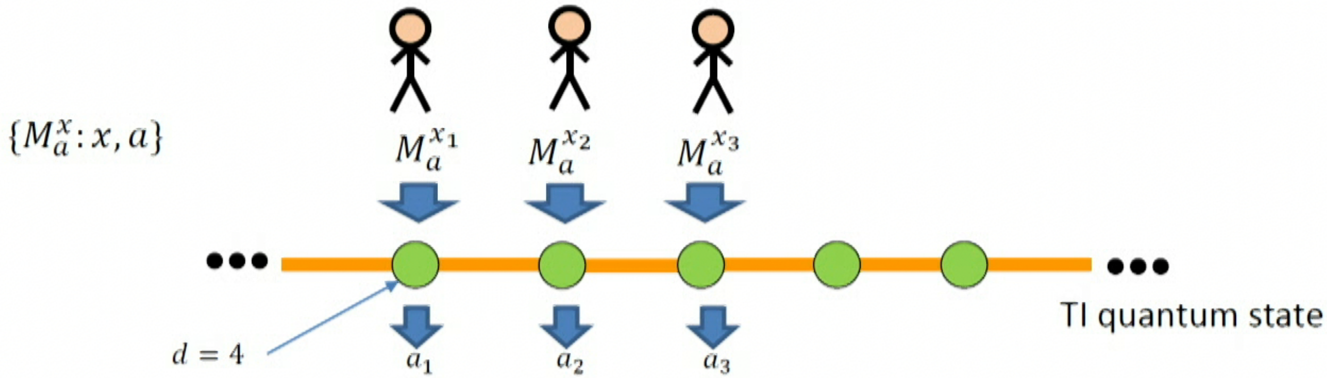
$$M_0^1 - M_1^1 = \quad A_1 = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 \\ \sin(\theta) & -\cos(\theta) & 0 & 0 \\ 0 & 0 & \cos(\phi) & \sin(\phi) \\ 0 & 0 & \sin(\phi) & -\cos(\phi) \end{pmatrix}$$

Optimization over quantum TI boxes



$$C_0 E_0 + C_1 E_1 + \sum_{x,y=0,1} C_{xy}^{1,2} E_{xy}^{1,2} + C_{xy}^{1,3} E_{xy}^{1,3} = \min_{\rho, TI} \langle h(\theta, \varphi) \rangle_\rho,$$

Optimization over quantum TI boxes

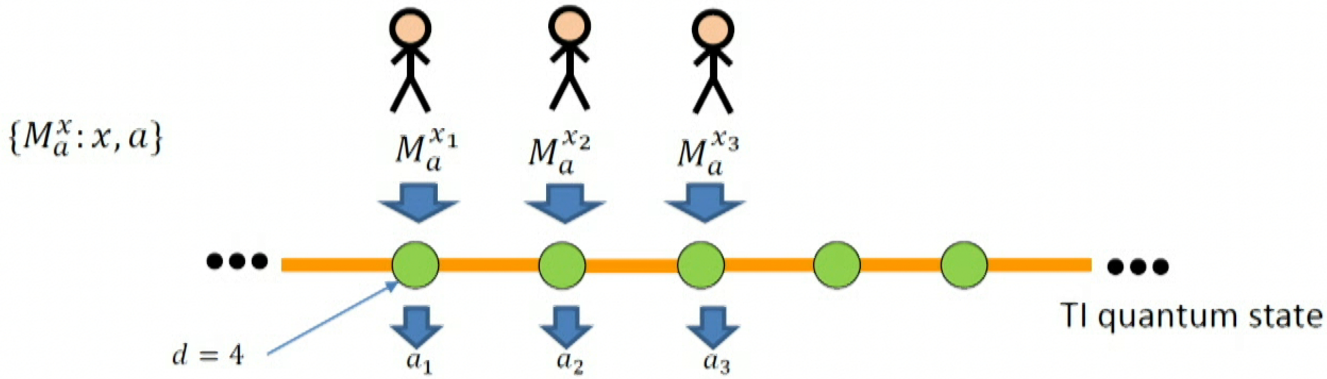


$$C_0 E_0 + C_1 E_1 + \sum_{x,y=0,1} C_{xy}^{1,2} E_{xy}^{1,2} + C_{xy}^{1,3} E_{xy}^{1,3} =$$

ground state energy per site of the 3-local 2-body TI Hamiltonian

$$H(\theta, \varphi) = \sum_i h_i(\theta, \varphi)$$

Optimization over quantum TI boxes

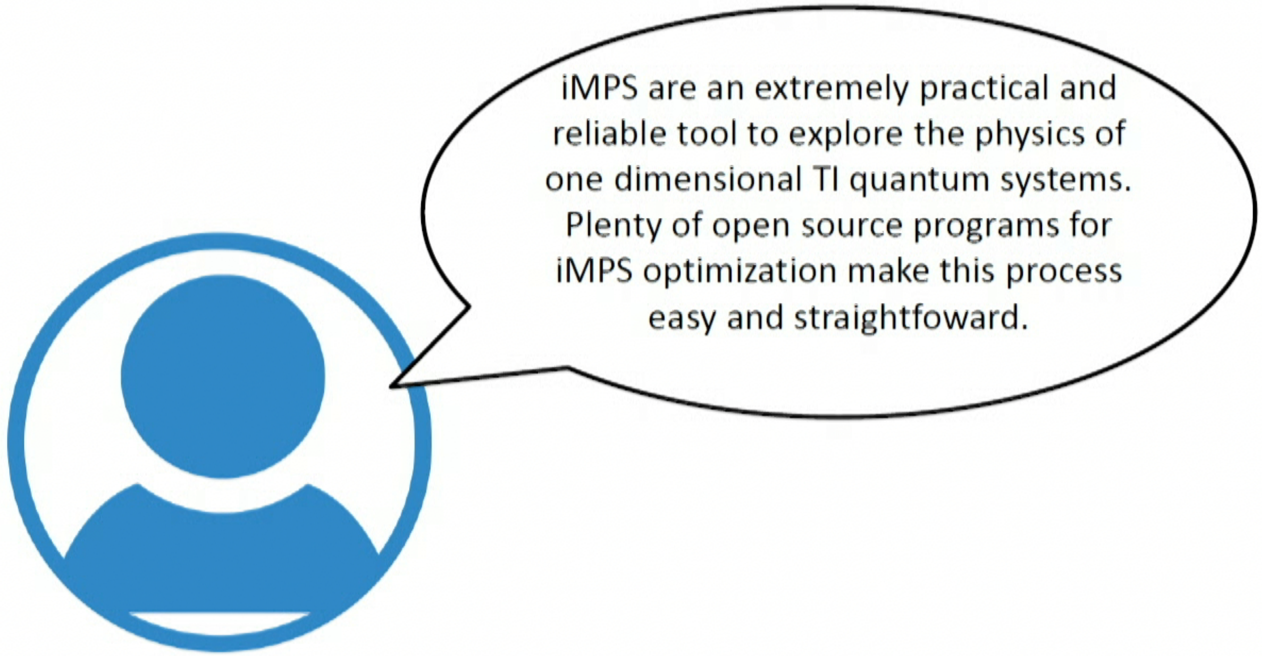


A problem suitable for iMPS!!!

$$\rho_{1,\dots,k} = \sum_{i_1,\dots,i_k} tr(A_{j_k}^\dagger \dots A_{j_1}^\dagger \sigma A_{i_1} \dots A_{i_k}) |i_1, \dots, i_k\rangle\langle j_1, \dots, j_k|$$

$$\sigma \geq 0, tr(\sigma) = 1, \sum_i A_i^\dagger \sigma A_i = \sigma, \sum_i A_i A_i^\dagger = 1$$

D. Perez-Garcia, F. Verstraete, M.M. Wolf and J.I. Cirac, Quantum Inf. Comput. 7, 401 (2007).



Generic condensed matter guy
with a quantum information background

Four days later…

Most open packages for MPS optimization do not allow the user to optimize over iMPS

