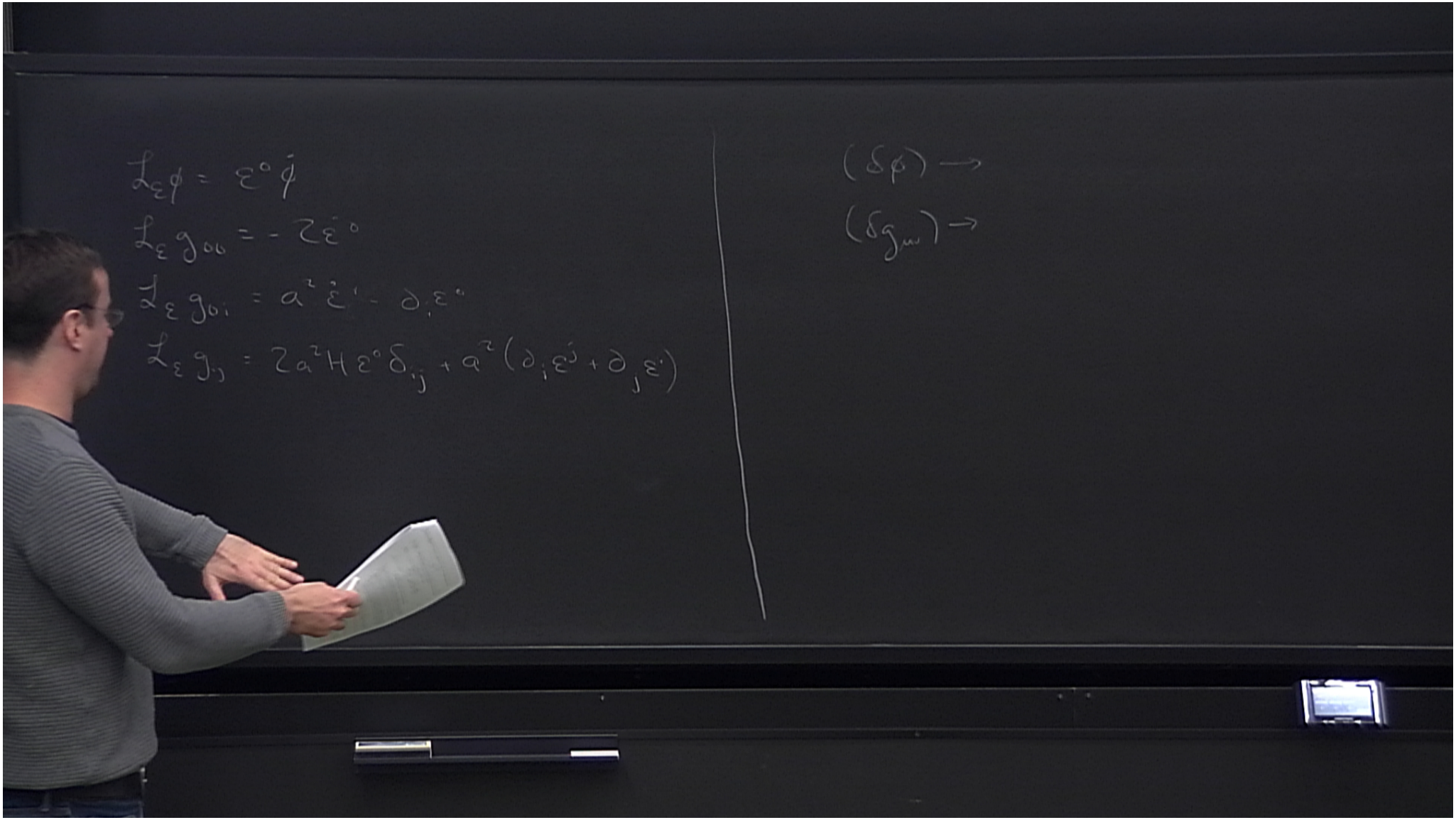
Title: PSI 16/17 Explorations in Cosmology (Kendrick Smith) - Lecture 11

Date: Apr 25, 2017 10:15 AM

URL: http://pirsa.org/17040064

Abstract:


$\mathcal{L}_\xi \phi = \xi^0 \dot{\phi}$
$\mathcal{L}_\xi g_{00} = -2\dot{\xi}^0$
$\mathcal{L}_\xi g_{0i} = a^2 \dot{\xi}^i - \partial_i \xi^0$
$\mathcal{L}_\xi g_{ij} = 2a^2 H \xi^0 \delta_{ij} + a^2 (\partial_i \xi^j + \partial_j \xi^i)$
$(\delta\phi) \rightarrow$
$(\delta g_{\mu\nu}) \rightarrow$

$$(\delta\phi) \to (\delta\phi) + \mathcal{L}_\varepsilon \phi + \cdots = (\delta\phi) + \dot{\phi}\,\varepsilon^0$$

$$(\delta g_{\mu\nu}) \to (\delta g_{\mu\nu}) + \mathcal{L}_\varepsilon g_{\mu\nu} + \cdots$$

GAUGE FIXING

$$\delta\phi = 0$$

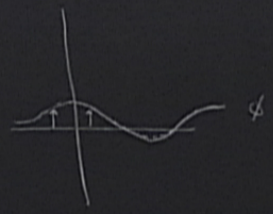


$$(\delta\phi) \to (\delta\phi) + \mathcal{L}_\varepsilon \phi + \cdots = (\delta\phi) + \dot{\phi}\,\varepsilon^0$$

$$(\delta g_{\mu\nu}) \to (\delta g_{\mu\nu}) + \mathcal{L}_\varepsilon g_{\mu\nu} + \cdots$$

GAUGE FIXING

$$\delta\phi = 0$$



$(\delta\phi) \to (\delta\phi) + \mathcal{L}_\xi \phi + \cdots = (\delta\phi) + \dot{\phi}\,\varepsilon^0$

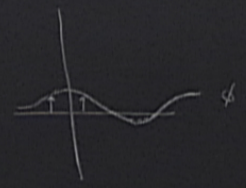
$(\delta g_{\mu\nu}) \to (\delta g_{\mu\nu}) + \mathcal{L}_\xi g_{\mu\nu} + \cdots$

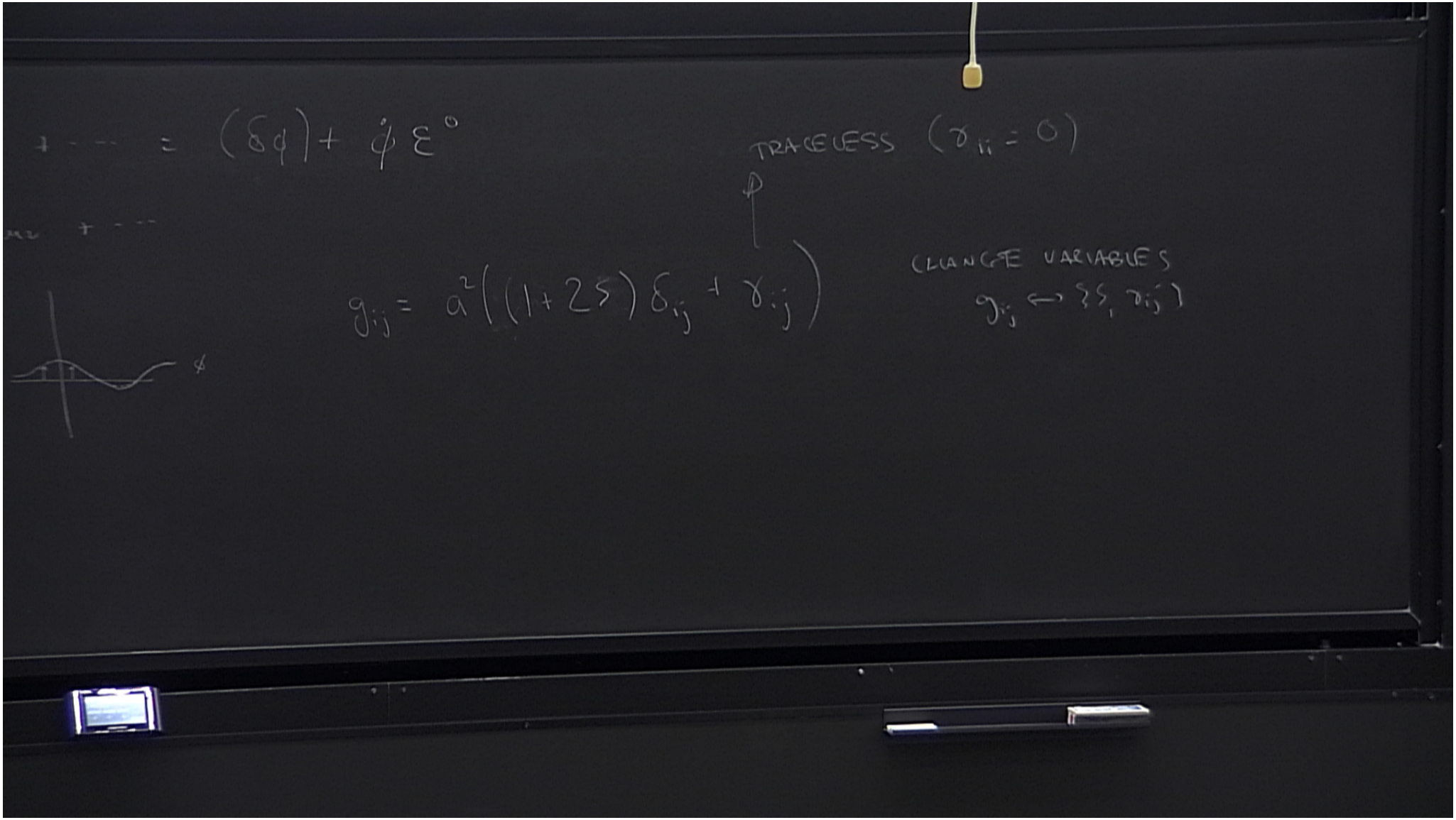
TRACELESS $(\gamma_{ii} = 0)$

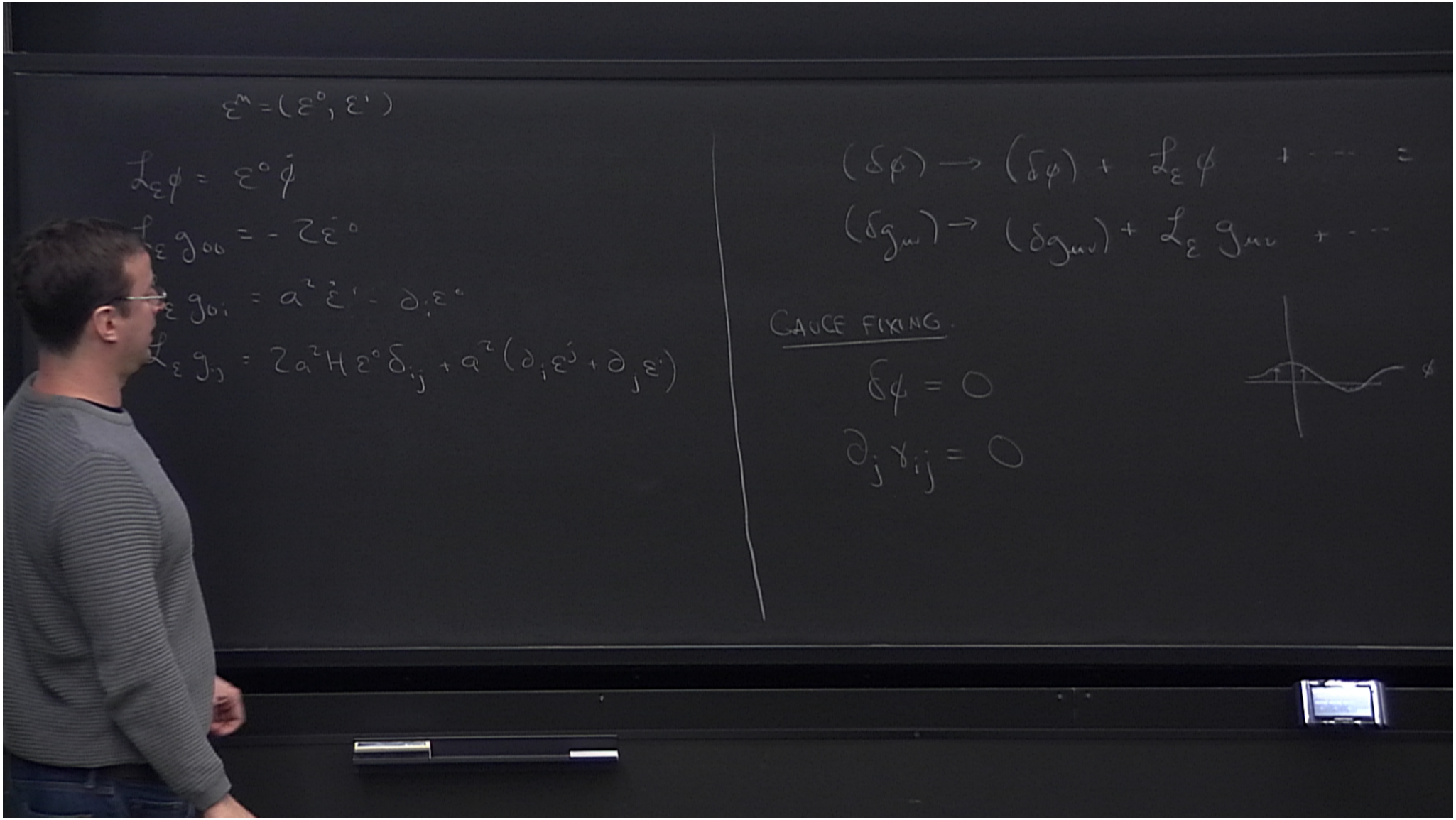
$g_{ij} = a^2\left((1+2\zeta)\delta_{ij} + \gamma_{ij}\right)$

GAUGE FIXING

$\delta\phi = 0$




$= (\delta\phi) + \dot{\phi}\,\varepsilon^0$
TRACELESS $(\gamma_{ii} = 0)$
$g_{ij} = a^2\left((1+2\zeta)\,\delta_{ij} + \gamma_{ij}\right)$
CHANGE VARIABLES
$g_{ij} \leftrightarrow \{\zeta, \gamma_{ij}\}$


$\xi^\mu = (\xi^0, \xi^i)$
$\mathcal{L}_\xi \phi = \xi^0 \dot{\phi}$
$\mathcal{L}_\xi g_{00} = -2\dot{\xi}^0$
$\mathcal{L}_\xi g_{0i} = a^2 \dot{\xi}^i - \partial_i \xi^0$
$\mathcal{L}_\xi g_{ij} = 2a^2 H \xi^0 \delta_{ij} + a^2(\partial_i \xi^j + \partial_j \xi^i)$
$(\delta\phi) \to (\delta\phi) + \mathcal{L}_\xi \phi + \cdots$
$(\delta g_{\mu\nu}) \to (\delta g_{\mu\nu}) + \mathcal{L}_\xi g_{\mu\nu} + \cdots$
GAUGE FIXING
$\delta\phi = 0$
$\partial_j \gamma_{ij} = 0$

$\varepsilon^\mu = (\varepsilon^0, \varepsilon^i)$

$\mathcal{L}_\varepsilon \phi = \varepsilon^0 \dot{\phi}$

$\mathcal{L}_\varepsilon g_{00} = -2\dot{\varepsilon}^0$

$\mathcal{L}_\varepsilon g_{0i} = a^2 \dot{\varepsilon}^i - \partial_i \varepsilon^0$

$\mathcal{L}_\varepsilon g_{ij} = 2a^2 H \varepsilon^0 \delta_{ij} + a^2 (\partial_i \varepsilon^j + \partial_j \varepsilon^i)$

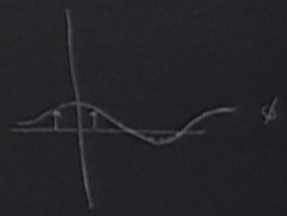
$(\delta\phi) \to (\delta\phi) + \mathcal{L}_\varepsilon \phi + \cdots$

