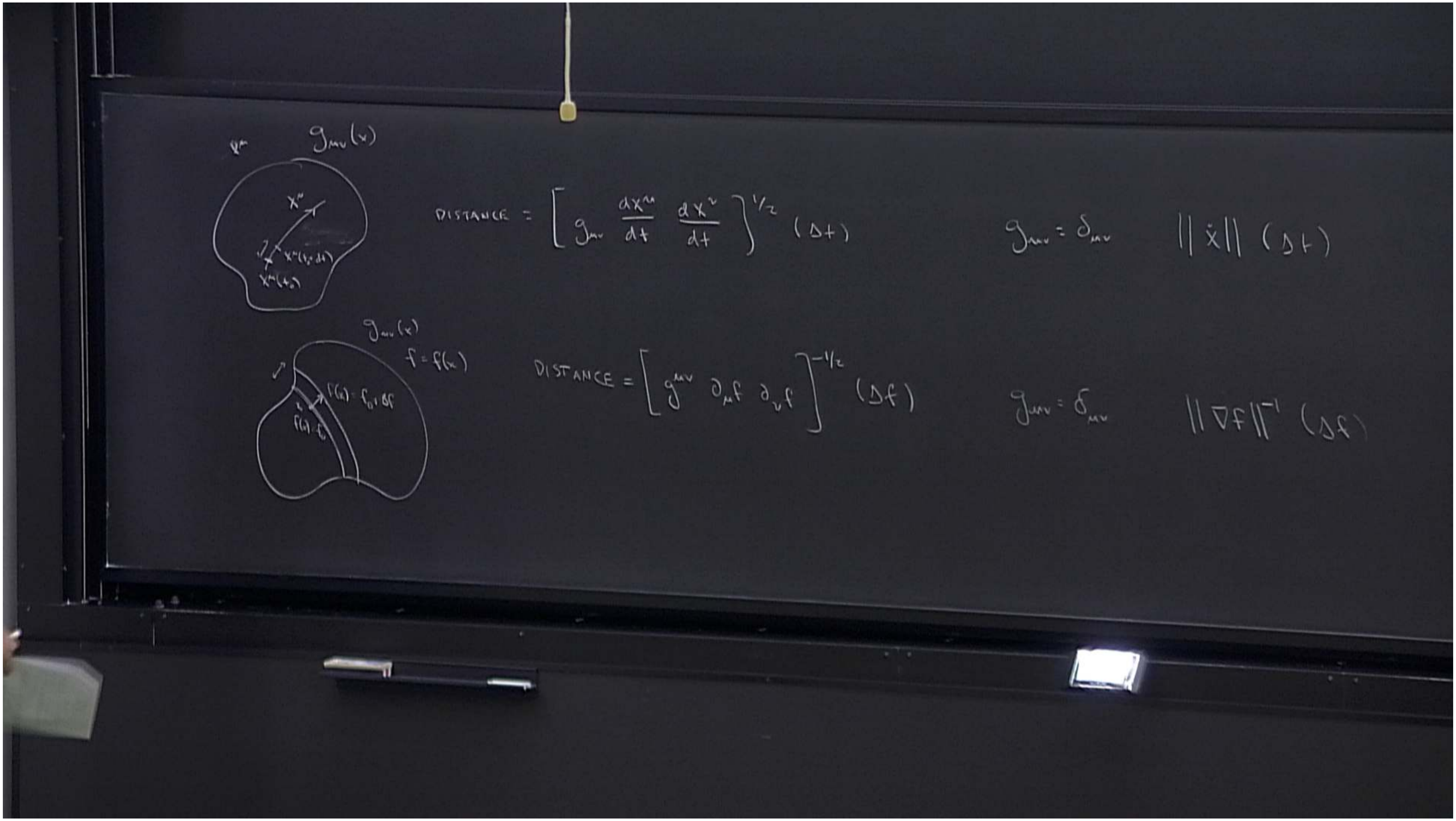
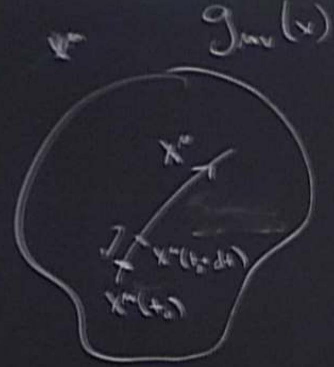
Title: PSI 16/17 Explorations in Cosmology (Kendrick Smith) - Lecture 8

Date: Apr 20, 2017 10:15 AM

URL: http://pirsa.org/17040061

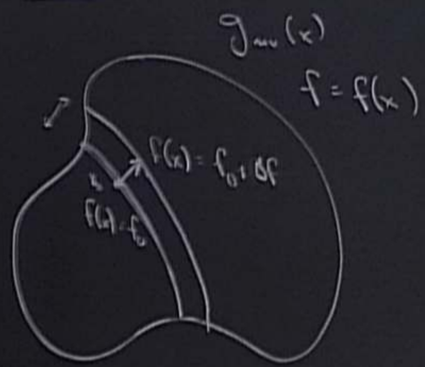
Abstract:


$g_{\mu\nu}(x)$
x^μ
$x^\mu(t_0+dt)$
$x^\mu(t_0)$
DISTANCE = $\left[g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \right]^{1/2} (\Delta t)$
$g_{\mu\nu} = \delta_{\mu\nu}$
$\|\dot{x}\| (\Delta t)$
$g_{\mu\nu}(x)$
$f = f(x)$
$f(x) = f_0 + \Delta f$
$f(x) = f_0$
DISTANCE = $\left[g^{\mu\nu} \partial_\mu f \, \partial_\nu f \right]^{-1/2} (\Delta f)$
$g_{\mu\nu} = \delta_{\mu\nu}$
$\|\nabla f\|^{-1} (\Delta f)$



DISTANCE $= \left[g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \right]^{1/2} (\Delta t)$

$\sim \|\dot{x}\| (\Delta t)$ IF $g_{\mu\nu} = \delta_{\mu\nu}$



DISTANCE $= \left[g^{\mu\nu} \partial_\mu f \, \partial_\nu f \right]^{-1/2} (\Delta f)$

$\sim \|\nabla f\|^{-1} (\Delta f)$ IF $g_{\mu\nu} = \delta_{\mu\nu}$

$f)$

$g_{\mu\nu} = \delta_{\mu\nu}$

Θ^a $F_{ab}(\Theta)$

LET $f = f(\Theta)$

$F_{ab} = \delta_{ab}$

$f = \Theta_1$

$\sigma(f) = \left[(F^{-1})_{11}\right]^{1/2}$

$$F^{prior}_{ab} = \begin{pmatrix} \sigma_f^{-2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

"FULLY MARGINALIZED" STATISTICAL ERROR

$$\sigma(f) = \left[\left(\frac{\partial f}{\partial \Theta_a}\right)\left(\frac{\partial f}{\partial \Theta_b}\right) \left(F^{-1}\right)^{ab} \right]^{1/2}$$

GAUSSIAN PRIOR ON f WITH ERROR σ_f

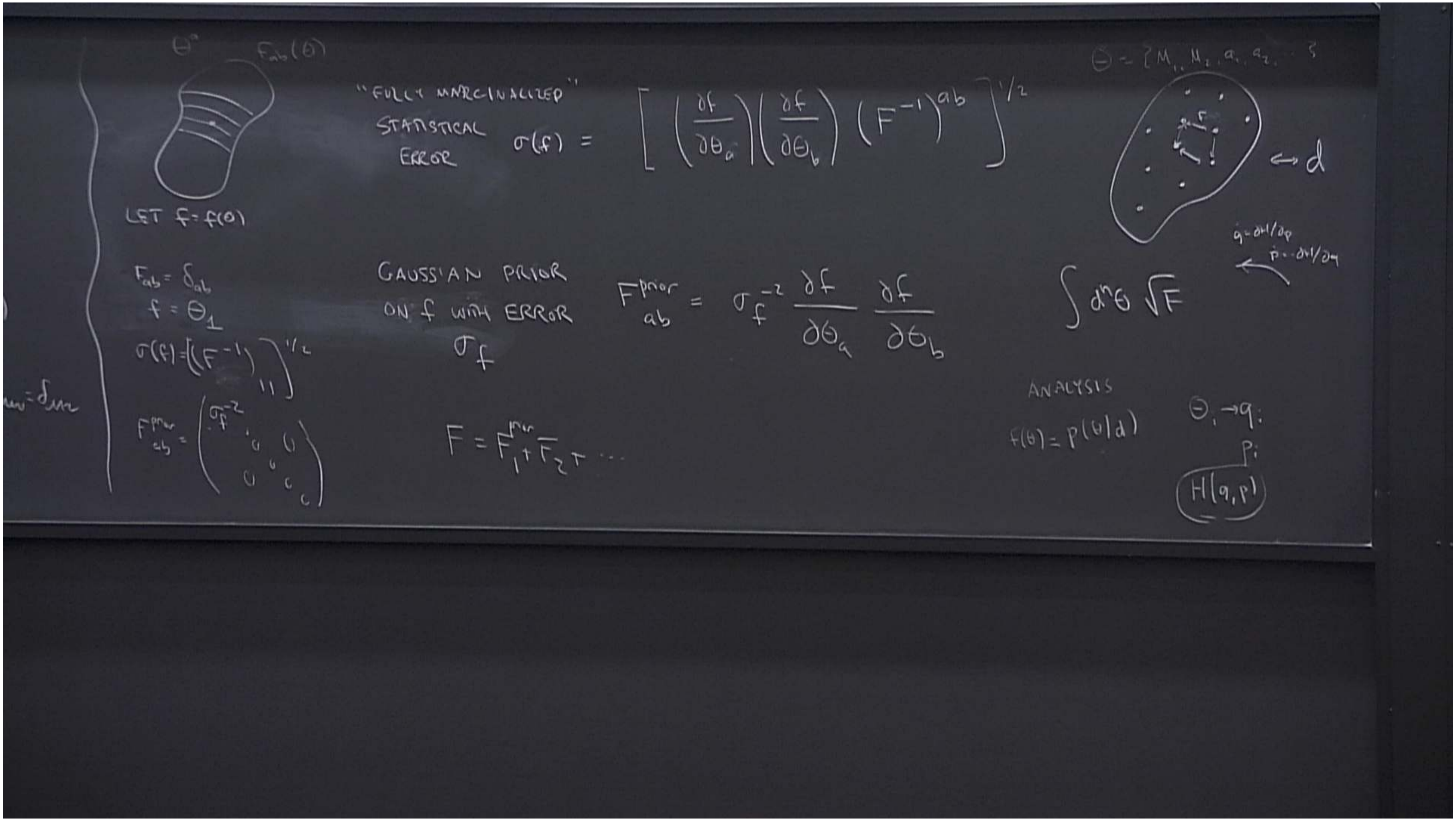
$$F^{prior}_{ab} = \sigma_f^{-2} \frac{\partial f}{\partial \Theta_a} \frac{\partial f}{\partial \Theta_b}$$

