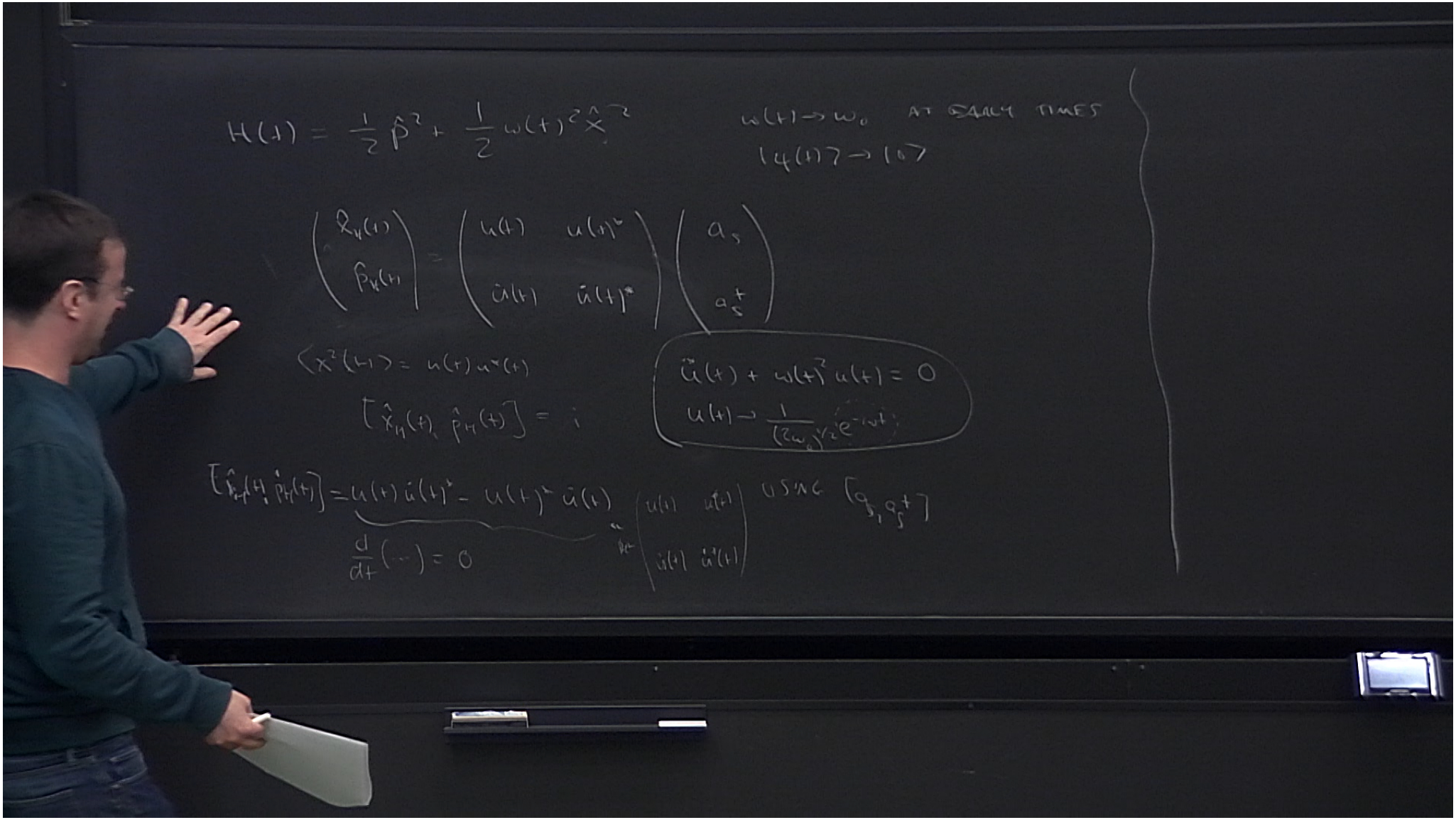
Title: PSI 16/17 Explorations in Cosmology (Kendrick Smith) - Lecture 14

Date: Apr 28, 2017 10:15 AM

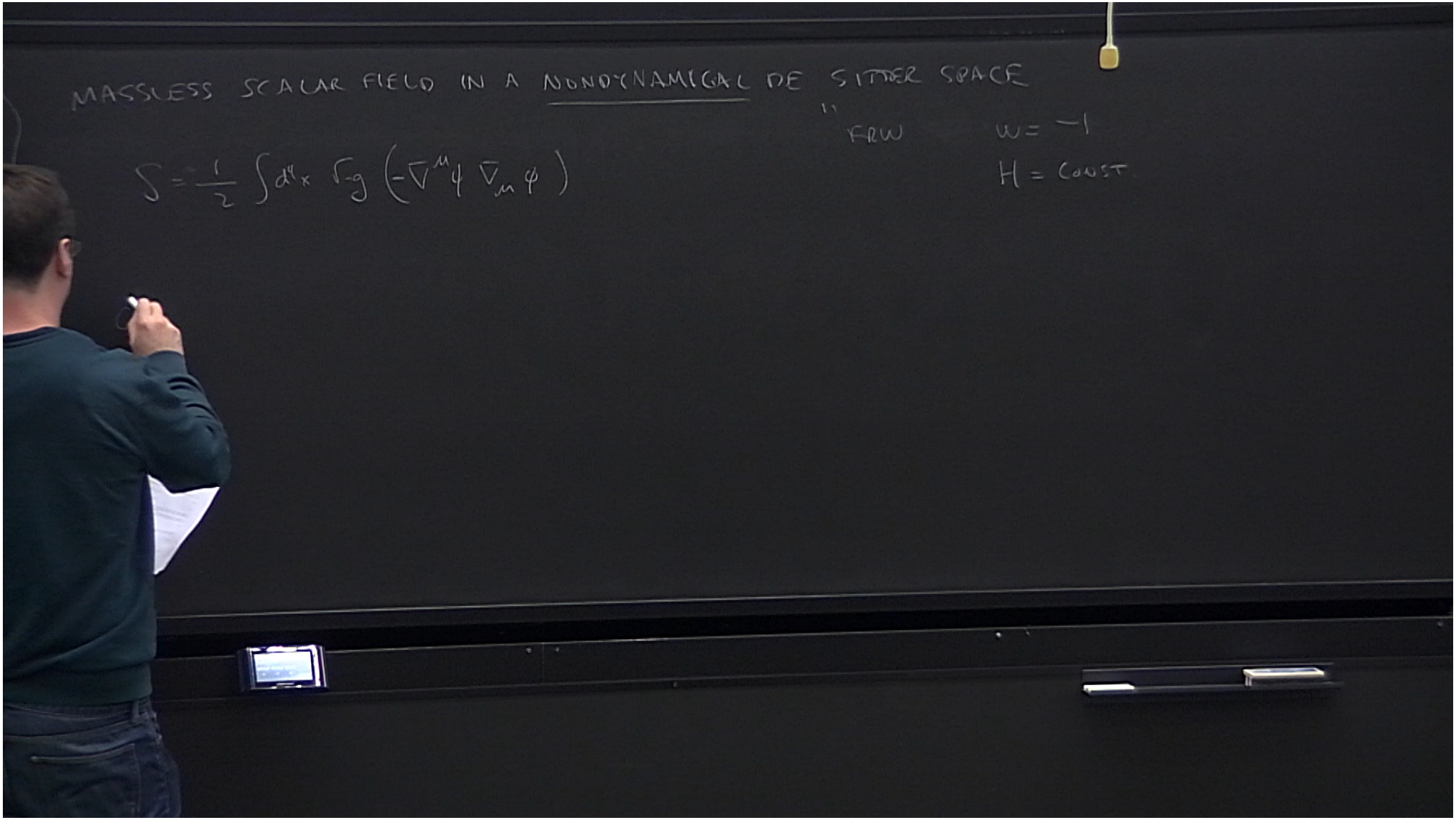
URL: http://pirsa.org/17040067

Abstract:


$H(t) = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\omega(t)^2\hat{x}^2$
$\omega(t) \to \omega_0$ AT EARLY TIMES
$|\psi(t)\rangle \to |0\rangle$
$\begin{pmatrix} \hat{x}_H(t) \\ \hat{p}_H(t) \end{pmatrix} = \begin{pmatrix} u(t) & u(t)^* \\ \dot{u}(t) & \dot{u}(t)^* \end{pmatrix} \begin{pmatrix} a_s \\ a_s^\dagger \end{pmatrix}$
$\langle x^2(t) \rangle = u(t)u^*(t)$
$[\hat{x}_H(t), \hat{p}_H(t)] = i$
$\ddot{u}(t) + \omega(t)^2 u(t) = 0$
$u(t) \to \frac{1}{(2\omega_0)^{1/2}} e^{-i\omega_0 t}$
$[\hat{x}_H(t), \hat{p}_H(t)] = u(t)\dot{u}(t)^* - u(t)^*\dot{u}(t)$
$\frac{d}{dt}(\dots) = 0$
$= \begin{vmatrix} u(t) & u(t)^* \\ \dot{u}(t) & \dot{u}^*(t) \end{vmatrix}$ USING $[a_s, a_s^\dagger]$

