Title: PSI 16/17 Explorations in Cosmology (Kendrick Smith) - Lecture 1

Date: Apr 10, 2017 10:15 AM

URL: http://pirsa.org/17040053

Abstract:

STANDARD COSMOLOGICAL MODEL

- BACKGROUND METRIC IS FRW
- UNIVERSE CONSISTS OF
  - "BARYONIC" MATTER (4.9%)
  - DARK MATTER (26%)
  - COSMOLOGICAL CONSTANT (69%)
  - PHOTONS (0.005%)
  - NEUTRINOS (0.1 – 0.5%)
- INITIAL PERTURBATIONS ARE
  - GAUSSIAN RANDOM
  - ADIABATIC SCALAR
  - WITH A NEARLY SCALE-INVARIANT POWER SPECTRUM

6 PARAMETERS

$\Omega_b = 0.0486 \pm 0.0007$

$\Omega_\Lambda = 0.691 \pm 0.006$

$h = 0.677 \pm 0.005$

$\Delta^2_S = (2.11 \pm 0.05) \times 10^{-9}$

$n_S = 0.967 \pm 0.004$

$\tau = 0.058 \pm 0.012$

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6 PARAME

$\Omega_b = 0.$

$\Omega_\Lambda = 0.$

$h = 0$

$\Delta_S^2 = ($

$n_S = ($

$\tau = 0.$

6 PARAMETERS

$\Omega_b = 0.0486 \pm 0.0007$

$\Omega_\Lambda = 0.691 \pm 0.006$

} BACKGROUND

$h = 0.677 \pm 0.005$

$\Delta^2_S = (2.11 \pm 0.05) \times 10^{-9}$

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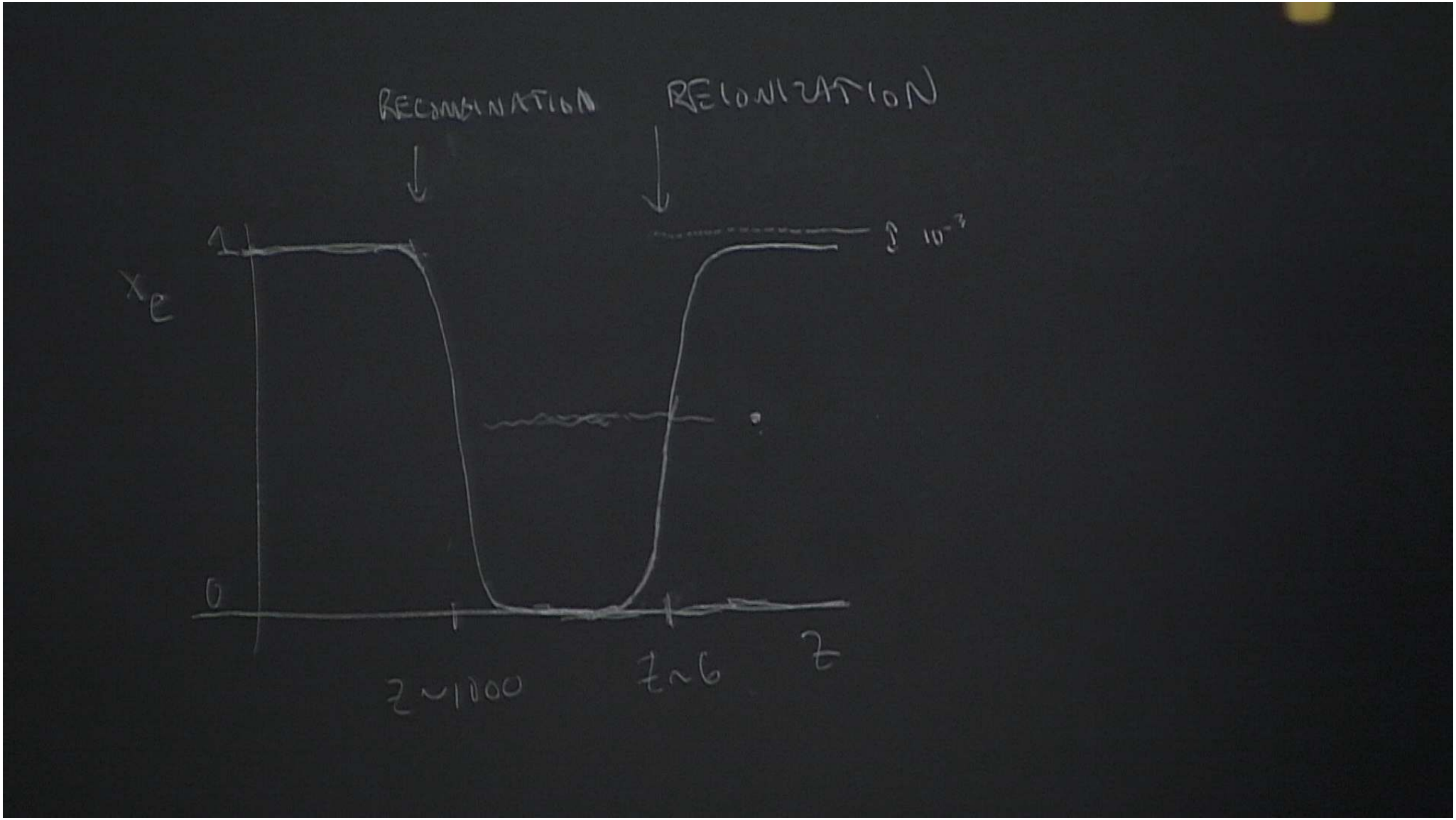
} PERTURBATIONS

