Title: Tensor network and (p-adic) AdS/CFT

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Abstract: We will describe how the reconstruction of a bulk operator can be organised systematically. With a suitable parametrisation, an analogue of the HKLL formula emerges, involving a smearing function satisfying a Klein Gordon equation in the graph. The parametrisation also allows us to read off interaction vertices, and build up loop diagrams systematically. When we interpret the Bruhat-Tits tree as a tensor network, we recover (partially) features of the p-adic AdS/CFT dictionary discussed recently in the literature.

Tensor Network and (p-adic) AdS/CFT

Perimeter Institute, April 21, 2017
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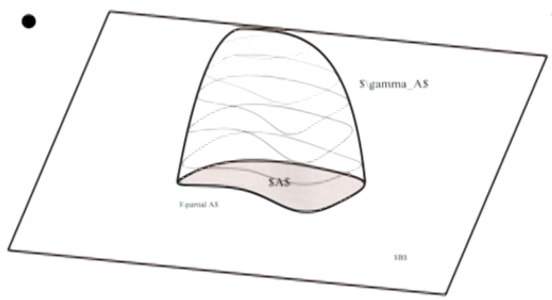
Work done in collaboration with Arpan Bhattacharya, Yang Lei, Wei Li

Overview

- Emergence of the HKLL relation in tensor networks
- An interacting theory in the bulk
- P-adic ads/cft and tensor network

Tensor network and AdS

- For MERA type networks, it recovers a Ryu-Takayanagi type entanglement entropy swingle

- 

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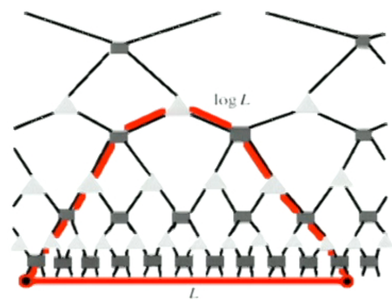


Picture courtesy Orus

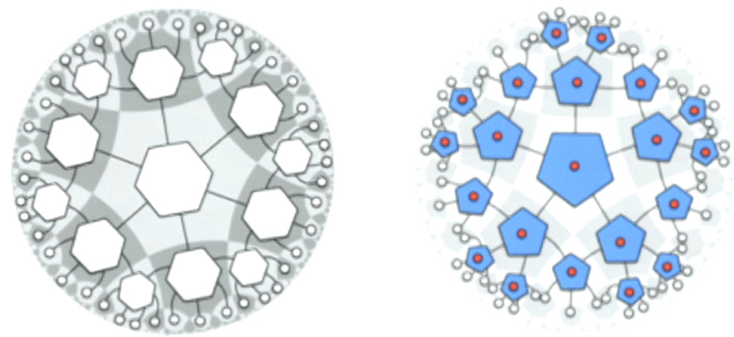
$$S_{EE} = \frac{A}{4G}$$

$$S_{EE} \leq \mathcal{N} \log L$$

Proposals — HAPPY Code, Bidirectional holographic codes Yang, Hayden, Qi , random tensor network…

Hayden, Nezami, Qi, Thomas, Walter, Yang

- Based on two sets of considerations —
- 1) RT formula
- 2) Inspirations from HKLL formula— subregion reconstruction



(a) Holographic hexagon state (b) Holographic pentagon code

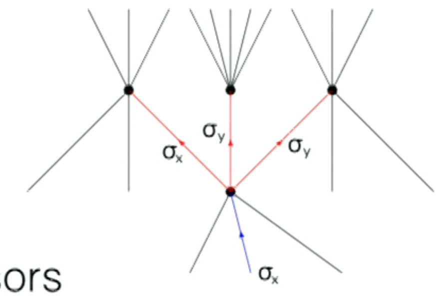
Figure 4. White dots represent physical legs on the boundary. Red dots represent logical input legs associated to each perfect tensor.

picture courtesy HAPPY

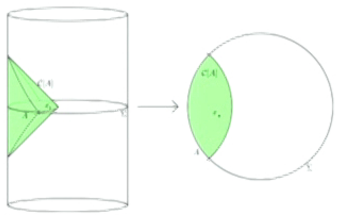
Almheiri, Dong, Harlow; Bousson, Freivogel, Leichenauer, Rosenhaus, Zukowski; Czech, Kaczmarek, Nogueira, Van Raamsdonk; Dong, Harlow, Wall; Cotler, Hayden, Salton, Swingle, Walter

Example:Perfect code

- The crucial ingredient == Perfect tensors
- Perfect tensors are “unitary maps in all directions”
- Bulk operators can be recreated as boundary operators — mimic HKLL relation
- Operator pushing:
- (Take a Pauli basis $\mathrm{tr}(O_i O_j) = \delta_{ij}$ When pushed to the other side — there is a big product involving exactly L legs if there are 2L legs for perfect tensors



HKLL Relation



Picture courtesy HAPPY

Figure 11. Bulk field reconstruction in the causal wedge. On the left is a spacetime diagram, showing the full spacetime extent of the causal wedge $\mathcal{C}[A]$ associated with a boundary subregion A that lies within a boundary time slice Σ. The point x lies within $\mathcal{C}[A]$ and thus any operator at x can be reconstructed on A. On the right is a bulk time slice containing x and Σ, which has a geometry similar to that of our tensor networks. The point x can simultaneously lie in distinct causal wedges, so $\phi(x)$ has multiple representations in the CFT.

