Title: Hyper-invariant tensor networks and holography

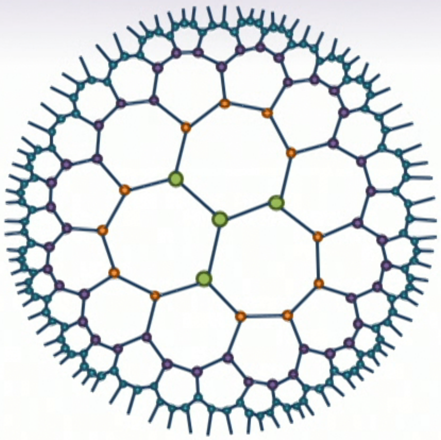
Date: Apr 21, 2017 11:50 AM

URL: http://pirsa.org/17040049

Abstract: I will propose a new class of tensor network state as a model for the AdS/CFT correspondence and holography. This class shall be demonstrated to retain key features of the multi-scale entanglement renormalization ansatz (MERA), in that they describe quantum states with algebraic correlation functions, have free variational parameters, and are efficiently contractible. Yet, unlike MERA, they are built according to a uniform tiling of hyperbolic space, without inherent directionality or preferred locations in the holographic bulk, and thus circumvent key arguments made against the MERA as a model for AdS/CFT. Novel holographic features of this tensor network class will be examined, such as an equivalence between the causal cone C[R] and the entanglement wedge E[R] of connected boundary regions R.

"Tensor Networks for Quantum Field Theories II", Perimeter Institute, April 2017

Hyper-invariant tensor networks and holography



Glen Evenbly

arXiv:1704.04229

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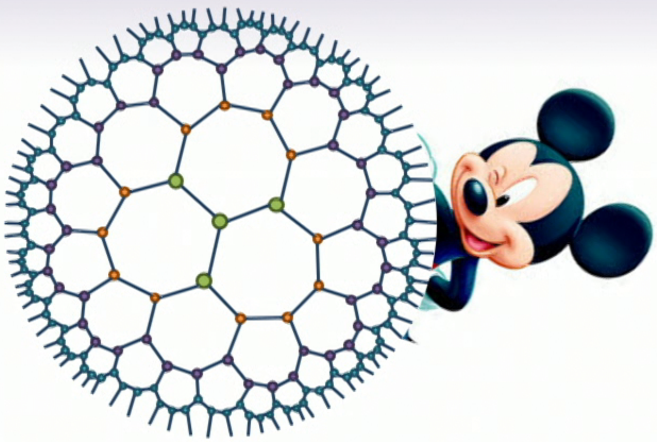
"Tensor Networks for Quantum Field Theories II", Perimeter Institute, April 2017

Hyper-invariant tensor networks and holography



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Overview:

Motivation:

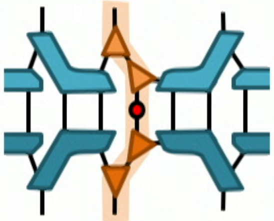
- MERA as models for holography
- Holographic codes as models for holography

Construction:

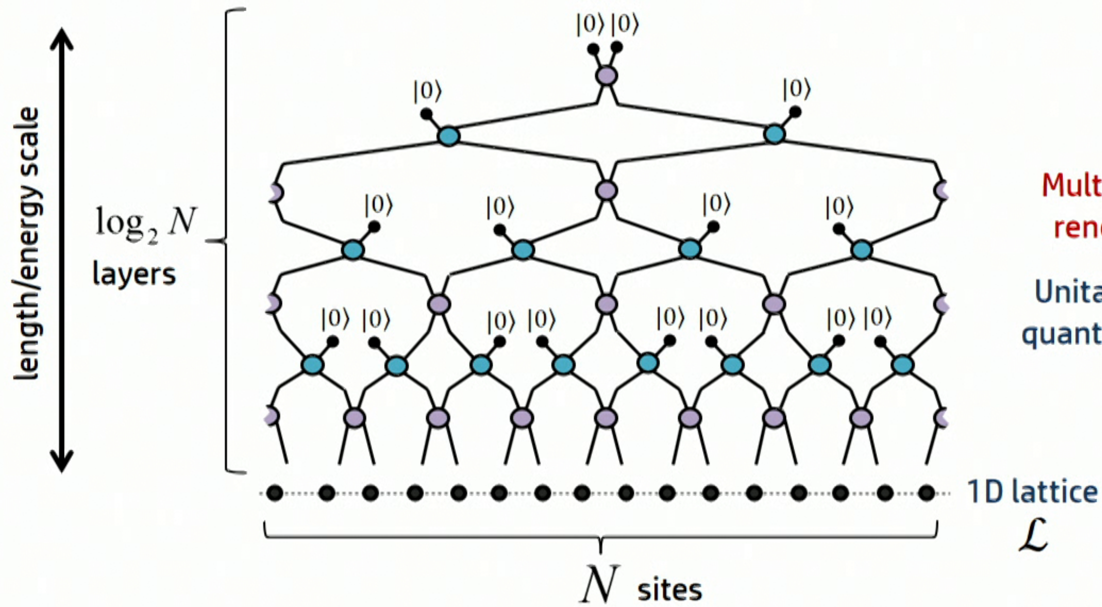
- How to build networks that are invariant on the hyperbolic disk?
- Want: efficient contractibility and non-trivial entanglement / correlations

Implications:

- Causal properties?



Intro to MERA

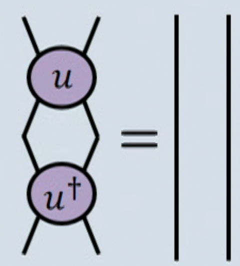


Multi-scale entanglement renormalization ansatz:

Unitary circuit for preparing quantum states on the lattice

- lower layers encode short-ranged properties of the state
- higher layers encode long-ranged properties of the state

Unitary gates:



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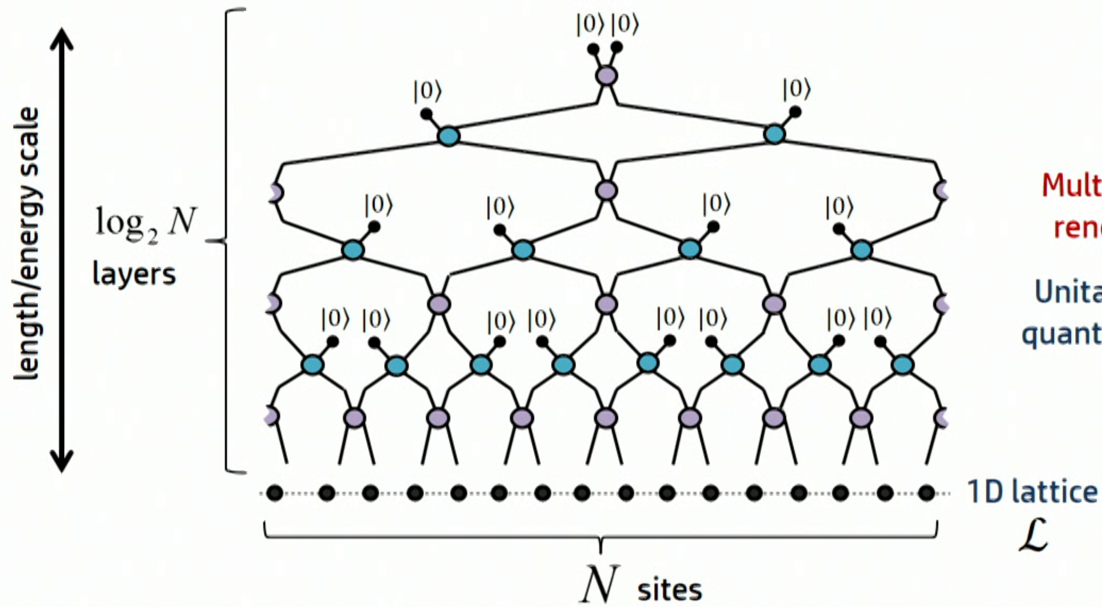

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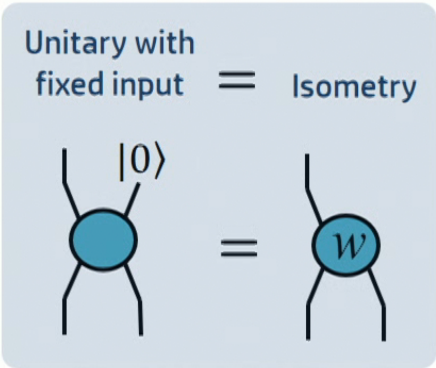

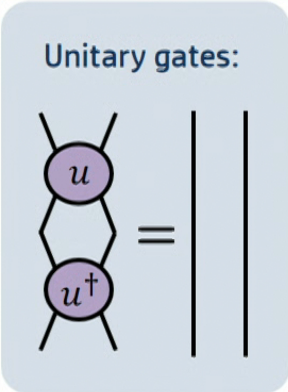

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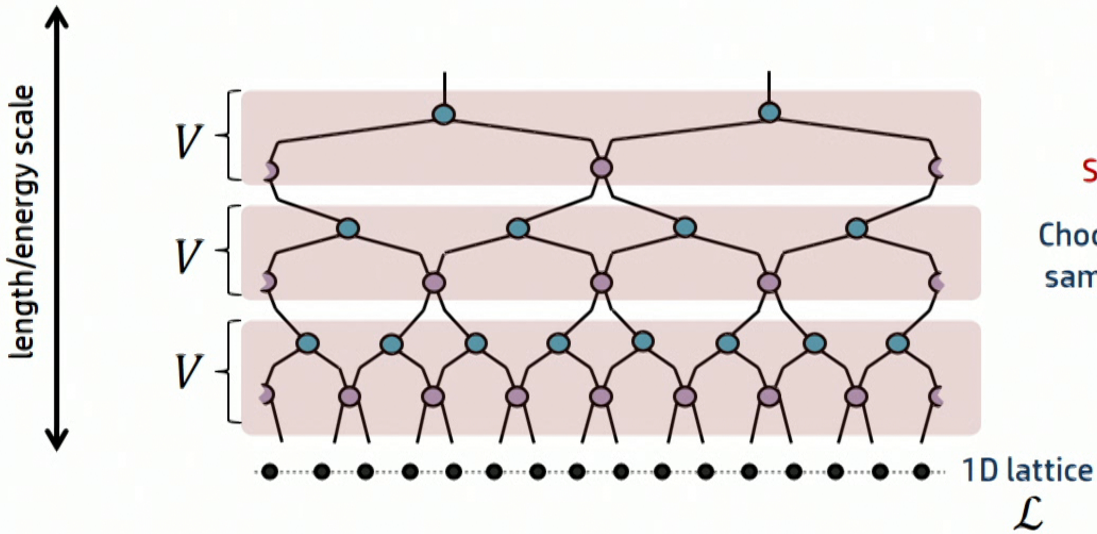
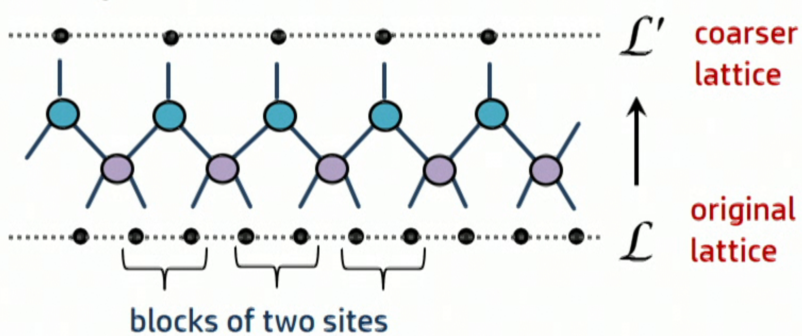
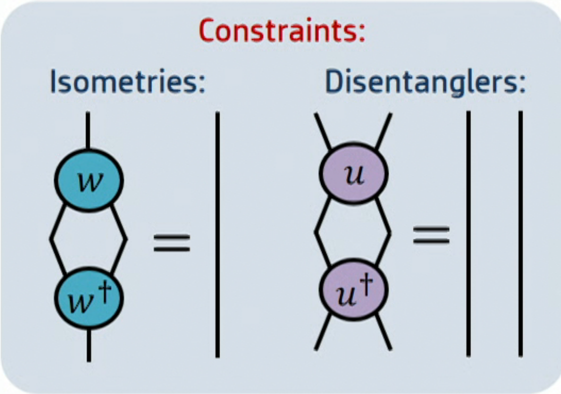


Scale-invariant MERA

Choose each layer to be the same (for RG fixed points!)

Entanglement Renormalization (ER)



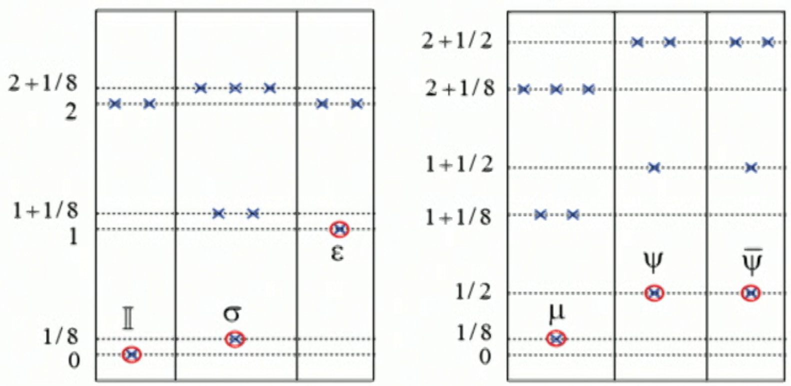


Numerical example: Scale-invariant MERA

1D critical Ising model: $H = \sum_r \left(-X(r)X(r+1) + Z(r)\right)$

- optimize a scale-invariant MERA for its ground state (bond dim: $\chi = 36$)
- extract the conformal data that characterizes the critical theory

Scaling Dimensions Δ



primary fields:	exact	MERA
spin:	$\sigma = 0.125$	0.1250003
energy:	$\varepsilon = 1$	1.0000200
disorder:	$\mu = 0.125$	0.1250002
fermion:	$\psi = 0.5$	0.5000004

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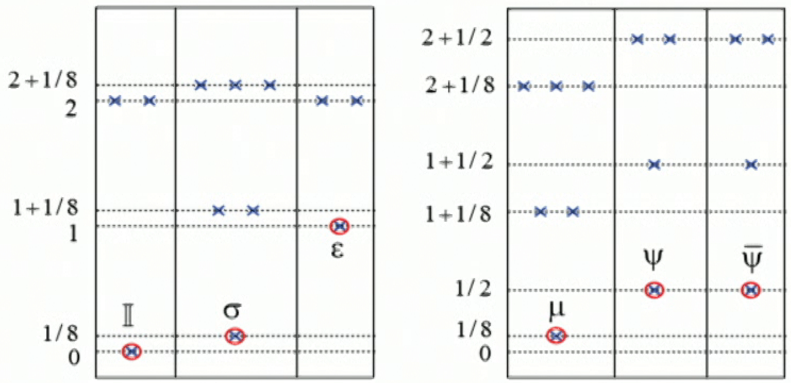


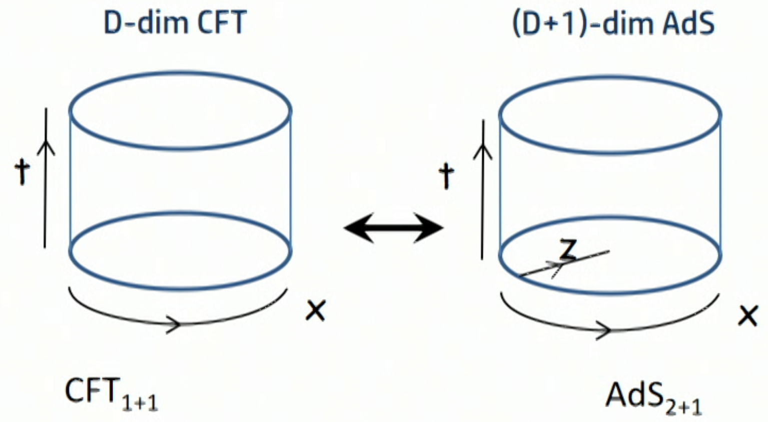
OPE Coefficients

C^{exact}	$C^{\text{MERA}}_{\chi=36}$	error
$C_{\epsilon,\sigma,\sigma} = 1/2$	0.50008	0.016%
$C_{\epsilon,\mu,\mu} = -1/2$	-0.49997	0.006%
$C_{\psi,\mu,\sigma} = \frac{e^{-i\pi/4}}{\sqrt{2}}$	$\frac{1.00068e^{-i\pi/4}}{\sqrt{2}}$	0.068%
$C_{\bar{\psi},\mu,\sigma} = \frac{e^{i\pi/4}}{\sqrt{2}}$	$\frac{1.00068e^{i\pi/4}}{\sqrt{2}}$	0.068%
$C_{\epsilon,\psi,\bar{\psi}} = i$	$1.0001i$	0.010%
$C_{\epsilon,\bar{\psi},\psi} = -i$	$-1.0001i$	0.010%

- Scale-invariant MERA accurately encodes ground states of lattice CFT's

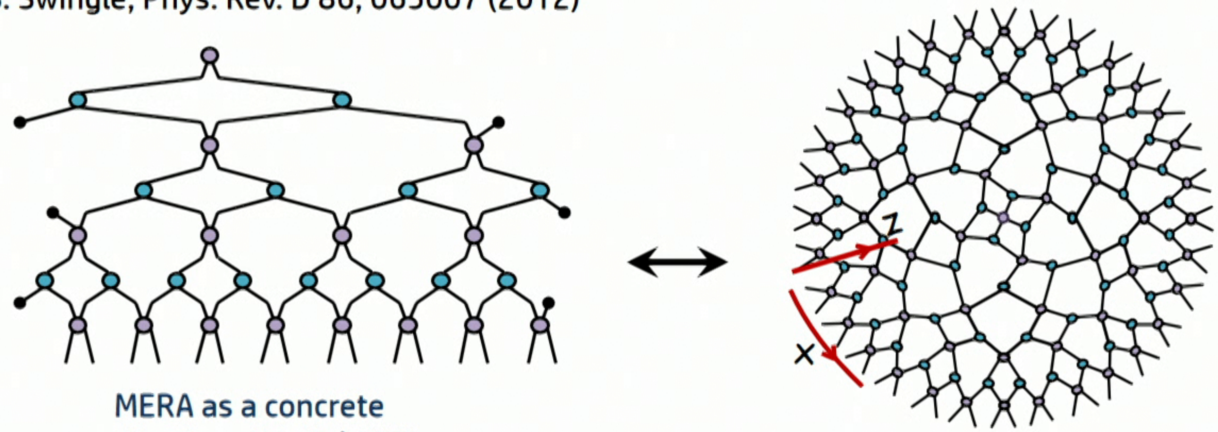
MERA as a model for holography

AdS / CFT correspondence:



Entanglement renormalization and holography:

B. Swingle, Phys. Rev. D 86, 065007 (2012)



MERA as a concrete realization of AdS / CFT?

MERA as a model for holography

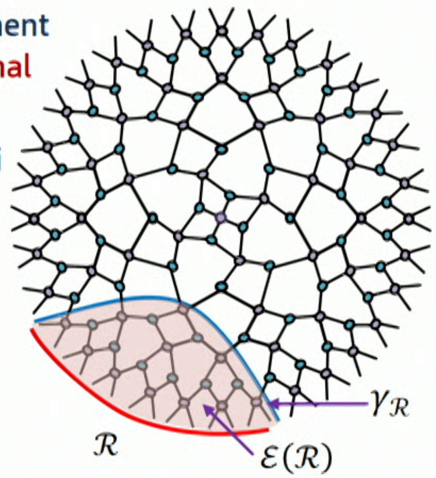
MERA as a realization of holography or AdS / CFT? Many different opinions...

However, this proposal has certainly been useful to tensor networks:

- many new developments in TN methods and algorithms
- we should think about properties of tensor networks geometrically

Scaling of entanglement entropy from minimal surfaces

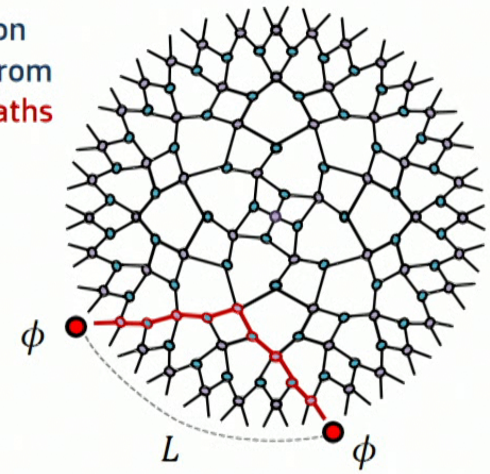
(like Ryu-Takayanagi holographic E.E)



Log correction to the area law

$$S_L = k_1 \log_2(L) + k_2$$

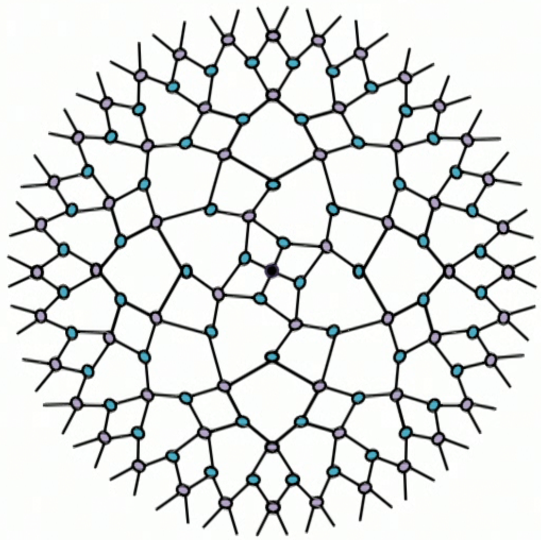
Correlation functions from geodesic paths



Polynomial decay of correlators

$$\langle \phi(x)\phi(x+L)\rangle \propto L^{-2\Delta}$$

MERA as a model for holography

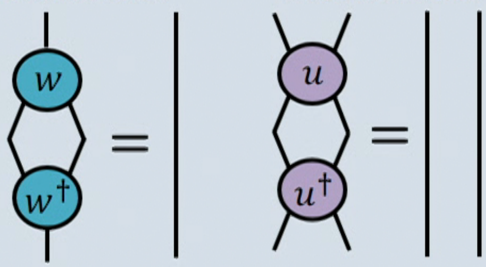


Scale-invariant MERA

- Efficiently contractible
- Entanglement and correlation functions compatible with critical ground states
- Scale-invariance....
  ...but preferred directions and locations in the holographic bulk

Constraints:

Isometries: Disentanglers:



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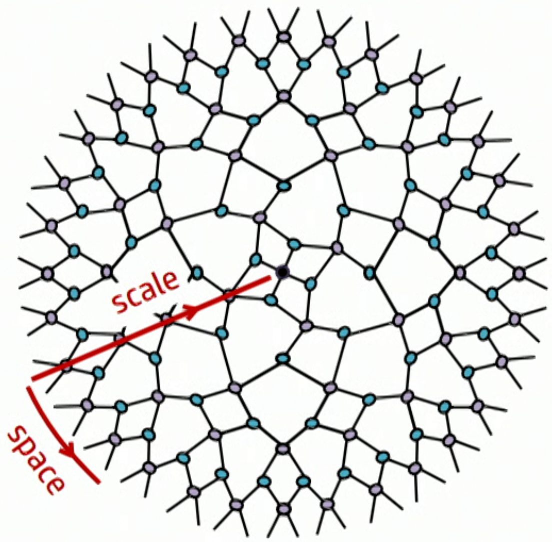

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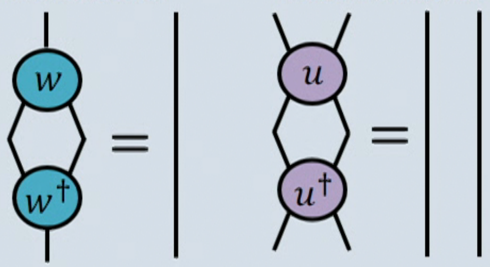


Preferred directions result from isometric / unitary constraints

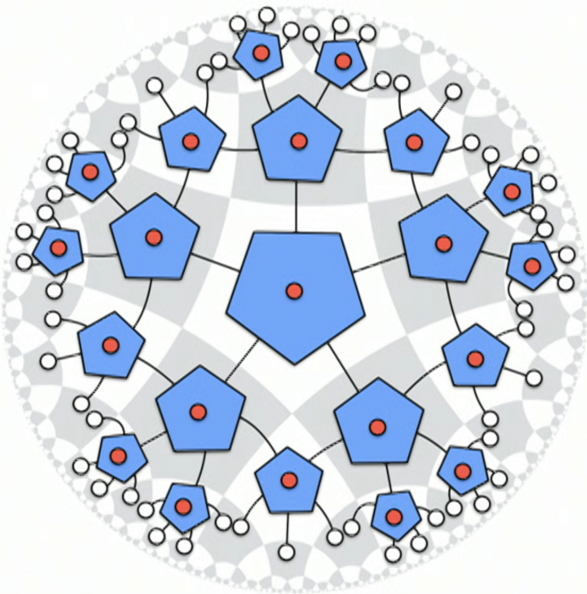
Basis for many arguments against MERA as a direct realization of AdS/CFT

Can we construct a tensor network that is uniform in the bulk?