$\tau = 0.058 \pm 0.012$

} NUISANCE
OPTICAL DEPTH TO CMB


RECOMBINATION
REIONIZATION
1
x_e
10^{-3}
0
$z \sim 1100$
$z \sim 6$
z

FRW METRIC

$$ds^2 = -dt^2 + a(t)^2 dx^2$$

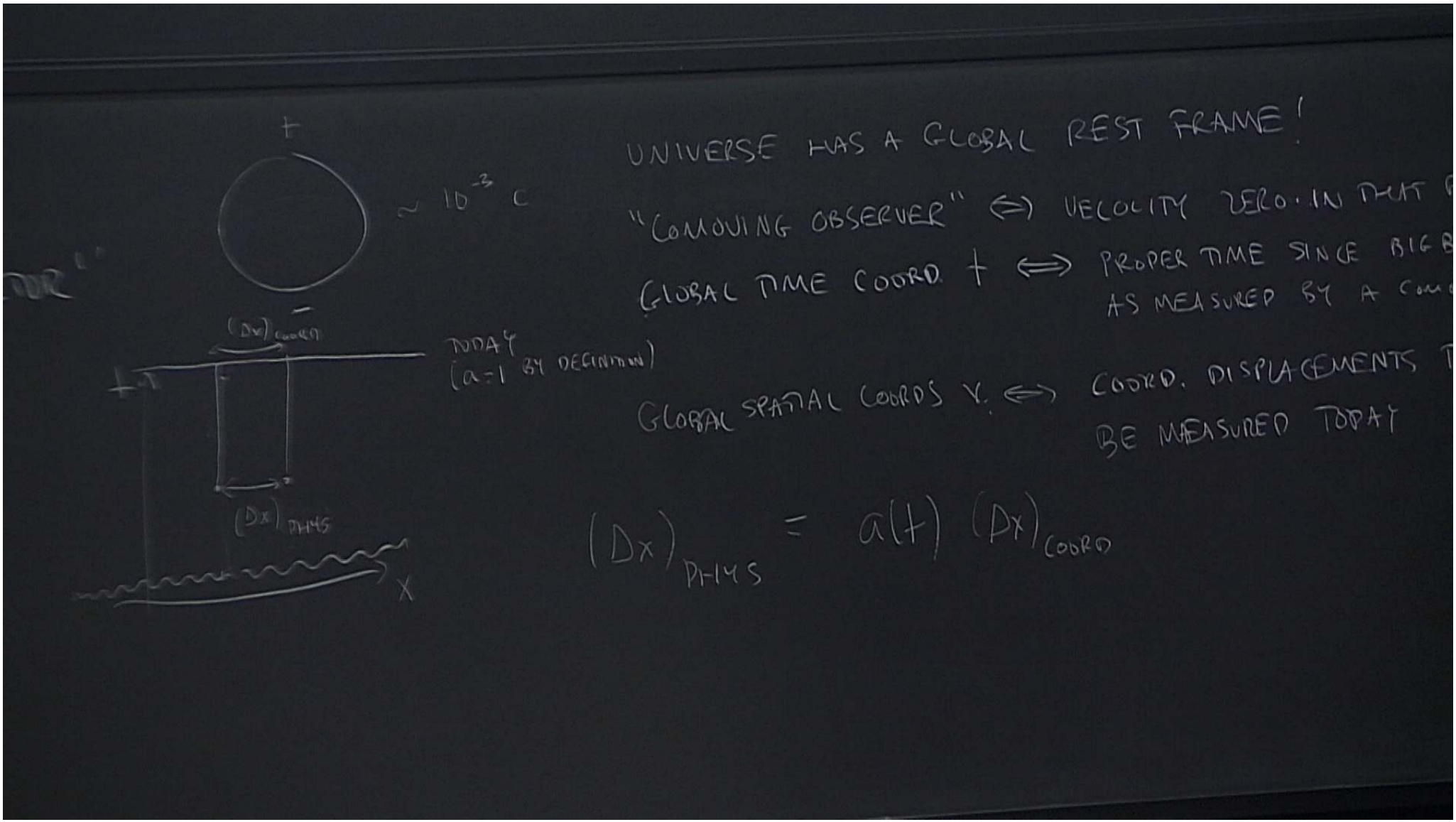
$a(t)$ = "SCALE FACTOR"

SYMMETRIES:

- ROTATIONS IN 3D
- SPATIAL TRANSLATIONS IN 3D
- NO BOOSTS ✱ ⇒ PREFERRED REST FRAME
- NO TIME TRANSLATION

MINKOWSKI $(a = \text{CONST.})$ HAS MORE SYMMETRY

$a = e^{Ht}$ "DE SITTER"



UNIVERSE HAS A GLOBAL REST FRAME!