MASSLESS SCALAR FIELD IN A NONDYNAMICAL DE SITTER SPACE

$$S = -\frac{1}{2}\int d^4x\, \sqrt{-g}\left(-\nabla^\mu\phi\,\nabla_\mu\phi\right)$$


MASSLESS SCALAR FIELD IN A NONDYNAMICAL DE SITTER SPACE
$S = -\frac{1}{2}\int d^4x \sqrt{-g}\left(-\nabla^\mu\varphi\,\nabla_\mu\varphi\right)$
FRW
$w = -1$
$H = \text{CONST}$

MASSLESS SCALAR FIELD IN A NONDYNAMICAL DE SITTER SPACE

$$S = \frac{1}{2}\int d^4x \sqrt{-g}\left(-\nabla^\mu \phi \nabla_\mu \phi\right)$$

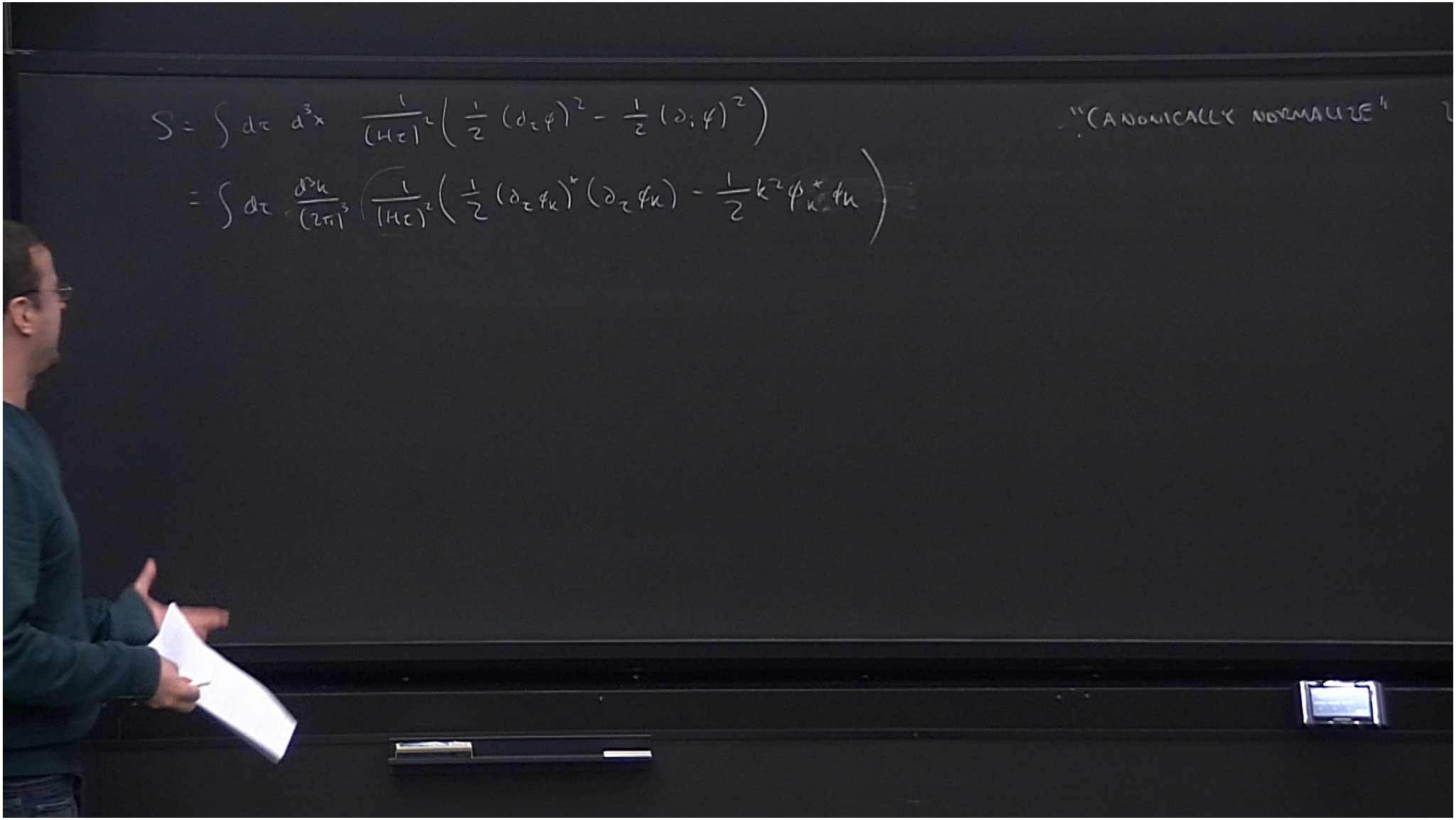
" FRW $\quad w = -1$

$H = \text{CONST}$

$$ds^2 = -dt^2 + e^{2Ht} dx^2 \qquad (-\infty < t < \infty)$$

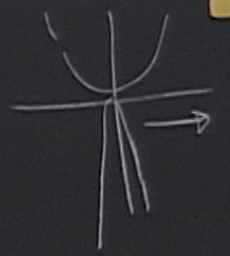
$$= \frac{1}{(H\tau)^2}\left(-d\tau^2 + dx^2\right) \qquad (-\infty < \tau < 0)$$

⚠ WE'LL USE CONFORMAL TIME


$S = \int d\tau\, d^3x\, \frac{1}{(H\tau)^2}\left(\frac{1}{2}(\partial_\tau\varphi)^2 - \frac{1}{2}(\partial_i\varphi)^2\right)$
"CANONICALLY NORMALIZE"
$= \int d\tau \frac{d^3k}{(2\pi)^3}\frac{1}{(H\tau)^2}\left(\frac{1}{2}(\partial_\tau\varphi_k)^*(\partial_\tau\varphi_k) - \frac{1}{2}k^2\varphi_k^*\varphi_k\right)$

"CANONICALLY NORMALIZE" $\psi = (H\tau)^{-1}\phi$

$$S = \int d\tau \frac{d^3k}{(2\pi)^3}\left(\frac{1}{2}(\partial_\tau \psi_k)^*(\partial_\tau \psi_k) - \frac{1}{2}\underbrace{\left(k^2 - \frac{2}{\tau^2}\right)}_{\omega(\tau)^2}\psi_k^* \psi_k\right)$$



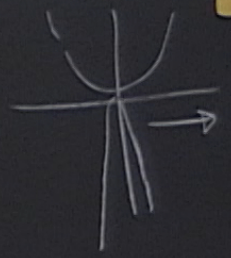
"CANONICALLY NORMALIZE" $\psi = (H\tau)^{-1}\phi$

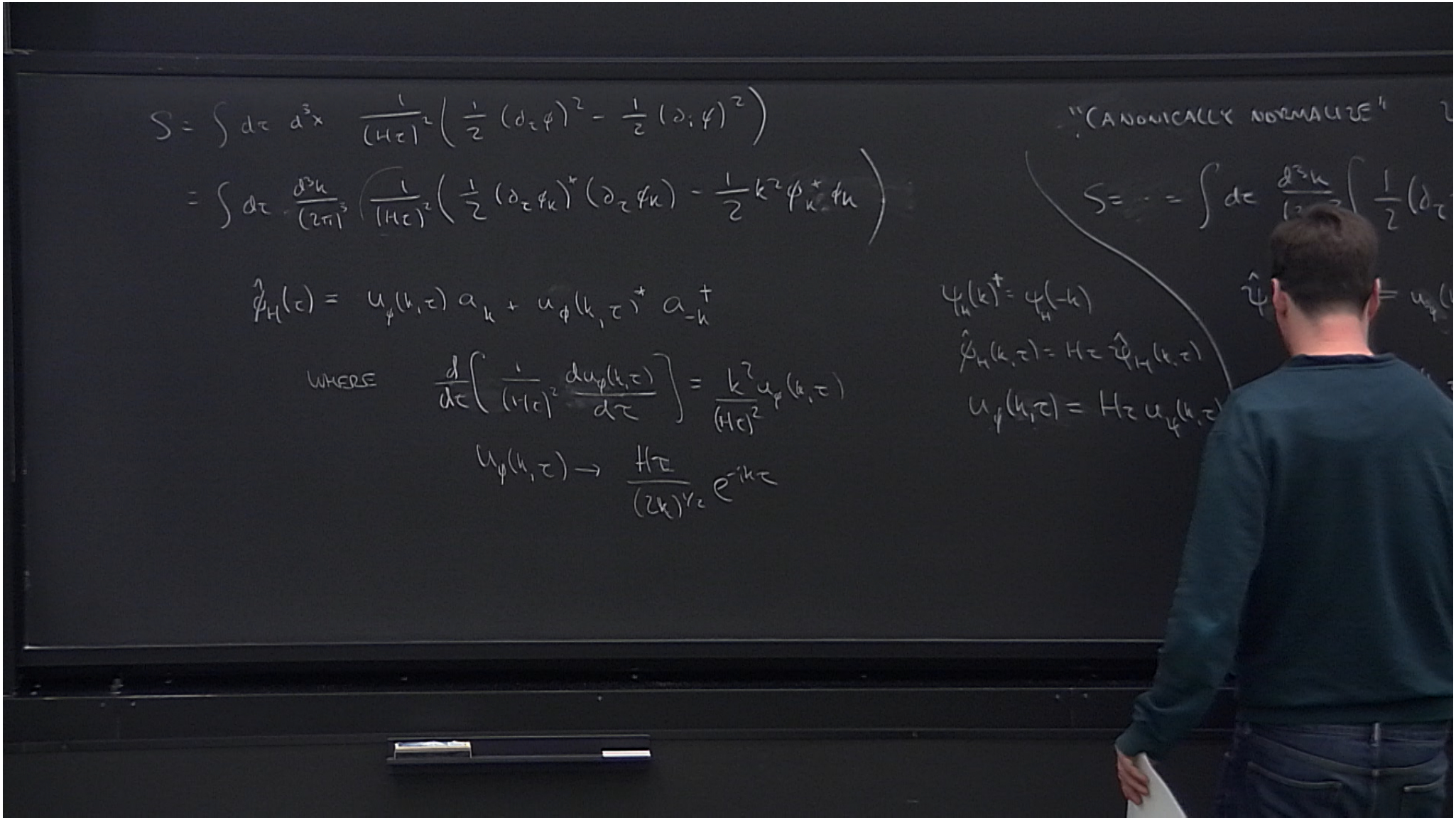
$$S = \int d\tau \frac{d^3k}{(2\pi)^3}\left[\frac{1}{2}(\partial_\tau \psi_k)^*(\partial_\tau \psi_k) - \frac{1}{2}\underbrace{\left(k^2 - \frac{2}{\tau^2}\right)}_{\omega(\tau)^2}\psi_k^* \psi_k\right]$$