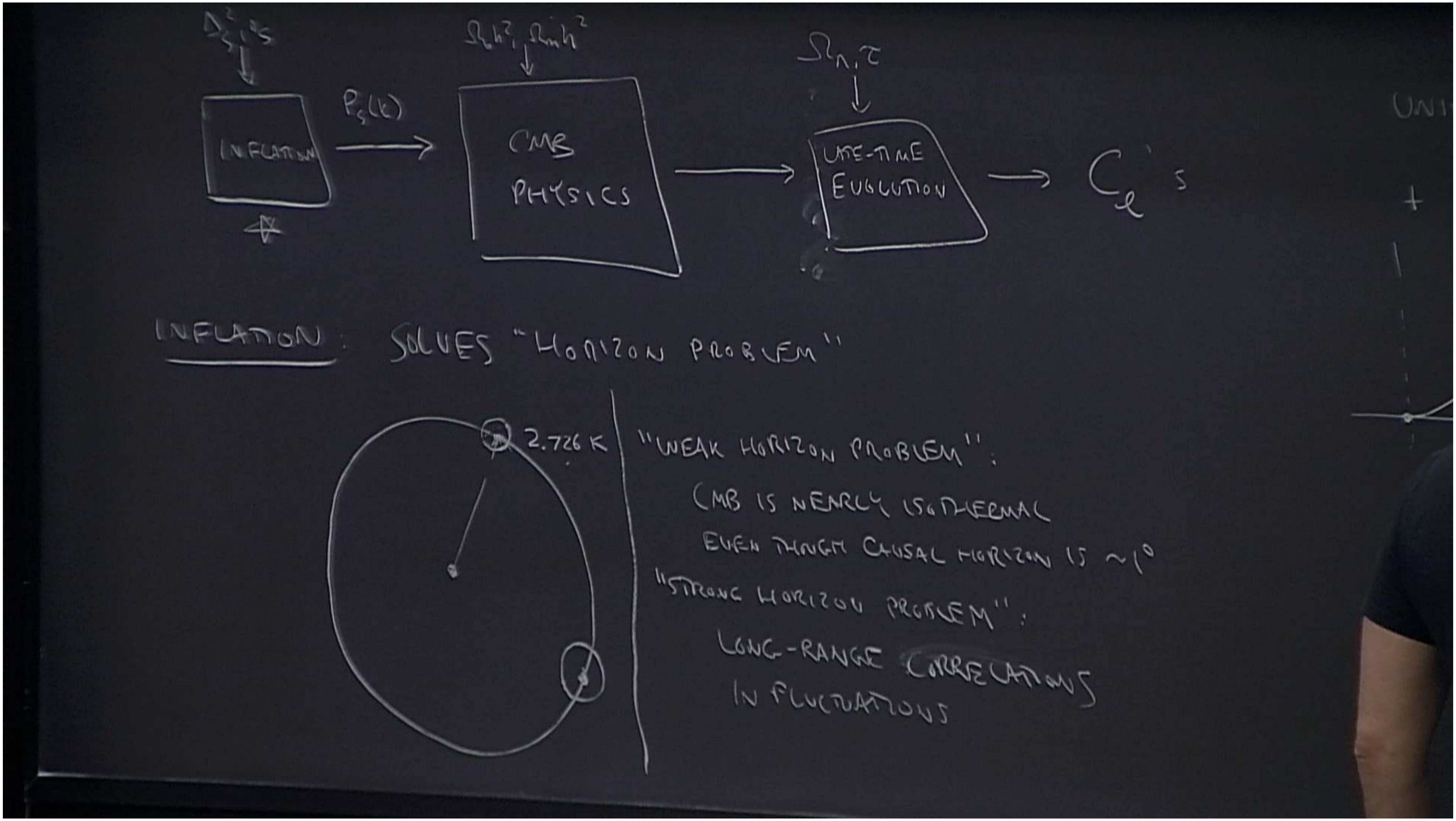
$$F = F_1^{prior} + F_2 + \cdots$$