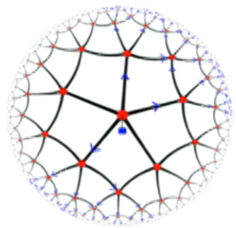
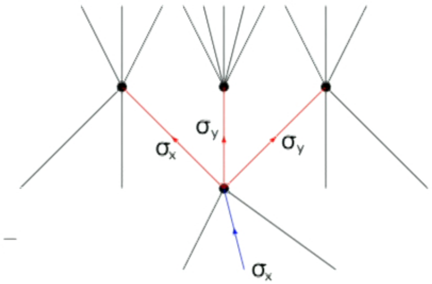


Figure 13. Disconnected reconstruction of a central operator beyond the causal wedge. Each of two separate connected boundary regions is too small for reconstruction of the central operator, yet the reconstruction is possible on the union of the two regions. In this example the greedy algorithm reaches the central tensor when applied to both connected components at once, but not when applied to either component by itself.

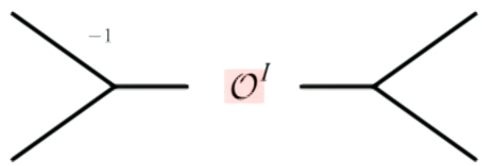
- HKLL relation :
- $\phi(x,z) = \int d^d y K(x,z|y)\mathcal{O}(y)$

$$[\Box + m^2]K(x,z|y) = 0 \qquad K(x,z|y) = \left(\frac{z}{(z^2+(x-y)^2)}\right)^{\Delta^-}$$

$$\Delta^{\pm} = \frac{d}{2} \pm \sqrt{d^2/4 + m^2L^2}$$

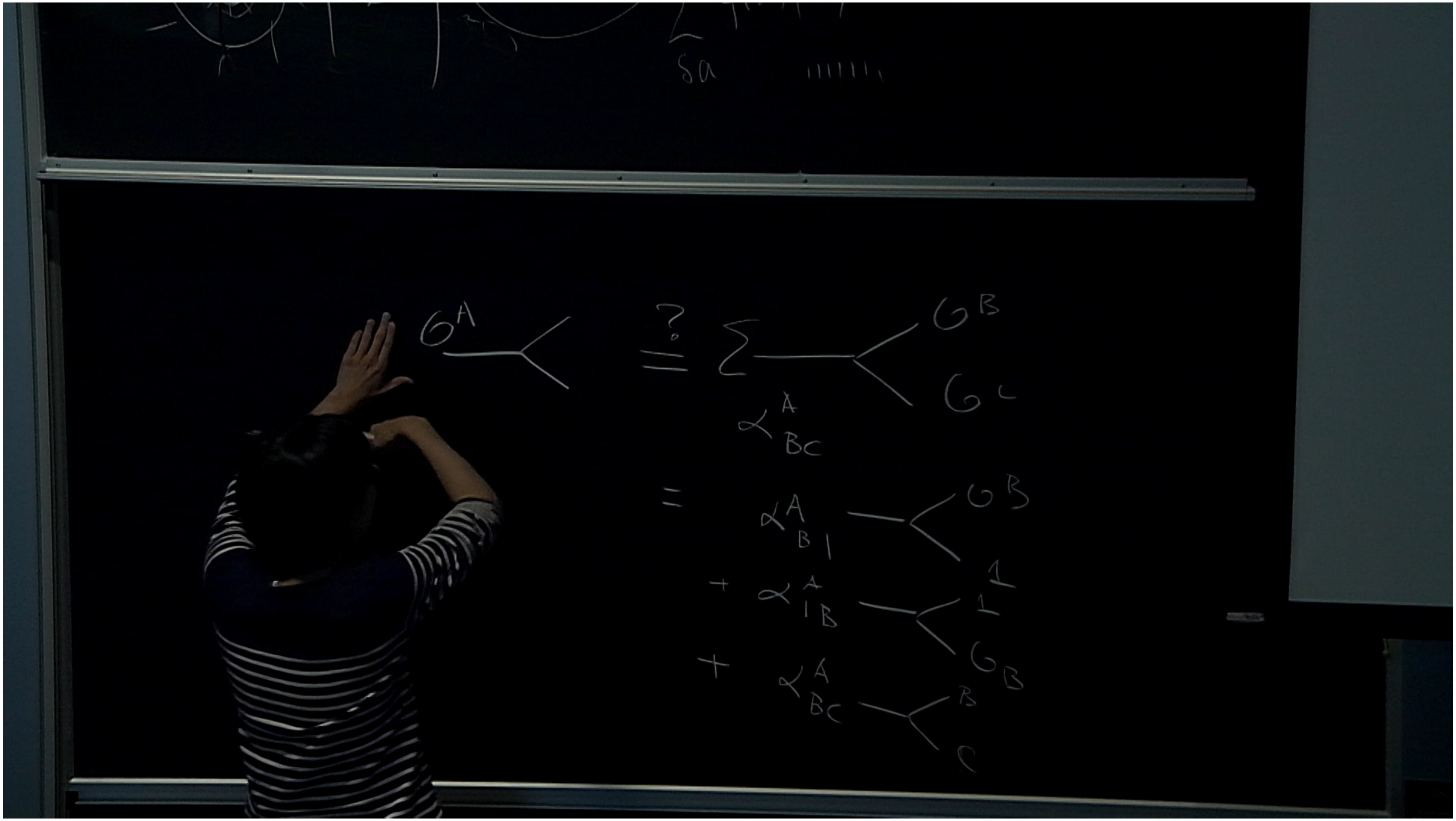


How to recover that?