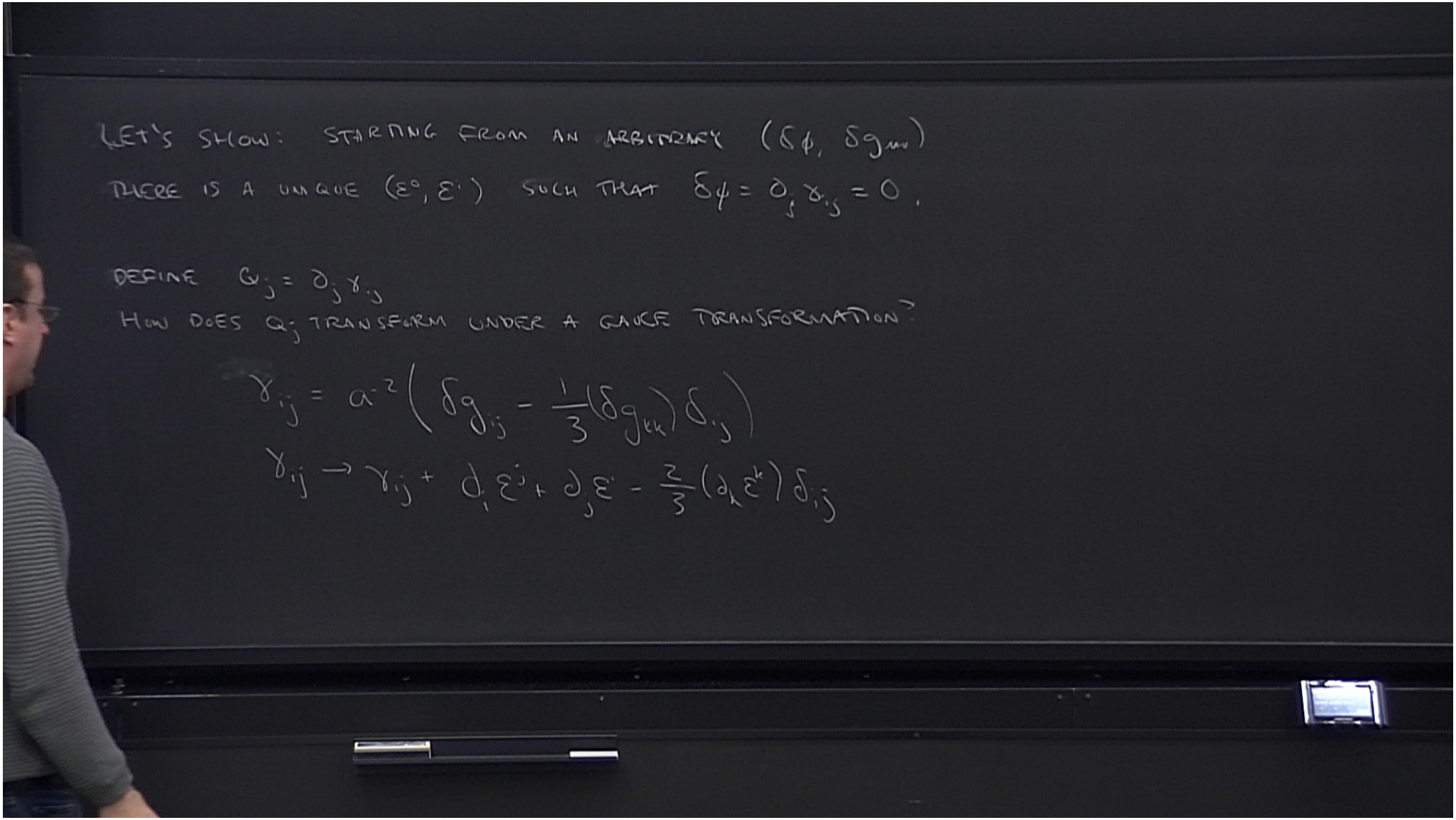
$(\delta g_{\mu\nu}) \to (\delta g_{\mu\nu}) + \mathcal{L}_\varepsilon g_{\mu\nu} + \cdots$

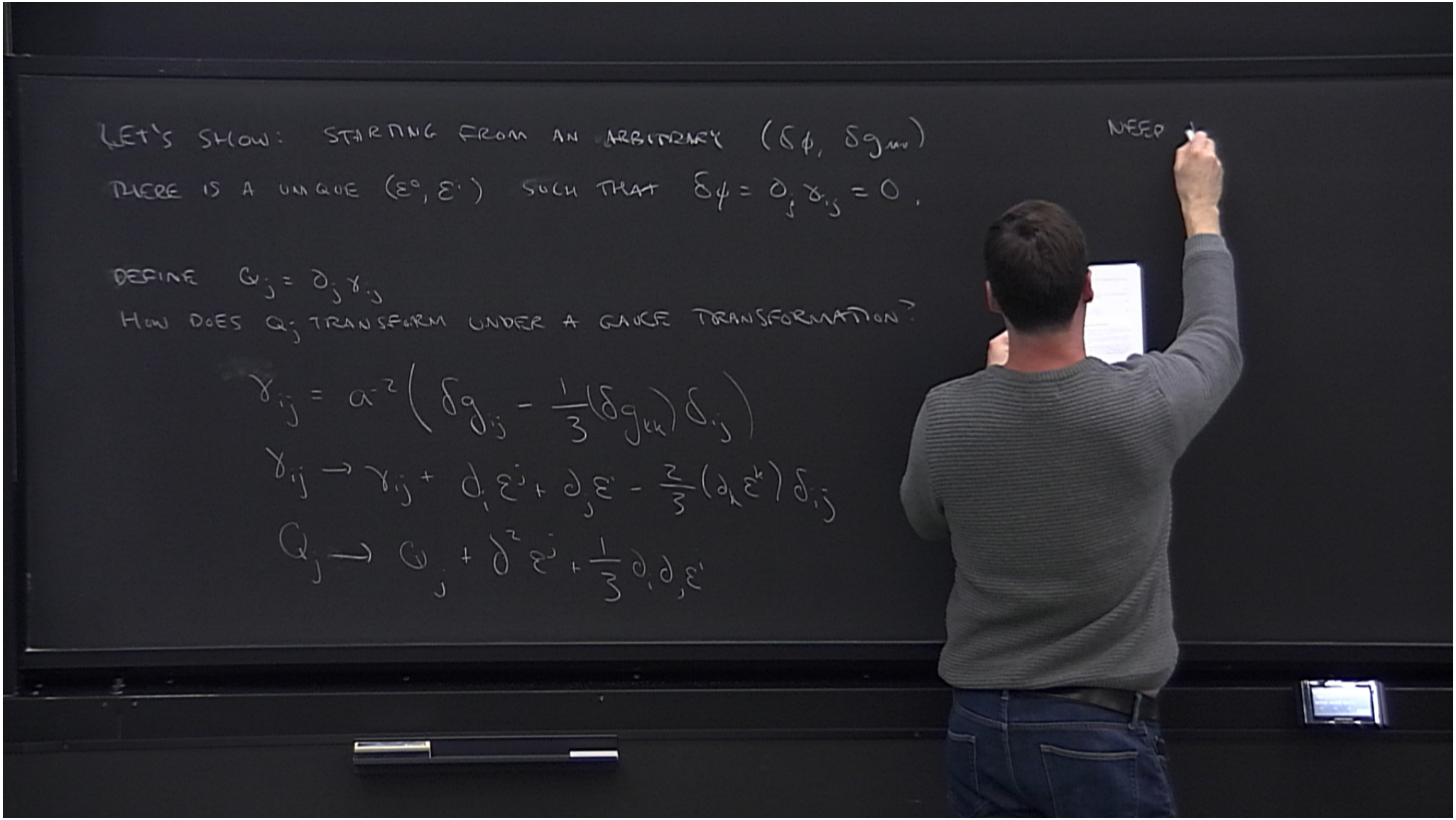
GAUGE FIXING

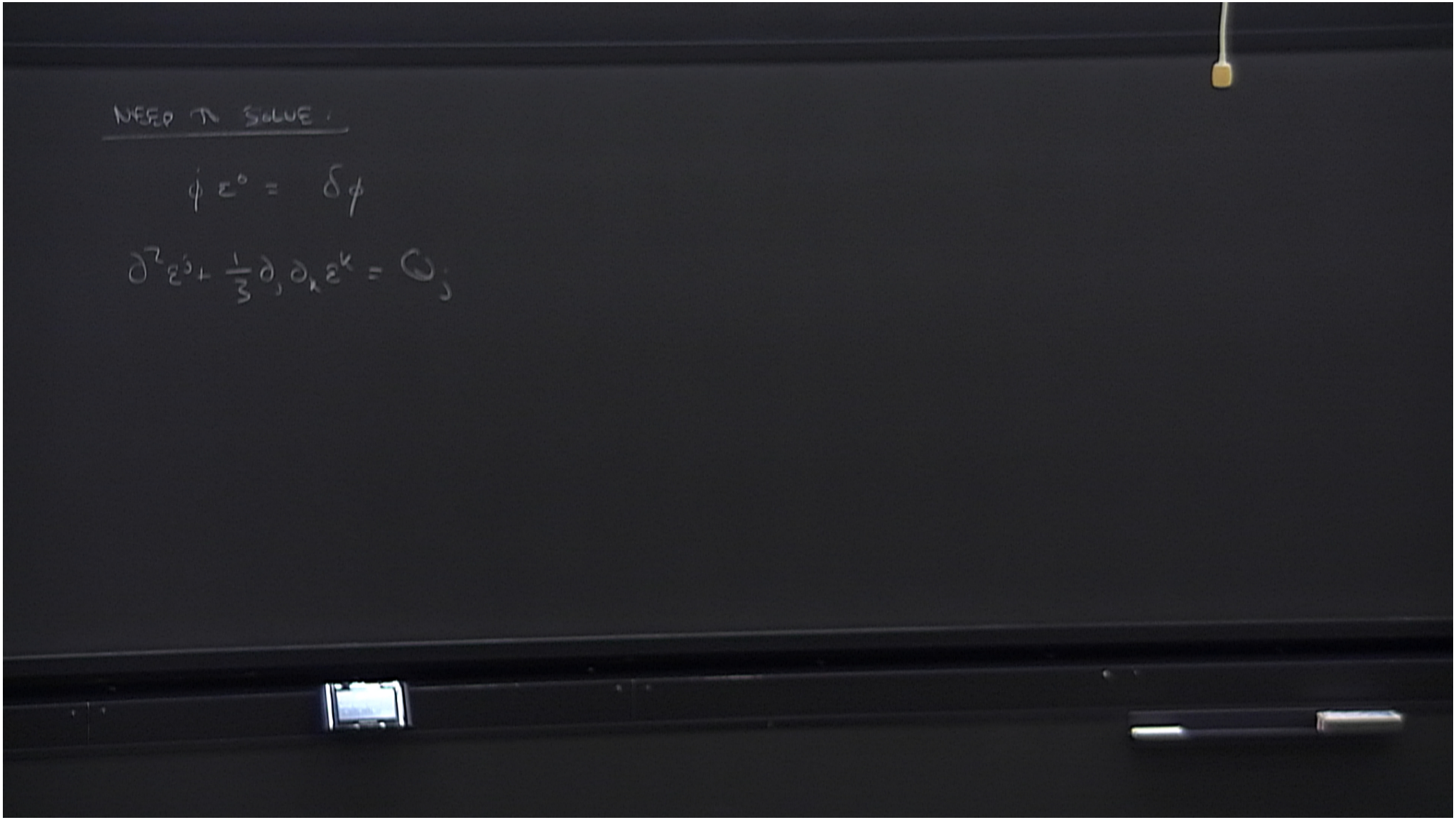
$\delta\phi = 0$

$\partial_j \gamma_{ij} = 0$




LET'S SHOW: STARTING FROM AN ARBITRARY $(\delta\phi, \delta g_{\mu\nu})$
THERE IS A UNIQUE $(\varepsilon^0, \varepsilon^i)$ SUCH THAT $\delta\phi = \partial_j \gamma_{ij} = 0$.
DEFINE $Q_j = \partial_j \gamma_{ij}$
HOW DOES Q_j TRANSFORM UNDER A GAUGE TRANSFORMATION?
$\gamma_{ij} = a^{-2}\left(\delta g_{ij} - \frac{1}{3}(\delta g_{kk})\delta_{ij}\right)$
$\gamma_{ij} \to \gamma_{ij} + \partial_i \varepsilon^j + \partial_j \varepsilon^i - \frac{2}{3}(\partial_k \varepsilon^k)\delta_{ij}$


LET'S SHOW: STARTING FROM AN ARBITRARY $(\delta\phi, \delta g_{\mu\nu})$
THERE IS A UNIQUE $(\varepsilon^0, \varepsilon^i)$ SUCH THAT $\delta\phi = \partial_j \gamma_{ij} = 0$.
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HOW DOES Q_j TRANSFORM UNDER A GAUGE TRANSFORMATION?
$\gamma_{ij} = a^{-2}\left(\delta g_{ij} - \frac{1}{3}(\delta g_{kk})\delta_{ij}\right)$
$\gamma_{ij} \to \gamma_{ij} + \partial_i \varepsilon_j + \partial_j \varepsilon_i - \frac{2}{3}(\partial_k \varepsilon^k)\delta_{ij}$
$Q_j \to Q_j + \nabla^2 \varepsilon_j + \frac{1}{3}\partial_j \partial_i \varepsilon_i$


NEED TO SOLVE:
$\dot{\phi}\,\varepsilon^0 = \delta\phi$
$\partial^2\varepsilon^j + \frac{1}{3}\partial_j\partial_k\varepsilon^k = \Omega_j$

NEED TO SOLVE:

$$\dot{\varphi}\,\varepsilon^0 = \delta\dot{\varphi} \quad \Rightarrow \quad \varepsilon^0 = \frac{\delta\varphi}{\dot{\varphi}}$$

$$\partial^2 \varepsilon^{ij} + \frac{1}{3}\partial_i \partial_j \varepsilon^k{}_k = \Omega_{ij}$$

$$-k^2 \varepsilon^{ij}(k) - \frac{1}{3} k_i k_j \varepsilon^k{}_k(k) = \Omega_{ij}(k)$$

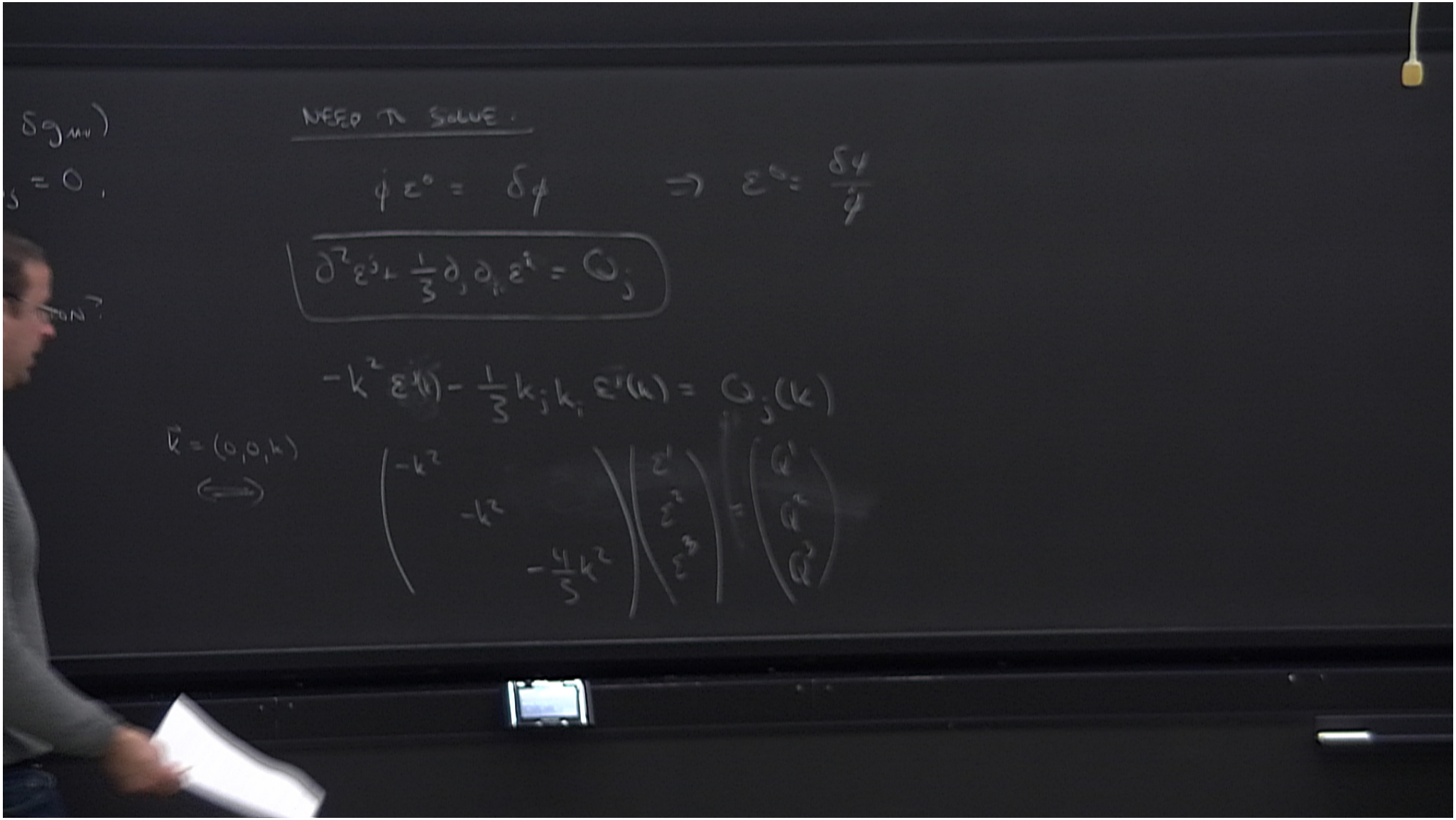
NEED TO SOLVE:

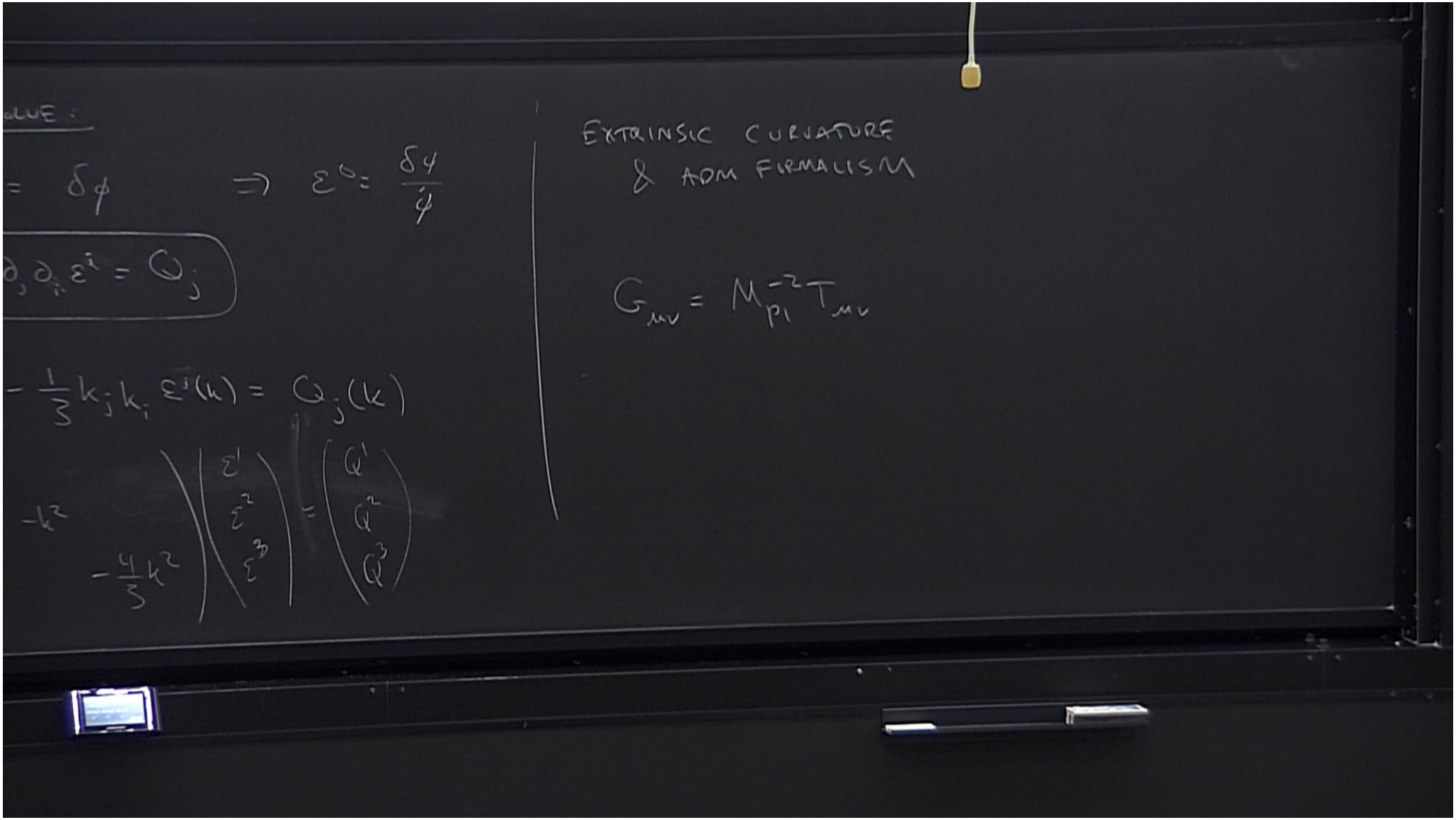
$$\dot{\varphi}\,\varepsilon^0 = \delta\dot{\varphi} \quad \Rightarrow \quad \varepsilon^0 = \frac{\delta\varphi}{\dot{\varphi}}$$

$$\partial^2 \varepsilon^i{}_j + \frac{1}{3}\partial_j \partial_i \varepsilon^k{}_k = \Omega_j$$

$$-k^2 \varepsilon^i{}_j(k) - \frac{1}{3} k_j k_i \varepsilon^k{}_k(k) = \Omega_j(k)$$

$$M^i{}_j(k)\,\varepsilon^j{}_i(k) = \Omega_j(k)$$


$\delta g_{\mu\nu}$)
$= 0$.
NEED TO SOLVE:
$\dot{\varphi}\,\varepsilon^0 = \delta\varphi \quad \Rightarrow \quad \varepsilon^0 = \frac{\delta\varphi}{\dot{\varphi}}$
$\partial^2 \varepsilon^j + \frac{1}{3}\partial_j \partial_i \varepsilon^i = Q_j$
$-k^2 \varepsilon^j(k) - \frac{1}{3} k_j k_i \varepsilon^i(k) = Q_j(k)$
$\vec{k} = (0,0,k)$
$\begin{pmatrix} -k^2 & & \\ & -k^2 & \\ & & -\frac{4}{3}k^2 \end{pmatrix} \begin{pmatrix} \varepsilon^1 \\ \varepsilon^2 \\ \varepsilon^3 \end{pmatrix} = \begin{pmatrix} Q^1 \\ Q^2 \\ Q^3 \end{pmatrix}$


= δφ ⇒ ε⁰ = δχ / φ̇
∂_j ∂_i ε^i = Q_j
- 1/3 k_j k_i ε^i(k) = Q_j(k)
-k²
-4/3 k²
ε¹ ε² ε³
Q¹ Q² Q³
EXTRINSIC CURVATURE
& ADM FORMALISM
G_μν = M_Pl^-2 T_μν

GLUE :

$= \delta\phi \quad \Rightarrow \quad \varepsilon^0 = \frac{\delta\chi}{\dot{\phi}}$

$\partial_j \partial_i \varepsilon^i = Q_j$

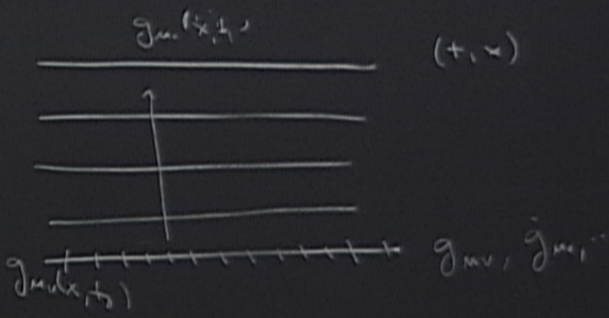
$-\frac{1}{3} k_j k_i \varepsilon^i(k) = Q_j(k)$

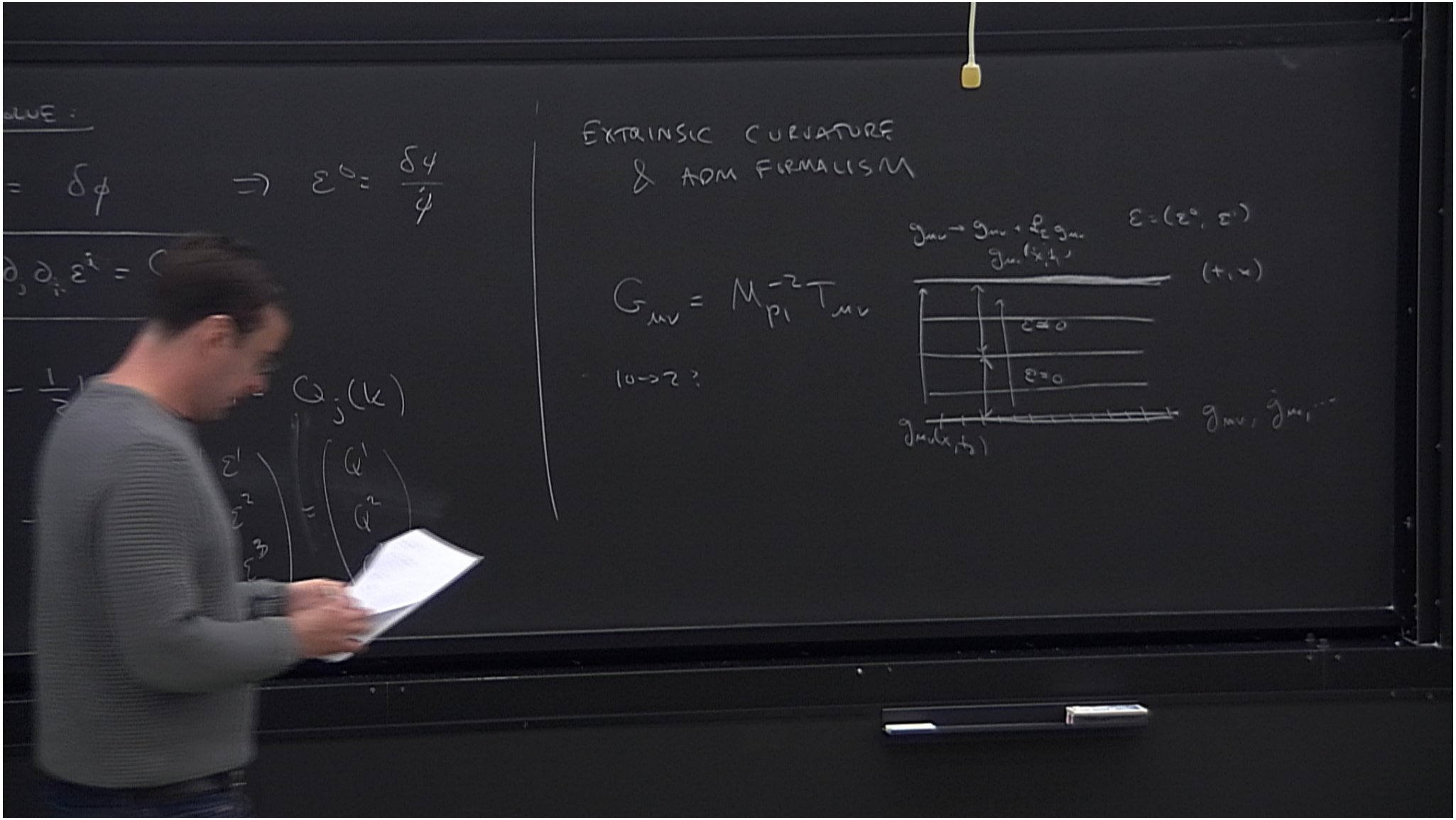
$$\begin{pmatrix} -k^2 & & \\ & -\frac{4}{3}k^2 & \\ & & \end{pmatrix} \begin{pmatrix} \varepsilon^1 \\ \varepsilon^2 \\ \varepsilon^3 \end{pmatrix} = \begin{pmatrix} Q^1 \\ Q^2 \\ Q^3 \end{pmatrix}$$

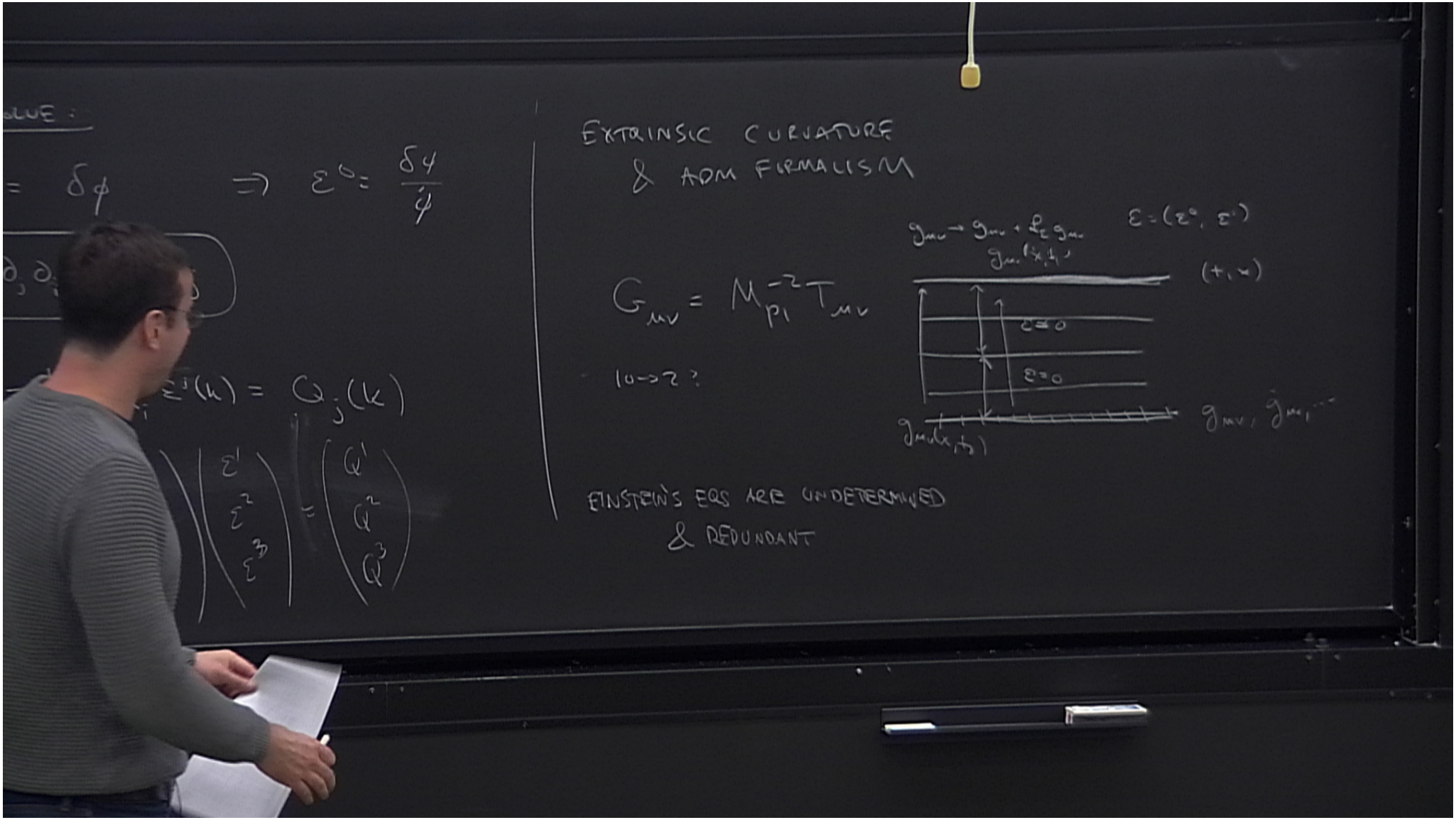
EXTRINSIC CURVATURE & ADM FORMALISM

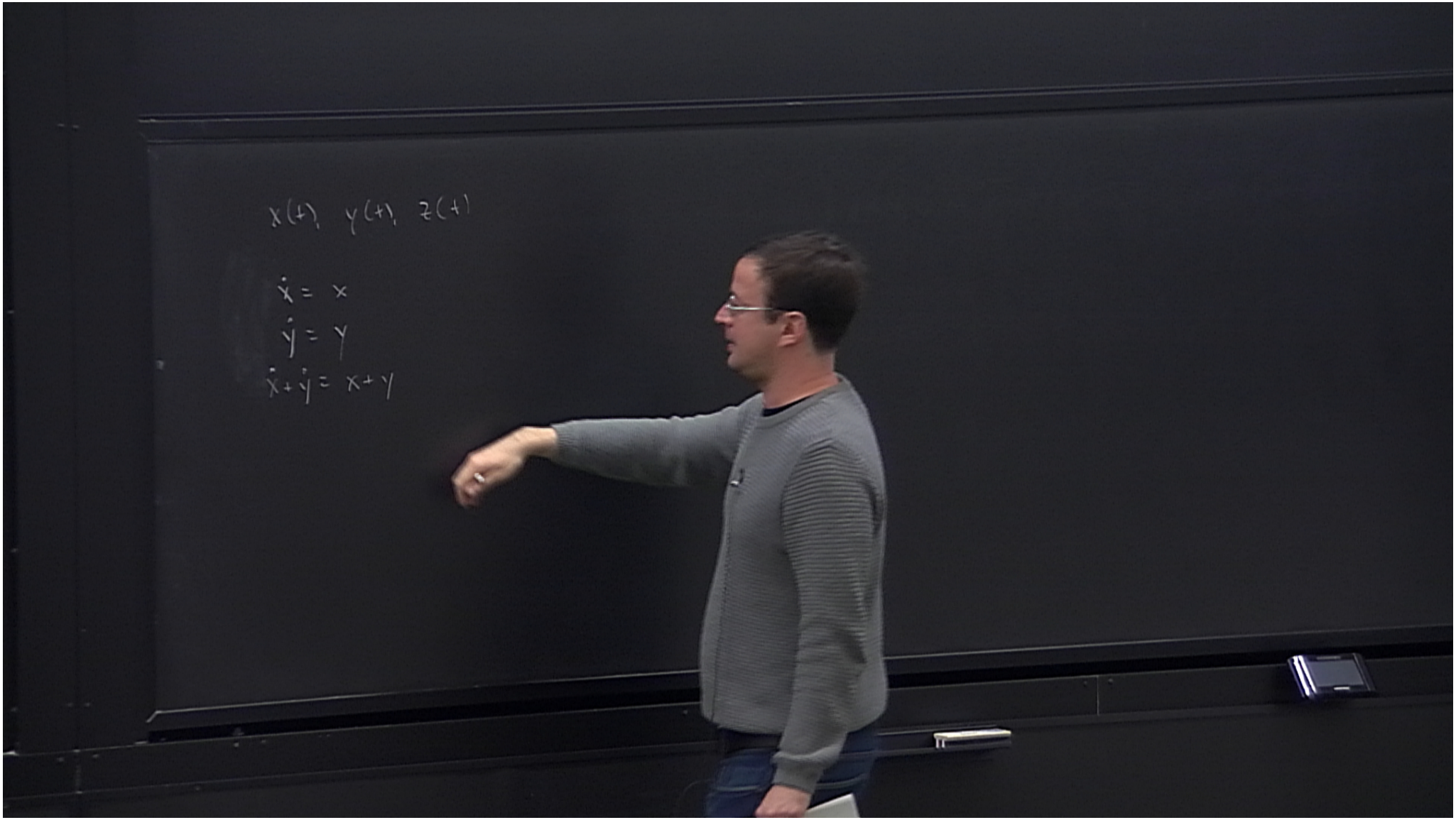
$G_{\mu\nu} = M_{Pl}^{-2} T_{\mu\nu}$

10 -> 2 ?