"COMOVING OBSERVER" ⇔ VELOCITY ZERO IN THAT

GLOBAL TIME COORD. t ⇔ PROPER TIME SINCE BIG B
AS MEASURED BY A COM

GLOBAL SPATIAL COORDS X ⇔ COORD. DISPLACEMENTS
BE MEASURED TODAY

$$(\Delta x)_{PHYS} = a(t)\,(\Delta x)_{COORD}$$

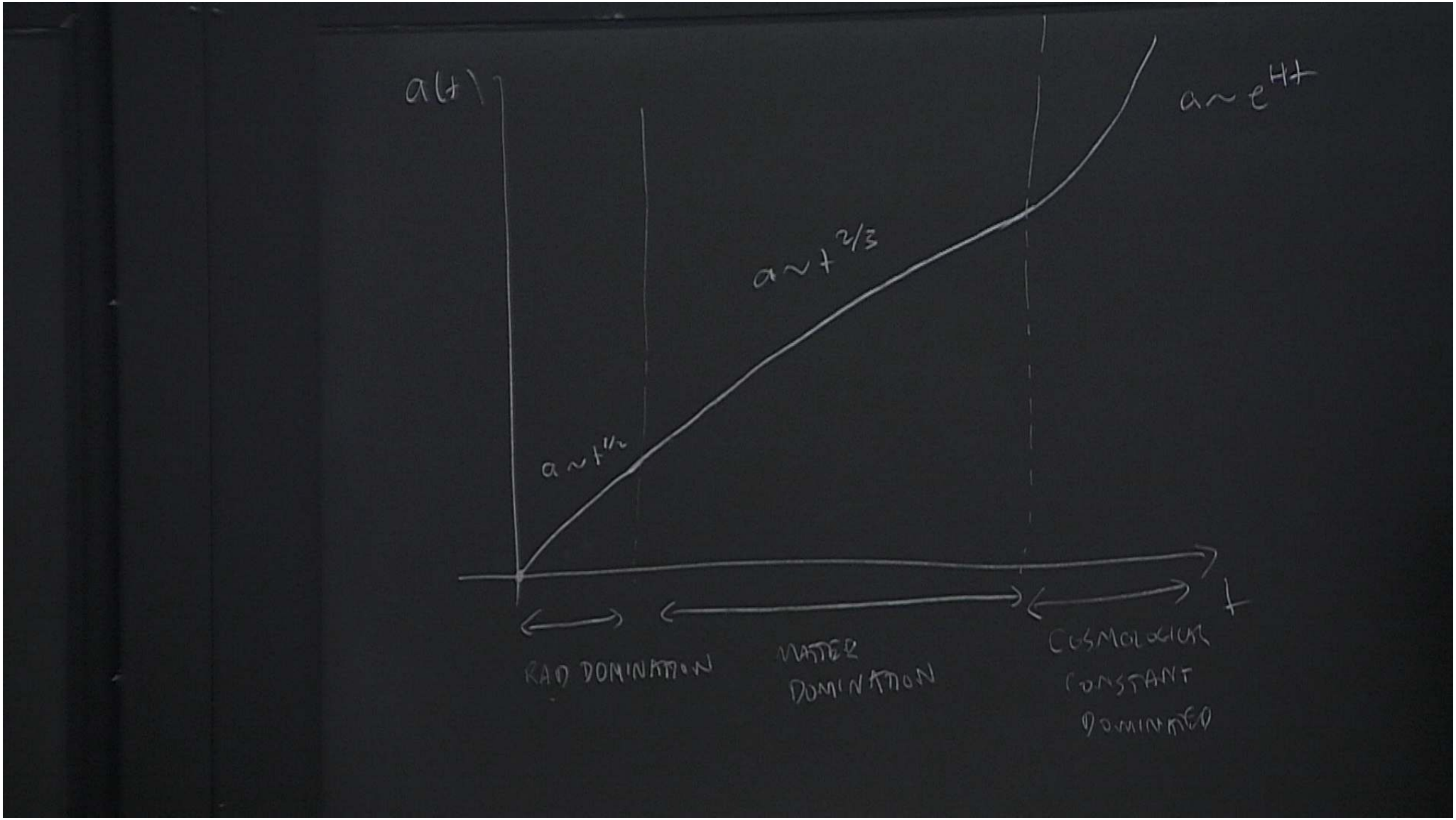
UNIVERSE HAS A GLOBAL REST FRAME!

"COMOVING OBSERVER" $\Leftrightarrow$ VELOCITY ZERO IN THAT FRAME

GLOBAL TIME COORD. t $\Leftrightarrow$ PROPER TIME SINCE BIG BANG, AS MEASURED BY A COMOVING OBSERVER

GLOBAL SPATIAL COORDS $\vec{x}$ $\Leftrightarrow$ COORD. DISPLACEMENTS THAT WOULD BE MEASURED TODAY ["COMOVING COORDINATES"]

$$(\Delta x)_{PHYS} = a(t)\,(\Delta x)_{COORD}$$


a(t)
$a \sim e^{Ht}$
$a \sim t^{2/3}$
$a \sim t^{1/2}$
t
RAD DOMINATION
MATTER DOMINATION
COSMOLOGICAL CONSTANT DOMINATED