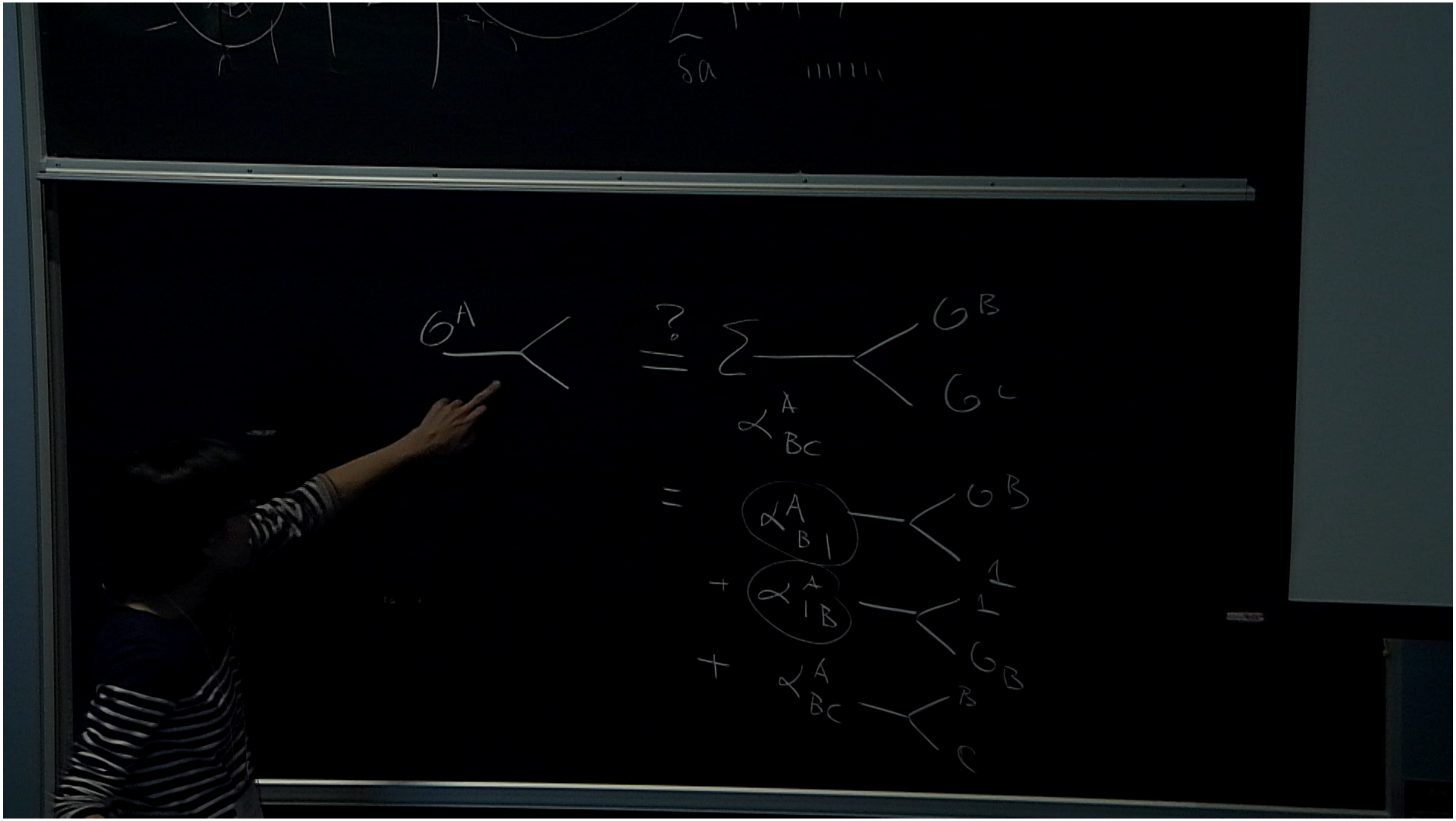


$$= \sum \alpha^{IJ} \mathcal{O}^J \otimes \mathbb{I} + \tilde{\alpha}^{IJ} \mathbb{I} \otimes \mathcal{O}^J$$

