Title: PSI 16/17 Explorations in Cosmology (Kendrick Smith) - Lecture 13

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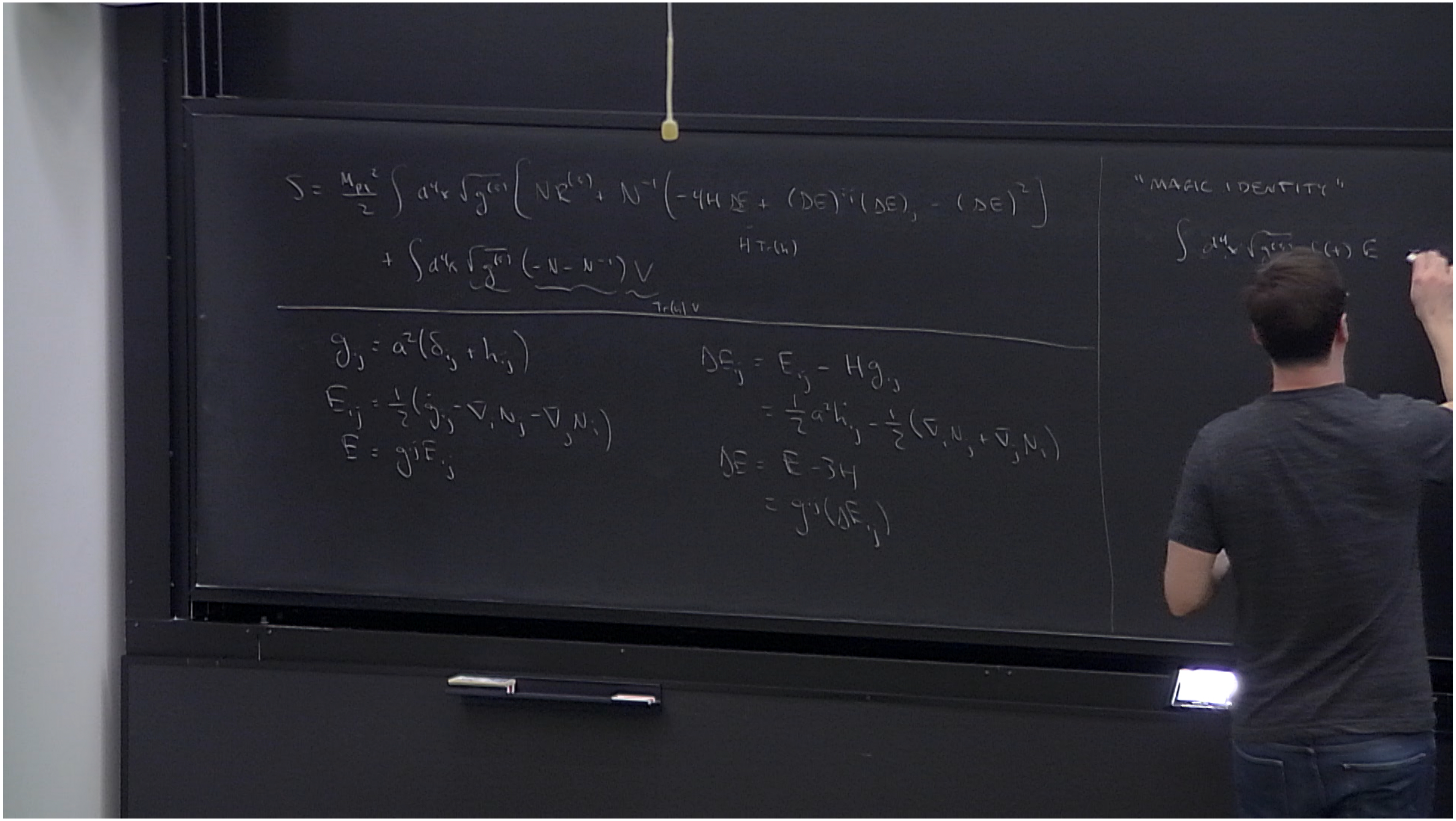
Abstract:


$S = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{g^{(3)}} \left[N R^{(3)} + N^{-1} \left(-4H \Delta E + (\Delta E)^{ij} (\Delta E)_{ij} - (\Delta E)^2 \right) \right]$
$\dot{H}\,Tr(h)$
$+ \int d^4x \sqrt{g^{(3)}} \left(-N - N^{-1} \right) V$
$Tr(h)\,V$
$g_{ij} = a^2 (\delta_{ij} + h_{ij})$
$E_{ij} = \frac{1}{2} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)$
$E = g^{ij} E_{ij}$
$\Delta E_{ij} = E_{ij} - H g_{ij}$
$= \frac{1}{2} a^2 \dot{h}_{ij} - \frac{1}{2} (\nabla_i N_j + \nabla_j N_i)$
$\Delta E = E - 3H$
$= g^{ij} (\Delta E_{ij})$
"MAGIC IDENTITY"
$\int d^4x \sqrt{g^{(3)}}\, f(t)\, E$

$(\Delta E)^2]$

$\ldots_j + \nabla_j N_i)$

"MAGIC IDENTITY"

WHERE f IS ANY FUNCTION OF t BUT NOT x

$$\int d^4x\, \sqrt{g^{(3)}}\, f(t)\, E = \int d^4x\, \sqrt{g^{(3)}}\, f(t)\, g^{ij}\left[\tfrac{1}{2}\dot{g}_{ij} - \tfrac{1}{2}\cancel{\nabla_i N_j}_{\,0} - \tfrac{1}{2}\cancel{\nabla_j N_i}_{\,0}\right]$$

$$= \int d^4x\, \underbrace{\sqrt{g^{(3)}}\left(\tfrac{1}{2} g^{ij}\dot{g}_{ij}\right)}_{\frac{d}{dt}\sqrt{g^{(3)}}}\, f(t)$$

$$= \int d^4x\, \sqrt{g^{(3)}}\left[-f'(t)\right]$$

$$\int d^4x\, \sqrt{g^{(3)}}\, f(t)\, \Delta E = \int d^4x\, \sqrt{g^{(3)}}\left[-f'(t) - 3H f(t)\right]$$

g_{00} g^{00}

$-dt^2 + a(t)^2 dx_i dx_i$ $\quad \delta_{ij} + h_{ij}$ $\quad R^{(3)} = \mathcal{O}(h_{ij})$ $\quad \int d^3x \sqrt{g^{(3)}}\, R^{(3)} = \mathcal{O}(h_{ij}^2)$

$N = 1 + \psi$ PERTURBATIONS ARE $\{\psi, N^i, h_{ij}\}$

$$S = \frac{M_{pl}^2}{2}\int d^4x \sqrt{g^{(3)}}\left[(1+\psi)R^{(3)} + (1-\psi)\left(-4H\,\Delta E + (DE)^i(DE)_j - (DE)^2\right)\right] + \int d^4x \sqrt{g^{(3)}}\left[-2-\psi^2\right]V$$

BOTH TERMS ARE SECOND ORDER!

APPLY MAGIC IDENTITY WITH $f = -4H$

$$\frac{M_{pl}^2}{2}\int d^4x \sqrt{g^{(3)}}\,(-4H\,\Delta E) = \frac{M_{pl}^2}{2}\int d^4x \sqrt{g^{(3)}}\,(4\dot{H} + 12H^2)$$

$$= \frac{M_{pl}^2}{2}\int d^4x \sqrt{g^{(3)}}\,(2V)$$

$$M_{pl}^2 H^2 = \frac{\rho}{3} = \frac{1}{6}\dot{\varphi}^2 + \frac{1}{3}V$$

$$M_{pl}^2 \dot{H} = -\frac{\rho + p}{2} = -\frac{1}{2}\dot{\varphi}^2$$

$$S = \frac{M_{pl}^2}{2}\int d^4x \sqrt{g^{(3)}}\left[(1+\psi)R^{(3)} + 4H\psi(DE) + (DE)^i_j (DE)^j_i - (DE)^2\right] + \int d^4x \sqrt{g^{(3)}}\left(-\psi^2 V\right)$$