STRESS-ENERGY

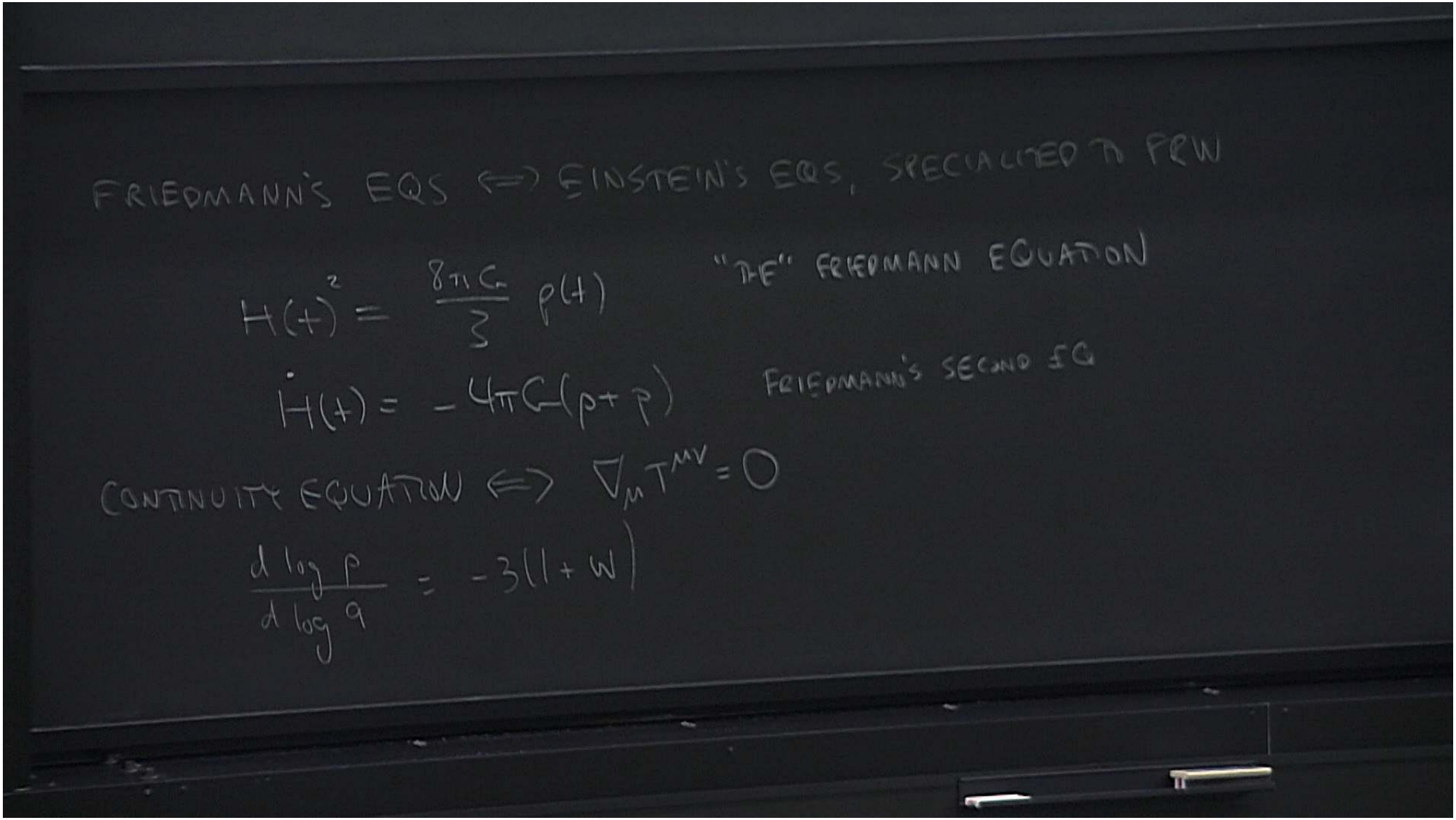
$$T_{\mu\nu} = \begin{pmatrix} \rho(t) & & \\ & a^2 p(t) \delta_{ij} & \end{pmatrix}$$