$$+ \sum \beta^{IJK} \mathcal{O}^J \otimes \mathcal{O}^K$$



σ^A
=?
Σ
σ^B
σ^C
α^A_BC
=
α^A_B1
+ α^A_1B
+ α^A_BC



σA
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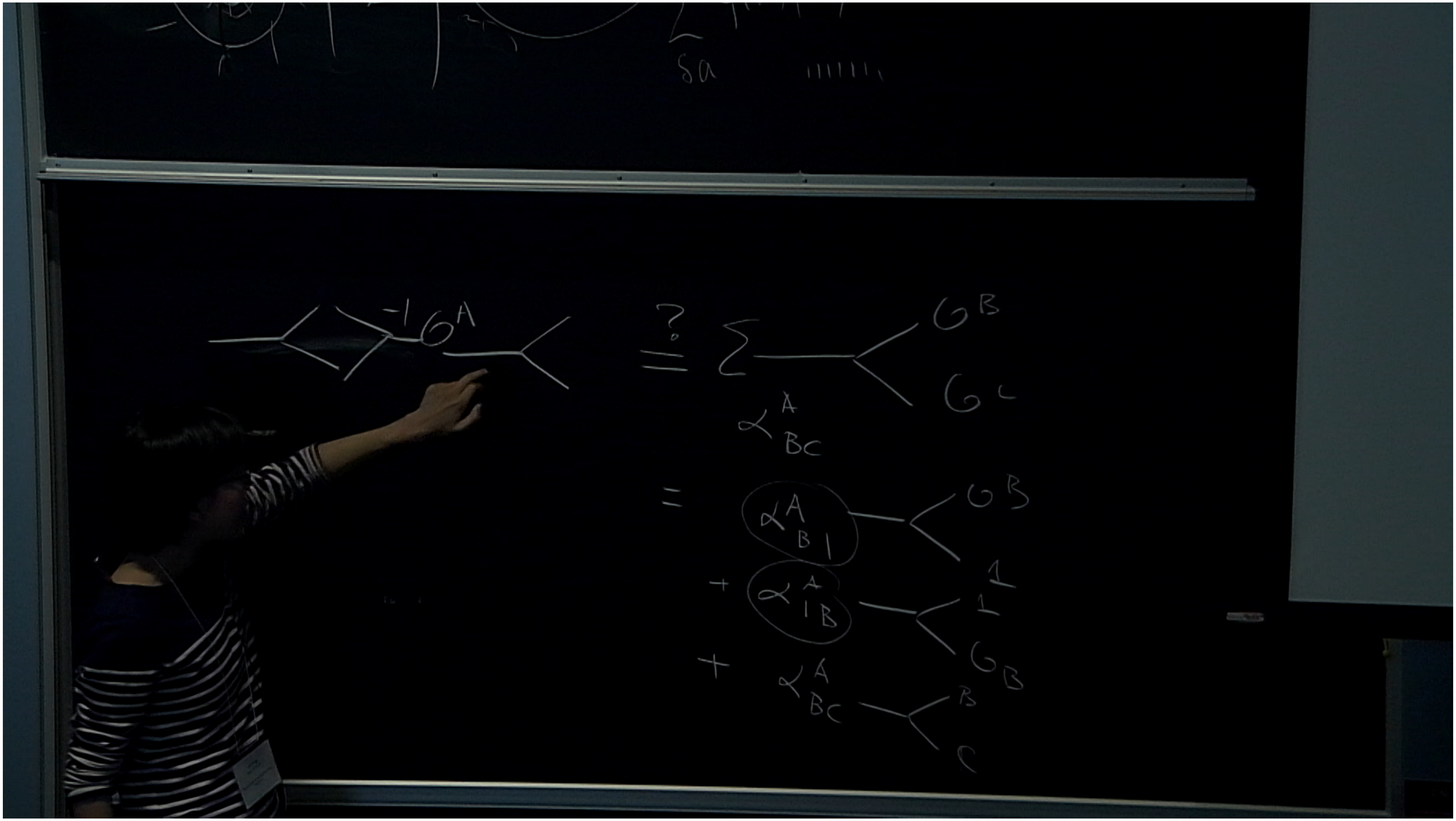