Holographic quantum error correcting codes

(Pastawski, Yoshida, Harlow, Preskill, arXiv:1503.06237)

Code based on {4,5} tessellation



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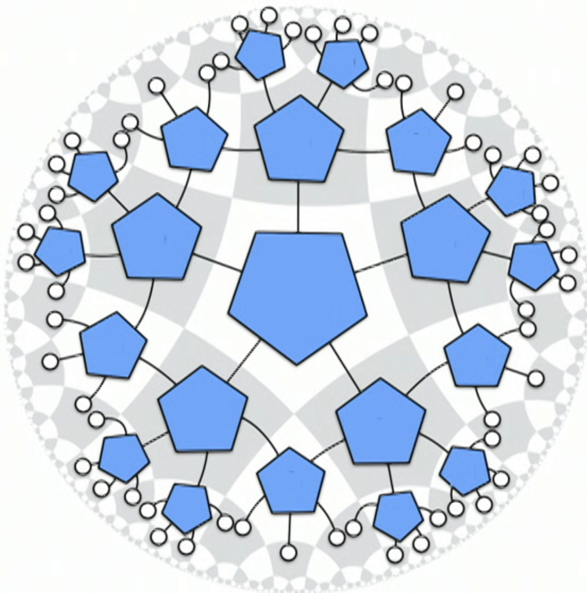

Can have free bulk indices (red circles)

Here shall consider the case with fixed bulk indices

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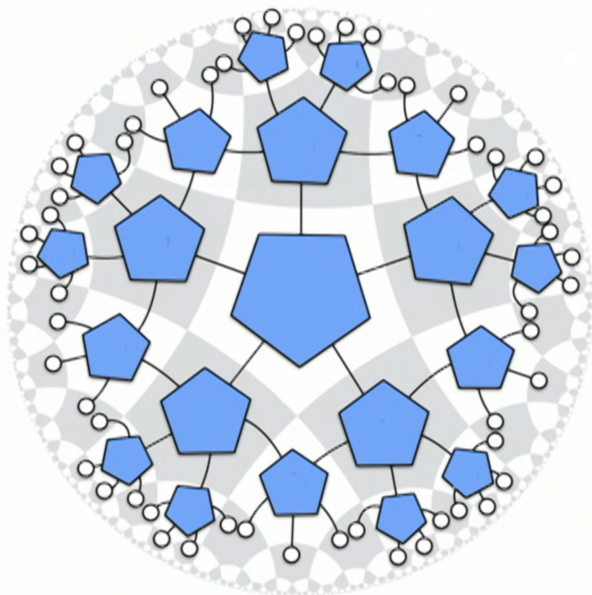
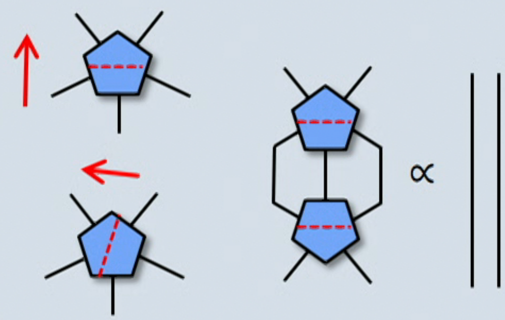


Can have free bulk indices (red circles)

Here shall consider the case with fixed bulk indices

Built from perfect tensors: isometric across all partitions



- Achieve bulk uniformity (no preferred locations or directions in holographic bulk)
- Properties are not compatible with CFTs (trivial correlation functions)

Holographic quantum error correcting codes

(Pastawski, Yoshida, Harlow, Preskill, arXiv:1503.06237)

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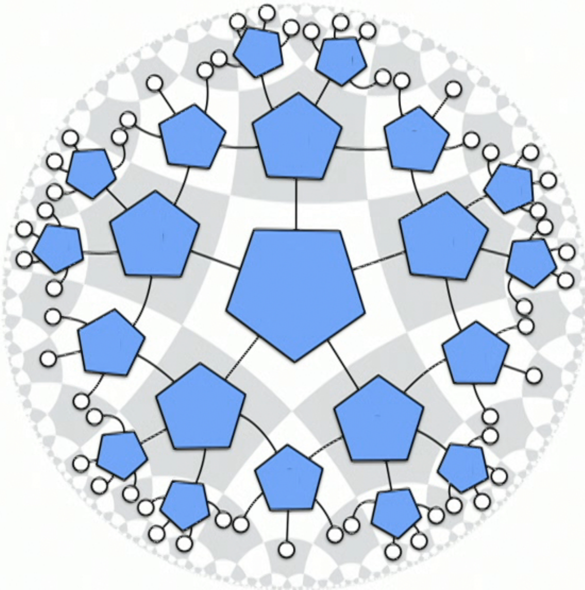
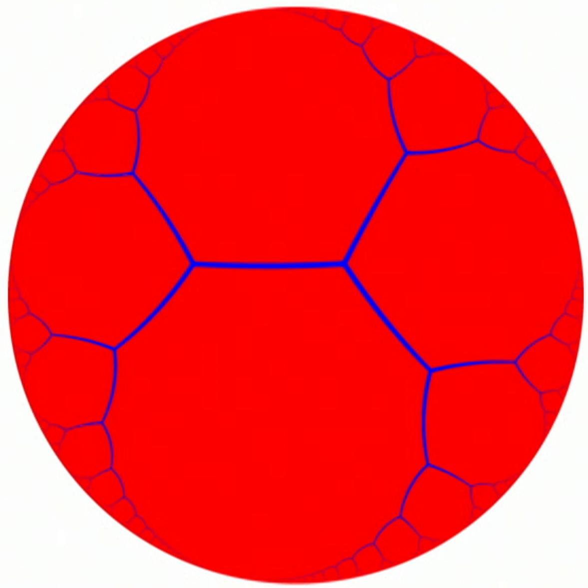


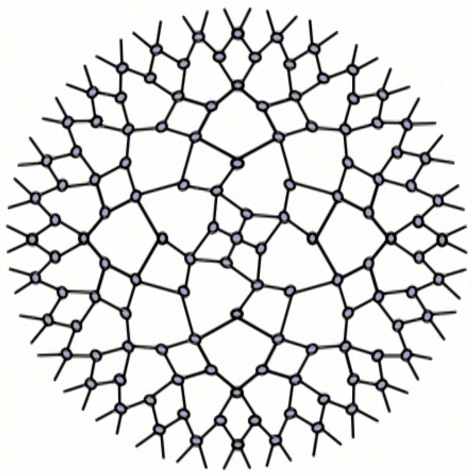
Network based on {∞,3} tessellation



- Holographic codes talked about by Tobias Osborne correspond to tree tensor networks
- Not suitable for representing ground states of CFT's (e.g. entanglement entropy not correct)

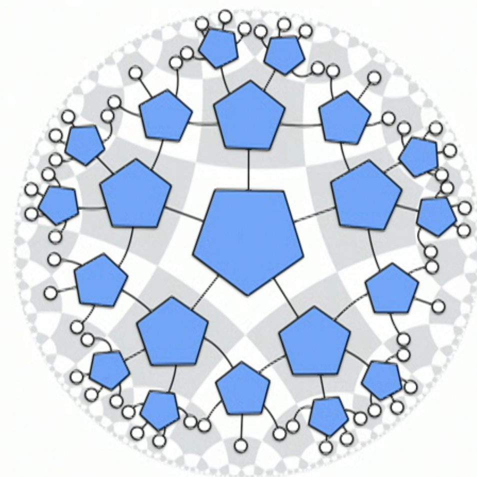
Tensor networks as models for holography

Scale-invariant MERA



- Efficiently contractible ✓
- Properties compatible with CFT's ✓
- Not uniform in the bulk ✗

Holographic codes



- Efficiently contractible ✓
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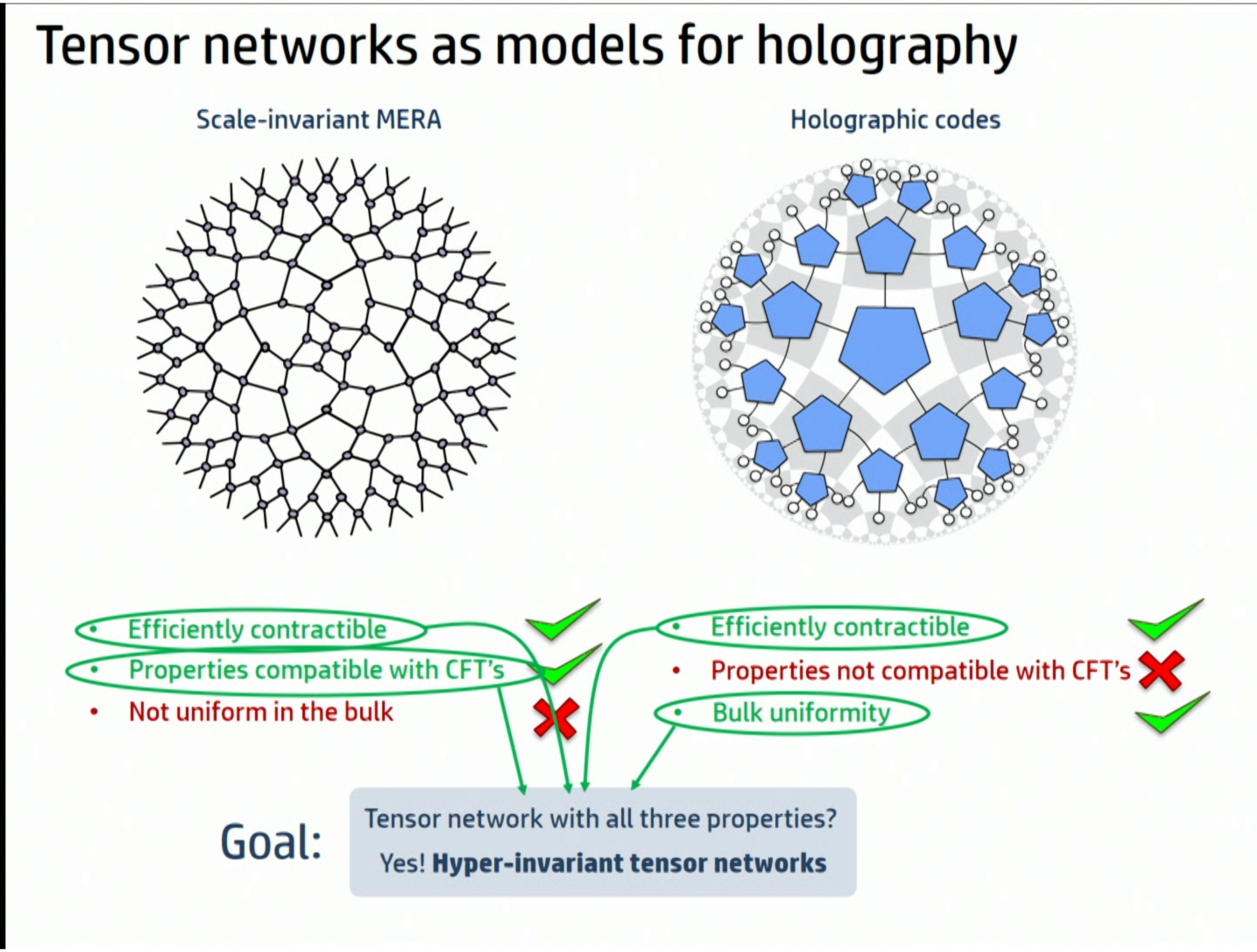
- Bulk uniformity ✓

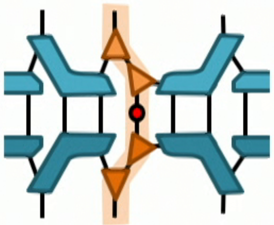

Tensor networks as models for holography
Scale-invariant MERA
Holographic codes
Efficiently contractible
Properties compatible with CFT's
Not uniform in the bulk
Efficiently contractible
Properties not compatible with CFT's
Bulk uniformity
Goal:
Tensor network with all three properties?
Yes! Hyper-invariant tensor networks

Overview:

Motivation:
- MERA as models for holography
- Holographic codes as models for holography

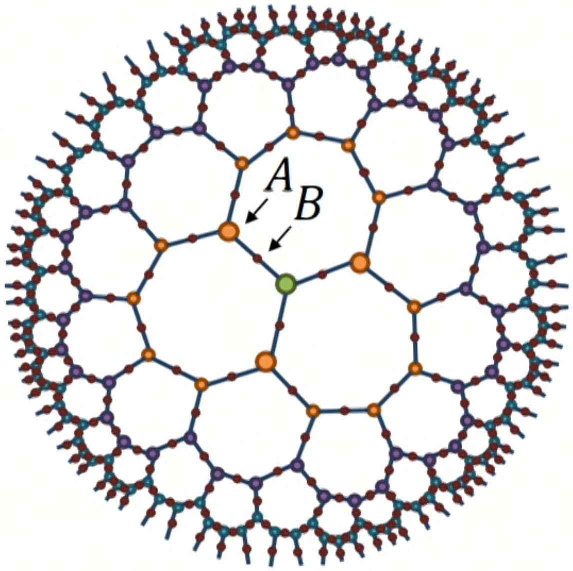
Construction:
- How to build networks that are invariant on the hyperbolic disk?
- Want: efficient contractibility and non-trivial entanglement / correlations

Implications:
- Causal properties?



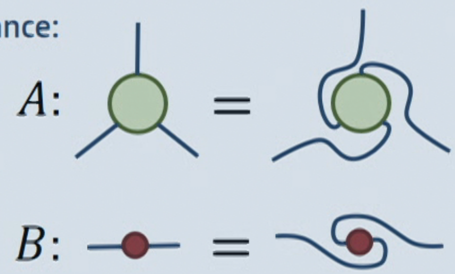
Hyper-invariant networks

Network from {7,3} hyperbolic tessellation



- plaquettes have 7 vertices, vertices have 3 edges
- 3-index tensor A placed on each vertex
- matrix B is placed on each edge between two vertices

Rotation invariance:



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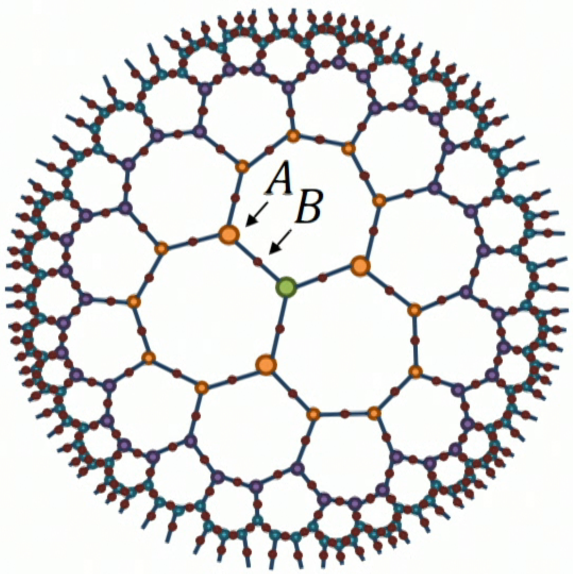

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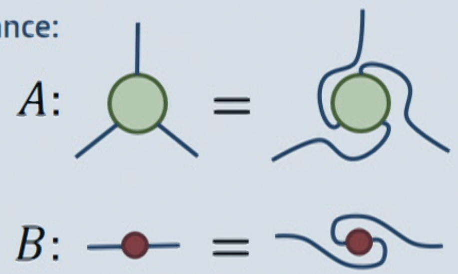
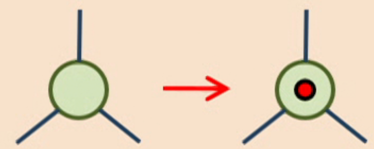


Note: we focus on networks without free bulk indices (though these could be added)



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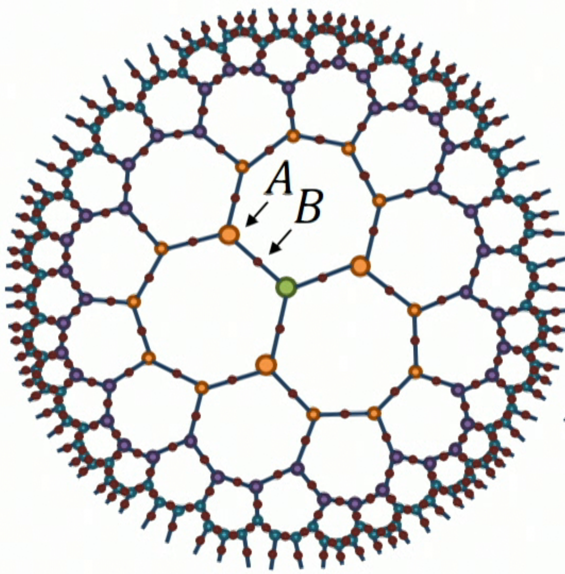

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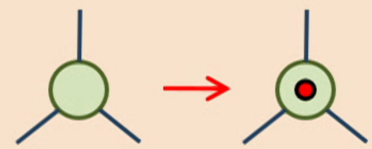


Copies of same A and B
tensors at each location

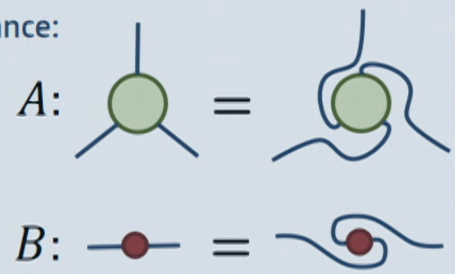
+

Rotation invariance

=

Bulk uniformity: no
preferred locations or
directions in the bulk (in the
limit of an infinite tiling)

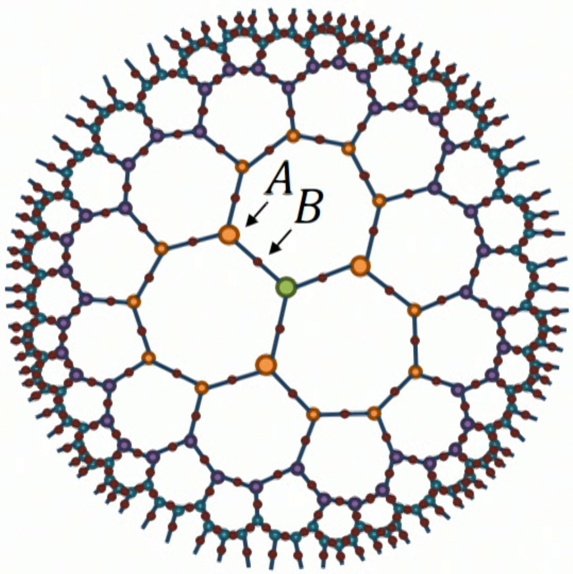
Rotation
invariance:



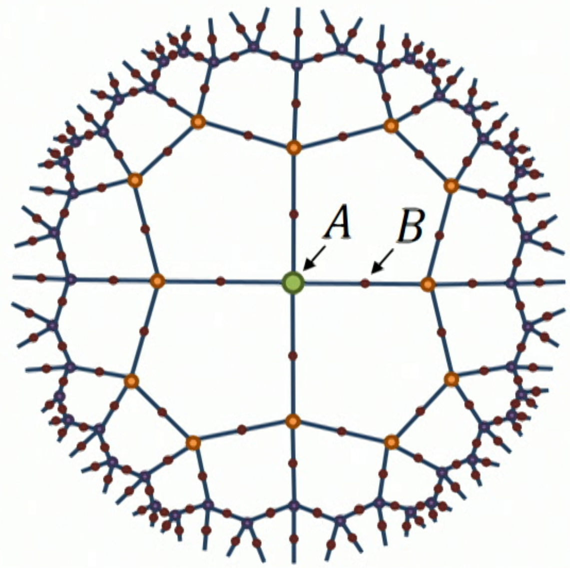
Hyper-invariant networks

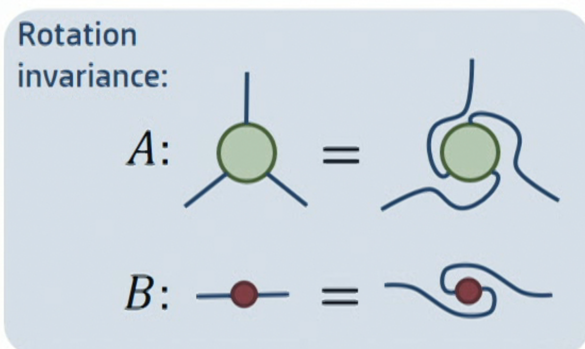
Many forms are possible!

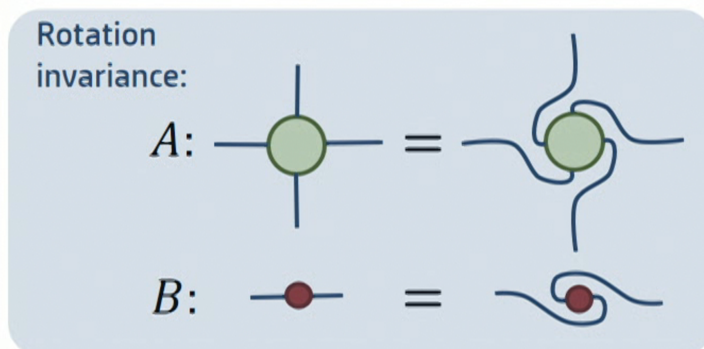
Network from {7,3} hyperbolic tessellation



Network from {5,4} hyperbolic tessellation







Hyper-invariant networks

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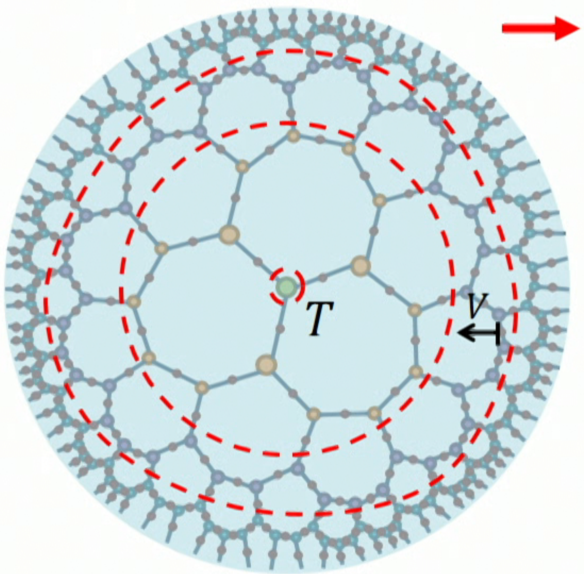
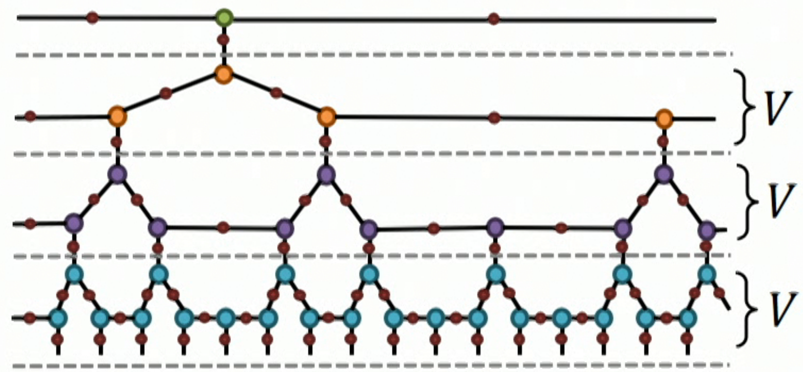
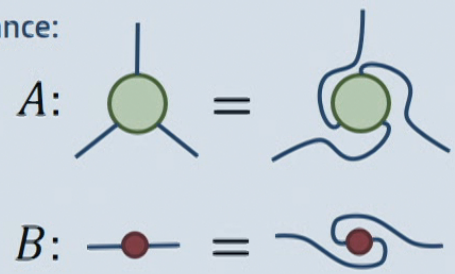


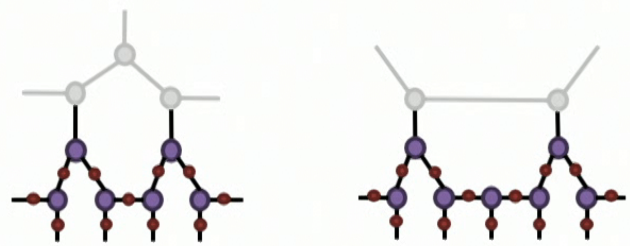
Unwrap into layers about chosen bulk point T:
(each layer is a string of alternating A and B tensors)



Rotation invariance:



Each layer composed of 2 types of unit cell:



2-site cell

3-site cell

Hyper-invariant networks

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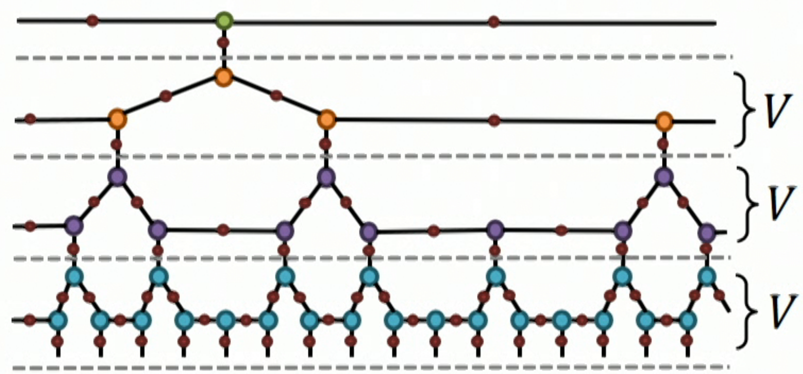


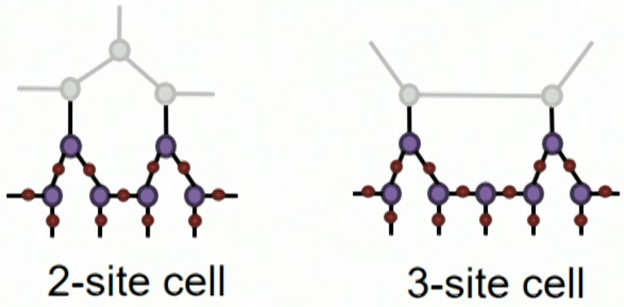
Notice: the pattern of cells is fractal

(no finite repeating pattern even in thermodynamic limit)

Scale factor is irrational:

$$s = \frac{3+\sqrt{5}}{2} \approx 2.618$$

Each layer composed of 2 types of unit cell:



Hyper-invariant networks

What additional constraints are needed for preservation of locality?