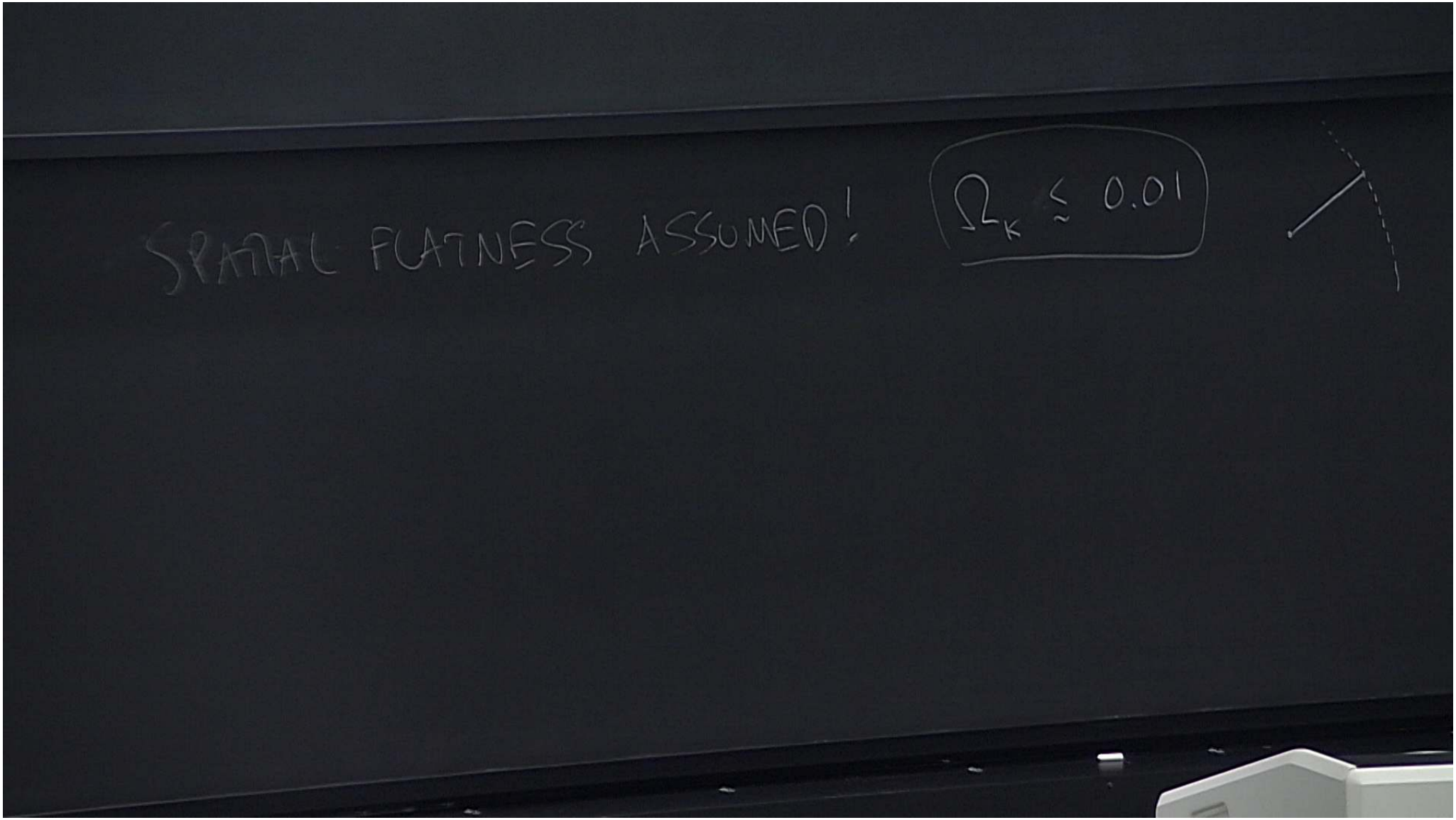
$\rho(t)$ = PHYSICAL ENERGY DENSITY

$p(t)$ = PRESSURE

$$w(t) = \frac{p(t)}{\rho(t)}$$ "EQUATION OF STATE"

$$H(t) = \frac{1}{a}\frac{da}{dt}$$ "HUBBLE PARAMETER"


FRIEDMANN'S EQS ⟺ EINSTEIN'S EQS, SPECIALIZED TO FRW
$H(t)^2 = \frac{8\pi G}{3}\rho(t)$
"1st" FRIEDMANN EQUATION
$\dot{H}(t) = -4\pi G(\rho + p)$
FRIEDMANN'S SECOND EQ
CONTINUITY EQUATION ⟺ $\nabla_\mu T^{\mu\nu} = 0$
$\frac{d\log\rho}{d\log a} = -3(1+w)$


SPATIAL FLATNESS ASSUMED!
$\Omega_k \lesssim 0.01$

FRIEDMANN'S EQS $\Leftrightarrow$ EINSTEIN'S EQS, SPECIALIZED TO FRW

$$H(t)^2 = \frac{8\pi G}{3}\rho(t)$$ "THE" FRIEDMANN EQUATION

$$\dot{H}(t) = -4\pi G(\rho + p)$$ FRIEDMANN'S SECOND EQ

CONTINUITY EQUATION $\Leftrightarrow \nabla_\mu T^{\mu\nu} = 0$

$$\frac{d\log\rho}{d\log a} = -3(1+w)$$

SPATIAL FLATNESS ASSUMED! $\Omega_k \lesssim 0.01$

TO CLOSE DIFFERENTIAL EQS, NEED TO KNOW HOW $p(t)$ EVOLVES

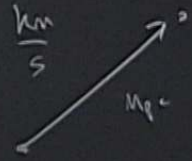
SIMPLEST EXAMPLE: "CONSTANT-w UNIVERSE", $w = \frac{p(t)}{\rho(t)}$ IS INDEPENDENT OF TIME

$$w = \begin{cases} \frac{1}{3} & \text{"RADIATION"} \\ 0 & \text{"MATTER"} \\ -1 & \text{COSMOLOGICAL CONSTANT} \end{cases} \Rightarrow \rho(a) \propto a^{-3(1+w)}$$