Finite chain, closed boundary conditions

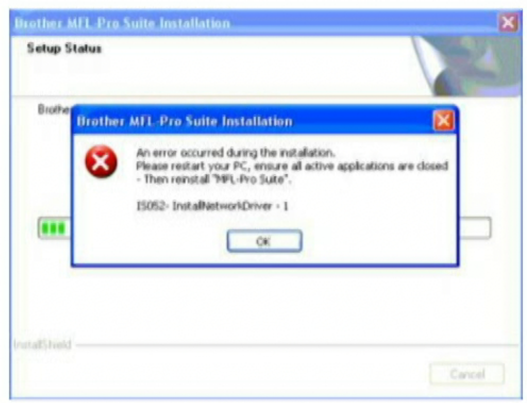
Finite chain, open boundary conditions

Most open sofware only allows the user to optimize a class of 2-local Hamiltonians

$$H = \sum_k J_x \sigma_x^k \sigma_x^{k+1} + J_y \sigma_y^k \sigma_y^{k+1} + J_z \sigma_z^k \sigma_z^{k+1}$$

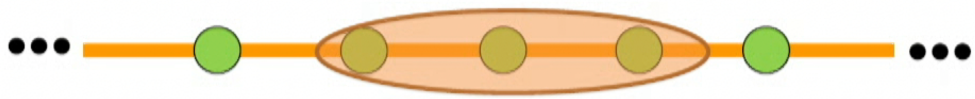
In order to operate, most open sofware for iMPS requires the user to install hundreds of packages for hours...

...and the last package always fails to install (in Windows)...



[A,E0]=minimizeHamiltonian(H,d)

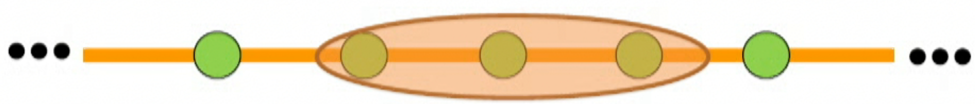
$$A_1, \dots, A_d \Rightarrow \rho_{1,2,3} = \sum_{i_1,\dots,i_3} tr(A^\dagger_{j_3} \dots A^\dagger_{j_1} \sigma A_{i_1} \dots A_{i_3}) |i_1, \dots, i_3\rangle\langle j_1, \dots, j_3|$$



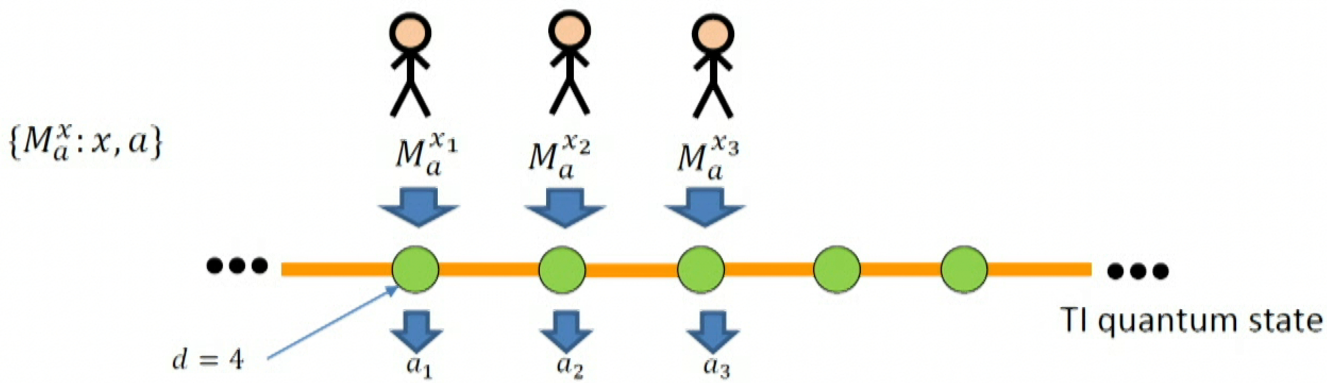
No single open package for iMPS optimization returns the tensor of the iMPS

[A,E0]=minimizeHamiltonian(H,d)

$$A_1, \dots, A_d \Rightarrow \rho_{1,2,3} = \sum_{i_1,\dots,i_3} tr(A^{\dagger}_{j_3} \dots A^{\dagger}_{j_1} \sigma A_{i_1} \dots A_{i_3}) |i_1, \dots, i_3\rangle\langle j_1, \dots, j_3|$$



Optimization over quantum TI boxes



$$\min\{\langle h(\theta,\varphi)\rangle_\rho : \rho, iMPS\}$$

Our tools
- Open Source MPS
- Our own TEBD implementation
  - Returns iMPS tensor
  - very slow (~6 hours/optimization)

Wall Michael L, and Lincoln D Carr, New Journal of Physics, 14 (12), 125015 (2012).
G. Vidal (2007), Phys. Rev. Lett., 98, 070201.

$$I_{\mathrm{T}} \equiv -2E_0 - 4E_1 - 2E_{00}^{1,2} + 2E_{01}^{1,2} + 2E_{10}^{1,2} + 2E_{11}^{1,2} + E_{00}^{1,3} + E_{11}^{1,3} \geq -4,$$

$$I_{\mathrm{G}} \equiv -4E_0 - 6E_1 - 3E_{00}^{1,2} + 2E_{01}^{1,2} + 3E_{10}^{1,2} + 2E_{11}^{1,2} + 2E_{00}^{1,3} + E_{10}^{1,3} + E_{11}^{1,3} \geq -6,$$

$$I_T \approx -4.1847$$

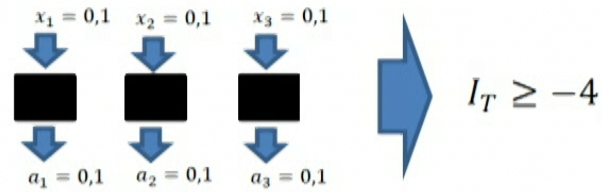
$$I_G \approx -6.1798$$

Z. Wang, S. Singh and MN, arXiv:1608.03485.

$$I_{\mathrm{T}} \equiv -2E_0 - 4E_1 - 2E_{00}^{1,2} + 2E_{01}^{1,2} + 2E_{10}^{1,2} + 2E_{11}^{1,2} + E_{00}^{1,3} + E_{11}^{1,3} \geq -4,$$

$$I_{\mathrm{G}} \equiv -4E_0 - 6E_1 - 3E_{00}^{1,2} + 2E_{01}^{1,2} + 3E_{10}^{1,2} + 2E_{11}^{1,2} + 2E_{00}^{1,3} + E_{10}^{1,3} + E_{11}^{1,3} \geq -6,$$

$I_T \approx -4.1847$



$P_{1,2,3}(a,b,c|\,x,y,z)$, tripartite local,

$P_{1,2}(a,b|\,x,y) = P_{2,3}(a,b|\,x,y)$

$I_G \approx -6.1798$

Z. Wang, S. Singh and MN, arXiv:1608.03485.

$$I_{\mathrm{T}} \equiv -2E_0 - 4E_1 - 2E_{00}^{1,2} + 2E_{01}^{1,2} + 2E_{10}^{1,2} + 2E_{11}^{1,2} + E_{00}^{1,3} + E_{11}^{1,3} \geq -4,$$

$$I_{\mathrm{G}} \equiv -4E_0 - 6E_1 - 3E_{00}^{1,2} + 2E_{01}^{1,2} + 3E_{10}^{1,2} + 2E_{11}^{1,2} + 2E_{00}^{1,3} + E_{10}^{1,3} + E_{11}^{1,3} \geq -6,$$

$I_T \approx -4.1847$ ⇒ Detects standard tripartite nonlocality, tells us nothing about TI