ALL TERMS MANIFESTLY SECOND ORDER

EXPAND TO SECOND ORDER + FIX GAUGE $h_{ij} = 2\zeta\delta_{ij} + \gamma_{ij}$ WHERE $\gamma_{ii} = 0$ AND $\partial_j \gamma_{ij} = 0$

$$S = \underbrace{S_0}_{\text{SCALAR}} + \underbrace{S_2}_{\text{TENSOR}}$$

$$S_2 = \frac{M_{pl}^2}{8}\int d^4x \left(a^3 \dot\gamma_{ij}\dot\gamma_{ij} - a(\partial_k \gamma_{ij})(\partial_k \gamma_{ij})\right)$$

$$S_0 = \frac{M_{pl}^2}{2}\int d^4x \left(-6a^3\dot\zeta^2 + 2a(\partial_i\zeta)^2\right) + \frac{M_{pl}^2}{2}\int d^4x \left[\psi\left(12a^3 H\dot\zeta - 4a\partial^2\zeta\right) + (\partial_i N^i)(4a^3\dot\zeta)\right]$$

$$+ \frac{M_{pl}^2}{2}\int d^4x\, a^3\left[-4H\psi(\partial_i N^i) + \frac{1}{2}(\partial_i N^j)(\partial_i N^j) - \frac{1}{2}(\partial_i N^i)^2\right] + \int d^4x\, a^3\left(-\psi^2 V\right)$$

ψ

ALL TERMS MANIFESTLY SECOND ORDER

$\partial_j \gamma_j = 0$

$$\psi = \frac{\dot{\zeta}}{H}$$

$$N^i = \partial_i\left[-\frac{1}{a^2 H}\zeta + \frac{\dot{\phi}^2}{2M_{pl}^2 H^2}\partial^{-2}\dot{\zeta}\right]$$

$$\ldots a\partial^2\zeta) + (\partial_i N^i)(4a^3\dot{\zeta})\Big]$$

$$\ldots)^2\Big] + \int d^4x\, a^3\left(-\psi^2 V\right)$$

$$S_0 = \frac{1}{2}\int d^4x \, \frac{\dot{\phi}^2}{H^2}\left(a^3\dot{\zeta}^2 - a(\partial_i\zeta)(\partial_i\zeta)\right)$$

