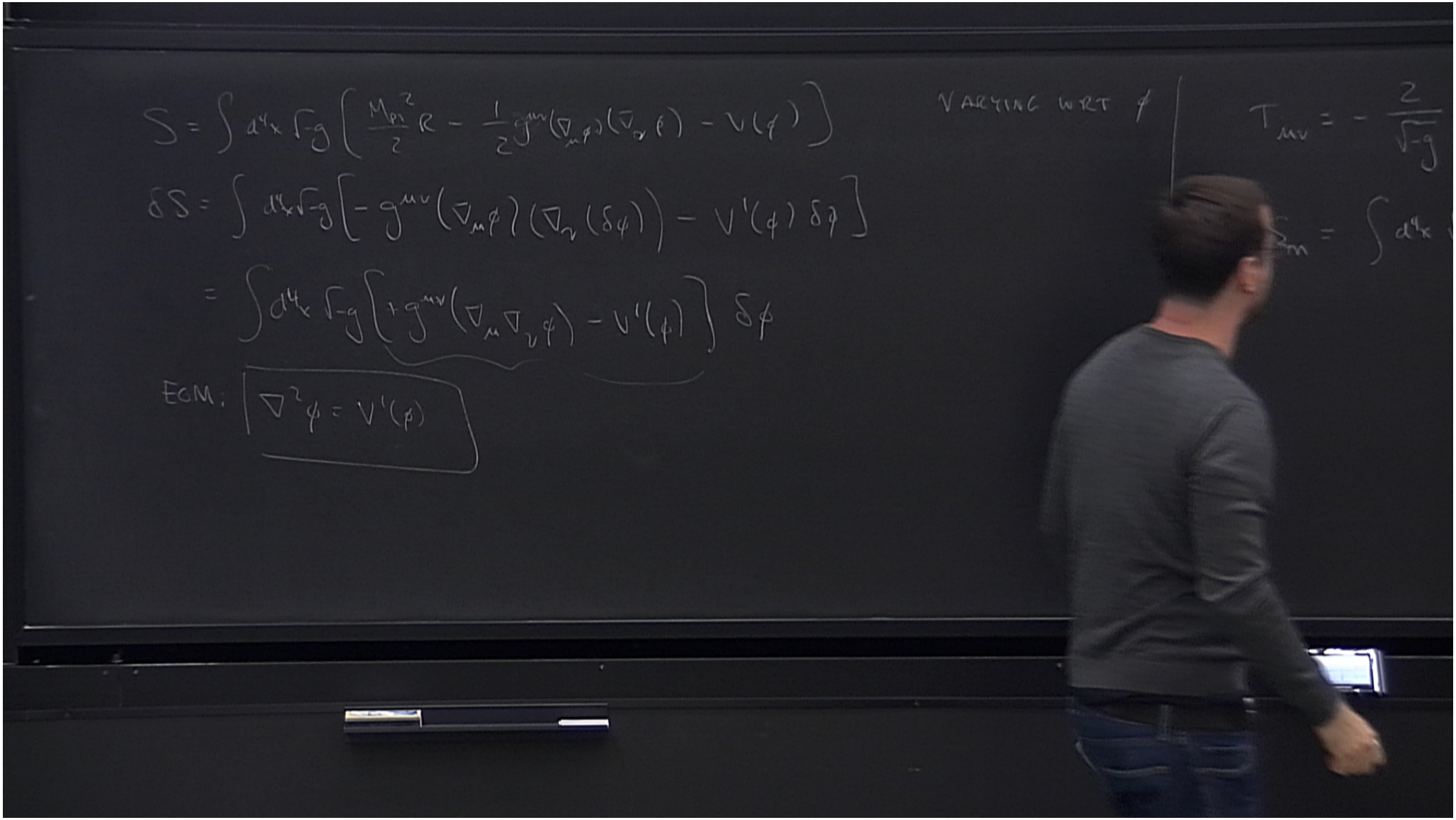
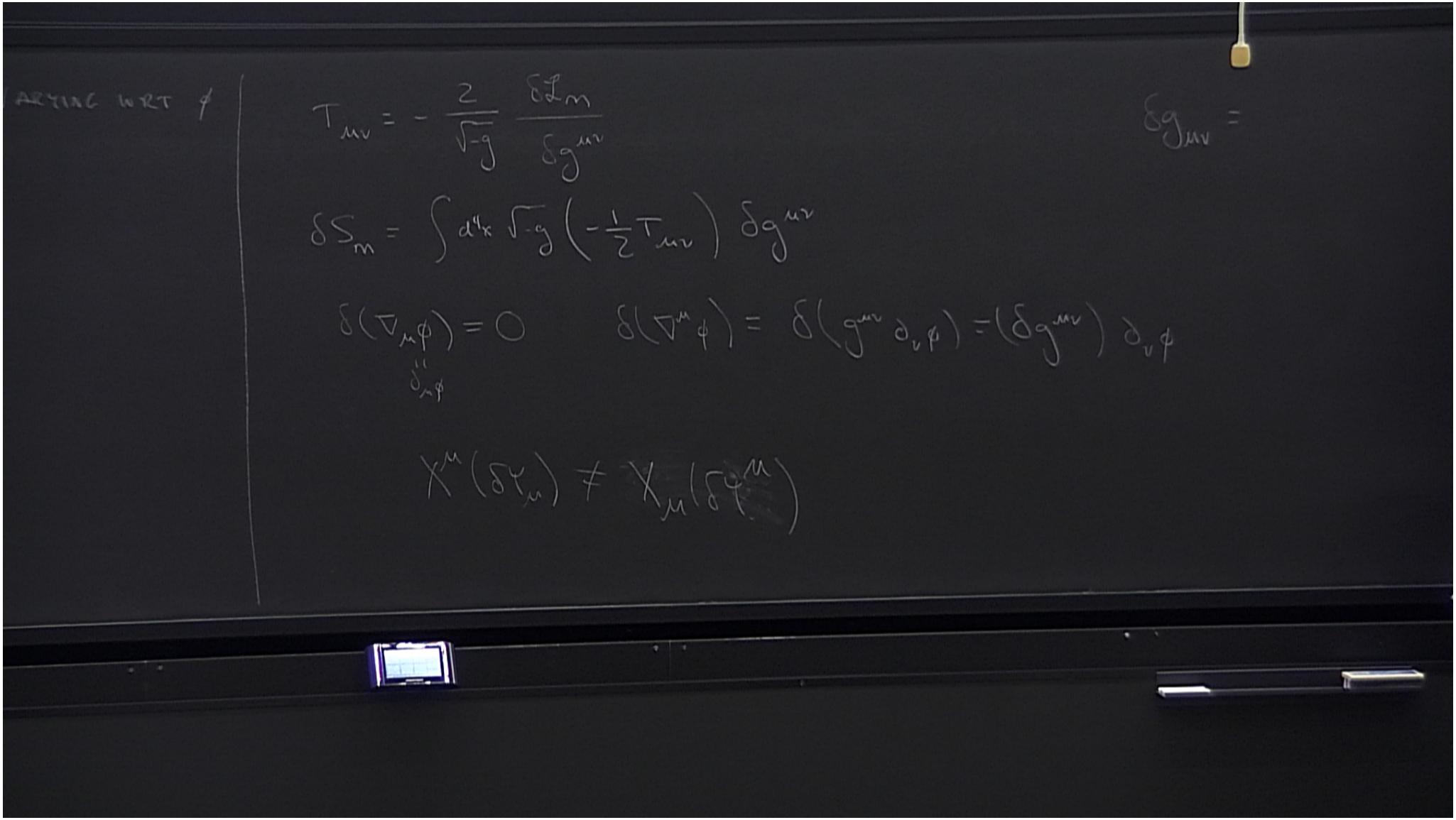
Title: PSI 16/17 Explorations in Cosmology (Kendrick Smith) - Lecture 9

