Title: PSI 16/17 Explorations in Cosmology (Kendrick Smith) - Lecture 3

Date: Apr 12, 2017 10:15 AM

URL: http://pirsa.org/17040055

Abstract:

SCALAR RANDOM VARIABLE $X$

MEAN $\bar{x} = \langle x \rangle$

VARIANCE $\langle (x - \bar{x})^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$

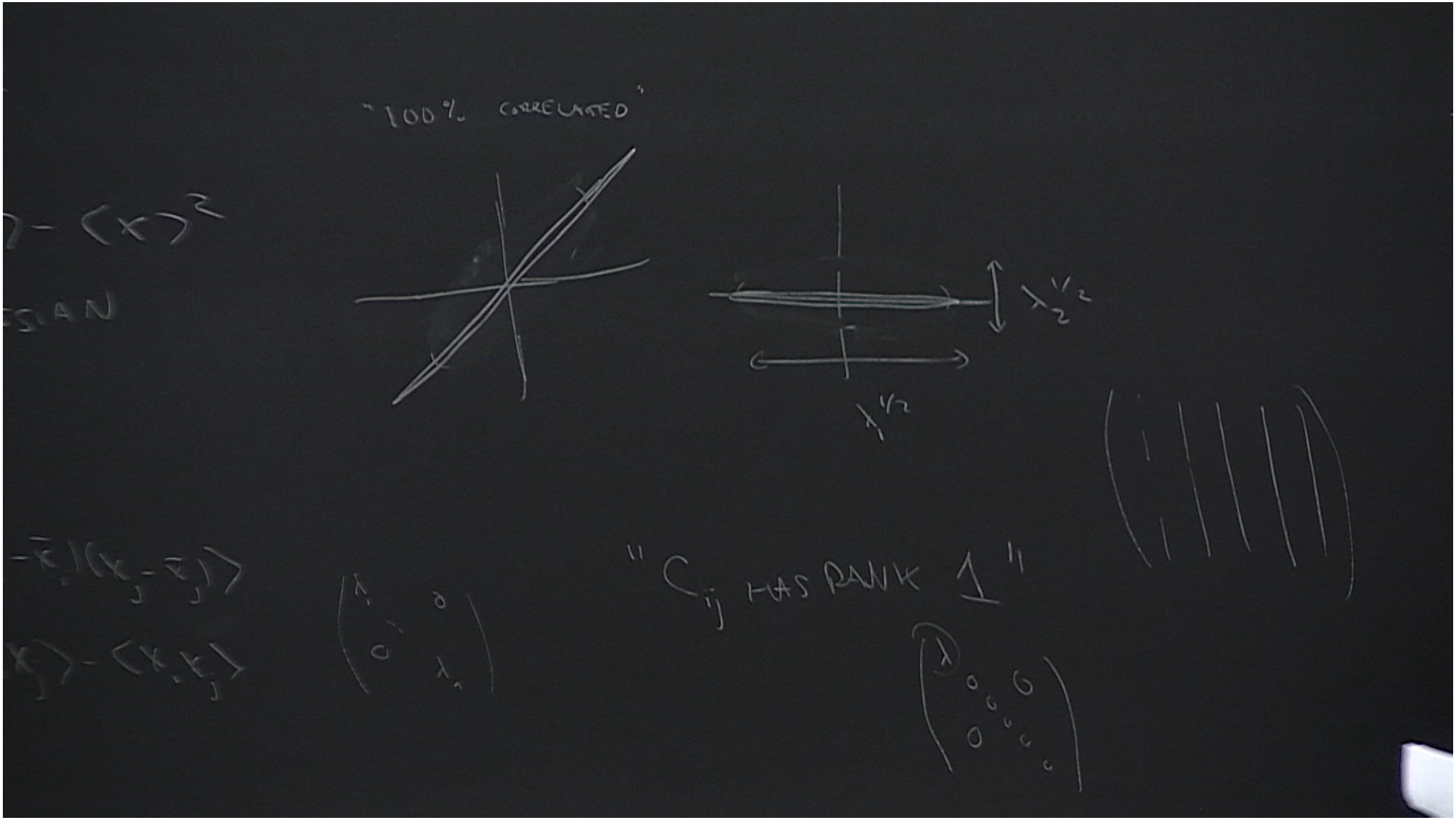
WICK'S THEOREM $\Leftrightarrow$ GAUSSIAN