$\xi \quad 1$

$\gamma_{ij} \quad 2$

$g_{\mu\nu}$	10
ϕ	+1
GAUGE	−4
NONDYNAMICAL	−4
	3

$\gamma_{ij} \quad 2$

$\dot{\phi} = \dot{\bar{\phi}}$

$V(\phi) = V(\bar{\phi})$

$\delta\phi \neq 0 \Rightarrow \delta\phi = 0$

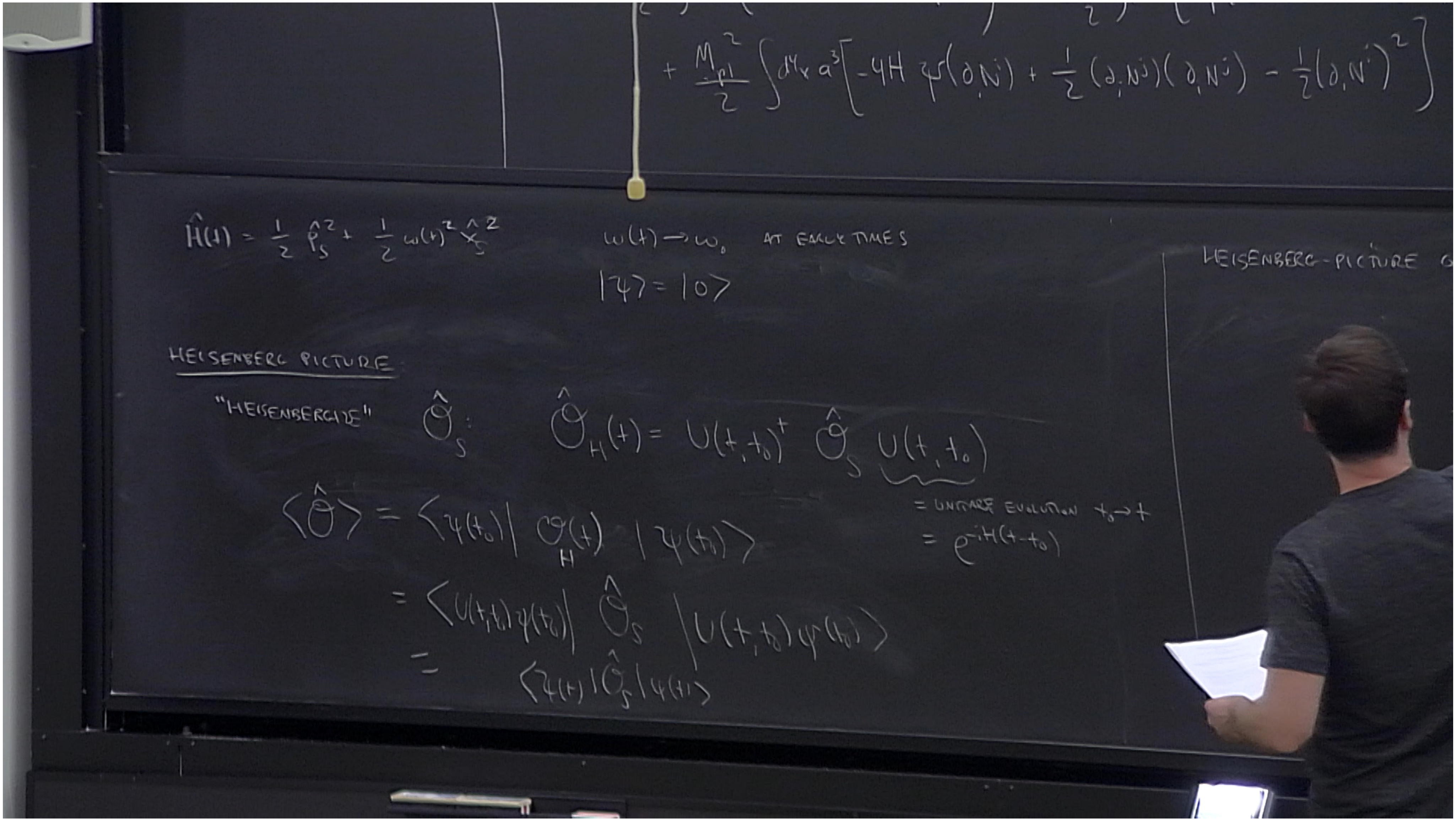
$\bar{\phi}$

ϕ

$\delta\phi = 0$

$\partial_i \gamma_{ij} = 0$

$g_{ii} = 0$


$+ \frac{M_{pl}^2}{2} \int d^4x\, a^3 \left[-4H\, \psi (\partial_i N^i) + \frac{1}{2} (\partial_i N^j)(\partial_i N^j) - \frac{1}{2} (\partial_i N^i)^2 \right]$
$\hat{H}(t) = \frac{1}{2} \hat{p}_S^2 + \frac{1}{2} \omega(t)^2 \hat{x}_S^2$
$\omega(t) \to \omega_0$ AT EARLY TIMES
$|\psi\rangle = |0\rangle$
HEISENBERG-PICTURE
HEISENBERG PICTURE:
"HEISENBERGIZE"
$\hat{O}_S$: $\hat{O}_H(t) = U(t,t_0)^\dagger\, \hat{O}_S\, U(t,t_0)$
= UNITARY EVOLUTION $t_0 \to t$
$= e^{-iH(t-t_0)}$
$\langle \hat{O} \rangle = \langle \psi(t_0) | \hat{O}_H(t) | \psi(t_0) \rangle$
$= \langle U(t,t_0)\psi(t_0) | \hat{O}_S | U(t,t_0)\psi(t_0) \rangle$
$= \langle \psi(t) | \hat{O}_S | \psi(t) \rangle$

HEISENBERG-PICTURE OPERATORS SATISFY "CLASSICAL" EQS OF MOTION.

