

Title: PSI 16/17 Explorations in Cosmology (Kendrick Smith) - Lecture 1

Date: Apr 10, 2017 10:15 AM

URL: <http://pirsa.org/17040053>

Abstract:

STANDARD COSMOLOGICAL MODEL

- BACKGROUND METRIC IS FRW
- UNIVERSE CONSISTS OF
 - "BARYONIC" MATTER (4.9%)
 - DARK MATTER (26%)
 - COSMOLOGICAL CONSTANT (69%)
 - PHOTONS (0.005%)
 - NEUTRINOS (0.1 - 0.5%)
- INITIAL PERTURBATIONS ARE
 - GAUSSIAN RANDOM
 - ADIABATIC SCALAR
 - WITH A NEARLY SCALE-INVARIANT POWER SPECTRUM

6 PARAMETERS

$$\Omega_b = 0.0486 \pm 0.0007$$

$$\Omega_m = 0.691 \pm 0.006$$

$$h = 0.677 \pm 0.005$$

$$\Delta_s^2 = (2.11 \pm 0.05) \times 10^{-9}$$

$$n_s = 0.967 \pm 0.004$$

$$\tau = 0.058 \pm 0.012$$

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Λ CDM PARAMETERS

$$\Omega_b = 0.049$$

$$\Omega_m = 0.26$$

$$h = 0.69$$

$$\Delta_S^2 = 1$$

$$n_s = 0.96$$

$$\tau = 0.08$$

6 PARAMETERS

$$\Omega_b = 0.0486 \pm 0.0007$$

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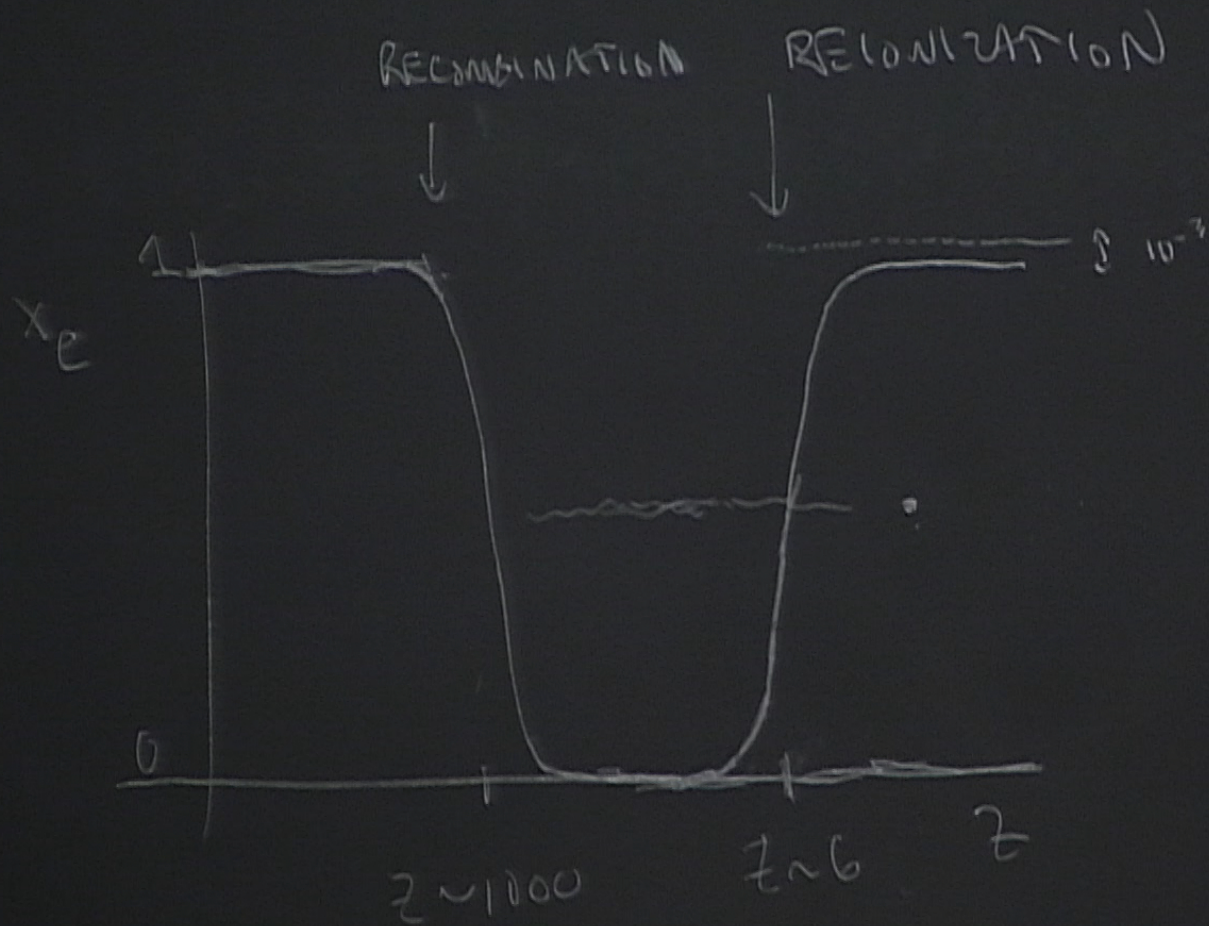
$$\tau = 0.058 \pm 0.012$$