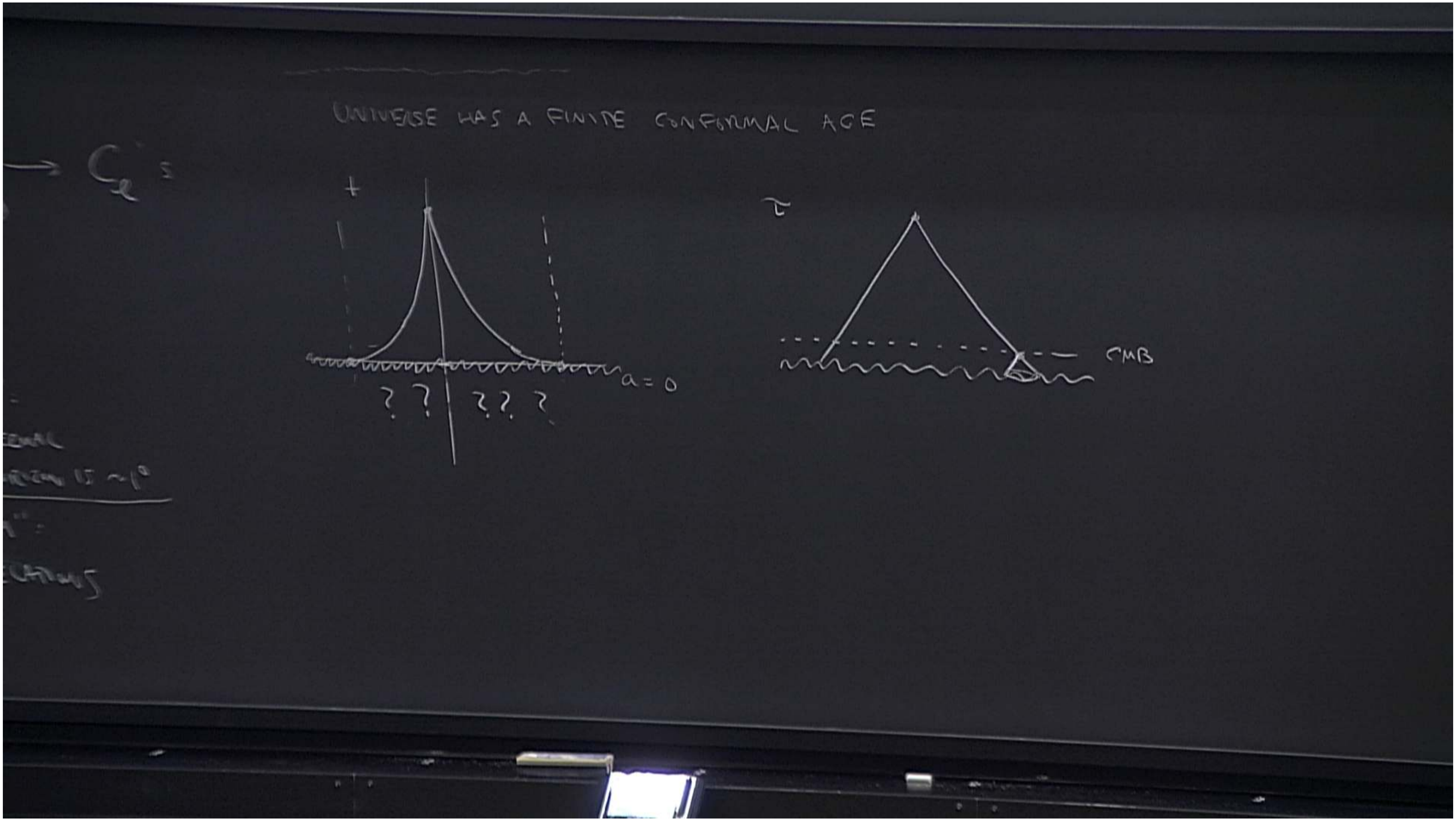
$\Theta = \{M_1, M_2, a_1, a_2, \ldots\}$