MULTIVARIATE R.V. $(X_1, \cdots, X_n)$

MEAN $\bar{X}_i = \langle X_i \rangle$

COVARIANCE MATRIX $C_{ij} = \langle (X_i - \bar{X}_i)(X_j - \bar{X}_j) \rangle$

$= \langle X_i X_j \rangle - \langle X_i \rangle \langle X_j \rangle$


"100% CORRELATED"
$\lambda_2^{1/2}$
$\lambda_1^{1/2}$
"$C_{ij}$ HAS RANK 1"

## RANDOM FIELDS IN 1D

$X(t)$ ASSUME $\langle X(t) \rangle = 0$

$\langle X(t) X(t') \rangle = \xi(t - t')$ "CORRELATION FUNCTION"

$$\tilde{X}(\omega) = \int dt \; X(t) e^{i\omega t}$$

$$X(t) = \int \frac{d\omega}{2\pi} \tilde{X}(\omega) e^{-i\omega t}$$

$X(t)$ REAL-VALUED $\iff \tilde{X}(-\omega) = \tilde{X}(\omega)^*$

FIELDS IN 1D

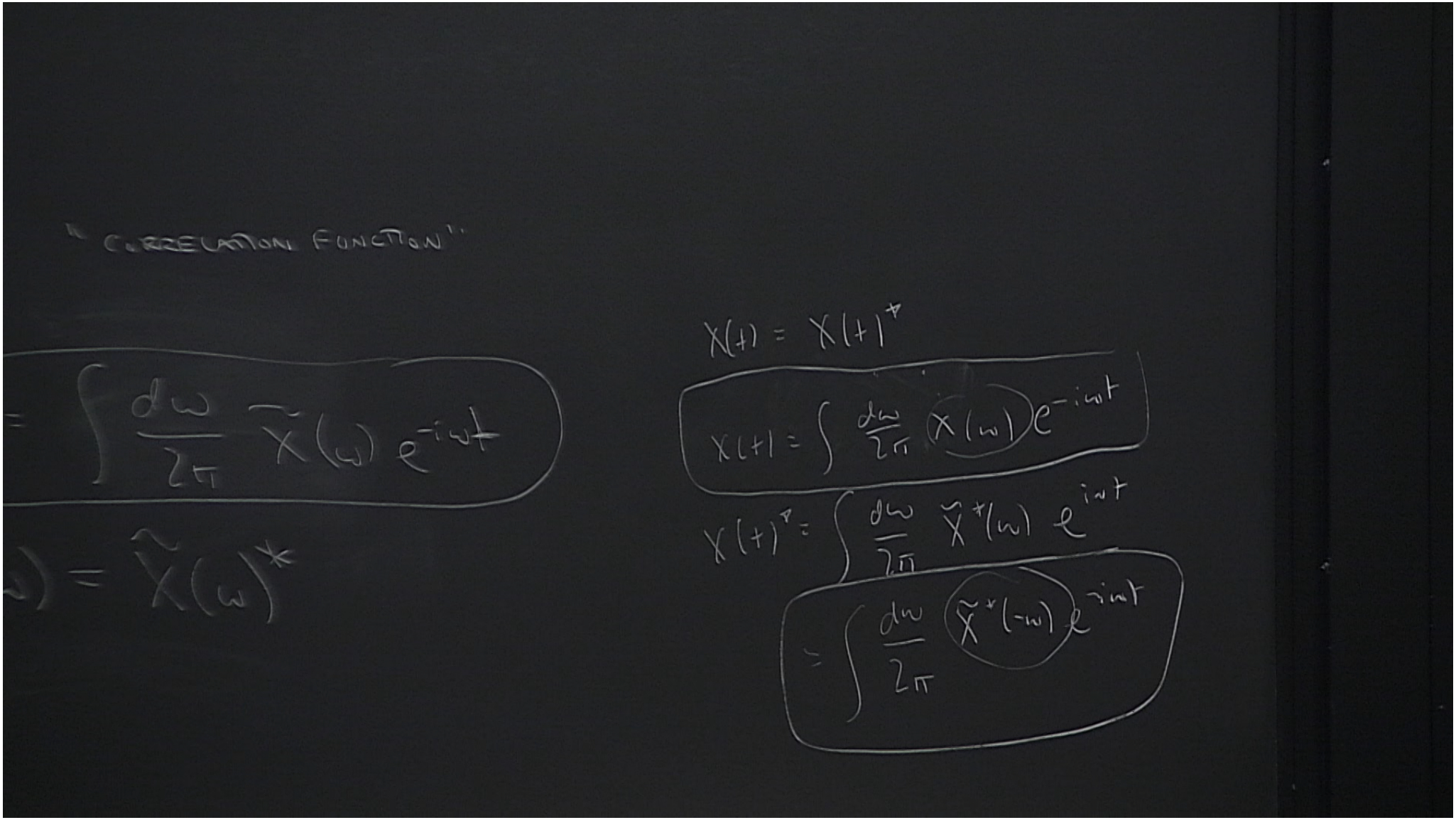
$X(t)$ ASSUME $\langle X(t) \rangle = 0$