EXTRINSIC CURVATURE
& ADM FORMALISM
$G_{\mu\nu} = M_{Pl}^{-2} T_{\mu\nu}$
10->2 ?
$\varepsilon = (\varepsilon^0, \varepsilon^i)$
(+,-)
$\varepsilon=0$
$\varepsilon=0$


$= \delta\phi \quad \Rightarrow \quad \varepsilon^0 = \frac{\delta\chi}{\dot{\phi}}$
$\varepsilon^j(k) = Q_j(k)$
EXTRINSIC CURVATURE
& ADM FORMALISM
$G_{\mu\nu} = M_{Pl}^{-2} T_{\mu\nu}$
10->2 ?
$g_{\mu\nu} \to g_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu}$
$\xi = (\xi^0, \xi^i)$
$(+,-)$
$\xi=0$
$\xi=0$
$g_{\mu\nu}, g_{\mu\nu}$
EINSTEIN'S EQS ARE UNDETERMINED
& REDUNDANT


x(t), y(t), z(t)
$\dot{x} = x$
$\dot{y} = y$
$\dot{x} + \dot{y} = x + y$

$\delta\phi \Rightarrow \varepsilon^0 = \frac{\delta\psi}{\dot{\phi}}$

$\partial_i \varepsilon^i = Q_j$

$\frac{1}{3} k_j k_i \varepsilon^i(k) = Q_j(k)$

$$-\frac{4}{3}k^2 \begin{pmatrix} \varepsilon^1 \\ \varepsilon^2 \\ \varepsilon^3 \end{pmatrix} = \begin{pmatrix} Q^1 \\ Q^2 \\ Q^3 \end{pmatrix}$$

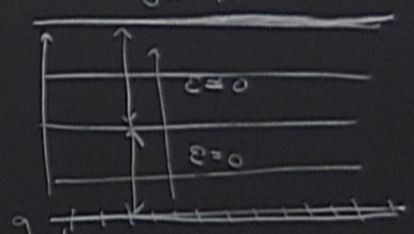
EXTRINSIC CURVATURE
& ADM FORMALISM

$G_{\mu\nu} = M_{Pl}^{-2} T_{\mu\nu}$

10 -> 2 ?

$g_{\mu\nu} \to g_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu}$ $\quad \xi = (\xi^0, \xi^i)$

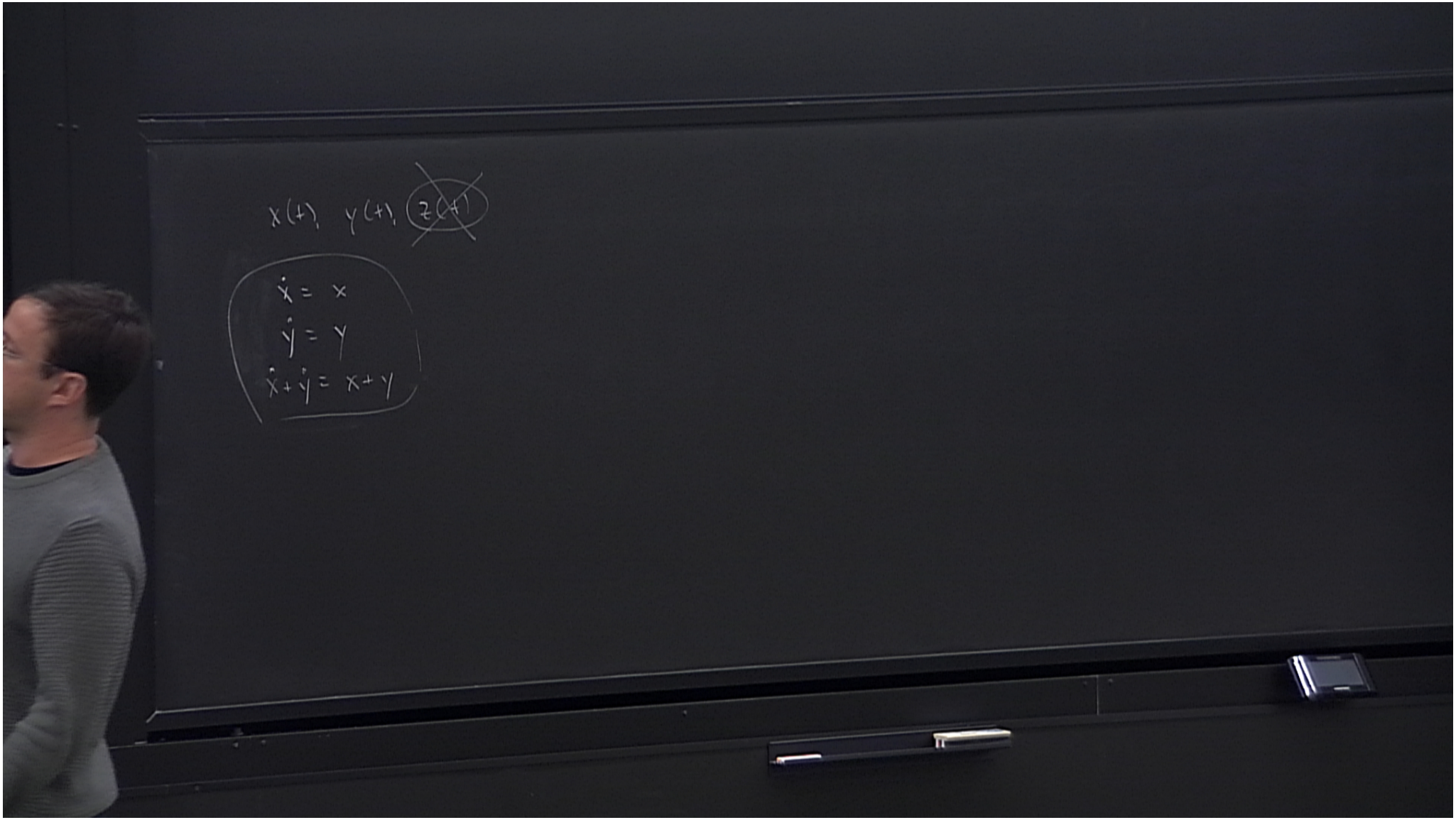
$g_{\mu\nu}(x,t)$



$g_{\mu\nu}, g_{\mu\nu}, \ldots$

EINSTEIN'S EQS ARE UNDETERMINED
& REDUNDANT

$0 = 0$


x(t), y(t), z(t)
$\dot{x} = x$
$\dot{y} = y$
$\dot{x} + \dot{y} = x + y$

$\delta\phi \Rightarrow \varepsilon^0 = \frac{\delta\phi}{\dot{\phi}}$

$\partial_i \varepsilon^i = Q_j$

$\frac{1}{3} k_j k_i \varepsilon^i(k) = Q_j(k)$

$$-\frac{4}{3}k^2 \begin{pmatrix} \varepsilon^1 \\ \varepsilon^2 \\ \varepsilon^3 \end{pmatrix} = \begin{pmatrix} Q^1 \\ Q^2 \\ Q^3 \end{pmatrix}$$

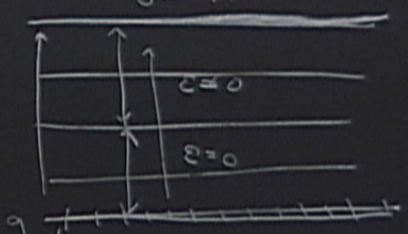
EXTRINSIC CURVATURE
& ADM FORMALISM

$G_{\mu\nu} = M_{Pl}^{-2} T_{\mu\nu}$

$10 \rightarrow 6 \rightarrow 2$

$g_{\mu\nu} \rightarrow g_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu}$ $\quad \xi = (\xi^0, \xi^i)$

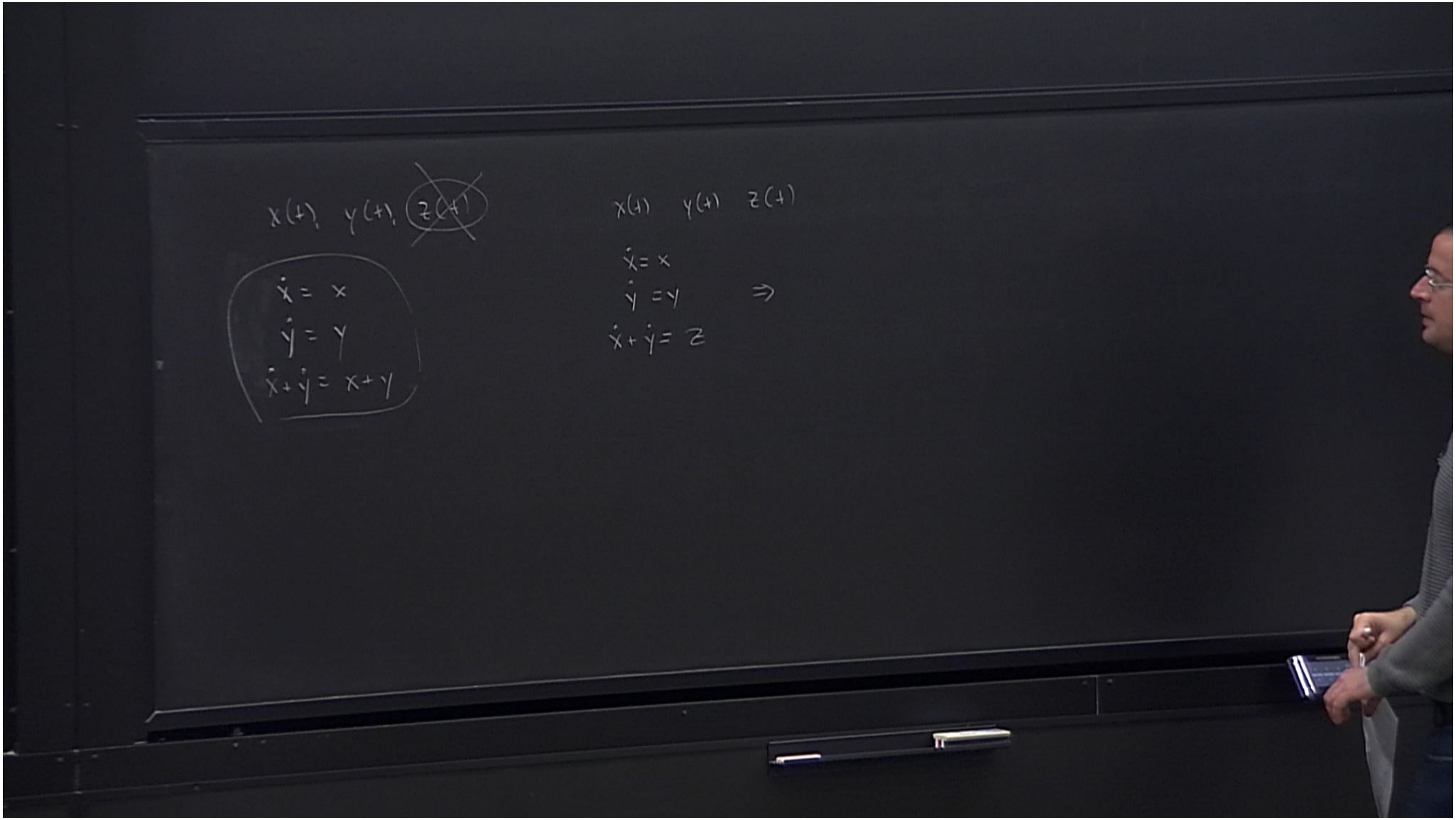
$g_{\mu\nu}(x, t)$

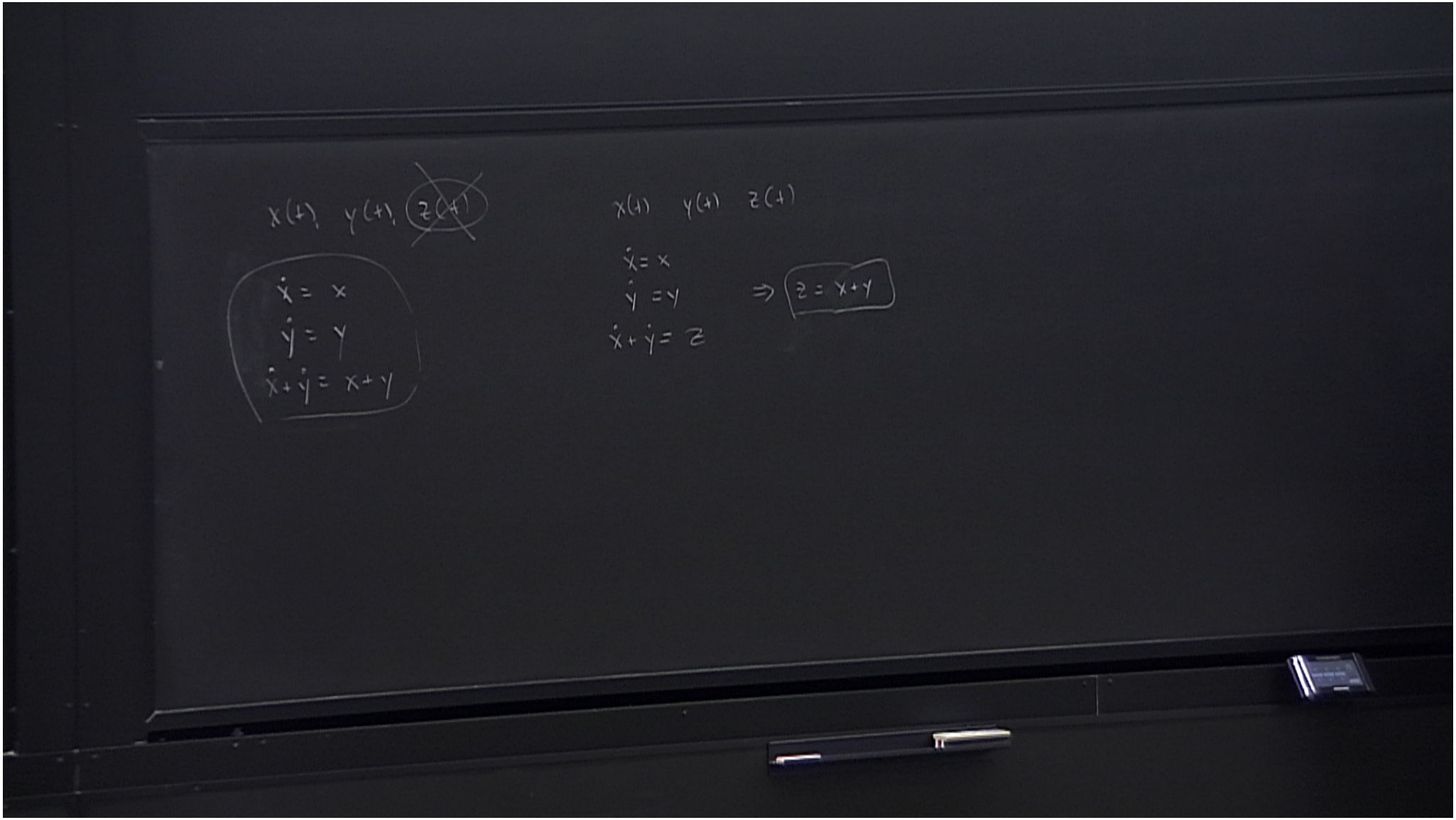


$g_{\mu\nu}, g_{\mu\nu}, \cdots$

EINSTEIN'S EQS ARE UNDETERMINED
& REDUNDANT

$0 = 0$


x(t), y(t), z(t)
ẋ = x
ẏ = y
ẋ + ẏ = x + y
x(t) y(t) z(t)
ẋ = x
ẏ = y ⇒
ẋ + ẏ = z


x(t), y(t), z(t)
ẋ = x
ẏ = y
ẋ + ẏ = x + y
x(t) y(t) z(t)
ẋ = x
ẏ = y
ẋ + ẏ = z
⇒ z = x + y

$\delta\phi \Rightarrow \varepsilon^0 = \frac{\delta\psi}{\dot{\phi}}$

$\partial_i \varepsilon^i = Q_j$

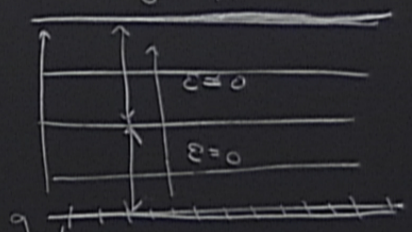
$\frac{1}{3} k_j k_i \varepsilon^i(k) = Q_j(k)$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & -\frac{4}{3}k^2 \end{pmatrix} \begin{pmatrix} \varepsilon^1 \\ \varepsilon^2 \\ \varepsilon^3 \end{pmatrix} = \begin{pmatrix} Q^1 \\ Q^2 \\ Q^3 \end{pmatrix}$$

EXTRINSIC CURVATURE
& ADM FORMALISM

$g_{\mu\nu} \to g_{\mu\nu} + \mathcal{L}_\varepsilon g_{\mu\nu}$ $\quad \varepsilon = (\varepsilon^0, \varepsilon^i)$