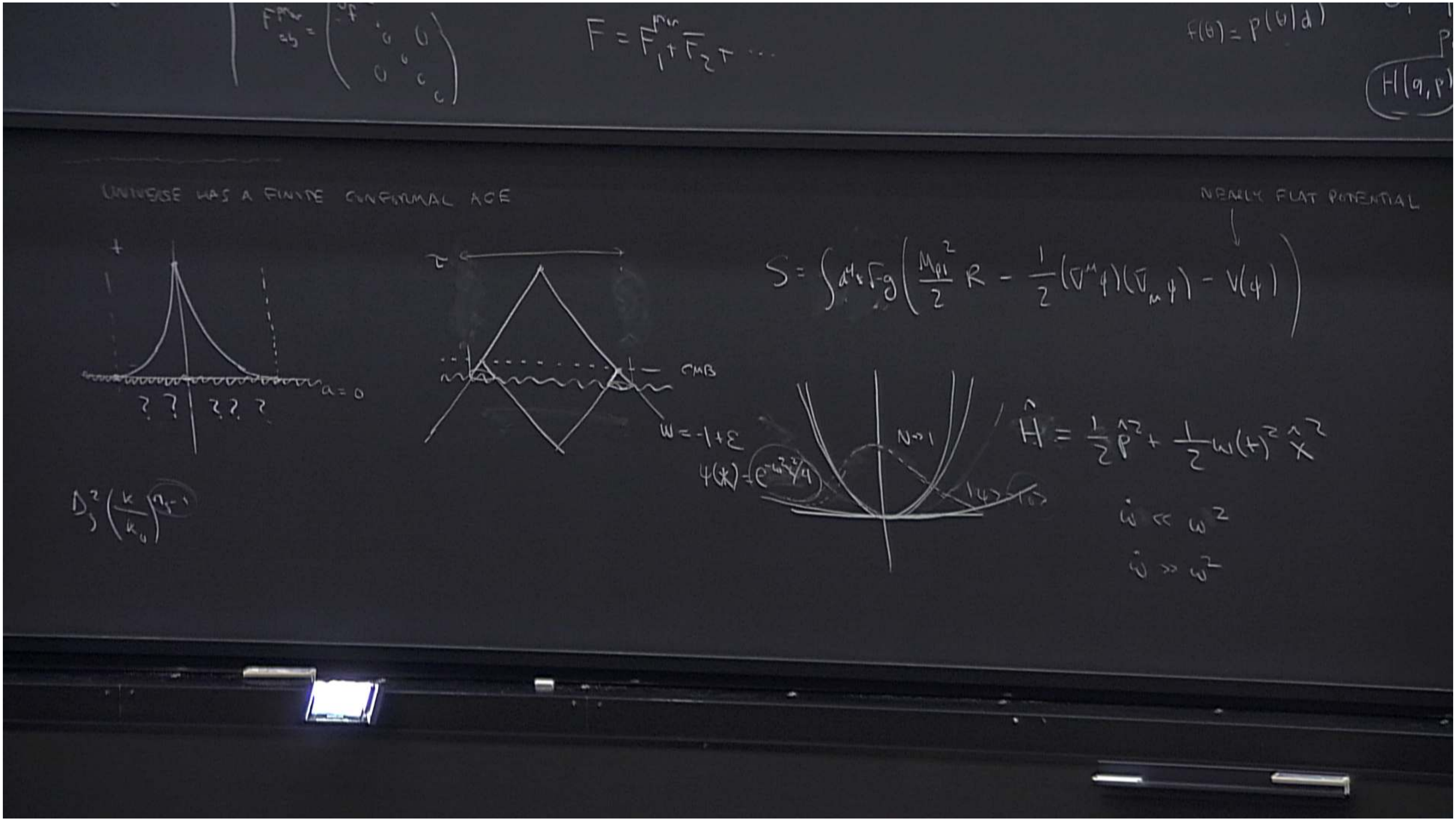
$\leftrightarrow d$

$\int d^n\Theta \sqrt{F}$


$F_{ab}(\theta)$
LET $f = f(\theta)$
$F_{ab} = \delta_{ab}$
$f = \theta_1$
$\sigma(f) = \left[(F^{-1})_{11}\right]^{1/2}$
"FULLY MARGINALIZED" STATISTICAL ERROR
$\sigma(f) = \left[\left(\frac{\partial f}{\partial \theta_a}\right)\left(\frac{\partial f}{\partial \theta_b}\right)(F^{-1})^{ab}\right]^{1/2}$
GAUSSIAN PRIOR ON f WITH ERROR σ_f
$F^{prior}_{ab} = \sigma_f^{-2} \frac{\partial f}{\partial \theta_a} \frac{\partial f}{\partial \theta_b}$
$F = F^{prior}_1 + F_2 + \cdots$
$\theta = \{M_1, M_2, a_1, a_2, \ldots\}$
$\leftrightarrow d$
$\dot q = \partial H/\partial p$
$\dot p = -\partial H/\partial q$
$\int d^n\theta \sqrt{F}$
ANALYSIS
$f(\theta) = p(\theta|d)$
$\theta_i \to q_i$
p_i
$H(q,p)$


INFLATION
$P_\zeta(k)$
CMB
PHYSICS
LATE-TIME
EVOLUTION
C_ℓ's
INFLATION: SOLVES "HORIZON PROBLEM"
2.726 K
"WEAK HORIZON PROBLEM":
CMB IS NEARLY ISOTHERMAL
EVEN THOUGH CAUSAL HORIZON IS ~1°
"STRONG HORIZON PROBLEM":
LONG-RANGE CORRELATIONS
IN FLUCTUATIONS


UNIVERSE HAS A FINITE CONFORMAL AGE
t
a = 0
? ? ?? ?
τ
CMB


UNIVERSE HAS A FINITE CONFORMAL AGE
NEARLY FLAT POTENTIAL
CMB
a = 0
w = -1 + ε

$\sim \|\nabla f\|^{-1} (\Delta f)$ IF $g_{\mu\nu} = \delta_{\mu\nu}$

$\sigma(f) = \left((F^{-1})_{11}\right)^{1/2}$ $\quad \sigma_f$

$F^{\mu\nu}_{ab} = \begin{pmatrix} \sigma_f^{-2} & & \\ & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad F = F_1 + F_2 + \dots$