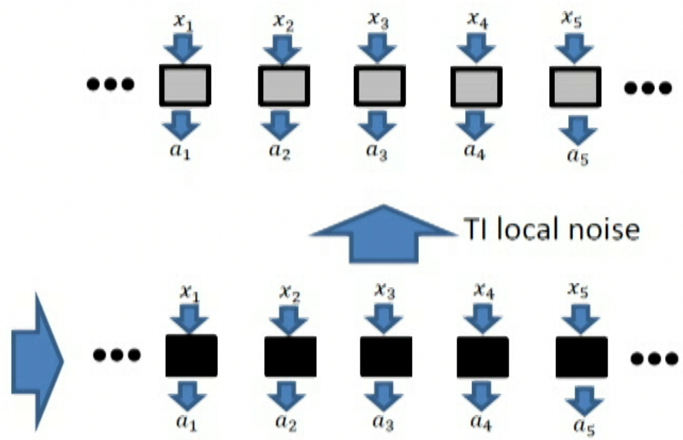
$I_G \approx -6.1798$

Z. Wang, S. Singh and MN, arXiv:1608.03485.

$$I_{\mathrm{T}} \equiv -2E_0 - 4E_1 - 2E_{00}^{1,2} + 2E_{01}^{1,2} + 2E_{10}^{1,2} + 2E_{11}^{1,2} + E_{00}^{1,3} + E_{11}^{1,3} \geq -4,$$

$$I_{\mathrm{G}} \equiv -4E_0 - 6E_1 - 3E_{00}^{1,2} + 2E_{01}^{1,2} + 3E_{10}^{1,2} + 2E_{11}^{1,2} + 2E_{00}^{1,3} + E_{10}^{1,3} + E_{11}^{1,3} \geq -6,$$

$I_T \approx -4.1847$



$I_G \approx -6.1798$

Z. Wang, S. Singh and MN, arXiv:1608.03485.

$$I_{\mathrm{T}} \equiv -2E_0 - 4E_1 - 2E_{00}^{1,2} + 2E_{01}^{1,2} + 2E_{10}^{1,2} + 2E_{11}^{1,2} + E_{00}^{1,3} + E_{11}^{1,3} \geq -4,$$

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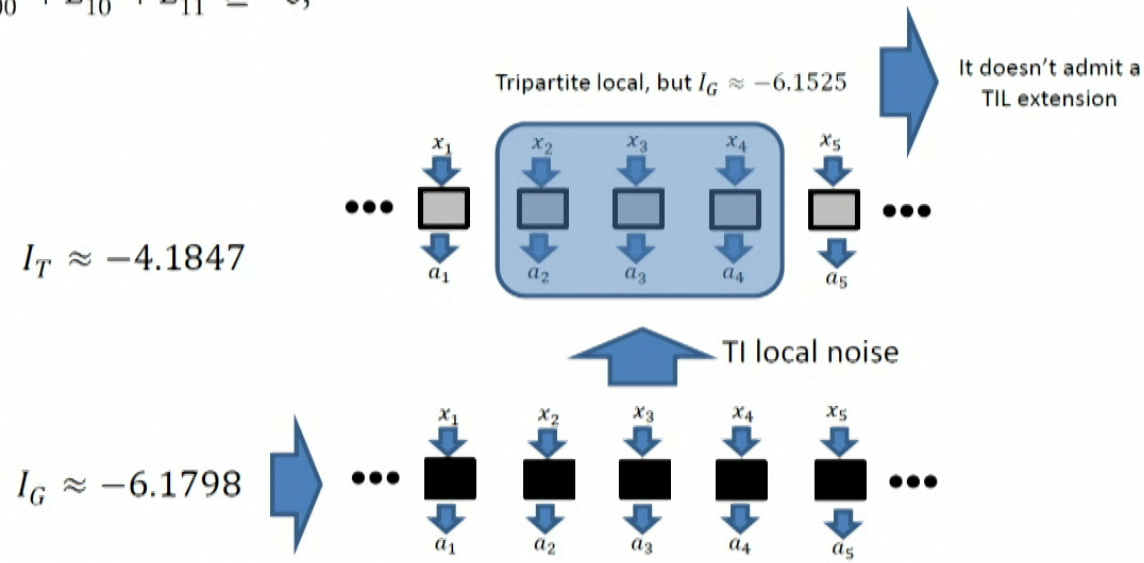


Z. Wang, S. Singh and MN, arXiv:1608.03485.

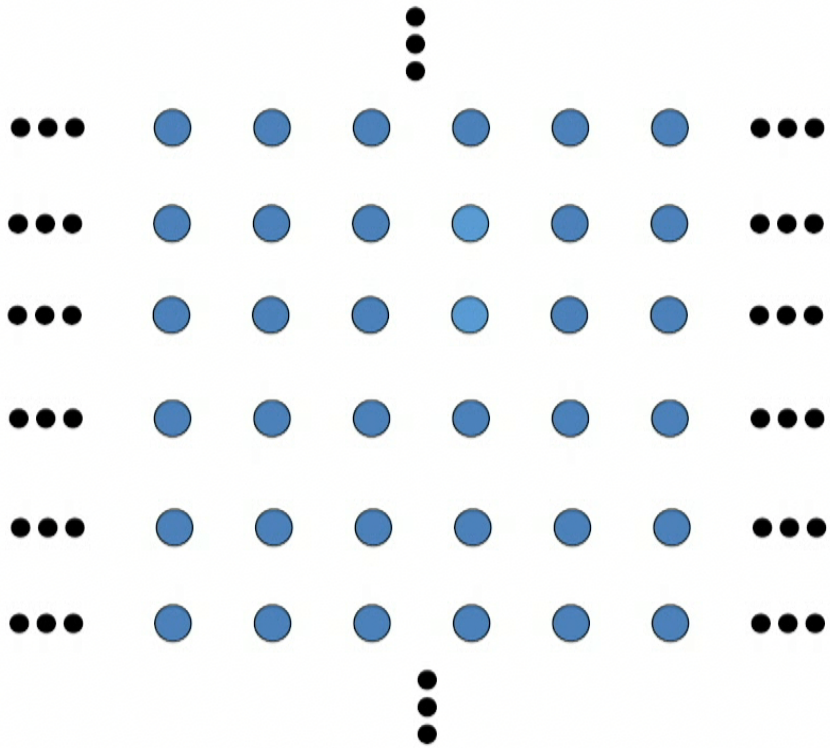
$$I_{\mathrm{T}} \equiv -2E_0 - 4E_1 - 2E_{00}^{1,2} + 2E_{01}^{1,2} + 2E_{10}^{1,2} + 2E_{11}^{1,2} + E_{00}^{1,3} + E_{11}^{1,3} \geq -4,$$

$$I_{\mathrm{G}} \equiv -4E_0 - 6E_1 - 3E_{00}^{1,2} + 2E_{01}^{1,2} + 3E_{10}^{1,2} + 2E_{11}^{1,2} + 2E_{00}^{1,3} + E_{10}^{1,3} + E_{11}^{1,3} \geq -6,$$

$I_T \approx -4.1847$ ⇒ Detects standard tripartite nonlocality

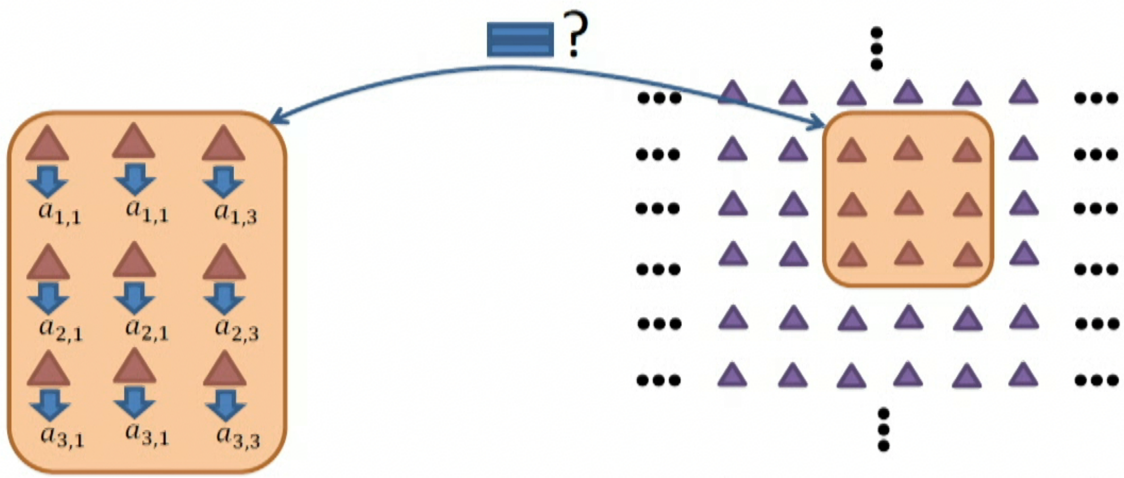
$I_G \approx -6.1798$ ⇒ Detects genuine TI nonlocality

Entanglement and nonlocality in higher spatial dimensions?