$$\frac{d\hat{x}_H(t)}{dt} = \hat{p}_H(t) \qquad (*)$$

$$\frac{d\hat{p}_H(t)}{dt} = -\omega(t)^2 \hat{x}_H(t)$$

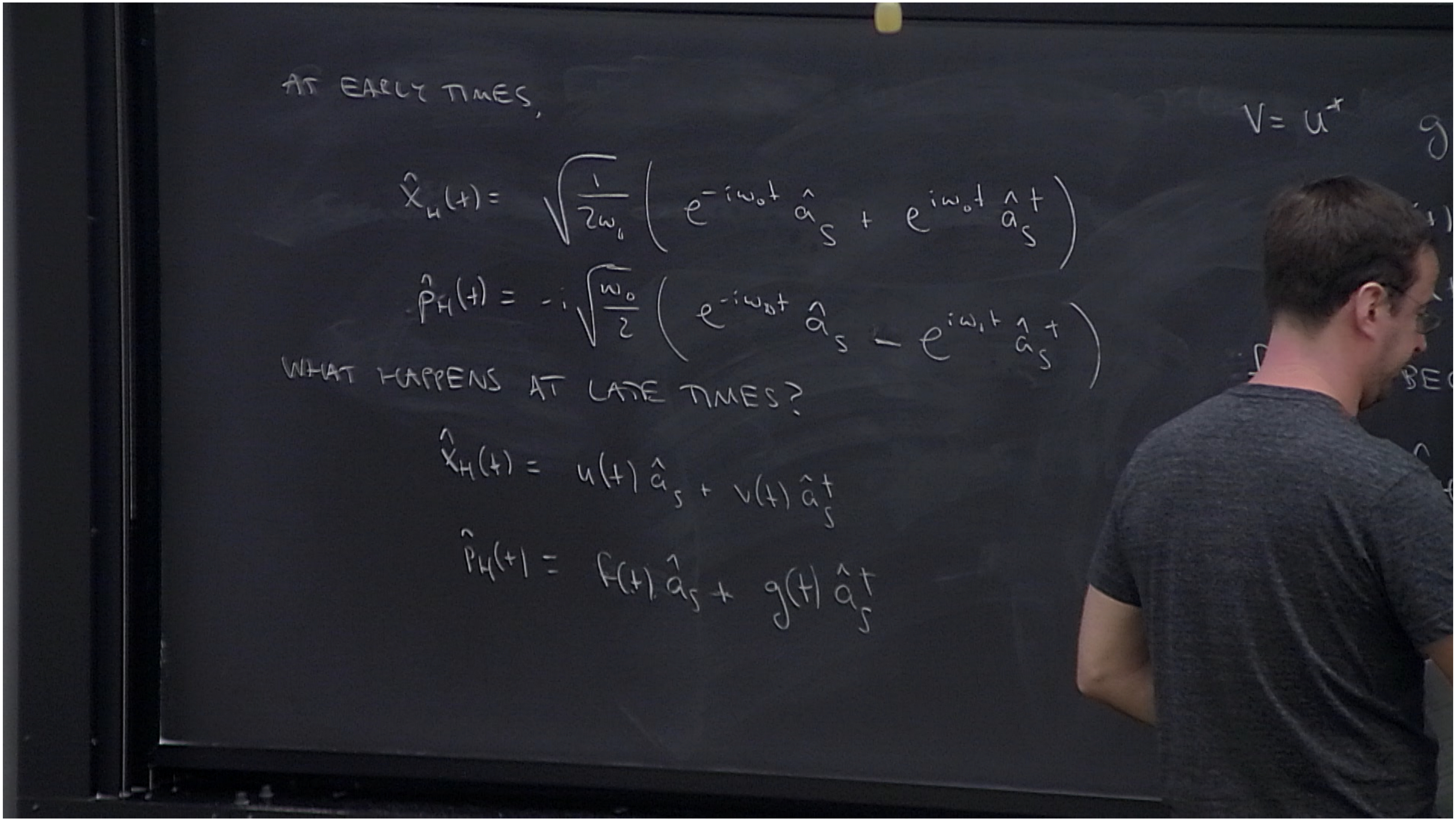
$t_0 \to t$

IN GENERAL, $\frac{d}{dt}\hat{O}_H(t) = i\left[\hat{H}_H(t), \hat{O}_H(t)\right]$

