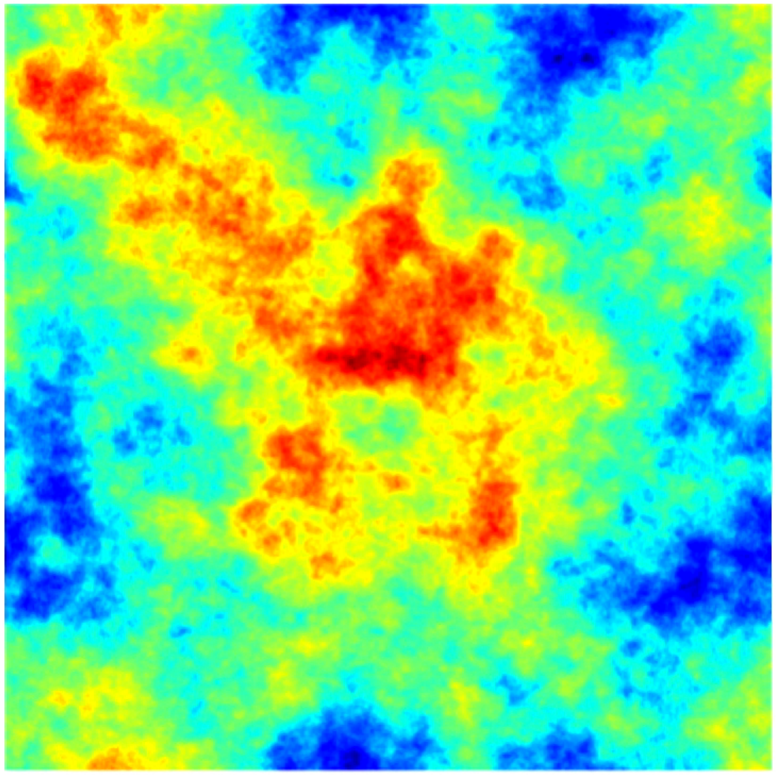
Title: PSI 16/17 Explorations in Cosmology (Kendrick Smith) - Lecture 4

Date: Apr 13, 2017 10:15 AM

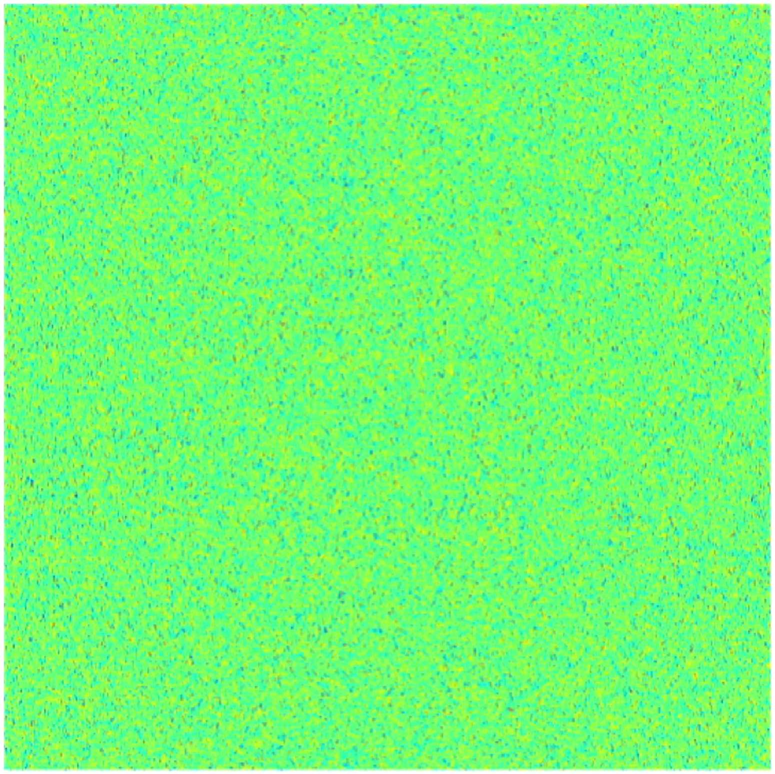
URL: http://pirsa.org/17040056

Abstract:

"Red" power law spectrum: $C(l) \propto l^{-3}$



White noise: C(l) = constant



No correlation between pixels in this case!

2D

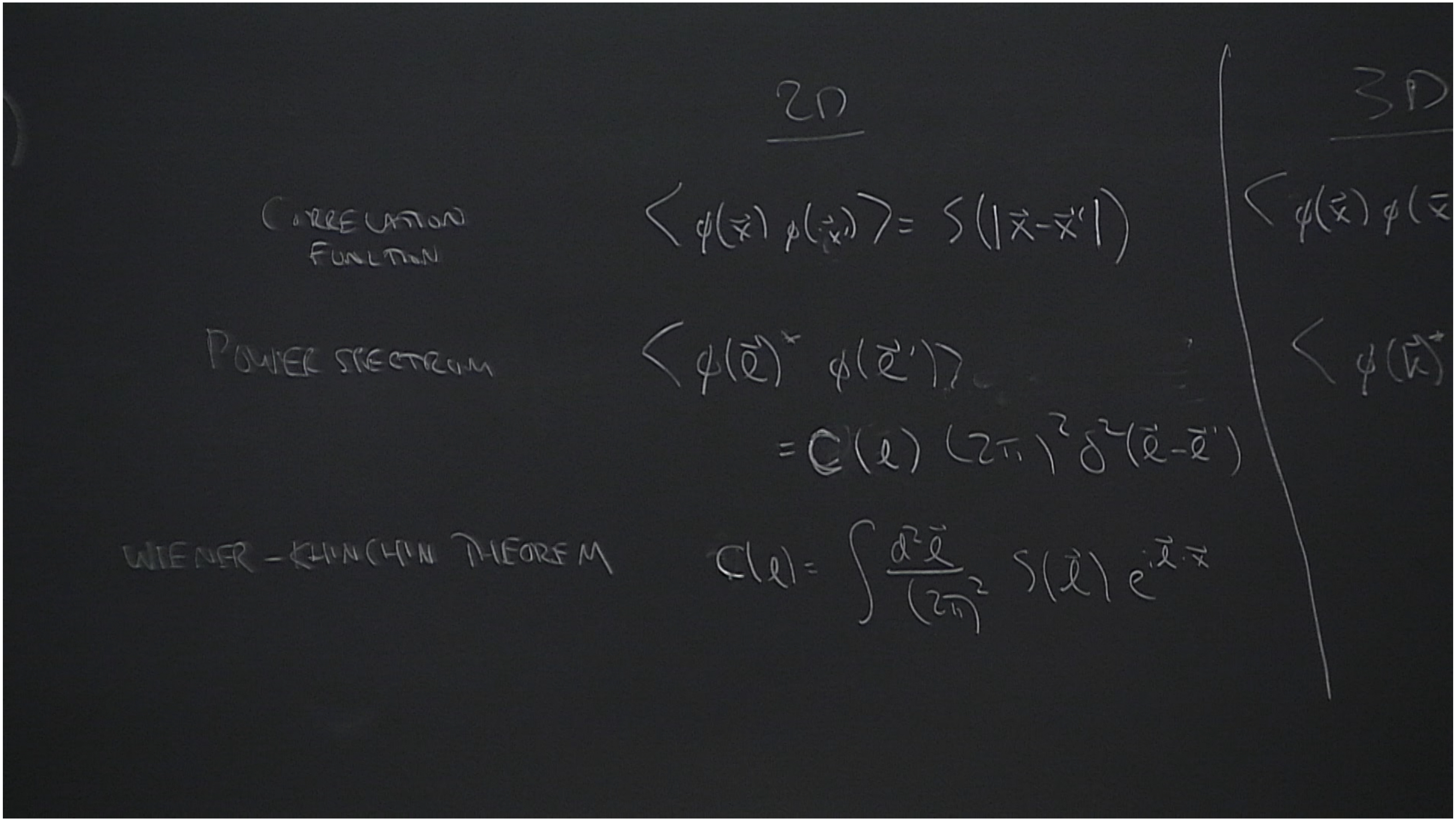
$$\langle \phi(\vec{x})\, \phi(\vec{x}') \rangle = \xi(|\vec{x} - \vec{x}'|)$$

$$\langle \phi(\vec{\ell})^{*}\, \phi(\vec{\ell}') \rangle = C(\ell)\, (2\pi)^2\, \delta^2(\vec{\ell} - \vec{\ell}')$$