Z. Wang and MN arXiv:1703.05640.

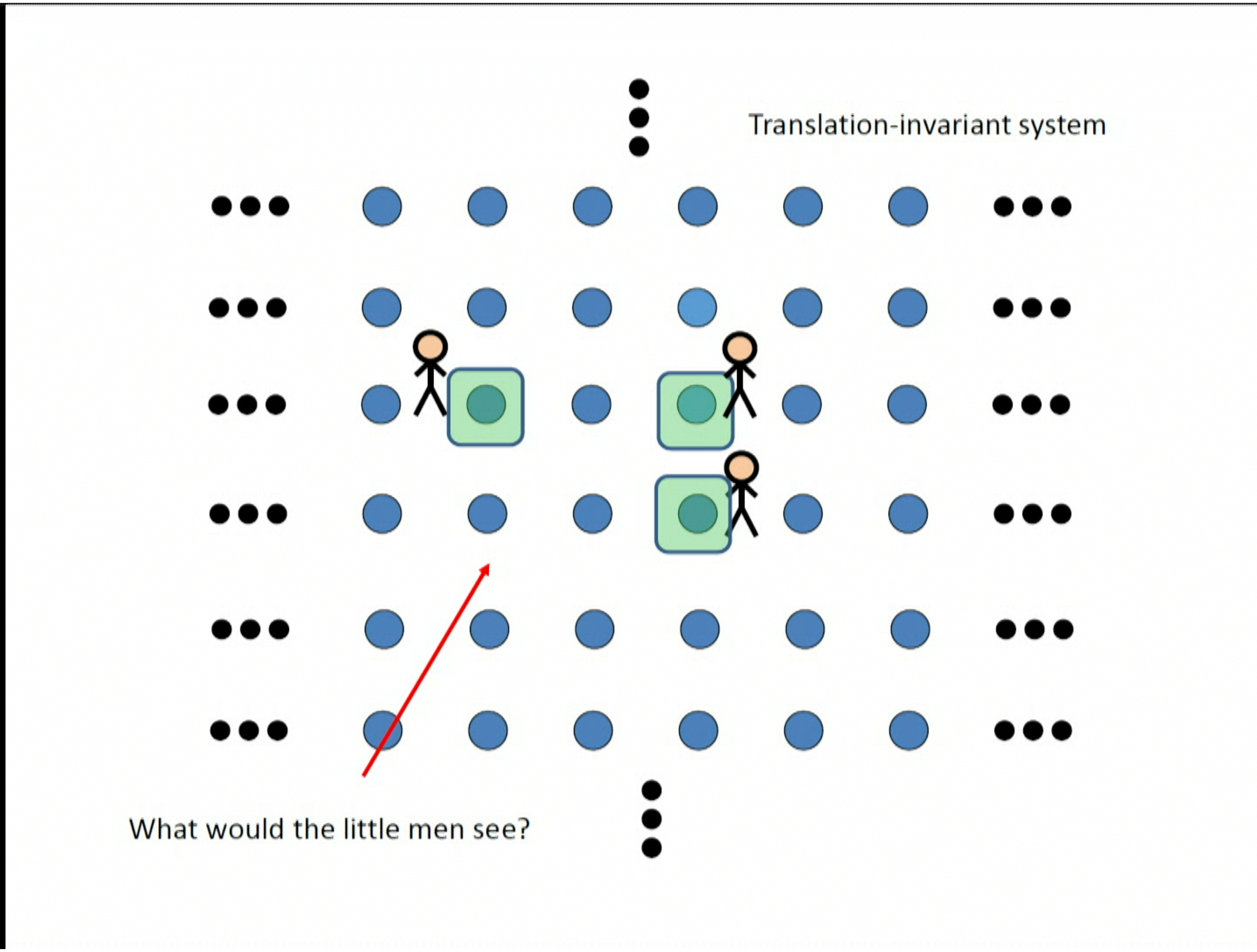
Marginal problem for 2-D classical TI distributions



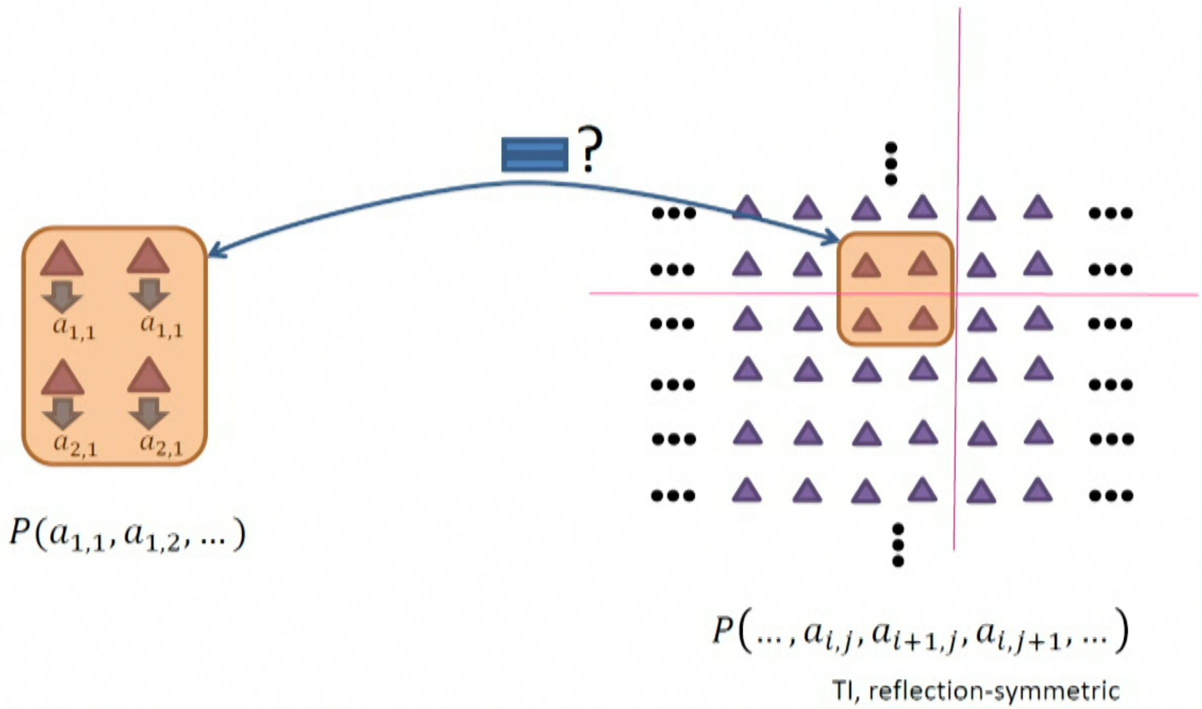
$P(a_{1,1}, a_{1,2}, \dots)$

$P(\dots, a_{i,j}, a_{i+1,j}, a_{i,j+1}, \dots), TI$

Z. Wang and MN arXiv:1703.05640.


Translation-invariant system
What would the little men see?