$\psi_H(k)^\dagger = \psi_H(-k)$

$$\hat{\psi}_H(k,\tau) = u(k,\tau)\, a_k + u(k,\tau)^* a_{-k}^\dagger$$

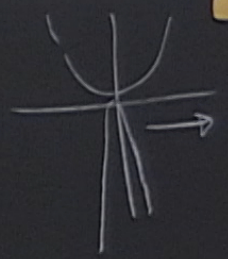
WHERE $\ddot{u}(k,\tau) = -\left(k^2 - \frac{2}{\tau^2}\right) u(k,\tau)$




$S = \int d\tau\, d^3x\, \frac{1}{(H\tau)^2}\left(\frac{1}{2}(\partial_\tau\varphi)^2 - \frac{1}{2}(\partial_i\varphi)^2\right)$
$= \int d\tau \frac{d^3k}{(2\pi)^3}\left(\frac{1}{(H\tau)^2}\left(\frac{1}{2}(\partial_\tau\varphi_k)^*(\partial_\tau\varphi_k) - \frac{1}{2}k^2\varphi_k^*\varphi_k\right)\right)$
"CANONICALLY NORMALIZE"
$\hat{\varphi}_H(\tau) = u_\varphi(k,\tau)\, a_k + u_\varphi(k,\tau)^*\, a_{-k}^\dagger$
$\psi_H(k)^\dagger = \psi_H(-k)$
$\hat{\varphi}_H(k,\tau) = H\tau\, \hat{\psi}_H(k,\tau)$
$u_\psi(k,\tau) = H\tau\, u_\varphi(k,\tau)$
WHERE $\frac{d}{d\tau}\left[\frac{1}{(H\tau)^2}\frac{du_\varphi(k,\tau)}{d\tau}\right] = \frac{k^2}{(H\tau)^2}u_\varphi(k,\tau)$
$u_\psi(k,\tau) \to \frac{H\tau}{(2k)^{1/2}}e^{-ik\tau}$

"CANONICALLY NORMALIZE" $\psi = (H\tau)^{-1}\phi$

$$S = \int d\tau \frac{d^3k}{(2\pi)^3}\left[\frac{1}{2}(\partial_\tau \psi_k)^*(\partial_\tau \psi_k) - \frac{1}{2}\underbrace{\left(k^2 - \frac{2}{\tau^2}\right)}_{\omega(k,\tau)^2}\psi_k^* \psi_k\right]$$



$\psi_H(k)^\dagger = \psi_H(-k)$

$\hat{\psi}_H(k,\tau) = H\tau\,\hat{\psi}_H(k,\tau)$

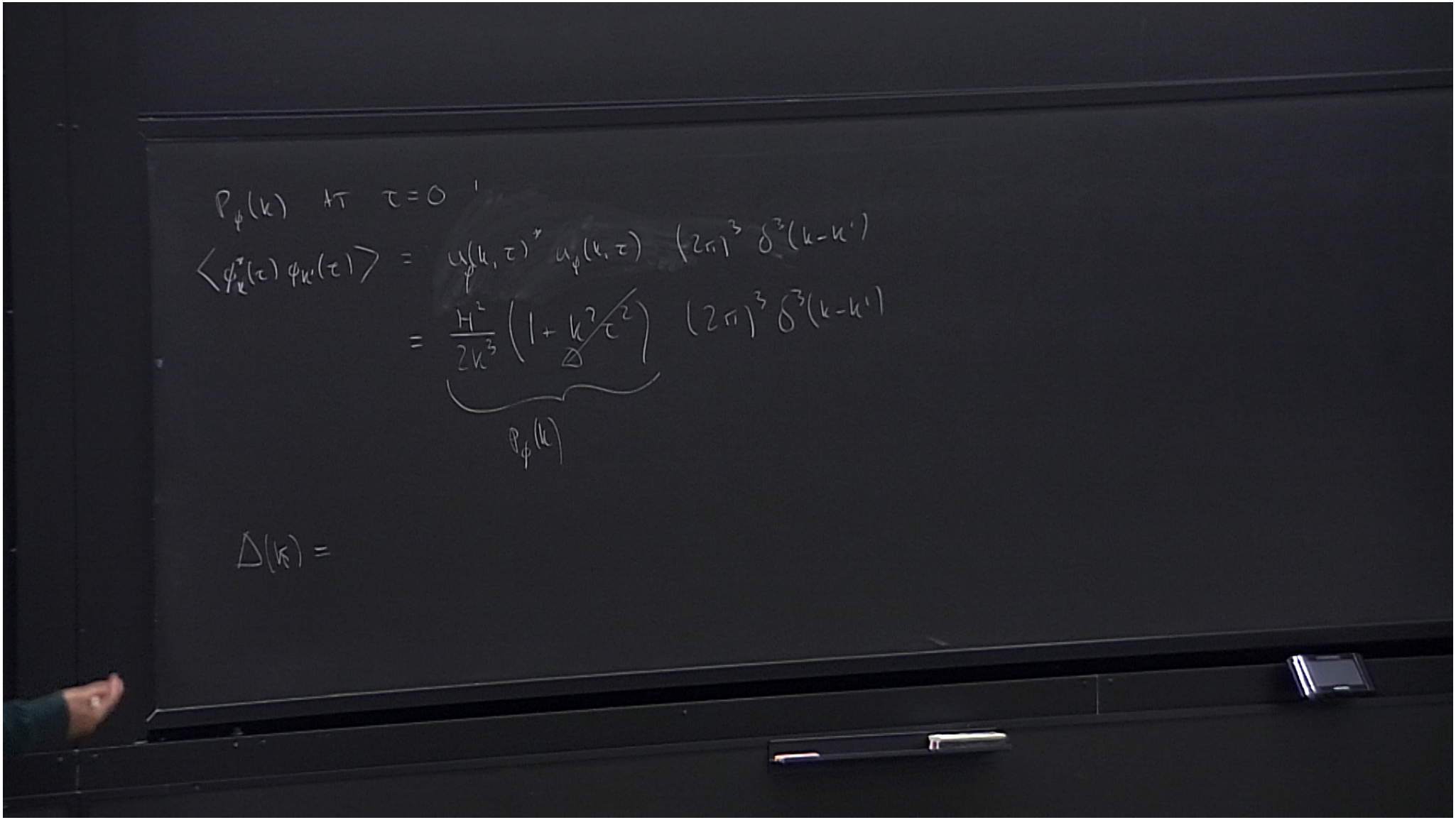
$u_\psi(k,\tau) = H\tau\, u_\phi(k,\tau)$