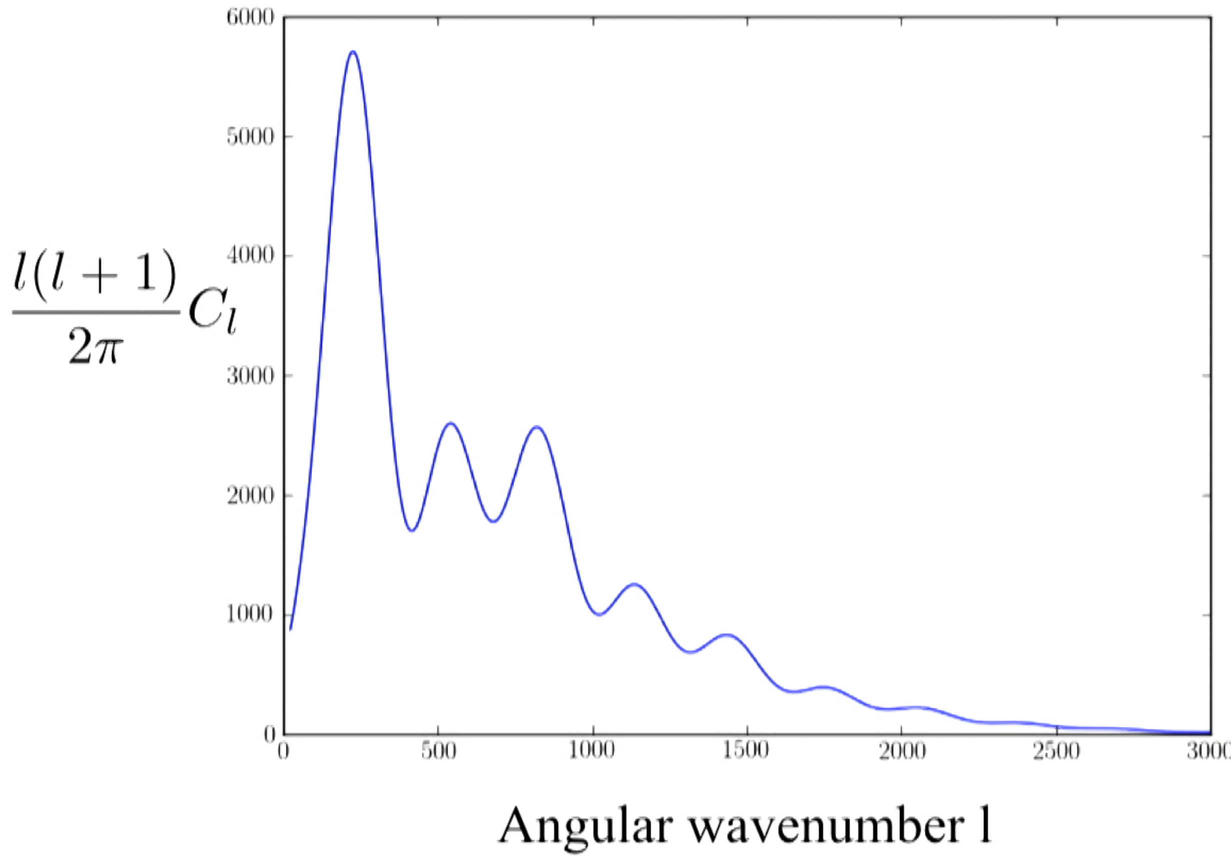
3D

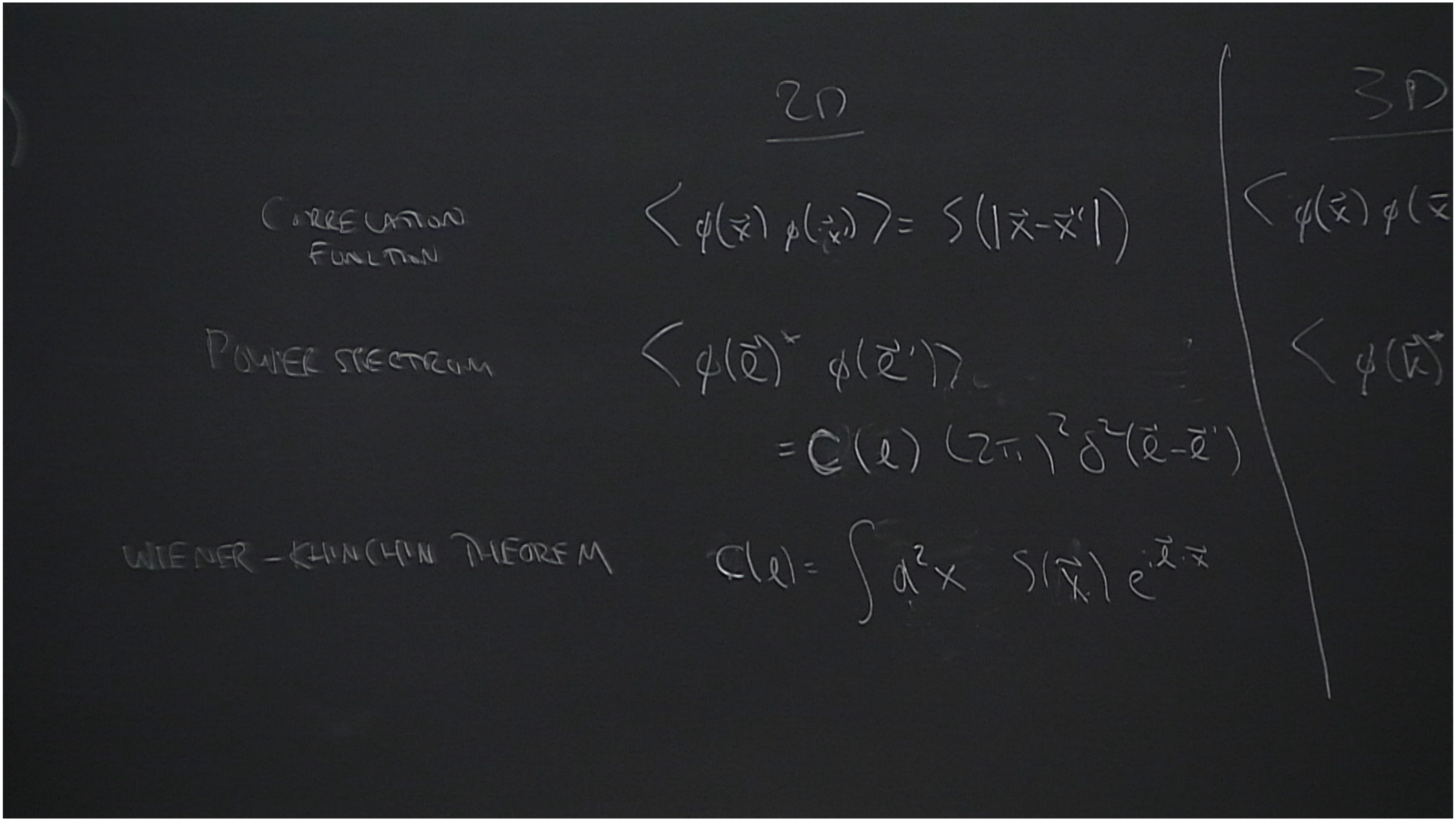
$$\langle \phi(\vec{x})\, \phi(\vec{x}') \rangle = \xi(|x - x'|)$$

$$\langle \phi(\vec{k})^{*}\, \phi(\vec{k}') \rangle = P(k)\, (2\pi)^3\, \delta^3(k - k')$$


2D
CORRELATION FUNCTION
$\langle \phi(\vec{x})\, \phi(\vec{x}') \rangle = S(|\vec{x}-\vec{x}'|)$
POWER SPECTRUM
$\langle \phi(\vec{\ell})^* \phi(\vec{\ell}') \rangle = C(\ell)\,(2\pi)^2 \delta^2(\vec{\ell}-\vec{\ell}')$
WIENER-KHINCHIN THEOREM
$C(\ell) = \int \frac{d^2\vec{\ell}}{(2\pi)^2} S(\vec{\ell})\, e^{i\vec{\ell}\cdot\vec{x}}$
3D
$\langle \phi(\vec{x})\, \phi(\vec{x}$
$\langle \phi(\vec{k})^*$

CMB power spectrum




2D
3D
CORRELATION FUNCTION
$\langle \phi(\vec{x})\, \phi(\vec{x}') \rangle = S(|\vec{x}-\vec{x}'|)$
POWER SPECTRUM
$\langle \phi(\vec{\ell})^* \phi(\vec{\ell}') \rangle = C(\ell)\,(2\pi)^2\,\delta^2(\vec{\ell}-\vec{\ell}')$
WIENER-KHINCHIN THEOREM
$C(\ell) = \int d^2x\; S(\vec{x})\, e^{i\vec{\ell}\cdot\vec{x}}$

$$= \xi(|\vec{x}-\vec{x}'|)$$

$$) \, (2\pi)^2 \delta^2(\vec{\ell}-\vec{\ell}')$$

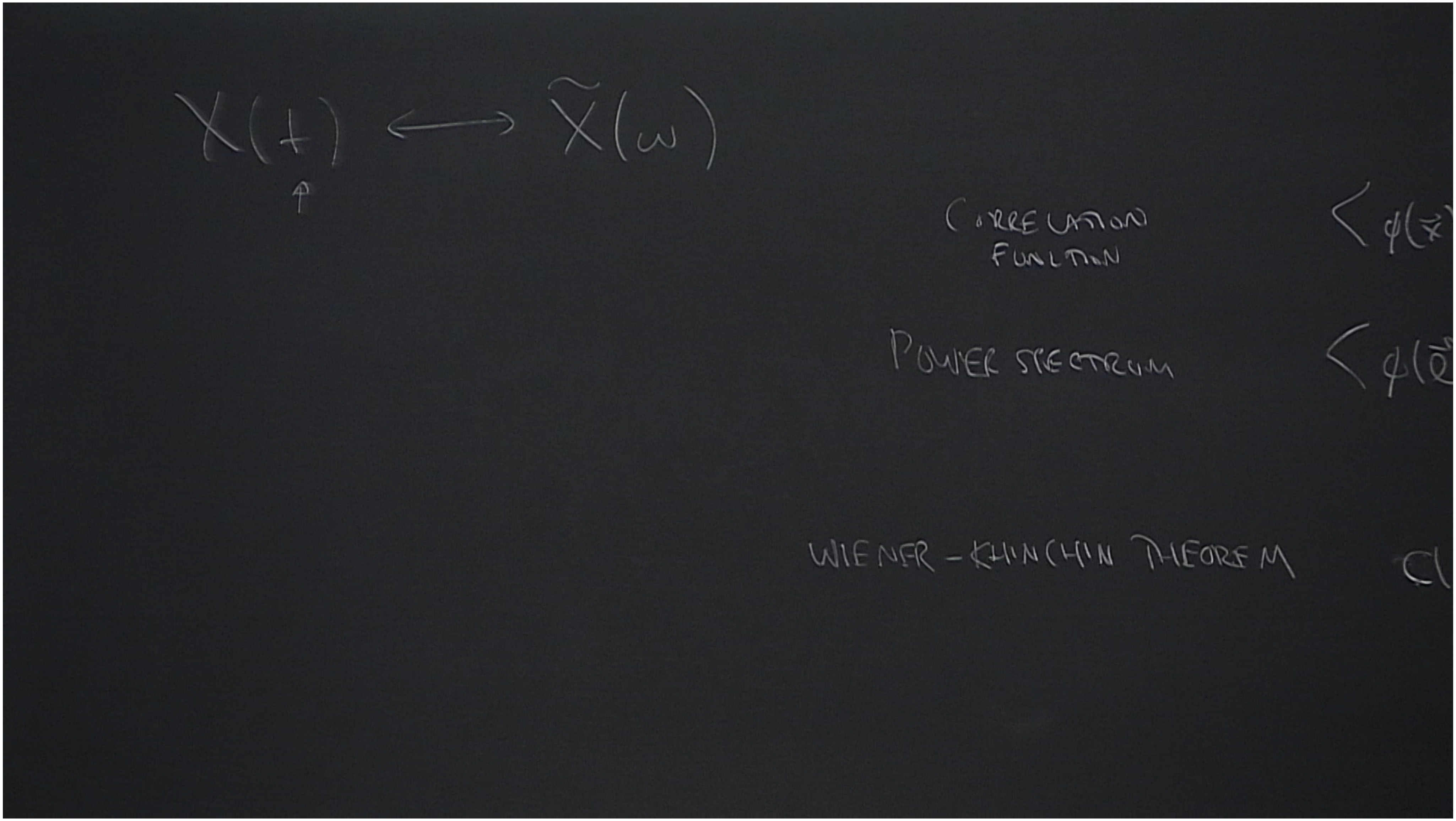
$$d^2x \; \xi(\vec{x}) \, e^{i\vec{\ell}\cdot\vec{x}}$$

3D

$$\langle \phi(\vec{x}) \phi(\vec{x}') \rangle = \xi(|x - x'|)$$

$$\langle \phi(\vec{k})^* \phi(\vec{k}') \rangle = P(k) \, (2\pi)^3 \, \delta^3(k - k')$$

$$P(k) = \int d^3\vec{x} \; \xi(x) \, e^{i\vec{k}\cdot\vec{x}}$$


X(t) ⟷ X̃(ω)
CORRELATION FUNCTION
POWER SPECTRUM
WIENER - KHINCHIN THEOREM

C(l)

$$\langle \phi(\vec{x})\, \phi(\vec{x}') \rangle = \xi(|\vec{x}-\vec{x}'|)$$

$$\langle \phi(\vec{\ell})^* \, \phi(\vec{\ell}') \rangle = C(\ell)\,(2\pi)^2\, \delta^2(\vec{\ell}-\vec{\ell}')$$