First: revisit MERA and holographic codes

Unwrap into layers about chosen bulk point T:

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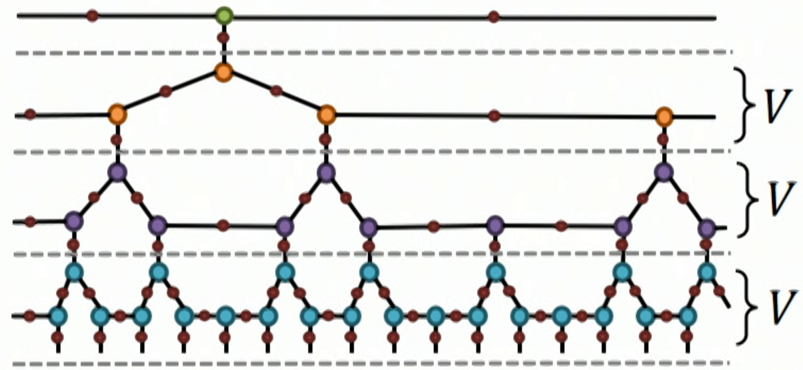

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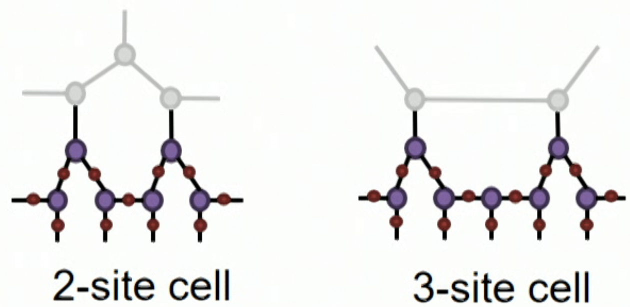

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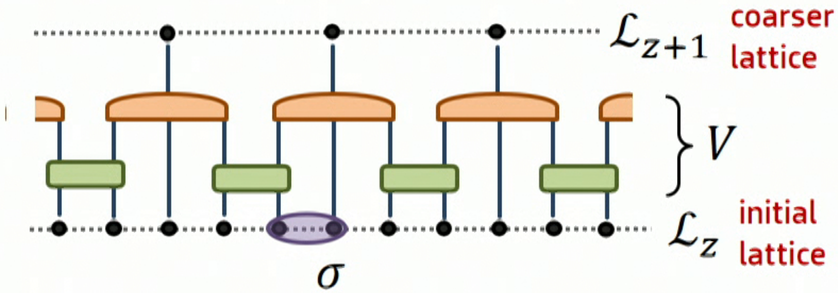
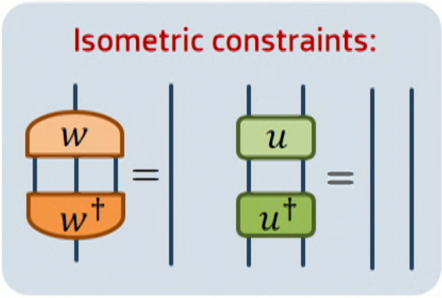
Constraints in MERA

Layer of MERA as a coarse-graining transformation (Entanglement Renormalization):





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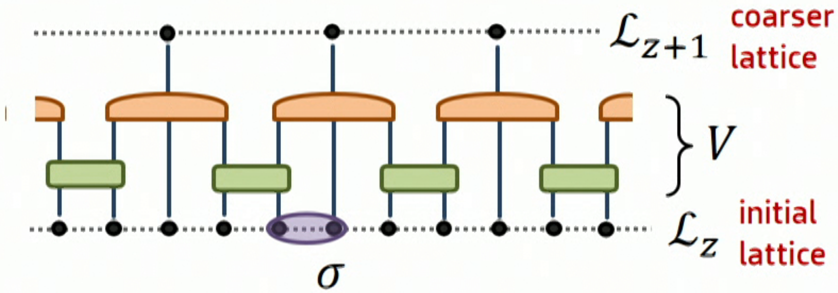
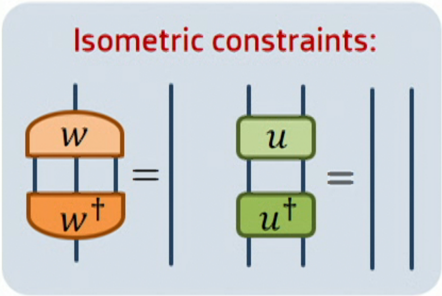
Layer of MERA as a coarse-graining transformation
(Entanglement Renormalization):





Local operators map to local operators: $\sigma' = V\sigma V^\dagger$

Constraints in MERA

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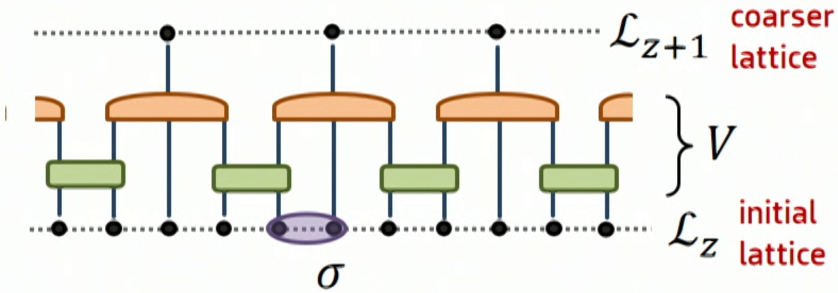

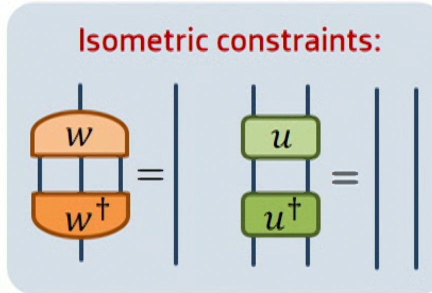

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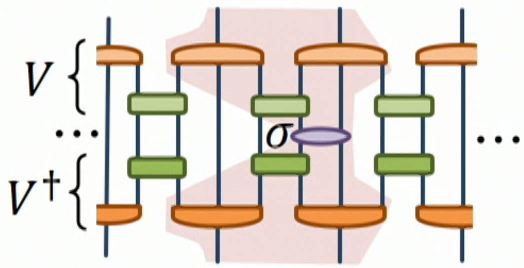

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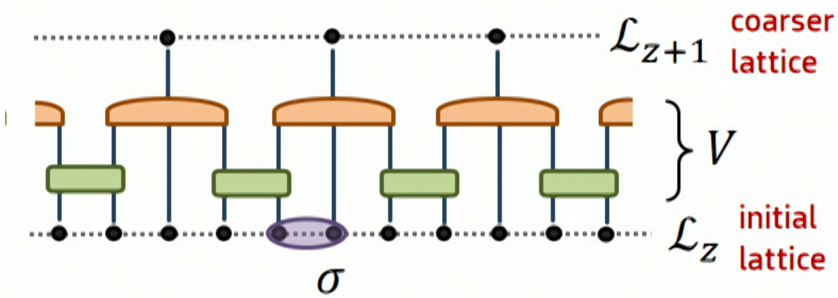

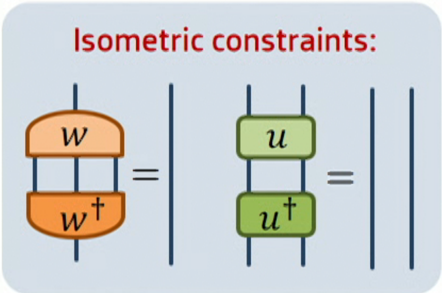

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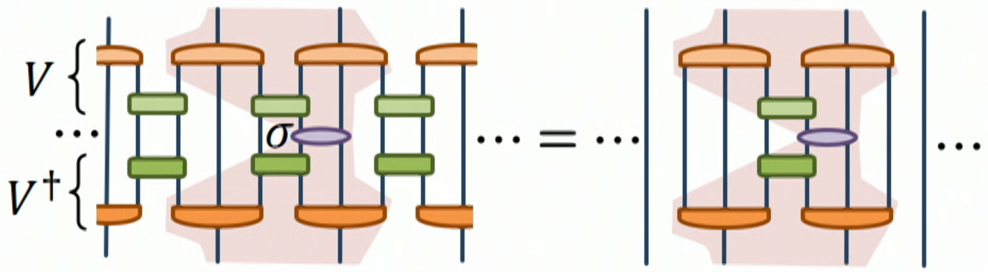

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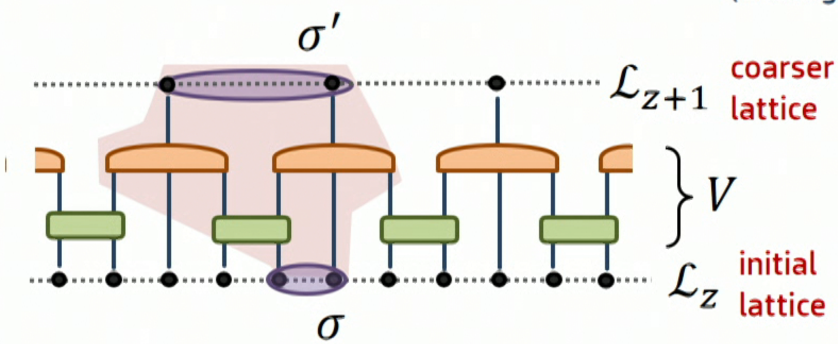

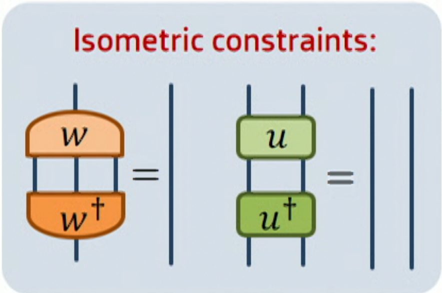

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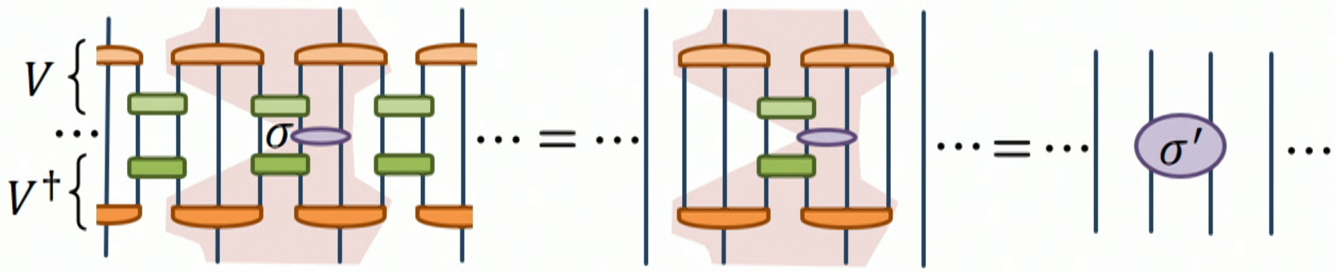


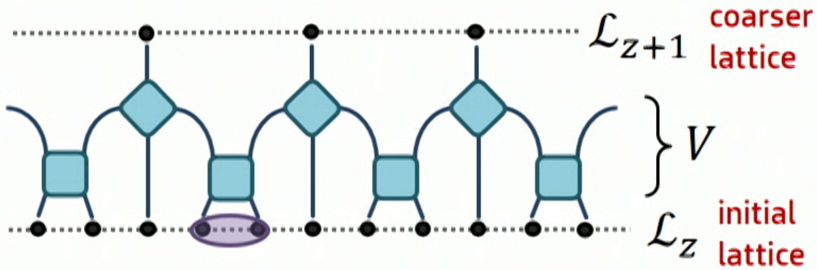
Preservation of locality is important!

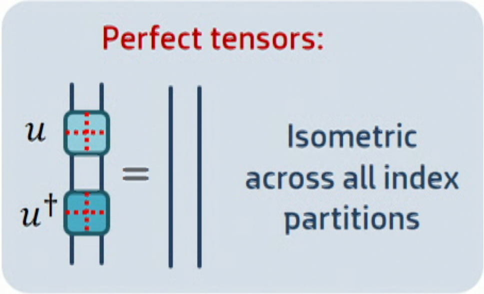
Practically: allows MERA to be efficiently contracted

Conceptually: to reproduce features of CFTs (like scaling operators)

Constraints in holographic codes

Layer of perfect tensors as a coarse-graining transformation:





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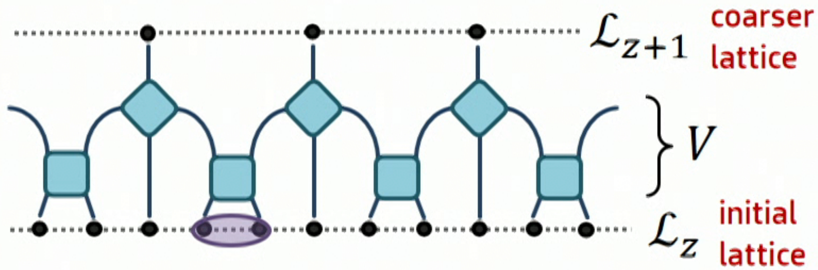

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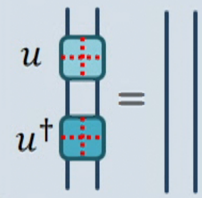

Isometric across all index partitions

Perfect tensors also preserve locality...

...but cause some operators to coarse-grain trivially

Constraints in holographic codes

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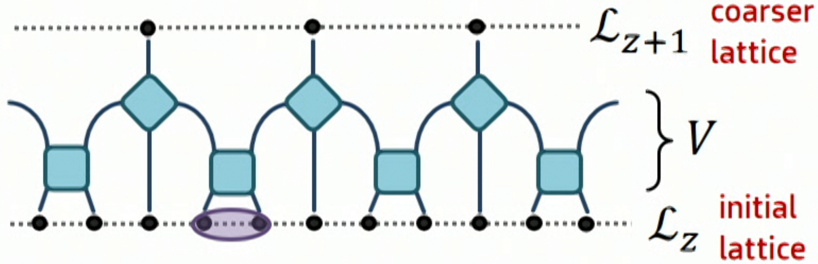

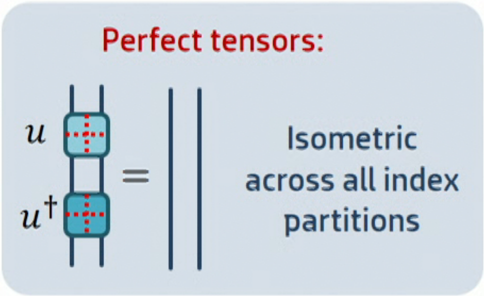

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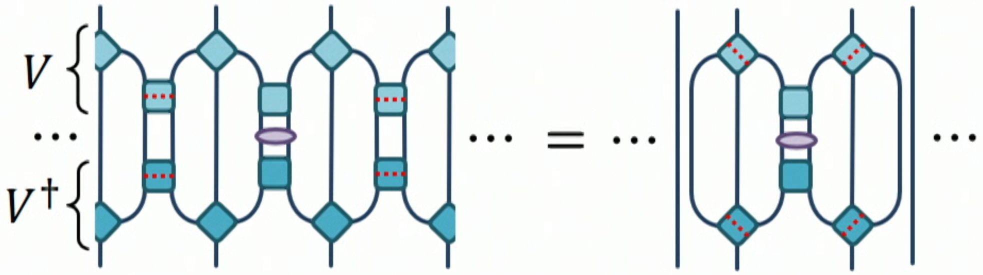
Coarse-graining of a local operator: $\sigma' = V\sigma V^\dagger$



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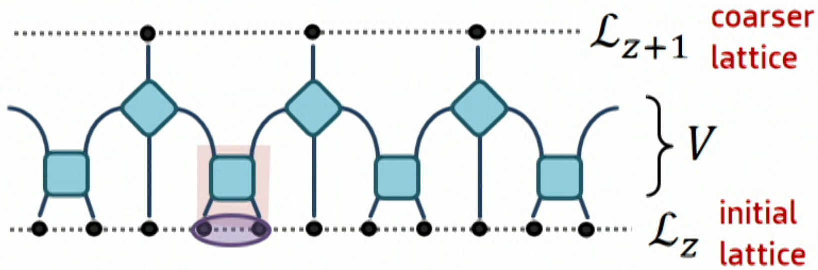

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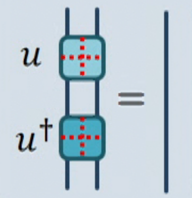

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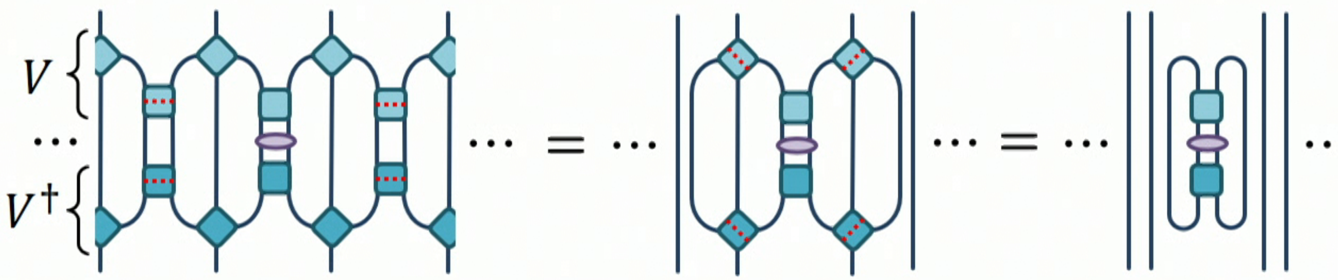
Coarse-graining of a local operator: $\sigma' = V\sigma V^\dagger$

trivial operator

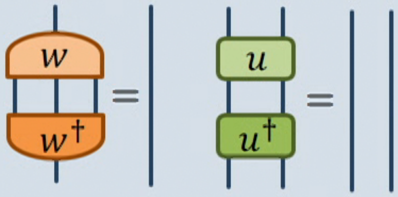
$\Rightarrow \sigma' \propto \mathbb{I}$



Hyper-invariant networks: constraints

Scale-invariant MERA

Isometric constraints:



Not strong enough!

(to be compatible with bulk uniformity)

Holographic codes

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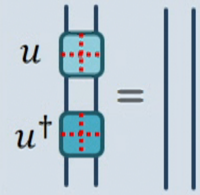

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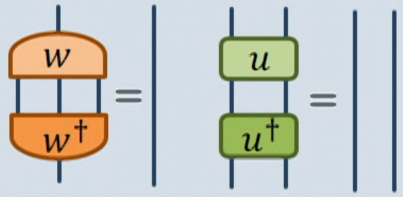

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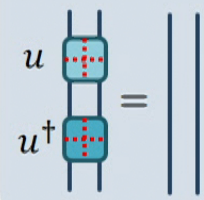

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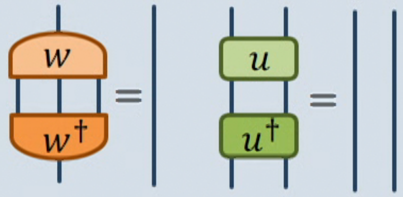

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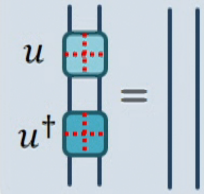

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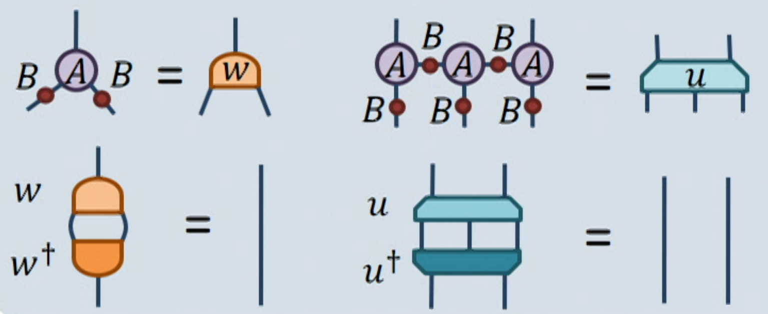
New idea: **Multi-tensor constraints.** Constrain certain products of tensors to be isometric

Multi-tensor constraints for {7,3} network:



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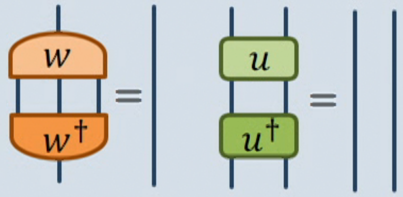

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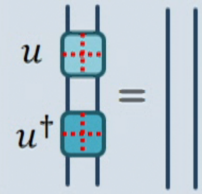

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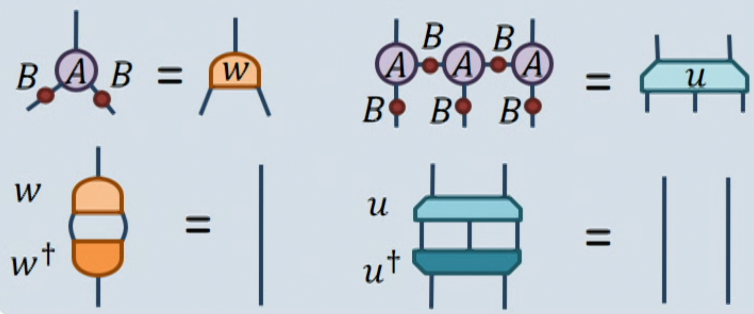
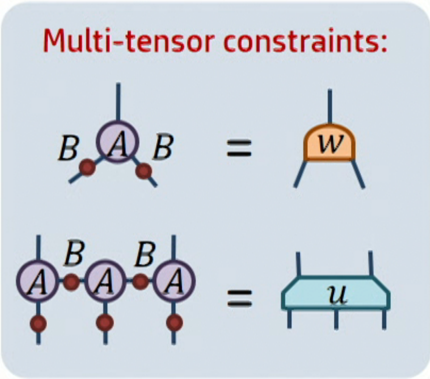
Multi-tensor constraints for {7,3} network:

2-to-1 isometry

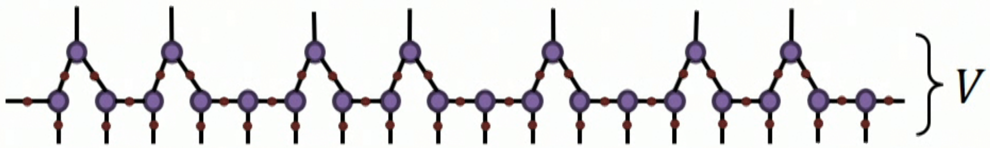
3-to-2 isometry



Hyper-invariant networks: constraints



Layer of the {7,3} network:



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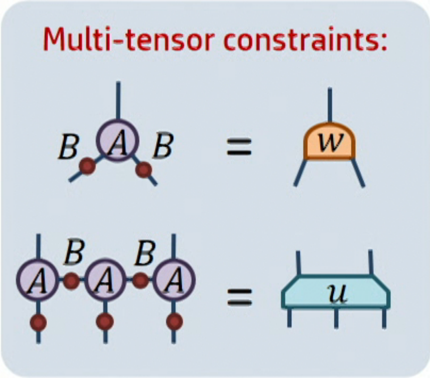