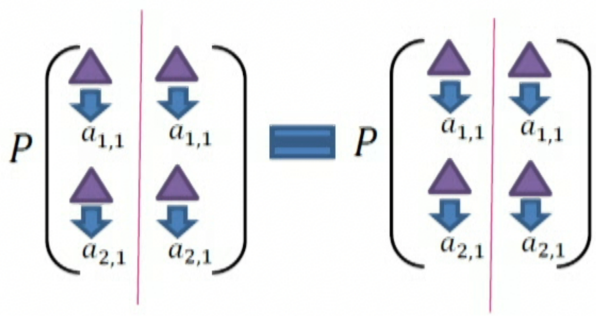
Marginal problem for 2-D classical TI distributions with reflection symmetry



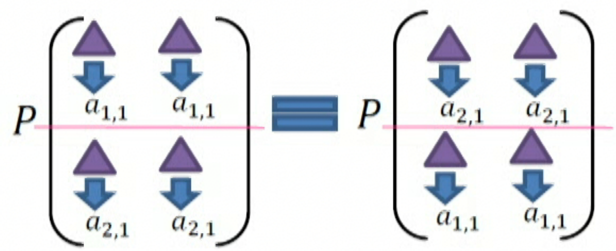
Z. Wang and MN arXiv:1703.05640.

Marginal problem for 2-D classical TI distributions with reflection symmetry

Solution:



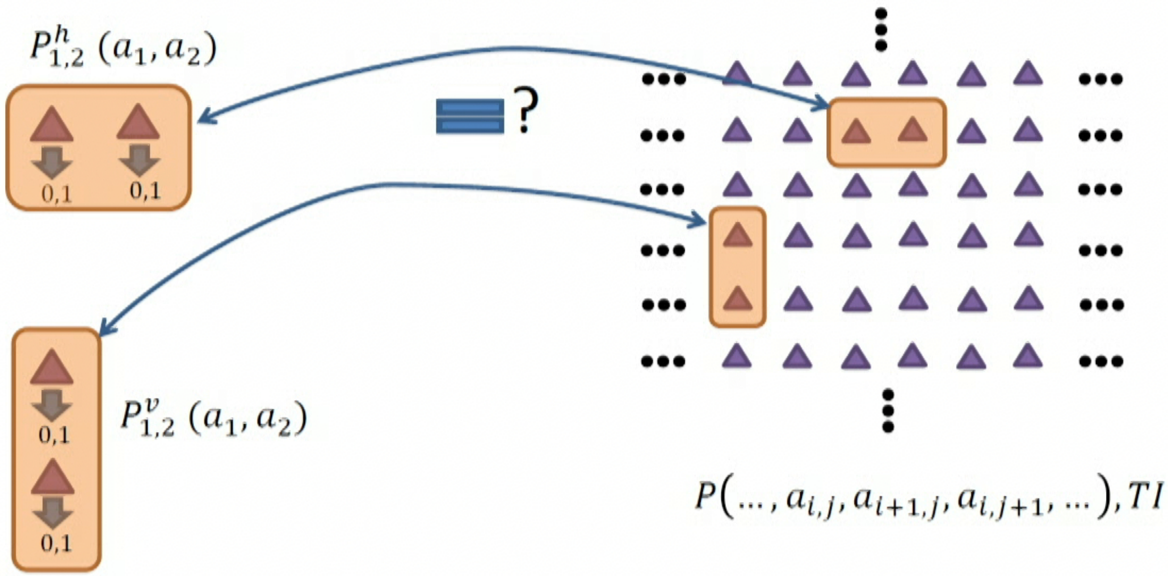
Vertical reflection symmetry



Horizontal reflection symmetry

Z. Wang and MN arXiv:1703.05640.

Marginal problem for 2-D classical TI bit distributions (nearest-neighbors)



Marginal problem for 2-D classical TI bit distributions (nearest-neighbors)

Solution:

$P^{v}_{1,2}\,(a_1, a_2)$ $P^{v}_{1,2}\,(a_1, a_2)$ $P^{h}_{1,2}\,(a_1, a_2)$ $P^{h}_{1,2}\,(a_1, a_2)$

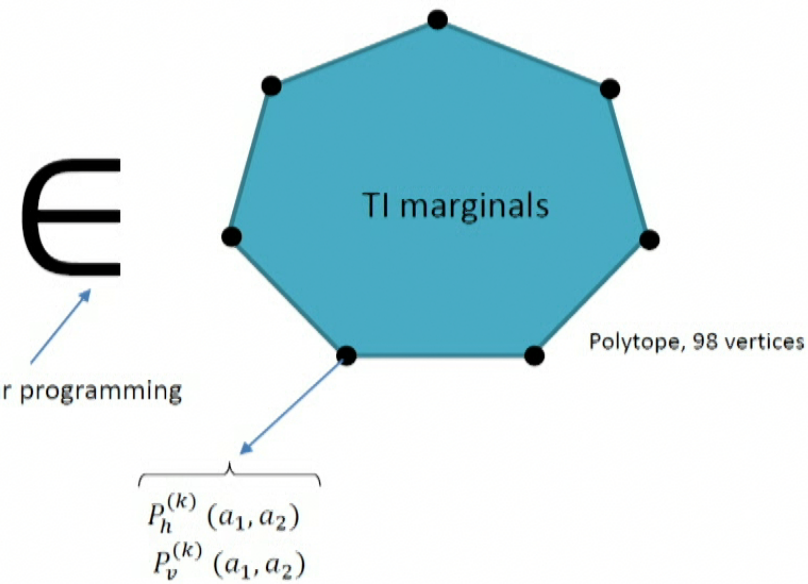
$$P_1^{v}\,(a) = P_2^{v}\,(a) = P_1^{h}\,(a) = P_2^{h}\,(a)$$

Z. Wang and MN arXiv:1703.05640.

Marginal problem for 2-D classical TI trit distributions (nearest-neighbors)

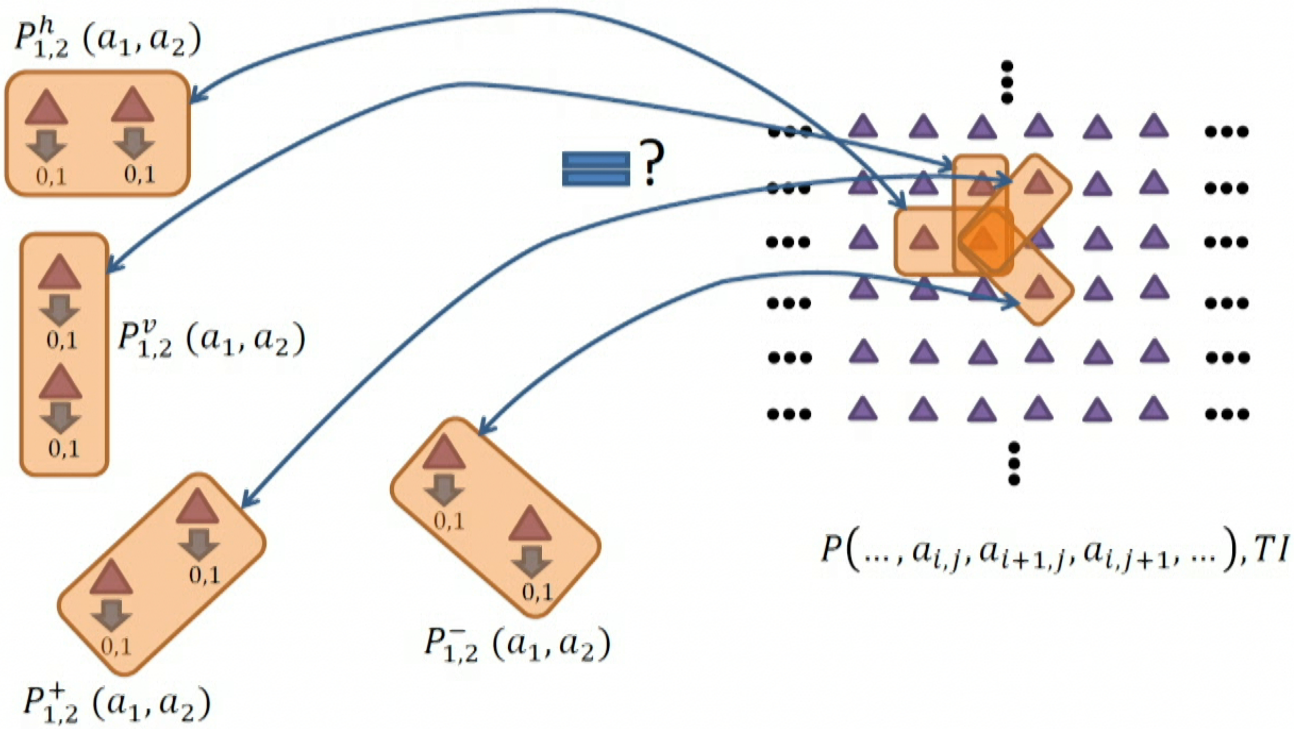
Solution:

$P^h_{1,2}\ (a_1, a_2), P^v_{1,2}\ (a_1, a_2)$



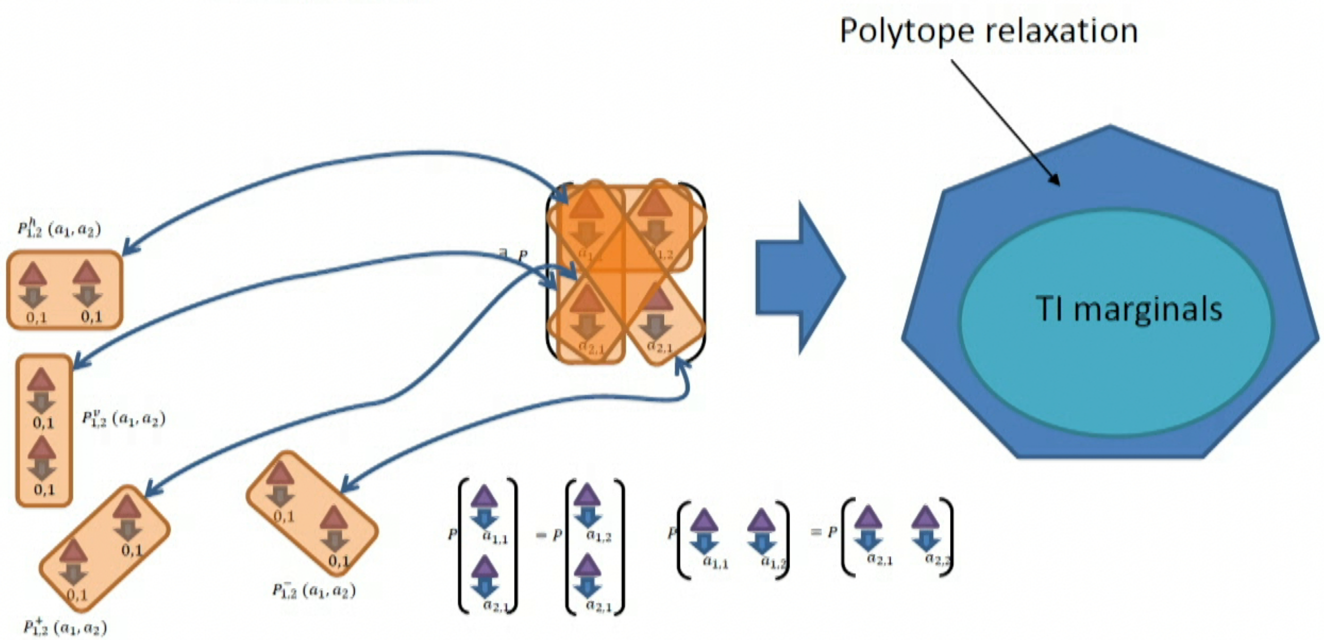
Z. Wang and MN arXiv:1703.05640.

Marginal problem for 2-D classical TI bit distributions (nearest and next-to nearest-neighbors)



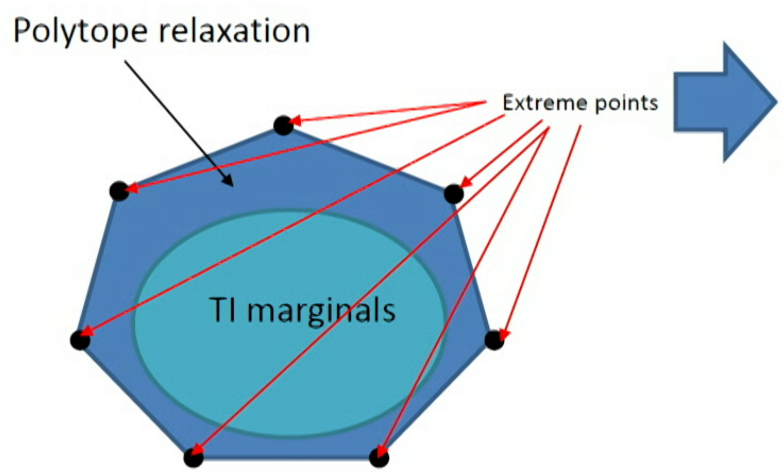
Marginal problem for 2-D classical TI bit distributions (nearest and next-to nearest-neighbors)

Proof (sketch)



Marginal problem for 2-D classical TI bit distributions (nearest and next-to nearest-neighbors)

Proof (sketch)



-13 extreme points

-We verified that all of them admit a TI extension

Lörwald Stefan, and Gerhard Reinelt (2015), "PANDA: a software for polyhedral transformations," EURO Journal on Computational Optimization, 1–12.

Marginal problem for 2-D classical TI bit distributions (nearest and next-to nearest-neighbors)

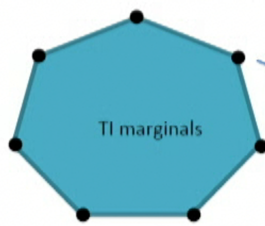
TI Hamiltonian

$$H=\sum_{i}\sum_{a,b=0,1} h_h^{(i,j),(i+1,j)}(a,b)+h_v^{(i,j),(i,j+1)}(a,b)+h_+^{(i,j),(i+1,j+1)}(a,b)+h_-^{(i,j+1),(i+1,j)}(a,b)$$

Minimum energy per site

$P^{v}_{1,2}(a_1,a_2)$

$$= \min_{k=1,\dots,13}\left(\sum_{a,b=0,1} h_h(a,b)P_h^{(k)}(a,b)+h_v(a,b)P_v^{(k)}(a,b)+ h_+(a,b)P_+^{(k)}(a,b)+h_-(a,b)P_-^{(k)}(a,b)\right)$$



$$\begin{cases} P_h^{(k)}(a_1,a_2) \\ P_v^{(k)}(a_1,a_2) \\ P_+^{(k)}(a_1,a_2) \\ P_-^{(k)}(a_1,a_2) \end{cases}$$