BACKGROUND

PERTURBATIONS

NUISANCE

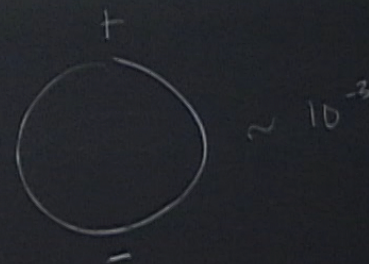
OPTICAL DEPTH TO CMB



FRW METRIC

$$ds^2 = -dt^2 + a(t)^2 dx^2$$

$a(t)$ = "SCALE FACTOR"



SYMMETRIES:

- ROTATIONS IN 3D
- SPATIAL TRANSLATIONS IN 3D
- NO BOOSTS $\nrightarrow$ PREFERRED REST FRAME
- NO TIME TRANSLATION

MINIKOWSKI | $\left(\begin{array}{l} a = \text{const.} \\ H \neq 0 \end{array} \right)$ HAS NO DE SYMMETRY
 $a = e^{Ht}$ "DE SITTER"



$\sim 10^{-3} c$

TODAY
($a=1$ BY DEFINITION)

UNIVERSE HAS A GLOBAL REST FRAME!

"COMOVING OBSERVER" $\Leftrightarrow$ VELOCITY ZERO IN THAT
GLOBAL TIME COORD. $t \Leftrightarrow$ PROPER TIME SINCE BIG BANG
AS MEASURED BY A COMOVING OBSERVER