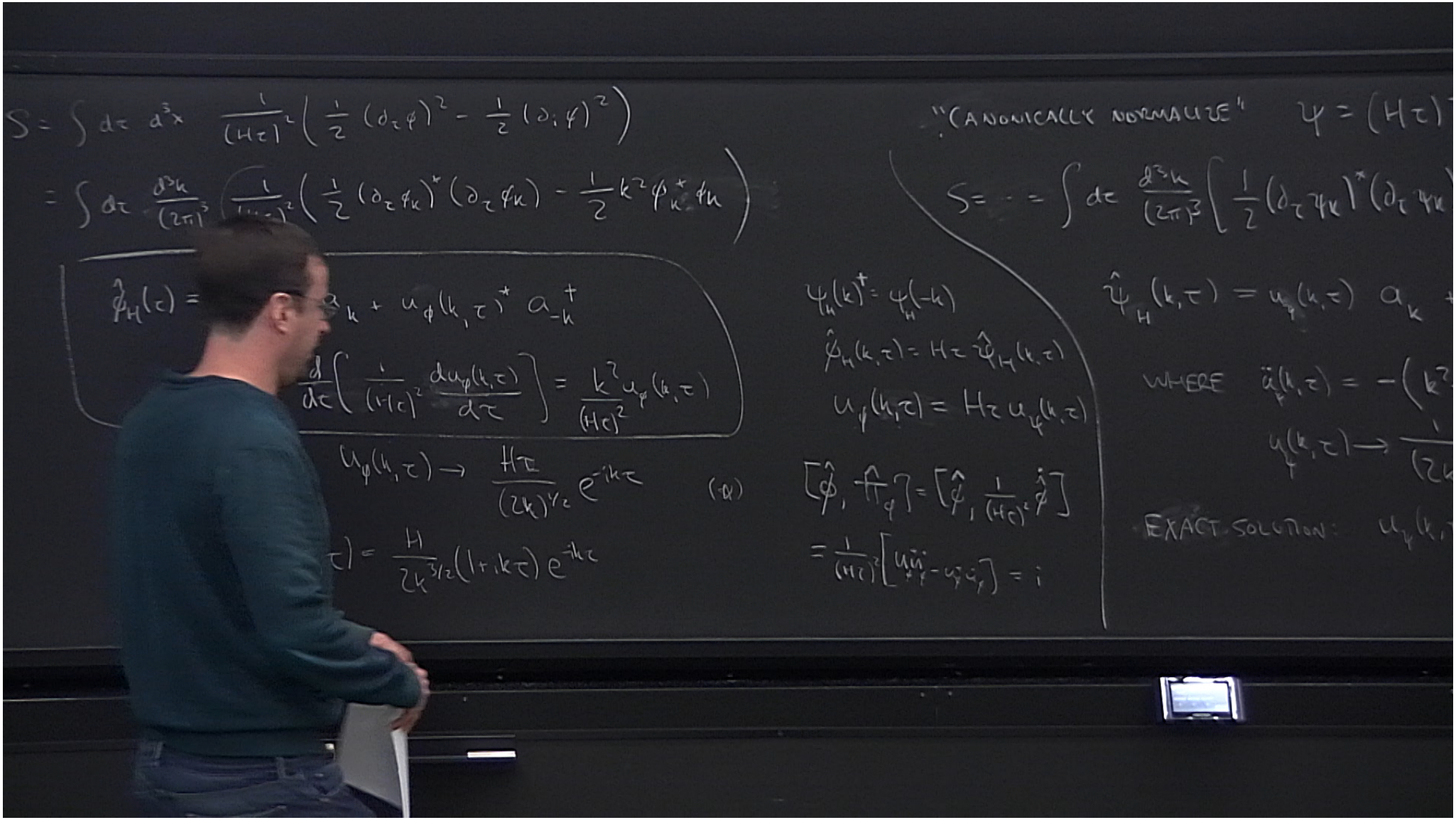
$$\hat{\psi}_H(k,\tau) = u_\psi(k,\tau)\, a_k + u_\psi(k,\tau)^* a_{-k}^\dagger$$

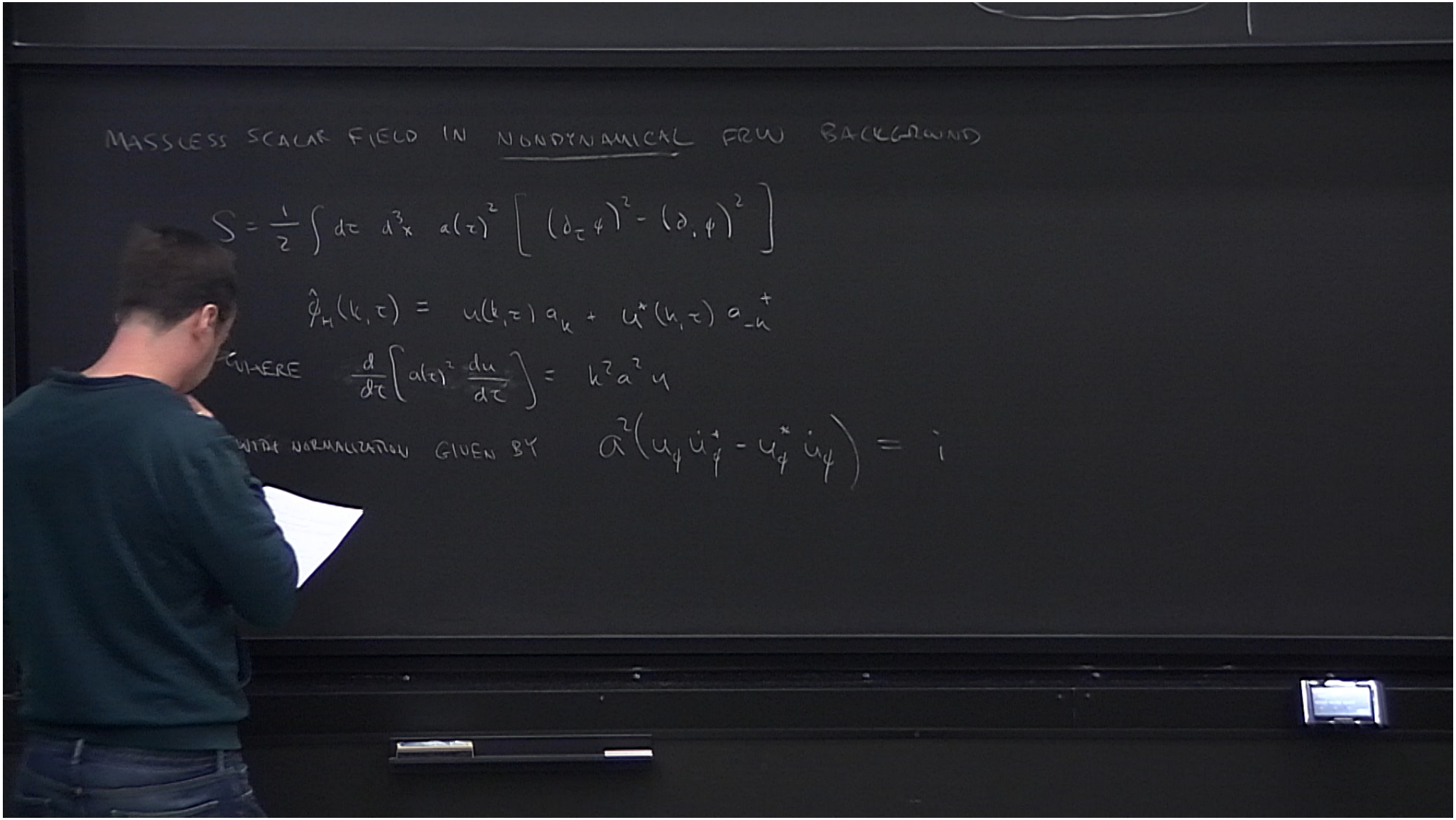
WHERE $\ddot{u}_\psi(k,\tau) = -\left(k^2 - \frac{2}{\tau^2}\right) u_\psi(k,\tau)$

$$u_\psi(k,\tau) \to \frac{1}{(2k)^{1/2}} e^{-ik\tau}$$

EXACT SOLUTION: $u_\psi(k,\tau) = \frac{1}{(2k)^{1/2}}\left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}$


$P_\phi(k)$ AT $\tau = 0$!
$\langle \phi_k^*(\tau)\, \phi_{k'}(\tau) \rangle = u_\phi(k,\tau)^*\, u_\phi(k,\tau)\, (2\pi)^3 \delta^3(k-k')$
$= \frac{H^2}{2k^3}\left(1 + k^2\tau^2\right) (2\pi)^3 \delta^3(k-k')$
$P_\phi(k)$
$\Delta(k) =$


"CANONICALLY NORMALIZE"
WHERE
EXACT SOLUTION:


MASSLESS SCALAR FIELD IN NONDYNAMICAL FRW BACKGROUND
$S = \frac{1}{2}\int d\tau\, d^3x\, a(\tau)^2 \left[(\partial_\tau \phi)^2 - (\partial_i \phi)^2 \right]$
$\hat{\phi}_H(k,\tau) = u(k,\tau)\, a_k + u^*(k,\tau)\, a^\dagger_{-k}$
WHERE $\frac{d}{d\tau}\left[a(\tau)^2 \frac{du}{d\tau} \right] = k^2 a^2 u$
WITH NORMALIZATION GIVEN BY $a^2 (u_q u_q'^* - u_q^* u_q') = i$

MASSLESS SCALAR FIELD IN NONDYNAMICAL FRW BACKGROUND

$$S = \frac{1}{2}\int d\tau\, d^3x\, a(\tau)^2 \left[(\partial_\tau \phi)^2 - (\partial_i \phi)^2 \right]$$