Z. Wang and MN arXiv:1703.05640.

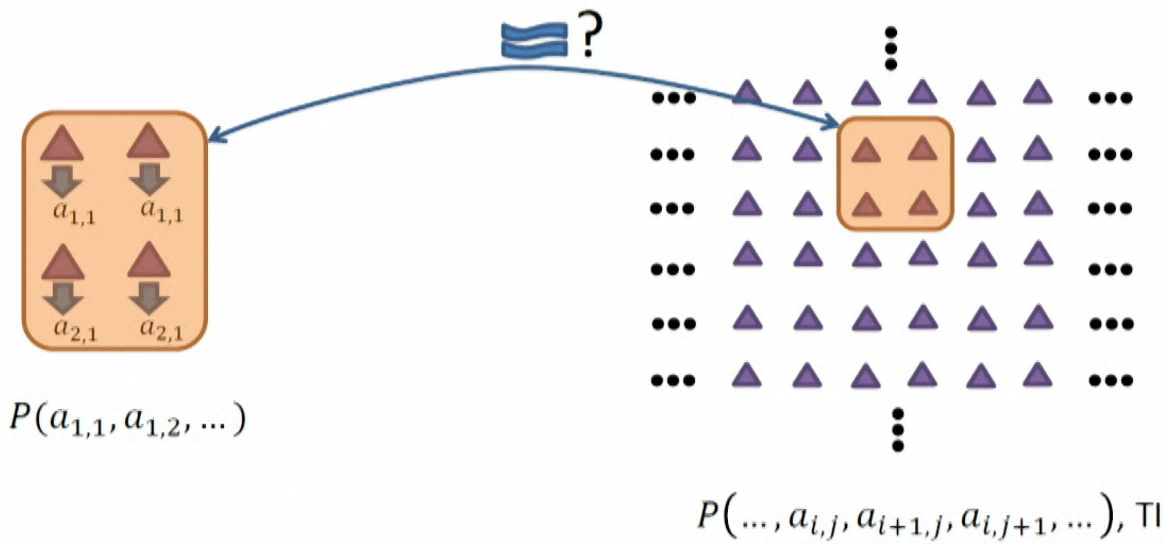
Exact solutions of the TI marginal problem

	bits	trits
2D	Nearest and next-to-nearest neighbors	Nearest neighbors
3D	Nearest neighbors	?

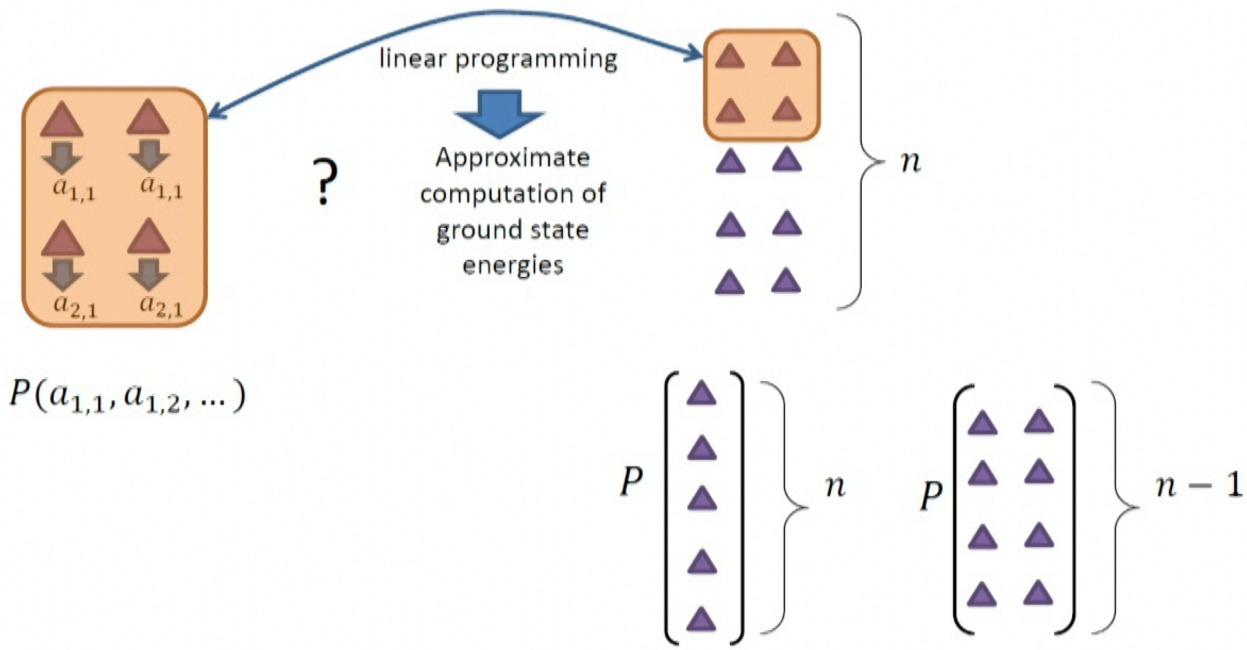
(and TI with reflexion symmetry for 2x2 squares)

Z. Wang and MN arXiv:1703.05640.

Algorithm to approximately solve the marginal problem in any scenario
(with a finite number of outcomes)

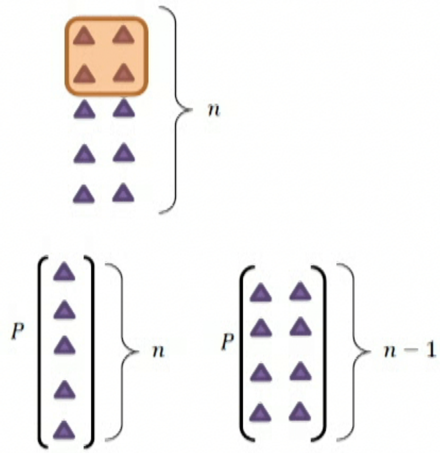


Algorithm to approximately solve the marginal problem in any scenario
(with a finite number of outcomes)

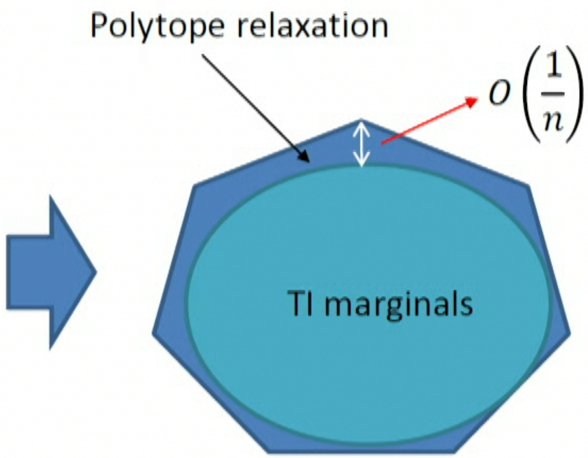


Independent on the position in the rectangle

Algorithm to approximately solve the marginal problem in any scenario
(with a finite number of outcomes)