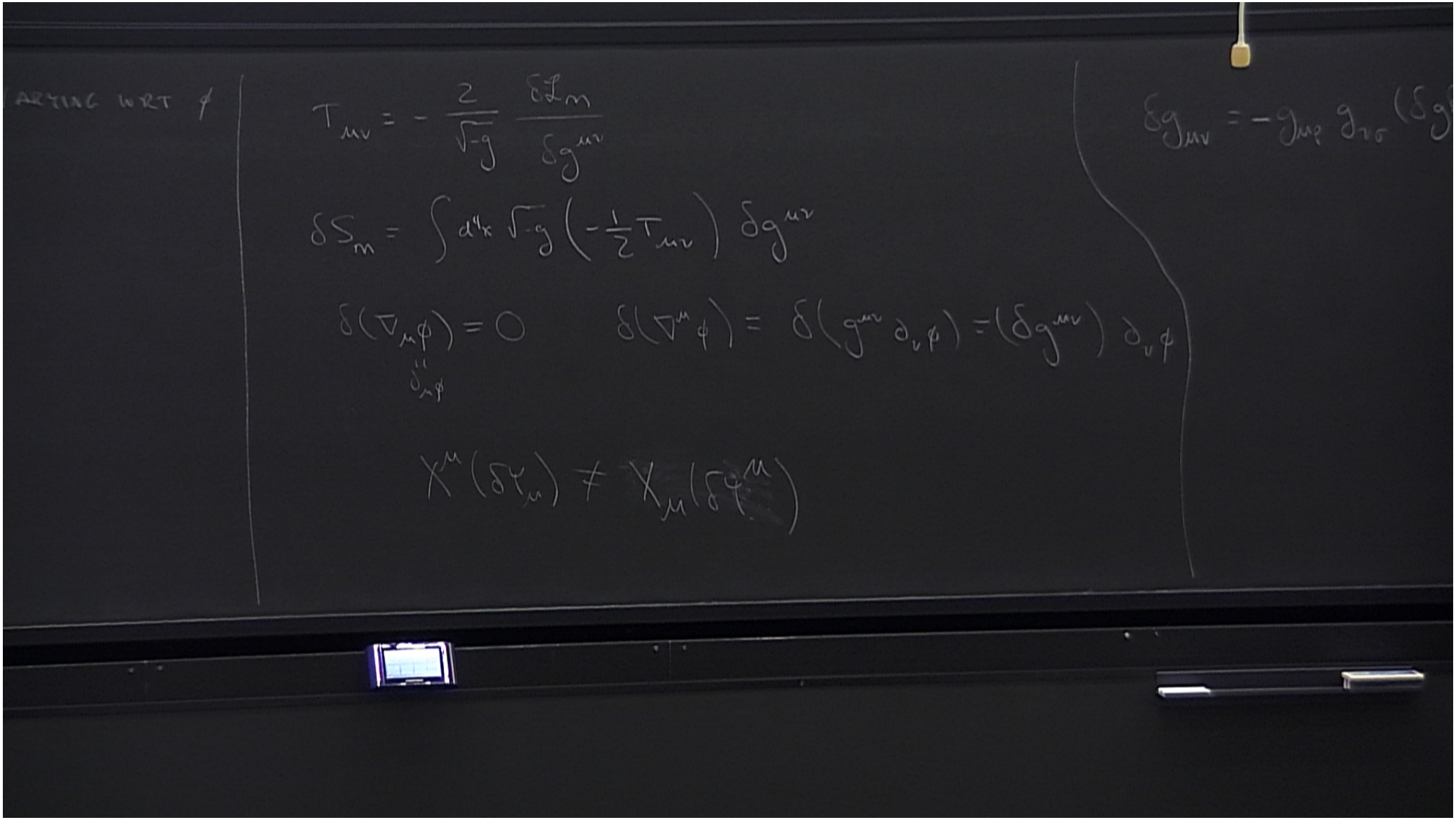
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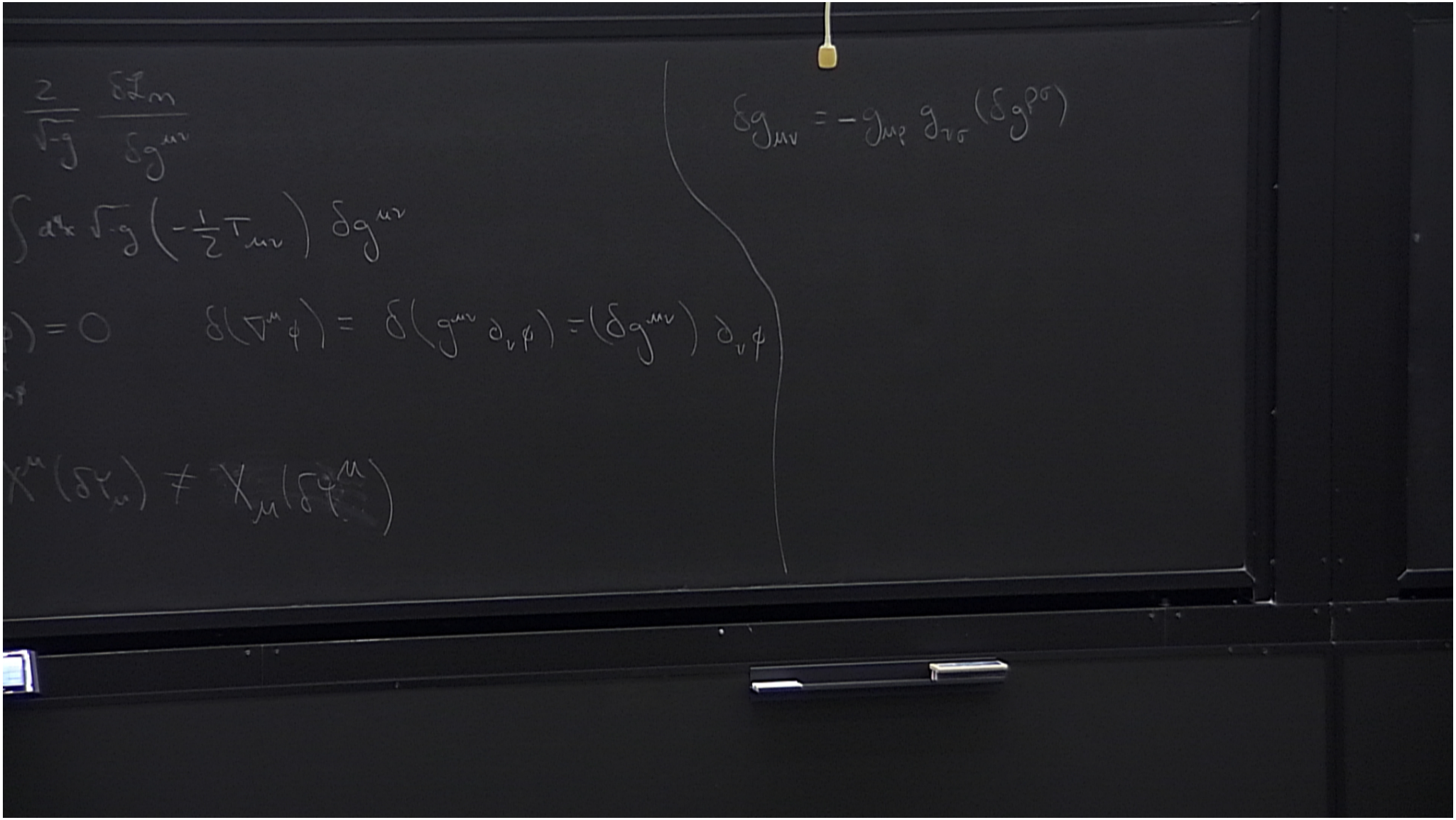
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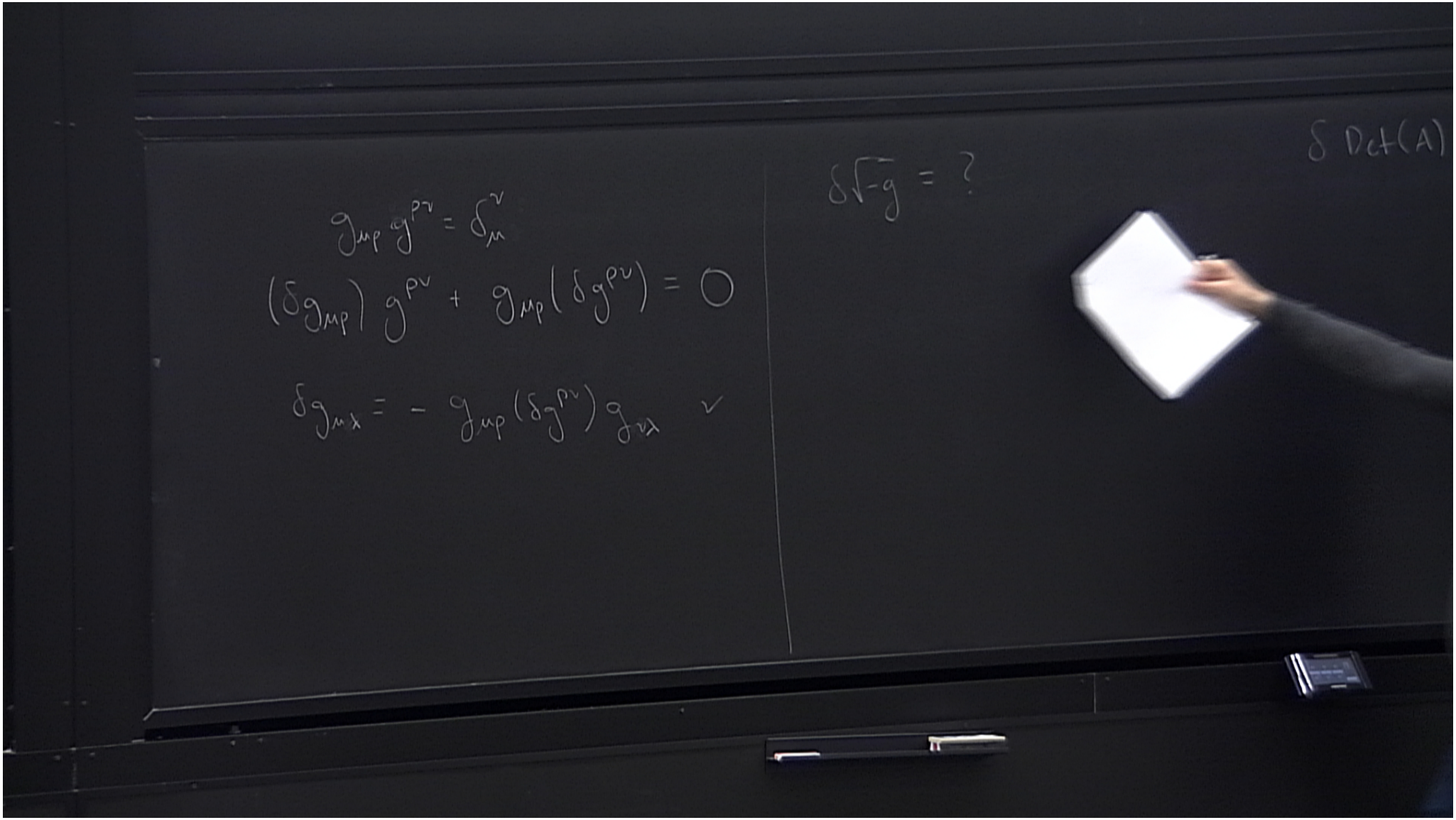
Abstract:

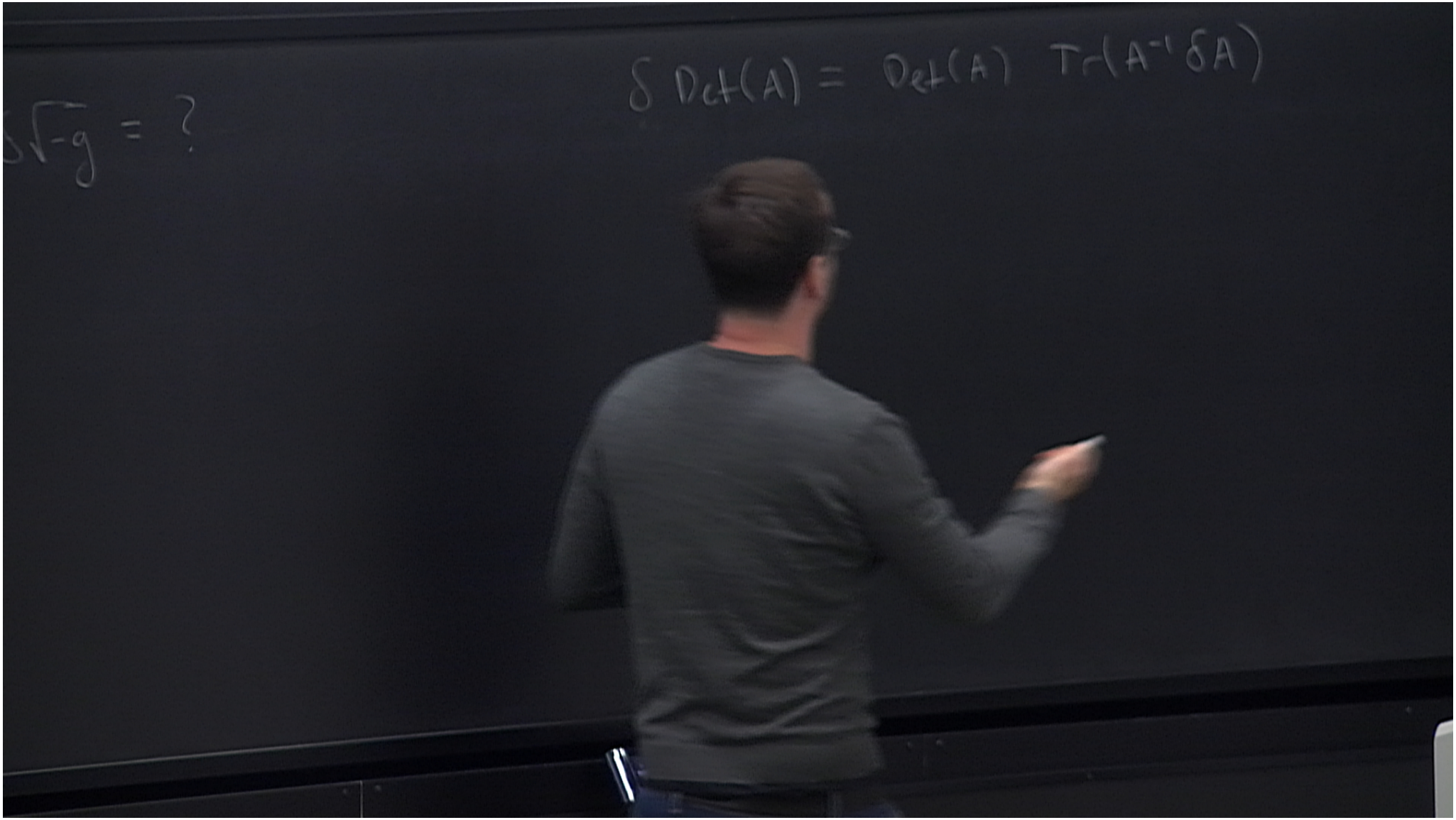

$S = \int d^4x \sqrt{-g} \left(\frac{M_{Pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} (\nabla_\mu \phi)(\nabla_\nu \phi) - V(\phi) \right)$
VARYING WRT ϕ
$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}$
$\delta S = \int d^4x \sqrt{-g} \left[-g^{\mu\nu} (\nabla_\mu \phi)(\nabla_\nu (\delta\phi)) - V'(\phi)\delta\phi \right]$
$= \int d^4x \sqrt{-g} \left[+g^{\mu\nu} (\nabla_\mu \nabla_\nu \phi) - V'(\phi) \right] \delta\phi$
EOM: $\nabla^2 \phi = V'(\phi)$

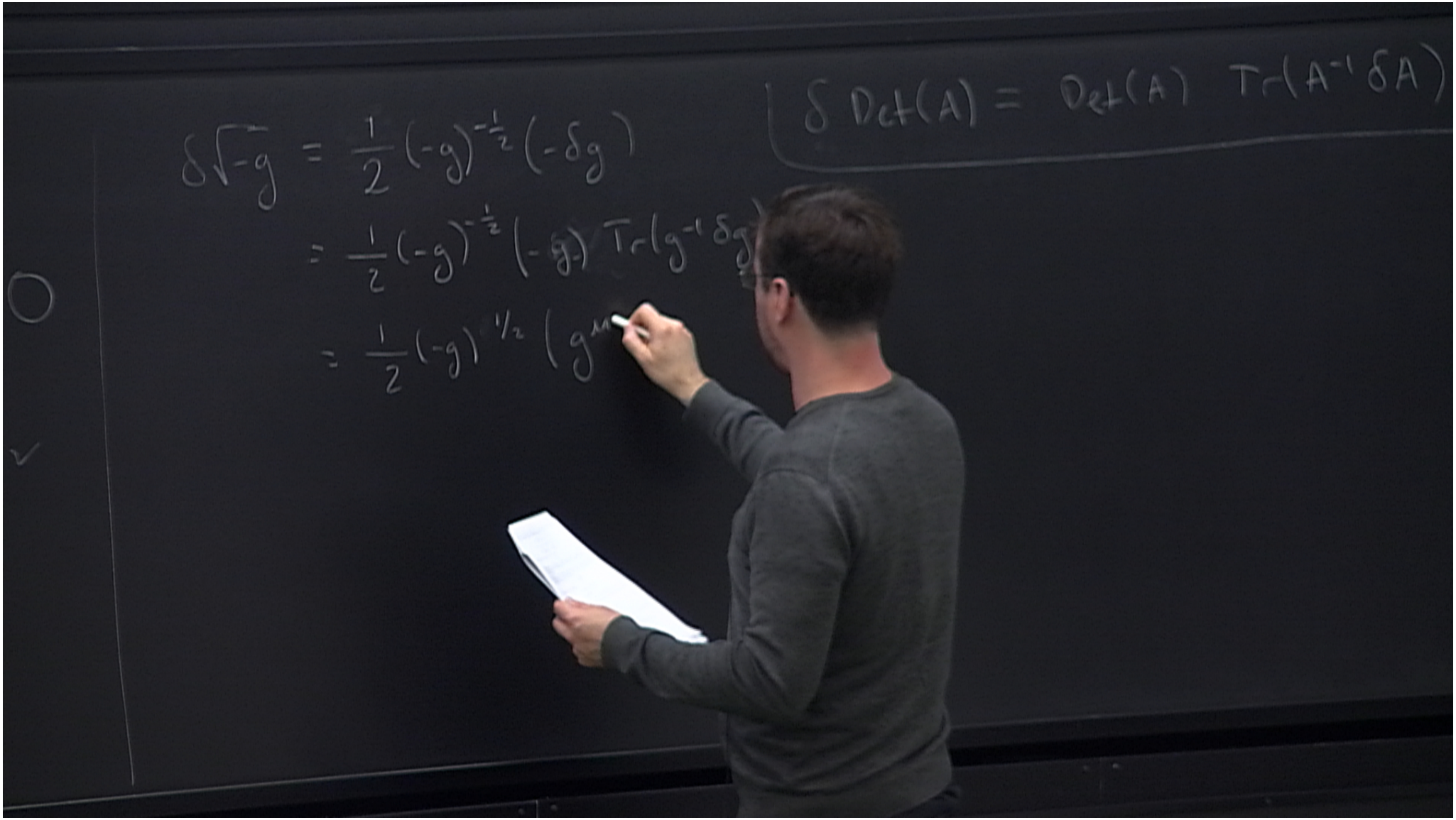

ARYING WRT φ
$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$\delta g_{\mu\nu} =$
$\delta S_m = \int d^4x \sqrt{-g} \left(-\frac{1}{2} T_{\mu\nu}\right) \delta g^{\mu\nu}$
$\delta(\nabla_\mu \phi) = 0$
$\delta_\mu \phi$
$\delta(\nabla^\mu \phi) = \delta(g^{\mu\nu} \partial_\nu \phi) = (\delta g^{\mu\nu}) \partial_\nu \phi$
$X^\mu(\delta x_\mu) \neq X_\mu(\delta x^\mu)$


ARYING WRT φ
$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$\delta S_m = \int d^4x \sqrt{-g} \left(-\frac{1}{2} T_{\mu\nu}\right) \delta g^{\mu\nu}$
$\delta(\nabla_\mu \phi) = 0$
$\delta(\nabla^\mu \phi) = \delta(g^{\mu\nu} \partial_\nu \phi) = (\delta g^{\mu\nu}) \partial_\nu \phi$
$X^\mu(\delta x_\mu) \neq X_\mu(\delta x^\mu)$
$\delta g_{\mu\nu} = -g_{\mu\rho} g_{\nu\sigma} (\delta g$


$\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{I}_m}{\delta g^{\mu\nu}}$
$\int d^4x \sqrt{-g} \left(-\frac{1}{2} T_{\mu\nu}\right) \delta g^{\mu\nu}$
$\delta(\nabla^\mu \phi) = \delta(g^{\mu\nu} \partial_\nu \phi) = (\delta g^{\mu\nu}) \partial_\nu \phi$
$\delta g_{\mu\nu} = -g_{\mu\rho} g_{\nu\sigma} (\delta g^{\rho\sigma})$


$g_{\mu\rho} g^{\rho\nu} = \delta^{\nu}_{\mu}$
$(\delta g_{\mu\rho})\, g^{\rho\nu} + g_{\mu\rho}(\delta g^{\rho\nu}) = 0$
$\delta g_{\mu\lambda} = - g_{\mu\rho}(\delta g^{\rho\nu}) g_{\nu\lambda}$ ✓
$\delta\sqrt{-g} = ?$
$\delta\, Det(A)$


$\delta \sqrt{-g} = ?$
$\delta \, \mathrm{Det}(A) = \mathrm{Det}(A) \, \mathrm{Tr}(A^{-1} \delta A)$


$\delta\sqrt{-g} = \frac{1}{2}(-g)^{-\frac{1}{2}}(-\delta g)$
$= \frac{1}{2}(-g)^{-\frac{1}{2}}(-g)\,\mathrm{Tr}(g^{-1}\delta g)$
$= \frac{1}{2}(-g)^{1/2}(g^{\mu}$
$\delta\,\mathrm{Det}(A) = \mathrm{Det}(A)\,\mathrm{Tr}(A^{-1}\delta A)$