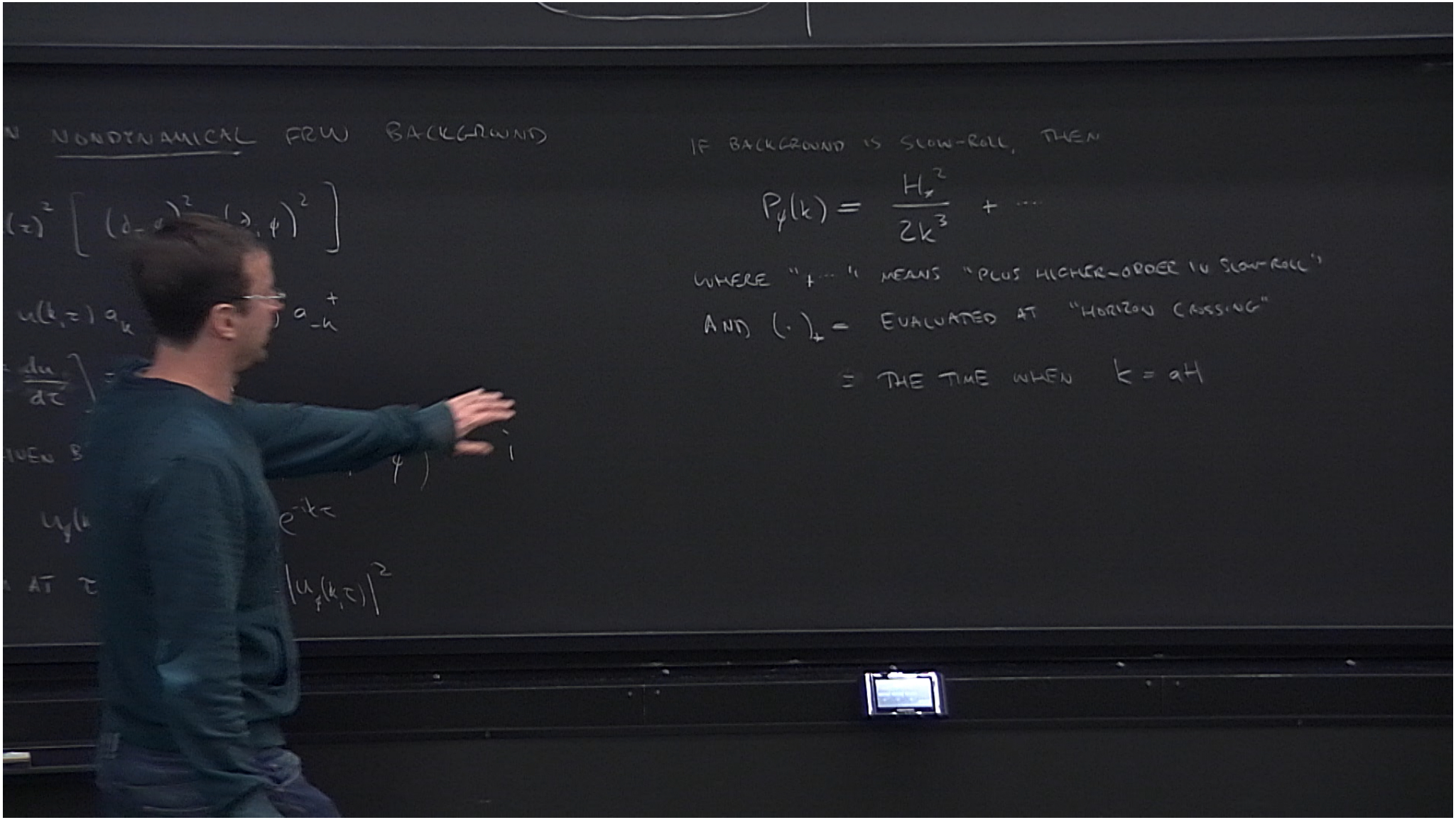
$$\hat{\phi}_H(k,\tau) = u(k,\tau)\, a_k + u^*(k,\tau)\, a^\dagger_{-k}$$

WHERE $\frac{d}{d\tau}\left[a(\tau)^2 \frac{du}{d\tau} \right] = k^2 a^2 u$

WITH NORMALIZATION GIVEN BY $a^2\left(u_q \dot{u}_q^* - u_q^* \dot{u}_q\right) = i$

AT EARLY TIMES, $u_q(k,\tau) \to \frac{1}{(2k)^{1/2} a} e^{-ik\tau}$

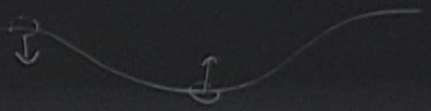
THE POWER SPECTRUM AT $\tau \to 0$ IS $P_\phi(k) = \left| u_q(k,\tau) \right|^2$


NONDYNAMICAL FRW BACKGROUND
IF BACKGROUND IS SLOW-ROLL, THEN
$P_\psi(k) = \frac{H_*^2}{2k^3} + \cdots$
WHERE "$+\cdots$" MEANS "PLUS HIGHER-ORDER IN SLOW-ROLL"
AND $(\cdot)_*$ = EVALUATED AT "HORIZON CROSSING"
= THE TIME WHEN $k = aH$

KGROUND

IF BACKGROUND IS SLOW-ROLL, THEN

$$P_\phi(k) = \frac{H_*^2}{2k^3} + \cdots$$



WHERE "$+\cdots$" MEANS "PLUS HIGHER-ORDER IN SLOW-ROLL"