$g_{\mu\nu}(x,t)$

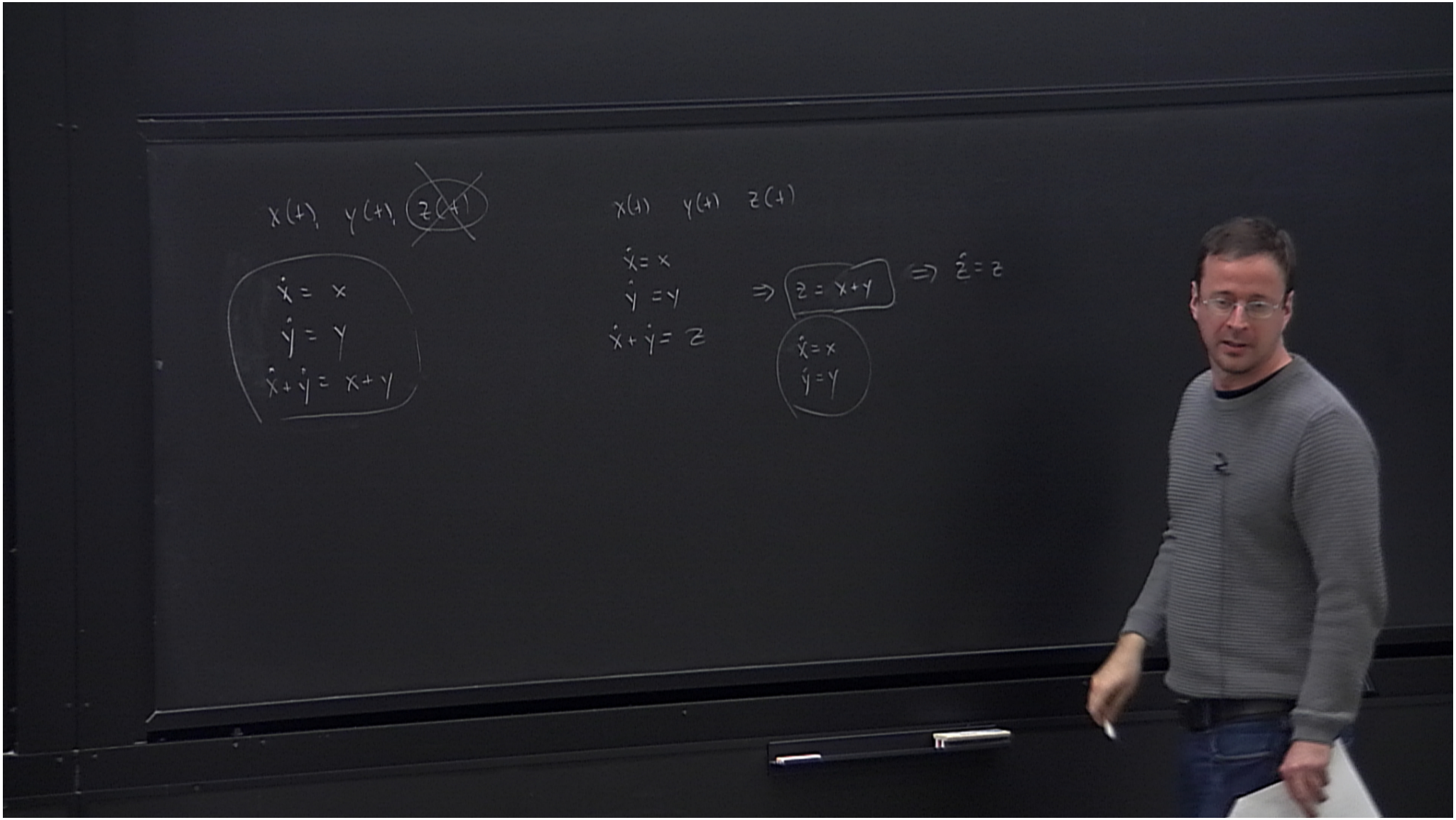


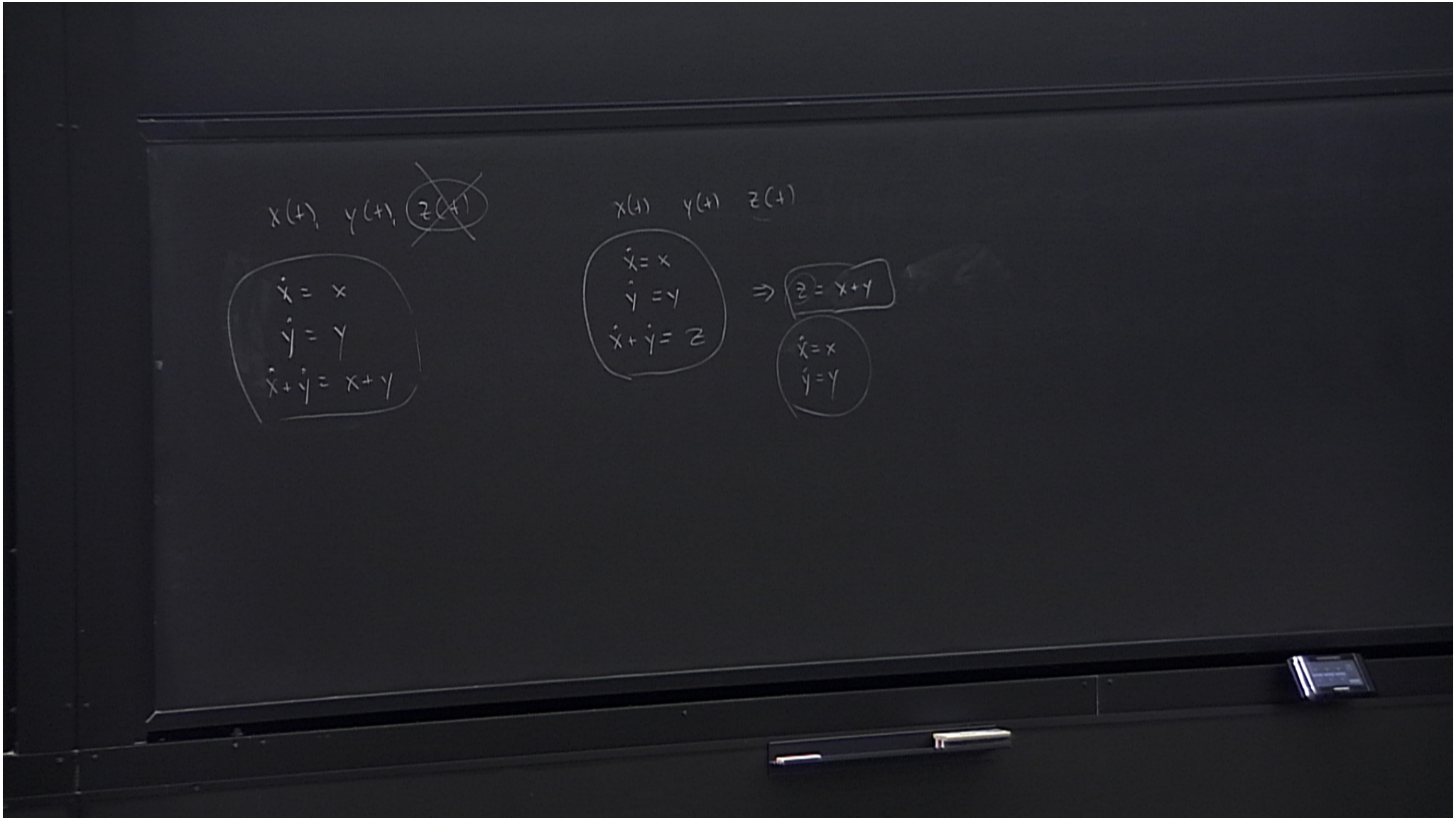
$G_{\mu\nu} = M_{Pl}^{-2} T_{\mu\nu}$

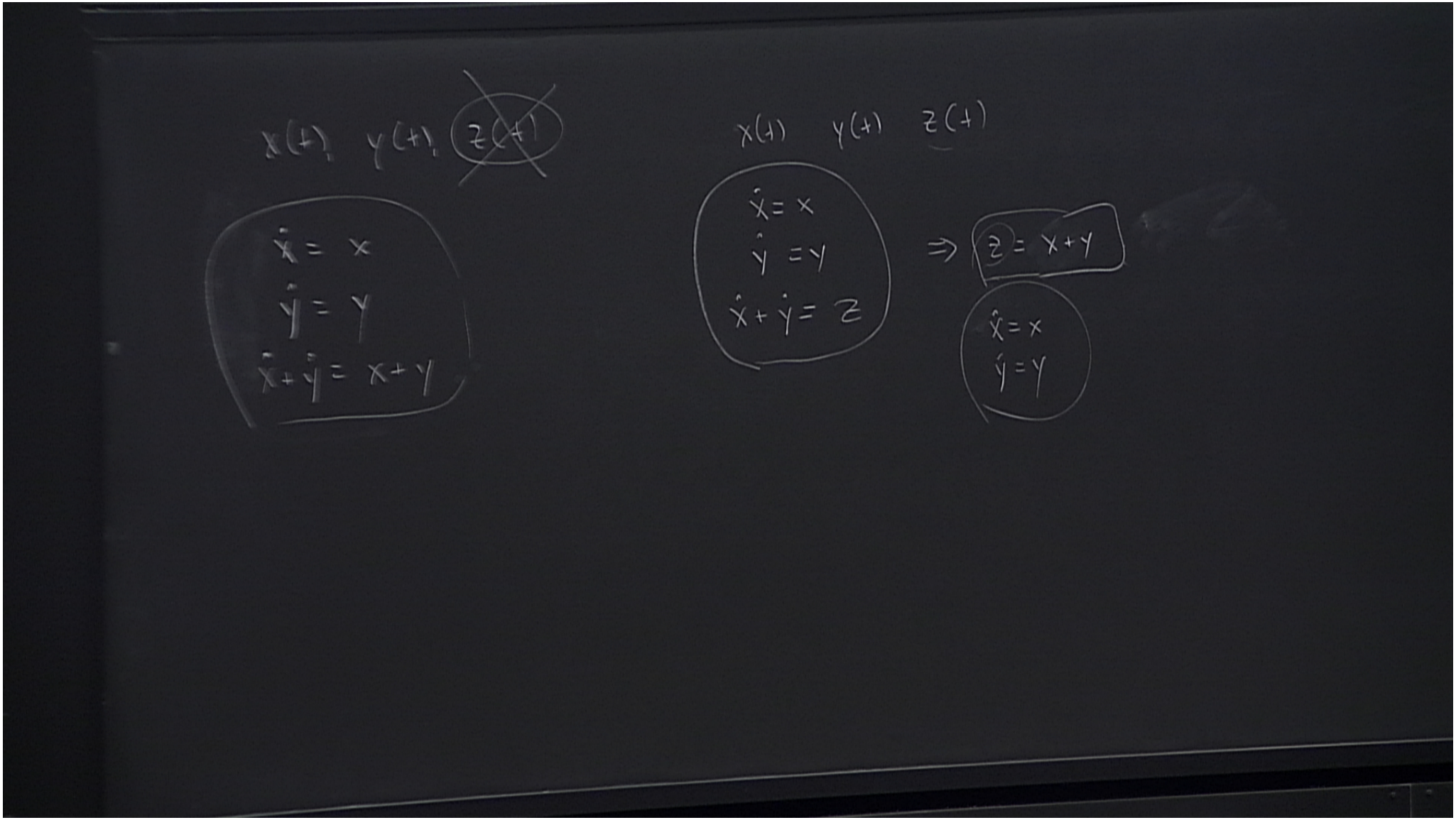
$10 \to 6 \to 2$
$\quad\quad * \quad 4$

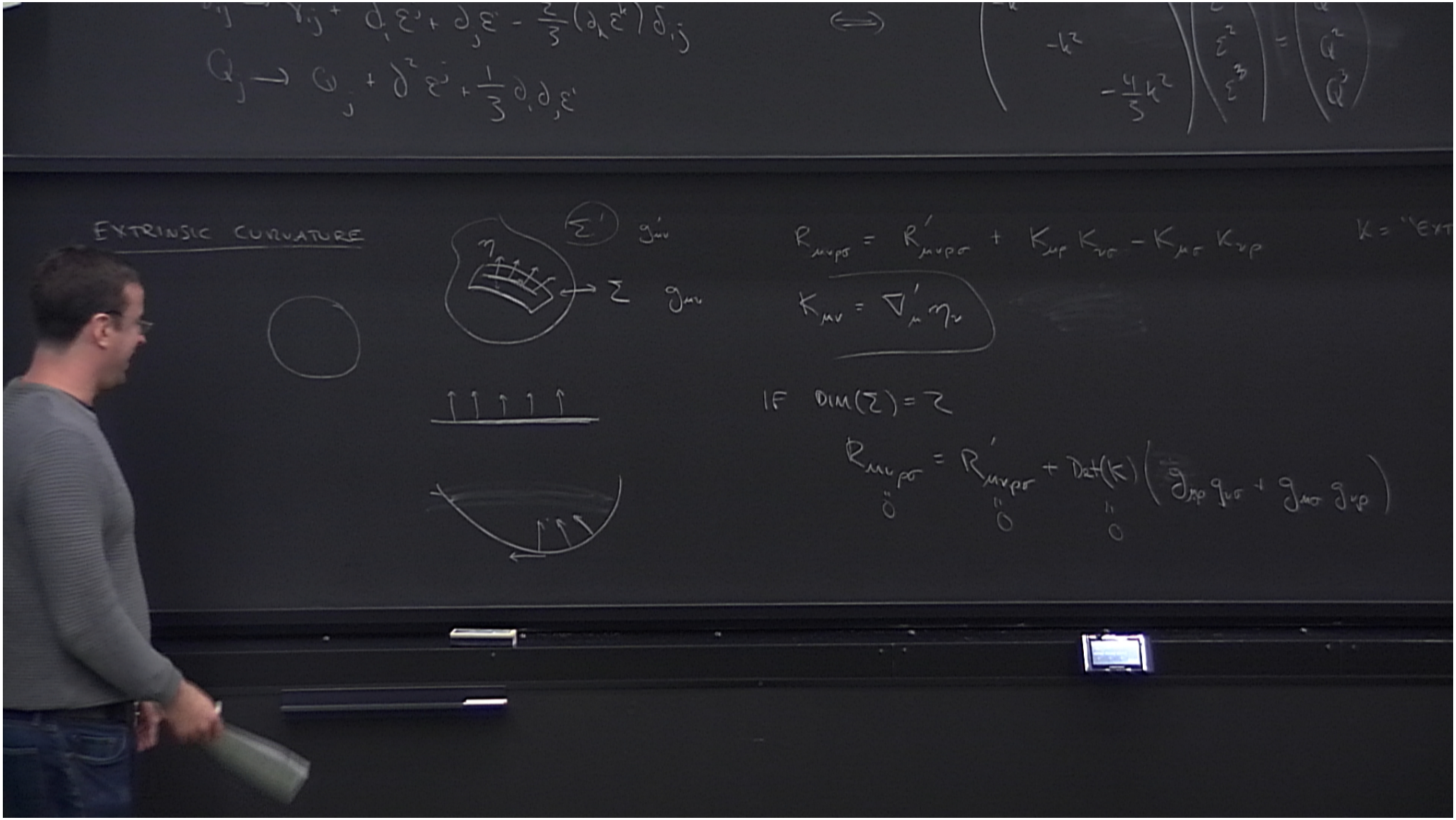
$0 = 0$

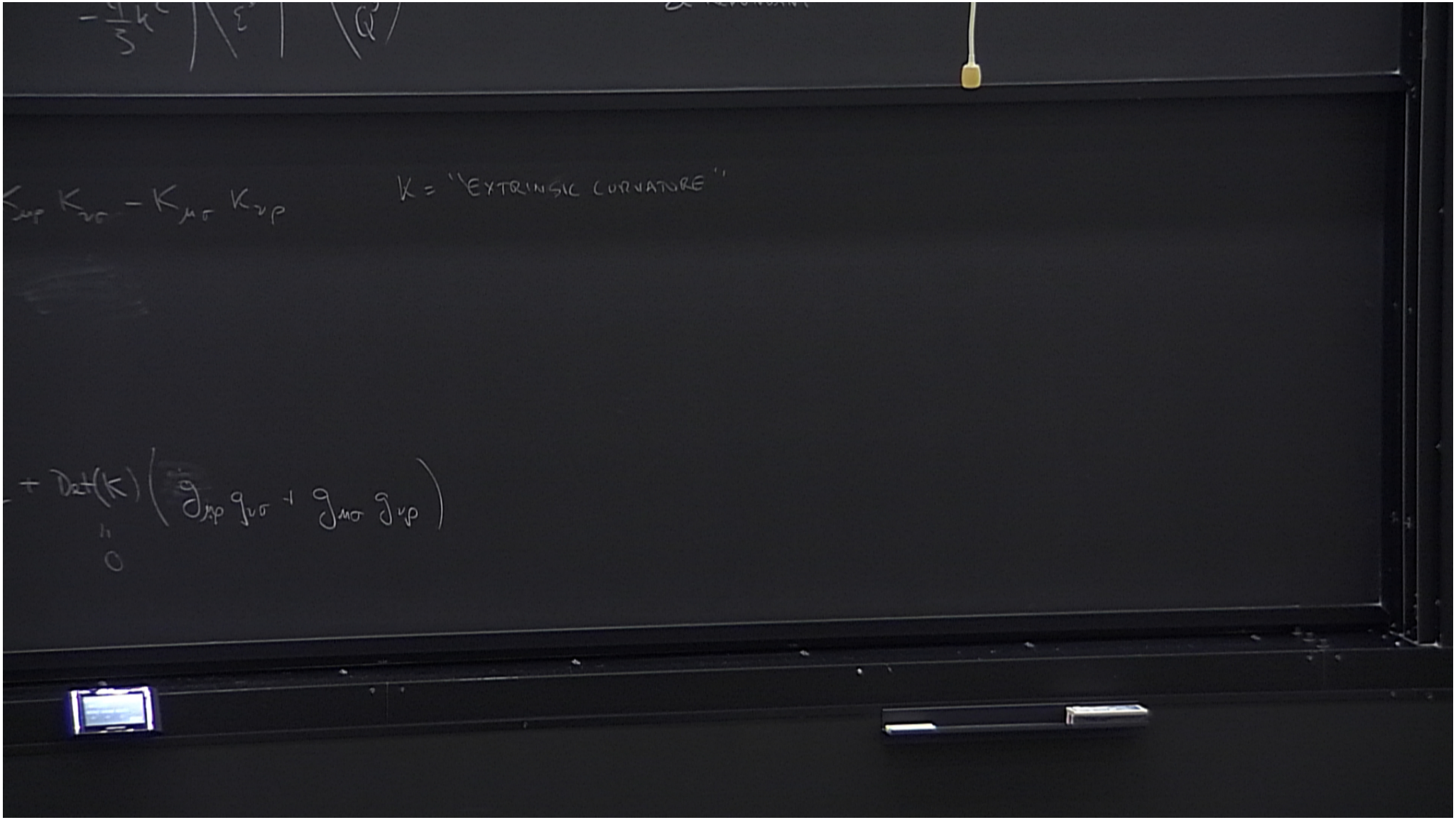
EINSTEIN'S EQS ARE UNDETERMINED
& REDUNDANT


x(t), y(t), z(t)
ẋ = x
ẏ = y
ẋ + ẏ = x + y
x(t) y(t) z(t)
ẋ = x
ẏ = y
ẋ + ẏ = z
⇒ z = x + y
⇒ ż = z
ẋ = x
ẏ = y

