Title: PSI 16/17 Explorations in Cosmology (Kendrick Smith) - Lecture 6

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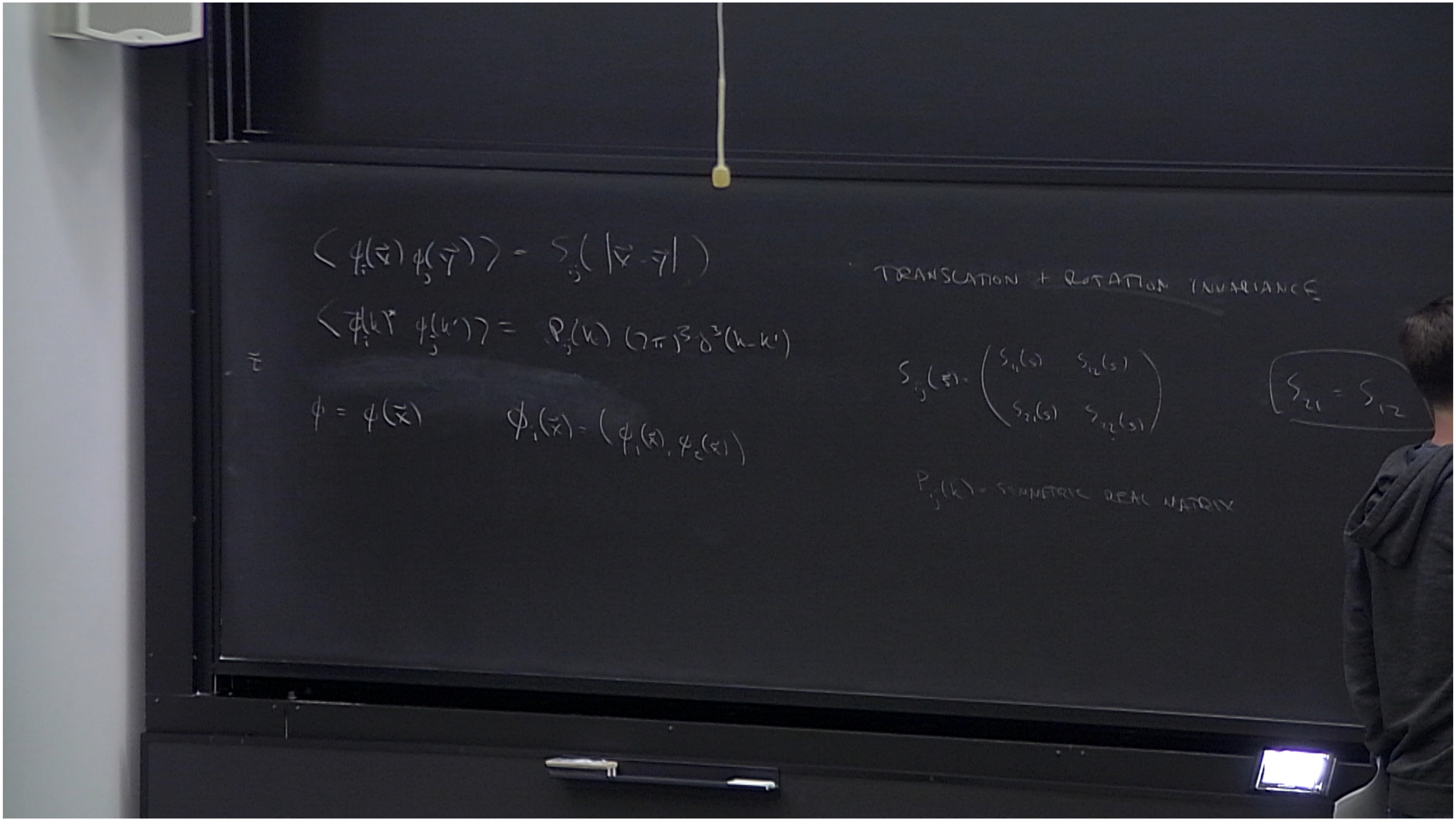
Abstract:


$\langle \phi_i(\vec{x})\, \phi_j(\vec{y}) \rangle = S_{ij}(|\vec{x}-\vec{y}|)$
$\langle \phi_i(k)^* \, \phi_j(k') \rangle = P_{ij}(k)\,(2\pi)^3 \delta^3(k-k')$
$\phi = \phi(\vec{x})$
$\phi_i(\vec{x}) = (\phi_1(\vec{x}), \phi_2(\vec{x}))$
TRANSLATION + ROTATION INVARIANCE
$S_{ij}(s) = \begin{pmatrix} S_{11}(s) & S_{12}(s) \\ S_{21}(s) & S_{22}(s) \end{pmatrix}$
$\boxed{S_{21} = S_{12}}$
$P_{ij}(k)$ = SYMMETRIC REAL MATRIX