ΛCDM: $\rho(a) = \rho_{r0}\, a^{-4} + \rho_{m0}\, a^{-3} + \Lambda$

$\rho(a) = \frac{1}{3}\rho_{r0}\, a^{-4} + O(a^{-3}) - \Lambda$

COSMOLOGIST'S PARAMETRIZATION

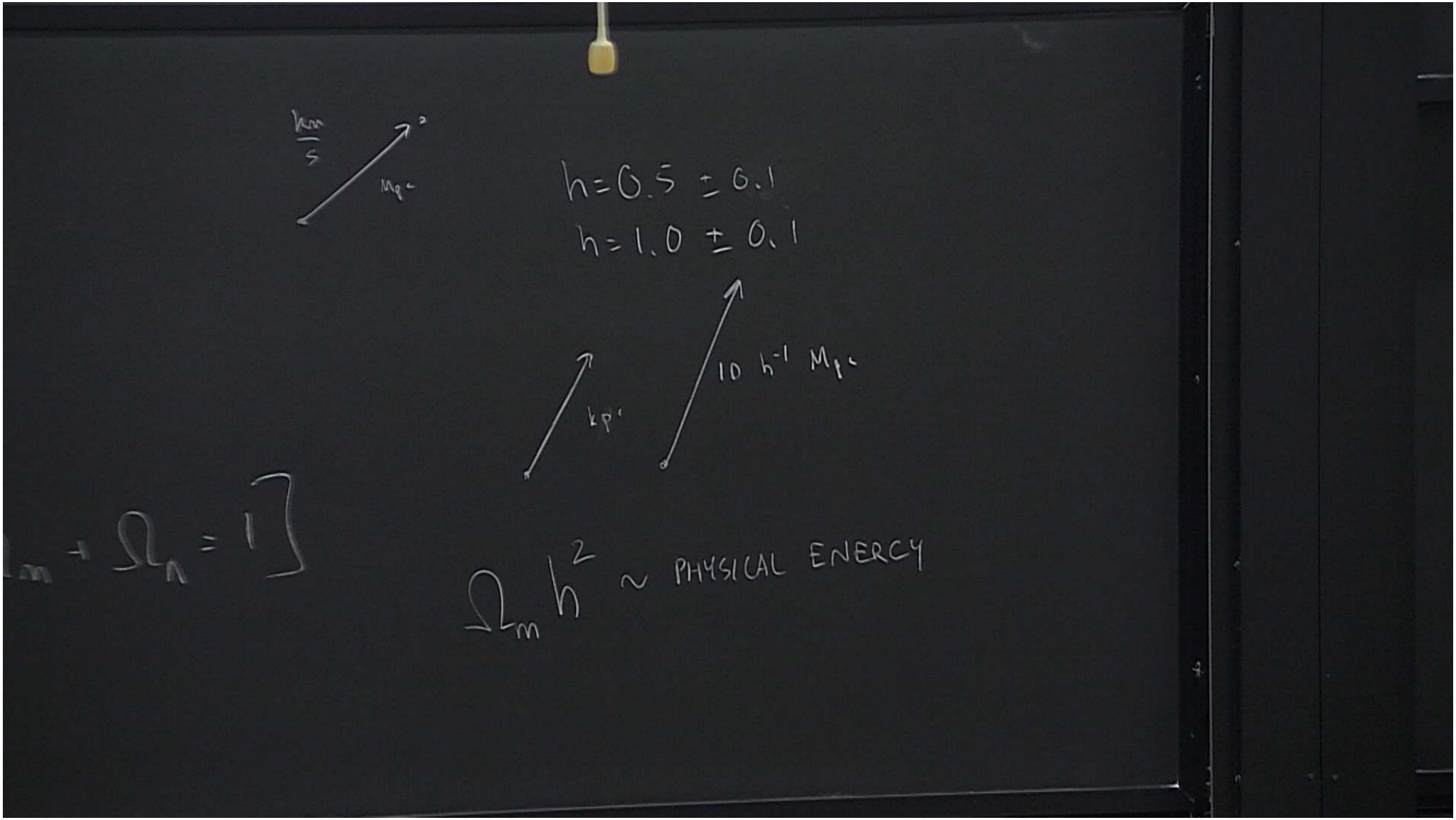


$$H_0 = 100h \ \text{km s}^{-1} \text{Mpc}^{-1}$$

$$\Longleftrightarrow \rho_{tot,0}$$

PARAMETERS $\Omega_{rad}, \Omega_m, \Omega_\Lambda$

$$\Omega_i \overset{DEF}{=} \frac{\rho_{i,0}}{\rho_{tot,0}} \qquad [\Omega_r + \Omega_m + \Omega_\Lambda = 1]$$


km/s
Mpc
h=0.5 ± 0.1
h=1.0 ± 0.1
kpc
10 h⁻¹ Mpc
Ωm + ΩΛ = 1
Ωm h² ~ PHYSICAL ENERGY