E.G. $\frac{d\hat{x}_H}{dt} = i\left[\frac{1}{2}\hat{p}_H(t)^2 + \frac{1}{2}\omega(t)^2\hat{x}_H(t)^2, \hat{x}_H(t)\right]$

$$\left[\hat{x}_H(t), \hat{p}_H(t)\right] = i$$

$$= i\left(\frac{1}{2}\right)\left(-2i\,\hat{p}_H(t)\right) = \hat{p}_H(t)$$


AT EARLY TIMES,
$\hat{x}_H(t) = \sqrt{\frac{1}{2\omega_0}}\left(e^{-i\omega_0 t}\hat{a}_S + e^{i\omega_0 t}\hat{a}_S^\dagger\right)$
$\hat{p}_H(t) = -i\sqrt{\frac{\omega_0}{2}}\left(e^{-i\omega_0 t}\hat{a}_S - e^{i\omega_0 t}\hat{a}_S^\dagger\right)$
WHAT HAPPENS AT LATE TIMES?
$\hat{x}_H(t) = u(t)\hat{a}_S + v(t)\hat{a}_S^\dagger$
$\hat{p}_H(t) = f(t)\hat{a}_S + g(t)\hat{a}_S^\dagger$
$V = u^*$

$v = u^*$ $\quad g = f^*$ BECAUSE $\hat{x}_H, \hat{p}_H$ ARE HERMITIAN

$$\hat{x}_H(t) = u(t)\,\hat{a}_s + u(t)^*\,\hat{a}_s^\dagger$$

$$\hat{p}_H(t) = f(t)\,\hat{a}_s + f(t)^*\,\hat{a}_s^\dagger$$

$f = \dot{u}$ BECAUSE $\dfrac{d\hat{x}_H}{dt} = \hat{p}_H$

$$\hat{x}_H(t) = u(t)\,\hat{a}_s + u(t)^*\,\hat{a}_s^\dagger \qquad \leftarrow \text{SOLUTION OF THE THEORY}$$

$$\hat{p}_H(t) = \dot{u}(t)\,\hat{a}_s + \dot{u}(t)^*\,\hat{a}_s^\dagger$$

EOM FOR $\hat{p}_H$: $\ddot{u}(t) = -\omega(t)^2\,u(t)$

INITIAL CONDITION: $u(t) = \dfrac{1}{(2\omega_0)^{1/2}} e^{-i\omega_0 t}$

$\ldots = u(t)\,\hat{a}_s + v(t)\,\hat{a}_s^\dagger$

$\hat{p}_H(t) = f(t)\,\hat{a}_s + g(t)\,\hat{a}_s^\dagger$

$\hat{x}_H(t) = u(t)\,\hat{a}_s + u(t)^*\,\hat{a}_s^\dagger$ ← SOLUTION OF THE THEORY

$\hat{p}_H(t) = \dot{u}(t)\,\hat{a}_s + \dot{u}(t)^*\,\hat{a}_s^\dagger$

EOM FOR $\hat{p}_H$: $\ddot{u}(t) = -\omega(t)^2\,u(t)$

INITIAL CONDITION: $u(t) = \frac{1}{(2\omega_0)^{1/2}} e^{-i\omega_0 t}$

CALCULATE ANY EXPECTATION VALUE, E.G $\langle \psi | \hat{x}^2 | \psi \rangle$ AT TIME t

$$\langle \psi(t_0) | \hat{x}_H(t)^2 | \psi(t_0) \rangle = \langle 0 | \big(u\,\hat{a}_s + u^* \underbrace{\hat{a}_s^\dagger}_{=0}\big)\big(u\,\underbrace{\hat{a}_s}_{=0} + u^*\,\hat{a}_s^\dagger\big) | 0 \rangle$$

$$[a, a^\dagger] = 1$$

$$= u(t)\,u(t)^* \underbrace{\langle 0 | \hat{a}_s \hat{a}_s^\dagger | 0 \rangle}_{=1}$$

$$= |u(t)|^2$$