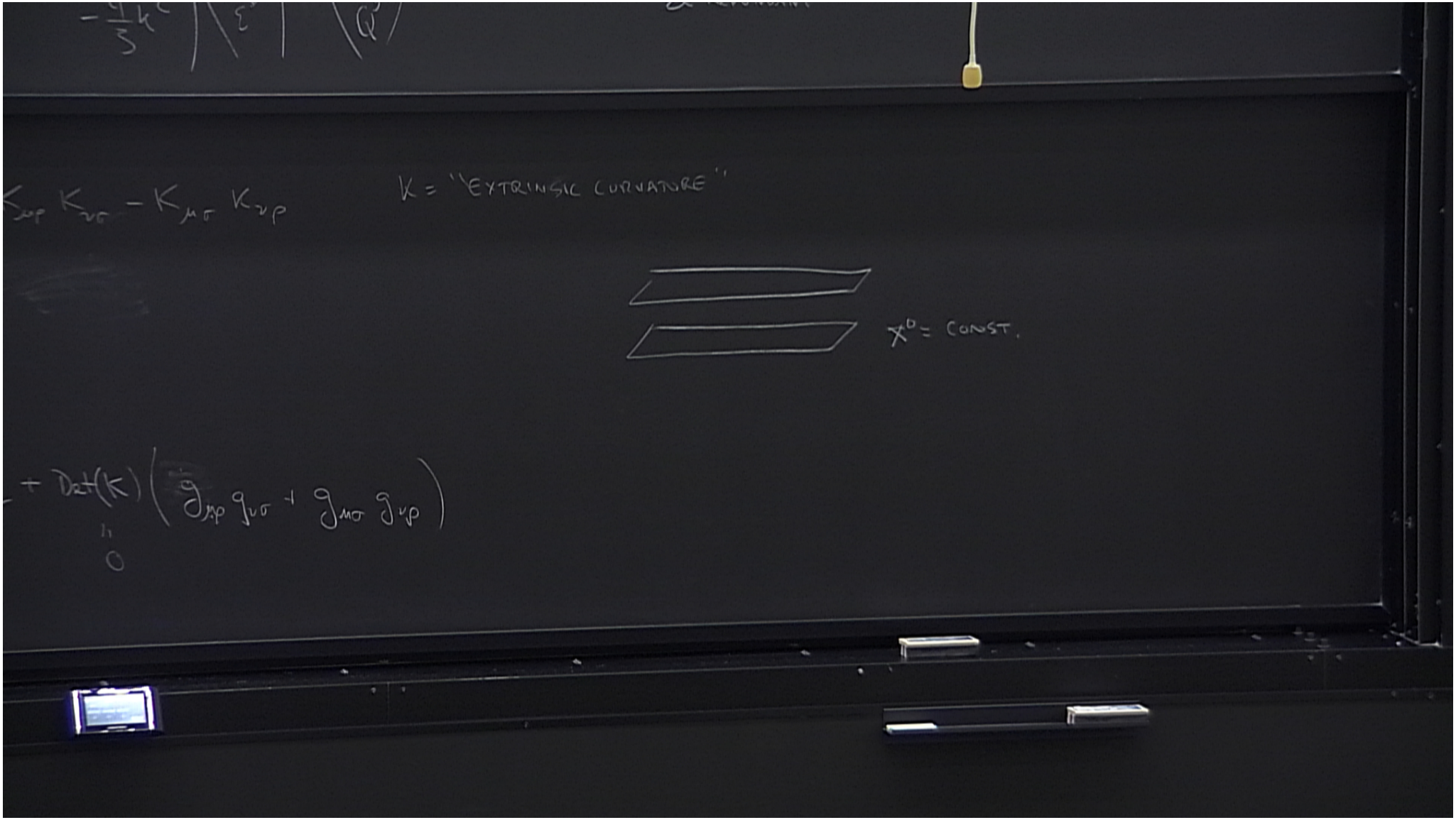

x(t), y(t), z(t)
ẋ = x
ẏ = y
ẋ + ẏ = x + y
x(t) y(t) z(t)
ẋ = x
ẏ = y
ẋ + ẏ = z
⇒ z = x + y
ẋ = x
ẏ = y


x(t), y(t), z(t)
$\dot{x} = x$
$\dot{y} = y$
$\dot{x} + \dot{y} = x + y$
x(t) y(t) z(t)
$\dot{x} = x$
$\dot{y} = y$
$\dot{x} + \dot{y} = z$
$\Rightarrow$
$z = x + y$
$\dot{x} = x$
$\dot{y} = y$


EXTRINSIC CURVATURE
Σ'
$g_{\mu\nu}$
Σ
$g_{\mu\nu}$
$R_{\mu\nu\rho\sigma} = R'_{\mu\nu\rho\sigma} + K_{\mu\rho}K_{\nu\sigma} - K_{\mu\sigma}K_{\nu\rho}$
$K_{\mu\nu} = \nabla'_\mu n_\nu$
IF $\dim(\Sigma) = 2$
$R_{\mu\nu\rho\sigma} = R'_{\mu\nu\rho\sigma} + \mathrm{Det}(K)\left(g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}\right)$
0 0 0


K = "EXTRINSIC CURVATURE"

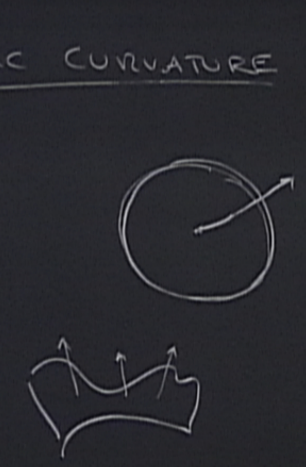

EXTRINSIC CURVATURE
Σ'
$g'_{\mu\nu}$
Σ
$g_{\mu\nu}$
$\lambda_k = 0$
$\lambda_k \neq 0$
$R_{\mu\nu\rho\sigma} = R'_{\mu\nu\rho\sigma} + K_{\mu\rho} K_{\nu\sigma} - K_{\mu\sigma}$
$K_{\mu\nu} = \nabla'_\mu n$
IF DIM(Σ)
Det(K)

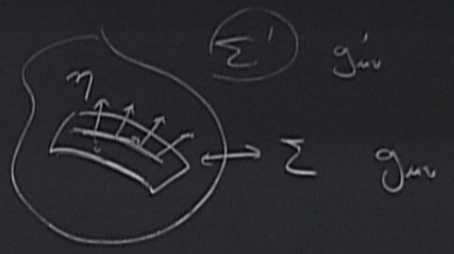

$K_{\mu\rho} K_{\nu\sigma} - K_{\mu\sigma} K_{\nu\rho}$
K = "EXTRINSIC CURVATURE"
x^0 = CONST.
$+ D_{2+}(k) \left(g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} \right)$
0

$$\frac{1}{3}\partial_i\partial_j\varepsilon'$$

$$-\frac{4}{3}k^2$$

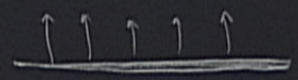
EXTRINSIC CURVATURE

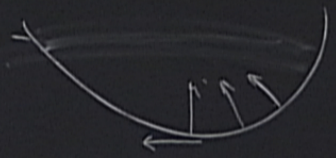




$$R_{\mu\nu\rho\sigma} = R'_{\mu\nu\rho\sigma} + K_{\mu\rho}K_{\nu\sigma} - K_{\mu\sigma}$$

$$K_{\mu\nu} = \nabla'_\mu n_\nu$$

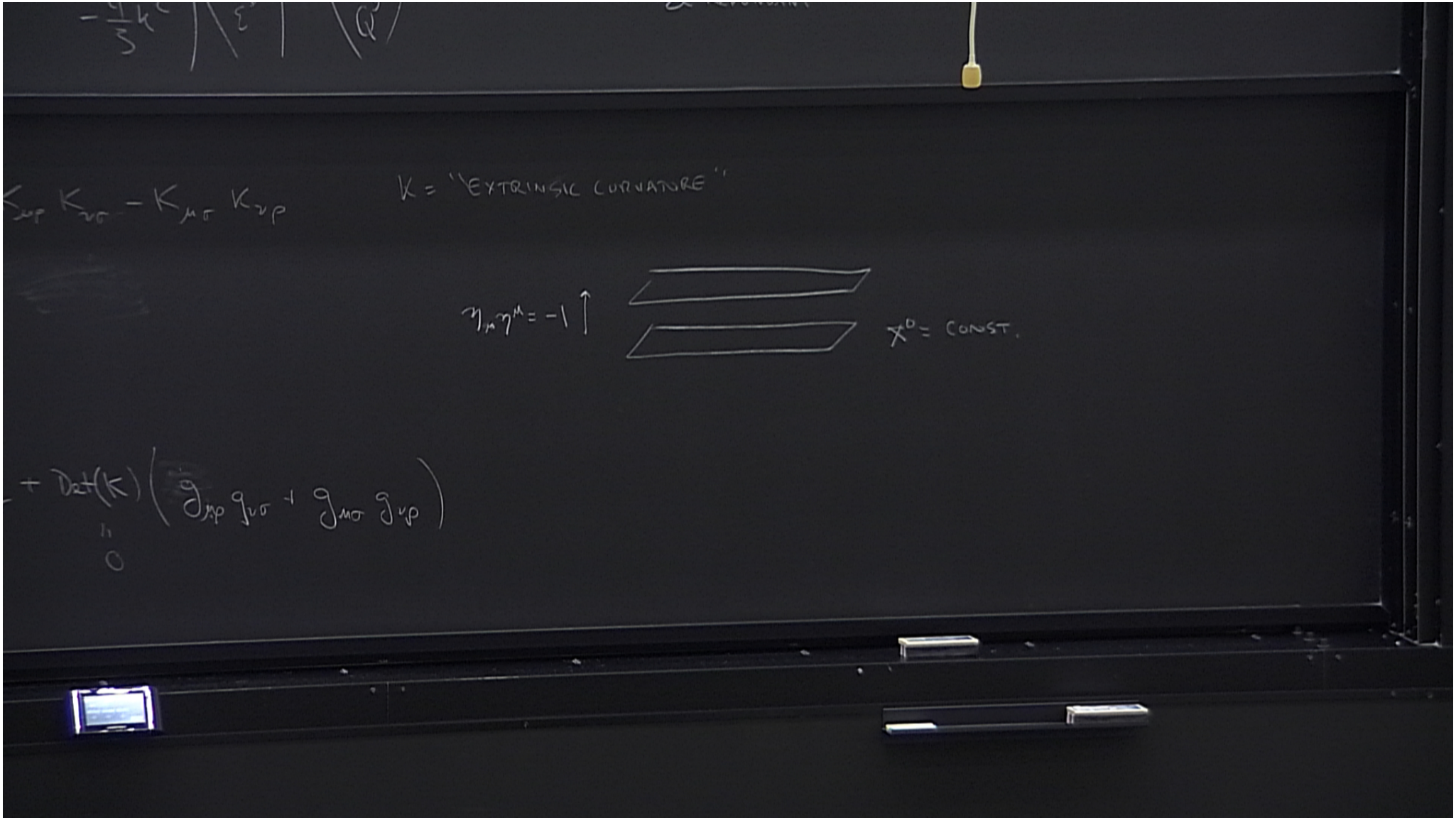




IF $\dim(\Sigma) = 2$

$$R_{\mu\nu\rho\sigma} = R'_{\mu\nu\rho\sigma} + \mathrm{Det}(K)\left(\bar{g}_{\mu\rho}\right.$$

$R_{\mu\nu\rho\sigma} = 0$ $\quad R'_{\mu\nu\rho\sigma} = 0$ $\quad \mathrm{Det}(K) = 0$


K = "EXTRINSIC CURVATURE"
$n_\mu n^\mu = -1$
$x^0 = \text{CONST.}$

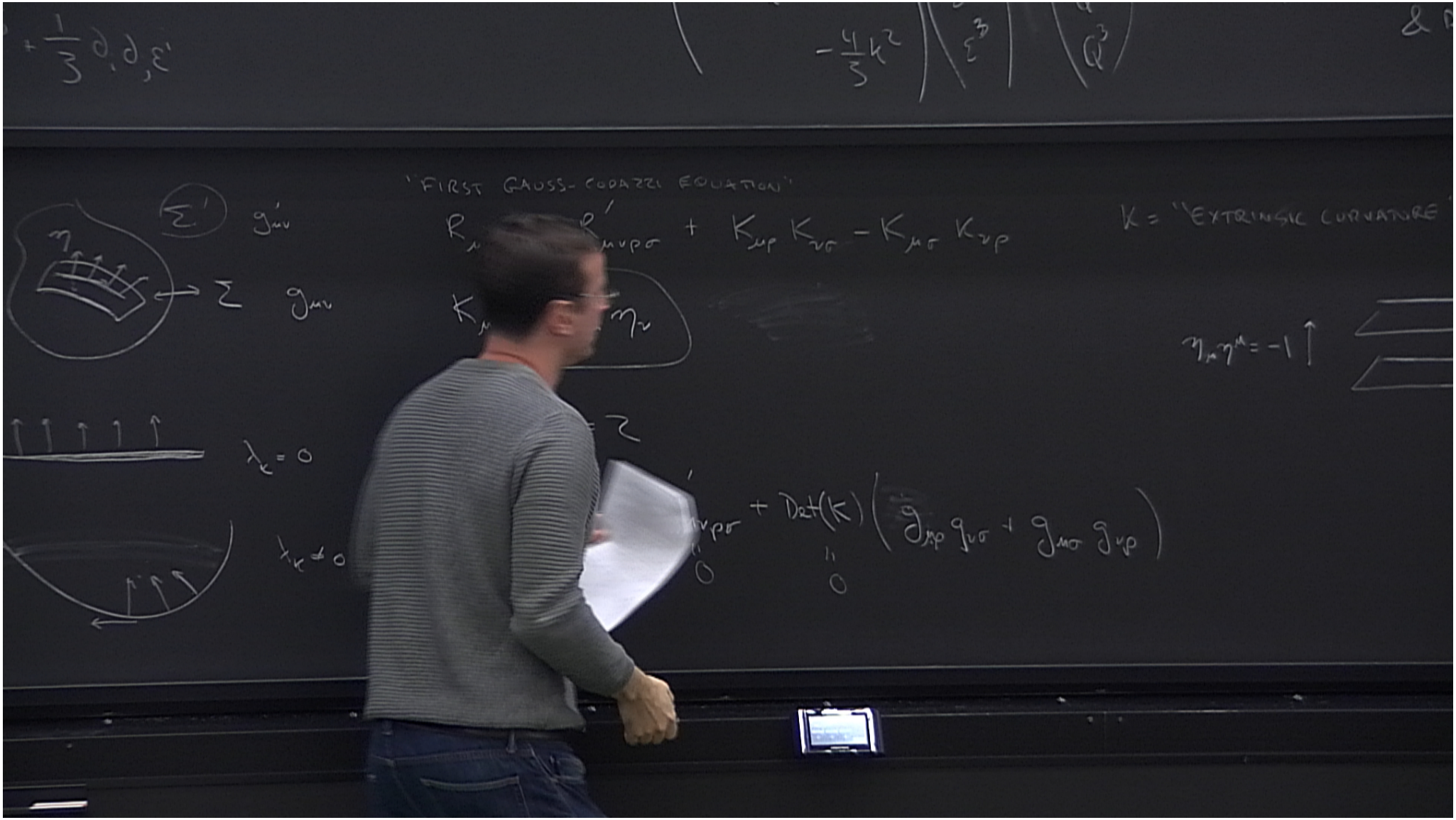
ADM VARIABLES $\quad g^{(4)}_{\mu\nu} \to (N, N^i, g^{(3)}_{ij})$

$$ds^2_{(4)} = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$g^{(4)}_{\mu\nu} = \left(\begin{array}{c|c} -N^2 + g^{(3)}_{ij} N^i N^j & g^{(3)}_{ij} N^i \\ \hline g^{(3)}_{ij} N^j & g^{(3)}_{ij} \end{array}\right)$$

N = "LAPSE";

N^i = "SHIFT";


'FIRST GAUSS-CODAZZI EQUATION'
K = 'EXTRINSIC CURVATURE'

ADM VARIABLES $\quad g^{(4)}_{\mu\nu} \to (N,\ N^i,\ g^{(3)}_{ij})$

$$ds^2_{(4)} = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$g^{(4)}_{\mu\nu} = \left(\begin{array}{c|c} -N^2 + g^{(3)}_{ij} N^i N^j & g^{(3)}_{ij} N^i \\ \hline g^{(3)}_{ij} N^j & g^{(3)}_{ij} \end{array}\right)$$

N = "LAPSE",

N^i = "SHIFT",

$$R^{(3)}_{ijkl} = R^{(4)}_{ijkl} + K_{il}K_{jk} - K_{ik}K_{jl}$$

$$K_{ij} = \frac{1}{2N}\left(\dot{g}^{(3)}_{ij} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right)$$

$$S_{EH} = \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g^{(4)}}\, R^{(4)}$$

$$= \frac{M_{Pl}^2}{2}\int \sqrt{g^{(3)}}\, N\left[R^{(3)} + K_{ij}K^{ij} - (K^i_i)^2\right]$$