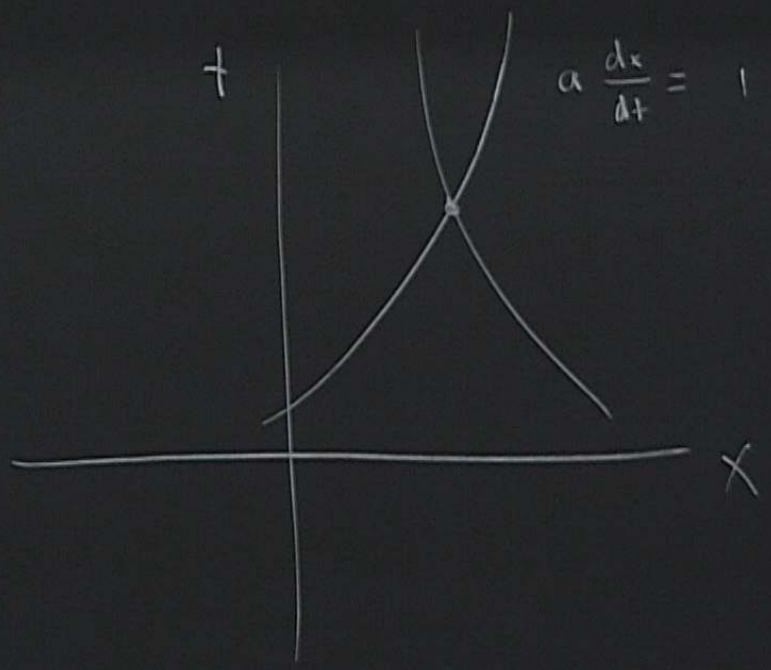
TIME COORDINATE $t \longleftrightarrow$ TIME COORD. a

$$H(a)^2 = \frac{3}{8\pi G}\left(\rho_{r0}\, a^{-4} + \rho_{m0}\, a^{-3} + \Lambda\right)$$

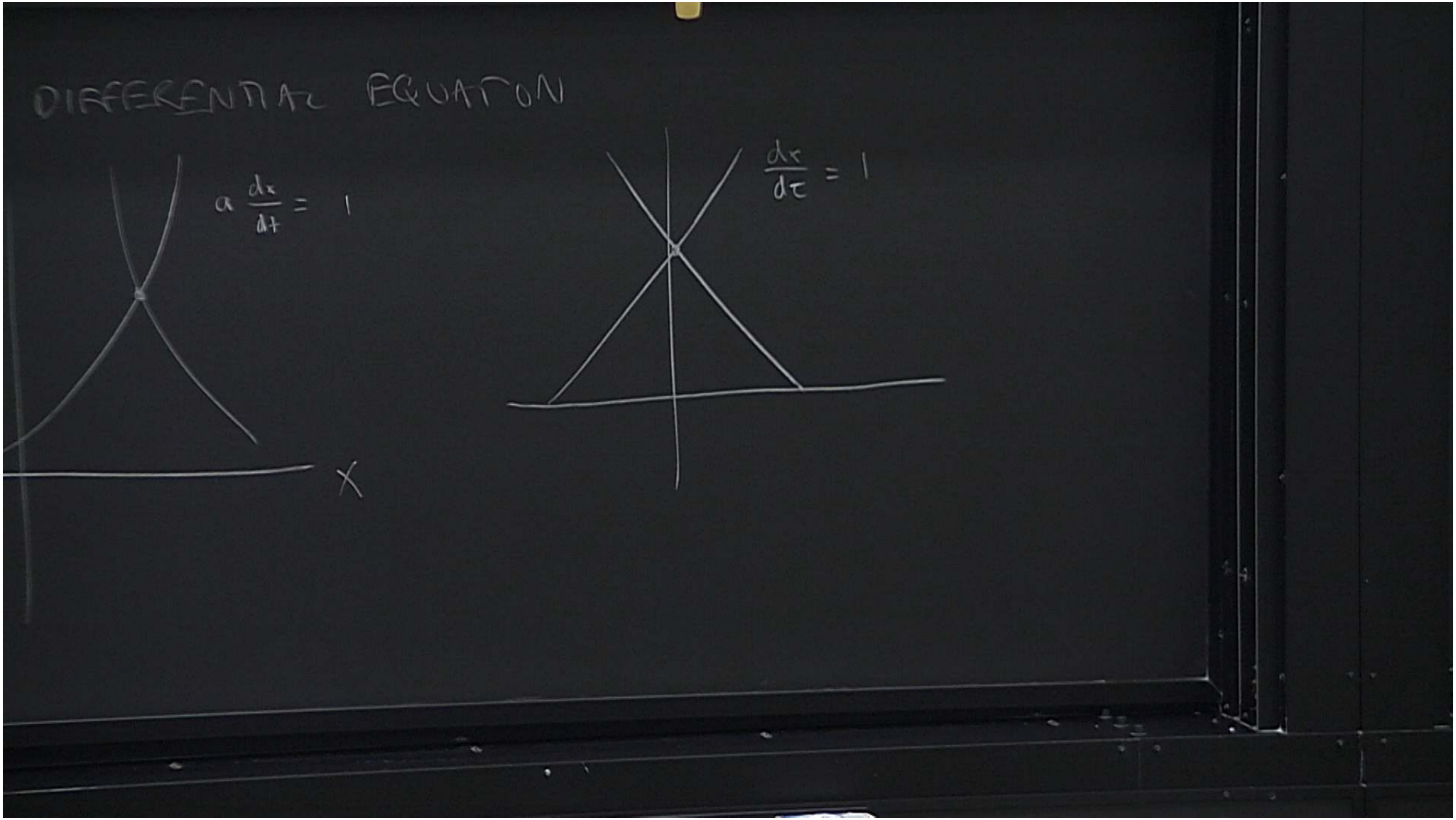
$$= H_0^2\left(\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda\right)$$