$$\delta\sqrt{-g} = \frac{1}{2}(-g)^{-\frac{1}{2}}(-\delta g)$$

$$= \frac{1}{2}(-g)^{-\frac{1}{2}}(-g)\,\mathrm{Tr}(g^{-1}\delta g)$$

$$= \frac{1}{2}(-g)^{1/2}\left(g^{\mu\nu}\delta g_{\mu\nu}\right)$$

$$= \frac{1}{2}(-g)^{1/2}\left(-g_{\mu\nu}\delta g^{\mu\nu}\right)$$

$$\delta\,\mathrm{Det}(A) = \mathrm{Det}(A)\,\mathrm{Tr}(A^{-1}\delta A)$$

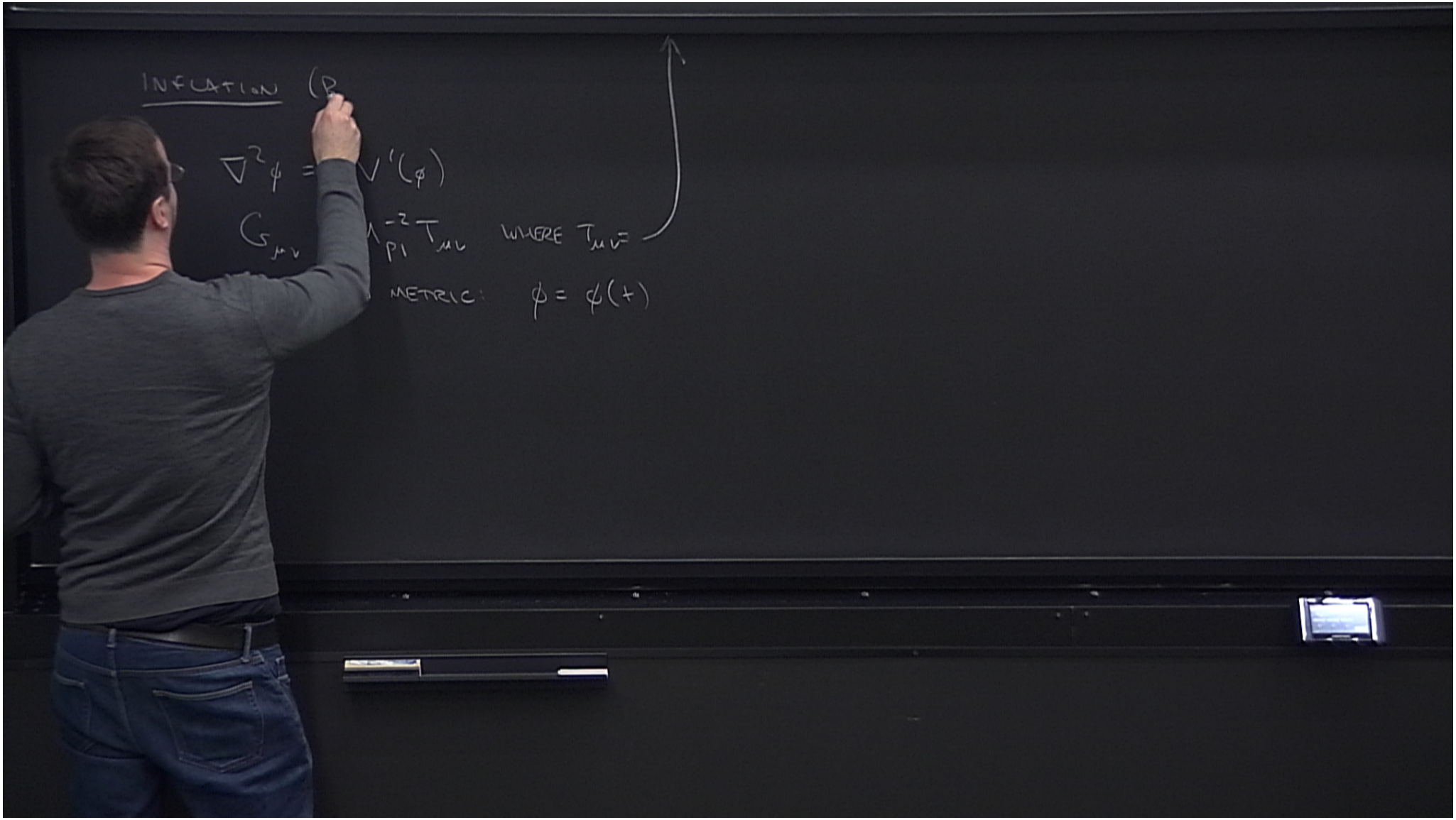
$$S_m = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} (\nabla_\mu \phi)(\nabla_\nu \phi) - V(\phi) \right)$$

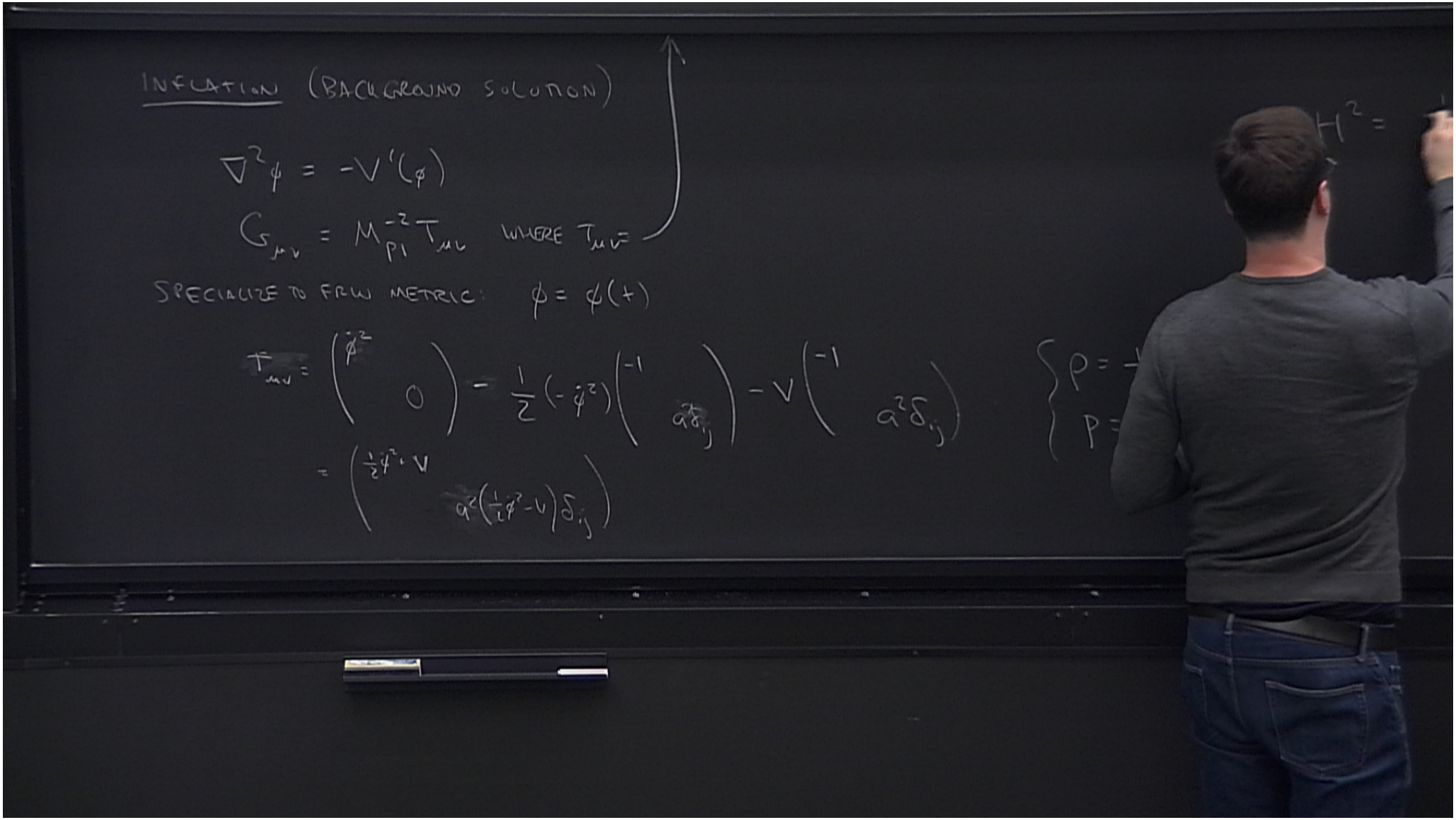
$$\delta S_m = \int d^4x \sqrt{-g} \left[\left(-\frac{1}{2} g_{\mu\nu} \delta g^{\mu\nu} \right) \left(-\frac{1}{2} (\nabla^\mu \phi)(\nabla_\mu \phi) - V(\phi) \right) - \frac{1}{2} (\delta g^{\mu\nu}) (\nabla_\mu \phi)(\nabla_\nu \phi) \right]$$

$$= \int d^4x \sqrt{-g} \underbrace{\left(-\frac{1}{2} (\nabla_\mu \phi)(\nabla_\nu \phi) + \frac{1}{4} (\nabla_\rho \phi)(\nabla^\rho \phi) g_{\mu\nu} + \frac{1}{2} V(\phi) g_{\mu\nu} \right)}_{(-1/2) T_{\mu\nu}} \delta g^{\mu\nu}$$

$$\boxed{T_{\mu\nu} = (\nabla_\mu \phi)(\nabla_\nu \phi) - \frac{1}{2} (\nabla_\rho \phi)(\nabla^\rho \phi) g_{\mu\nu} - V(\phi) g_{\mu\nu}}$$

INFLATION


INFLATION
$\nabla^2\phi = V'(\phi)$
$G_{\mu\nu} = \Lambda_{Pl}^{-2} T_{\mu\nu}$ WHERE $T_{\mu\nu} =$
METRIC: $\phi = \phi(t)$


INFLATION (BACKGROUND SOLUTION)
$\nabla^2\phi = -V'(\phi)$
$G_{\mu\nu} = M_{Pl}^{-2} T_{\mu\nu}$ WHERE $T_{\mu\nu} =$
SPECIALIZE TO FRW METRIC: $\phi = \phi(t)$
$T_{\mu\nu} = \begin{pmatrix} \dot\phi^2 & \\ & 0 \end{pmatrix} - \frac{1}{2}(-\dot\phi^2)\begin{pmatrix} -1 & \\ & a^2\delta_{ij} \end{pmatrix} - V\begin{pmatrix} -1 & \\ & a^2\delta_{ij} \end{pmatrix}$
$= \begin{pmatrix} \frac{1}{2}\dot\phi^2 + V & \\ & a^2\left(\frac{1}{2}\dot\phi^2 - V\right)\delta_{ij} \end{pmatrix}$
$H^2 =$
$\rho =$
$p =$