EM

$$C(\ell) = \int d^2x \; \xi(\vec{x})\, e^{i\vec{\ell}\cdot\vec{x}}$$

$$\xi(\vec{x}) = \int \frac{d^2\ell}{(2\pi)^2}\, C(\ell)\, e^{-i\vec{\ell}\cdot\vec{x}}$$

P(k)

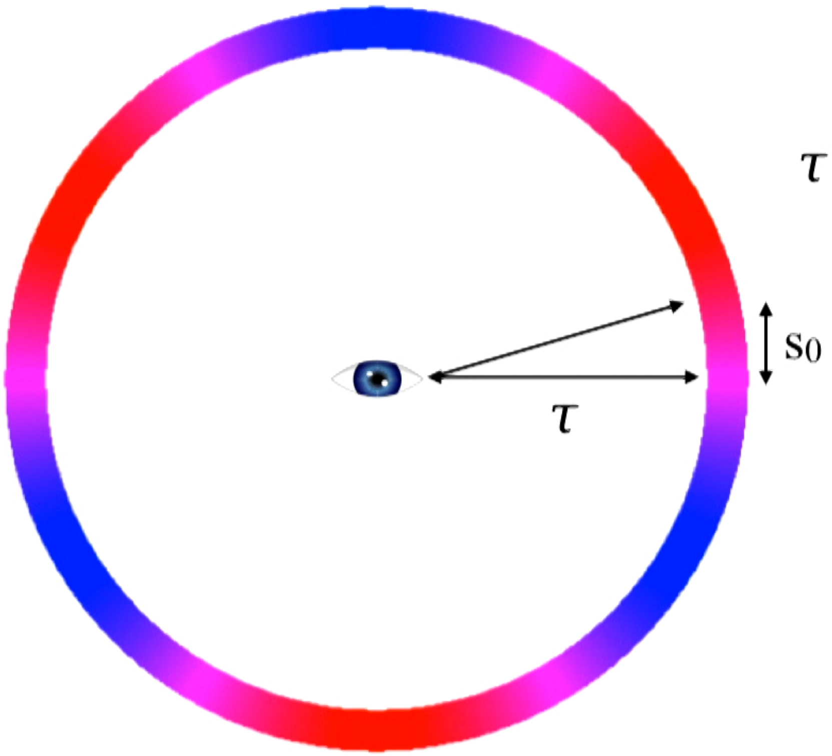
$$\langle \phi(\vec{x})\, \phi(\vec{x}') \rangle = \xi(|x-x'|)$$

$$\langle \phi(\vec{k})^* \phi(\vec{k}') \rangle = P(k)\,(2\pi)^3\, \delta$$

$$P(k) = \int d^3\vec{x} \;\; \xi(x)\; e^{i\vec{k}\cdot}$$

We see the sound horizon as a characteristic angle on the sky

$$\Delta\theta = s_0/\tau$$



s_0 = sound horizon
τ = conformal time to CMB

POWER SPECTRUM

$$\langle \phi(\vec{\ell})^* \, \phi(\vec{\ell}') \rangle = C(\ell)\,(2\pi)^2\,\delta^2(\vec{\ell}-\vec{\ell}')$$

WIENER-KHINCHIN THEOREM

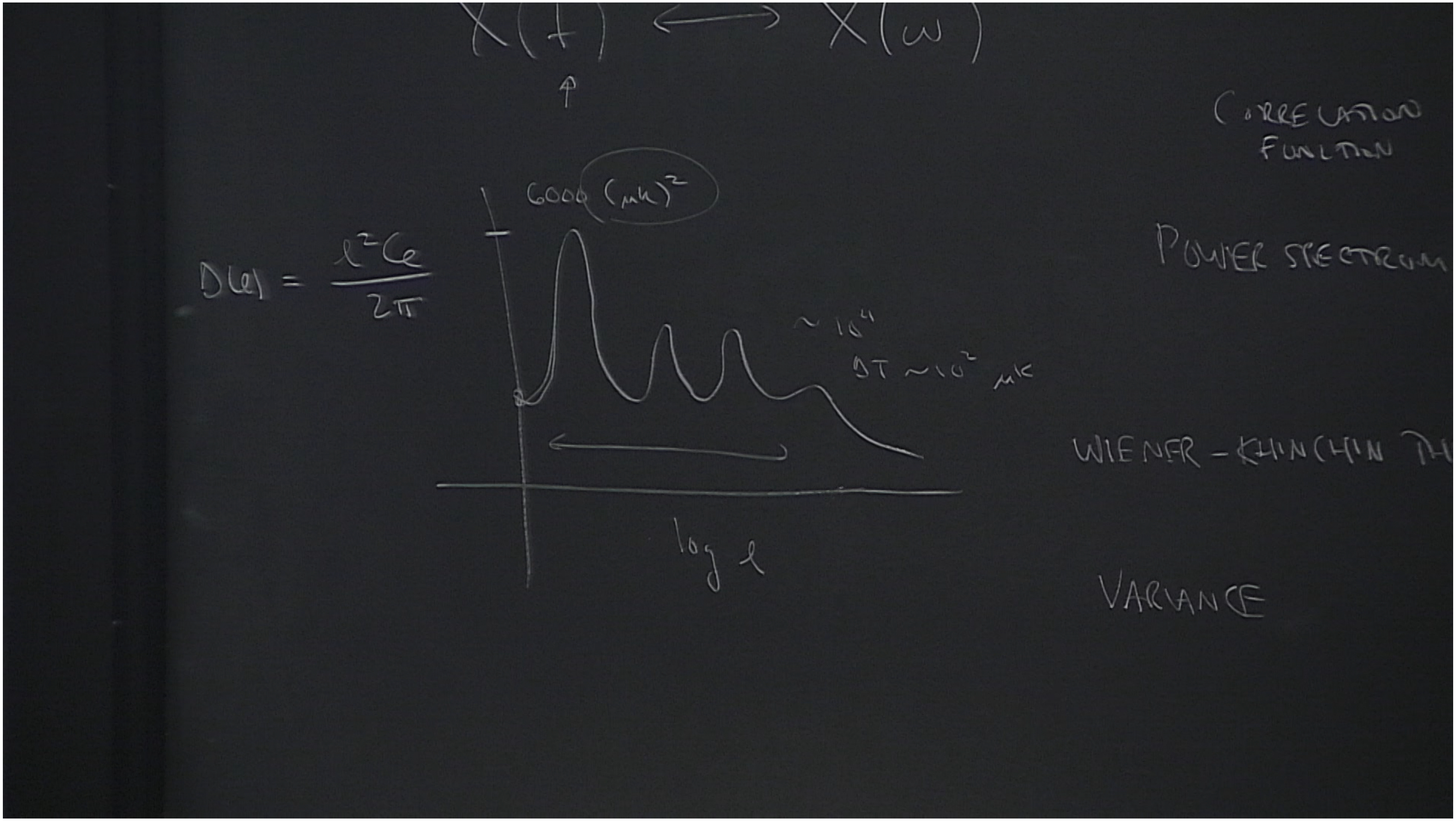
$$C(\ell) = \int d^2x \; S(\vec{x})\, e^{i\vec{\ell}\cdot\vec{x}}$$

VARIANCE

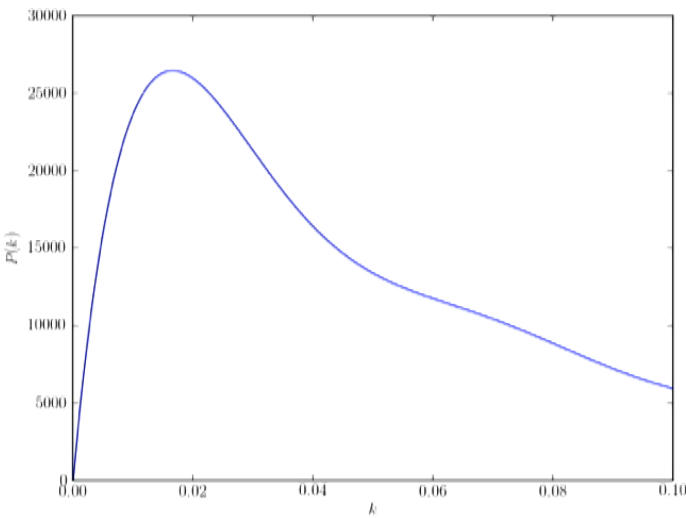
$$\langle \phi(x)^2 \rangle = \int \frac{d^2\ell}{(2\pi)^2}\, C(\ell) = \int d(\log \ell)\, \underbrace{\frac{\ell^2 C(\ell)}{2\pi}}_{= D(\ell)}$$

$$\langle \phi(\vec{k})^* \phi($$

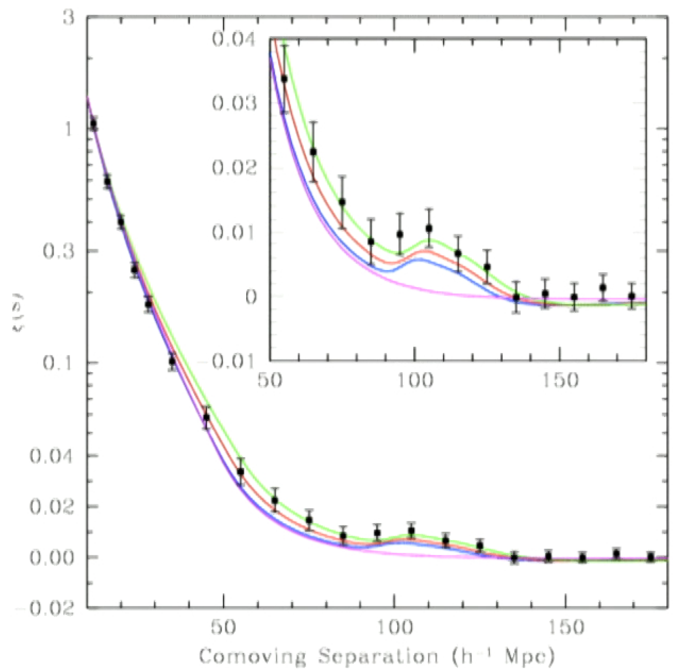
$$P(k)$$


X(t) ⟷ X(ω)
6000 (μK)²
D(l) = l² C_l / 2π
~10⁴
ΔT ~ 10² μK
log l
CORRELATION
FUNCTION
POWER SPECTRUM
WIENER - KHINCHIN TH
VARIANCE