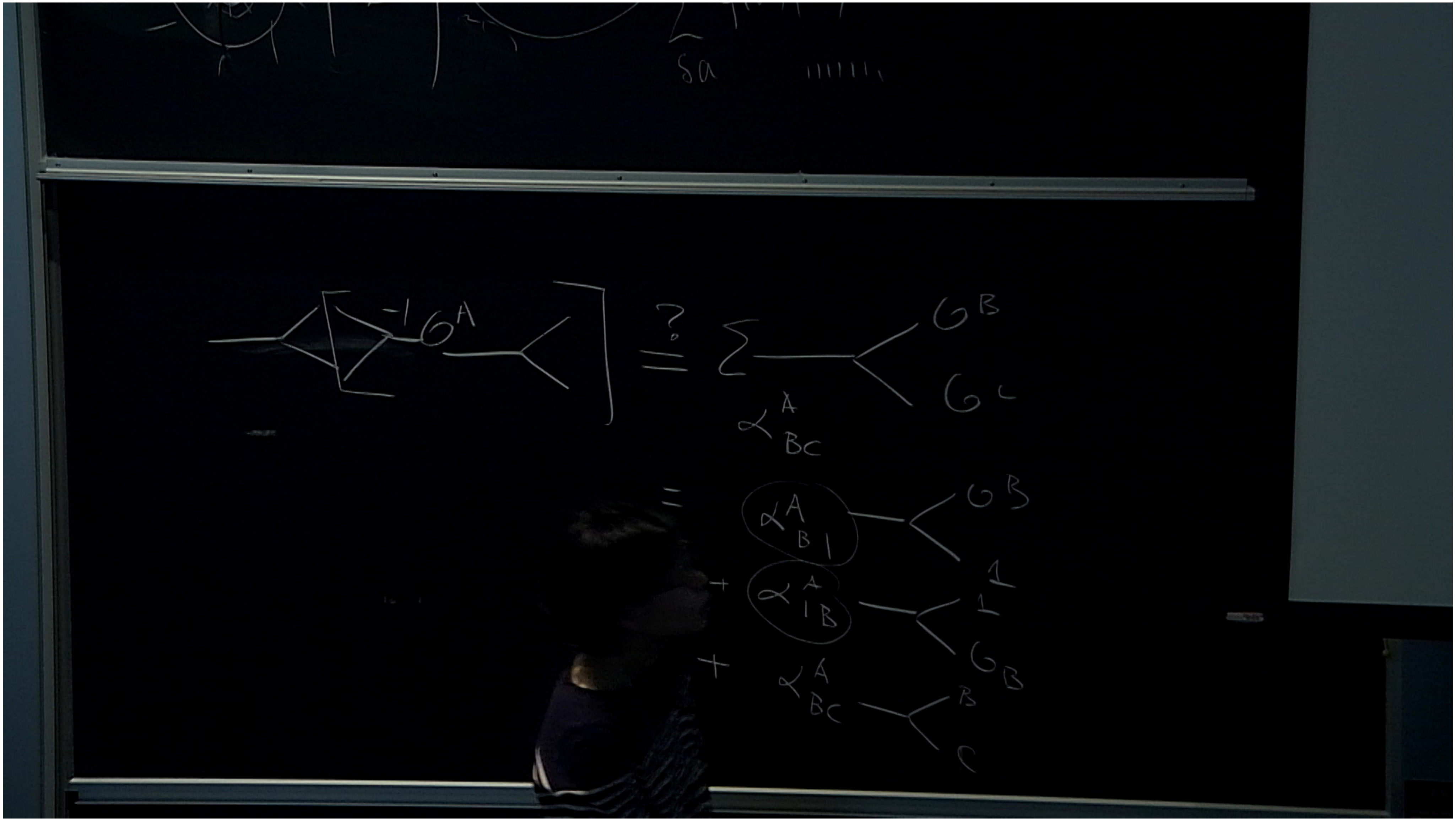

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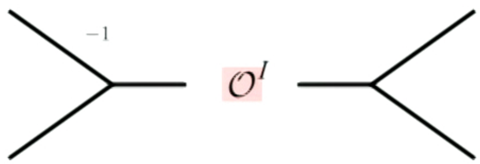

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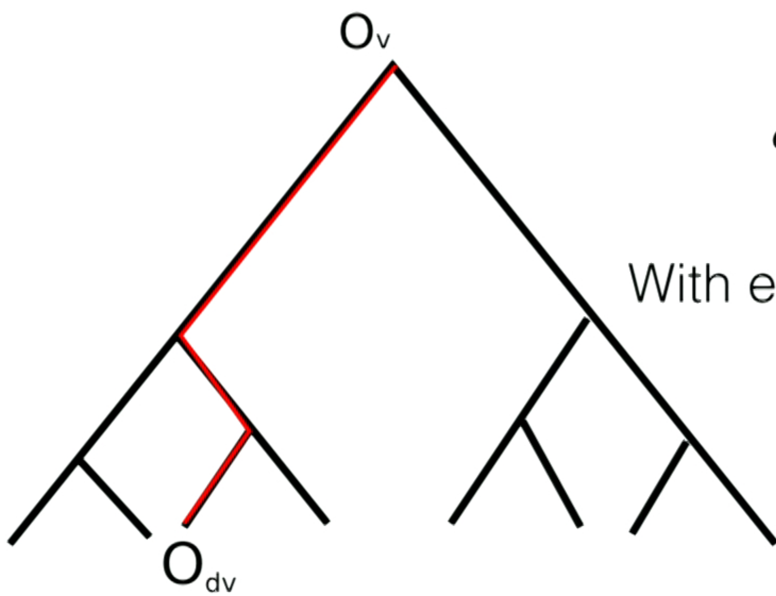
Linear term



$$\alpha(v, \partial v) = \prod_{v \in \mathcal{P}} \alpha(v)$$

With eigenvalues λ_I

Graph Laplacian

$$\Box\phi(v) = \sum_{u\sim v}(\phi(u) - \phi(v))$$

- consider $\quad G(v_1, v_2) = p^{-\Delta d(v_1,v_2)}$
- p+1 = valancy of graph
- We have $\quad (\Box_{v_1} + m^2)G(v_1, v_2) = \mathcal{N}\delta_{v_1,v_2}$

$$m^2 = -\frac{1}{\zeta_p(\Delta-1)\zeta(\Delta)}, \qquad \zeta_p(s) \equiv \frac{1}{1-p^{-s}}$$

Green's function from tenors

$$G_I(v, \partial v) = \lambda_I^{d(v,\partial v)} \qquad\qquad p^{-\bar{\Delta}_i} = \lambda_i$$

Field content recovers from values of the tensors.