Independent on the position in the rectangle

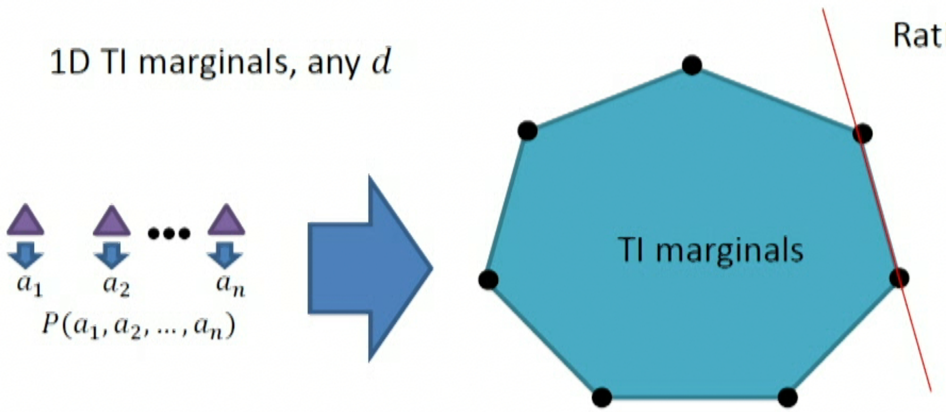


Z. Wang and MN arXiv:1703.05640.

Solved cases

1D TI marginals, any d

Rational convex polytope



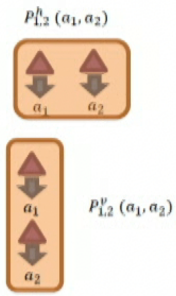
$\vec{F} \cdot \vec{P} \geq K$

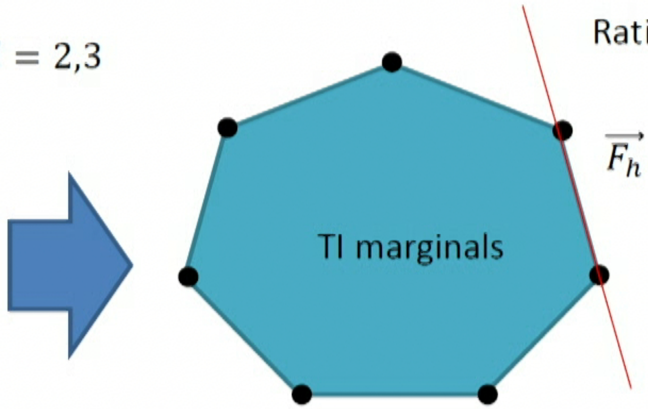
$\vec{F}, K$, integers

Exact computation of ground state energies

Solved cases

2D TI marginals, $d = 2{,}3$





Rational convex polytope

$$\overrightarrow{F_h} \cdot \overrightarrow{P_h} + \overrightarrow{F_v} \cdot \overrightarrow{P_v} \geq K$$

$\overrightarrow{F_h}$, $\overrightarrow{F_v}$, K, integers

Exact computation of ground state energies

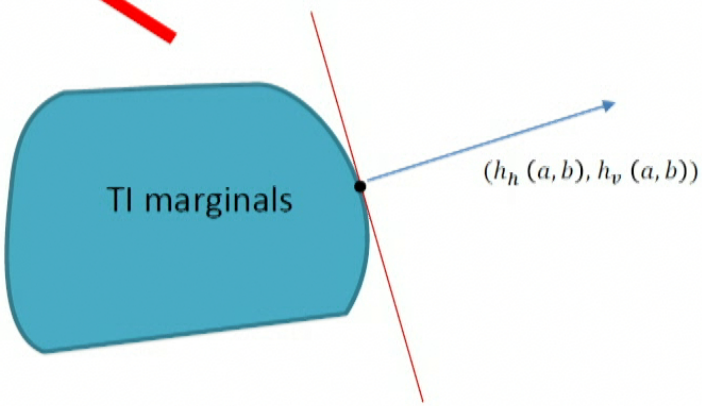
Solved cases

Properties

-Exact computation of ground state energies (per site)

-Simple shape (rational convex polytopes)

Exact computation of ground state energies (per site)



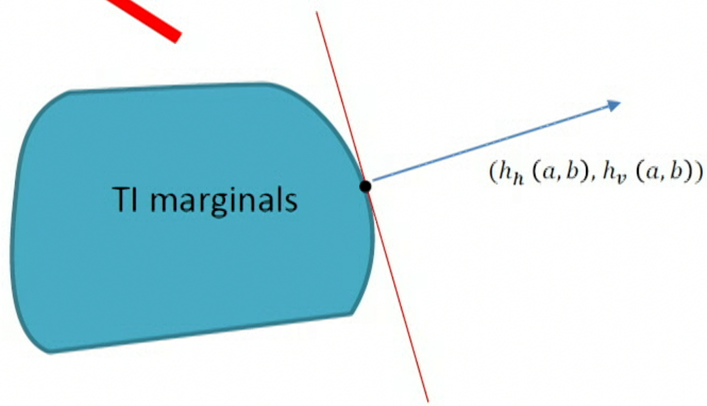
There exists no algorithm to solve the problem

$$\min_{(P_h(a,b), P_v(a,b)), TI} \sum_{a,b} h_h\ (a,b) P_h(a,b) + h_v\ (a,b) P_v(a,b)$$

for arbitrary {0,1}-valued $h_h\ (a,b)$, $h_v\ (a,b)$.

Z. Wang and MN arXiv:1703.05640.

Exact computation of ground state energies (per site)



There exists no algorithm to solve the problem

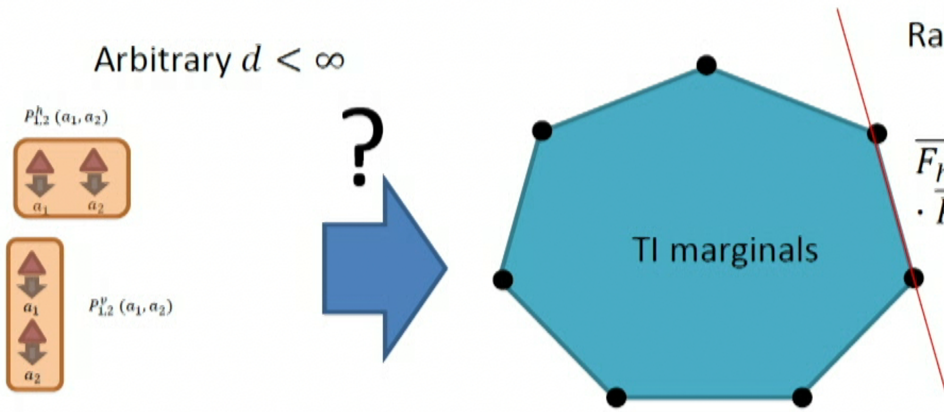
$$\min_{(P_h(a,b),P_v(a,b)),TI} \sum_{a,b} h_h(a,b)P_h(a,b) + h_v(a,b)P_v(a,b)$$

for arbitrary {0,1}-valued $h_h(a,b)$, $h_v(a,b)$.

Z. Wang and MN arXiv:1703.05640.

Simple shape

Arbitrary $d < \infty$



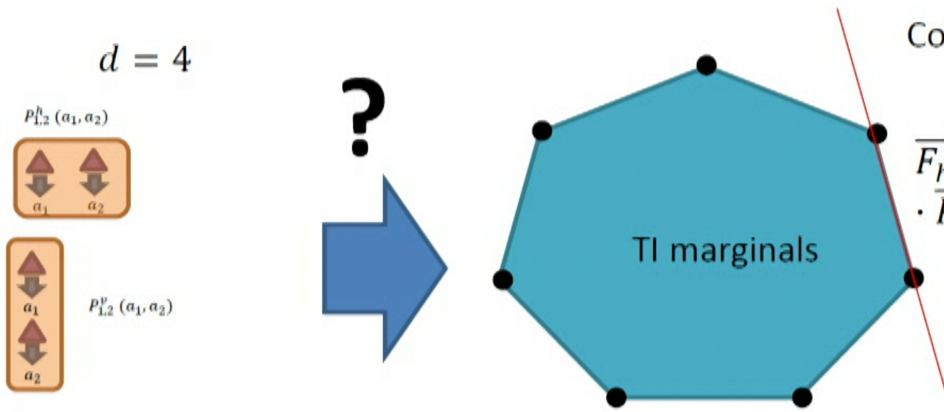
Rational convex polytope

$$\overrightarrow{F_h} \cdot \overrightarrow{P_h} + \overrightarrow{F_v} \cdot \overrightarrow{P_v} + \overrightarrow{F_+} \cdot \overrightarrow{P_+} + \overrightarrow{F_-} \cdot \overrightarrow{P_-} \geq K$$

Z. Wang and MN arXiv:1703.05640.

Current/Future work

Goal: solve the marginal problem for $d = 4$



Convex polytope

$$\overrightarrow{F_h} \cdot \overrightarrow{P_h} + \overrightarrow{F_v} \cdot \overrightarrow{P_v} + \overrightarrow{F_+} \cdot \overrightarrow{P_+} + \overrightarrow{F_-} \cdot \overrightarrow{P_-} \geq K$$

$\overrightarrow{F_h}, \overrightarrow{F_v}, \overrightarrow{F_+}, \overrightarrow{F_-}, K$, real

Z. Wang and MN, work in progress


Sukhi Singh
Zizhu Wang
The demiurge
fatboy