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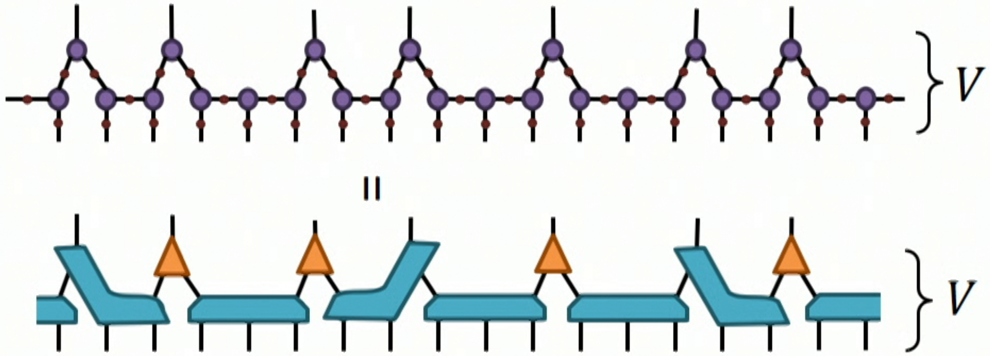

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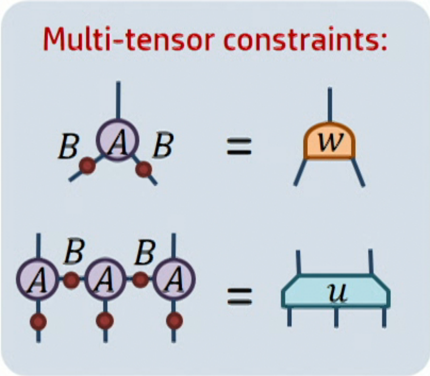

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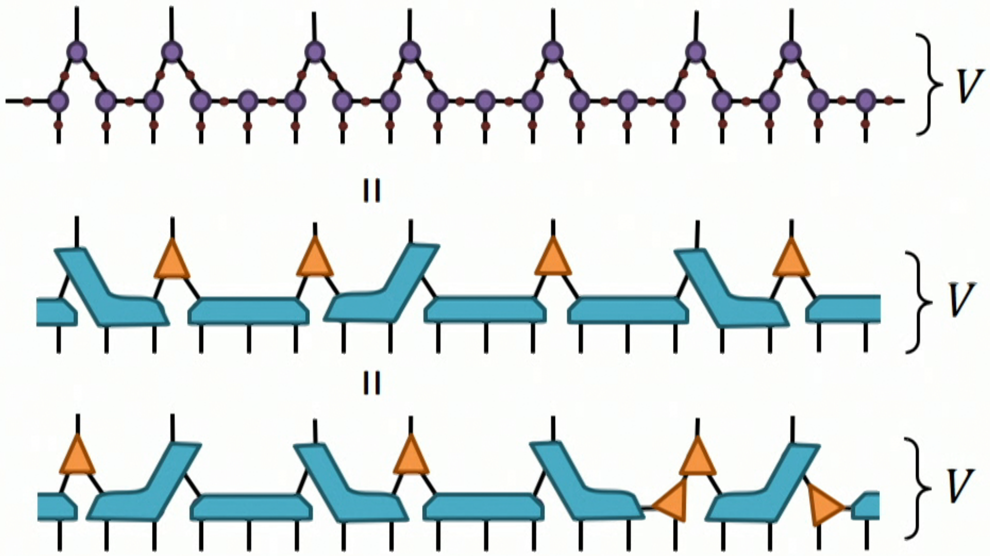
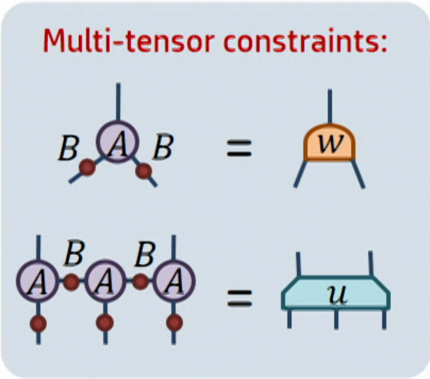
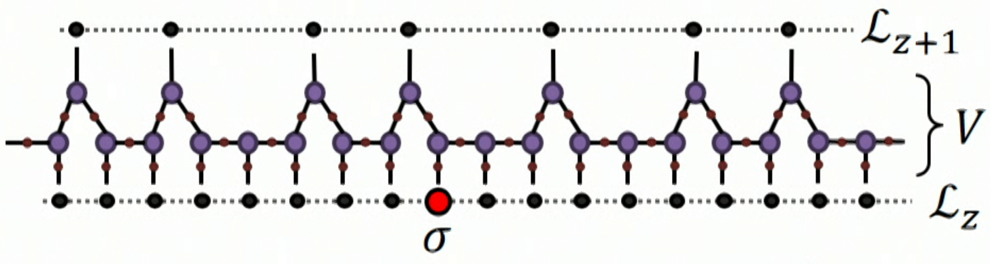


- Many ways to group tensors into isometries *w* and *u*
- Each layer "V" is an isometric mapping

Hyper-invariant networks: constraints

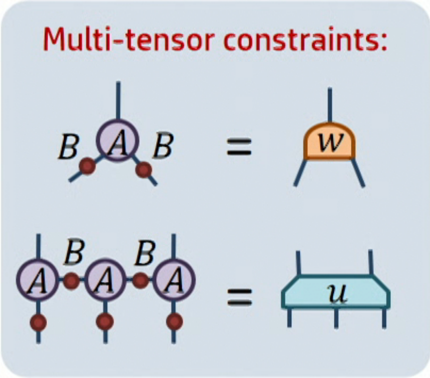


Layer of the {7,3} network as a coarse-graining transformation:



Coarse-graining of local operators? $\sigma' = V\sigma V^\dagger$

Hyper-invariant networks: constraints



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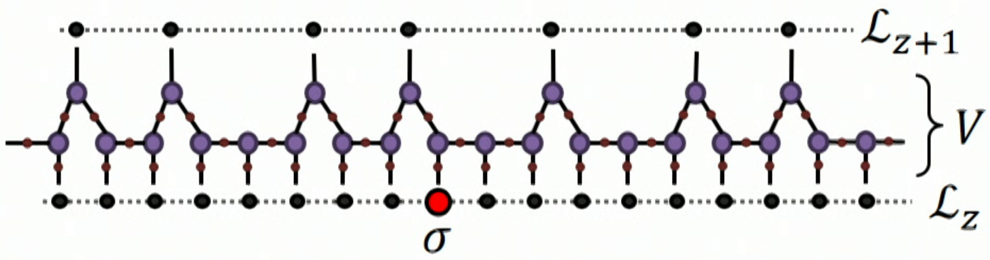

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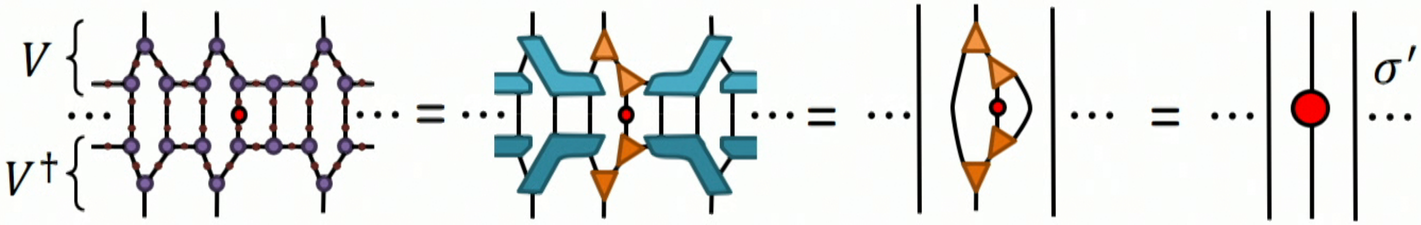

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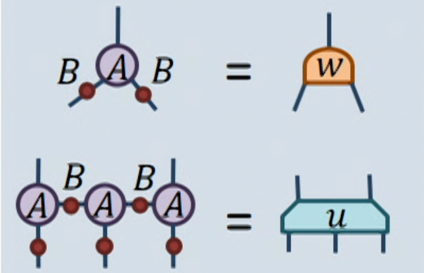

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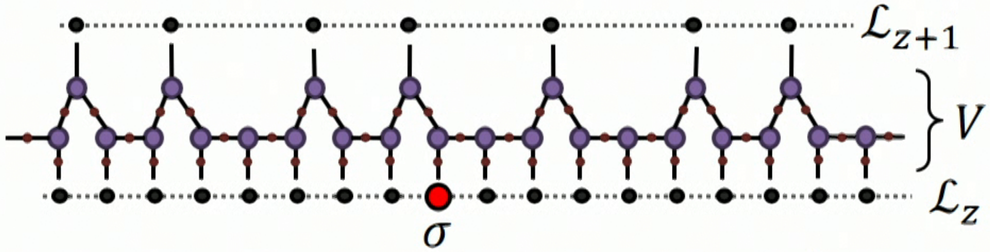

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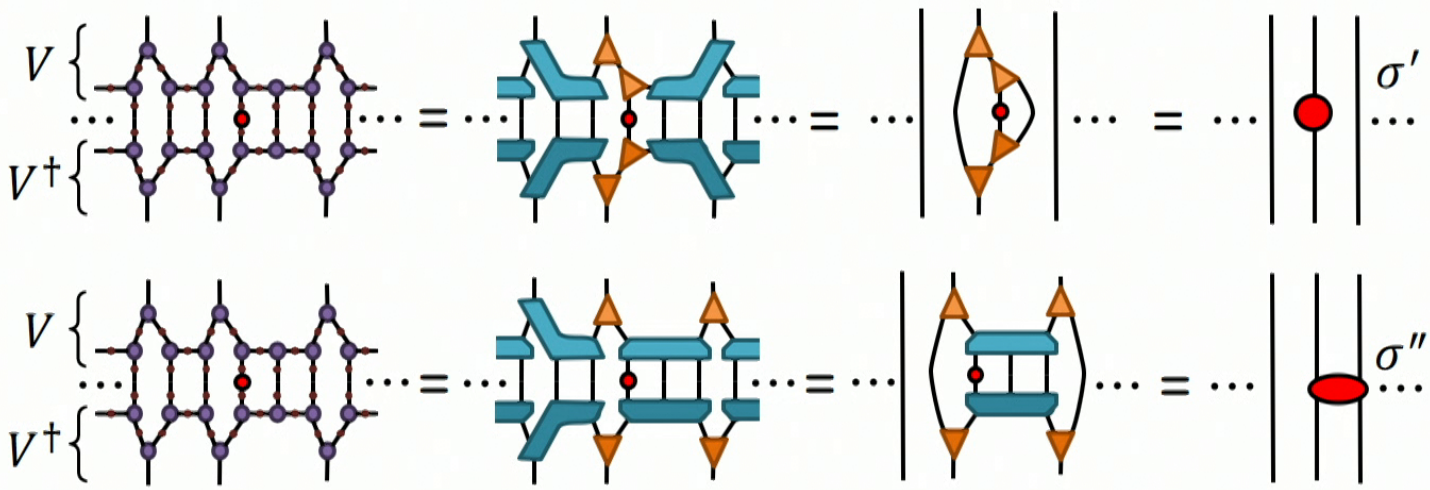
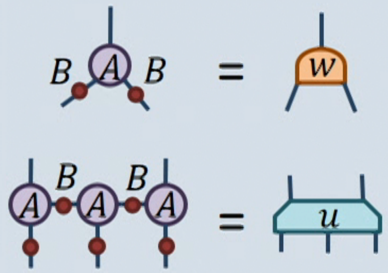


Hyper-invariant networks: constraints

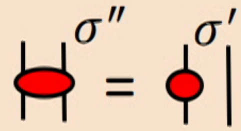
Multi-tensor constraints:



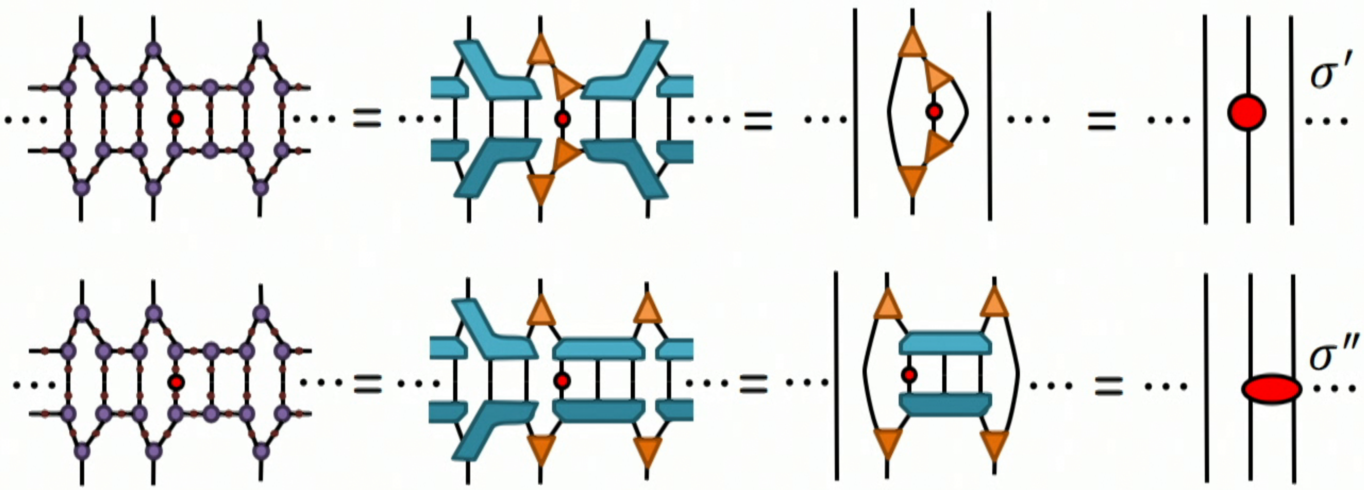
Coarse-grained operator depends on choice of grouping?

No!

Properties cannot be changed by which grouping is "imagined"

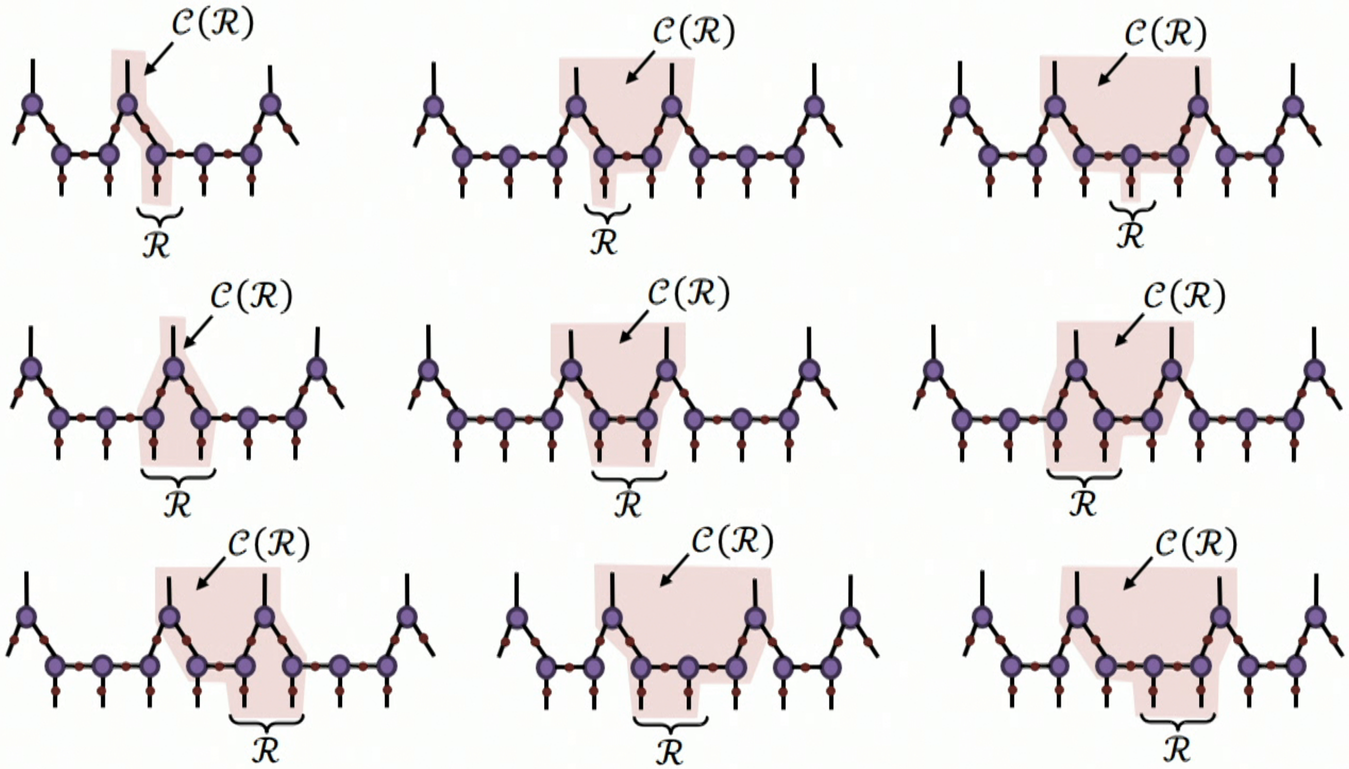


Non-trivial part of coarse-grained operators are understood through grouping that yields minimal support



Hyper-invariant networks: constraints

Causal cones through a single layer (from minimal support grouping):



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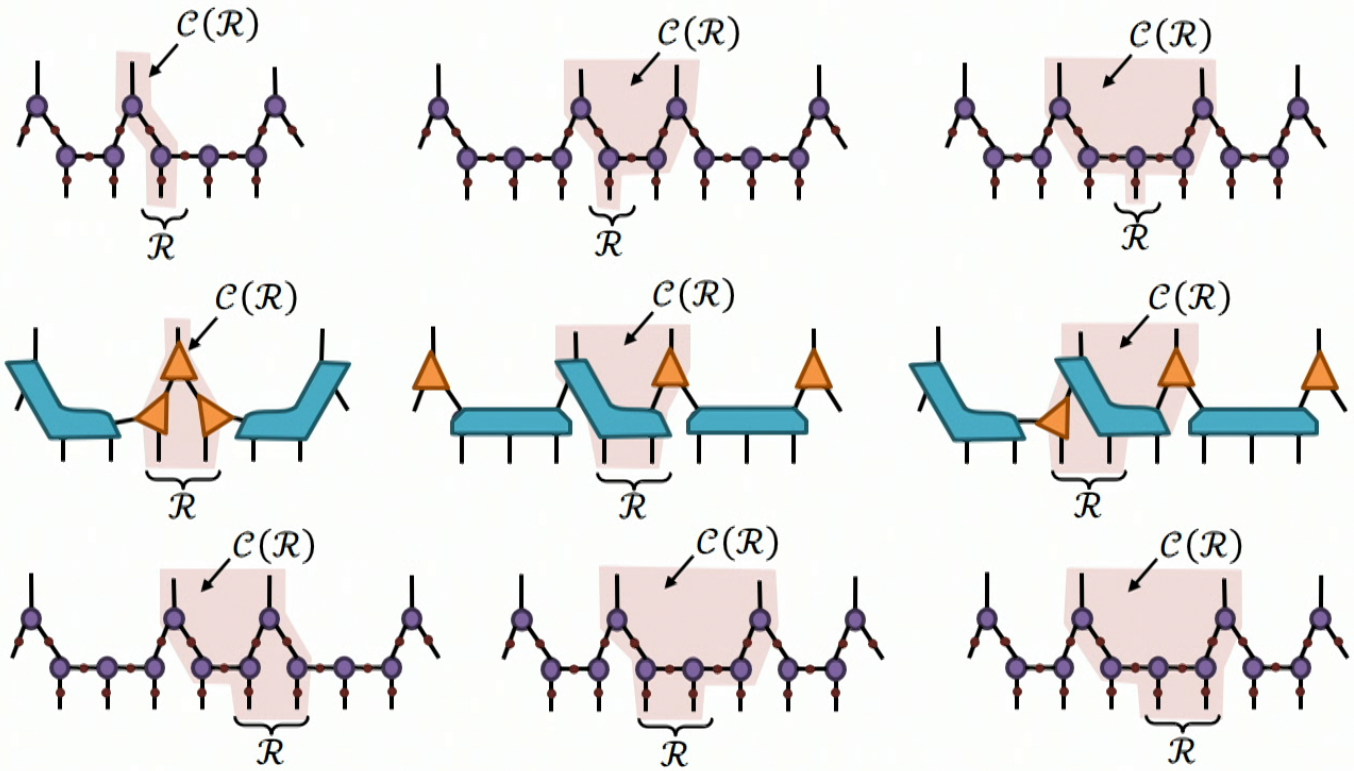


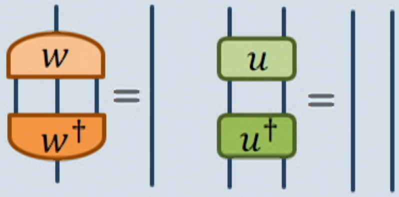
Locality: operators with support $L \leq 2$ sites mapped to operators with support $L \leq 2$ sites

No trivial coarse-grainings: support remains at $L > 0$ sites

Hyper-invariant networks: constraints

Scale-invariant MERA

Isometric constraints:



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(to be compatible with bulk uniformity)

Goldilocks constraints???

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Holographic codes

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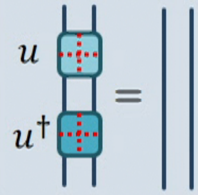

Isometric across all index partitions

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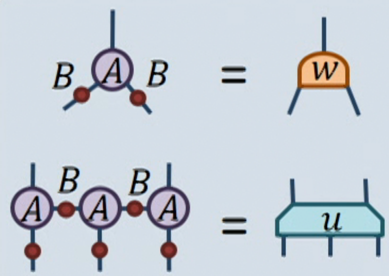
New idea: Multi-tensor constraints. Constrain certain products of tensors to be isometric

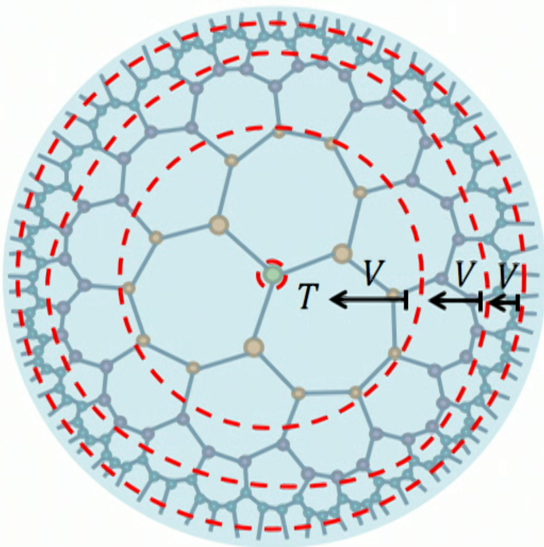


Just right!