$\ddot{x} + \mu\dot{x} + V' = 0 \qquad \mu\dot{x} + V' \approx 0$

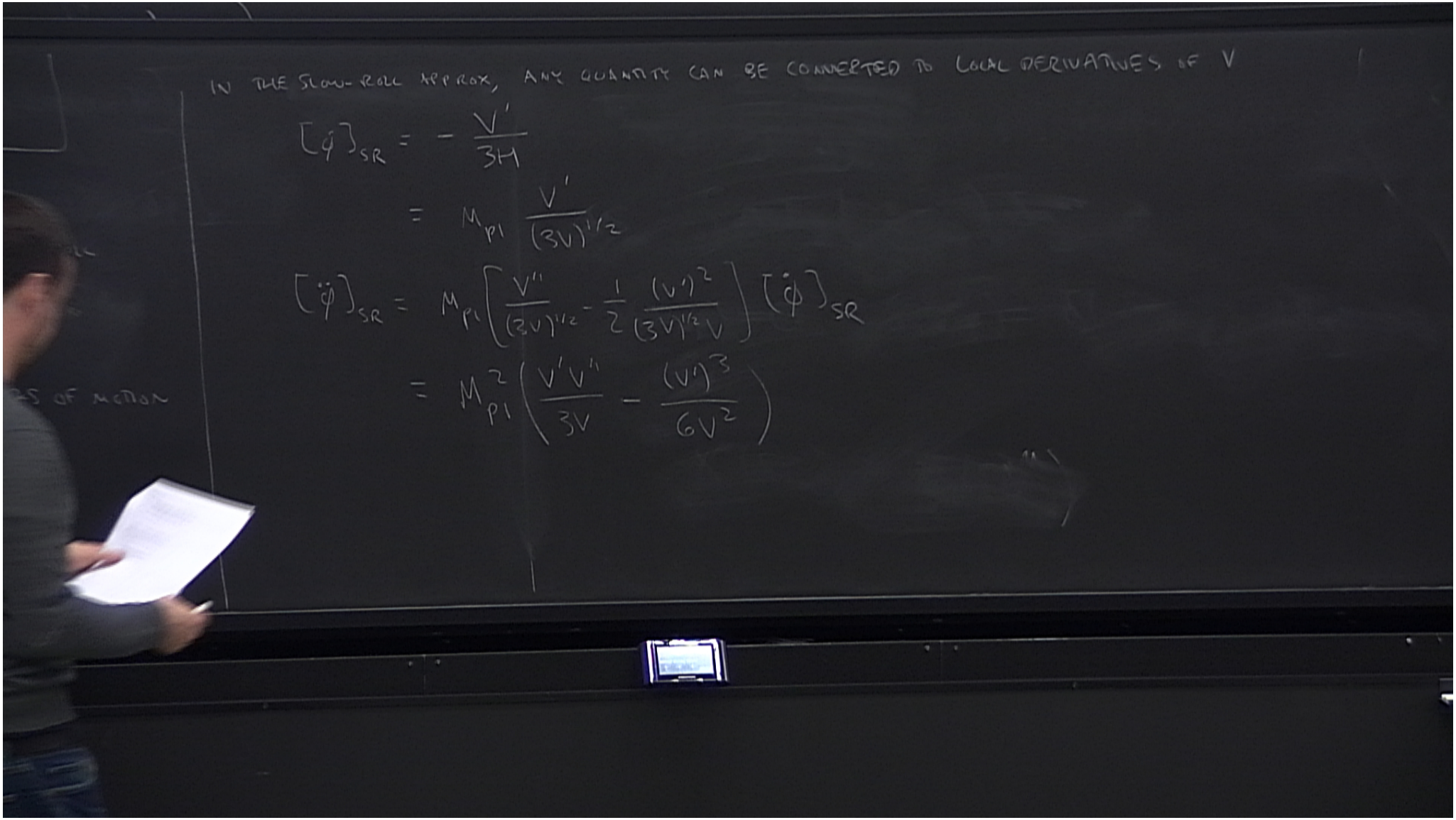
IN THE SLOW-ROLL APPROX, ANY QUANTITY

$$3H\dot{\phi} + V'(\phi) \approx 0$$
$$H^2 \approx \frac{1}{3M_{Pl}^2} V(\phi)$$

SLOW-ROLL EQS

ϕ_{SR} = EXACT SOLUTION TO THE SLOW-ROLL EQS OF MOTION

UNDER WHAT CONDITIONS IS $\phi \approx \phi_{SR}$?


IN THE SLOW-ROLL APPROX, ANY QUANTITY CAN BE CONVERTED TO LOCAL DERIVATIVES OF V
$[\dot\phi]_{SR} = -\frac{V'}{3H}$
$= M_{Pl}\frac{V'}{(3V)^{1/2}}$
$[\ddot\phi]_{SR} = M_{Pl}\left[\frac{V''}{(3V)^{1/2}} - \frac{1}{2}\frac{(V')^2}{(3V)^{1/2}V}\right][\dot\phi]_{SR}$
$= M_{Pl}^2\left(\frac{V'V''}{3V} - \frac{(V')^3}{6V^2}\right)$

IN THE SLOW-ROLL APPROX, ANY QUANTITY CAN BE CONVERTED TO LOCAL DERIVATIVES OF V

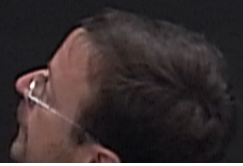
$$[\dot{\varphi}]_{SR} = -\frac{V'}{3H}$$

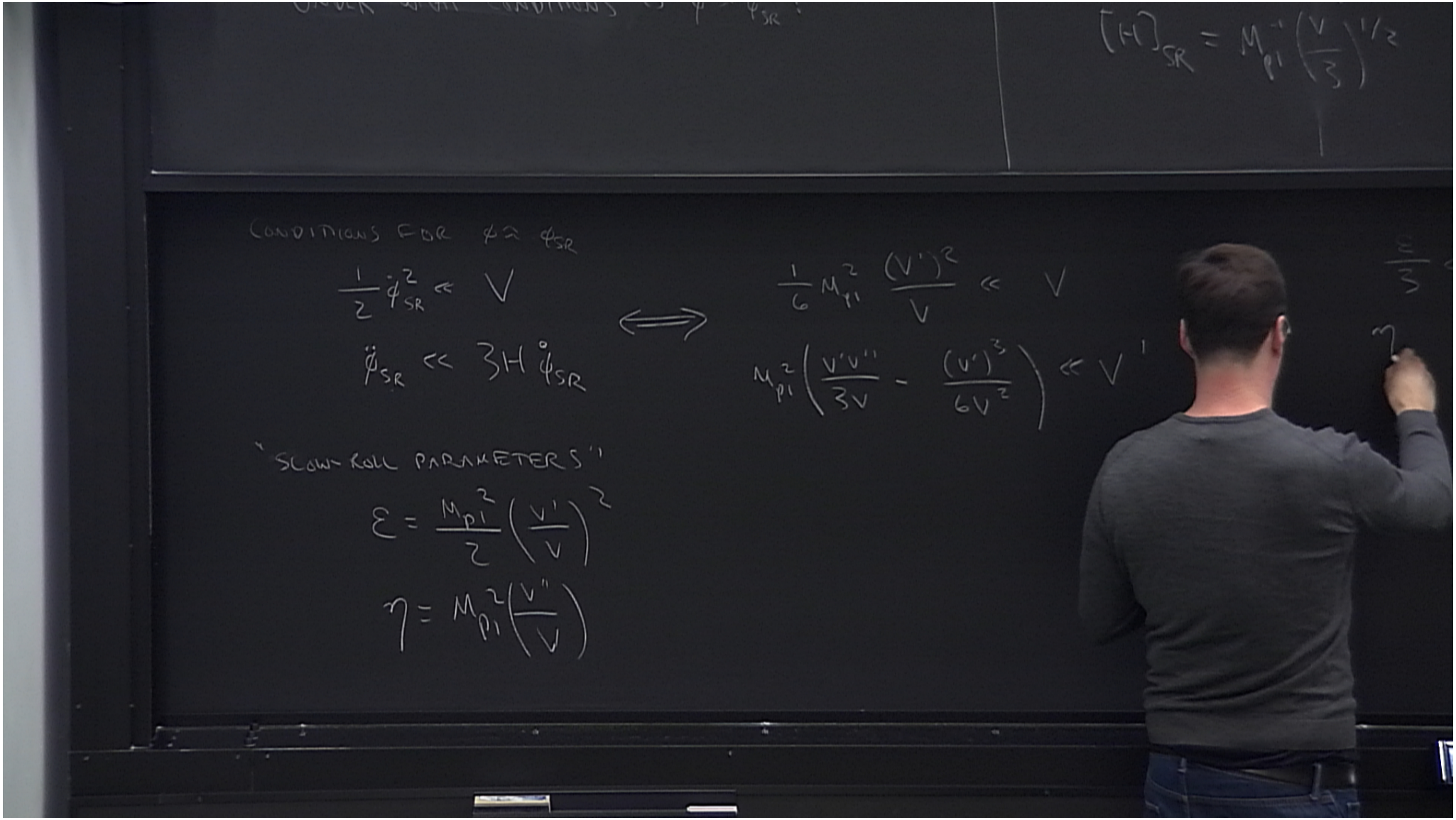
$$= M_{Pl}\frac{V'}{(3V)^{1/2}}$$

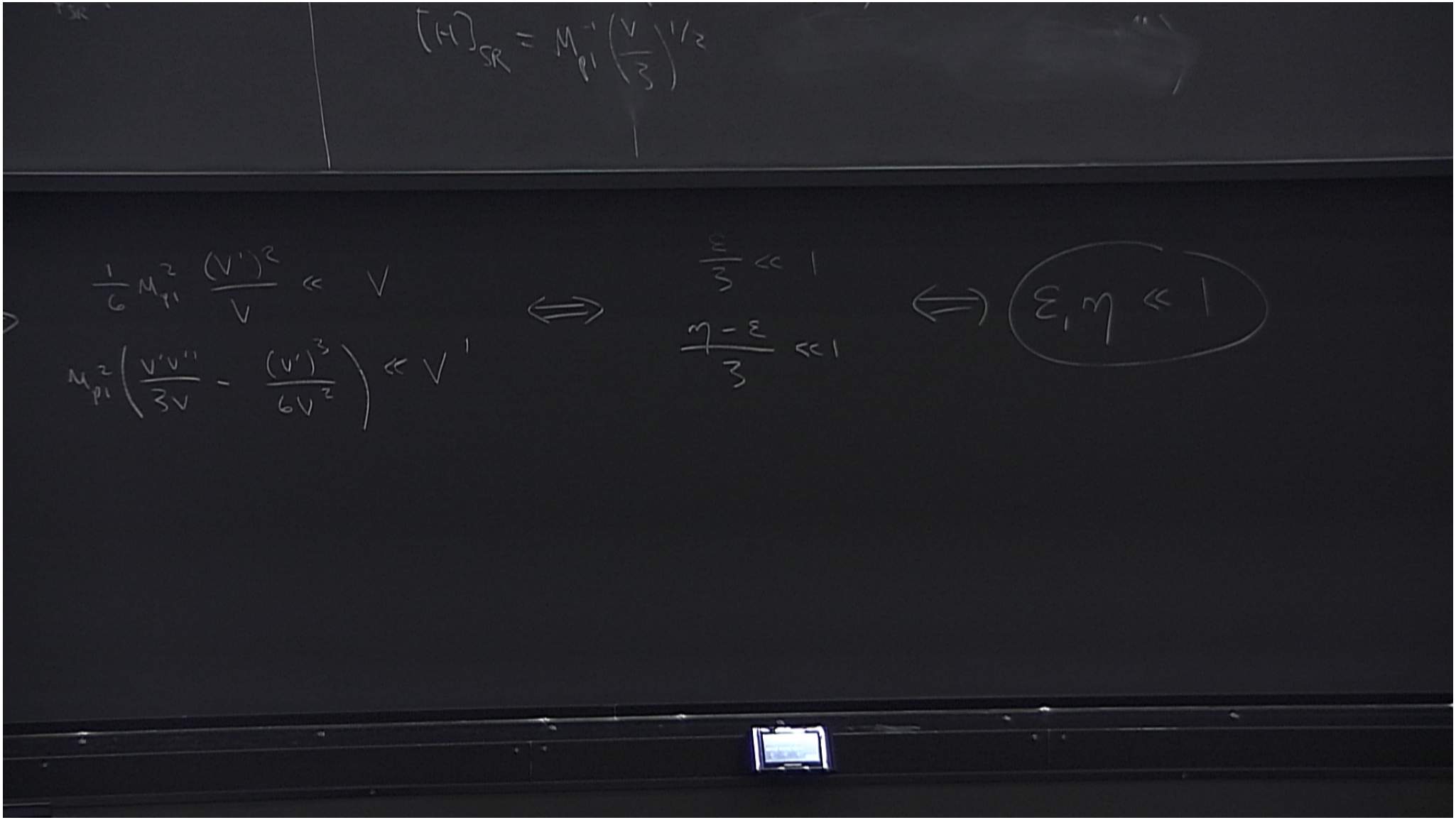
$$[\ddot{\varphi}]_{SR} = M_{Pl}\left(\frac{V''}{(3V)^{1/2}} - \frac{1}{2}\frac{(V')^2}{(3V)^{1/2}V}\right)[\dot{\varphi}]_{SR}$$