S_{ADM}

ADM VARIABLES

$g^{(4)}_{\mu\nu} \to (N,\ N^i,\ g^{(3)}_{ij})$

N, N^i: NONDYNAMICAL; $g^{(3)}_{ij}$: DYNAMICAL

$$ds^2_{(4)} = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$g^{(4)}_{\mu\nu} = \left(\begin{array}{c|c} -N^2 + g^{(3)}_{ij} N^i N^j & g^{(3)}_{ij} N^i \\ \hline g^{(3)}_{ij} N^j & g^{(3)}_{ij} \end{array}\right)$$

N = "LAPSE",

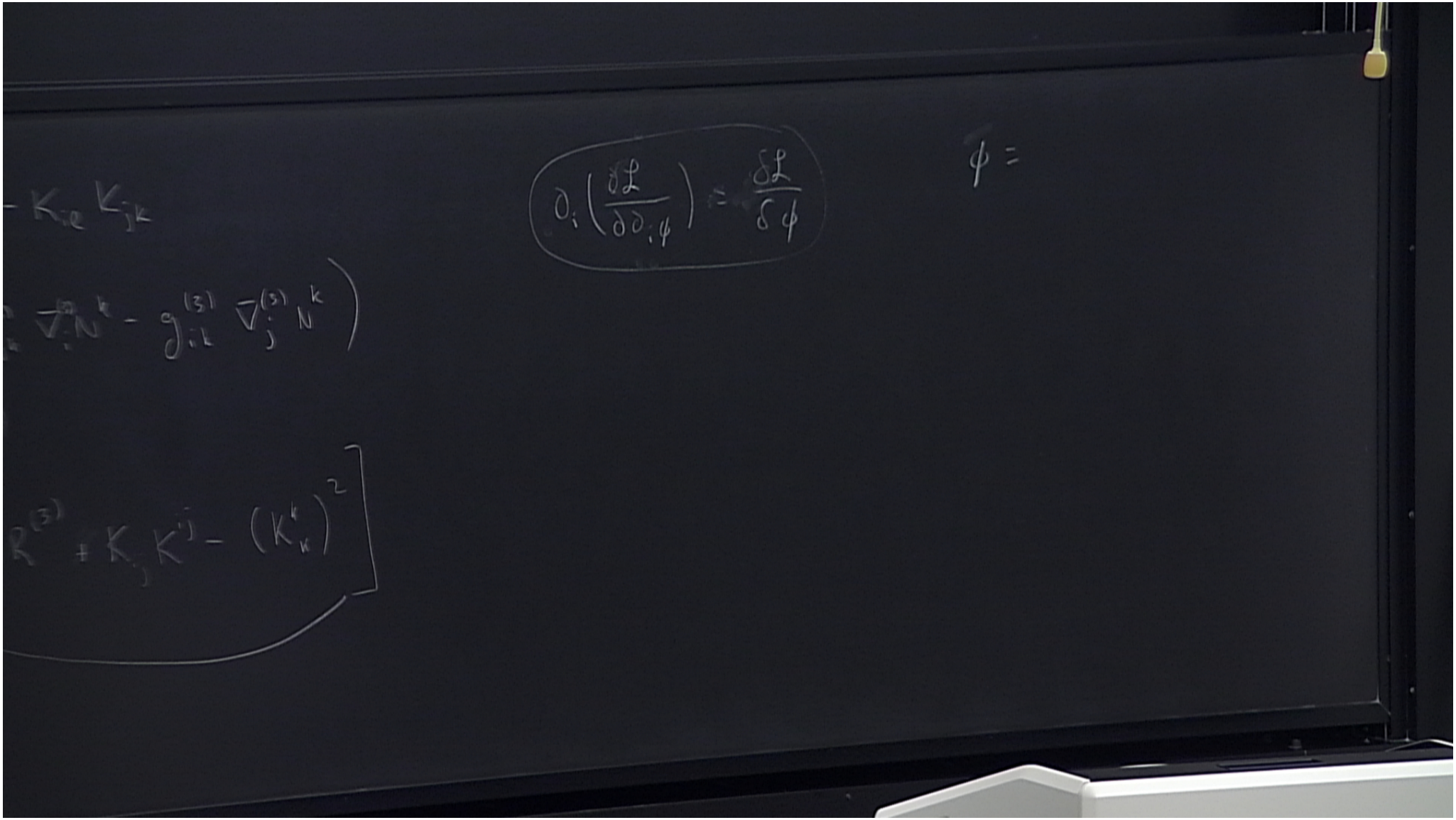
N^i = "SHIFT",

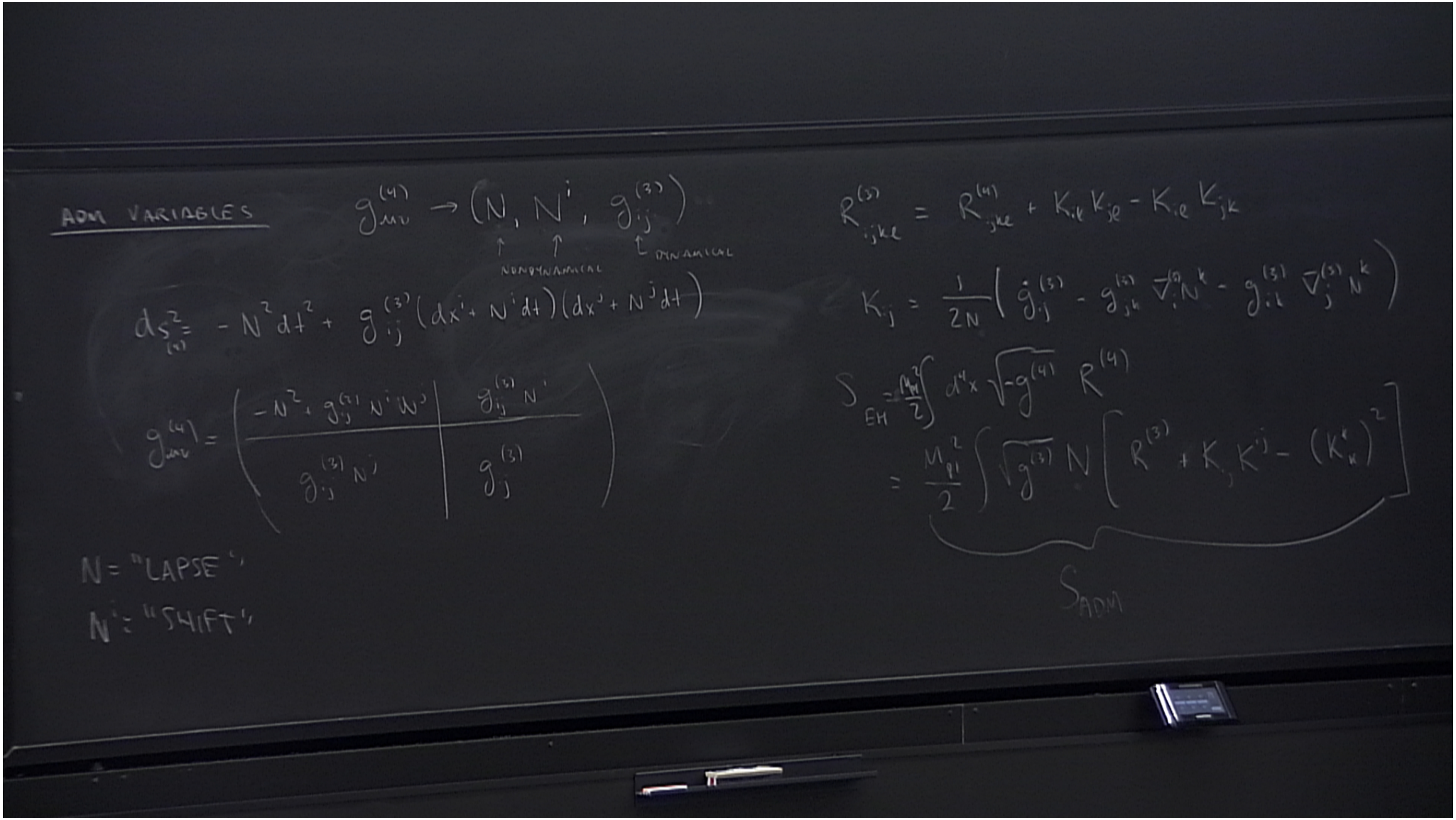
$$R^{(3)}_{ijkl} = R^{(4)}_{ijkl} + K_{il}K_{jk} - K_{ik}K_{jl}$$

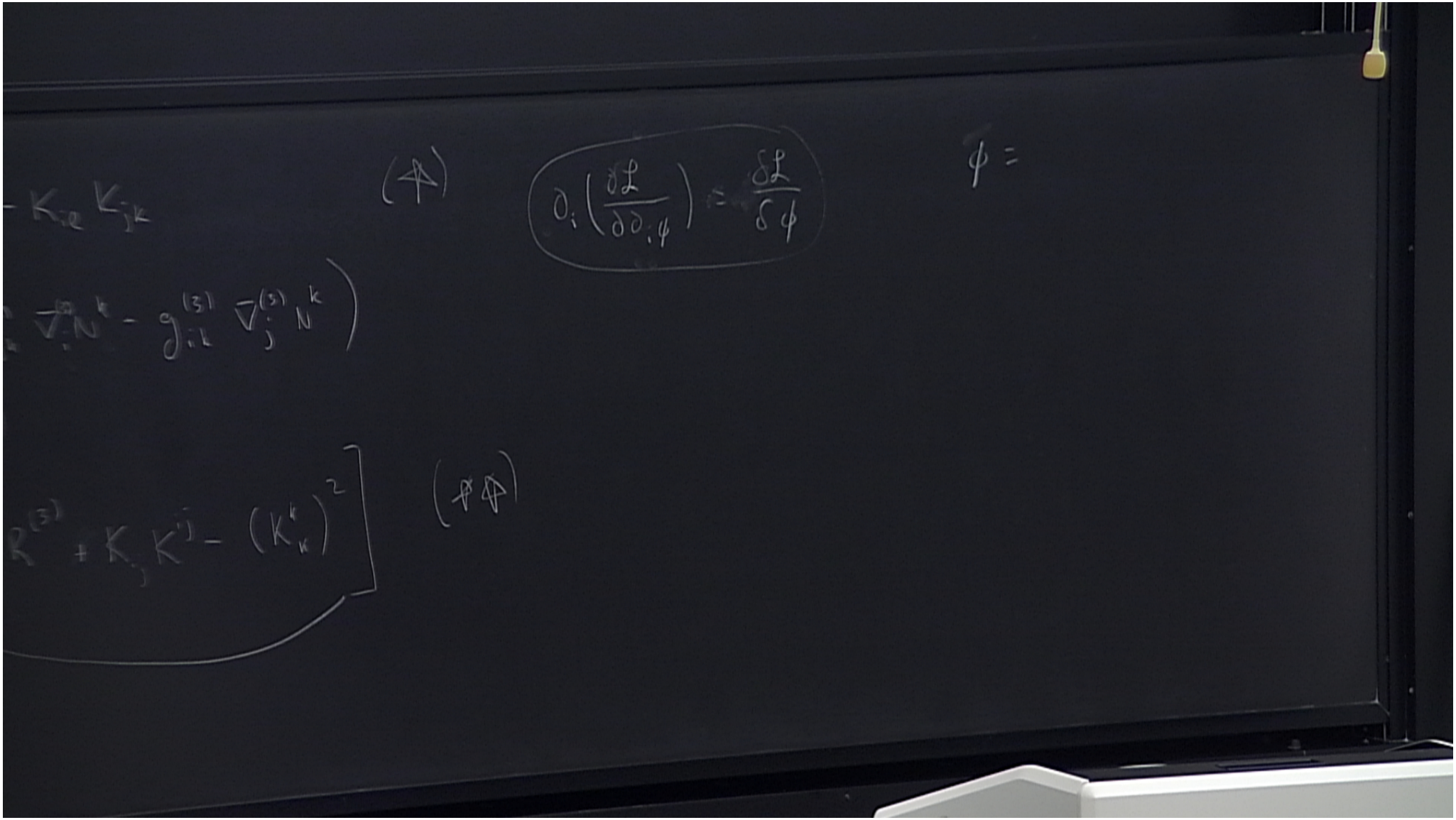
$$K_{ij} = \frac{1}{2N}\left(\dot{g}^{(3)}_{ij} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right)$$

$$S_{EH} = \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g^{(4)}}\, R^{(4)}$$

$$= \frac{M_{Pl}^2}{2}\int \sqrt{g^{(3)}} N \underbrace{\left[R^{(3)} + K_{ij}K^{ij} - (K^i_i)^2\right]}_{S_{ADM}}$$


$\partial_i\left(\frac{\partial \mathcal{L}}{\partial \partial_i \phi}\right) = \frac{\delta \mathcal{L}}{\delta \phi}$
$\phi =$


ADM VARIABLES
$g^{(4)}_{\mu\nu} \to (N, N^i, g^{(3)}_{ij})$
NONDYNAMICAL
DYNAMICAL
$ds^2_{(4)} = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$g^{(4)}_{\mu\nu} = \left(\begin{array}{c|c} -N^2 + g^{(3)}_{ij} N^i N^j & g^{(3)}_{ij} N^i \\ \hline g^{(3)}_{ij} N^j & g^{(3)}_{ij} \end{array}\right)$
N = "LAPSE"
N^i = "SHIFT"
$R^{(3)}_{ijke} = R^{(4)}_{ijke} + K_{ik}K_{je} - K_{ie}K_{jk}$
$K_{ij} = \frac{1}{2N}\left(\dot{g}^{(3)}_{ij} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right)$
$S_{EH} = \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g^{(4)}}\, R^{(4)}$
$= \frac{M_{Pl}^2}{2}\int \sqrt{g^{(3)}} N \left[R^{(3)} + K_{ij}K^{ij} - (K^k_k)^2\right]$
S_{ADM}

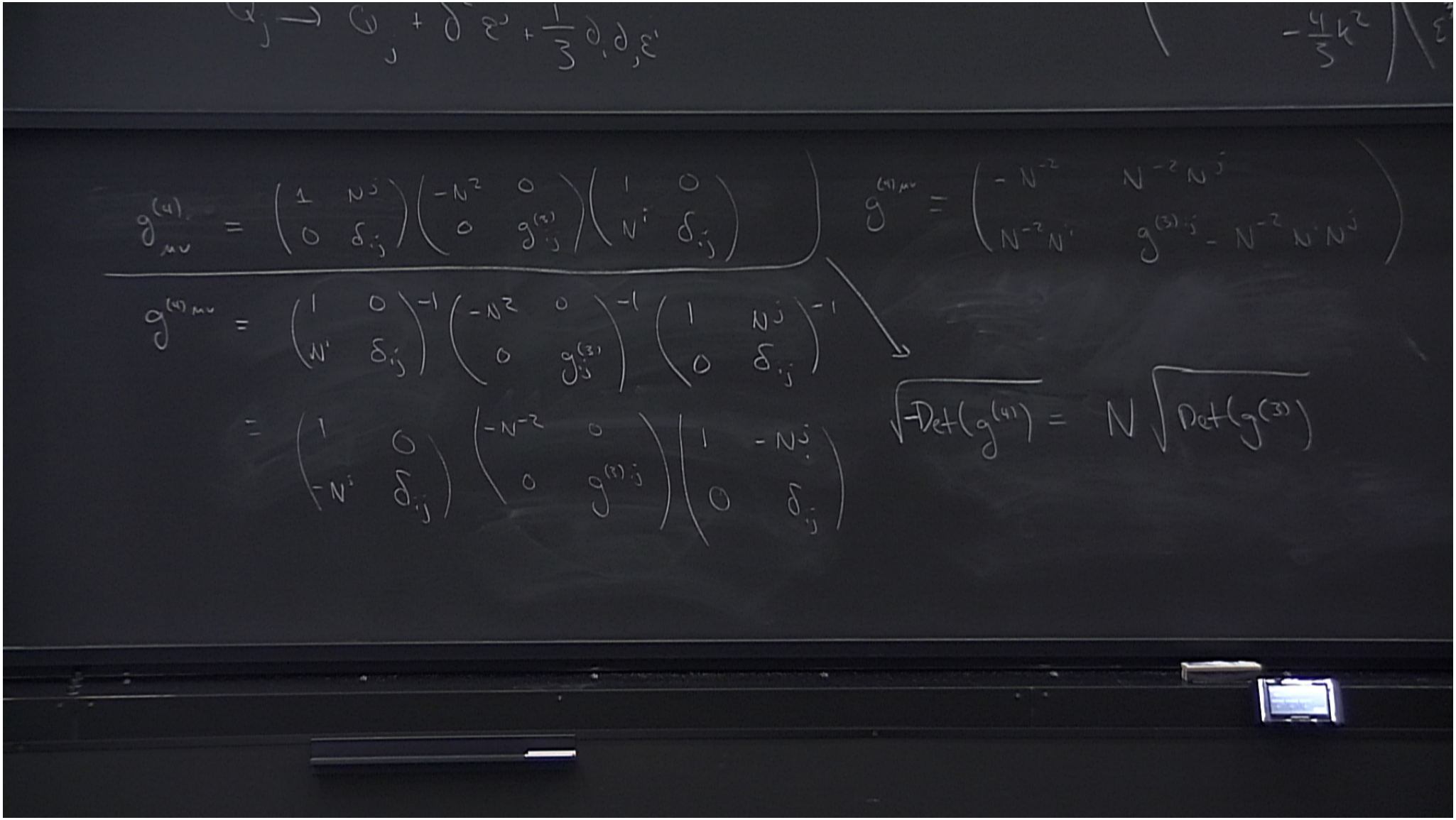

$-K_{ic}K_{jk}$
$\nabla_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k$
$R^{(3)} + K_j K^j - (K^k_k)^2$
(A)
$\partial_i\left(\frac{\delta\mathcal{L}}{\delta\partial_i\phi}\right) = \frac{\delta\mathcal{L}}{\delta\phi}$
$\phi =$