(compatible with bulk uniformity, but also give non-trivial correlations)

Hyper-invariant networks: causal properties

Can choose "center of orthogonality" at any bulk point:



- For any bulk point T, the network can be organised into concentric layers of isometric mappings V

Hyper-invariant networks: causal properties

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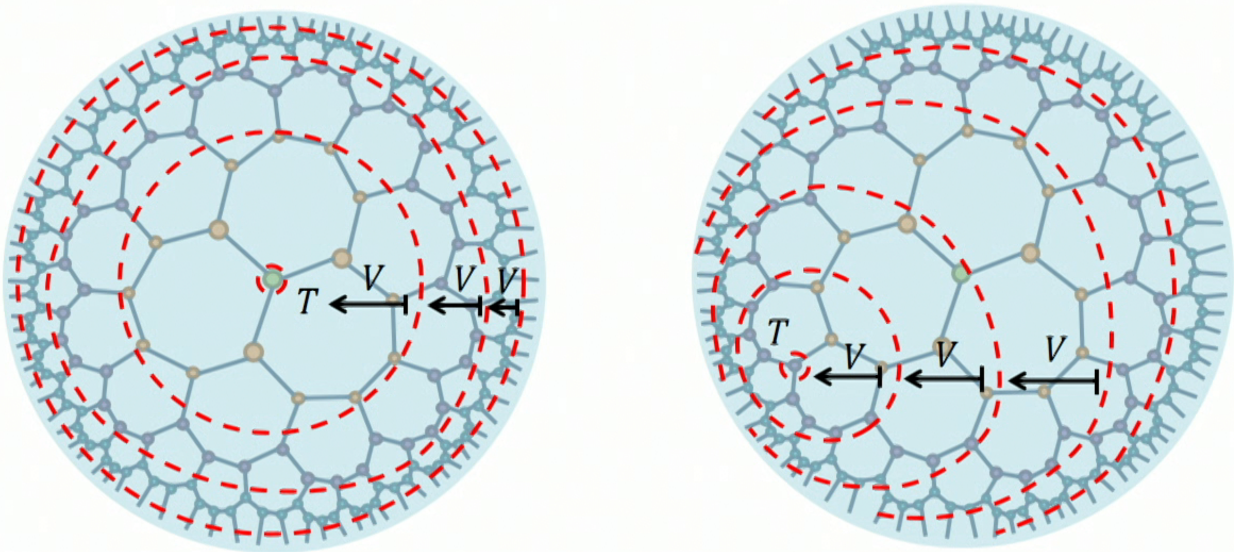

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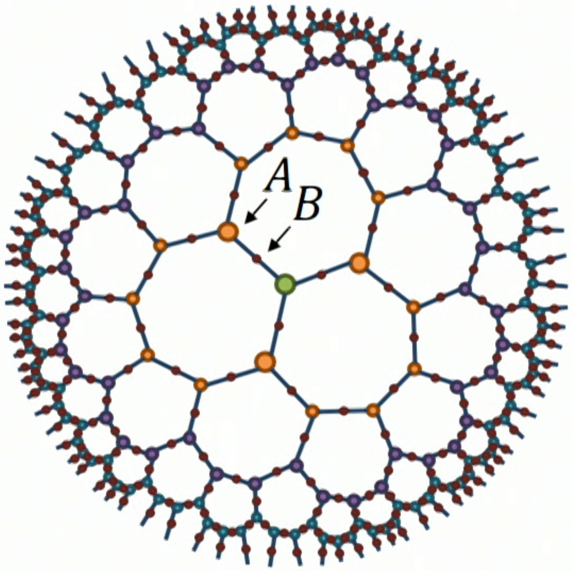
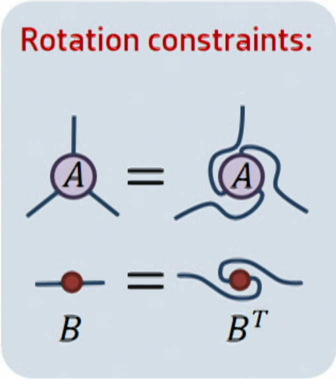
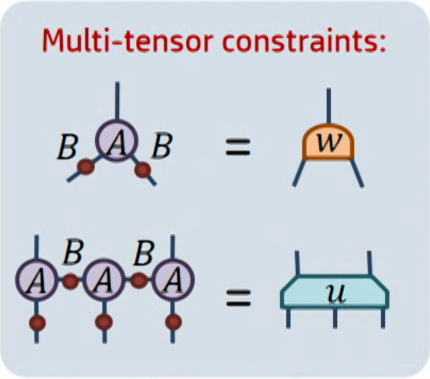
Hyper-invariant networks

{7,3} hyper-invariant network





Bulk uniformity

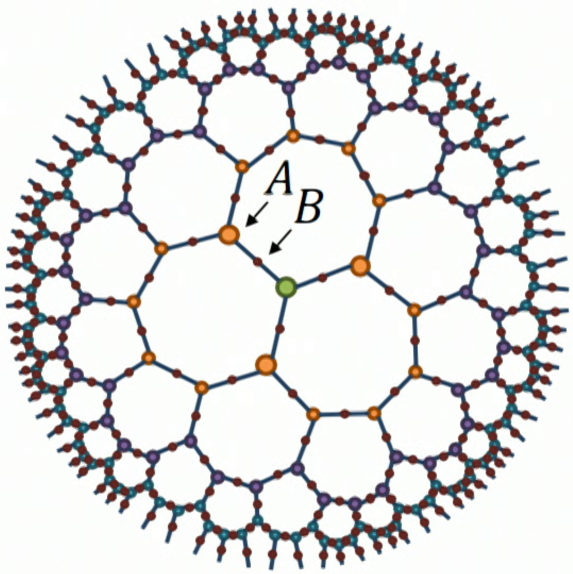


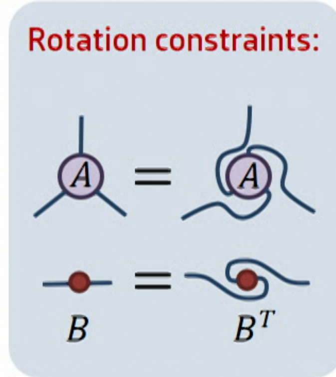
Preservation of locality

- Network **does not** have any **structurally trivial** correlation functions (between pairs of 2-site boundary regions)
- **Algebraic decay** of correlations follows geometrically (geodesic path lengths are same as MERA)

Hyper-invariant networks

{7,3} hyper-invariant network





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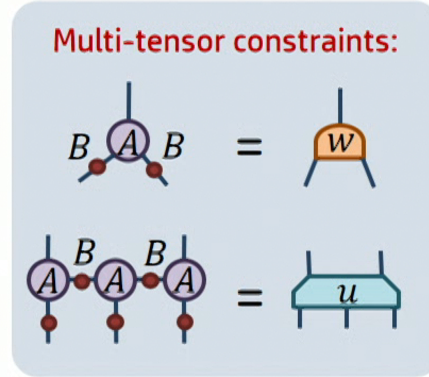

Preservation of locality

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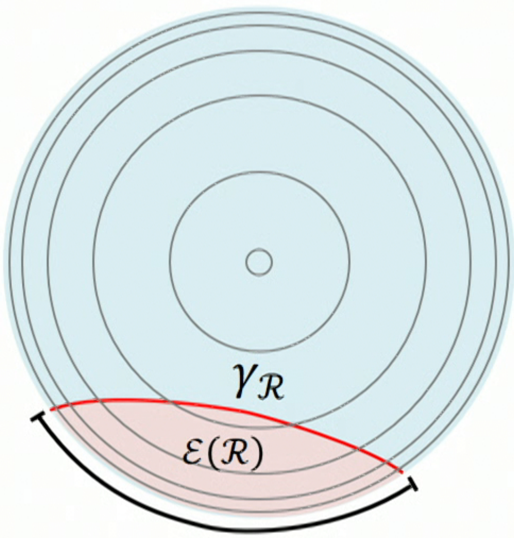
- Algebraic decay of correlations follows geometrically (geodesic path lengths are same as MERA)

- What are the implications of these constraints? (causal properties?)
- How can we find solutions to the constraints for tensors A and B?

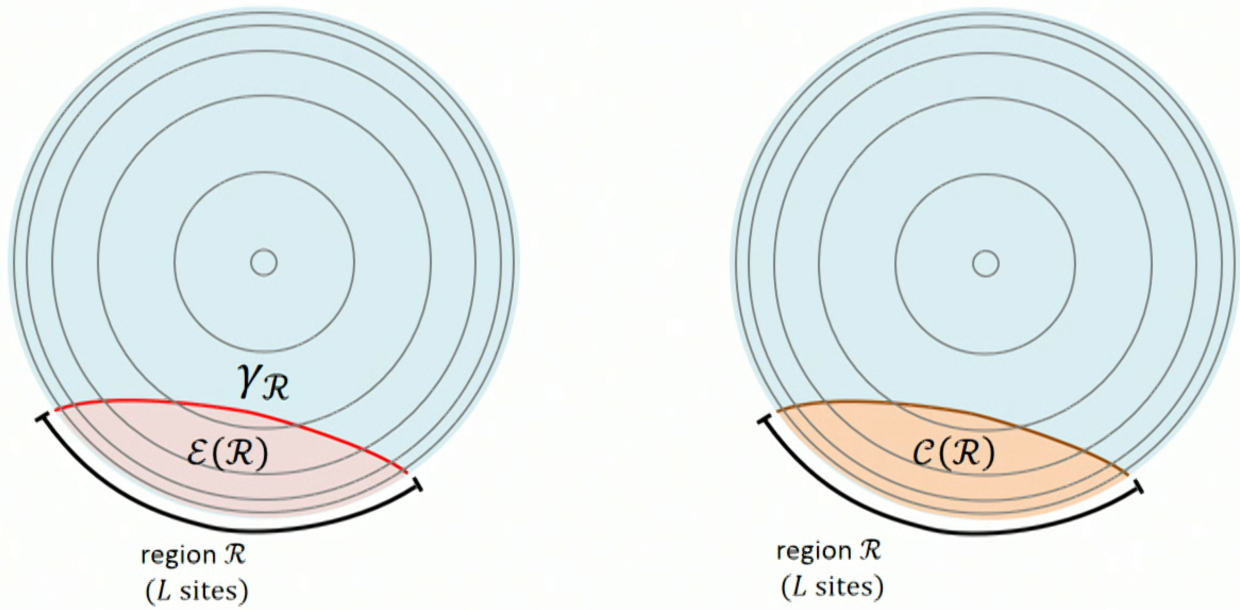
Hyper-invariant networks: causal properties



$\gamma_{\mathcal{R}}$ **minimal surface:** surface with same boundary as R that transects minimum number of bulk indices

$\mathcal{E}(\mathcal{R})$ **entanglement wedge:** set of tensors in the region bounded by $\gamma_{\mathcal{R}}$ and R

Hyper-invariant networks: causal properties



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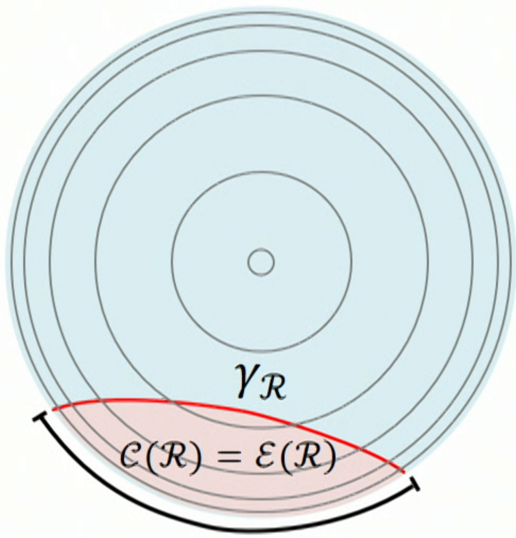
Hyper-invariant networks: causal properties



Holographic causality: for a continuous boundary region R of a hyper-invariant network, the causal cone $\mathcal{C}(\mathcal{R})$ is approximately coincident* with the entanglement wedge $\mathcal{E}(\mathcal{R})$

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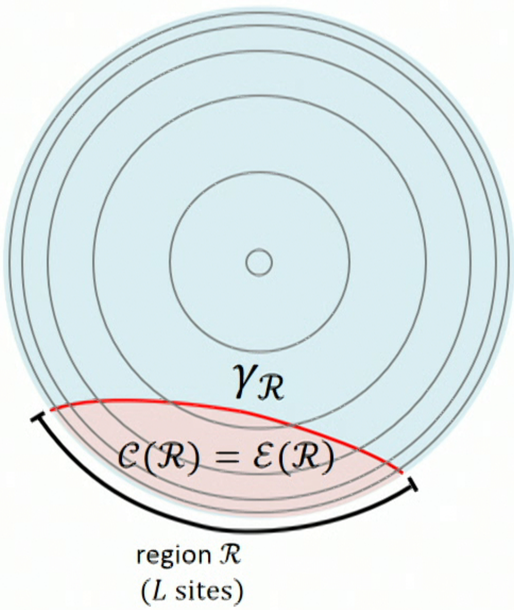

Holographic causality: for a continuous boundary region R of a hyper-invariant network, the causal cone $\mathcal{C}(\mathcal{R})$ is approximately coincident* with the entanglement wedge $\mathcal{E}(\mathcal{R})$

*Disclaimer: for most regions $\mathcal{C}(\mathcal{R}) = \mathcal{E}(\mathcal{R})$, however there exists some regions R for which $\mathcal{C}(\mathcal{R})$ is slightly larger than $\mathcal{E}(\mathcal{R})$.

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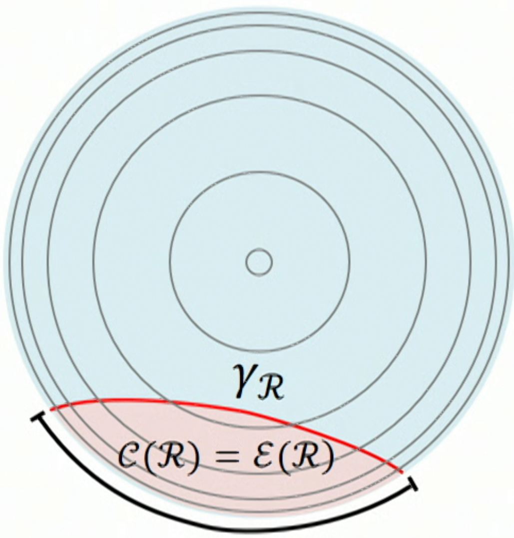
Hyper-invariant networks: causal properties



Holographic causality: for a continuous boundary region R of a hyper-invariant network, the causal cone $\mathcal{C}(\mathcal{R})$ is approximately coincident* with the entanglement wedge $\mathcal{E}(\mathcal{R})$

Causal cones are geometric!

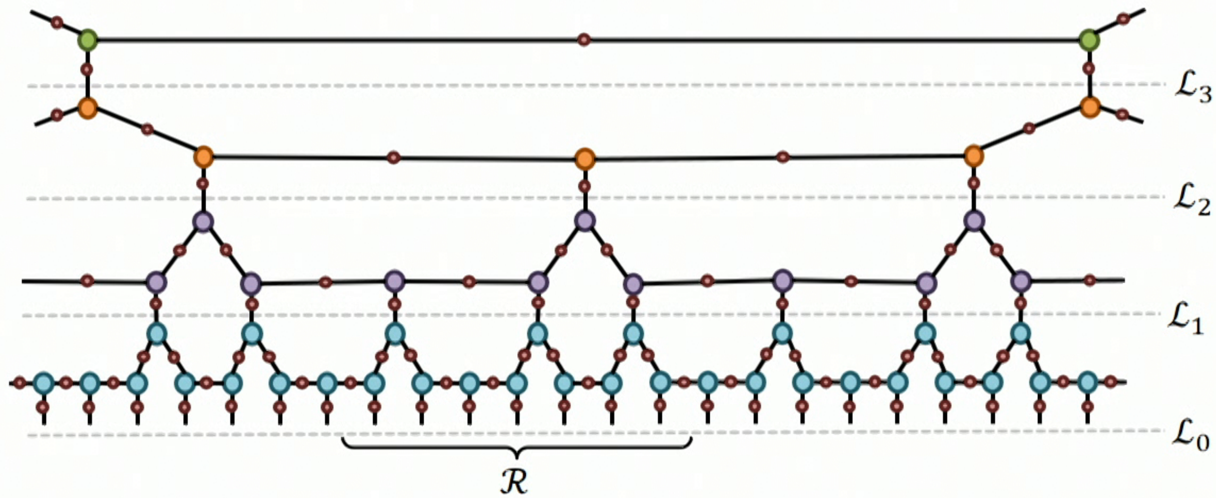
Consequence of:

Bulk uniformity + Shiftable centre of orthogonality

Not true for MERA!

Hyper-invariant networks: causal properties

Example of holographic causality:



Hyper-invariant networks: causal properties

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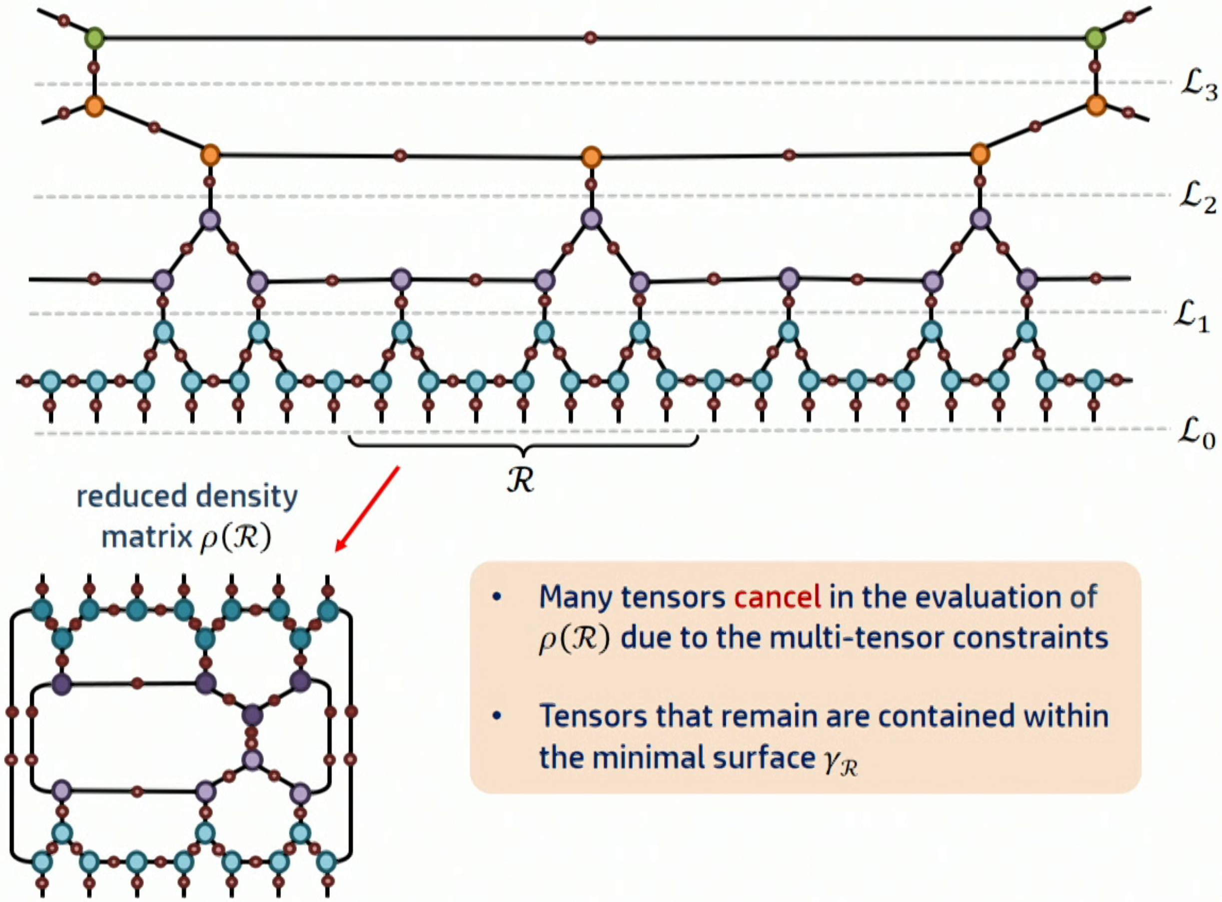

- Many tensors cancel in the evaluation of $\rho(\mathcal{R})$ due to the multi-tensor constraints
- Tensors that remain are contained within the minimal surface $\gamma_{\mathcal{R}}$

Hyper-invariant networks: causal properties

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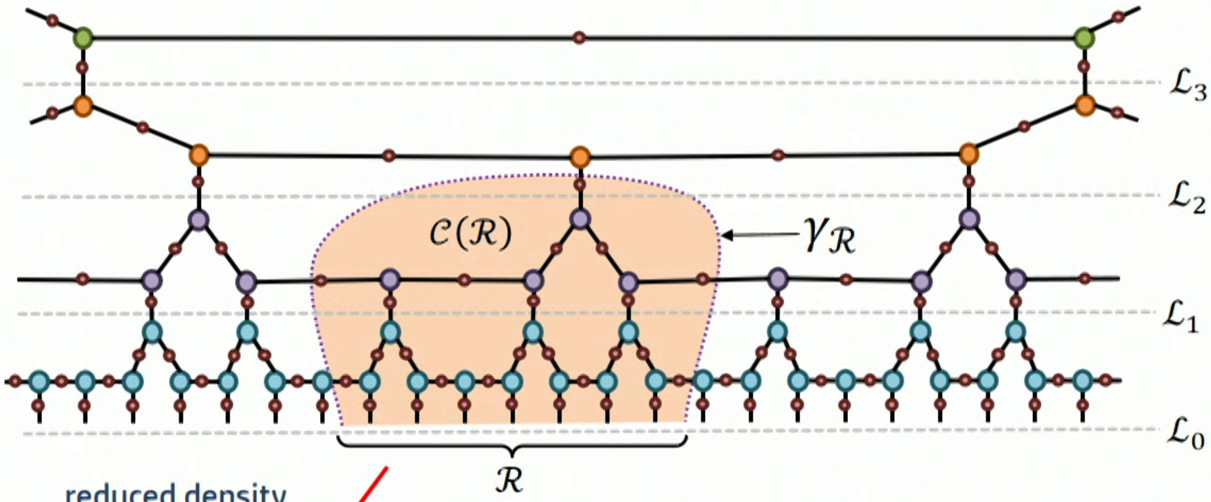
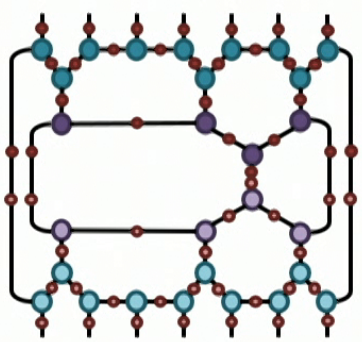


reduced density matrix $\rho(\mathcal{R})$



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Hyper-invariant networks: causal properties

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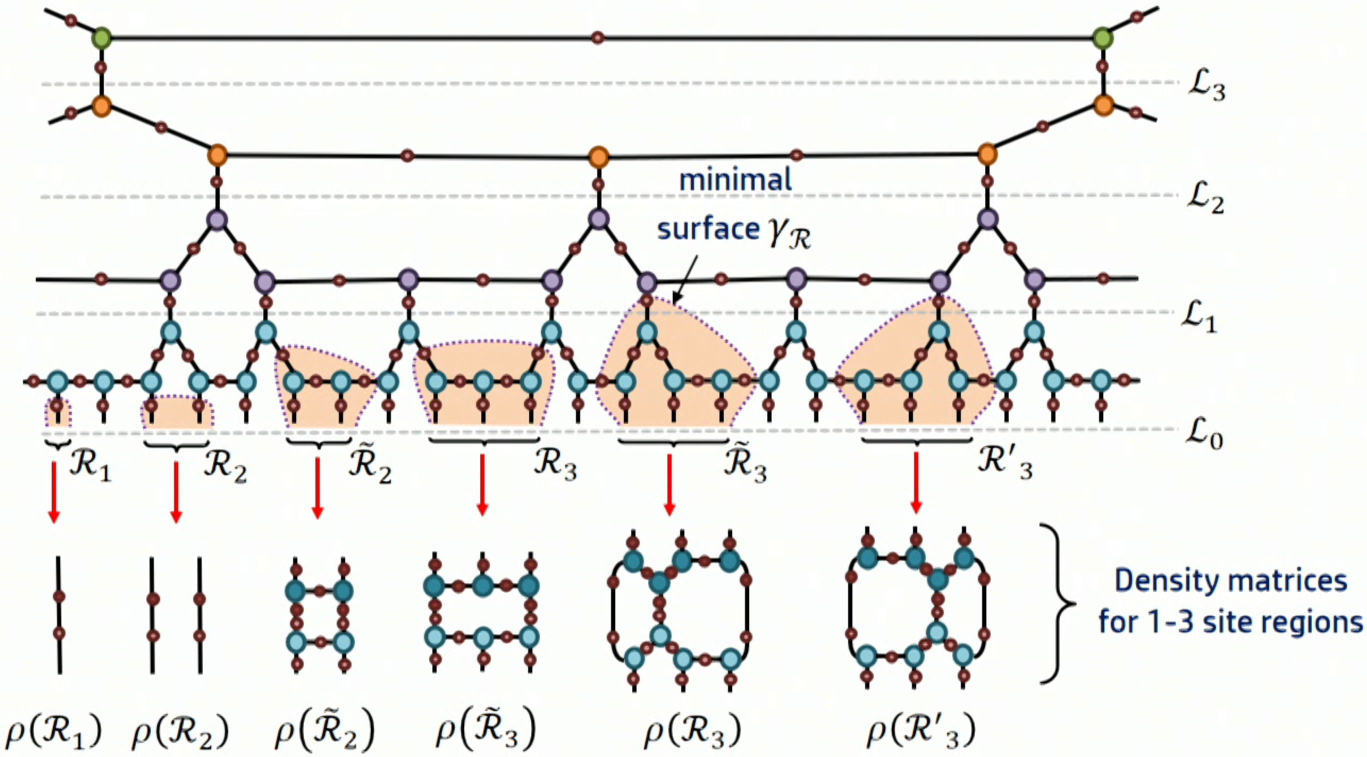
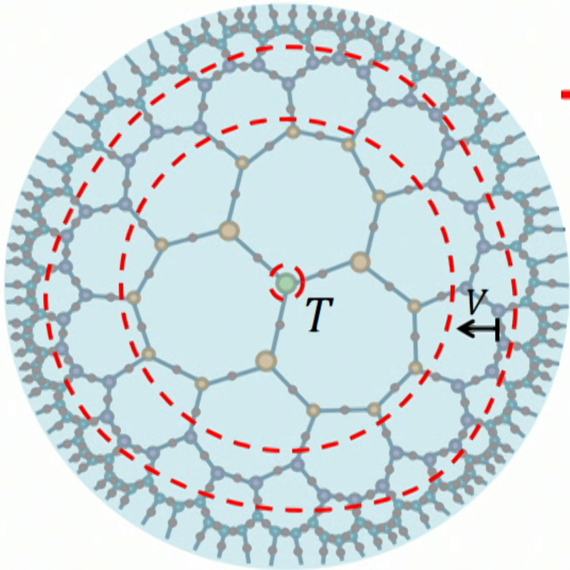


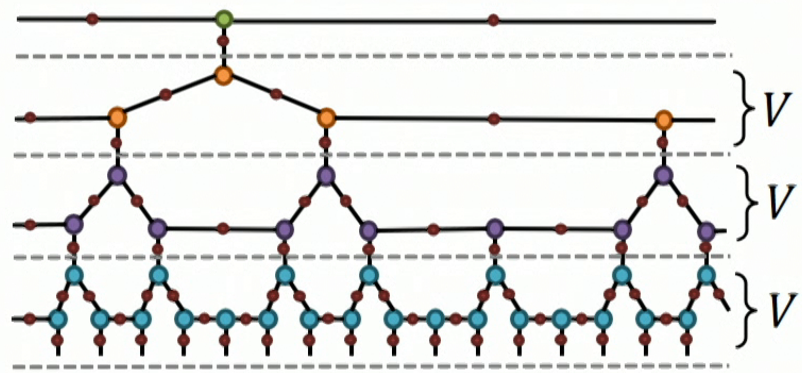
All obey holographic causality!