$\langle X(t) X(t') \rangle = \xi(t - t')$ "CORRELATION FUNCTION"

$= \int dt\, X(t) e^{i\omega t}$

$$X(t) = \int \frac{d\omega}{2\pi} \tilde{X}(\omega) e^{-i\omega t}$$

$X(t)$ REAL-VALUED $\iff$ $\tilde{X}(-\omega) = \tilde{X}(\omega)^*$


"CORRELATION FUNCTION"
$= \int \frac{d\omega}{2\pi} \tilde{X}(\omega) e^{-i\omega t}$
$\omega) = \tilde{X}(\omega)^*$
$X(t) = X(t)^*$
$X(t) = \int \frac{d\omega}{2\pi} X(\omega) e^{-i\omega t}$
$X(t)^* = \int \frac{d\omega}{2\pi} \tilde{X}^*(\omega) e^{i\omega t}$
$= \int \frac{d\omega}{2\pi} \tilde{X}^*(-\omega) e^{i\omega t}$

$$x(t) \to x(t+\tau)$$

$$\tilde{x}(\omega) \to e^{i\omega\tau}\,\tilde{x}(\omega)$$

$$\langle \tilde{x}(\omega)\,\tilde{x}(\omega') \rangle \to e^{i\omega\tau} e^{i\omega'\tau} \langle \tilde{x}(\omega)\,\tilde{x}(\omega') \rangle$$

$$= e^{i(\omega+\omega')\tau} \langle \tilde{x}(\omega)\,\tilde{x}(\omega') \rangle$$

$$\boxed{\langle \tilde{x}(\omega)\,\tilde{x}(\omega') \rangle = P(\omega)\,(2\pi)\,\delta(\omega+\omega')}$$

$P(\omega)$ → POWER SPECTRUM

MOST GENERAL TIME TRANSLATION INVARIANT 2PF

$\omega)\ (2\pi)\ \delta(\omega-\omega')$

$C_{ij} = \lambda_i \delta_{ij}$

WER SPECTRUM $P(\omega)$.