Matter power spectrum

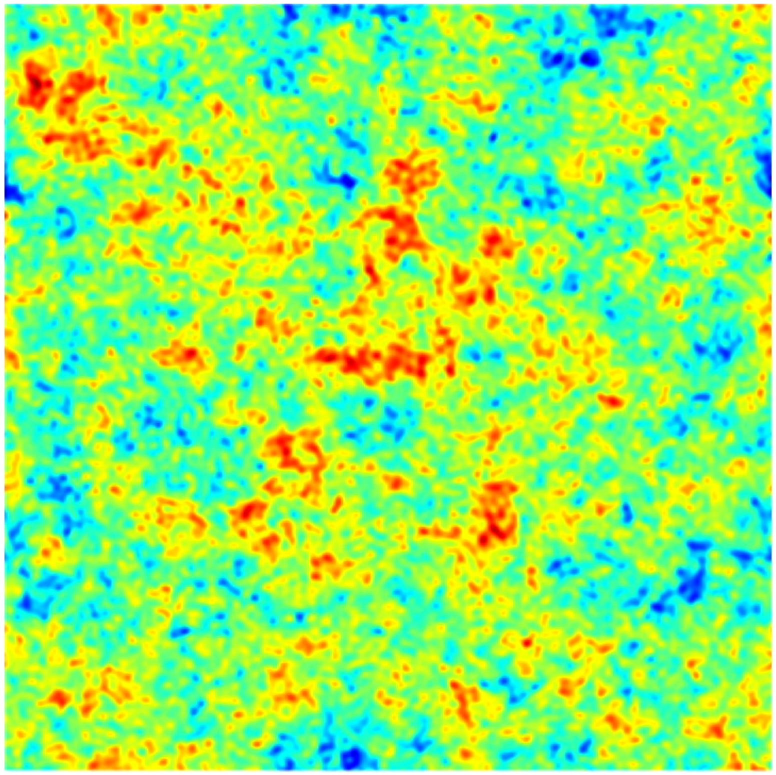


Matter correlation function



(Credit: SDSS)

Simulated Gaussian field with same power spectrum as the density field in the late universe



$$\langle \phi(\vec{k})\,\phi(\vec{k}')\rangle = P(k)\,(2\pi)^3\,\delta^3(k-k')$$

$$P(k) = \int d^3\vec{x}\;\xi(x)\,e^{i\vec{k}\cdot\vec{x}}$$

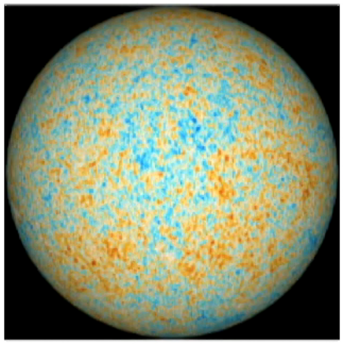
$$\langle \phi(x)^2\rangle = \int \frac{d^3k}{(2\pi)^3}\,P(k)$$

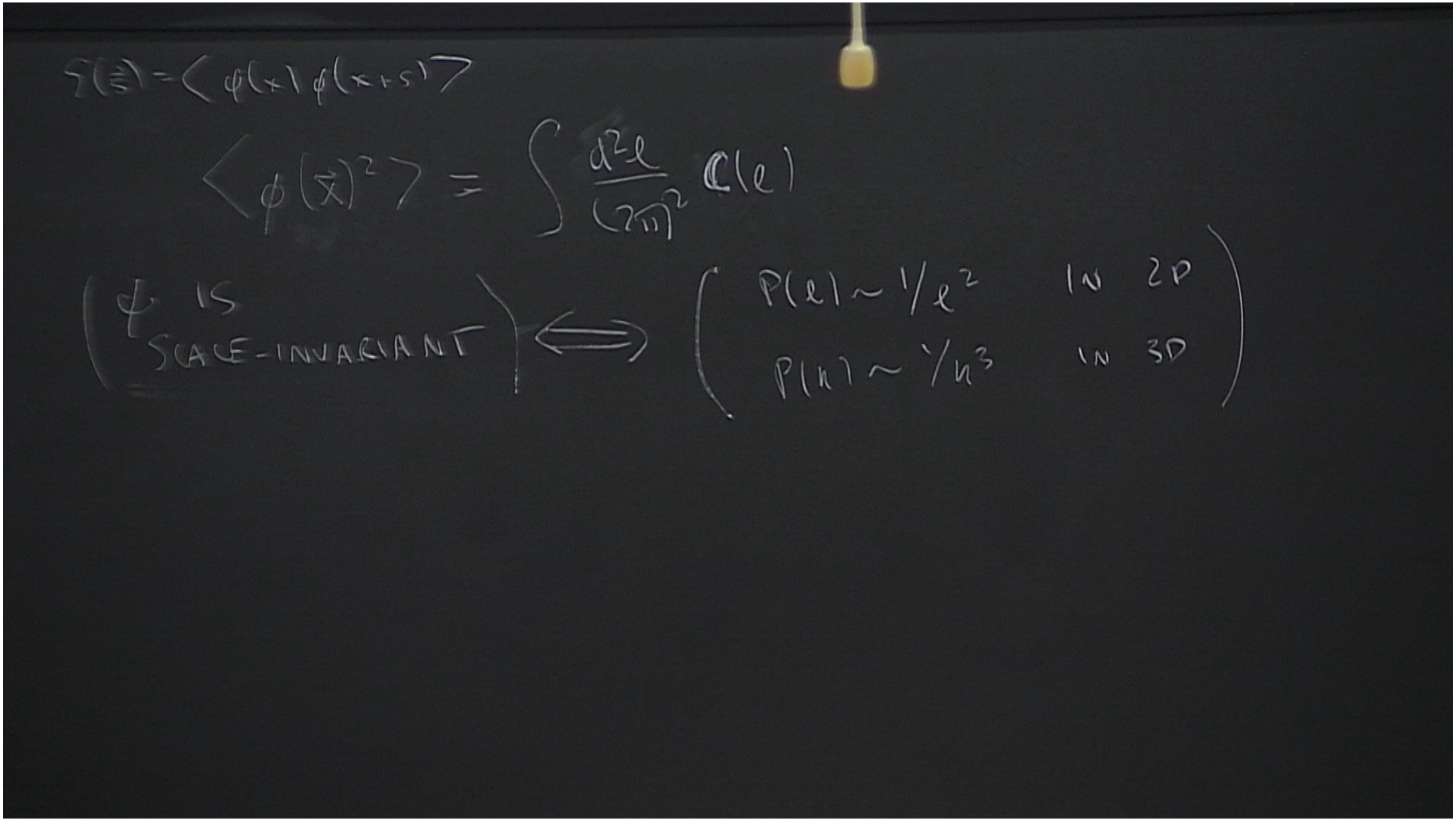
$$= \int d(\log k)\,\underbrace{\frac{k^3 P(k)}{2\pi^2}} = \Delta(k)$$

One last comment (a technical point). We've been working with 2D fields T(x,y) defined on the x-y plane, with the usual Fourier expansion in plane waves:

$$T(\mathbf{x}) = \int \frac{d^2\mathbf{l}}{(2\pi)^2}\, \tilde{T}(\mathbf{l}) e^{i\mathbf{l}\cdot\mathbf{x}}$$

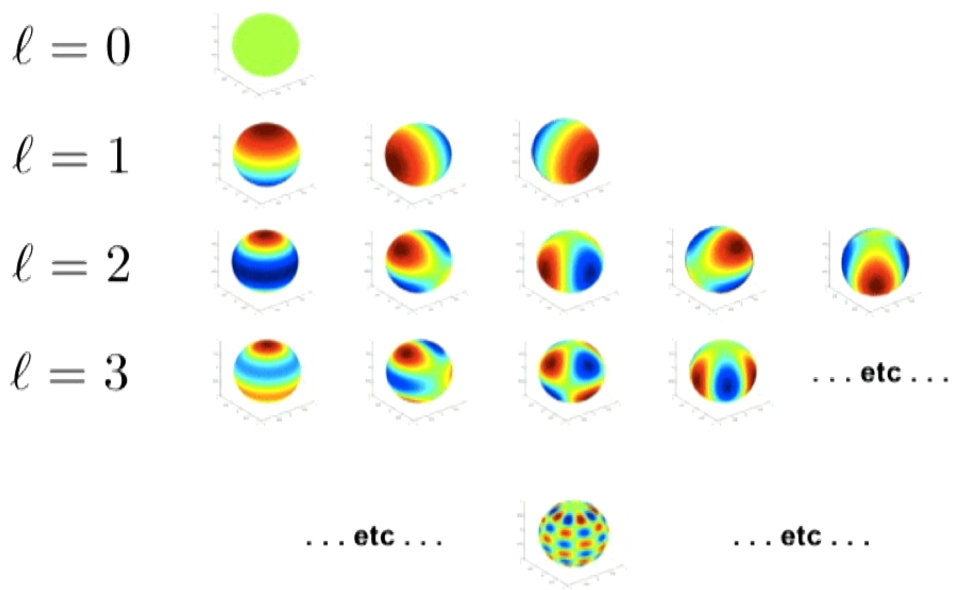
However, the CMB and many other 2D fields in cosmology are defined on a sphere: $T = T(\theta, \phi)$ or $T = T(\mathbf{n})$ where **n** is a unit 3-vector.




ξ(s) = ⟨ φ(x) φ(x+s) ⟩
⟨ φ(x)² ⟩ = ∫ d²l/(2π)² C(l)
φ IS SCALE-INVARIANT ⟺
P(l) ~ 1/l² IN 2D
P(k) ~ 1/k³ IN 3D

$\ell = 0$ harmonic: monopole (i.e. constant)
$\ell = 1$ harmonics: dipoles
$\ell = 2$ harmonics: quadrupoles, etc.

In general, spherical harmonics have characteristic wavelength $2\pi/\ell$



$\phi(x)$

$$\tilde{\phi}(k) = \int \underbrace{d^3x}_{\text{VOLUME}} \, \phi(x)\, e^{ik\cdot x}$$