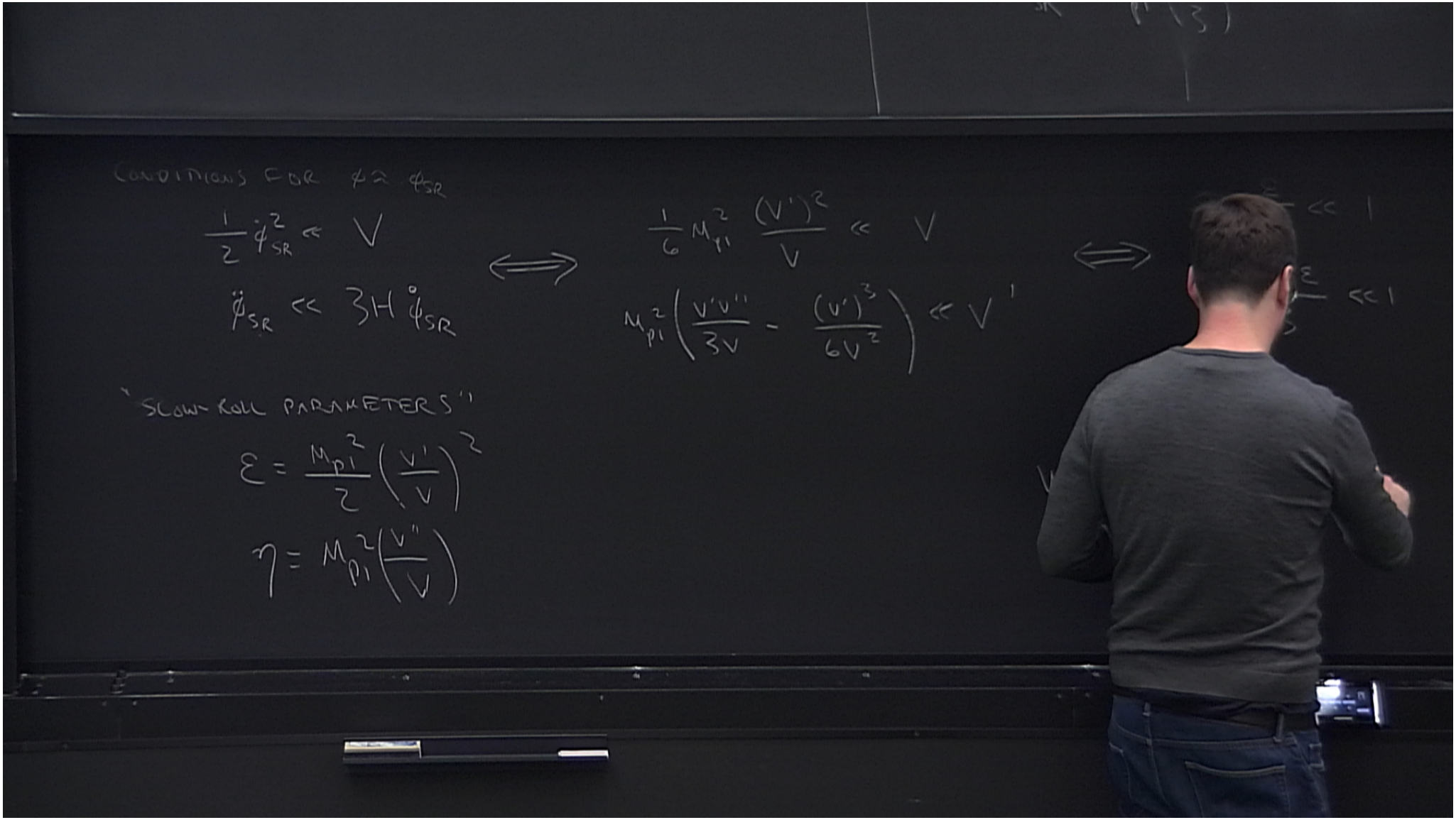
$$= M_{Pl}^2\left(\frac{V'V''}{3V} - \frac{(V')^3}{6V^2}\right)$$

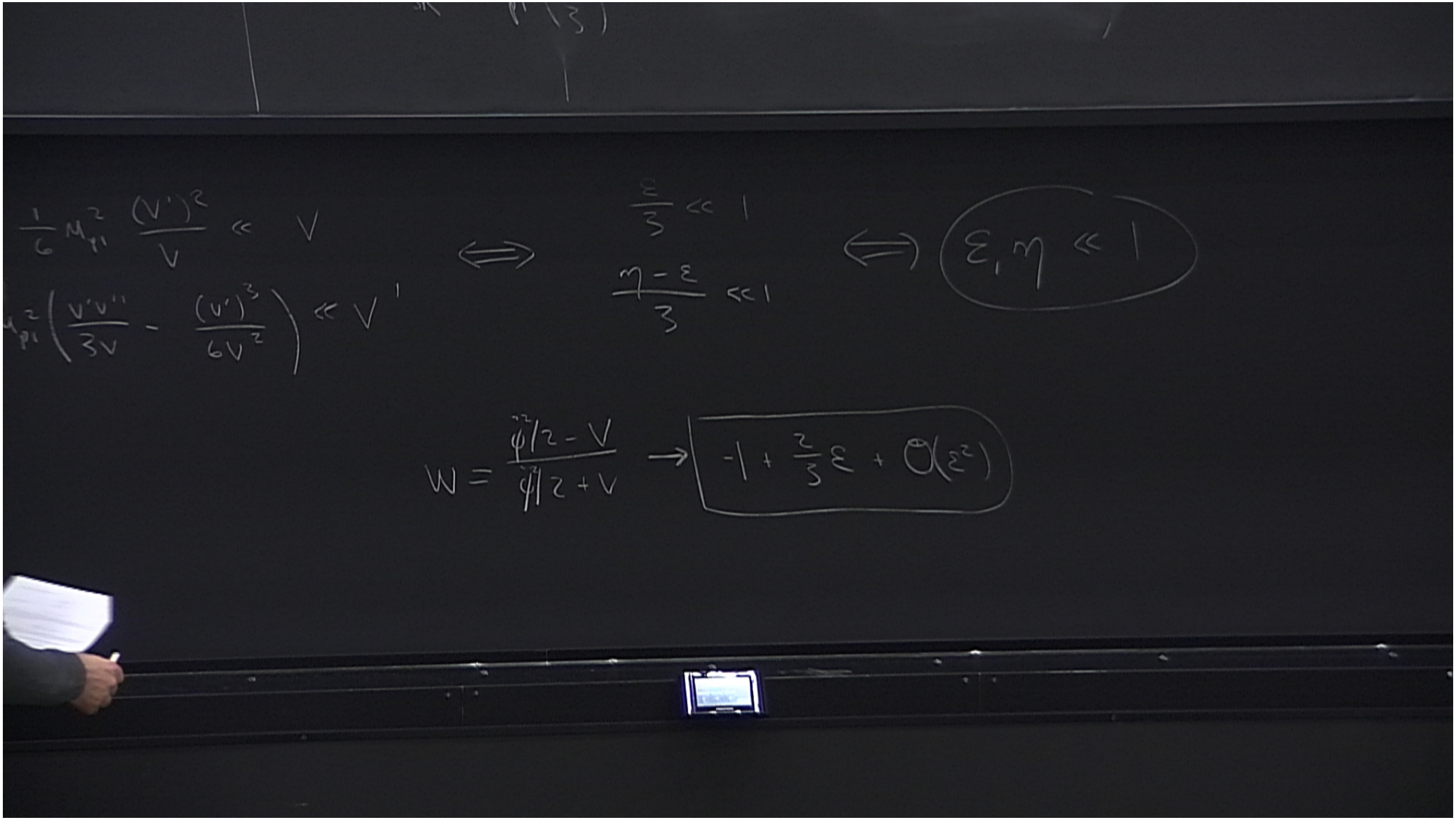
$$[H]_{SR} = M_{Pl}^{-1}\left(\frac{V}{3}\right)^{1/2}$$




$[-H]_{SR} = M_{Pl}^{-1}\left(\frac{V}{3}\right)^{1/2}$
CONDITIONS FOR $\phi \approx \phi_{SR}$
$\frac{1}{2}\dot{\phi}_{SR}^2 \ll V$
$\ddot{\phi}_{SR} \ll 3H\dot{\phi}_{SR}$
$\frac{1}{6}M_{Pl}^2\frac{(V')^2}{V} \ll V$
$M_{Pl}^2\left(\frac{V'V''}{3V} - \frac{(V')^3}{6V^2}\right) \ll V'$
"SLOW-ROLL PARAMETERS"
$\varepsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2$
$\eta = M_{Pl}^2\left(\frac{V''}{V}\right)$


$[H]_{SR} = M_{Pl}^{-1}\left(\frac{V}{3}\right)^{1/2}$
$\frac{1}{6}M_{Pl}^2\frac{(V')^2}{V} \ll V$
$M_{Pl}^2\left(\frac{V'V''}{3V} - \frac{(V')^3}{6V^2}\right) \ll V'$
$\Leftrightarrow$
$\frac{\varepsilon}{3} \ll 1$
$\frac{\eta-\varepsilon}{3} \ll 1$
$\Leftrightarrow$
$\varepsilon, \eta \ll 1$


CONDITIONS FOR $\phi \simeq \phi_{SR}$
$\frac{1}{2}\dot{\phi}_{SR}^2 \ll V$
$\ddot{\phi}_{SR} \ll 3H\dot{\phi}_{SR}$
$\Longleftrightarrow$
$\frac{1}{6}M_{Pl}^2\frac{(V')^2}{V} \ll V$
$M_{Pl}^2\left(\frac{V'V''}{3V} - \frac{(V')^3}{6V^2}\right) \ll V'$
$\Longleftrightarrow$
$\ll 1$
$\ll 1$
"SLOW-ROLL PARAMETERS"
$\varepsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2$
$\eta = M_{Pl}^2\left(\frac{V''}{V}\right)$