... INVARIANCE

$$\xi_{21} = \xi_{12}$$

... MATRIX

$\phi(\vec{x})$ GRF $P(k)$

$\phi(x_1)$ $\phi(x_2)$ $\phi(x_3)$ $\phi(x_4)$ $\phi(x_5)$ $|x_1 - x_5|$

$\phi(x_1)$

$\phi(x_1)$ $\phi(x_2)$

$$C_{ij} = \begin{pmatrix} \xi(0) & \xi(x_{12}) \\ \xi(x_{12}) & \xi(0) \end{pmatrix}$$

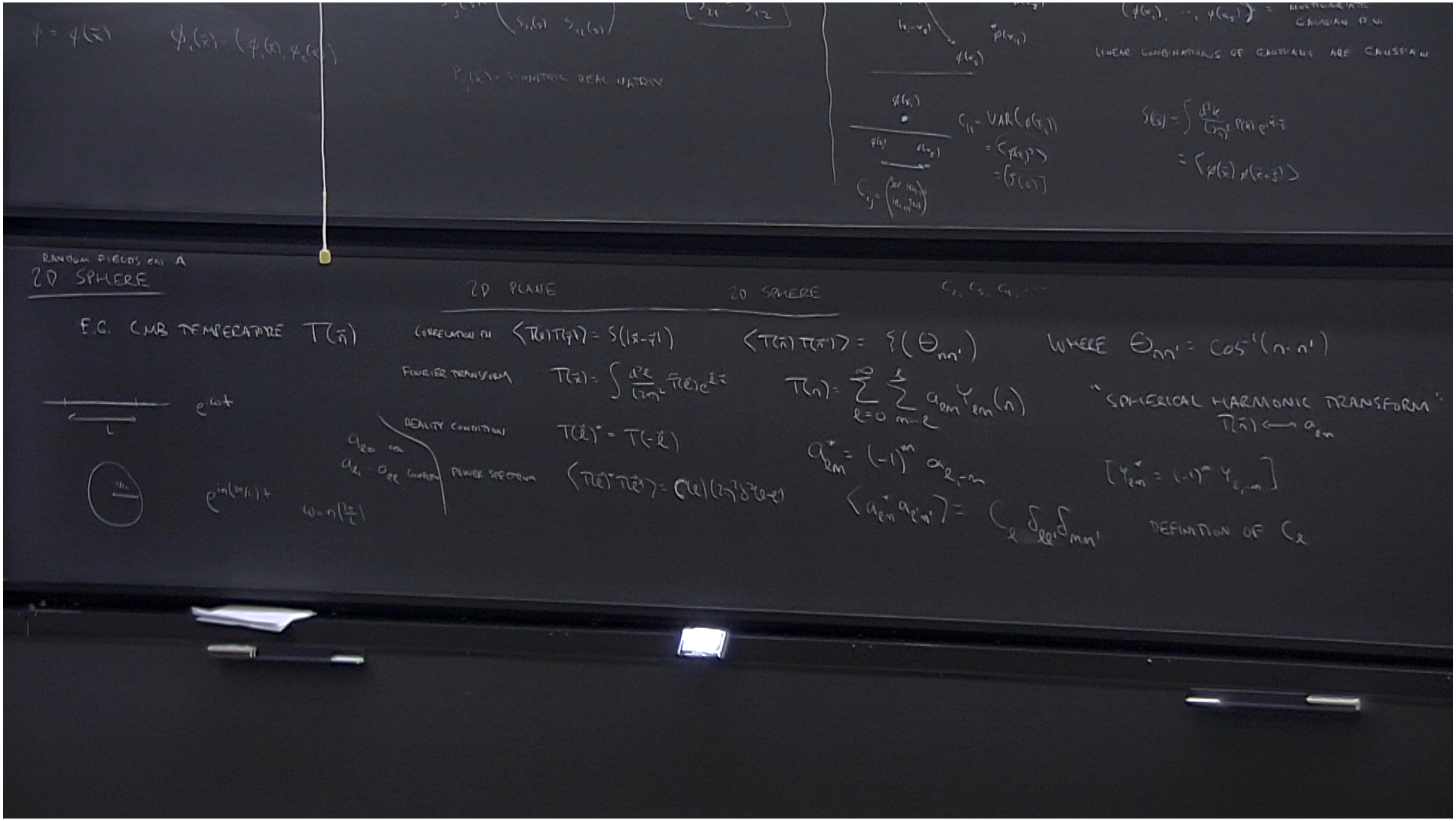
$$C_{11} = \mathrm{VAR}(\phi(x_1)) = \langle \phi(x_1)^2 \rangle = \xi(0)$$