$\int X(\omega) e^{i\omega t}\Big)\Big(\int \frac{d\omega'}{2\pi} X(\omega') e^{i\omega' t'}\Big) >$

$\frac{d\omega'}{2\pi} e^{i\omega t} e^{i\omega' t'} \langle X(\omega) X(\omega') \rangle$

$\frac{d\omega'}{2\pi} e^{i\omega t} e^{i\omega' t'} (2\pi)\delta(\omega+\omega') P(\omega) = \int \frac{d\omega}{2\pi} e^{i\omega t} e^{-i\omega t'} P(\omega)$

$= \int \frac{d\omega}{2\pi} e^{i\omega(t-t')} P(\omega) = \delta(t-t')$

$$\xi(t) = \int \frac{d\omega}{2\pi} P(\omega) e^{i\omega t}$$

"WIENER – KHINCHIN THEOREM": CORR FN $\xi$ AND POWER SPECTRUM $P(\omega)$ ARE FOURIER TRANSFORMS

WHITE NOISE: $\langle X(t) X(t') \rangle = \eta \, \delta(t - t')$

$$\xi(\tau) = \eta \, \delta(\tau)$$

$$P(\omega) = \eta$$

$X(t)$ GAUSSIAN w/ POWER SPECTRUM $P(\omega)$

$X'(t)$ IS ALSO A GAUSSIAN RANDOM FIELD.

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$= \langle X_i X_j \rangle - \langle X_i \rangle \langle X_j \rangle$