CONFORMAL TIME τ $\Leftrightarrow$ DEFINED BY DIFFERENTIAL EQUATION

$$\frac{d\tau}{dt} = \frac{1}{a(t)}$$



$$a\,\frac{dx}{dt} = 1$$


DIFFERENTIAL EQUATION
$a\frac{dx}{dt} = 1$
X
$\frac{dx}{dt} = 1$

TIME COORDINATE $t \longleftrightarrow$ TIME COORD. a CONFORMA

$$H(a)^2 = \frac{3}{8\pi G}\left(\rho_{r0}\, a^{-4} + \rho_{m0}\, a^{-3} + \Lambda\right)$$

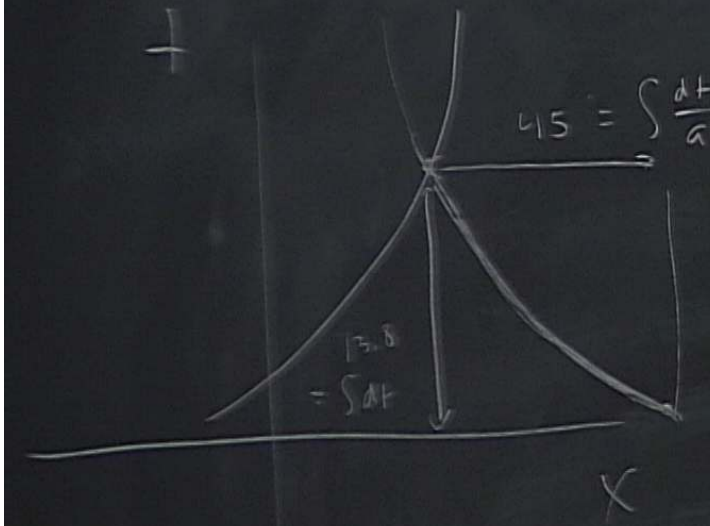
$$= H_0^2\left(\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda\right)$$

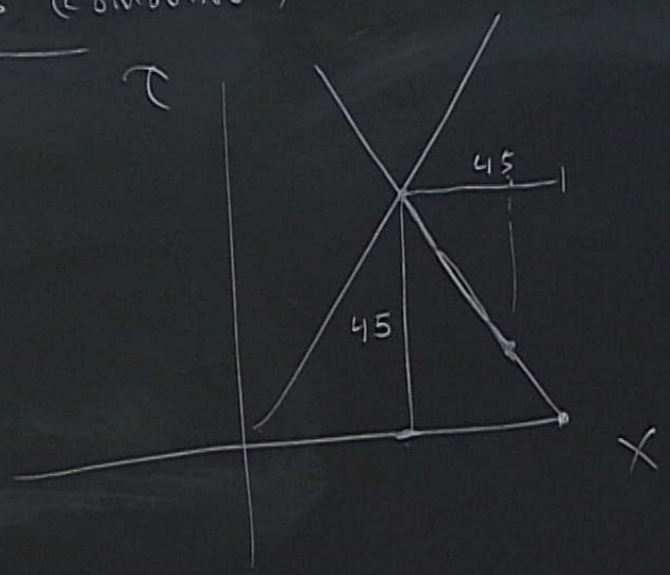
$$ds^2 = -dt^2 + a(t)^2 dx^2$$

$$= a(\tau)^2\left[-d\tau^2 + dx^2\right]$$

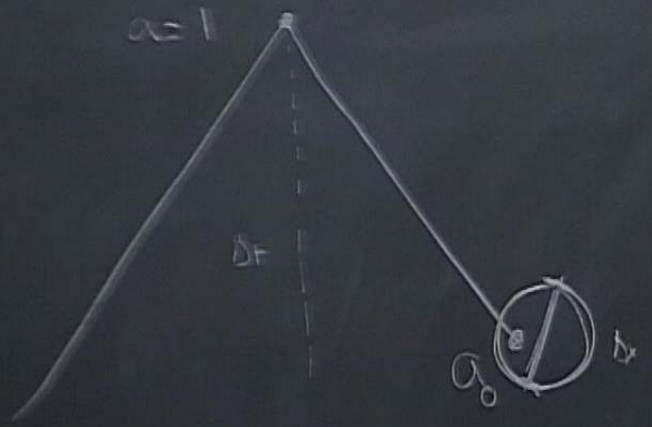
AGE = 13.8 BILLION YEARS

SIZE = 45 BILLION LIGHT YEARS (COMOVING) = CONFORMAL AGE OF UNIVERSE





$\rho(t)$ = PHYSICAL ENERGY DENSITY



$(\Delta x)_{PHYS}$ = PHYSICAL SIZE OF STANDARD RULER

$(\Delta x)_{COORD} = a^{-1}(\Delta x)_{PHYS}$

(Δt) = ELAPSED TIME

$(\Delta \tau)$ = ELAPSED CONFORMAL TIME

WHAT ANGLE $(\Delta\theta)$ DOES THE RULER SPAN ON THE SKY?

$$\Delta\theta = \frac{(\Delta x)_{PHYS}}{c\Delta t} \quad \frac{(\Delta x)_{COORD}}{c\Delta t} \quad \frac{(\Delta x)_{PHYS}}{c(\Delta\tau)} \quad \frac{(\Delta x)_{COORD}}{c(\Delta\tau)}$$