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\lambda}\left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\lambda g_{\mu\nu}\right)$$

$$R_{\mu\nu\lambda}{}^{\rho} = \partial_\nu \Gamma^\rho_{\mu\lambda} - \partial_\mu \Gamma^\rho_{\nu\lambda} + \Gamma^\sigma_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\sigma_{\nu\lambda}\Gamma^\rho_{\mu\sigma}$$

$$R_{\mu\nu} = R_{\mu\lambda\nu}{}^{\lambda}$$

$$= \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\mu \Gamma^\lambda_{\nu\lambda} + \Gamma^\lambda_{\mu\nu}\Gamma^\rho_{\lambda\rho} - \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\rho}$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$S = \underbrace{\left(\frac{M_{pl}^2}{2}\int d^4x \sqrt{-g}\, R\right)}_{S_{EH}} + S_m \qquad \left[M_{pl} = (8\pi G)^{-1/2}\right]$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}$$

$$G_{\mu\nu} = M_{pl}^{-2} T_{\mu\nu}$$

$\sim \|\nabla f\|^{-1} (\Delta f)$ IF $g_{\mu\nu} = \delta_{\mu\nu}$

$\sigma(f) = [(F^{-1})_{11}]^{1/2}$

$F^{\mu\nu}_{ab} = \begin{pmatrix} \sigma_f^{-2} & & \\ & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

σ_f

$F = F_1^{\mu\nu} + F_2 + \cdots$

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\lambda} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\lambda g_{\mu\nu} \right)$$

$$R_{\mu\nu\lambda}{}^{\rho} = \partial_\nu \Gamma^\rho_{\mu\lambda} - \partial_\mu \Gamma^\rho_{\nu\lambda} + \Gamma^\sigma_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\sigma_{\nu\lambda} \Gamma^\rho_{\mu\sigma}$$

$$R_{\mu\nu} = R_{\mu\lambda\nu}{}^{\lambda}$$

$$= \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\mu \Gamma^\lambda_{\nu\lambda} + \Gamma^\lambda_{\mu\nu} \Gamma^\rho_{\lambda\rho} - \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\rho}$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

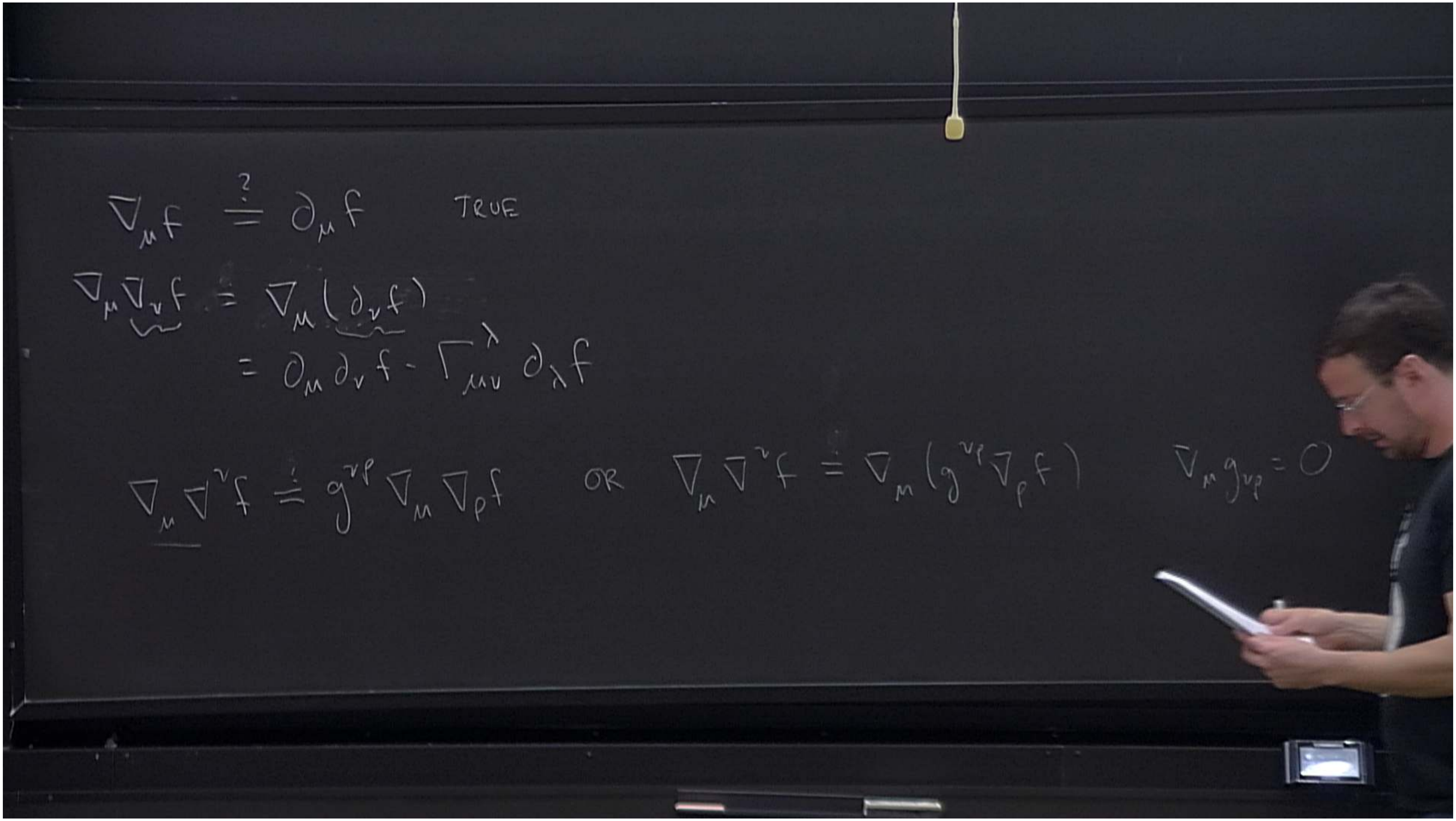
$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$S = \underbrace{\left(\frac{M_{pl}^2}{2} \int d^4x \sqrt{-g}\, R \right)}_{S_{EH}} + S_m$$