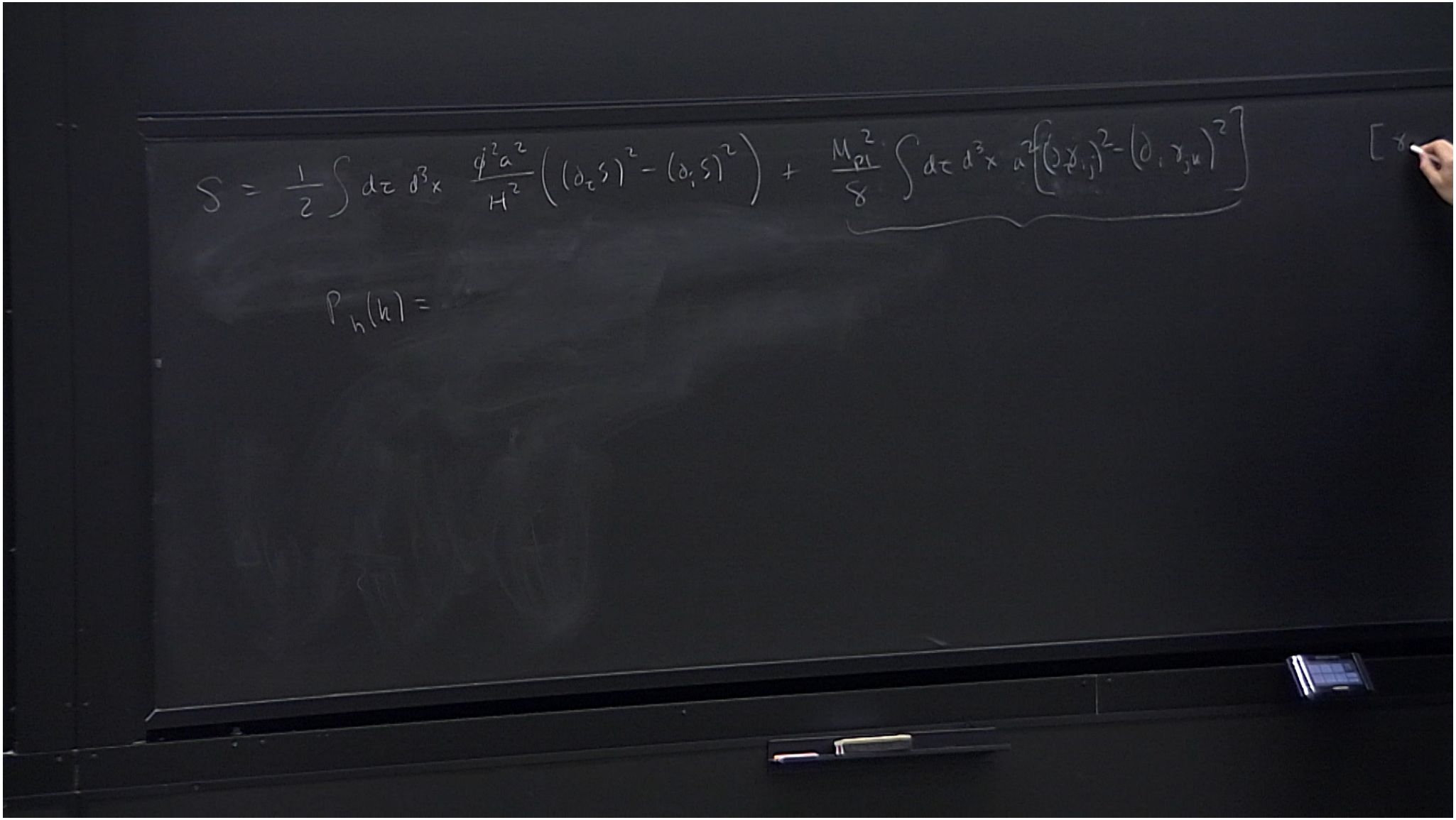
$\dot{\phi} = 0$

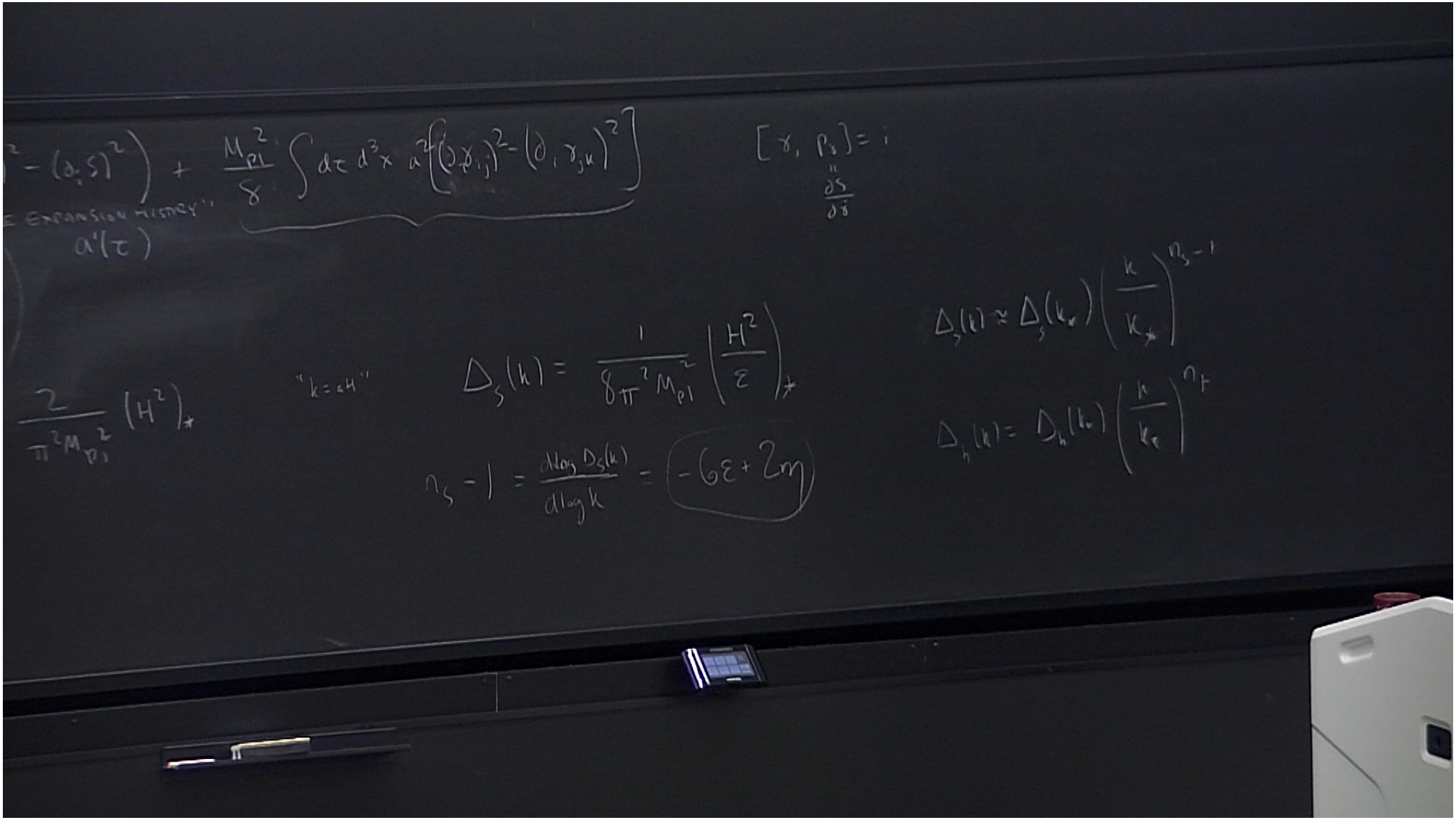
AND $(\cdot)_*$ = EVALUATED AT "HORIZON CROSSING"