3D

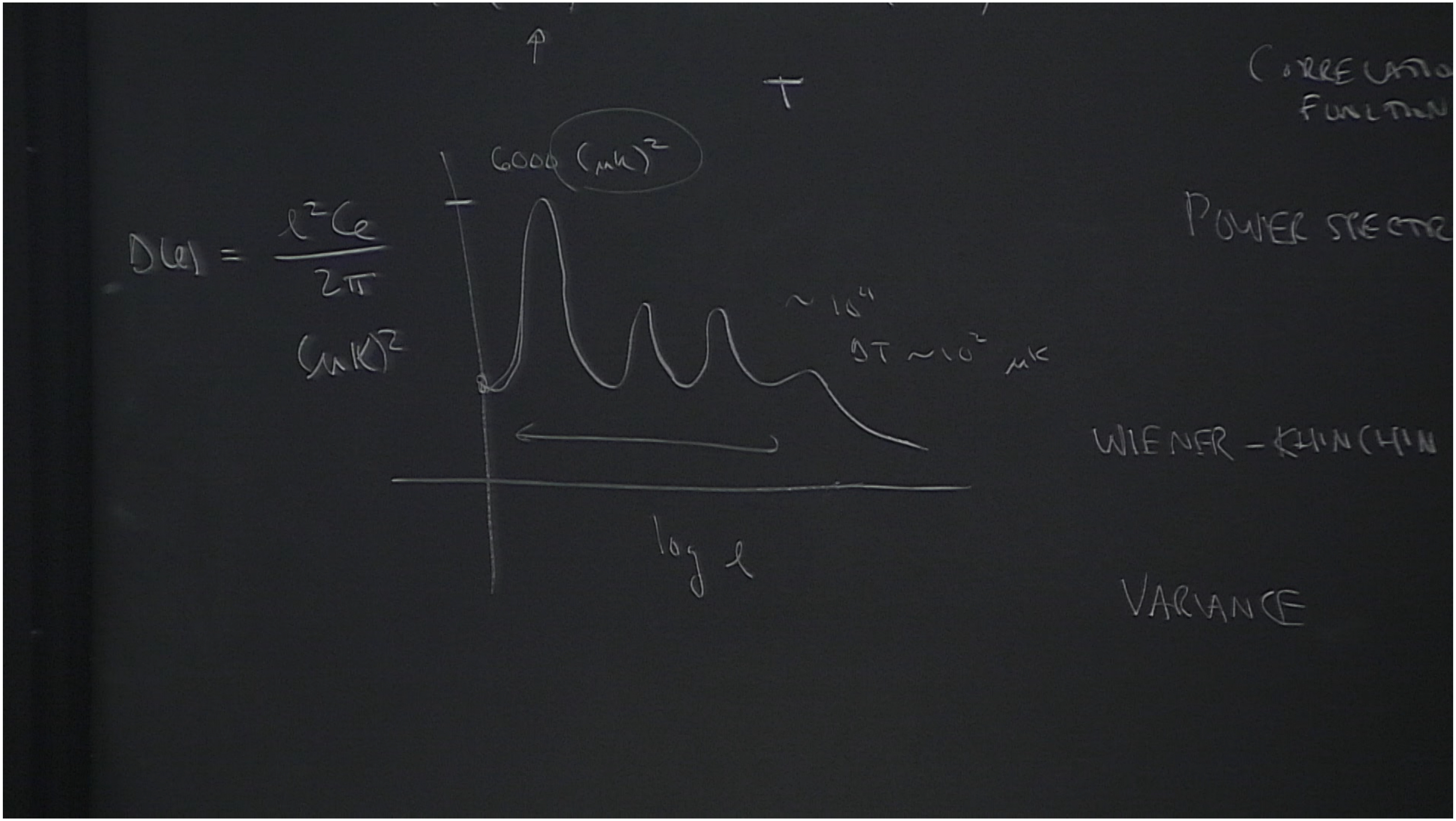
$$\langle \phi(\vec{x})\,\phi(\vec{x}\,') \rangle = \xi(|x - x'|)$$

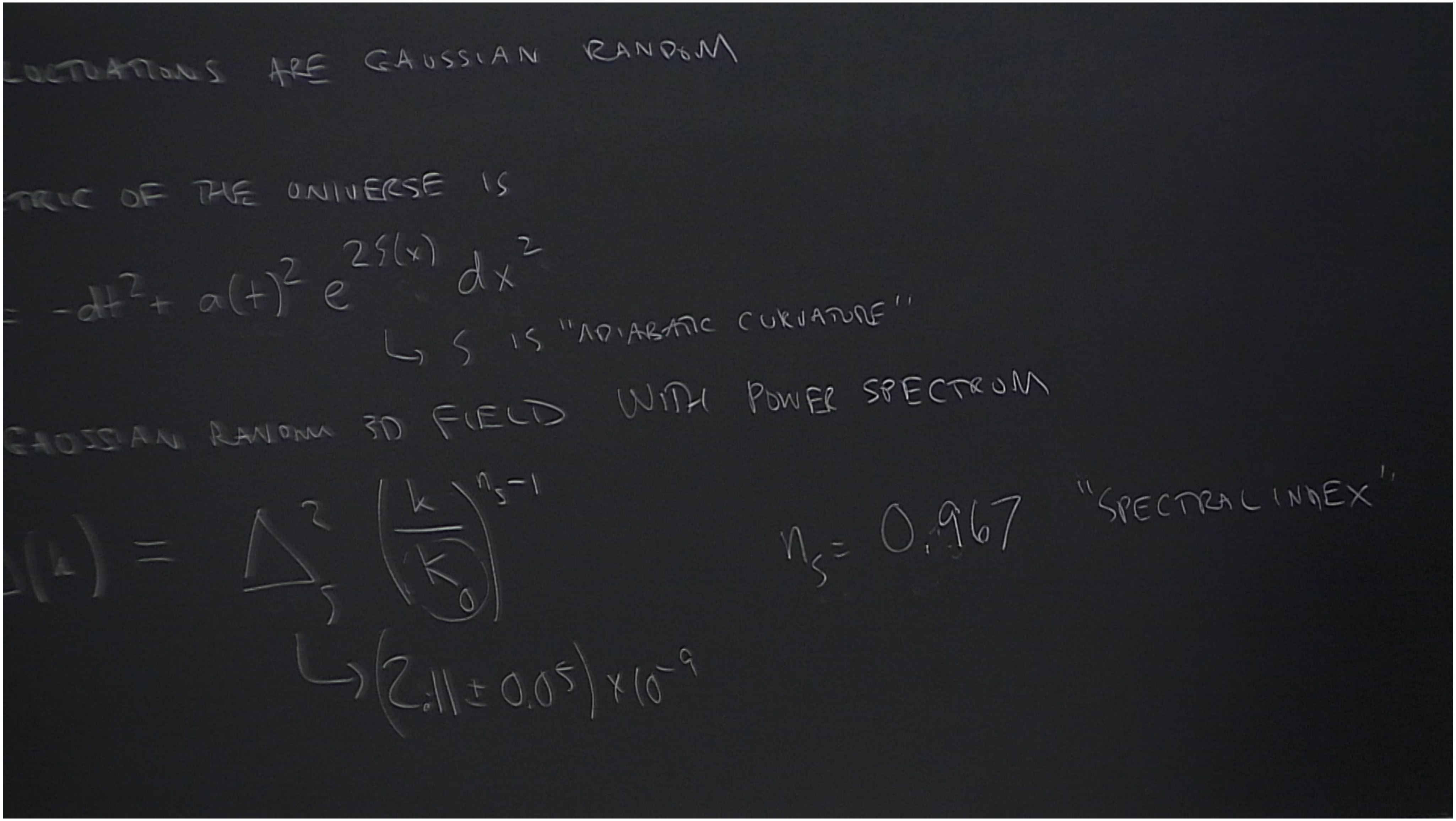
$$\langle \phi(\vec{k})^* \phi(\vec{k}\,') \rangle = P(k)\,(2\pi)^3\,\delta^3(k - k')$$