$$\left[M_{pl} = (8\pi G)^{-1/2} \right]$$

$$T_{\mu\nu} \overset{\text{DEF}}{=} -\frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}$$

$$\boxed{G_{\mu\nu} = M_{pl}^{-2} T_{\mu\nu}}$$


$\nabla_\mu f \overset{?}{=} \partial_\mu f$ TRUE
$\nabla_\mu \nabla_\nu f \overset{?}{=} \nabla_\mu (\partial_\nu f)$
$= \partial_\mu \partial_\nu f - \Gamma^\lambda_{\mu\nu} \partial_\lambda f$
$\nabla_\mu \nabla^\nu f \overset{?}{=} g^{\nu\rho} \nabla_\mu \nabla_\rho f$ OR $\nabla_\mu \nabla^\nu f \overset{?}{=} \nabla_\mu (g^{\nu\rho} \nabla_\rho f)$
$\nabla_\mu g_{\nu\rho} = 0$

$$\nabla_\mu f \overset{?}{=} \partial_\mu f \qquad \text{TRUE}$$

$$\nabla_\mu \underbrace{\nabla_\nu f} \overset{?}{=} \nabla_\mu (\underbrace{\partial_\nu f})$$

$$= \partial_\mu \partial_\nu f - \Gamma^\lambda_{\mu\nu} \partial_\lambda f$$

"METRIC COMPATIBILITY"

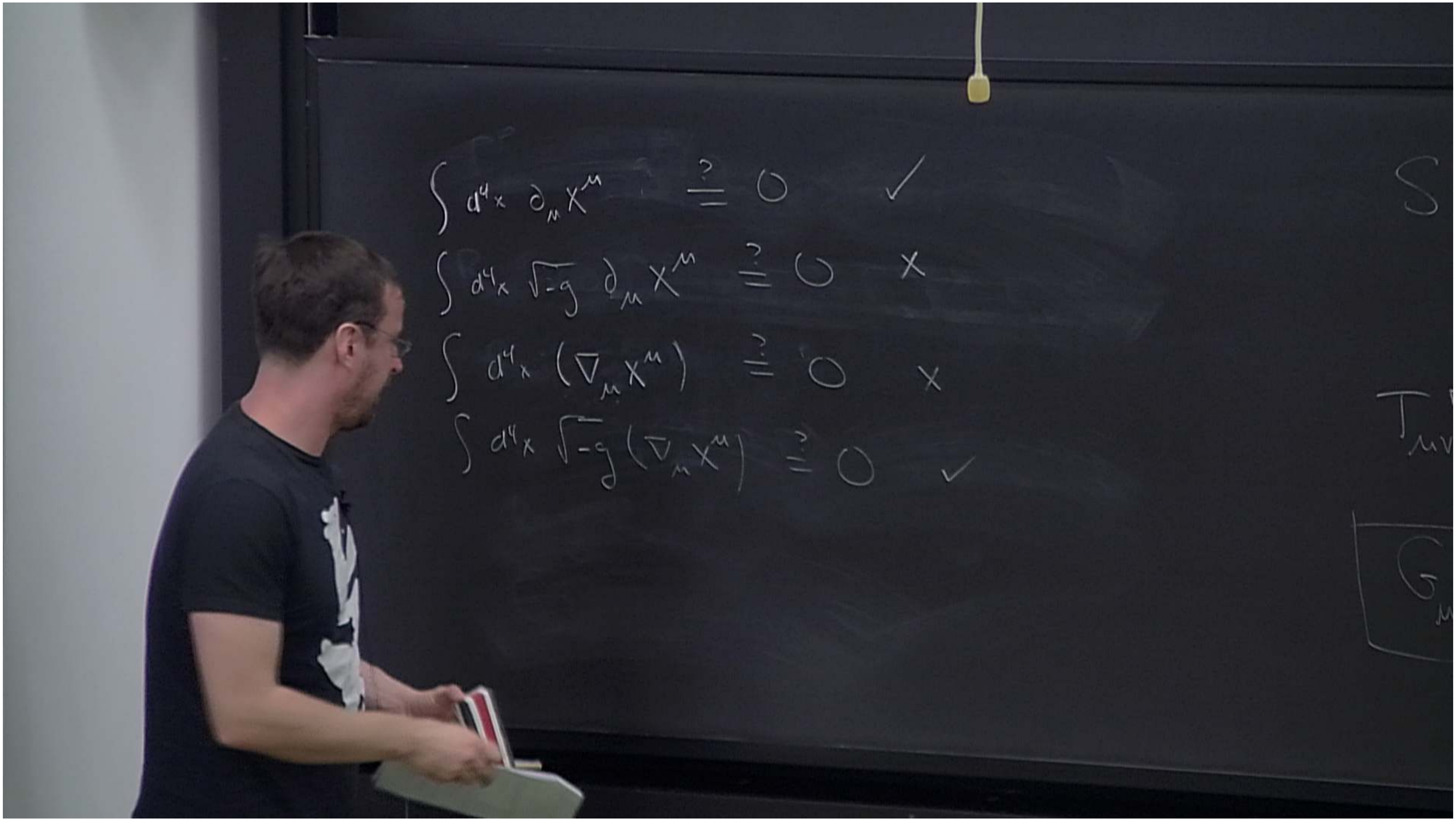
$$\nabla_\mu \nabla^\nu f \overset{?}{=} g^{\nu\rho} \nabla_\mu \nabla_\rho f \quad \text{OR} \quad \nabla_\mu \nabla^\nu f \overset{?}{=} \nabla_\mu (g^{\nu\rho} \nabla_\rho f) \qquad \nabla_\mu g_{\nu\rho} = 0$$