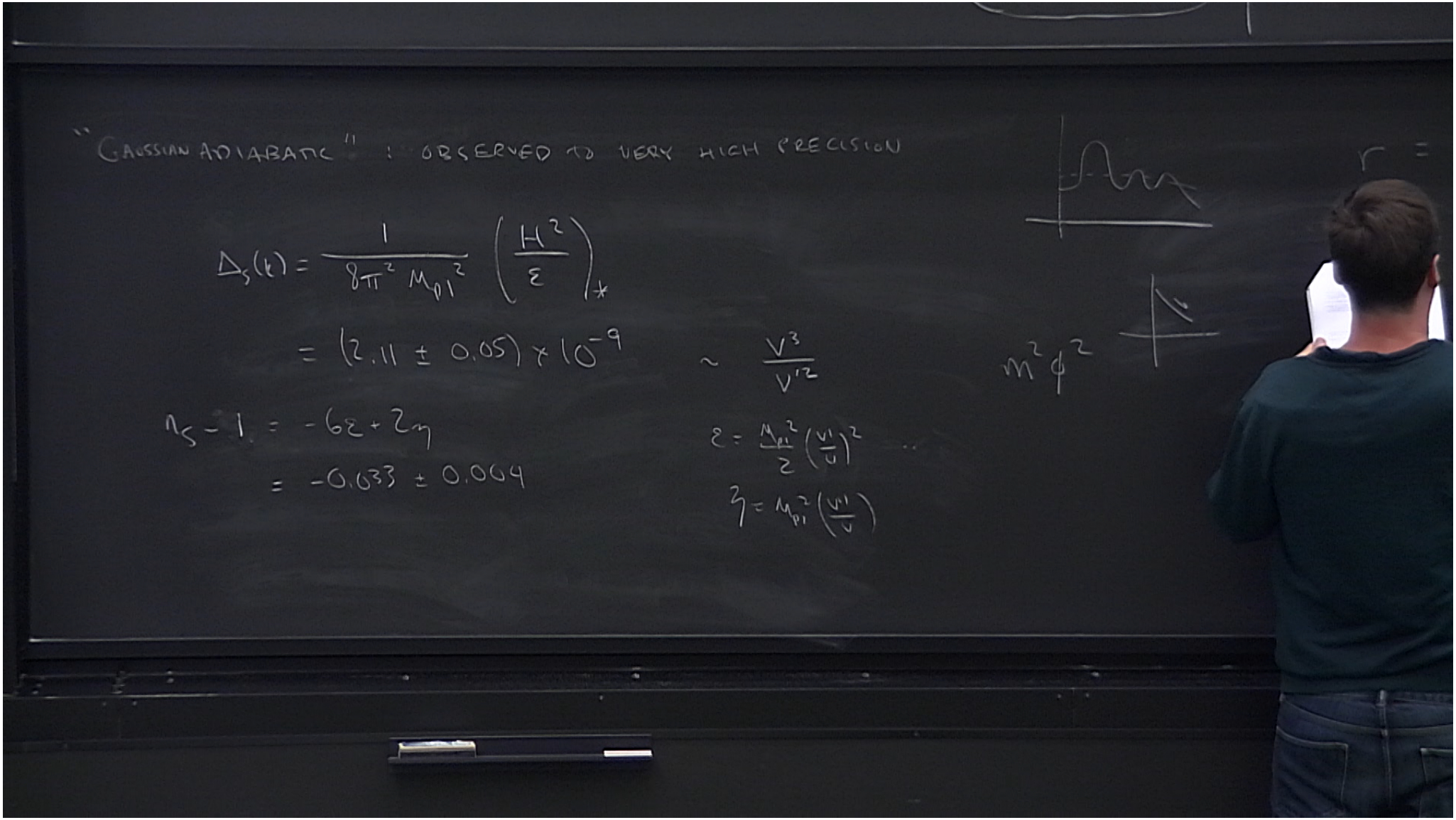
= THE TIME WHEN $k = aH$

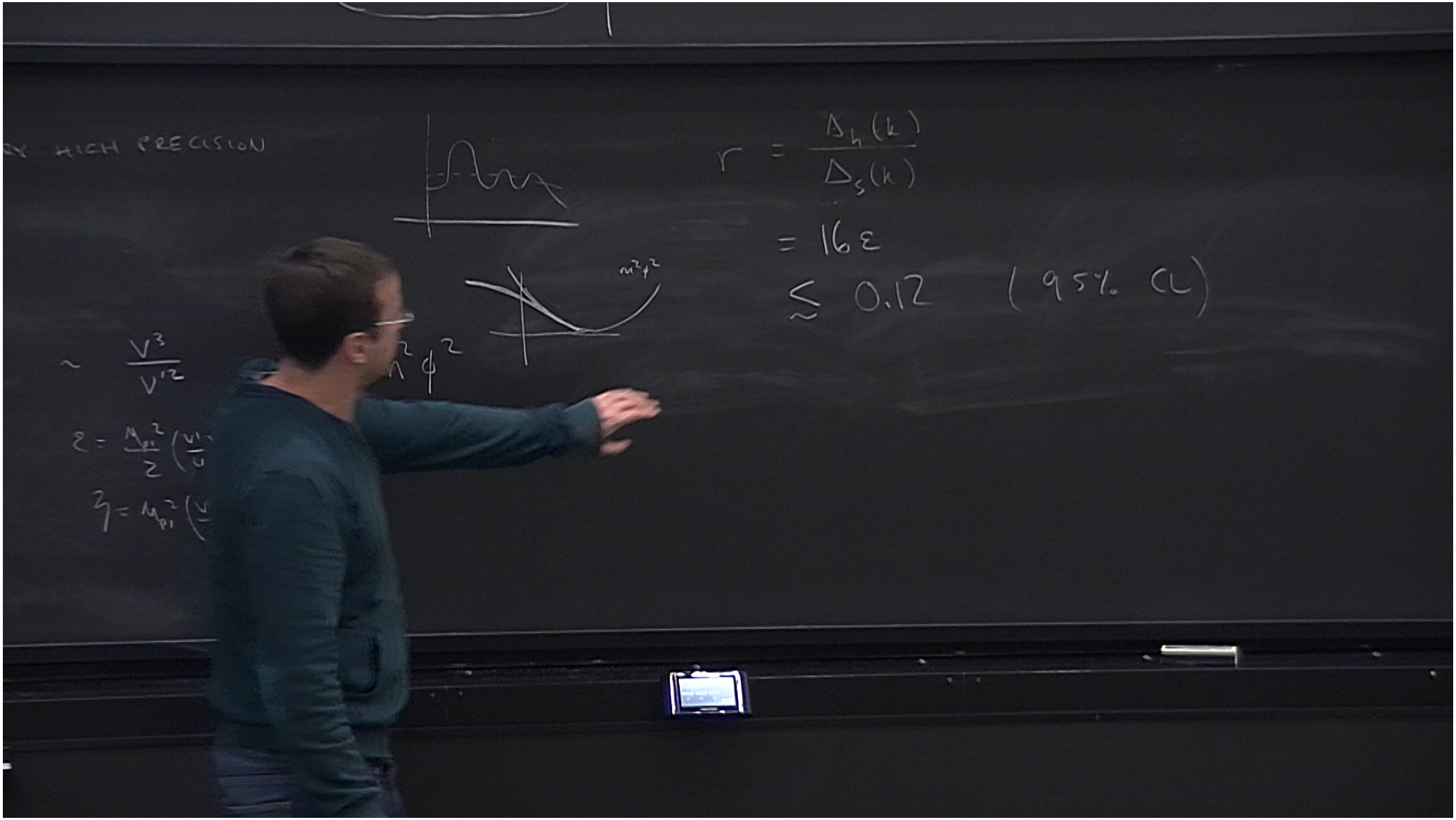
$\frac{1}{aH}$ = "COMOVING HORIZON"