GLOBAL SPATIAL COORDS $x \Leftrightarrow$ COORD. DISPLACEMENTS
BE MEASURED TODAY

$$(Dx)_{\text{PHYS}} = a(t) (Dx)_{\text{COORD}}$$

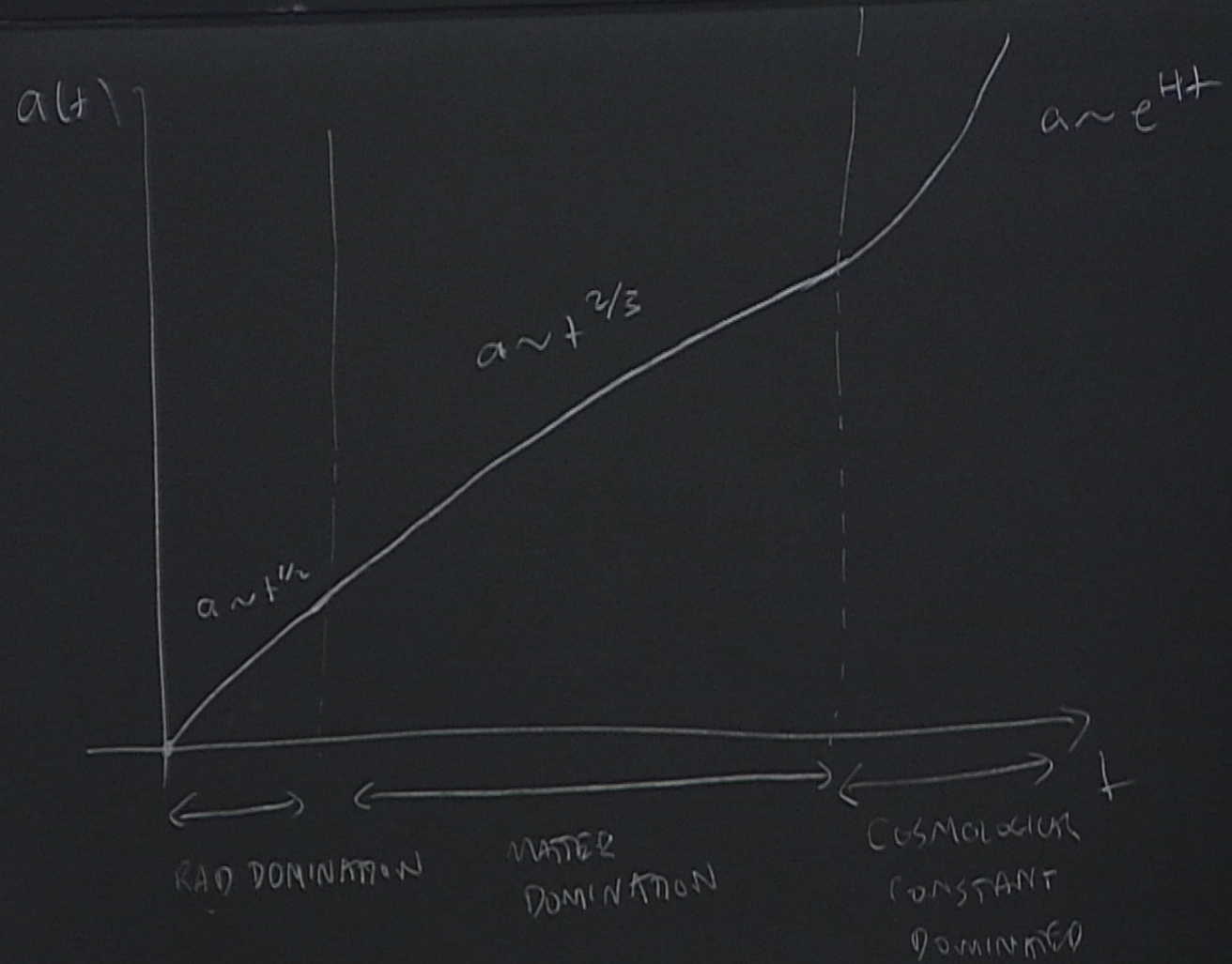
UNIVERSE HAS A GLOBAL REST FRAME!

"COMOVING OBSERVER" $\Leftrightarrow$ VELOCITY ZERO IN THAT FRAME

GLOBAL TIME COORD. $t \Leftrightarrow$ PROPER TIME SINCE BIG BANG,
AS MEASURED BY A COMOVING OBSERVER

GLOBAL SPATIAL COORDS $x \Leftrightarrow$ COORD. DISPLACEMENTS THAT WOULD
BE MEASURED TODAY ["COMOVING COORDINATES"]

$$(Dx)_{\text{PHYS}} = a(t) (Dx)_{\text{COORD}}$$



STRESS-ENERGY

$$T_{\mu\nu} = \begin{pmatrix} \rho(t) & \\ & a^2 p(t) \delta_{ij} \end{pmatrix}$$

$\rho(t)$ = PHYSICAL ENERGY DENSITY

$p(t)$ = PRESSURE

$$w(t) = \frac{\rho(t)}{p(t)}$$

"EQUATION OF STATE"

$$H(t) = \frac{1}{a} \frac{da}{dt}$$

"HUBBLE PARAMETER"

FRIEDMANN'S EQS $\Leftrightarrow$ EINSTEIN'S EQS, SPECIALIZED TO FRW

$$H(t)^2 = \frac{8\pi G}{3} \rho(t)$$

"1-F" FRIEDMANN EQUATION

$$\dot{H}(t) = -4\pi G(\rho + p)$$

FRIEDMANN'S SECOND EQ

CONTINUITY EQUATION $\Leftrightarrow \nabla_\mu T^{\mu\nu} = 0$

$$\frac{d \log \rho}{d \log a} = -3(1+w)$$

SPATIAL FLATNESS ASSUMED!