$$\nabla_\mu \nabla_\nu f = \nabla_\nu \nabla_\mu f \quad \text{TRUE} \quad \text{"TORSION FREE"}$$

$$\nabla_\mu \nabla_\nu X_\rho = \nabla_\nu \nabla_\mu X_\rho + R_{\mu\nu\rho}{}^{\lambda} X_\lambda$$

"DIVERGENCE FORMULA":

$$\nabla_\mu X^\mu = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, X^\mu \right)$$


$\int d^4x\ \partial_\mu X^\mu \stackrel{?}{=} 0$ ✓
$\int d^4x\ \sqrt{-g}\ \partial_\mu X^\mu \stackrel{?}{=} 0$ ✗
$\int d^4x\ (\nabla_\mu X^\mu) \stackrel{?}{=} 0$ ✗
$\int d^4x\ \sqrt{-g}\,(\nabla_\mu X^\mu) \stackrel{?}{=} 0$ ✓

$$F^{pmr}_{ab} = \begin{pmatrix} 0 & f & & \\ & & 0 & 0 \\ & 0 & 0 & \\ & & 0 & 0 \\ & & & 0 \end{pmatrix} \qquad F = F_1^{pmr} + F_2 + \cdots$$

$$\int d^4x \sqrt{-g}\,(\nabla_\mu X^\mu) f = \int d^4x \sqrt{-g} \left[X^\mu(-\nabla_\mu f) + \cancel{\nabla_\mu (f X^\mu)} \right]$$

$$\int d^4x \sqrt{-g}\,(\nabla_\mu \phi)(\nabla^\mu \psi) = -\int \sqrt{-g}\,\phi\,(\nabla_\mu \nabla^\mu \psi)$$

$$\int d^4x \sqrt{-g}\,(\nabla_\mu X_\nu)(\nabla^\nu Y^\mu) \neq \int \sqrt{-g}\,(\nabla^\nu X_\nu)(\nabla_\mu Y^\mu)$$

$$\int d^4x\,\sqrt{-g}\,(\nabla_\mu X^\mu)\,f = \int d^4x\,\sqrt{-g}\left[X^\mu(-\nabla_\mu f) + \cancel{\nabla_\mu (f X^\mu)}\right]$$

$$\int d^4x\,\sqrt{-g}\,(\nabla_\mu \phi)(\nabla^\mu \psi) = -\int \sqrt{-g}\,\phi\,(\nabla_\mu \nabla^\mu \psi)$$

$$\int d^4x\,\sqrt{-g}\,(\nabla_\mu X_\nu)(\nabla^\nu Y^\mu) = -\int d^4x\,\sqrt{-g}\,X_\nu\,(\nabla_\mu \nabla^\nu Y^\mu)$$

$$= \int d^4x\,\sqrt{-g}\,X^\nu\left[\nabla_\nu \nabla_\mu Y^\mu - R_{\mu\nu\lambda}{}^{\mu} Y^\lambda\right]$$

$$= \int d^4x\,\sqrt{-g}\,(\nabla_\nu X^\nu)(\nabla_\mu Y^\mu) + \int d^4x\,\sqrt{-g}\,R_{\nu\lambda} X^\nu Y^\lambda \,]$$