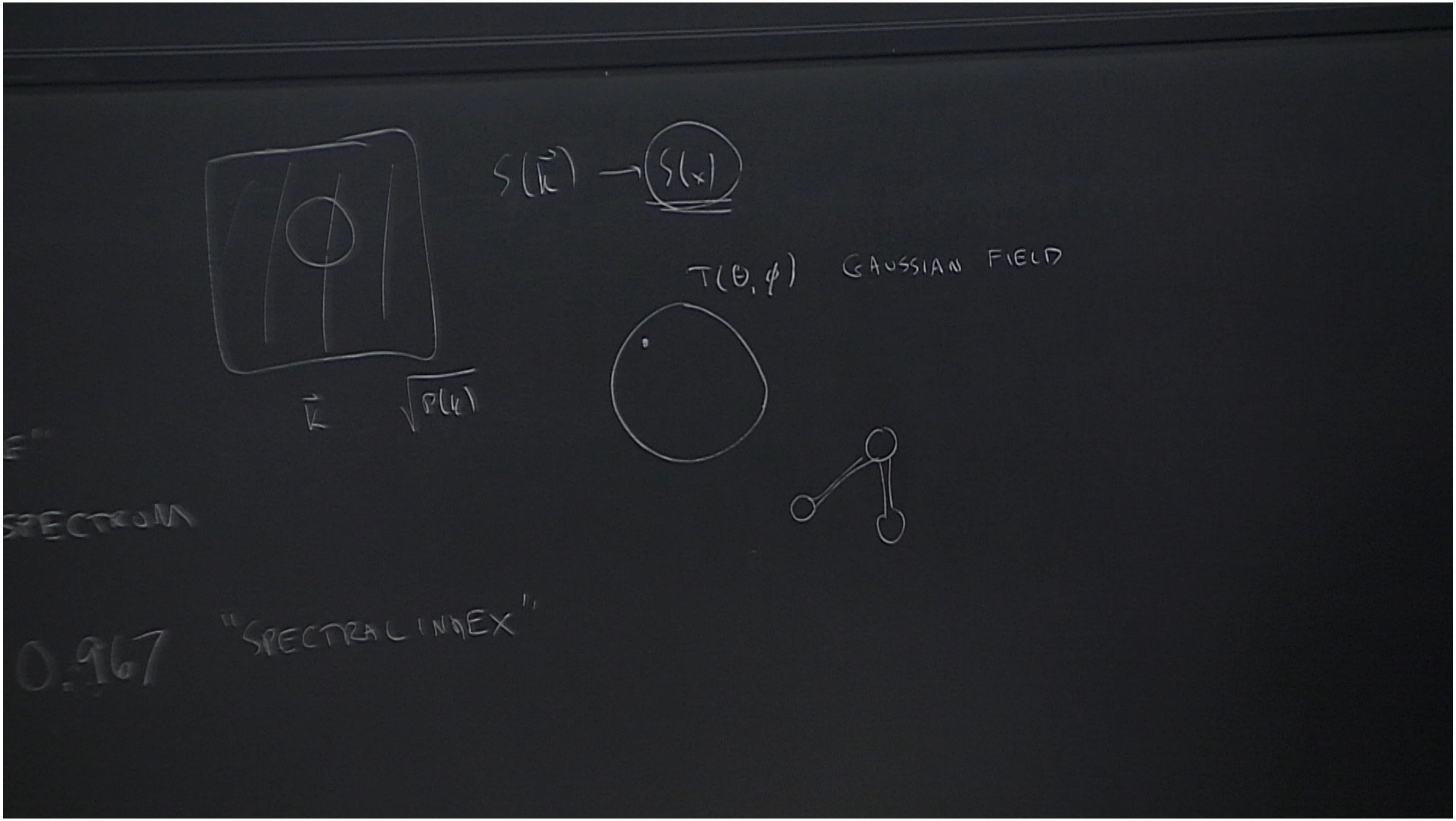
$P(k) \rightarrow$ VOLUME

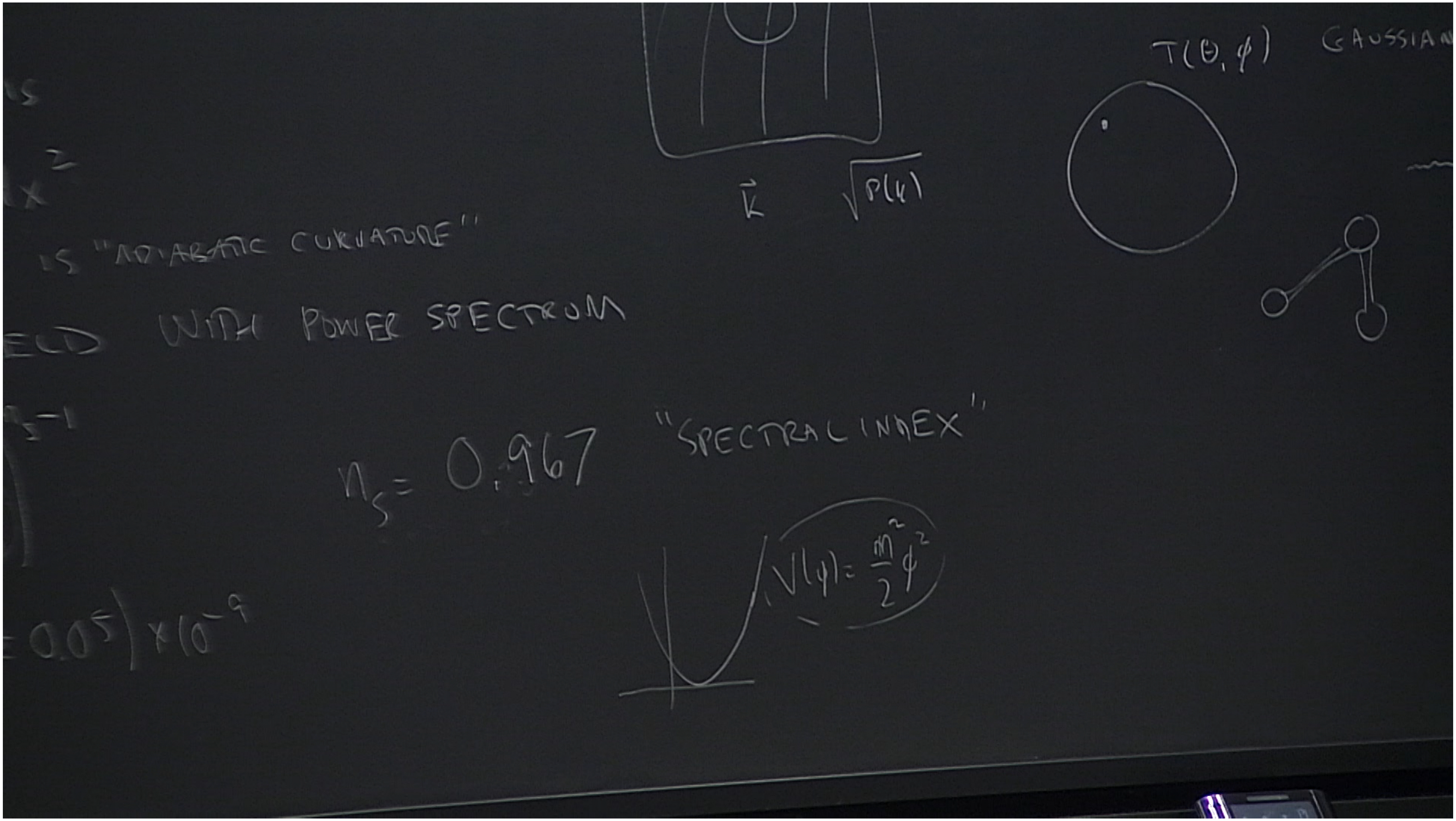
$$P(k) = \int d^3\vec{x}\;\xi(x)\, e^{i\vec{k}\cdot\vec{x}}$$

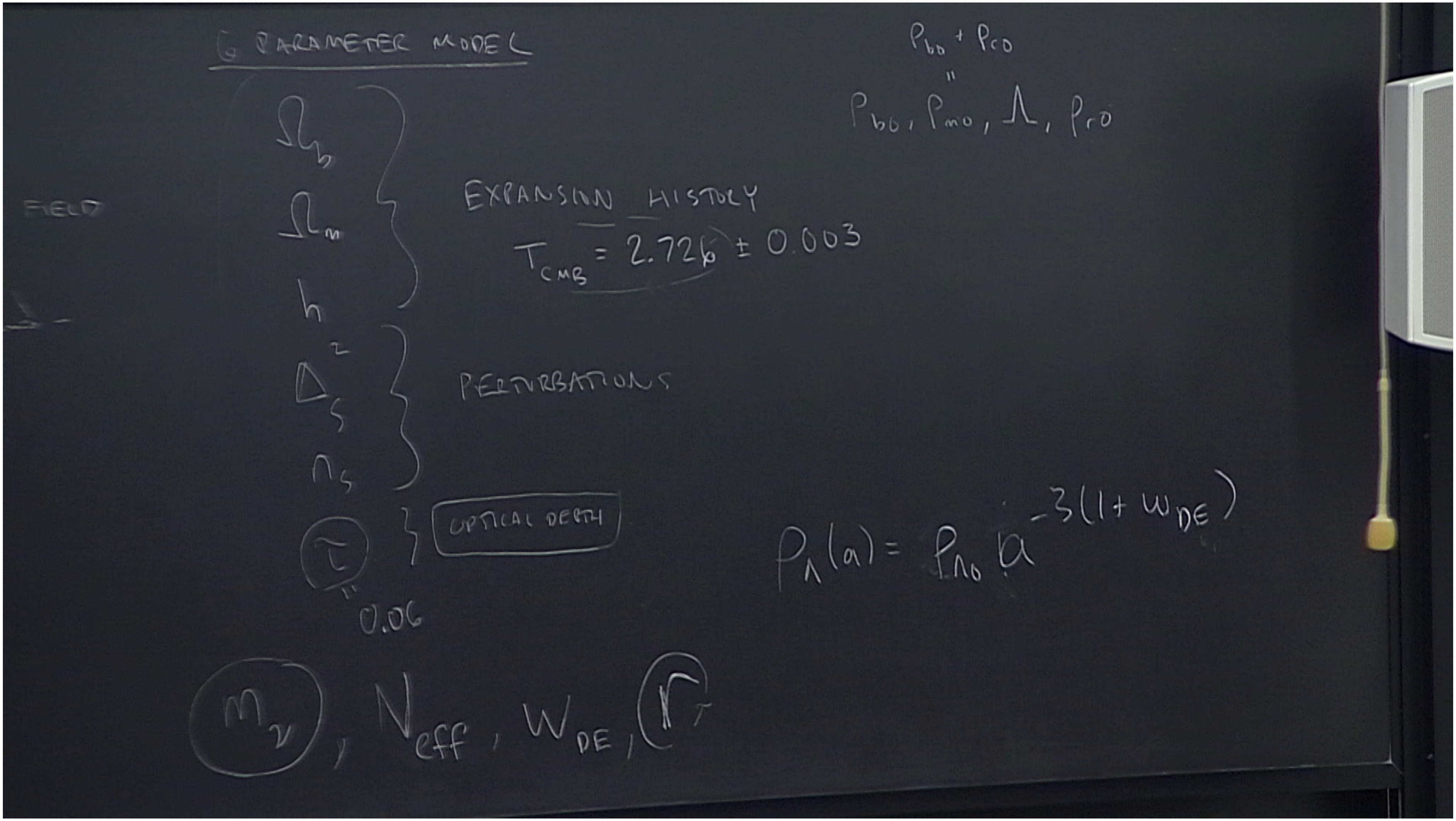
$$\langle \phi(x)^2 \rangle = \int \frac{d^3k}{\ldots}\, P(k)$$

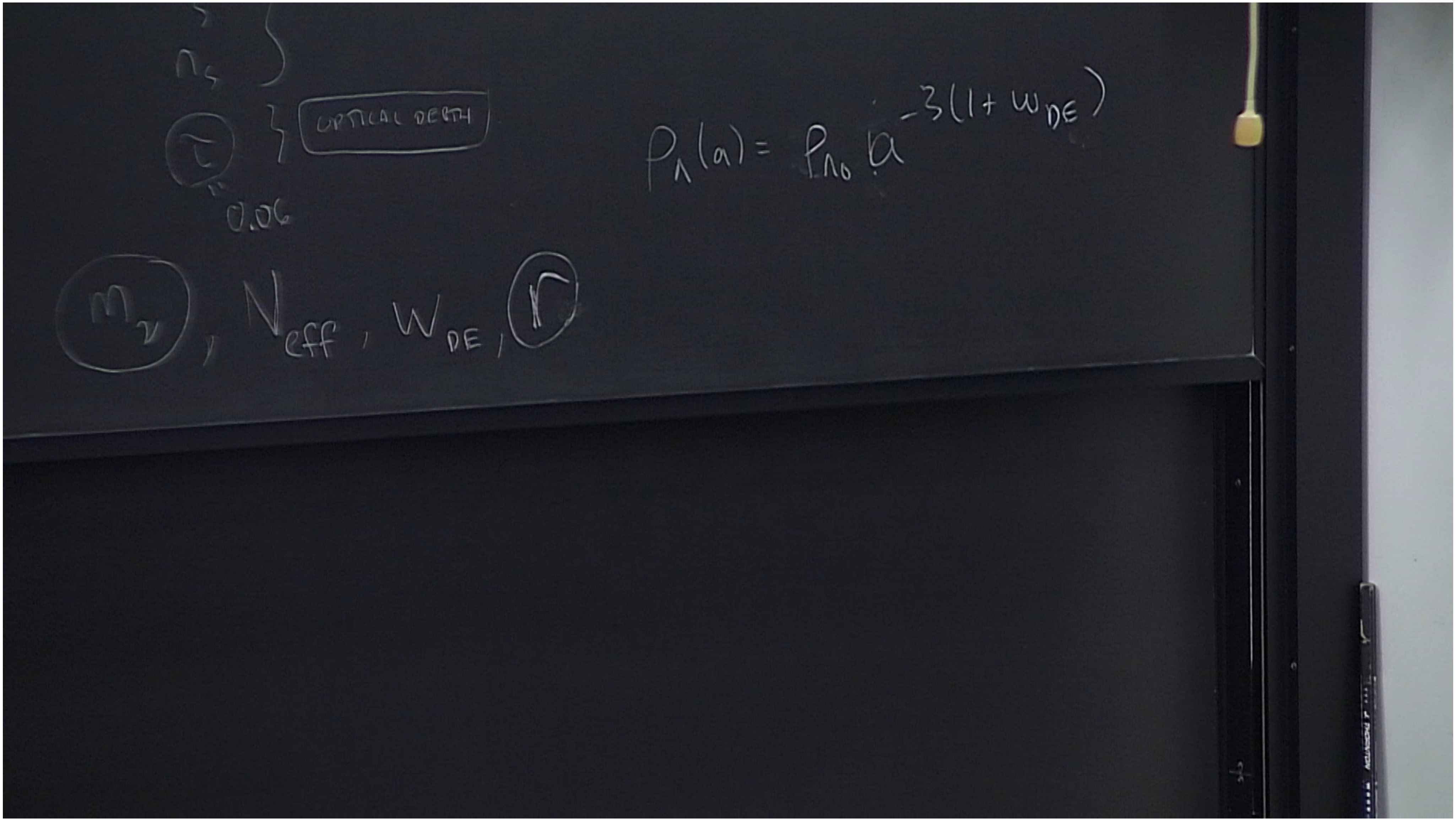

CORRELATION
FUNCTION
6000 (μK)²
D(l) = l²C_l / 2π
(μK)²
POWER SPECTR
~10⁴
ΔT ~ 10² μK
WIENER - KHINCHIN
log l
VARIANCE


ARE GAUSSIAN RANDOM
OF THE UNIVERSE IS
$-dt^2 + a(t)^2 e^{2\zeta(x)} dx^2$
ζ IS "ADIABATIC CURVATURE"
WITH POWER SPECTRUM
RANDOM 3D FIELD
$= \Delta_\zeta^2 \left(\frac{k}{k_0}\right)^{n_s - 1}$
$n_s = 0.967$ "SPECTRAL INDEX"
$(2.11 \pm 0.05) \times 10^{-9}$