$\frac{1}{6} M_{Pl}^2 \frac{(V')^2}{V} \ll V$
$\Leftrightarrow$
$\frac{\varepsilon}{3} \ll 1$
$\frac{\eta - \varepsilon}{3} \ll 1$
$\Leftrightarrow$
$\varepsilon, \eta \ll 1$
$M_{Pl}^2 \left(\frac{V'V''}{3V} - \frac{(V')^3}{6V^2} \right) \ll V'$
$w = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} \rightarrow -1 + \frac{2}{3}\varepsilon + \mathcal{O}(\varepsilon^2)$

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\nabla^\mu\varphi)(\nabla_\mu\varphi) - V(\varphi)\right)$$

$\delta g^{\mu\nu}$, $\delta\phi$

TO SECOND ORDER

$$S = \frac{M_{Pl}^2}{2}\int d^4x\,\frac{\dot\phi^2}{H^2}\left(a^3\dot\zeta^2 - a(\partial_i\zeta)(\partial_i\zeta)\right)$$

$$+ \frac{M_{Pl}^2}{8}\int d^4x\left[a^3\dot\gamma_{ij}\dot\gamma_{ij} - a(\partial_k\gamma_{ij})(\partial_k\gamma_{ij})\right]$$

WHERE ζ, γ_{ij} WILL BE DEFINED LATER!

$$S = \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g}\left(-\frac{1}{2}(\nabla^\mu\varphi)(\nabla_\mu\varphi) - V(\varphi)\right)$$

$\delta g^{\mu\nu}$, $\delta\phi$

TO SECOND ORDER

$$S = \frac{M_{Pl}^2}{2}\int d^4x \frac{\dot\phi^2}{H^2}\left(a^3\dot\zeta^2 - a(\partial_i\zeta)(\partial_i\zeta)\right)$$

$$+ \frac{M_{Pl}^2}{8}\int d^4x \left[a^3\dot\gamma_{ij}\dot\gamma_{ij} - a(\partial_k\gamma_{ij})(\partial_k\gamma_{ij})\right]$$

WHERE ζ, γ_{ij} WILL BE DEFINED LATER!

$$S = \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g}\left(-\frac{1}{2}(\nabla^\mu\varphi)(\nabla_\mu\varphi) - V(\varphi)\right)$$