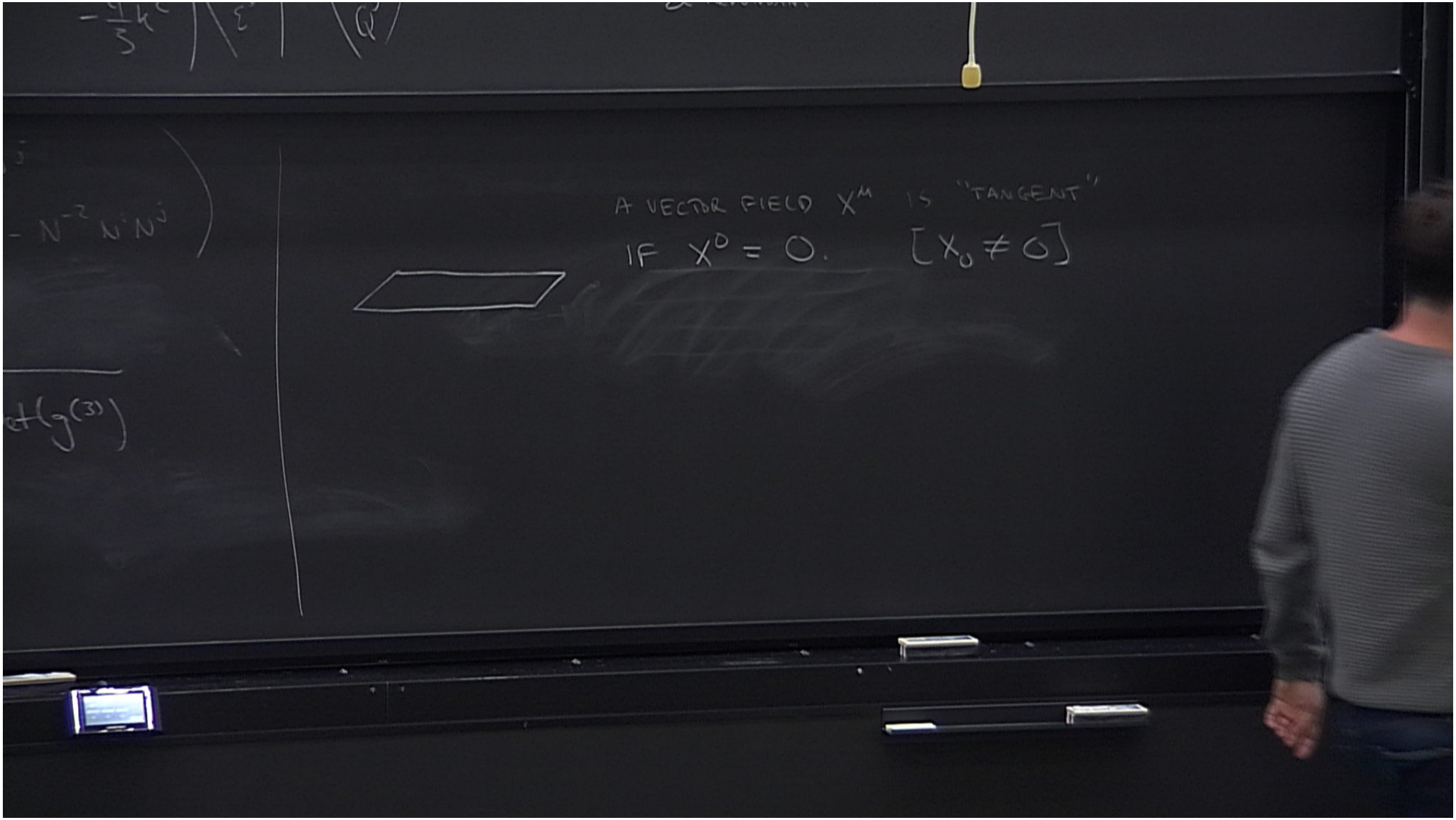
$$g^{(4)}_{\mu\nu} = \begin{pmatrix} 1 & N^j \\ 0 & \delta_{ij} \end{pmatrix} \begin{pmatrix} -N^2 & 0 \\ 0 & g^{(3)}_{ij} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ N^i & \delta_{ij} \end{pmatrix}$$

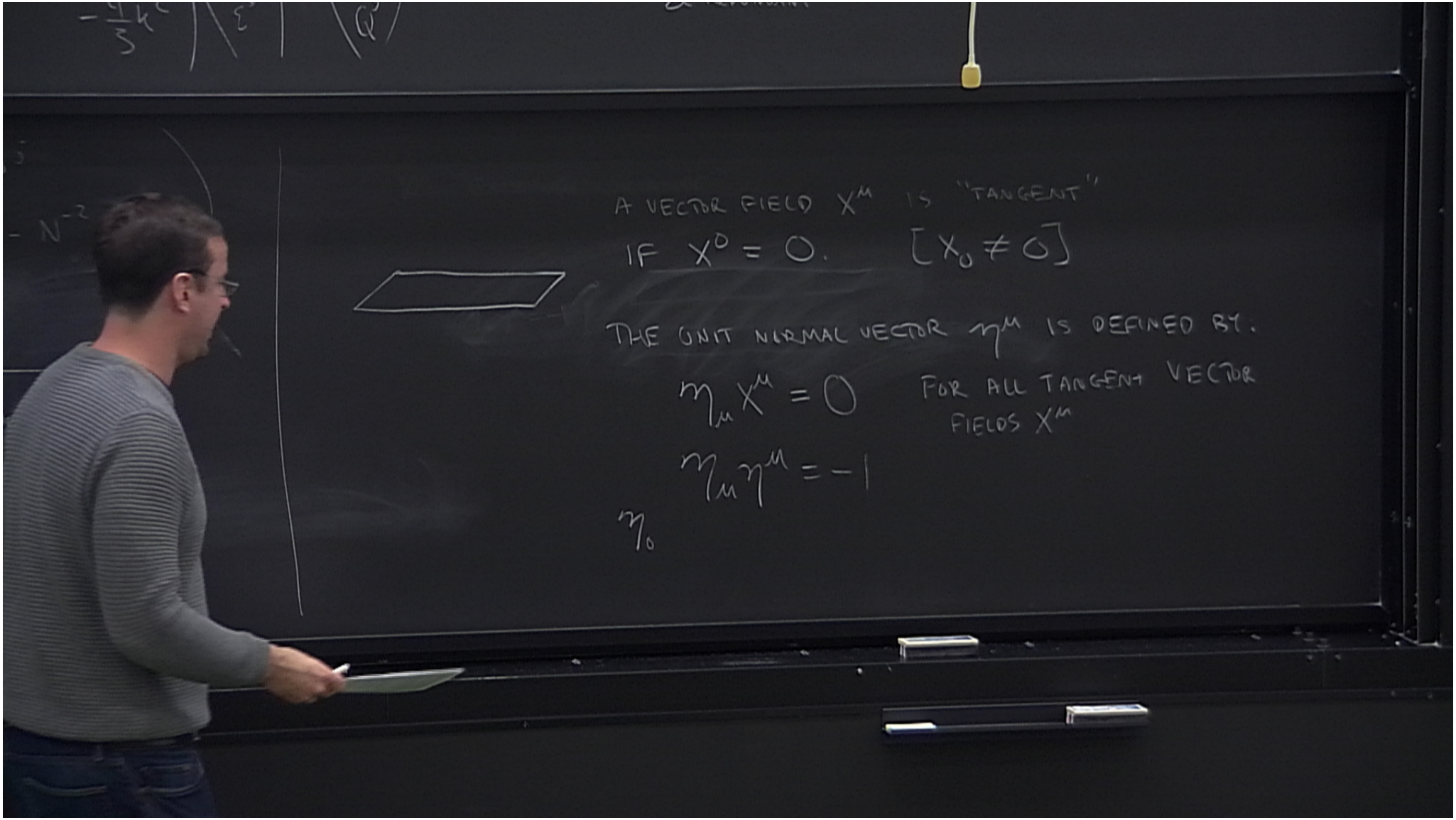
$$g^{(4)}_{\mu\nu} = \begin{pmatrix} 1 & N^j \\ 0 & \delta_{ij} \end{pmatrix} \begin{pmatrix} -N^2 & 0 \\ 0 & g^{(3)}_{ij} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ N^i & \delta_{ij} \end{pmatrix} = \begin{pmatrix} -N^{-2} & N^{-2} N^j \\ N^{-2} N^i & g^{(3)ij} - N^{-2} N^i N^j \end{pmatrix}$$

$$g^{(4)\mu\nu} = \begin{pmatrix} 1 & 0 \\ N^i & \delta_{ij} \end{pmatrix}^{-1} \begin{pmatrix} -N^2 & 0 \\ 0 & g^{(3)}_{ij} \end{pmatrix}^{-1} \begin{pmatrix} 1 & N^j \\ 0 & \delta_{ij} \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1 & 0 \\ -N^i & \delta_{ij} \end{pmatrix} \begin{pmatrix} -N^{-2} & 0 \\ 0 & g^{(3)ij} \end{pmatrix} \begin{pmatrix} 1 & -N^j \\ 0 & \delta_{ij} \end{pmatrix}$$

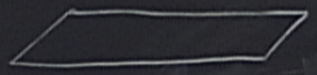

$g^{(4)}_{\mu\nu} = \begin{pmatrix} 1 & N^j \\ 0 & \delta_{ij} \end{pmatrix} \begin{pmatrix} -N^2 & 0 \\ 0 & g^{(3)}_{ij} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ N^i & \delta_{ij} \end{pmatrix}$
$g^{(4)\mu\nu} = \begin{pmatrix} -N^{-2} & N^{-2}N^j \\ N^{-2}N^i & g^{(3)ij} - N^{-2}N^iN^j \end{pmatrix}$
$g^{(4)\mu\nu} = \begin{pmatrix} 1 & 0 \\ N^i & \delta_{ij} \end{pmatrix}^{-1} \begin{pmatrix} -N^2 & 0 \\ 0 & g^{(3)}_{ij} \end{pmatrix}^{-1} \begin{pmatrix} 1 & N^j \\ 0 & \delta_{ij} \end{pmatrix}^{-1}$
$= \begin{pmatrix} 1 & 0 \\ -N^i & \delta_{ij} \end{pmatrix} \begin{pmatrix} -N^{-2} & 0 \\ 0 & g^{(3)ij} \end{pmatrix} \begin{pmatrix} 1 & -N^j \\ 0 & \delta_{ij} \end{pmatrix}$
$\sqrt{-Det(g^{(4)})} = N\sqrt{Det(g^{(3)})}$


A VECTOR FIELD X^M IS "TANGENT"
IF $X^0 = 0$. $[X_0 \neq 0]$


A VECTOR FIELD X^μ IS "TANGENT"
IF $X^0 = 0$. $[X_0 \neq 0]$
THE UNIT NORMAL VECTOR n^μ IS DEFINED BY.
$n_\mu X^\mu = 0$ FOR ALL TANGENT VECTOR FIELDS X^μ
$n_\mu n^\mu = -1$
n_0

$X^\mu = (0, x^i)$

A VECTOR FIELD X^μ IS "TANGENT" IF $X^0 = 0$. $[X_0 \neq 0]$



THE UNIT NORMAL VECTOR n^μ IS DEFINED BY.

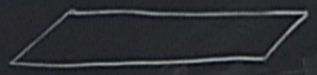
$n_\mu X^\mu = 0$ FOR ALL TANGENT VECTOR FIELDS X^μ

$n_\mu n^\mu = -1$

$\Rightarrow \boxed{n_0 = 0}$

$X^\mu = (0, x^i)$

A VECTOR FIELD X^μ IS "TANGENT" IF $X^0 = 0$. $[X_0 \neq 0]$



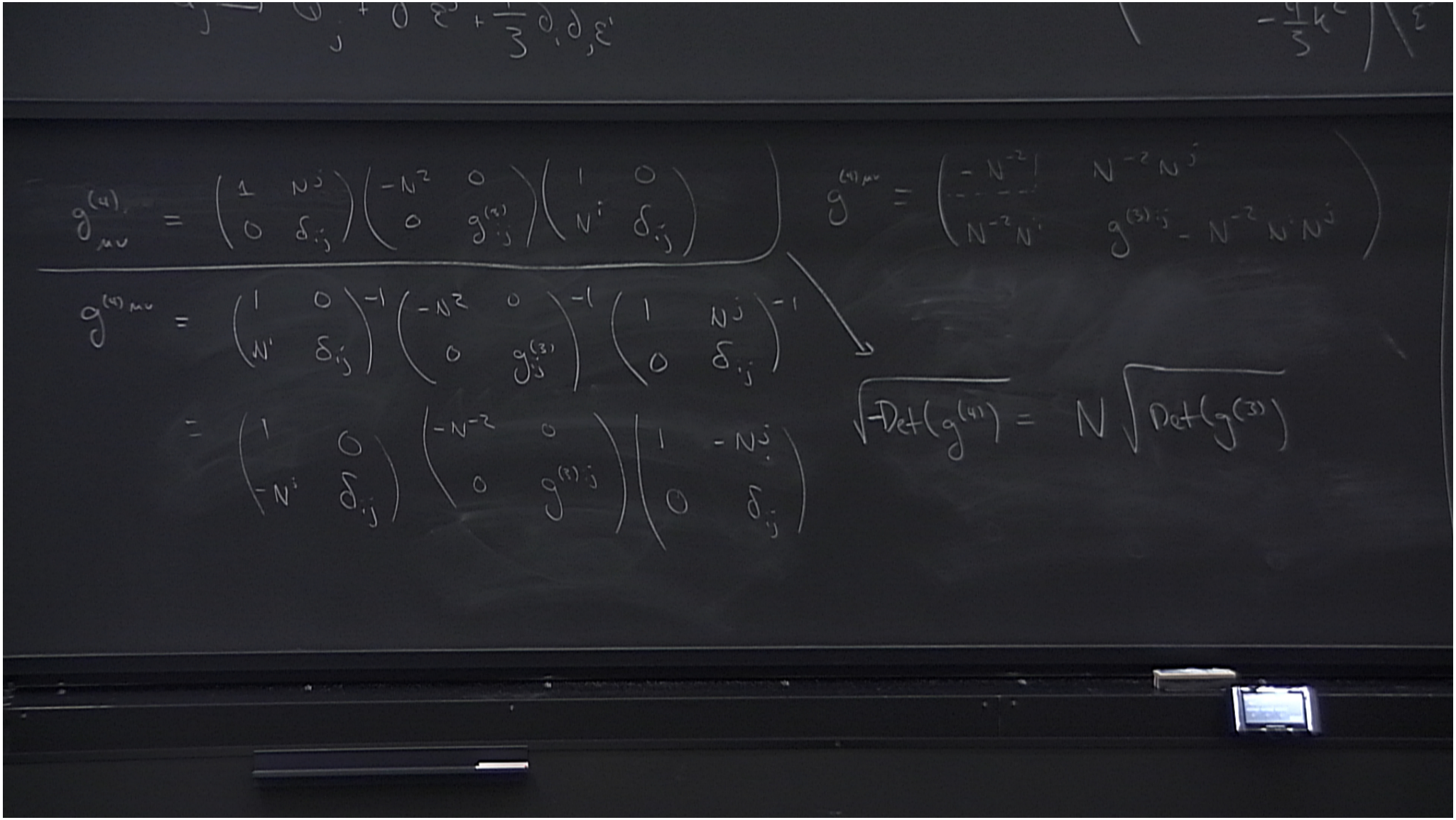
THE UNIT NORMAL VECTOR n^μ IS DEFINED BY:

$n_\mu X^\mu = 0$ FOR ALL TANGENT VECTOR FIELDS X^μ

$n_\mu n^\mu = -1$

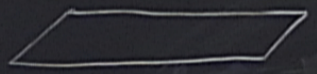
$\Leftrightarrow \boxed{n_i = 0}$

$n_\mu = (N, 0)$


$g^{(4)}_{\mu\nu} = \begin{pmatrix} 1 & N^j \\ 0 & \delta_{ij} \end{pmatrix} \begin{pmatrix} -N^2 & 0 \\ 0 & g^{(3)}_{ij} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ N^i & \delta_{ij} \end{pmatrix}$
$g^{(4)\mu\nu} = \begin{pmatrix} -N^{-2} & N^{-2} N^j \\ N^{-2} N^i & g^{(3)ij} - N^{-2} N^i N^j \end{pmatrix}$
$g^{(4)\mu\nu} = \begin{pmatrix} 1 & 0 \\ N^i & \delta_{ij} \end{pmatrix}^{-1} \begin{pmatrix} -N^2 & 0 \\ 0 & g^{(3)}_{ij} \end{pmatrix}^{-1} \begin{pmatrix} 1 & N^j \\ 0 & \delta_{ij} \end{pmatrix}^{-1}$
$= \begin{pmatrix} 1 & 0 \\ -N^i & \delta_{ij} \end{pmatrix} \begin{pmatrix} -N^{-2} & 0 \\ 0 & g^{(3)ij} \end{pmatrix} \begin{pmatrix} 1 & -N^j \\ 0 & \delta_{ij} \end{pmatrix}$
$\sqrt{-Det(g^{(4)})} = N \sqrt{Det(g^{(3)})}$

$X^\mu = (0, X^i)$

A VECTOR FIELD X^μ IS "TANGENT" IF $X^0 = 0$. $[X_0 \neq 0]$



THE UNIT NORMAL VECTOR n^μ IS DEFINED BY:

$n_\mu X^\mu = 0$ FOR ALL TANGENT VECTOR FIELDS X^μ

$n_\mu n^\mu = -1$

$\Leftrightarrow n_i = 0$

$n_\mu = (N, 0)$

$n^\mu = (-N^{-1}, N^{-1} N^i)$