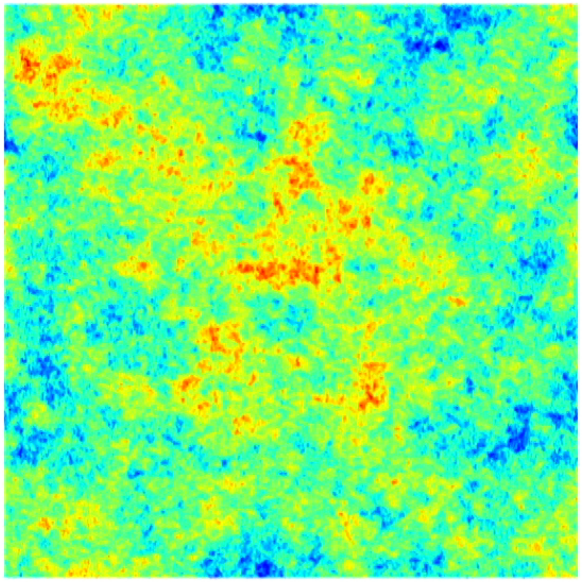

S(k) → S(x)
k
√P(k)
T(θ, φ) GAUSSIAN FIELD
PECTRUM
0.967 "SPECTRAL INDEX"


T(θ, φ)
GAUSSIA
IS "ADIABATIC CURVATURE"
WITH POWER SPECTRUM
√P(k)
n_s = 0.967
"SPECTRAL INDEX"
V(φ) = m²/2 φ²

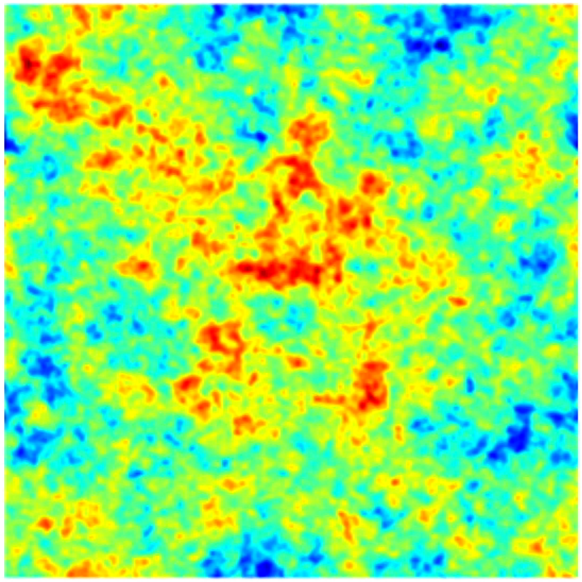

6 PARAMETER MODEL
EXPANSION HISTORY
T_CMB = 2.726 ± 0.003
PERTURBATIONS
OPTICAL DEPTH
0.06
ρ_b0 + ρ_r0
ρ_b0, ρ_m0, Λ, ρ_r0
ρ_Λ(a) = ρ_Λ0 a^{-3(1+w_DE)}
m_ν, N_eff, W_DE, r


n_s
τ
OPTICAL DEPTH
0.06
m_ν, N_{eff}, w_{DE}, r
$\rho_\Lambda(a) = \rho_{\Lambda 0}\, a^{-3(1+w_{DE})}$

Not self-similar: Scale invariant on scales larger than characteristic scale l_0^{-1}, fluctuations are smoothed out below this scale

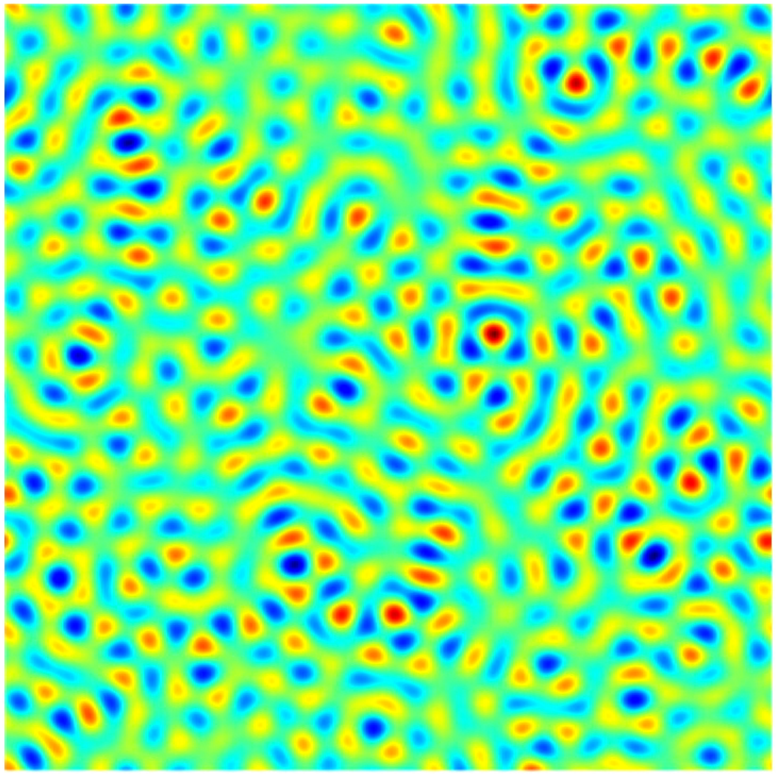


Scale invariant power spectrum: $C(l) \propto l^{-2}$



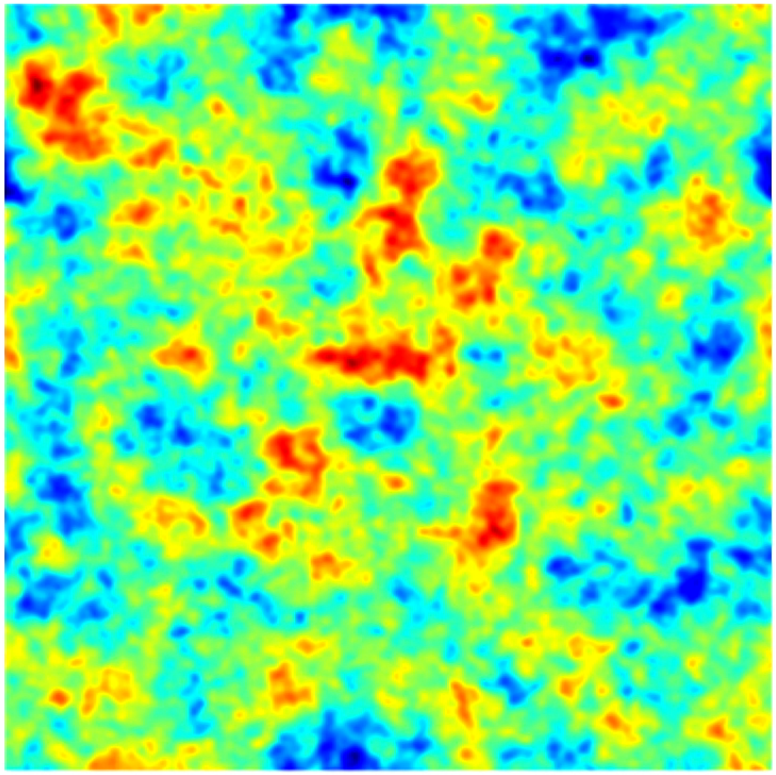
Scale invariant power spectrum with cutoff: $C(l) \propto l^{-2} \exp(-(l/l_0)^2)$

Delta function power spectrum: $C(l) \propto \delta(l-l_0)$



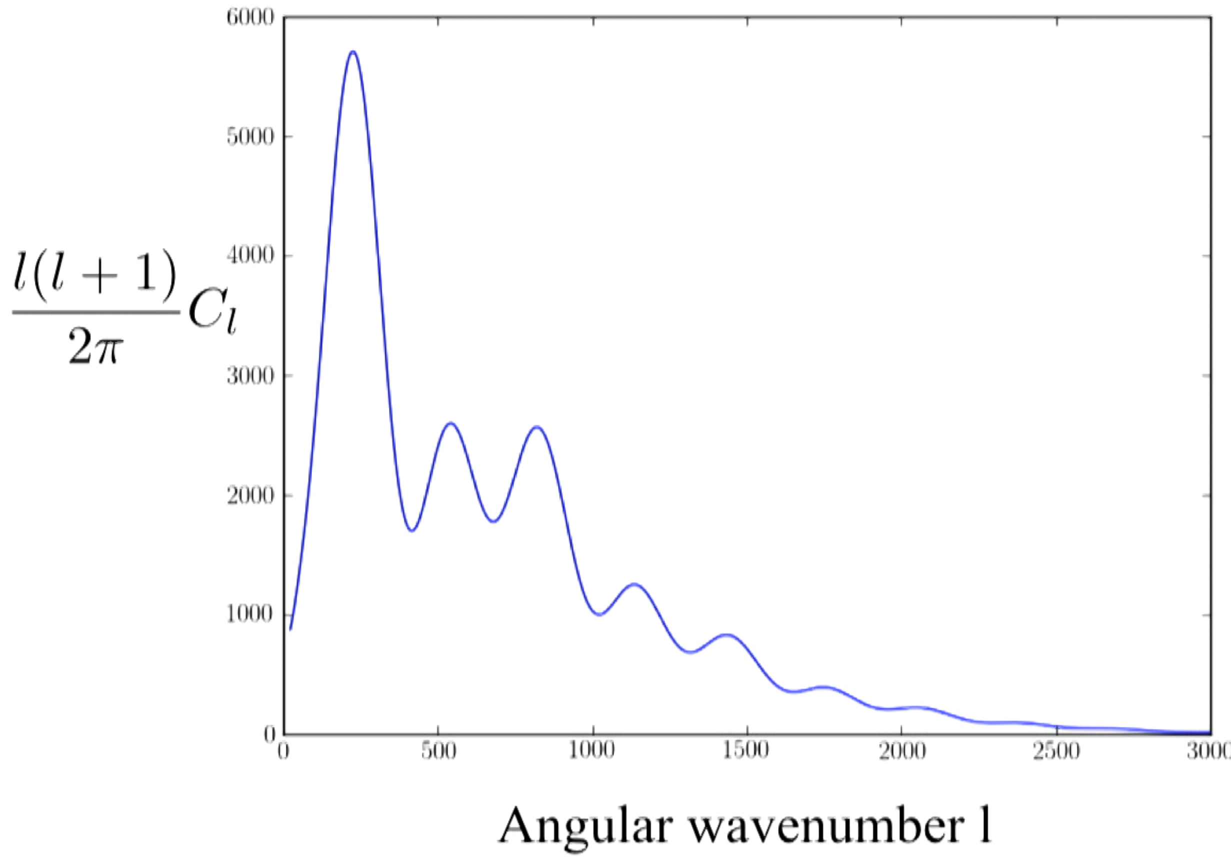
Very strongly defined characteristic scale