$\begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$

LINEAR COMBINATIONS OF GAUSSIAN R.V.S ARE ALSO GAUSSIAN RANDOM

$Y_a = A_{ai} X_i$

$$\xi(\tau) = \eta\,\delta(\tau)$$

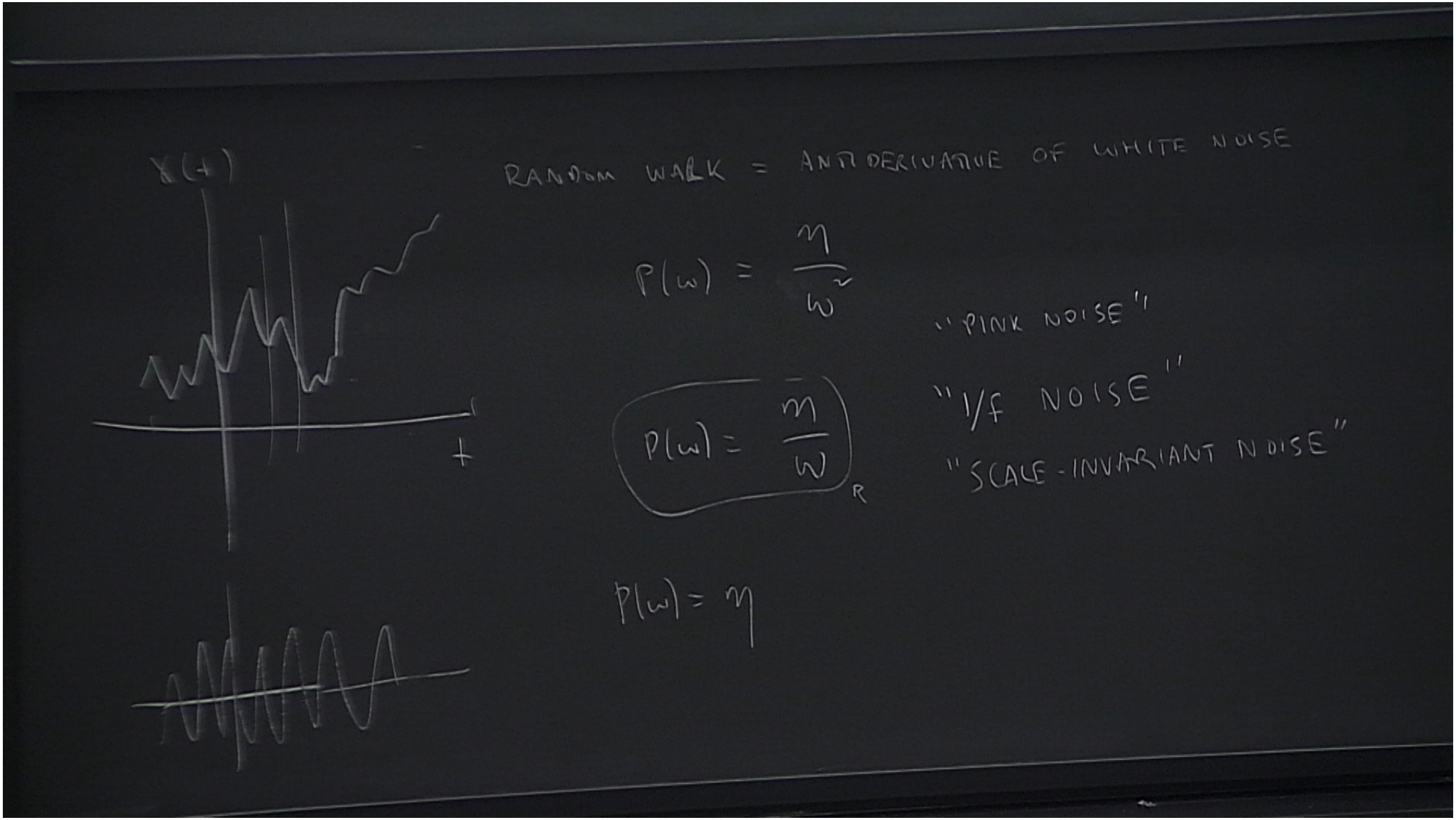
$$P(\omega) = \eta$$

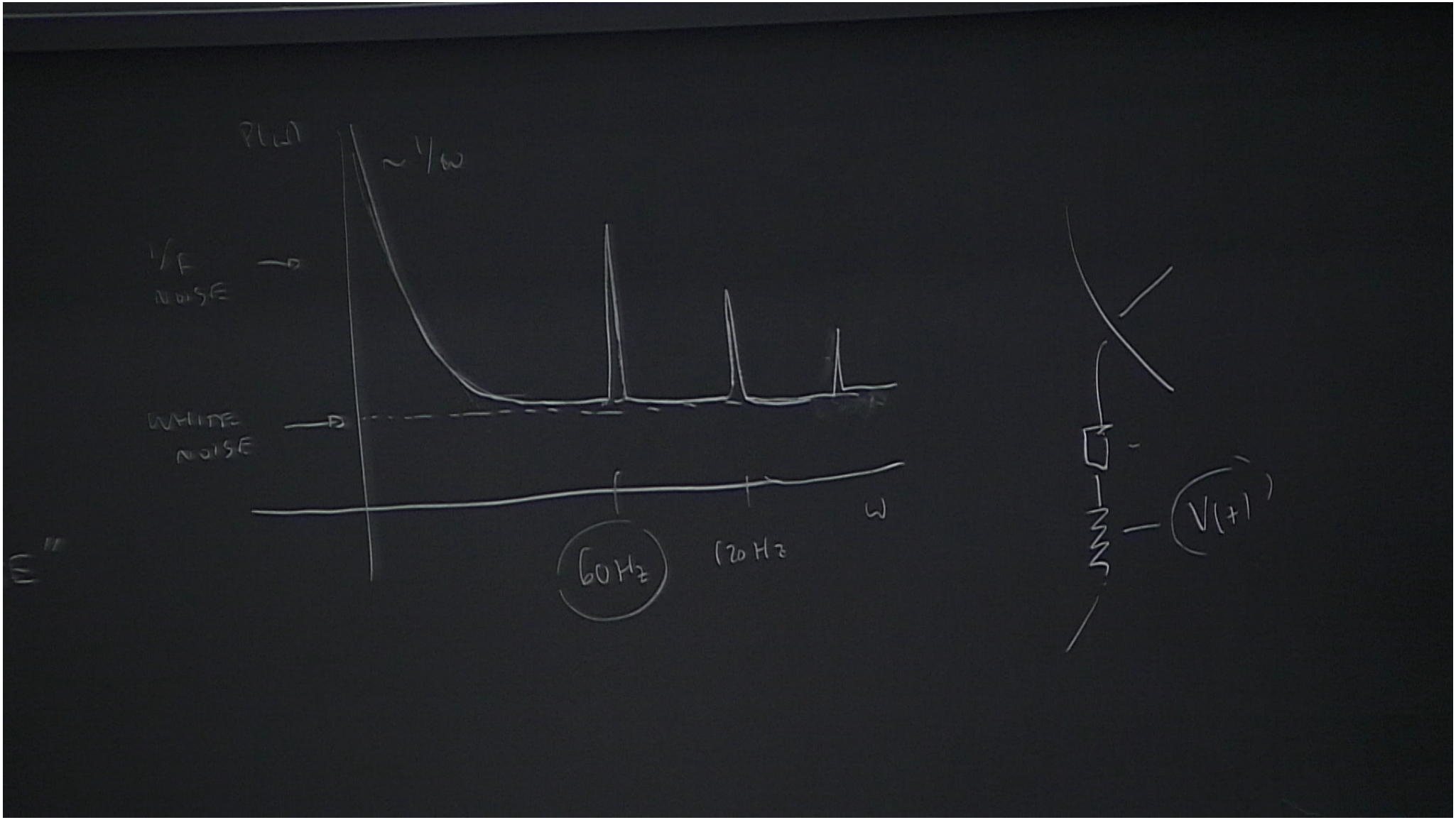
$X(t)$ GAUSSIAN w/ POWER SPECTRUM $P(\omega)$