This discussion was partially inspired by construction of the holographic dual based on the Hair Wavelet of Qi et al

Graph with loops — requires introduction of a conserved and irrotational flow. HAPPY; Yang, Hayden, Qi

$$G(v, \partial v) = \sum_{P(v,\partial v)} p^{-\bar{\Delta} d_P(v,\partial v)}$$

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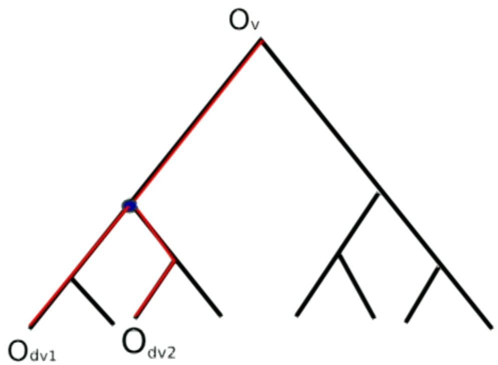
What about non-linear terms?



Splitting at some node, picking up

$$\beta^{IJK}$$

Altogether we have:

$$G_I(v,s)\beta^{IJK}G_J(s,\partial_{v_1})G_K(s,\partial v_2)$$

A summary of what we saw

- Field content and their interaction vertices are determined by the values of the tensors
- These considerations can be generalised to graphs with loops. This can be done by first specifying a flow diagram
- we can now ask questions about what constitutes a weakly coupled holographic theory from the perspective of tensor networks
- what are time-like vs space-like directions?

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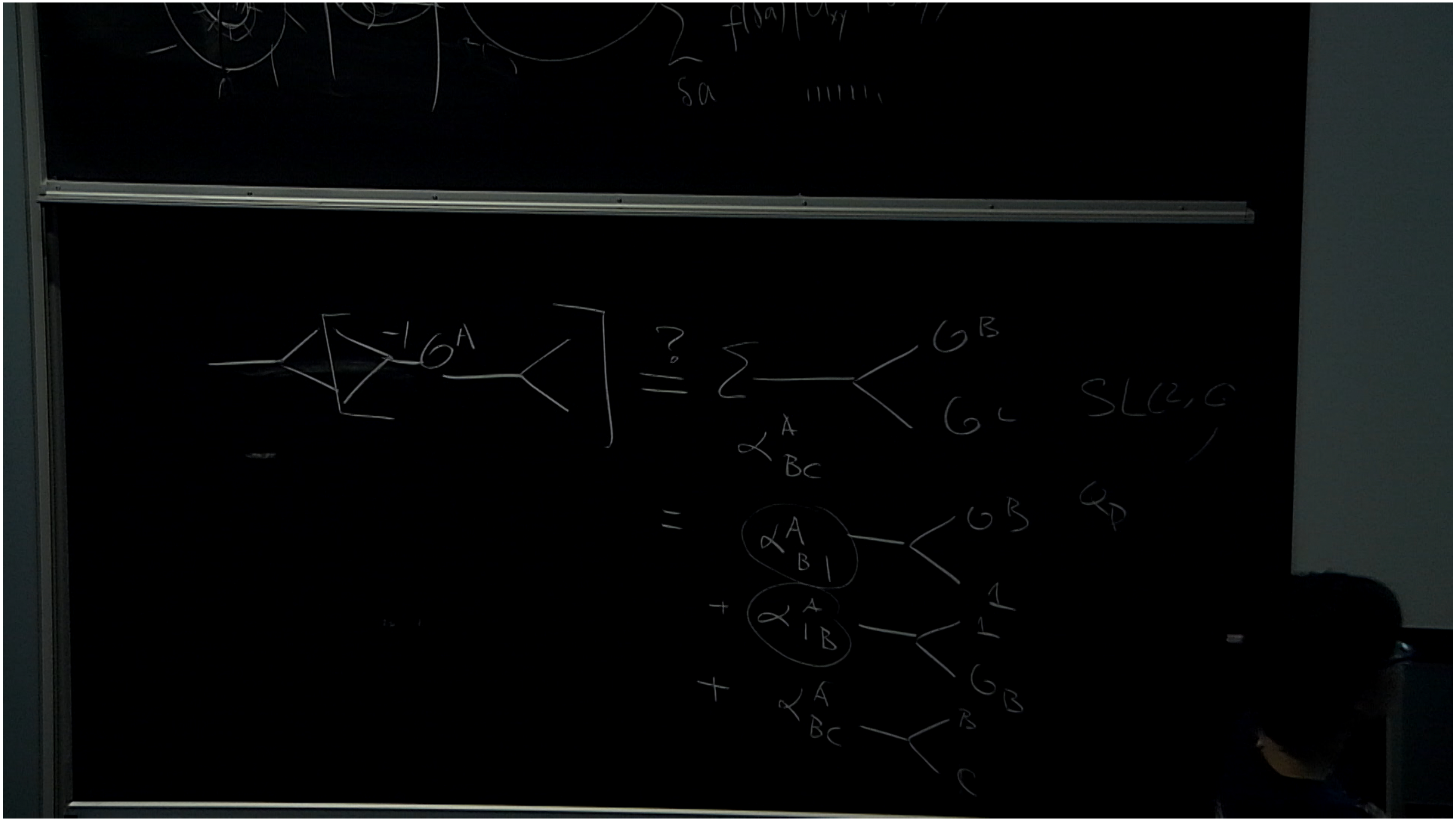
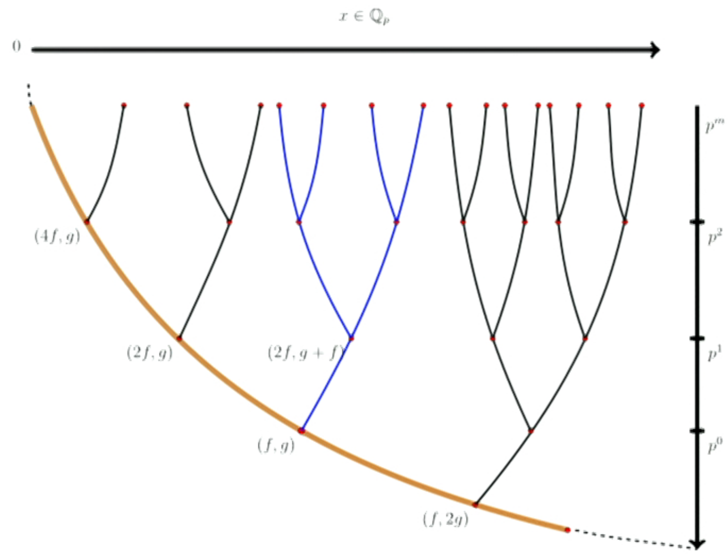
P-adic AdS/CFT