$$\Omega_k \lesssim 0.01$$

FRIEDMANN'S EQS $\Leftrightarrow$ EINSTEIN'S EQS, SPECIALIZED TO FRW

$$\star \quad H(t)^2 = \frac{8\pi G}{3} \rho(t)$$

"1st" FRIEDMANN EQUATION

$$\dot{H}(t) = -4\pi G(\rho + p)$$

FRIEDMANN'S SECOND EQ

CONTINUITY EQUATION $\Leftrightarrow \nabla_\mu T^{\mu\nu} = 0$

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SPATIAL FLATNESS ASSUMED!

$$\Omega_k \lesssim 0.01$$

TO SOLVE DIFFERENTIAL EOS, NEED TO KNOW HOW $p(t)$ EVOLVES

SIMPLEST EXAMPLE: "CONSTANT- w UNIVERSE",

$$w = \frac{p(t)}{\rho(t)} \text{ IS INDEPENDENT OF TIME}$$

$$w = \begin{cases} \frac{1}{3} \\ 0 \\ -1 \end{cases}$$

"RADIATION"

"MATTER"

"COSMOLOGICAL CONSTANT"

$$\Rightarrow \rho(a) \propto a^{-3(1+w)}$$

$$\Lambda\text{CDM:} \quad p(a) = \overset{\downarrow}{\rho_{r0}} a^{-4} + \overset{\downarrow}{\rho_{m0}} a^{-3} + \overset{\downarrow}{\Lambda}$$

$$p(a) = \frac{1}{3} \rho_{r0} a^{-4} + O(a^{-3}) - \Lambda$$

COSMOLOGIST'S PARAMETRIZATION

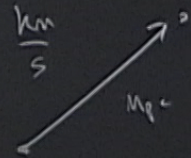
$$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$\Leftrightarrow \rho_{\text{tot},0}$$

PARAMETERS $\Omega_{\text{rad}}, \Omega_m, \Omega_\Lambda$

$$\Omega_i \stackrel{\text{DEF}}{=} \frac{\rho_{i,0}}{\rho_{\text{tot},0}}$$

$$[\Omega_r + \Omega_m + \Omega_\Lambda = 1]$$



$$\frac{km}{s} \rightarrow Mpc$$

$$h = 0.5 \pm 0.1$$

$$h = 1.0 \pm 0.1$$

$$kp' \rightarrow 10 h^{-1} Mpc$$

$$\Omega_m + \Omega_\Lambda = 1$$

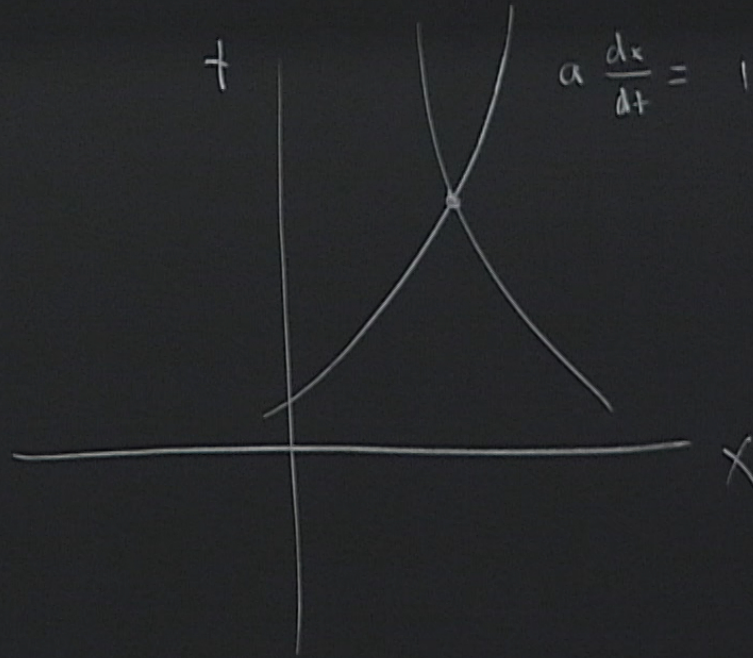
$$\Omega_m h^2 \sim \text{PHYSICAL ENERGY}$$