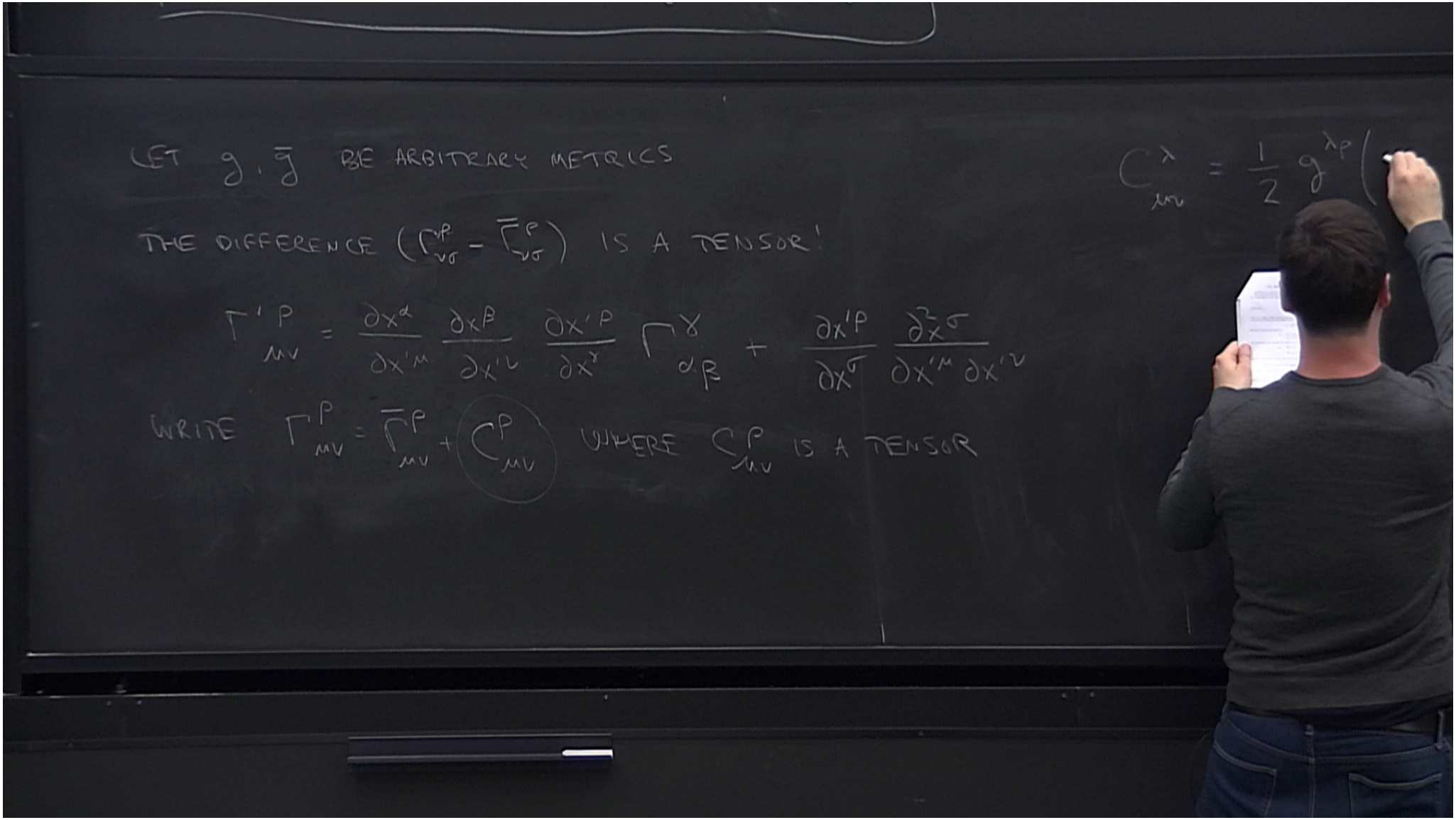
$\delta g^{\mu\nu}, \delta\phi$

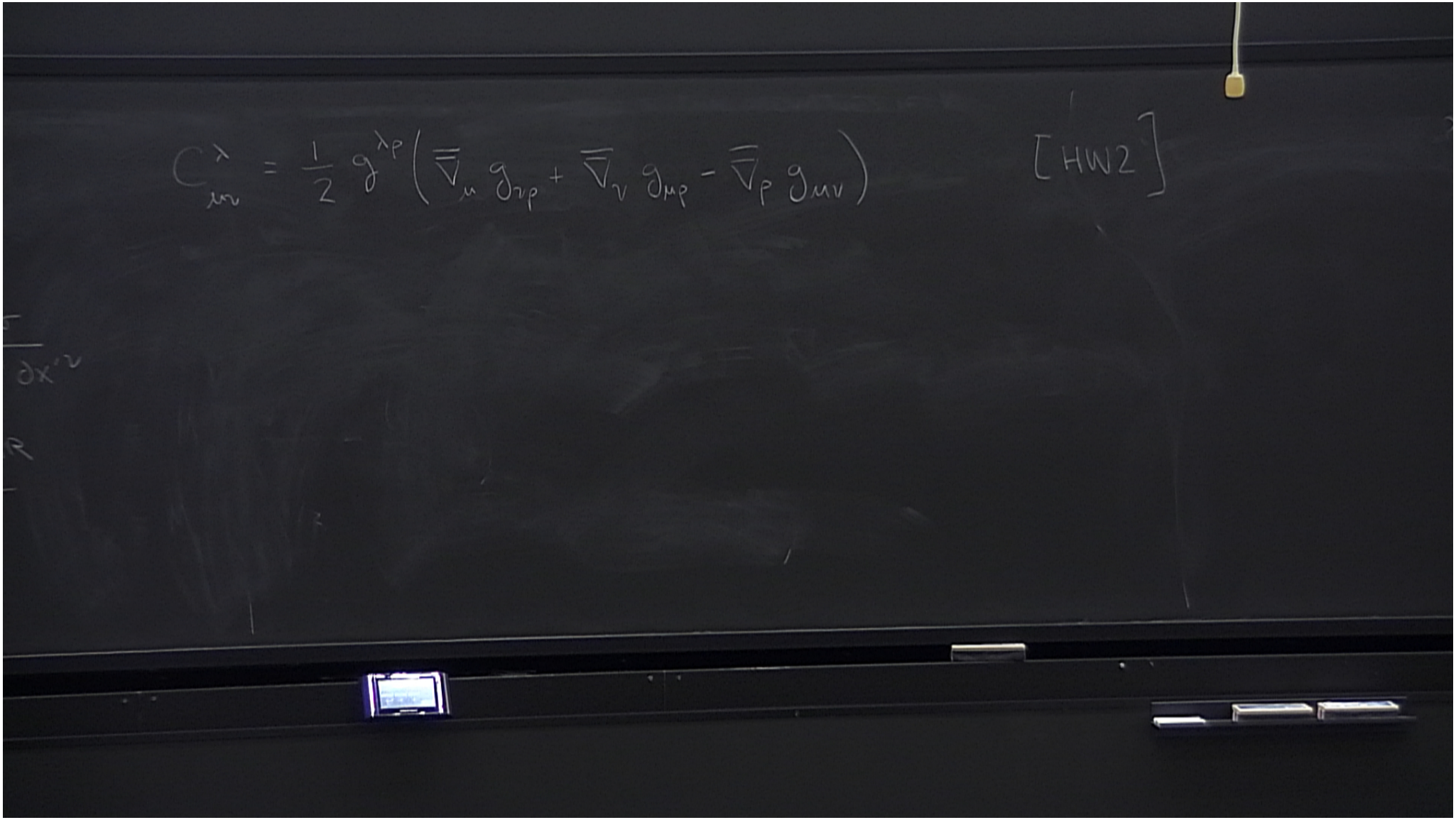
TO SECOND ORDER

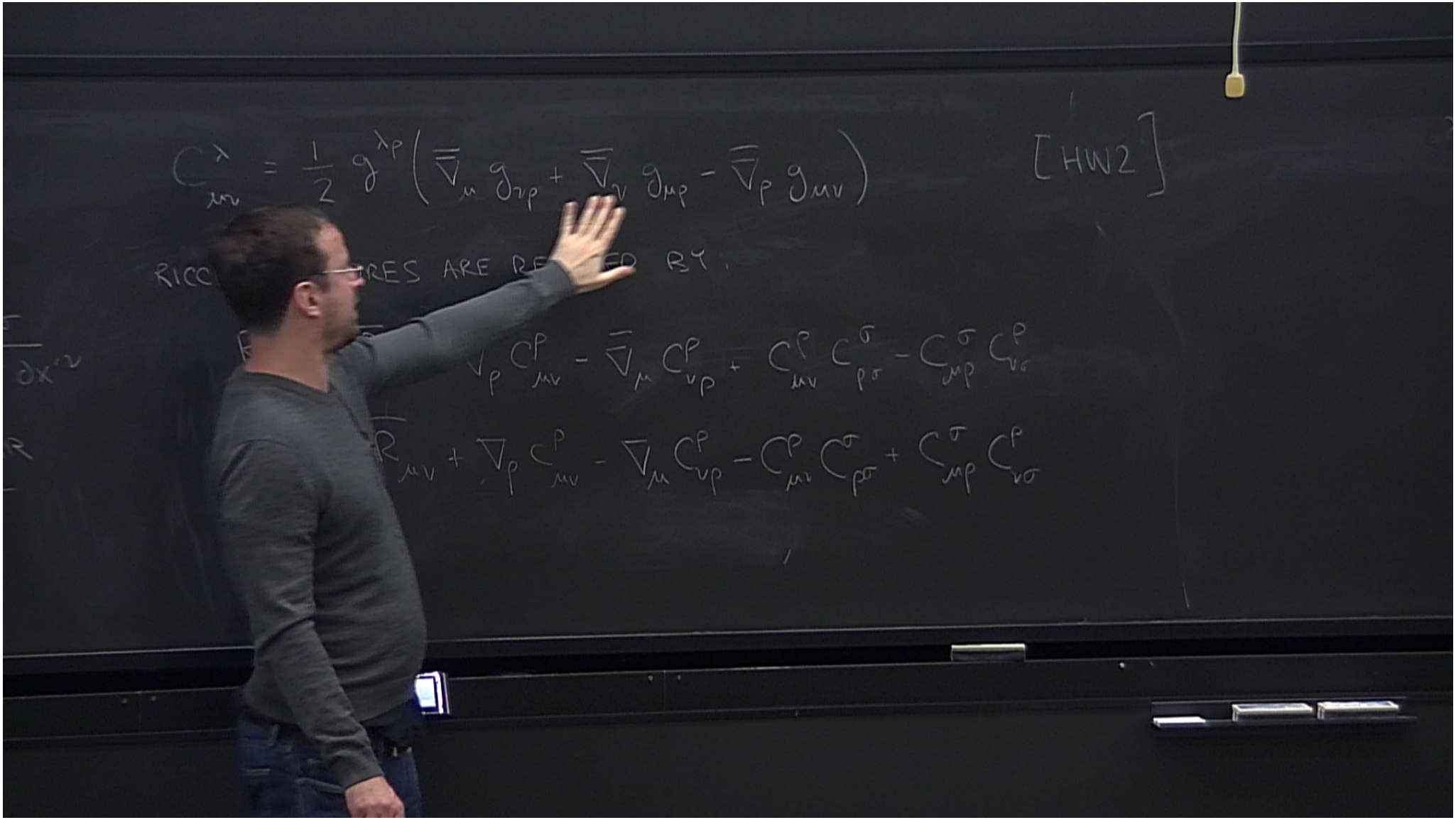
$$S = \frac{M_{Pl}^2}{2}\int d^4x \frac{\dot\phi^2}{H^2}\left(a^3\dot\zeta^2 - a(\partial_i\zeta)(\partial_i\zeta)\right)$$

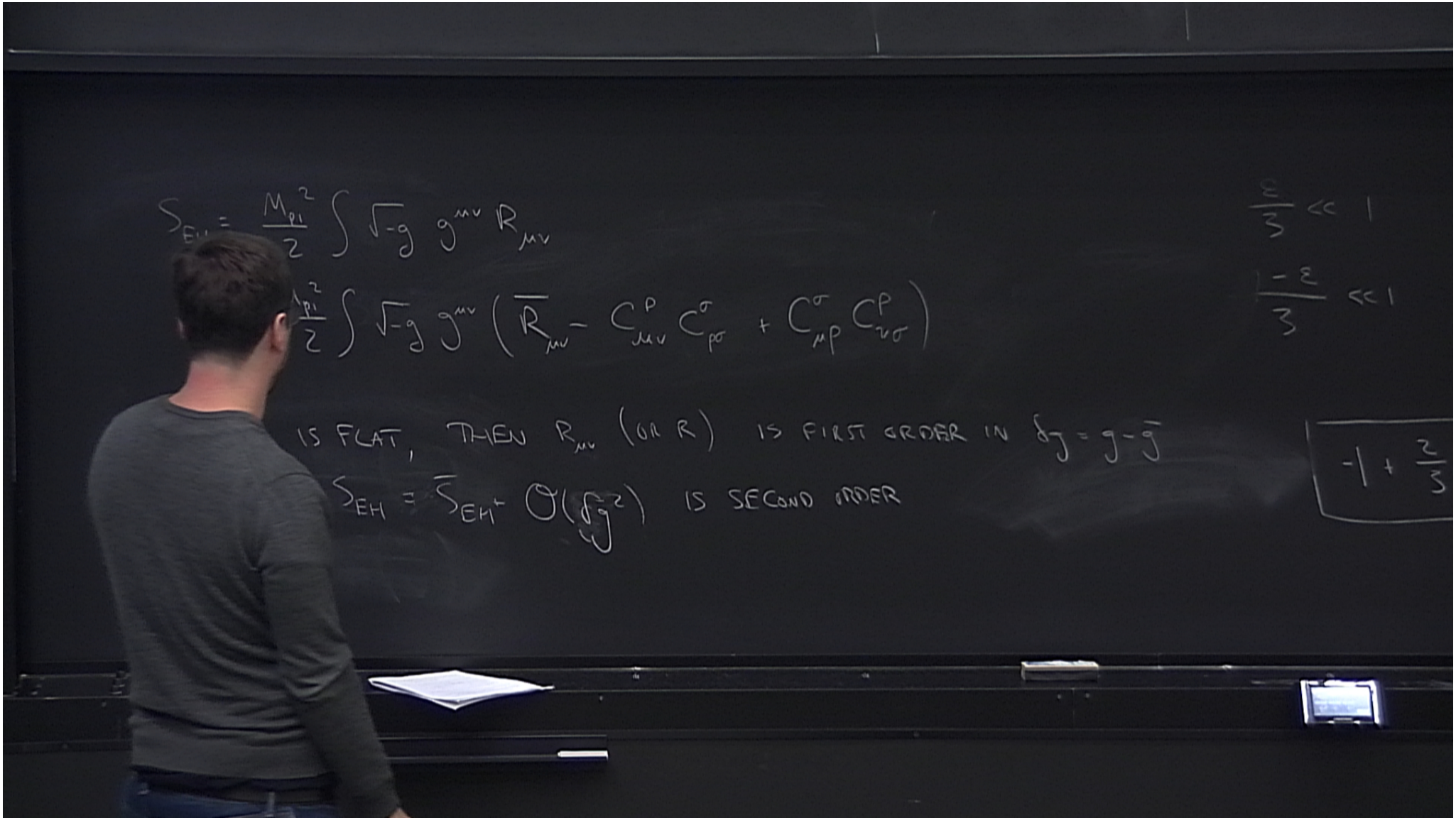
$$+ \frac{M_{Pl}^2}{8}\int d^4x \left[a^3\dot\gamma_{ij}\dot\gamma_{ij} - a(\partial_k\gamma_{ij})(\partial_k\gamma_{ij})\right]$$

WHERE ζ, γ_{ij} WILL BE DEFINED LATER!


LET $g, \bar{g}$ BE ARBITRARY METRICS
THE DIFFERENCE $(\Gamma^{\rho}_{\nu\sigma} - \bar{\Gamma}^{\rho}_{\nu\sigma})$ IS A TENSOR!
$$\Gamma'^{\rho}_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} \frac{\partial x'^{\rho}}{\partial x^{\gamma}} \Gamma^{\gamma}_{\alpha\beta} + \frac{\partial x'^{\rho}}{\partial x^{\sigma}} \frac{\partial^2 x^{\sigma}}{\partial x'^{\mu} \partial x'^{\nu}}$$
WRITE $\Gamma^{\rho}_{\mu\nu} = \bar{\Gamma}^{\rho}_{\mu\nu} + C^{\rho}_{\mu\nu}$ WHERE $C^{\rho}_{\mu\nu}$ IS A TENSOR
$C^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}$


$C^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}\left(\bar{\nabla}_{\mu} g_{\nu\rho} + \bar{\nabla}_{\nu} g_{\mu\rho} - \bar{\nabla}_{\rho} g_{\mu\nu}\right)$
[HW2]


$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}\left(\bar{\nabla}_\mu g_{\nu\rho} + \bar{\nabla}_\nu g_{\mu\rho} - \bar{\nabla}_\rho g_{\mu\nu}\right)$
[HW2]
RICC RES ARE RE ED BY:
$\nabla_\rho \Gamma^{\rho}_{\mu\nu} - \bar{\nabla}_\mu \Gamma^{\rho}_{\nu\rho} + \Gamma^{\rho}_{\mu\nu}\Gamma^{\sigma}_{\rho\sigma} - \Gamma^{\sigma}_{\mu\rho}\Gamma^{\rho}_{\nu\sigma}$
$R_{\mu\nu} + \nabla_\rho \Gamma^{\rho}_{\mu\nu} - \nabla_\mu \Gamma^{\rho}_{\nu\rho} - \Gamma^{\rho}_{\mu\nu}\Gamma^{\sigma}_{\rho\sigma} + \Gamma^{\sigma}_{\mu\rho}\Gamma^{\rho}_{\nu\sigma}$


$S_{EH} = \frac{M_{Pl}^2}{2} \int \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}$
$\frac{M_{Pl}^2}{2} \int \sqrt{-g}\, g^{\mu\nu} \left(\overline{R}_{\mu\nu} - C^{\rho}_{\mu\nu} C^{\sigma}_{\rho\sigma} + C^{\sigma}_{\mu\rho} C^{\rho}_{\nu\sigma} \right)$
IS FLAT, THEN $R_{\mu\nu}$ (OR R) IS FIRST ORDER IN $\delta g = g - \bar{g}$
$S_{EH} = \bar{S}_{EH} + O(\delta g^2)$ IS SECOND ORDER
$\frac{\varepsilon}{3} \ll 1$
$\frac{1-\varepsilon}{3} \ll 1$
$-1 + \frac{2}{3}$