- symmetries and the graph… Coxeter group seems like a difficult starting point.
- Bruhat-Tits Tree and the p-adic field
- $x = p^v(\sum_{m=0}^{\infty} a_m p^m)$, $|x|_p = p^{-v}$
- Adelic structure: $\prod_p^{\infty} |x|_p = 1$

String amplitudes admitting a p-adic decomposition Witten



Bruhat-Tits Tree



Recover HKLL relations as if they came from the action

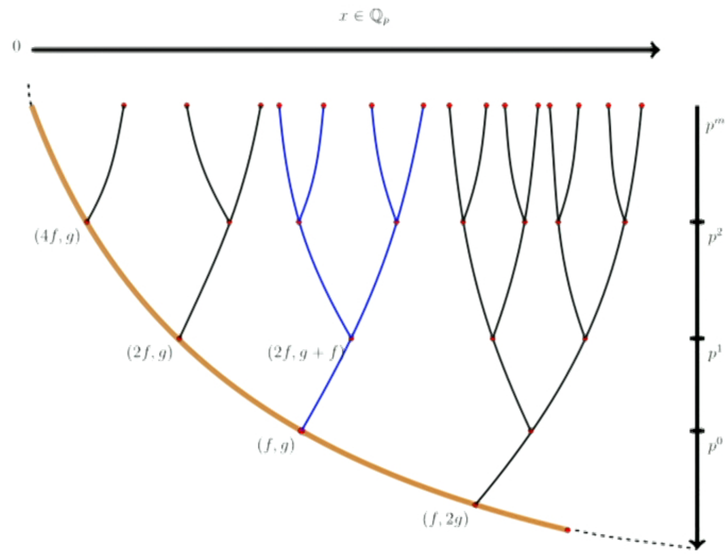
$$I = \sum_{v} -\frac{1}{2}\phi(v)(\Box + m_p^2)\phi(v) + \eta_3 \frac{\phi^3(v)}{3!} + \eta_4 \frac{\phi^4(v)}{4!} + \cdots ,$$

Bruhat-Tits Tree

	upper half plane $\mathbb{H}$	Bruhat-Tits tree $\mathbb{H}_p$
Isometry group G	$\mathrm{SL}(2,\mathbb{R})$	$\mathrm{PGL}(2,\mathbb{Q}_p)$
Isotopy group K	$\mathrm{SO}(2,\mathbb{R})$	$\mathrm{PGL}(2,\mathbb{Z}_p)$
Boundary	$\mathbb{R}$	$\mathbb{Q}_p$

$\mathbb{H} \equiv \mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2,\mathbb{R})$. vs $\mathbb{H}_p \equiv \dfrac{\mathrm{PGL}(2,\mathbb{Q}_p)}{\mathrm{PGL}(2,\mathbb{Z}_p)}$

Bruhat-Tits Tree



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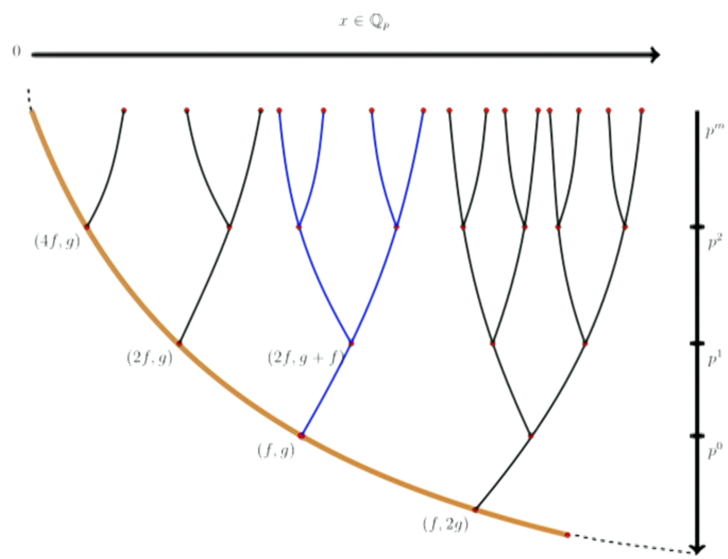