TIME COORDINATE $\leftrightarrow$ TIME COORD. a

$$H(a)^2 = \frac{3}{8\pi G} (p_{ro} a^{-4} + p_{mo} a^{-3} + \Lambda)$$
$$= H_0^2 (\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda)$$

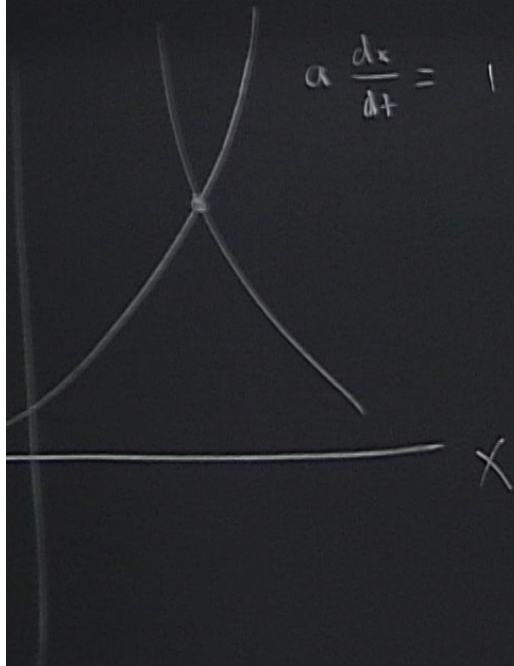
CONFORMAL TIME $\tau \Leftrightarrow$ DEFINED BY DIFFERENTIAL EQUATION