$$C_{ij} = \langle \phi(x_i)\,\phi(x_j) \rangle$$

$(\phi(x_1), \dots, \phi(x_5)) =$ MULTIVARIATE GAUSSIAN R.V.

LINEAR COMBINATIONS OF GAUSSIANS ARE GAUSSIAN

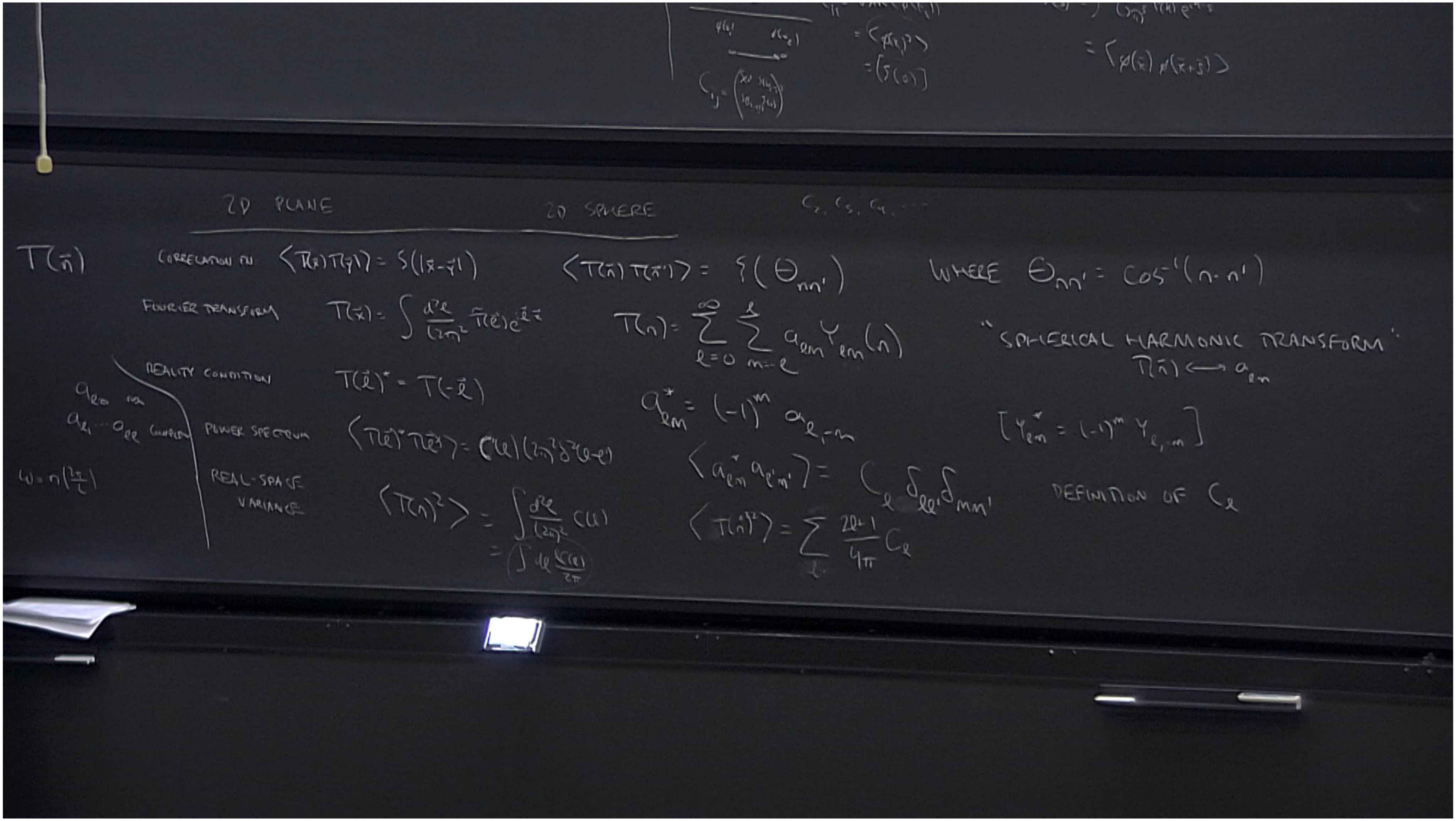
$$\xi(\vec{s}) = \int \frac{d^3k}{(2\pi)^3} P(k)\, e^{i\vec{k}\cdot\vec{s}} = \langle \phi(\vec{x})\, \phi(\vec{x}+\vec{s}) \rangle$$