How can we understand this?

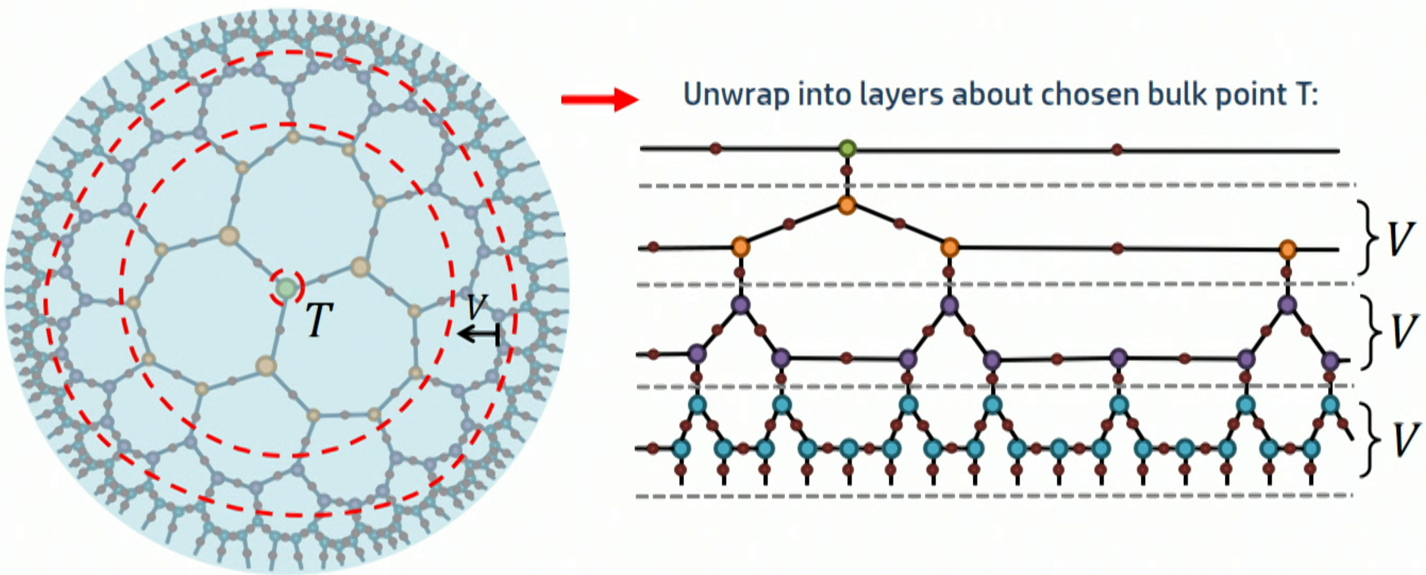
Hyper-invariant networks: causal properties



Unwrap into layers about chosen bulk point T:



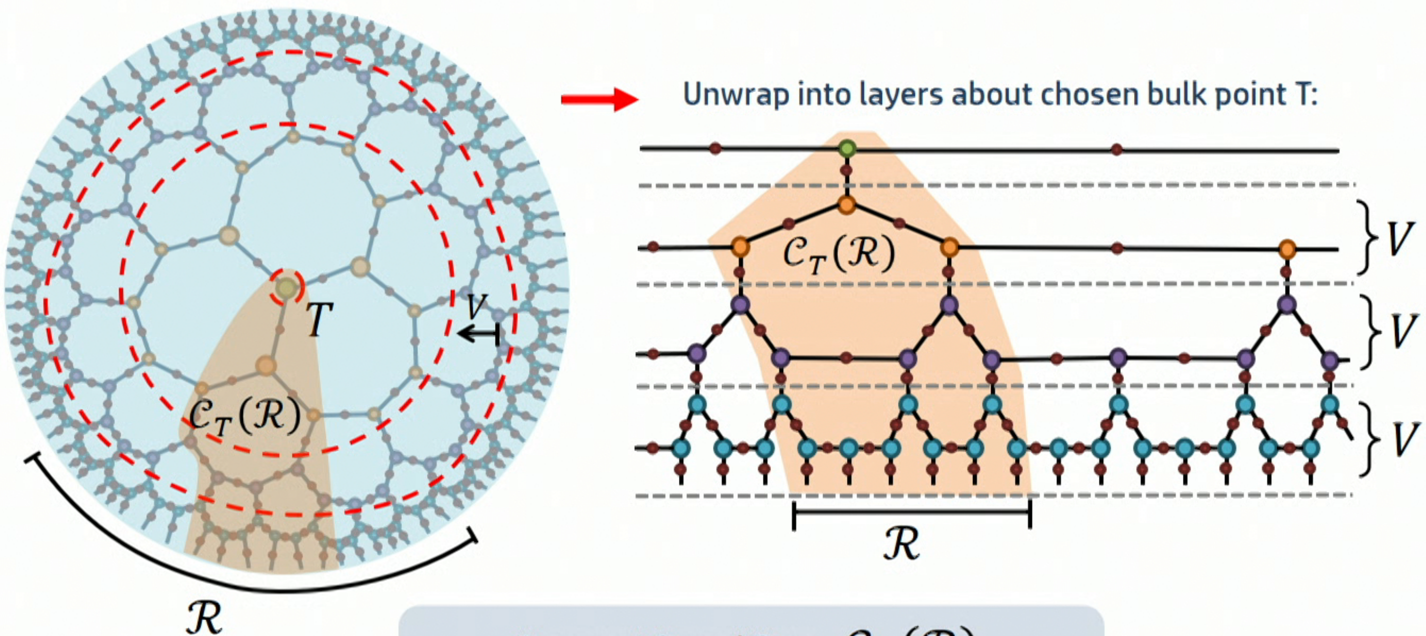
Hyper-invariant networks: causal properties



Apparent Causal Cone $\mathcal{C}_T(\mathcal{R})$

Causal cone that arises from a layer-by-layer analysis (around bulk point T)

Hyper-invariant networks: causal properties



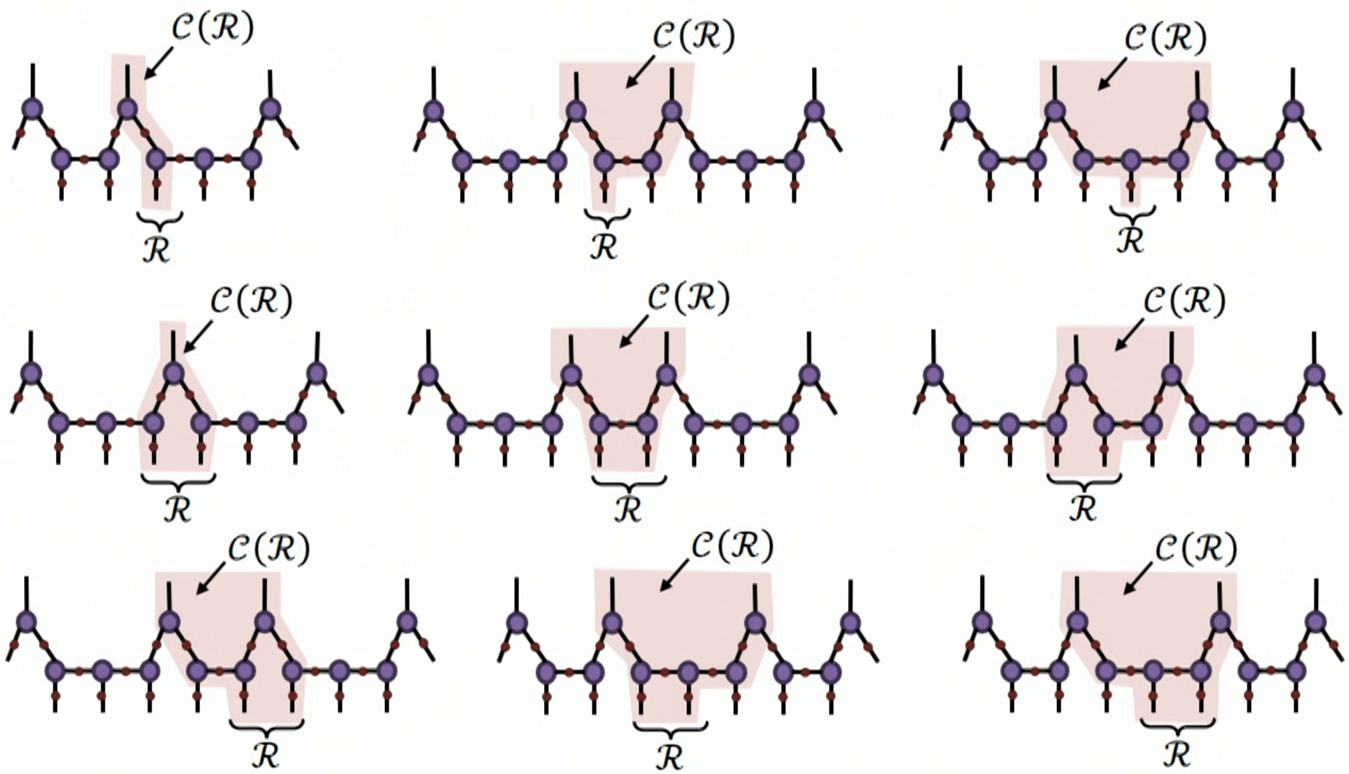
Apparent Causal Cone $\mathcal{C}_T(\mathcal{R})$

Causal cone that arises from a layer-by-layer analysis (around bulk point T)

- Apparent causal cone $\mathcal{C}_T(\mathcal{R})$ depends on choice of T
- For all T: $\mathcal{C}(\mathcal{R}) \subseteq \mathcal{C}_T(\mathcal{R})$

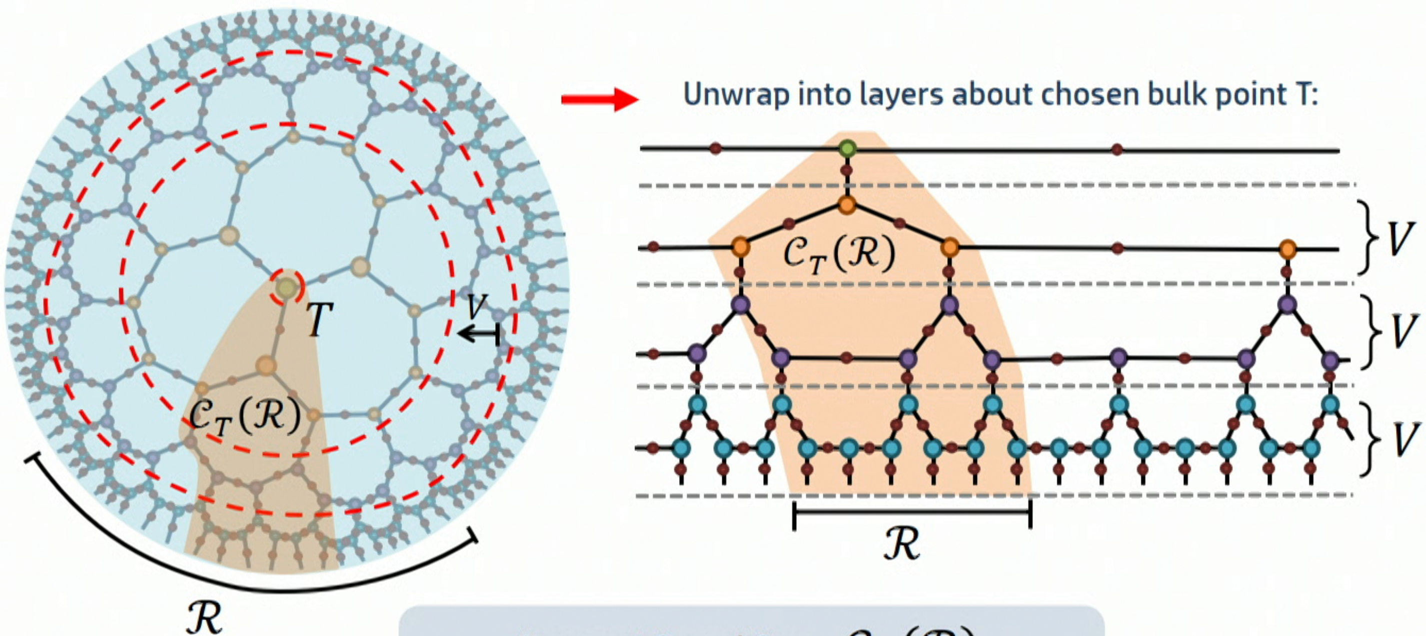
Hyper-invariant networks: constraints

Causal cones through a single layer (from minimal support grouping):



Causal cones through a layer of the hyper-invariant network look like causal cones through a layer of MERA

Hyper-invariant networks: causal properties



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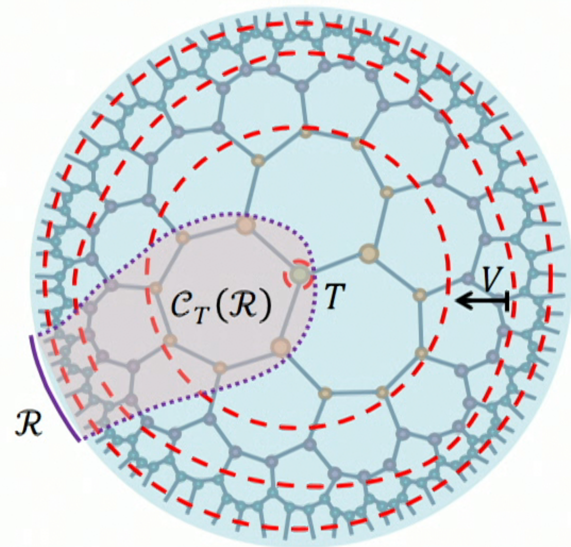
Hyper-invariant networks: causal properties

Argument for holographic causality:



Hyper-invariant networks: causal properties

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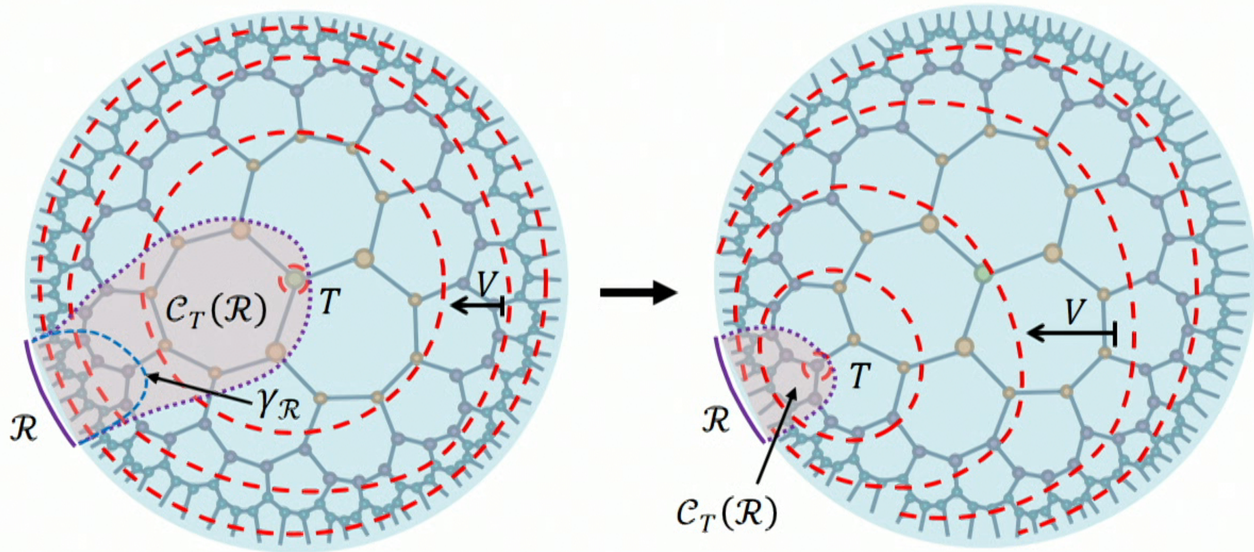

- Choose center point of layering T at the apex of the minimal surface
- Apparent causal cone reduces to true causal cone (equals entanglement wedge)

$$\mathcal{C}_T(\mathcal{R}) = \mathcal{C}(\mathcal{R}) \approx \mathcal{E}(\mathcal{R})$$

Hyper-invariant networks: causal properties

Freedom to chose center in hyper-invariant MERA is the same as choice of orthogonality center in MPS!

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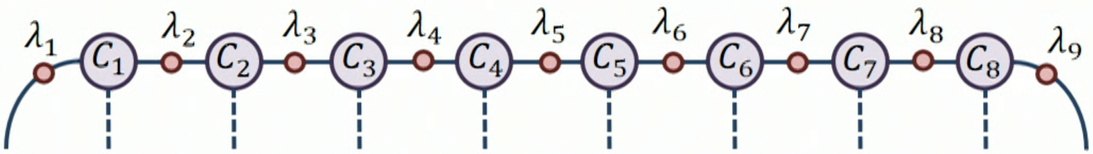
Consider a canonical form MPS (Schmidt form across all L/R partitions):



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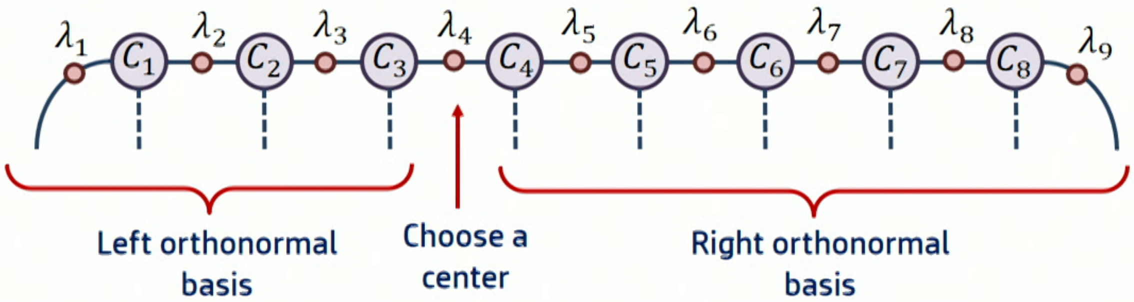

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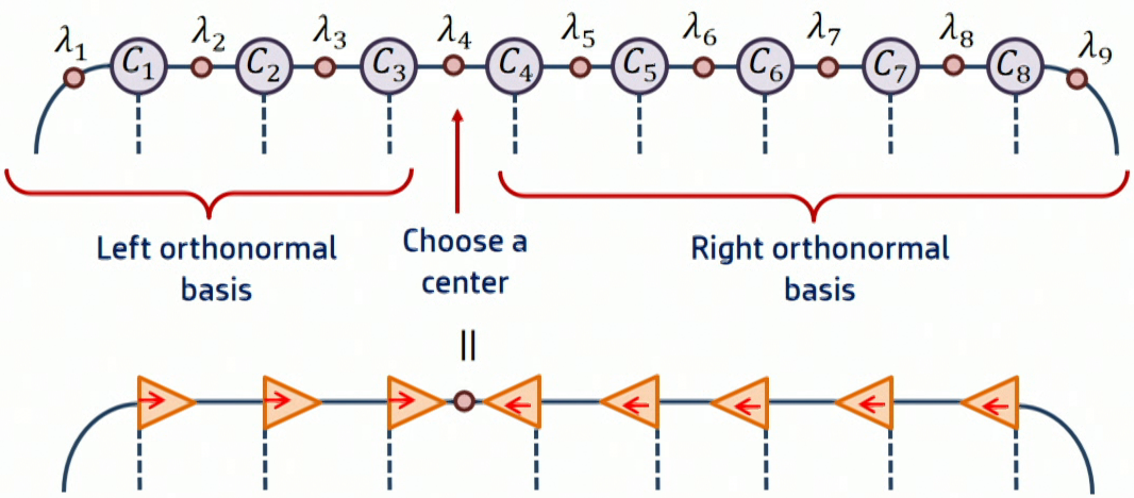
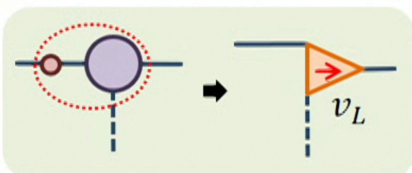
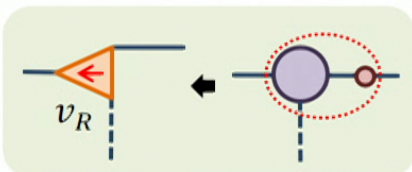




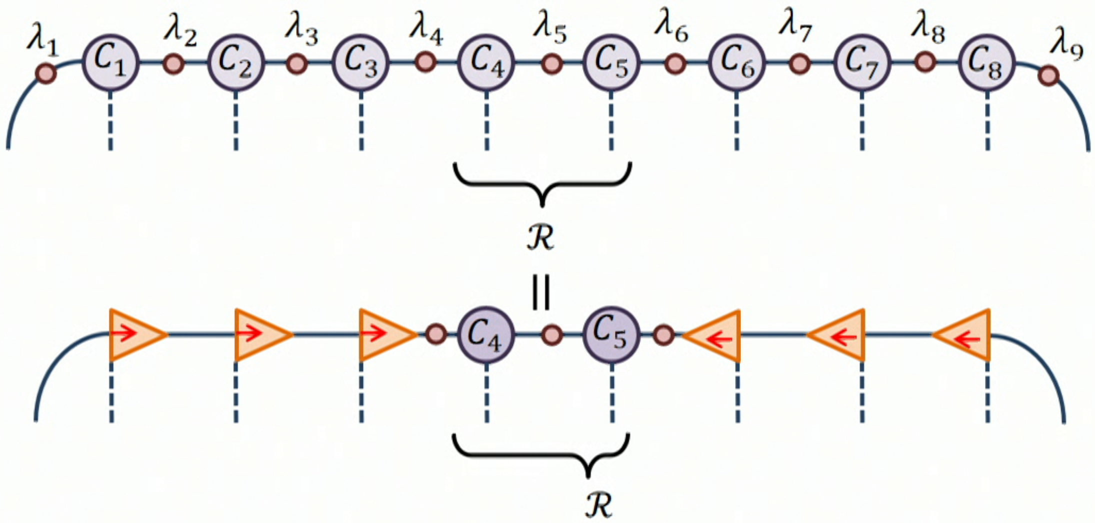
Multiplying weights from the left gives right facing isometry



Multiplying weights from the right gives left facing isometry

Hyper-invariant networks: causal properties

Canonical form MPS:



Hyper-invariant networks: causal properties

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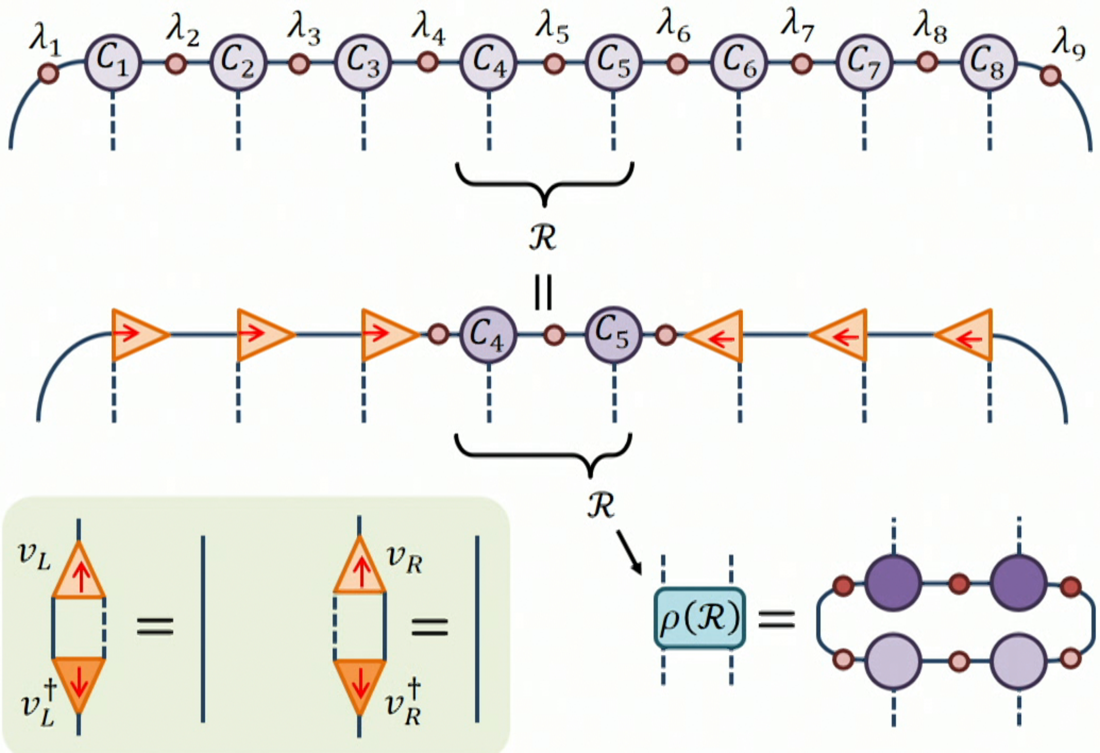
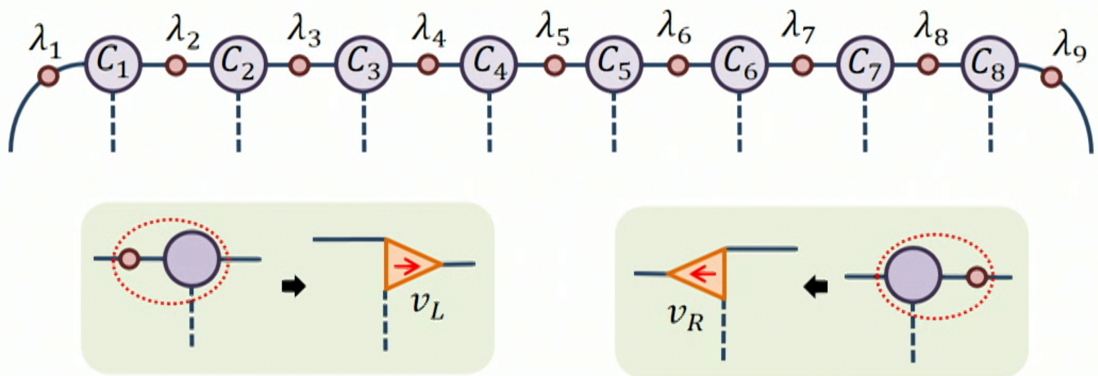


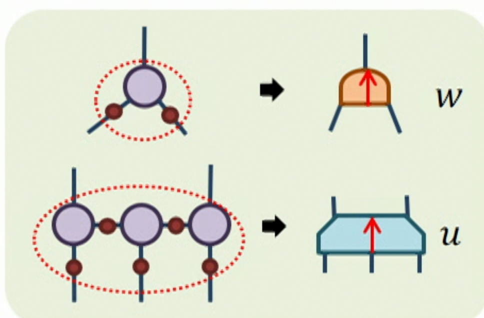
Reduced density matrix only depends on local tensors

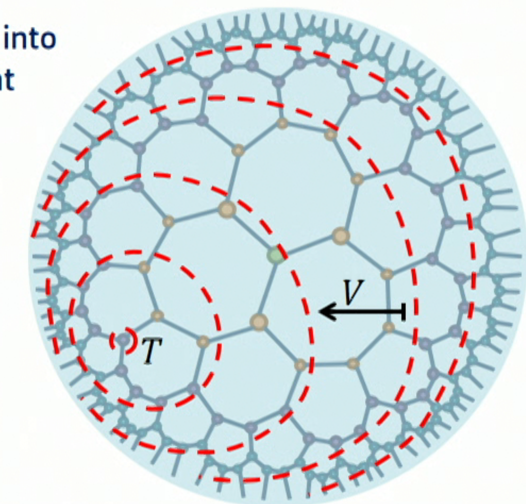
Hyper-invariant networks: causal properties

Canonical form MPS: can organise into isometries about any chosen point



Hyper-invariant network: can organise into isometric layers about any chosen point

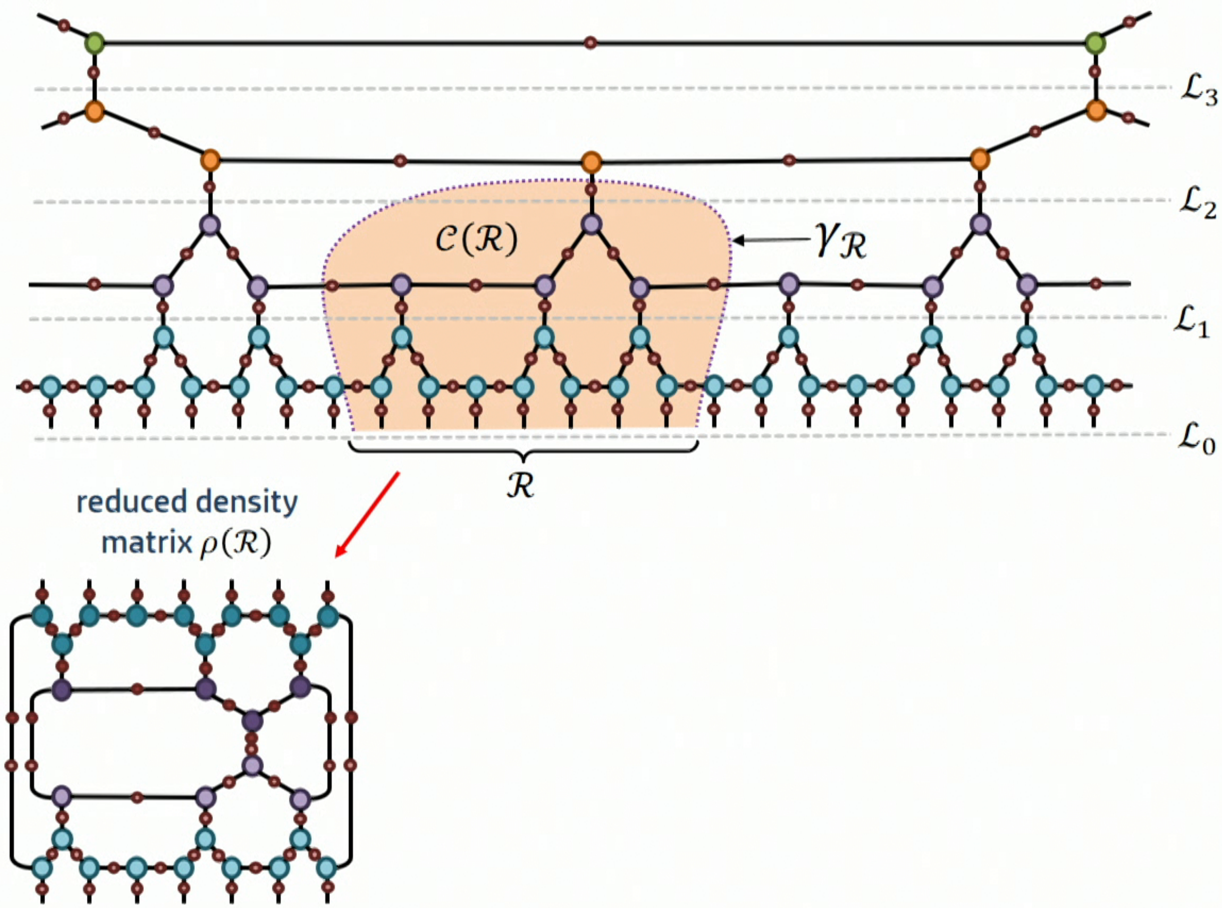




Generalization of canonical form from 1D line to hyperbolic disk!

Hyper-invariant networks: causal properties

Other implications of holographic causality?



Hyper-invariant networks: causal properties

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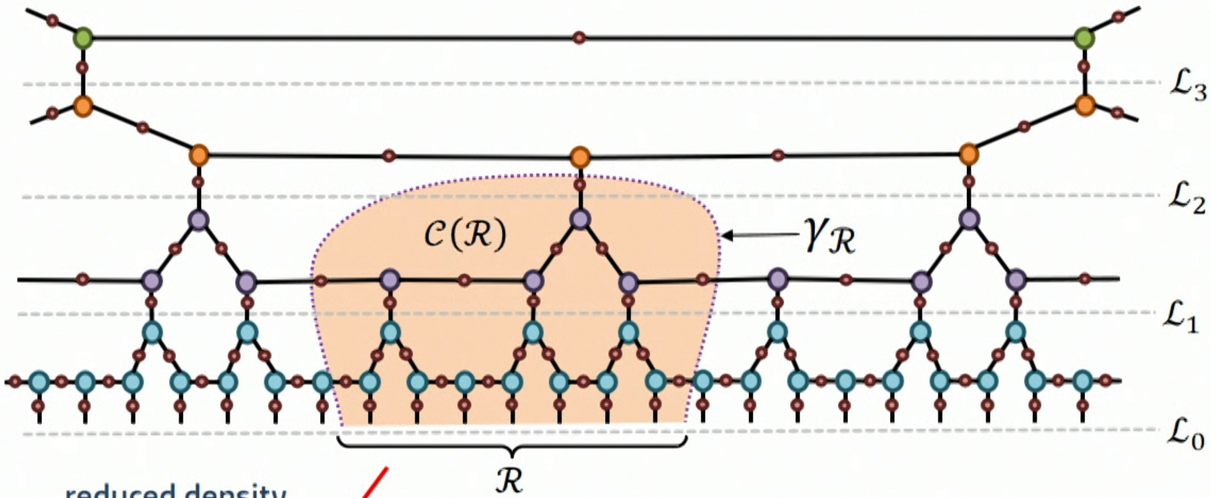
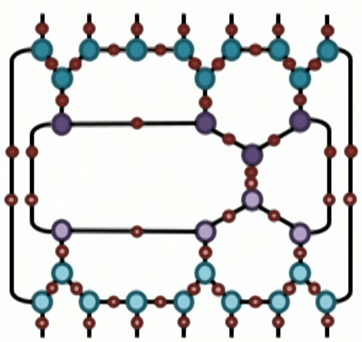


reduced density matrix $\rho(\mathcal{R})$



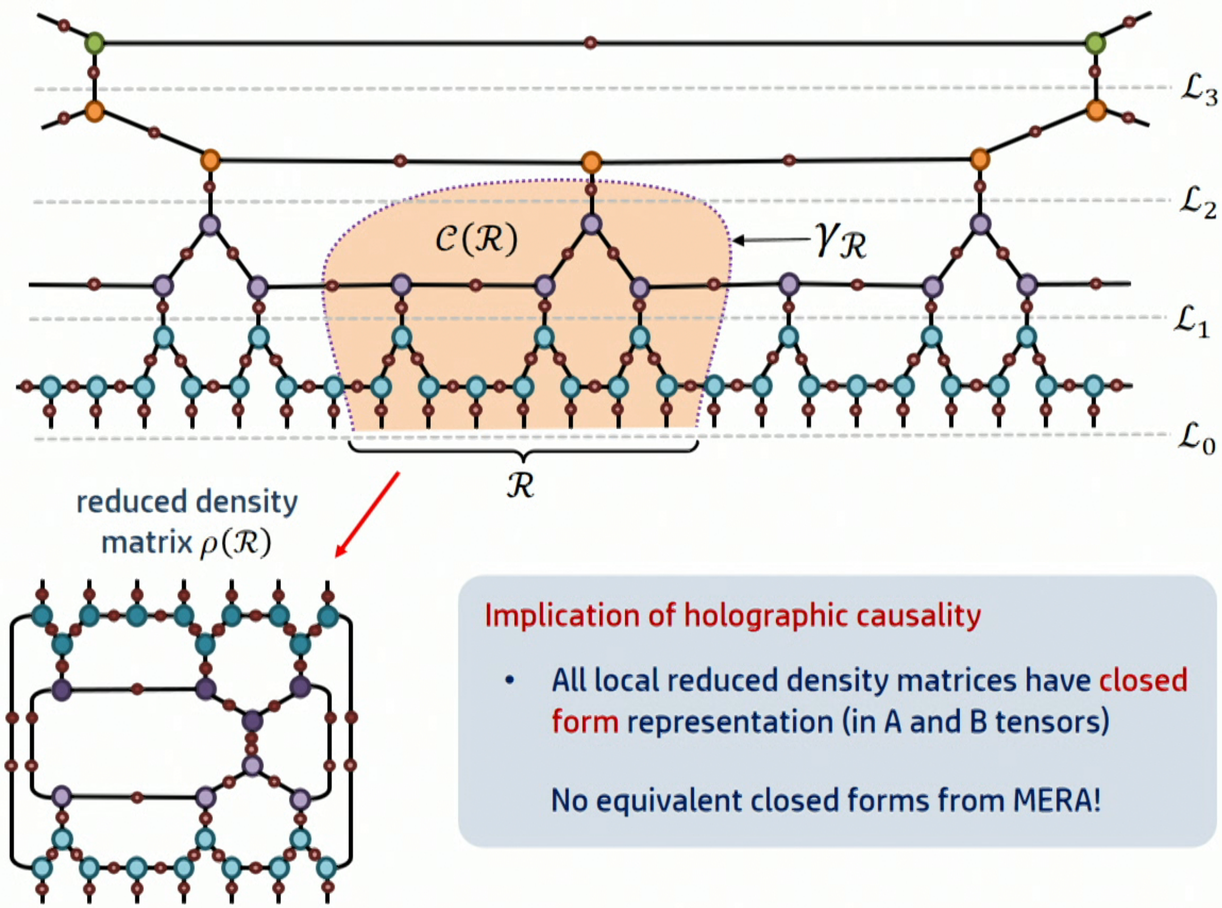
Implication of holographic causality

- All local reduced density matrices have closed form representation (in A and B tensors)

No equivalent closed forms from MERA!

Hyper-invariant networks: causal properties

Other implications of holographic causality?



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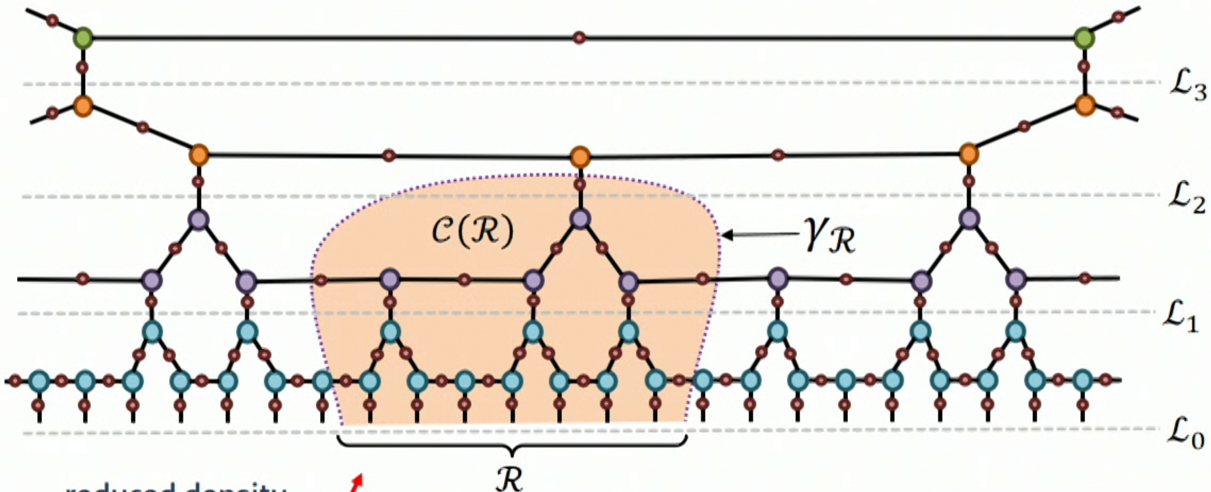
Hyper-invariant networks: causal properties



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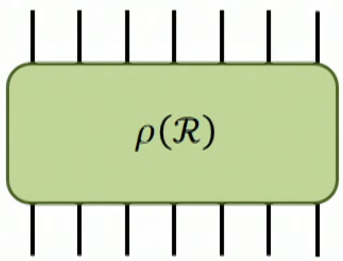
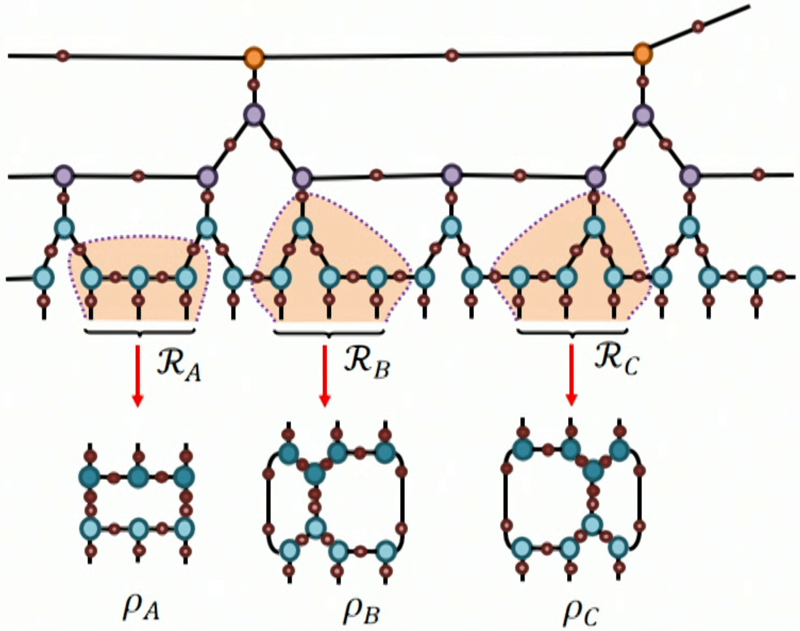


Sub-region duality: give reduced density matrix $\rho(\mathcal{R})$, can we recover the bulk tensors A, B from within the entanglement wedge?

Hyper-invariant networks: causal properties

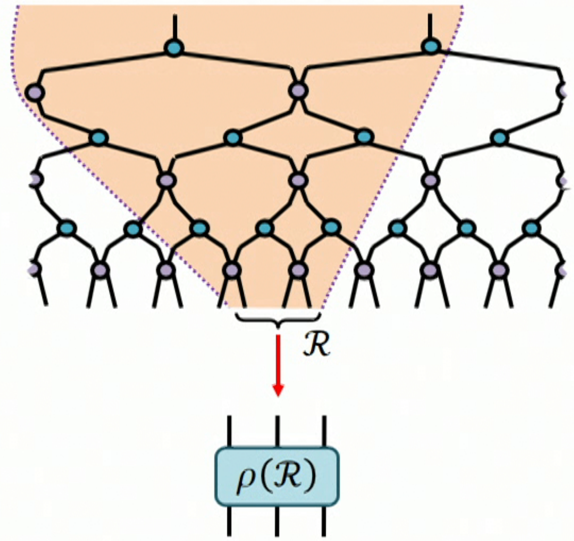
Hyper-invariant tensor network



- The 3-site density matrix is one of three possibilities

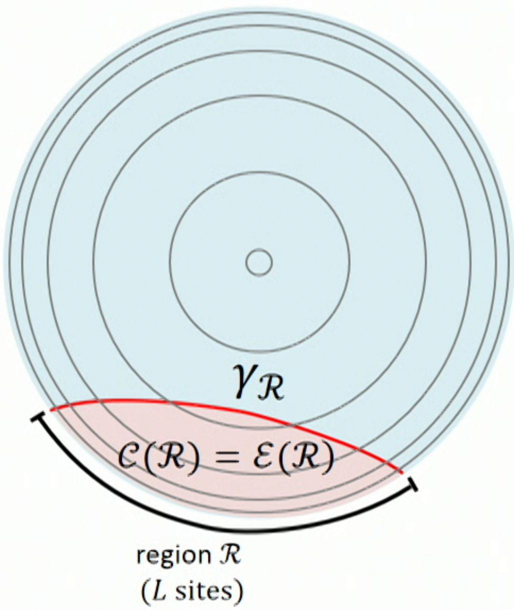
Hyper-invariant tensor network is better for describing translation invariant quantum states than MERA?

Scale-invariant MERA



- Each boundary region R has unique bulk causal cone
- Different 3-site density matrix $\rho(\mathcal{R})$ for each 3-site region R

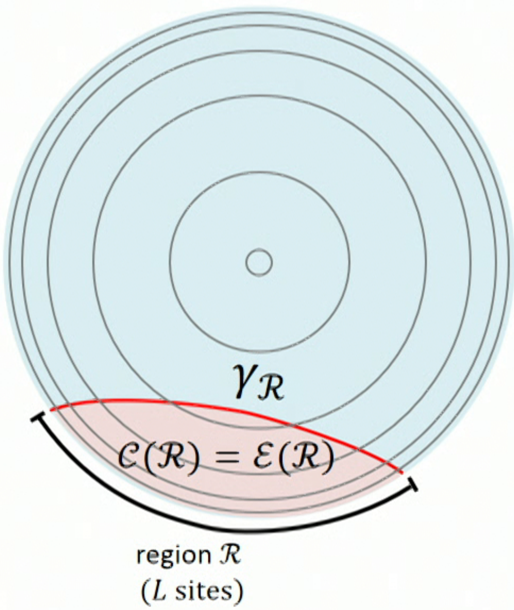
Hyper-invariant networks: causal properties



Holographic causality: for a continuous boundary region R of a hyper-invariant network, the causal cone $\mathcal{C}(\mathcal{R})$ is approximately coincident* with the entanglement wedge $\mathcal{E}(\mathcal{R})$

Other properties of hyper-invariant networks???

Hyper-invariant networks: causal properties



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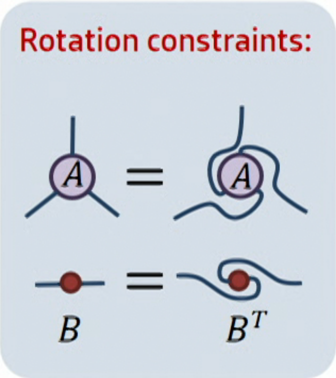
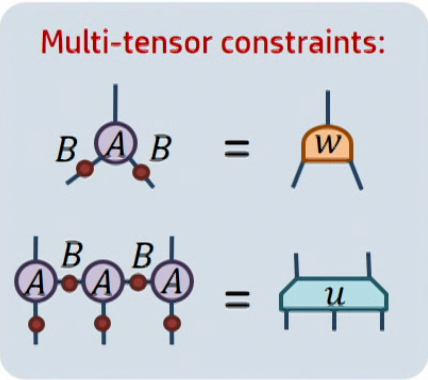
Other properties of hyper-invariant networks???

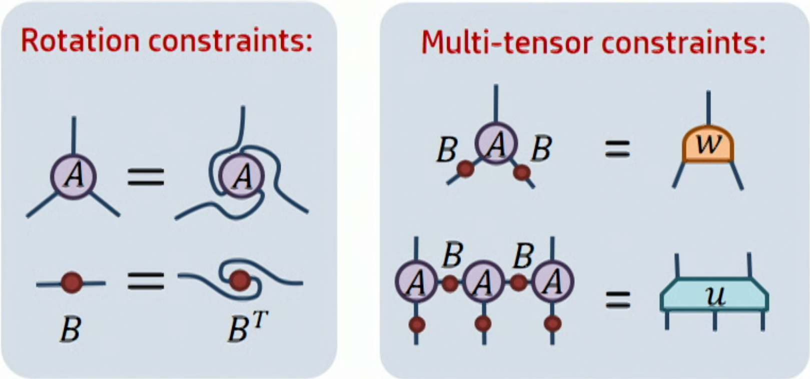
Hyper-invariant networks: parameterizations





Difficult set of constraints! How to solve?

Hyper-invariant networks: parameterizations



Difficult set of constraints! How to solve?