CMB map (simulated)

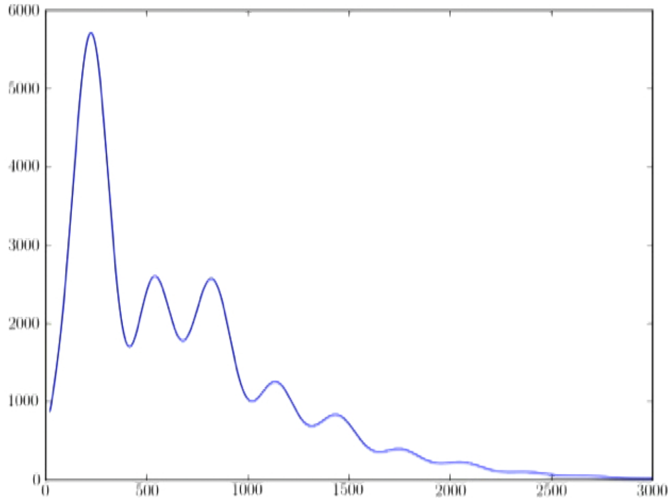


A Gaussian map with the CMB power spectrum

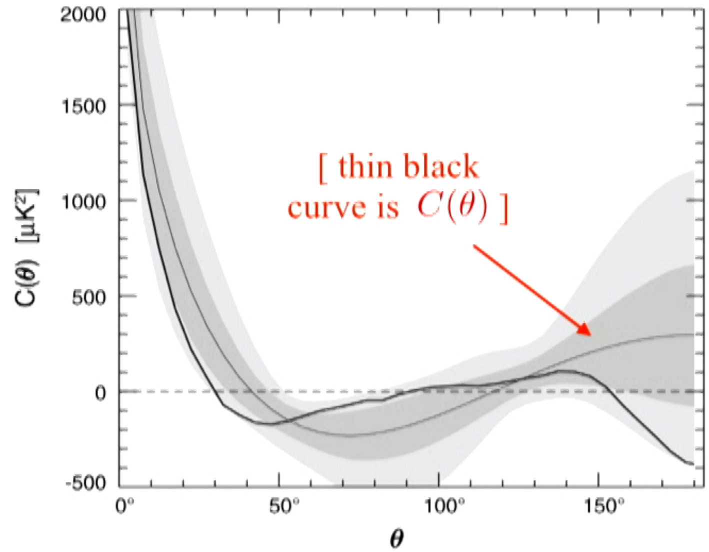
CMB power spectrum



CMB power spectrum C_L



CMB correlation function $C(\theta)$.



(Credit: WMAP/NASA)

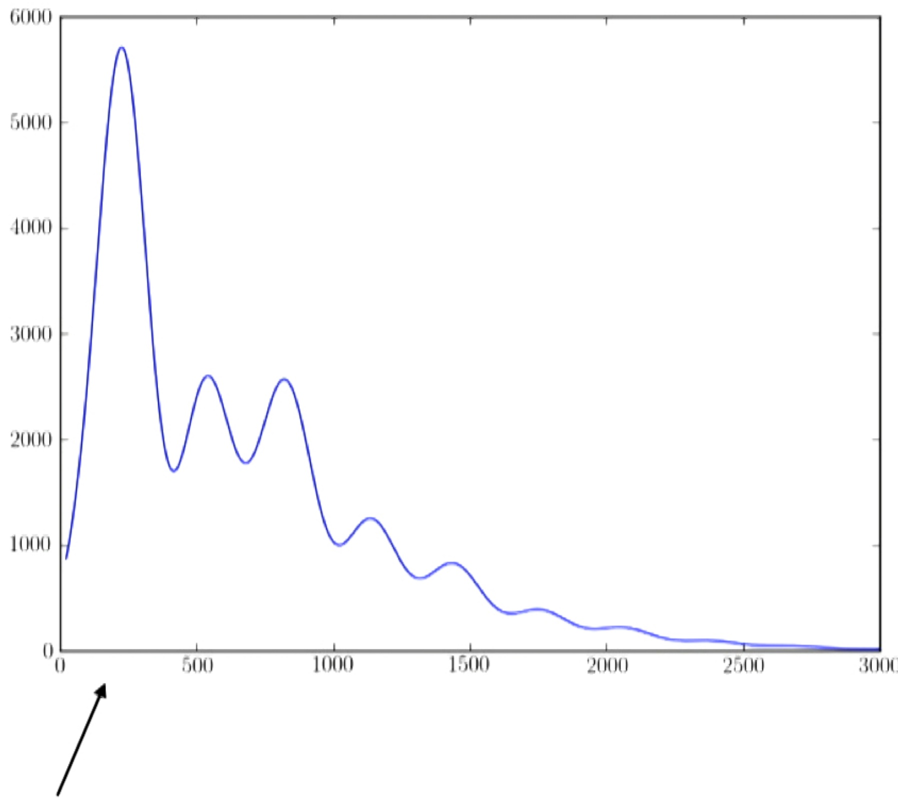
The two curves are Fourier transforms of each other (Wiener-Khinchin theorem)

Just for fun: characteristic scales in the CMB
(derivation will come later in the class!)

Before the CMB was formed, the universe is a plasma, inhomogeneities propagate as sound waves.

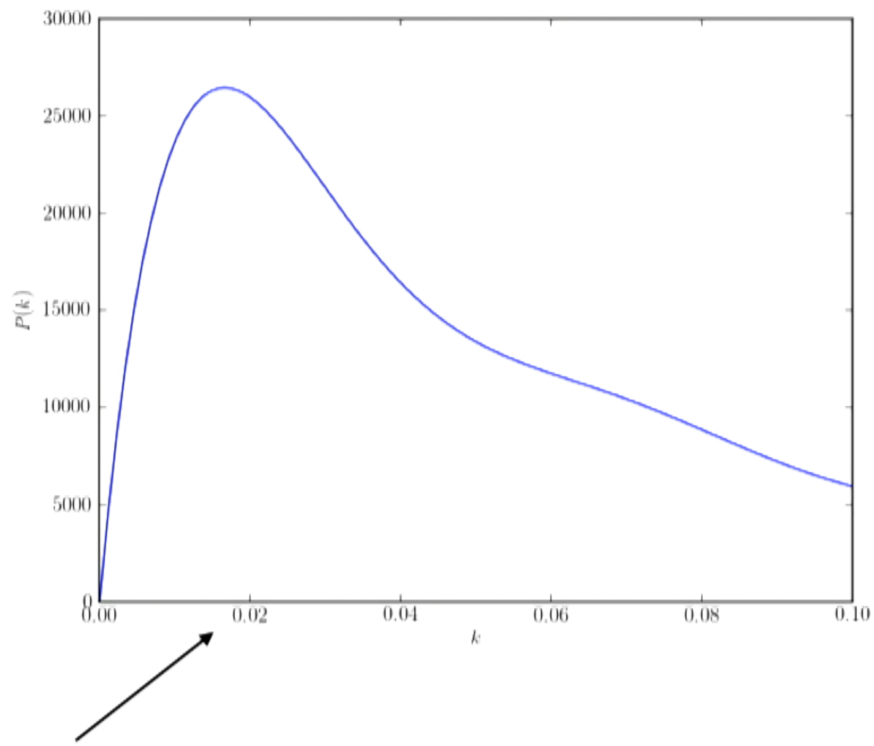
Define the "sound horizon" s_0 to be the maximum distance that a sound wave can travel between the big bang and the formation of the CMB.

The acoustic peak scale is determined by $\Delta\theta$



Acoustic peak scale $l_a \sim \pi / \Delta\theta = \pi\,(s_0/\tau)$

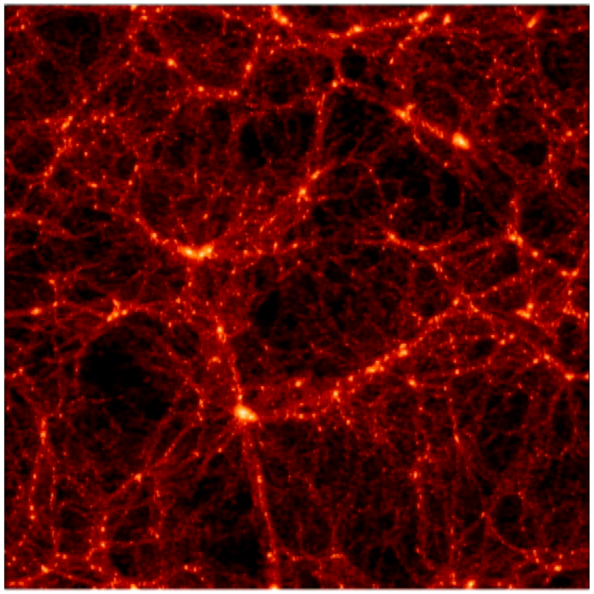
Matter power spectrum in the late universe (a 3D field)

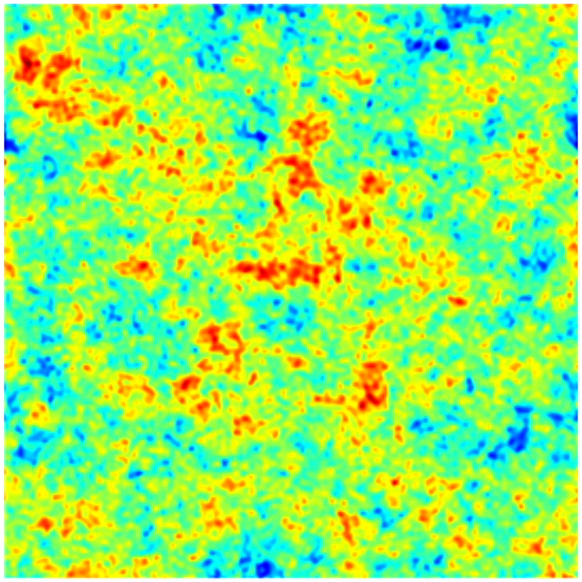


Characteristic scale! (Turns out to be the Hubble length evaluated at matter radiation equality)

The density field in the late universe is nonlinear, and modelling it as a Gaussian field is generally not a good approximation

These two maps have roughly the same power spectrum!





Perhaps surprisingly, Gaussian statistics are a good approximation on large scales (low k)

On a 2D sphere, the analogue of a plane wave $e^{i\mathbf{l}\cdot\mathbf{x}}$ is a spherical harmonic $Y_{lm}(\theta,\phi)$. This a special function defined for l=0,1,2,… and m=-l,…,l.