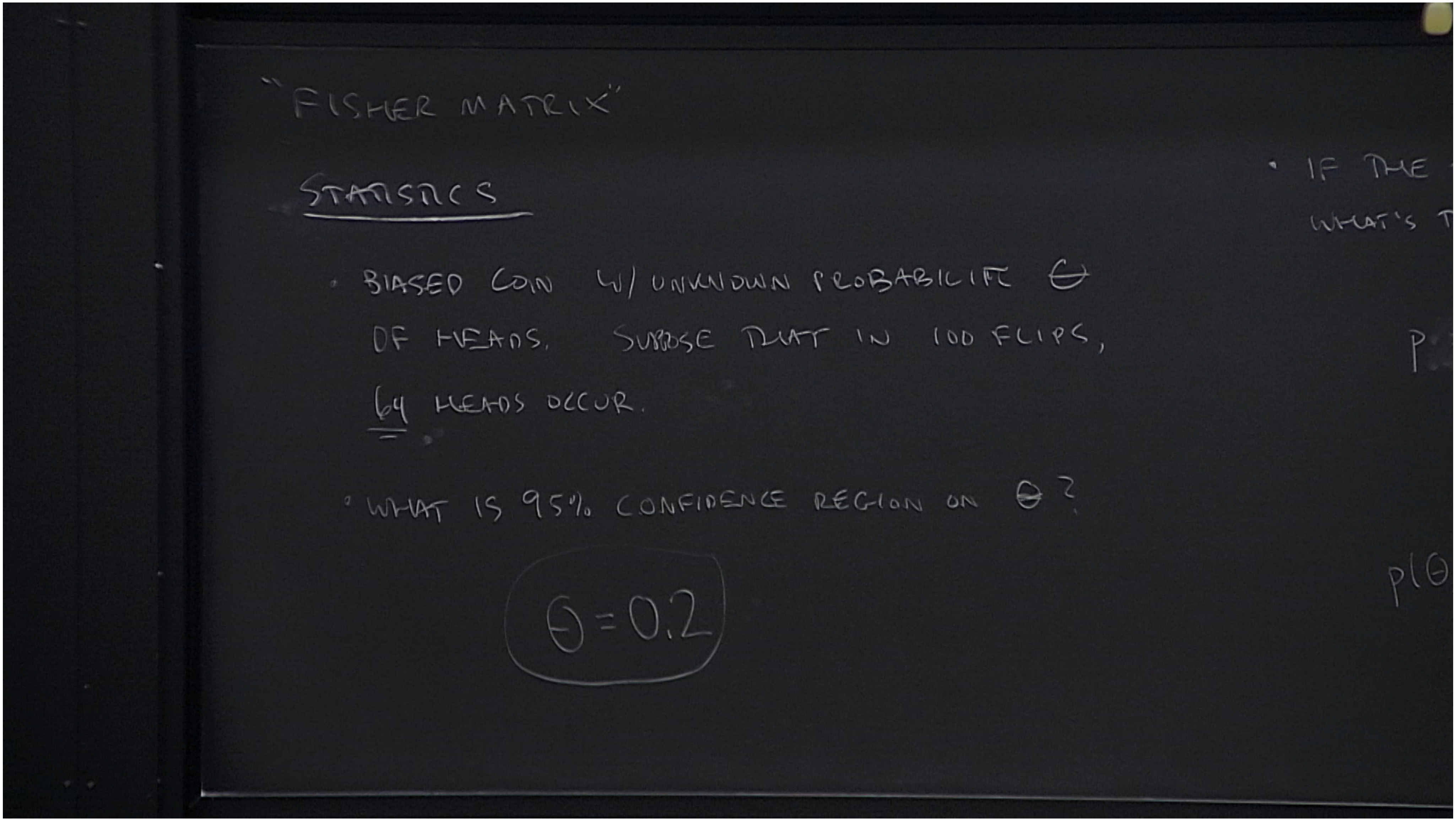

LINEAR COMBINATIONS OF GAUSSIANS ARE GAUSSIAN
$\xi(r) = \int \frac{d^2k}{(2\pi)^2} P(k) e^{ik\cdot r}$
$= \langle \phi(\vec{x}) \phi(\vec{x}+\vec{r}) \rangle$
RANDOM FIELDS ON A
2D SPHERE
E.G. CMB TEMPERATURE $T(\hat{n})$
$e^{i\omega t}$
L
$e^{in(2\pi/L)t}$
$\omega = n\left(\frac{2\pi}{L}\right)$
2D PLANE
CORRELATION FN $\langle T(\vec{x}) T(\vec{y}) \rangle = \xi(|\vec{x}-\vec{y}|)$
FOURIER TRANSFORM $T(\vec{x}) = \int \frac{d^2\ell}{(2\pi)^2} \tilde{T}(\vec{\ell}) e^{i\vec{\ell}\cdot\vec{x}}$
REALITY CONDITION $T(\vec{\ell})^* = T(-\vec{\ell})$