$Y(t) = X'(t)$ IS ALSO A GAUSSIAN RANDOM FIELD.

$$\tilde{Y}(\omega) = i\omega\, X(\omega)$$

$$P_Y(\omega) = \omega^2 P_X(\omega)$$


x(t)
t
RANDOM WALK = ANTIDERIVATIVE OF WHITE NOISE
$P(\omega) = \frac{\eta}{\omega^2}$
"PINK NOISE"
$P(\omega) = \frac{\eta}{\omega}$
"1/f NOISE"
"SCALE-INVARIANT NOISE"
R
$P(\omega) = \eta$

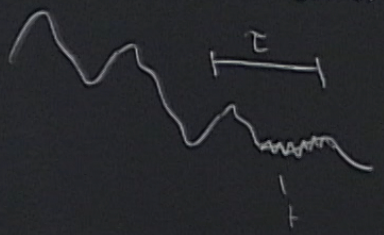

P(ω)
~ 1/ω
1/f NOISE
WHITE NOISE
ω
60 Hz
120 Hz
V(t)

$X(t)$ GAUSSIAN R.F. W/ POWER SPECTRUM $P(\omega)$

$Y(t)$ = "BOXCAR-AVERAGE" OF $X$

$$= \int_{t-\tau/2}^{t+\tau/2} dt' \; X(t')$$

WHAT IS $P_Y(\omega)$ IN TERMS OF $P_X(\omega)$?



$$Y(t) = \int_{t-\tau/2}^{t+\tau/2} dt' \; X(t')$$

$$= \int_{t-\tau/2}^{t+\tau/2} dt' \; \frac{d\omega}{2\pi} \tilde{X}(\omega) \, e^{i\omega t'}$$

$$= \int \frac{d\omega}{2\pi} \tilde{X}(\omega) \int_{t-\tau/2}^{t+\tau/2} dt' \; e^{i\omega t'}$$

$$= \int \frac{d\omega}{2\pi} \tilde{X}(\omega) \frac{\sin(\omega\tau/2)}{(\omega\tau/2)} e^{i\omega t}$$

$$\tilde{Y}(\omega) = \frac{\sin(\omega\tau/2)}{(\omega\tau/2)} \tilde{X}(\omega)$$

$$P_y(\omega) = \left[\frac{\sin(\omega\tau/2)}{(\omega\tau/2)}\right]^2 P_x(\omega)$$

$\to 1 \quad \omega \ll \tau^{-1}$

$\to 0 \quad \omega \gg \tau^{-1}$

$$\frac{d\omega}{2\pi}\,\tilde{X}(\omega)\,e^{i\omega t'}$$

$$\int_{t-\tau/2}^{t+\tau/2} dt'\, e^{i\omega t'}$$

$$\frac{\sin(\omega\tau/2)}{(\omega\tau/2)}\,e^{i\omega t}$$

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