There exists many families of solutions that satisfy the following properties:

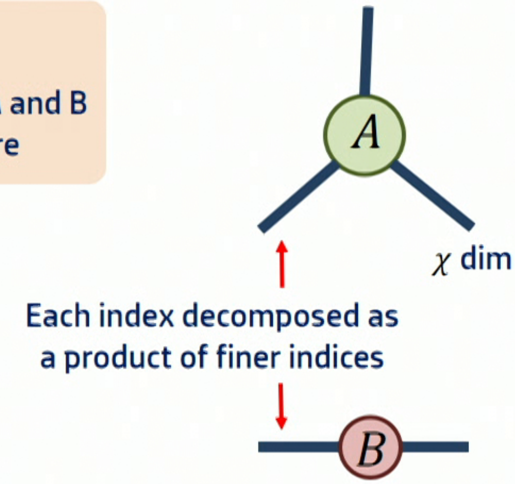
- Parameterized by a set of continuous variables $\{\theta_1, \theta_2, \theta_3, \dots, \theta_n\}$
- Number n of variables increases with bond dimension
- Entanglement / correlations are θ-dependent (and non-trivial)

Most general solution???
Best parameterization for practical purposes???

Hyper-invariant networks: parameterizations

Example solution family:

Strategy: build tensors A and B from some finer structure



Hyper-invariant networks: parameterizations

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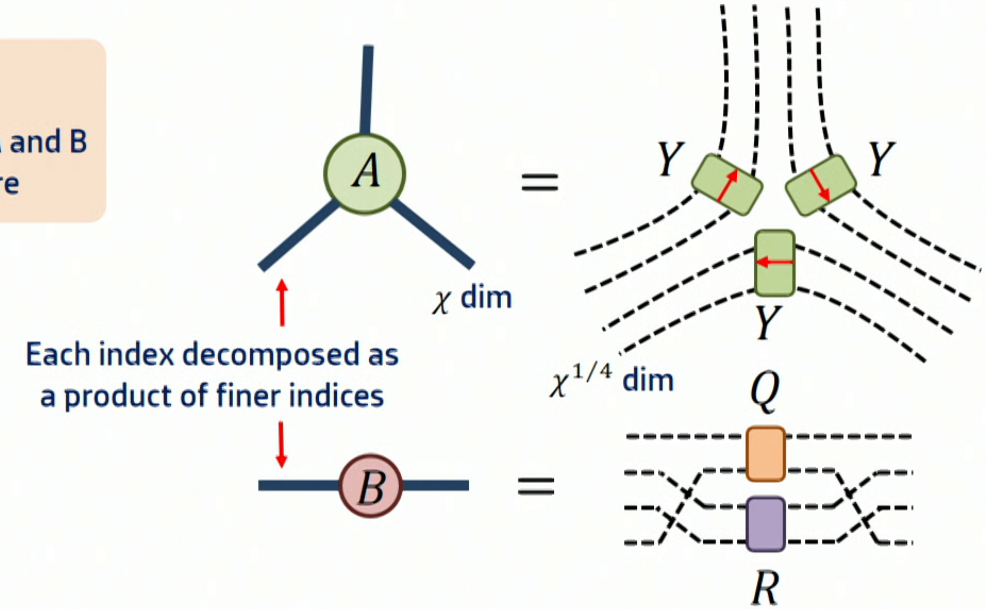
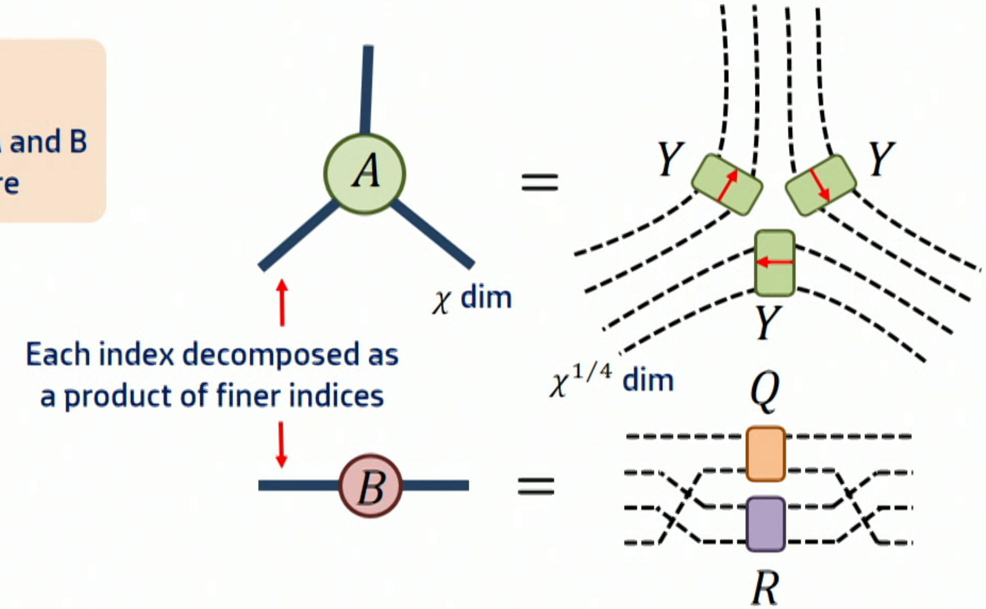


Hyper-invariant networks: parameterizations

Example solution family:

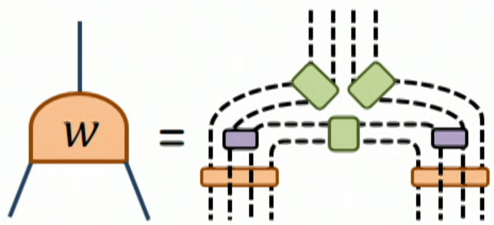
Strategy: build tensors A and B from some finer structure

Arrive at **single tensor** constraints on finer tensors Y, Q and R (doubly - unitary)

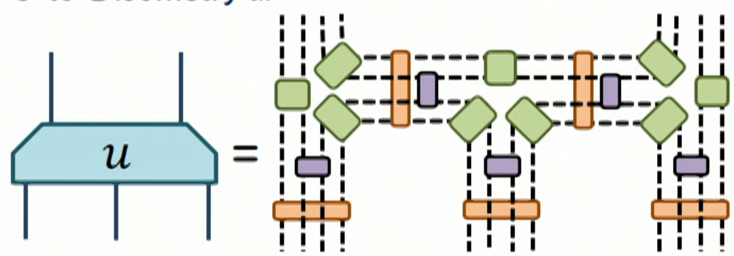


Multi-tensor constraints:

2-to-1 isometry *w:*



3-to-2 isometry *u:*



Hyper-invariant networks: parameterizations

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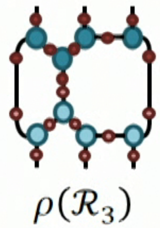

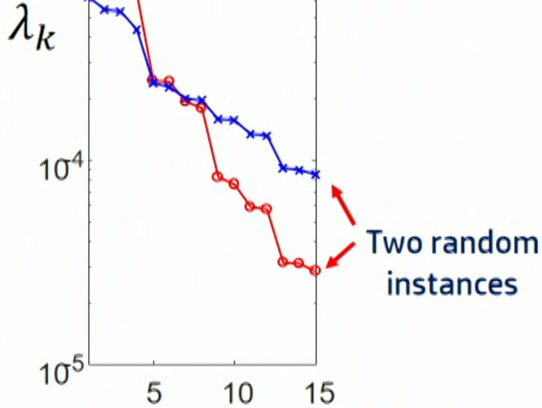

Hyper-invariant networks: parameterizations

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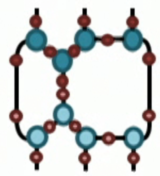
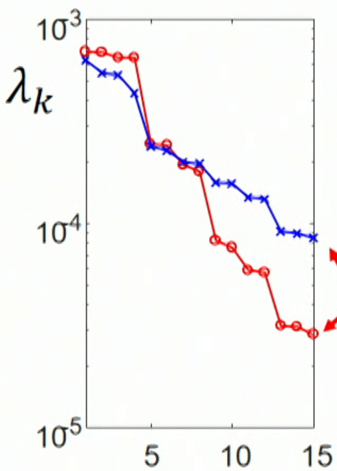
Spectrum of 3-site reduced density matrix

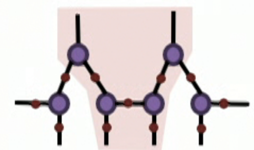


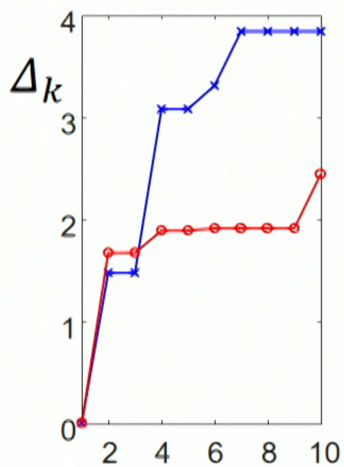
$\rho(\mathcal{R}_3)$



Two random instances

Scaling dimensions (diagonalise superoperator)

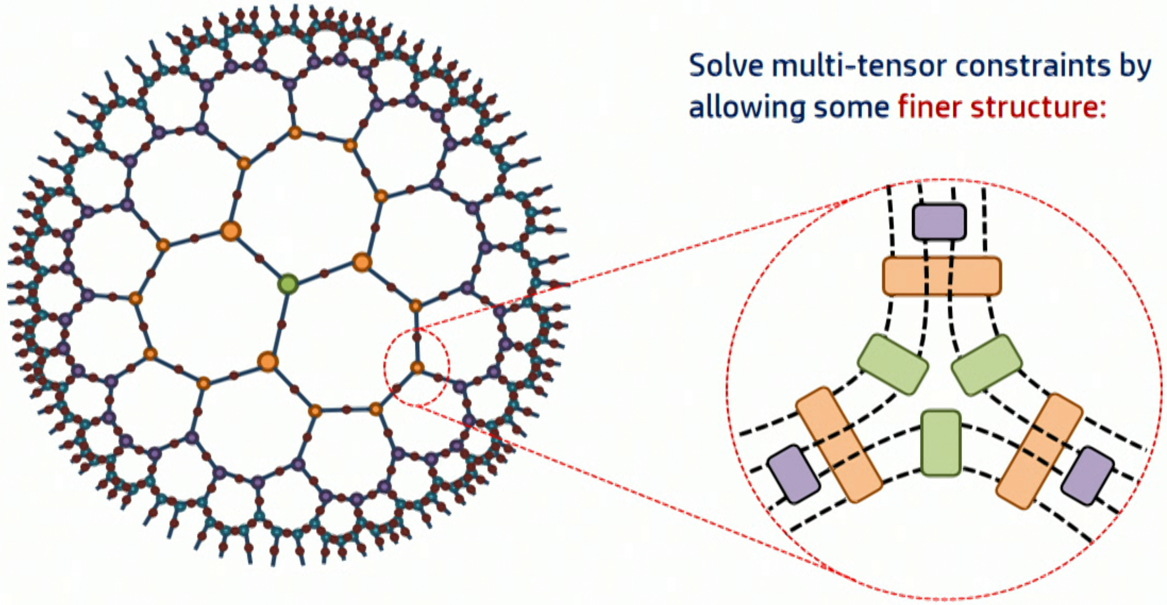




Polynomial decay of correlations:
$\langle \phi_i(0)\phi_j(r)\rangle \propto r^{-(\Delta_i+\Delta_j)}$

All superoperators found to have a non-trivial spectrum

Hyper-invariant networks: parameterizations

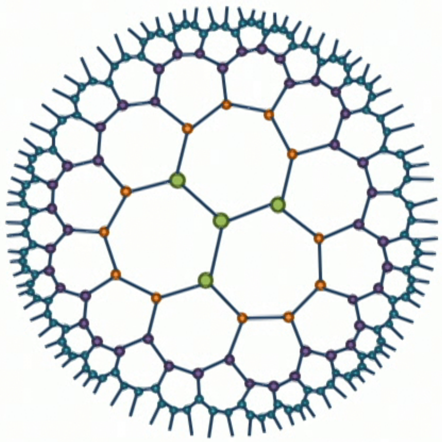


Solve multi-tensor constraints by allowing some finer structure:

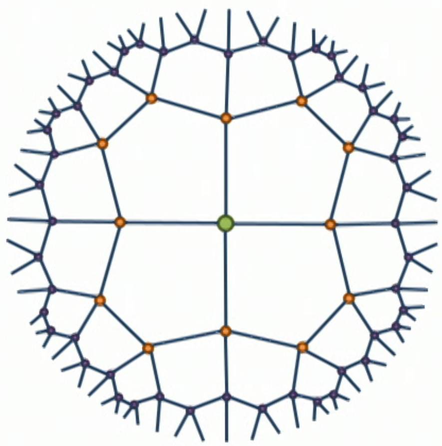
Consider more seriously? Bulk invariance as an emergent symmetry that is broken at short scales...

Hyper-invariant networks: summary

{7,3} Hyper-invariant network



{5,4} Hyper-invariant network



Hyper-invariant networks: summary

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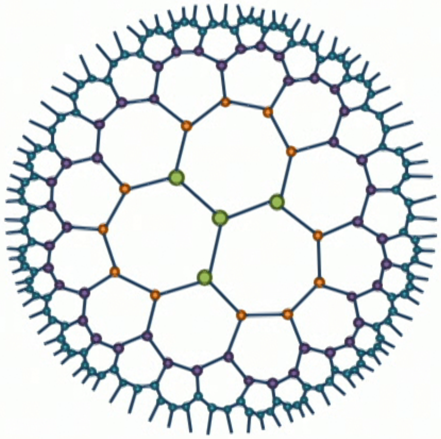
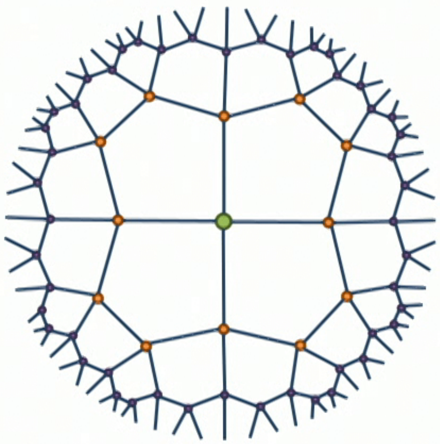
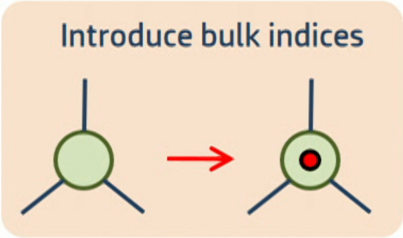


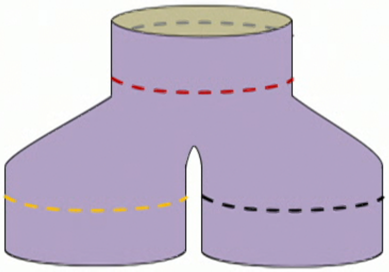
{5,4} Hyper-invariant network



Stuff to do:



Other geometries?



Hyper-invariant networks: summary

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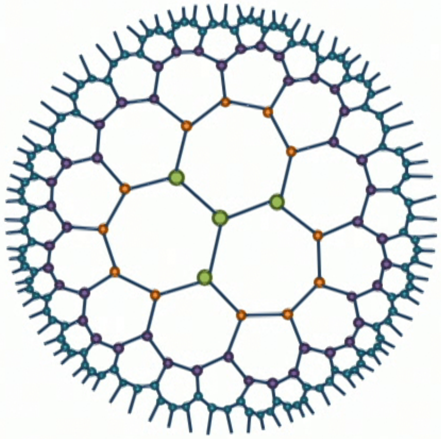
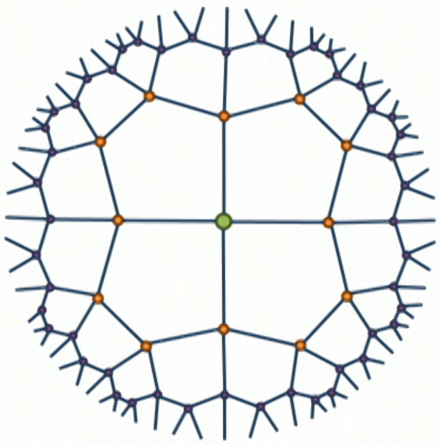
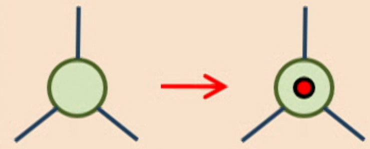


{5,4} Hyper-invariant network

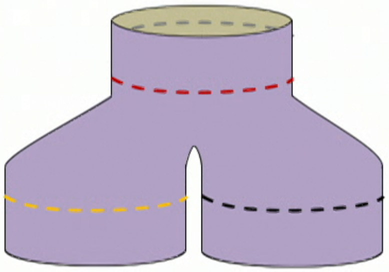


Stuff to do:

Introduce bulk indices



Other geometries?



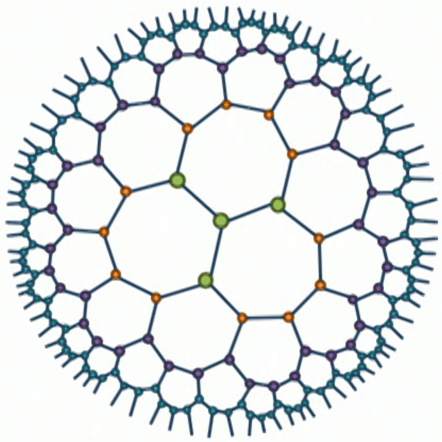
Characterization:

- other implications of bulk uniformity?
- interpretation in terms of holography?
- what class of quantum states can they describe?

Notice: perfect tensor codes can be understood as specific instances of hyper-invariant networks

Hyper-invariant networks: summary

{7,3} Hyper-invariant network



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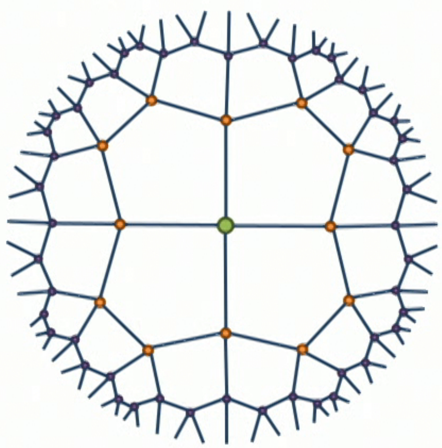
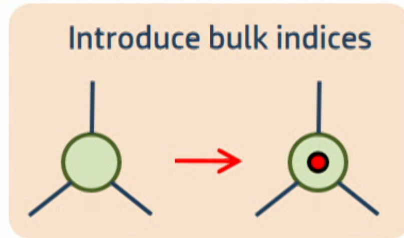


Stuff to do:



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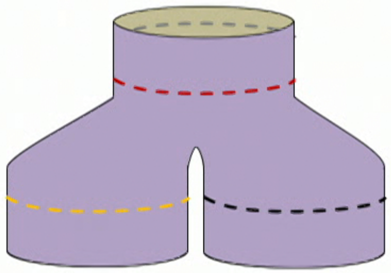

Characterization:

- other implications of bulk uniformity?
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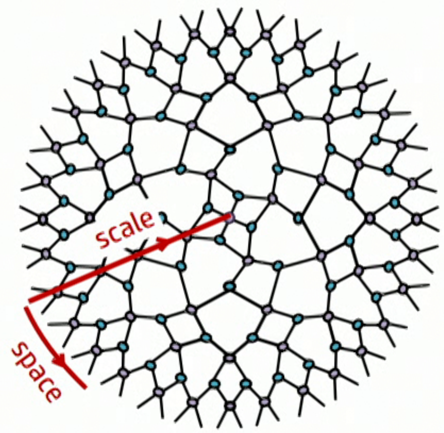
Notice: perfect tensor codes can be understood as specific instances of hyper-invariant networks

Practical:

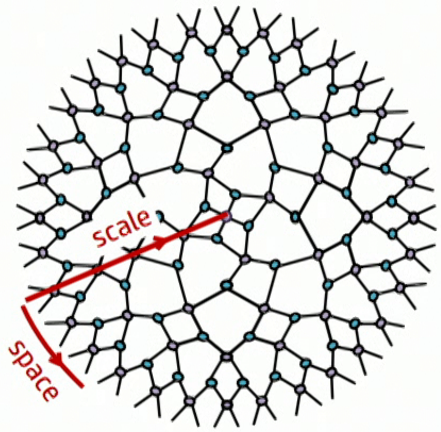
- best way to solve multi-tensor constraints?
- how to optimise numerically?
- ideas useful for other tensor network algorithms?

Hyper-invariant networks: summary



Scale-invariant MERA

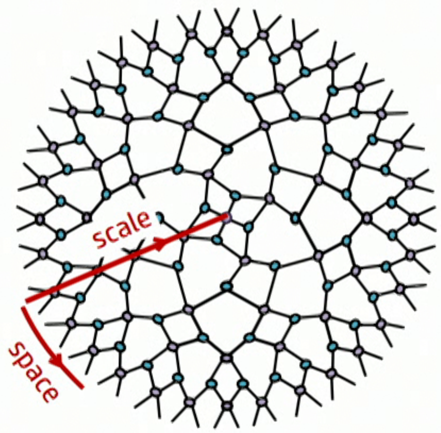
Hyper-invariant networks: summary



Scale-invariant MERA

- keep locality (and efficiency)
- keep interesting correlations
- add bulk uniformity

Hyper-invariant networks: summary



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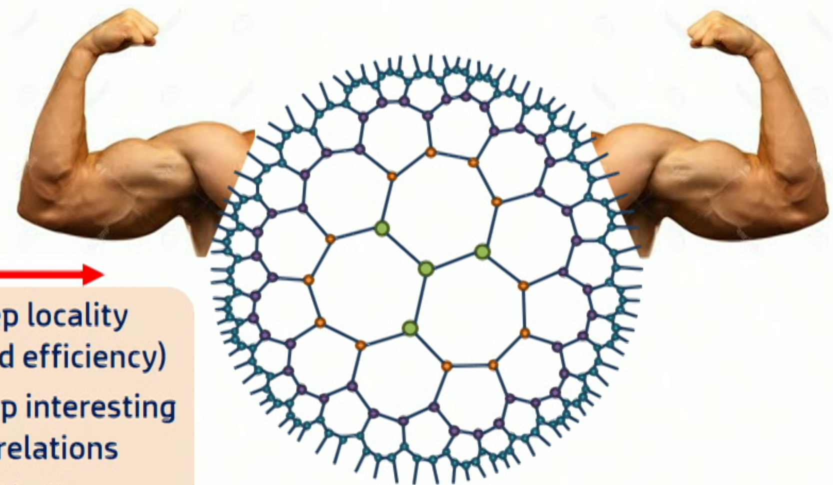
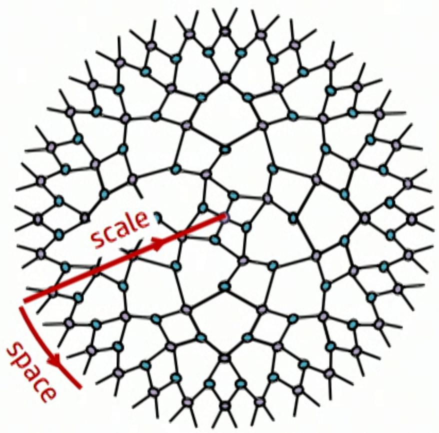


Hyper-invariant network
(aka MERA on steroids)

Hyper-invariant networks: summary



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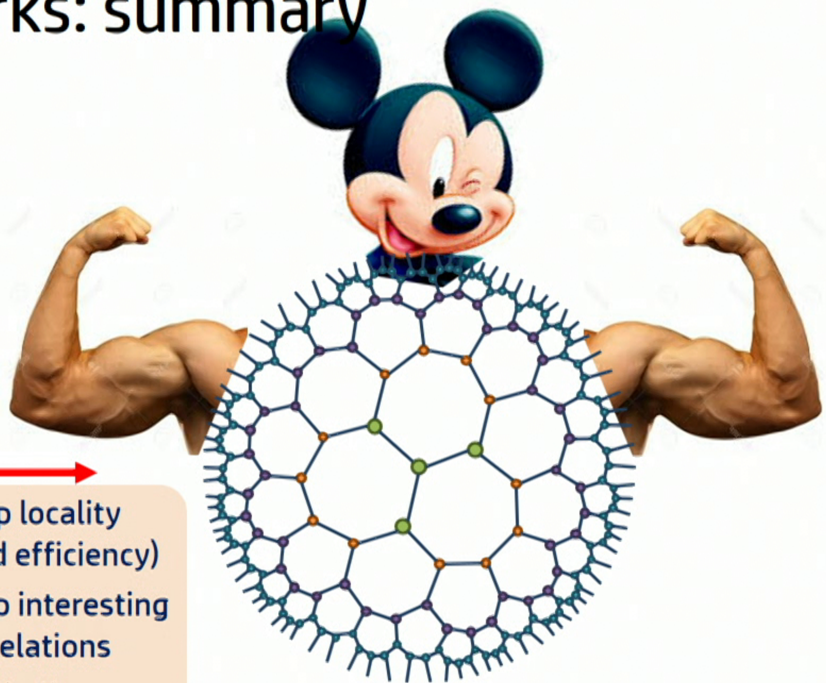

Hyper-invariant network
(aka MERA on steroids)

Thanks!