POWER SPECTRUM $\langle T(\vec{\ell})^* T(\vec{\ell}') \rangle = C(\ell)(2\pi)^2 \delta^2(\ell-\ell')$
2D SPHERE
$C_2, C_3, C_4, \ldots$
$\langle T(\hat{n}) T(\hat{n}') \rangle = \xi(\Theta_{nn'})$
WHERE $\Theta_{nn'} = \cos^{-1}(n \cdot n')$
$T(n) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(n)$
"SPHERICAL HARMONIC TRANSFORM"
$T(\hat{n}) \longleftrightarrow a_{\ell m}$
$a^*_{\ell m} = (-1)^m a_{\ell,-m}$
$[Y^*_{\ell m} = (-1)^m Y_{\ell,-m}]$
$\langle a^*_{\ell m} a_{\ell' m'} \rangle = C_\ell \delta_{\ell\ell'} \delta_{mm'}$
DEFINITION OF C_ℓ

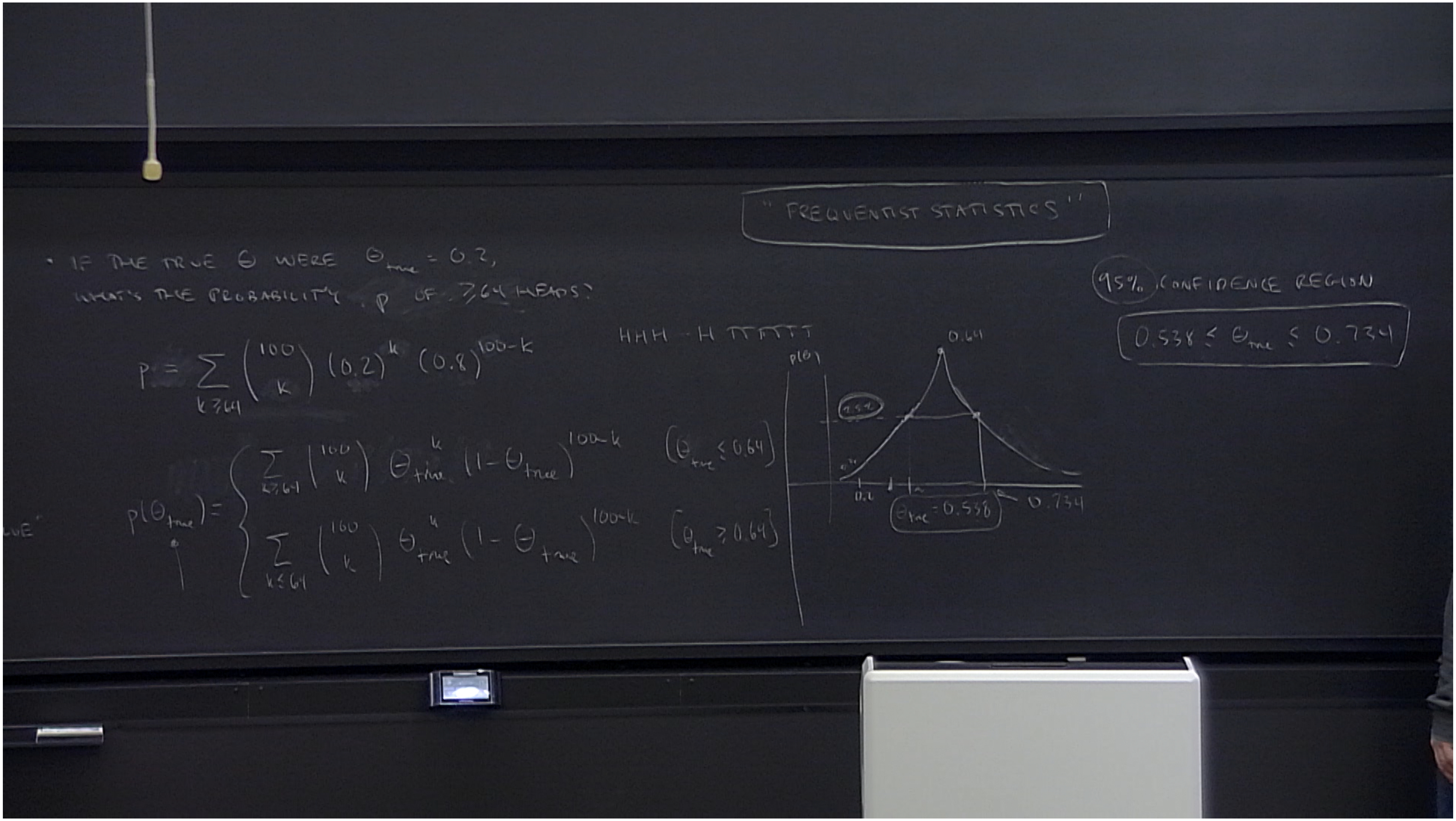
$a_{00} = 2.726\ \mathrm{K}$ $\quad (\Omega_{rad} h^2)$