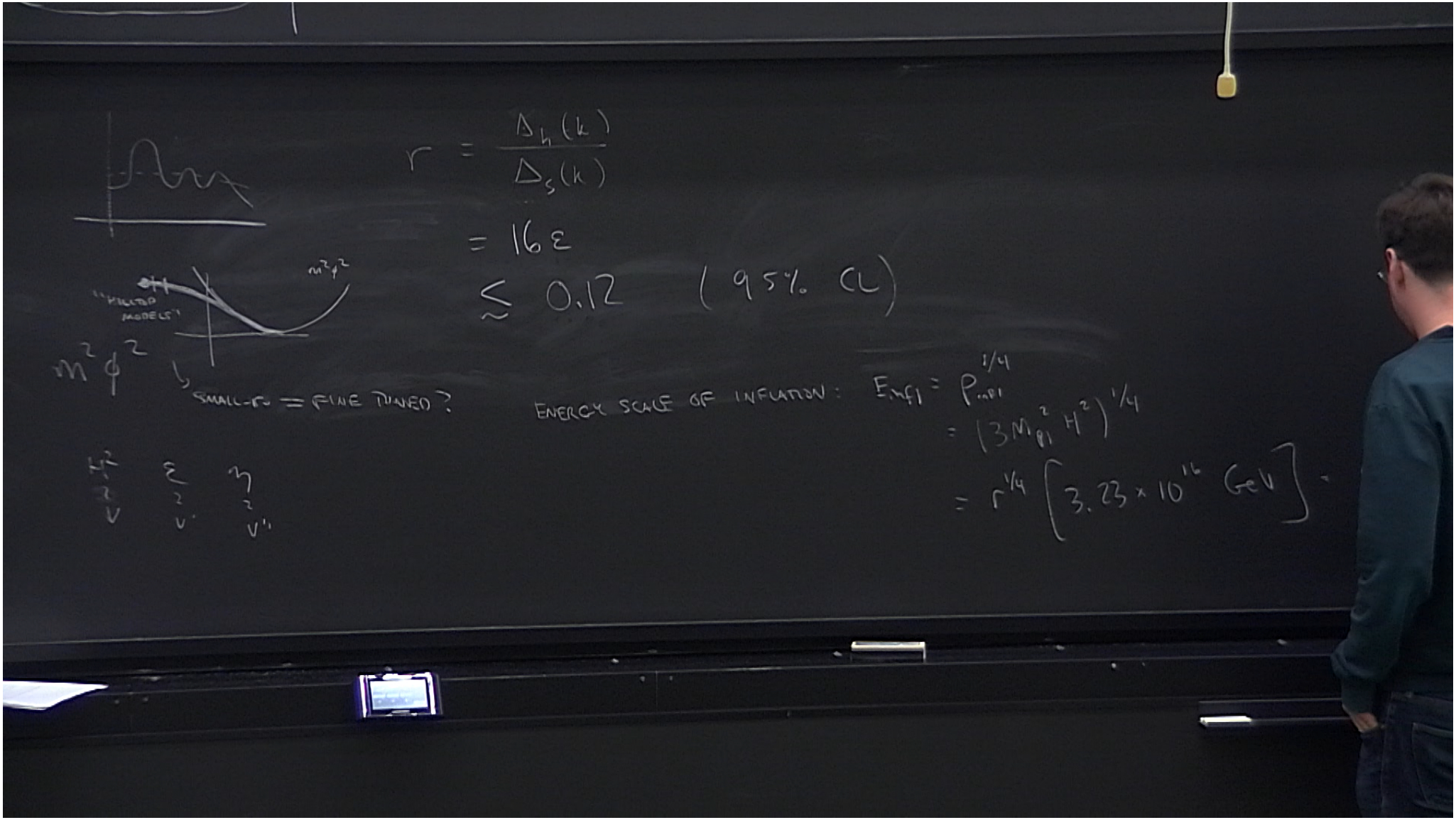
$= \dot{}$

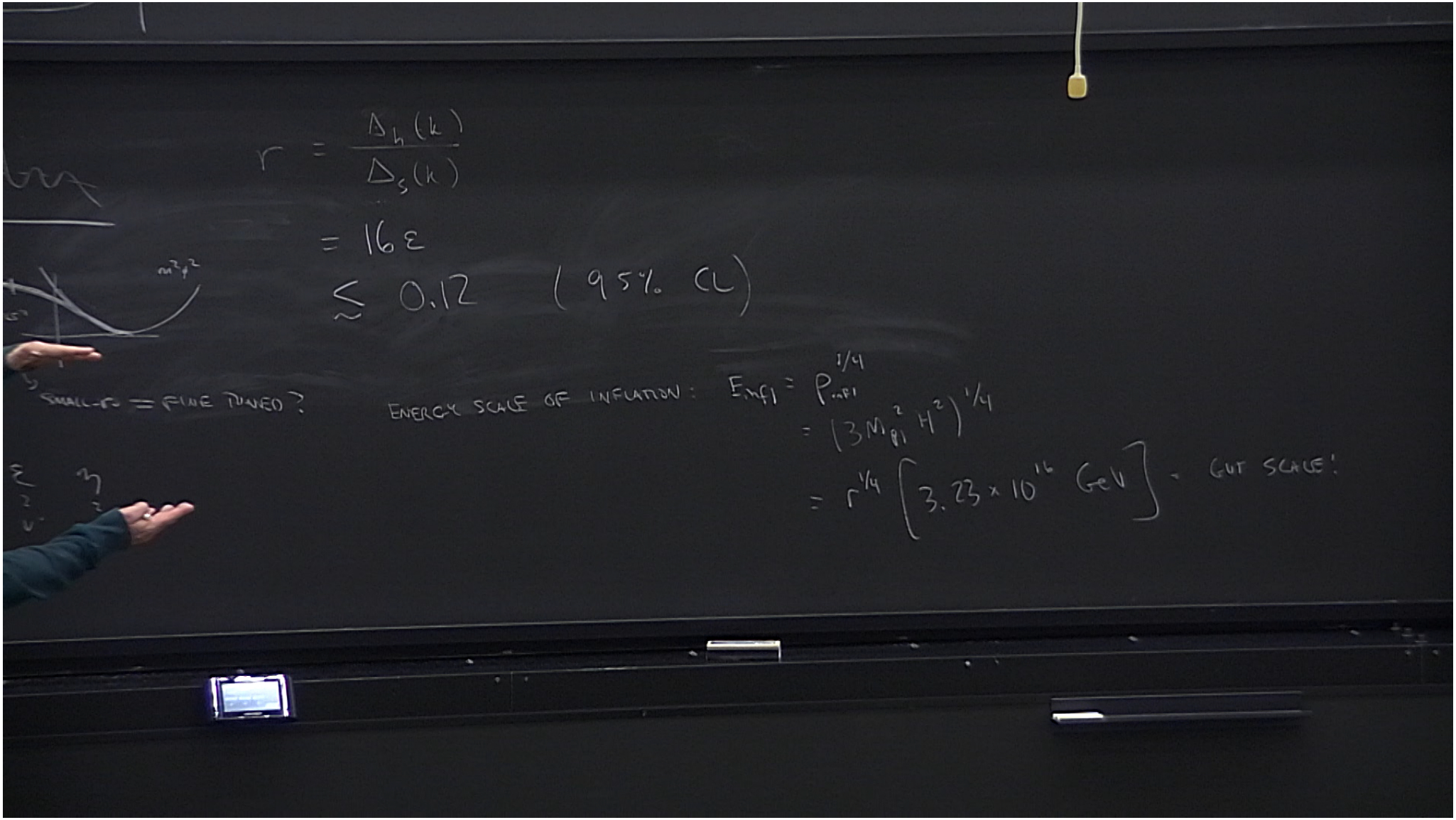

$S = \frac{1}{2}\int d\tau\, d^3x\, \frac{\phi'^2 a^2}{H^2}\left((\partial_\tau \zeta)^2 - (\partial_i \zeta)^2\right) + \frac{M_{Pl}^2}{8}\int d\tau\, d^3x\, a^2\left[(\partial_\tau \gamma_{ij})^2 - (\partial_i \gamma_{jk})^2\right]$
$P_h(k) =$

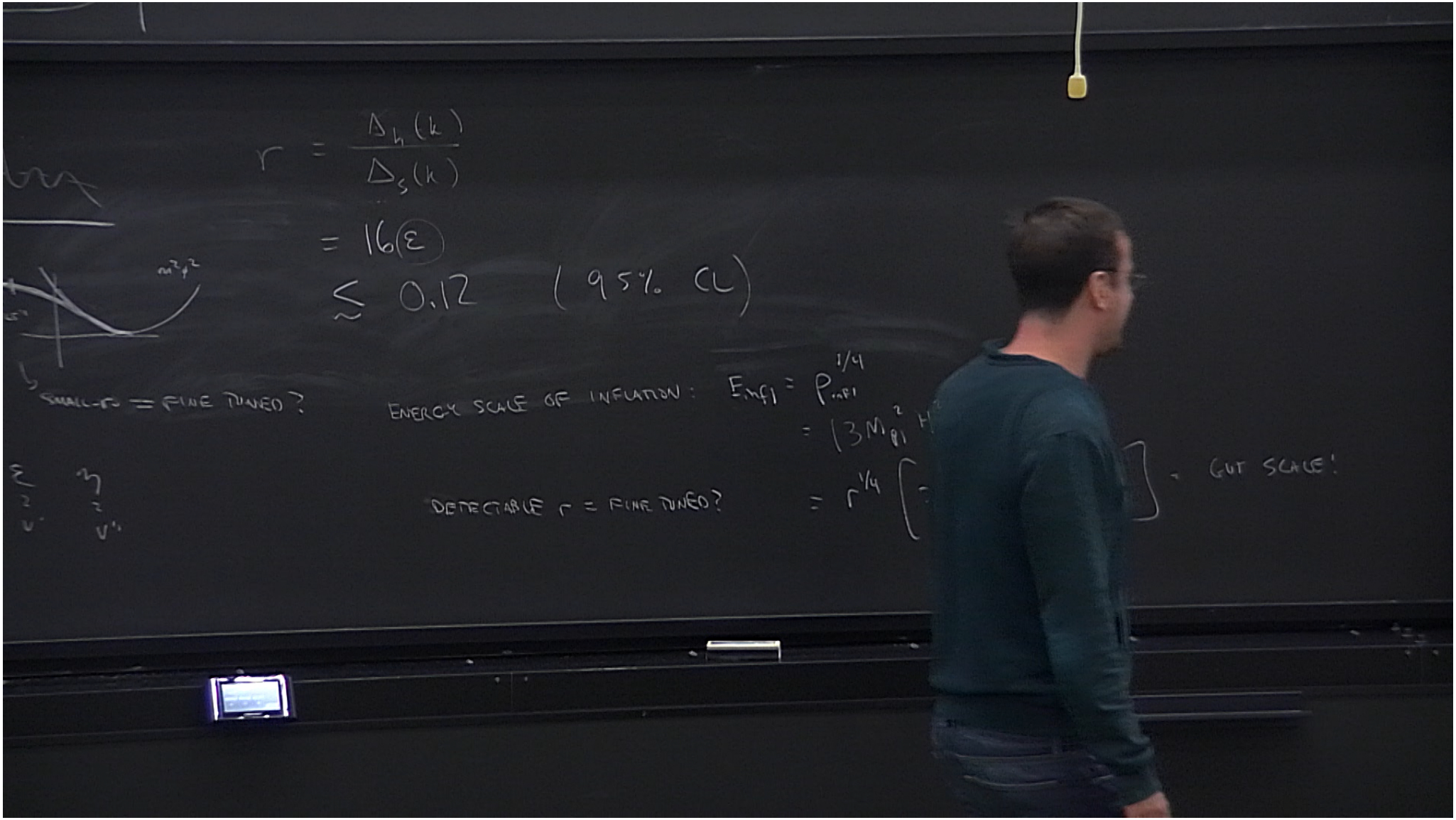

$-(\partial_i S)^2\Big) + \frac{M_{Pl}^2}{8}\int d\tau\, d^3x\, a^2\left[(\partial_\tau \gamma_{ij})^2 - (\partial_i \gamma_{jk})^2\right]$
$[\gamma, p_\gamma] = i$
$p_\gamma = \frac{\partial S}{\partial \dot{\gamma}}$
E EXPANSION HISTORY":
$a'(\tau)$
$\frac{2}{\pi^2 M_{Pl}^2}(H^2)_*$
"$k = aH$"
$\Delta_S(k) = \frac{1}{8\pi^2 M_{Pl}^2}\left(\frac{H^2}{\varepsilon}\right)_*$
$\Delta_S(k) \approx \Delta_S(k_*)\left(\frac{k}{k_*}\right)^{n_S - 1}$
$\Delta_h(k) = \Delta_h(k_*)\left(\frac{k}{k_*}\right)^{n_T}$
$n_S - 1 = \frac{d\log \Delta_S(k)}{d\log k} = -6\varepsilon + 2\eta$


"GAUSSIAN ADIABATIC" : OBSERVED TO VERY HIGH PRECISION
$\Delta_s(k) = \frac{1}{8\pi^2 M_{Pl}^2} \left(\frac{H^2}{\varepsilon}\right)_*$
$= (2.11 \pm 0.05) \times 10^{-9}$
$\sim \frac{V^3}{V'^2}$
$n_s - 1 = -6\varepsilon + 2\eta$
$= -0.033 \pm 0.004$
$\varepsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2$
$\eta = M_{Pl}^2\left(\frac{V'}{V}\right)$
$m^2\phi^2$
r =


HIGH PRECISION
$r = \frac{\Delta_h(k)}{\Delta_s(k)}$
$= 16\varepsilon$
$\lesssim 0.12$ (95% CL)
$m^2\phi^2$
$\sim \frac{V^3}{V'^2}$
$\varepsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right.$
$\eta = M_{Pl}^2\left(\frac{V''}{V}\right.$


$r = \frac{\Delta_h(k)}{\Delta_s(k)}$
$= 16\varepsilon$
$\lesssim 0.12$ (95% CL)
"HILLTOP MODELS"
$m^2\phi^2$
$m^2\phi^2$
SMALL-r = FINE TUNED?
ENERGY SCALE OF INFLATION: $E_{Infl} = P_{mPl}^{1/4}$
$= (3M_{Pl}^2 H^2)^{1/4}$
$= r^{1/4}\left[3.23 \times 10^{16}\ \text{GeV}\right]$.


$r = \frac{\Delta_h(k)}{\Delta_s(k)}$
$= 16\varepsilon$
$\lesssim 0.12$ (95% CL)
$m^2\phi^2$
SMALL-r = FINE TUNED?
ENERGY SCALE OF INFLATION: $E_{infl} = P_{infl}^{1/4}$
$= (3M_{Pl}^2 H^2)^{1/4}$
$= r^{1/4}\left[3.23 \times 10^{16}\ \text{GeV}\right]$ - GUT SCALE!


$r = \frac{\Delta_h(k)}{\Delta_s(k)}$
$= 16\varepsilon$
$\lesssim 0.12$ (95% CL)
$m^2\phi^2$
SMALL-r = FINE TUNED?
ENERGY SCALE OF INFLATION: $E_{infl} = P_{infl}^{1/4}$
$= (3M_{Pl}^2 H^2$
DETECTABLE r = FINE TUNED?
$= r^{1/4}$
GUT SCALE!