$$\frac{d\tau}{dt} = \frac{1}{a(t)}$$

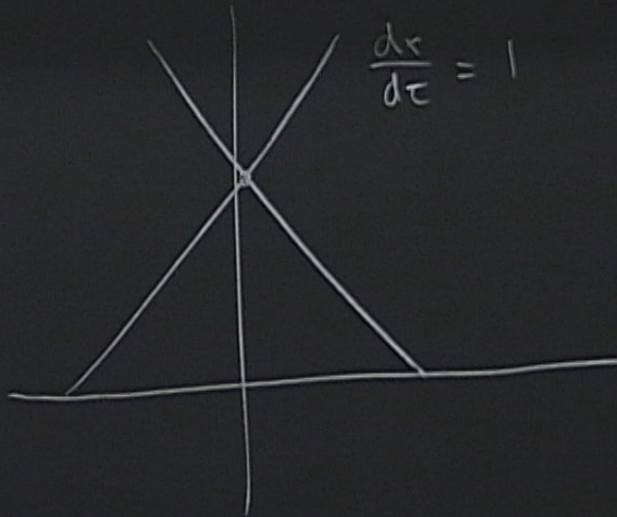


DIFFERENTIAL EQUATION

$$a \frac{dx}{dt} = 1$$



$$\frac{dx}{dt} = 1$$



TIME COORDINATE $\leftrightarrow$ TIME COORD. a

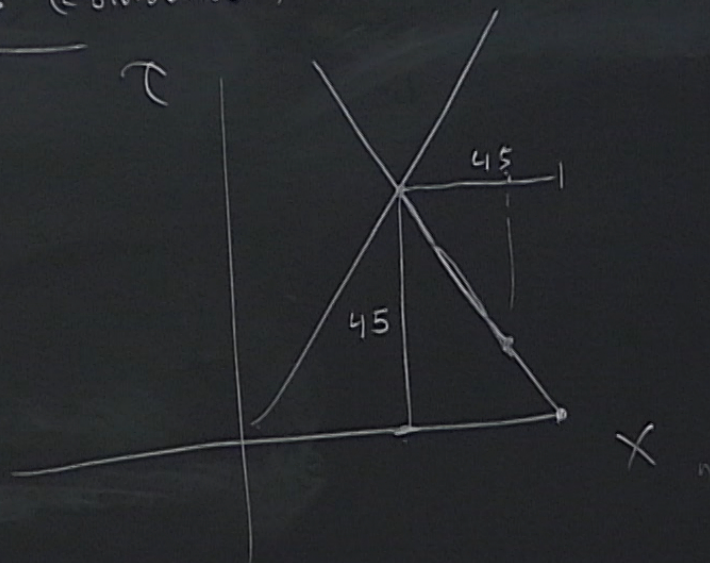
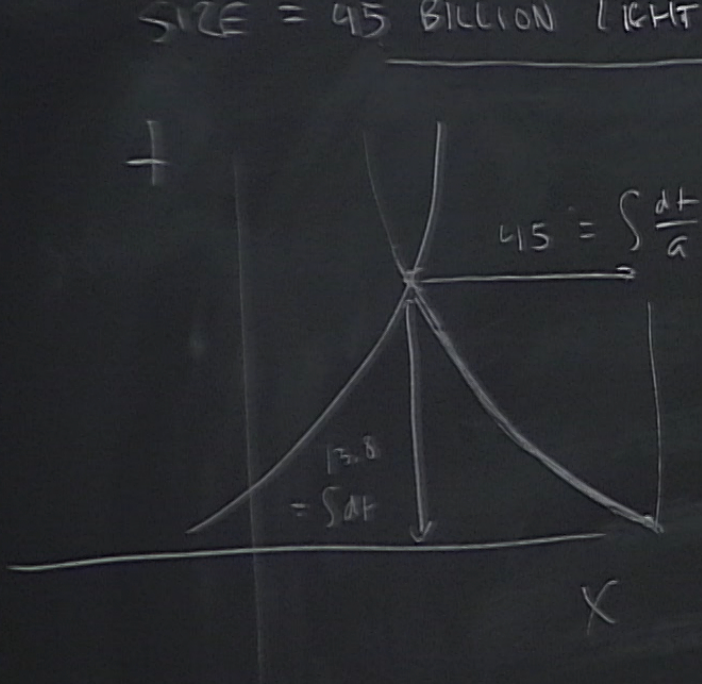
CONFORMAL

$$\begin{aligned} H(a)^2 &= \frac{3}{8\pi G} (p_{r0} a^{-4} + p_{m0} a^{-3} + \Lambda) \\ &= H_0^2 (\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda) \end{aligned}$$

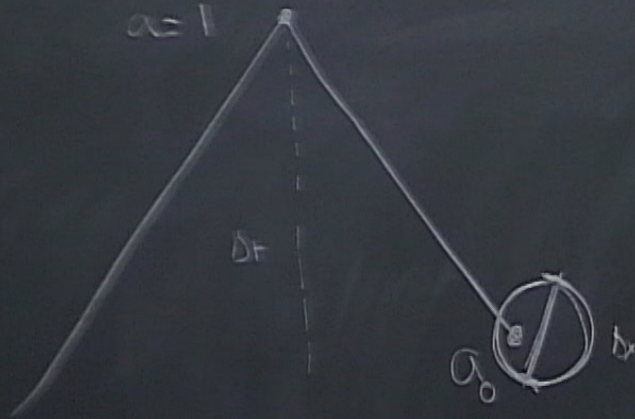
$$\begin{aligned} ds^2 &= -dt^2 + a(t)^2 dx^2 \\ &= a(\tau)^2 [-d\tau^2 + dx^2] \end{aligned}$$

AGE - 13.8 BILLION YEARS

SIZE = 45 BILLION LIGHT YEARS (COMOVING) = CONFORMAL AGE OF UNIVERSE



$\rho(t) = \text{PHYSICAL ENERGY DENSITY}$



$(\Delta x)_{\text{PHYS}} = \text{PHYSICAL SIZE OF STANDARD RULER}$

$$(\Delta x)_{\text{COORD}} = a^{-1} (\Delta x)_{\text{PHYS}}$$

$(\Delta t) = \text{ELAPSED TIME}$

$(\Delta \tau) = \text{ELAPSED CONFORMAL TIME}$

WHAT ANGLE $(\Delta \theta)$ DOES THE RULER SPAN ON THE SKY?

$$\Delta \theta = \frac{(\Delta x)_{\text{PHYS}}}{c \Delta t} = \frac{(\Delta x)_{\text{COORD}}}{c \Delta t} = \frac{(\Delta x)_{\text{PHYS}}}{c (\Delta \tau)} = \frac{(\Delta x)_{\text{COORD}}}{c (\Delta \tau)}$$