$a_{1m} \sim 3\ \mathrm{mK}$ $\quad$ (DIPOLE MODES: MOTION OF EARTH)

a_{2m} $\quad l \geq 2$ $\quad$ (COSMOLOGICAL FLUCTUATIONS)


2D PLANE
2D SPHERE
T(n̂)
CORRELATION FN ⟨T(x)T(y)⟩ = ξ(|x − y|)
⟨T(n̂) T(n̂')⟩ = ξ(Θ_nn')
WHERE Θ_nn' = cos⁻¹(n · n')
FOURIER TRANSFORM T(x) = ∫ d²ℓ/(2π)² T(ℓ) e^{iℓ·x}
T(n) = Σ_{ℓ=0}^{∞} Σ_{m=−ℓ}^{ℓ} a_ℓm Y_ℓm(n)
"SPHERICAL HARMONIC TRANSFORM"
T(n̂) ⟷ a_ℓm
REALITY CONDITION T(ℓ)* = T(−ℓ)
a*_ℓm = (−1)^m a_ℓ,−m
[Y*_ℓm = (−1)^m Y_ℓ,−m]
POWER SPECTRUM ⟨T(ℓ)* T(ℓ')⟩ = C(ℓ)(2π)² δ²(ℓ−ℓ')
⟨a*_ℓm a_ℓ'm'⟩ = C_ℓ δ_ℓℓ' δ_mm'
DEFINITION OF C_ℓ
REAL-SPACE VARIANCE
⟨T(n)²⟩ = ∫ d²ℓ/(2π)² C(ℓ)
= ∫ dℓ ℓC(ℓ)/2π
⟨T(n̂)²⟩ = Σ_ℓ (2ℓ+1)/4π C_ℓ
C₂, C₃, C₄, ...
= ⟨ϕ(x) ϕ(x+s)⟩


"FISHER MATRIX"
STATISTICS
· BIASED COIN W/ UNKNOWN PROBABILITY Θ OF HEADS. SUPPOSE THAT IN 100 FLIPS, 64 HEADS OCCUR.
· WHAT IS 95% CONFIDENCE REGION ON Θ?
Θ = 0.2
· IF THE
WHAT'S


"FREQUENTIST STATISTICS"
• IF THE TRUE θ WERE θ_true = 0.2,
WHAT'S THE PROBABILITY p OF ≥ 64 HEADS?
p = Σ_{k≥64} (100 choose k) (0.2)^k (0.8)^(100−k)
HHH ⋯ H TTTTTT
p(θ_true) = { Σ_{k≥64} (100 choose k) θ_true^k (1−θ_true)^(100−k) (θ_true ≤ 0.64); Σ_{k≤64} (100 choose k) θ_true^k (1−θ_true)^(100−k) (θ_true ≥ 0.64) }
p(θ)
0.64
θ_true = 0.538
0.734
95% CONFIDENCE REGION
0.538 ≤ θ_true ≤ 0.734