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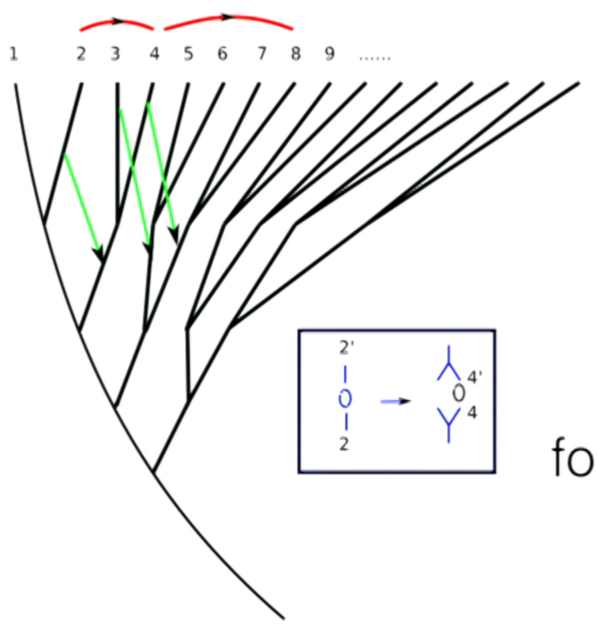
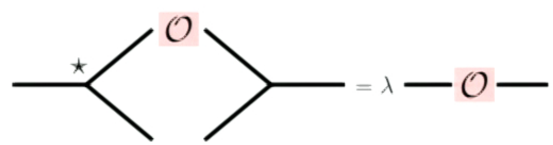
Isometry

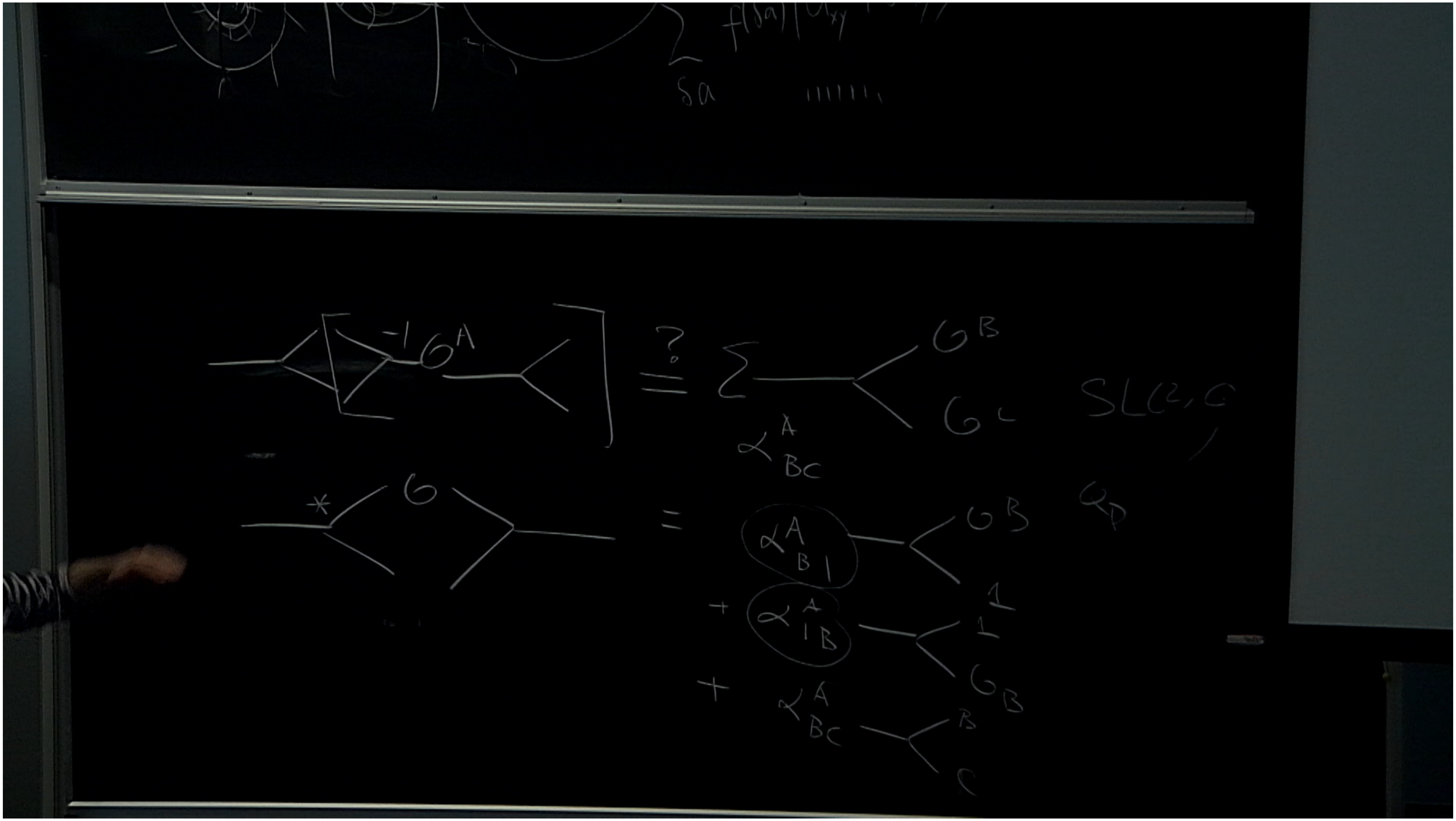


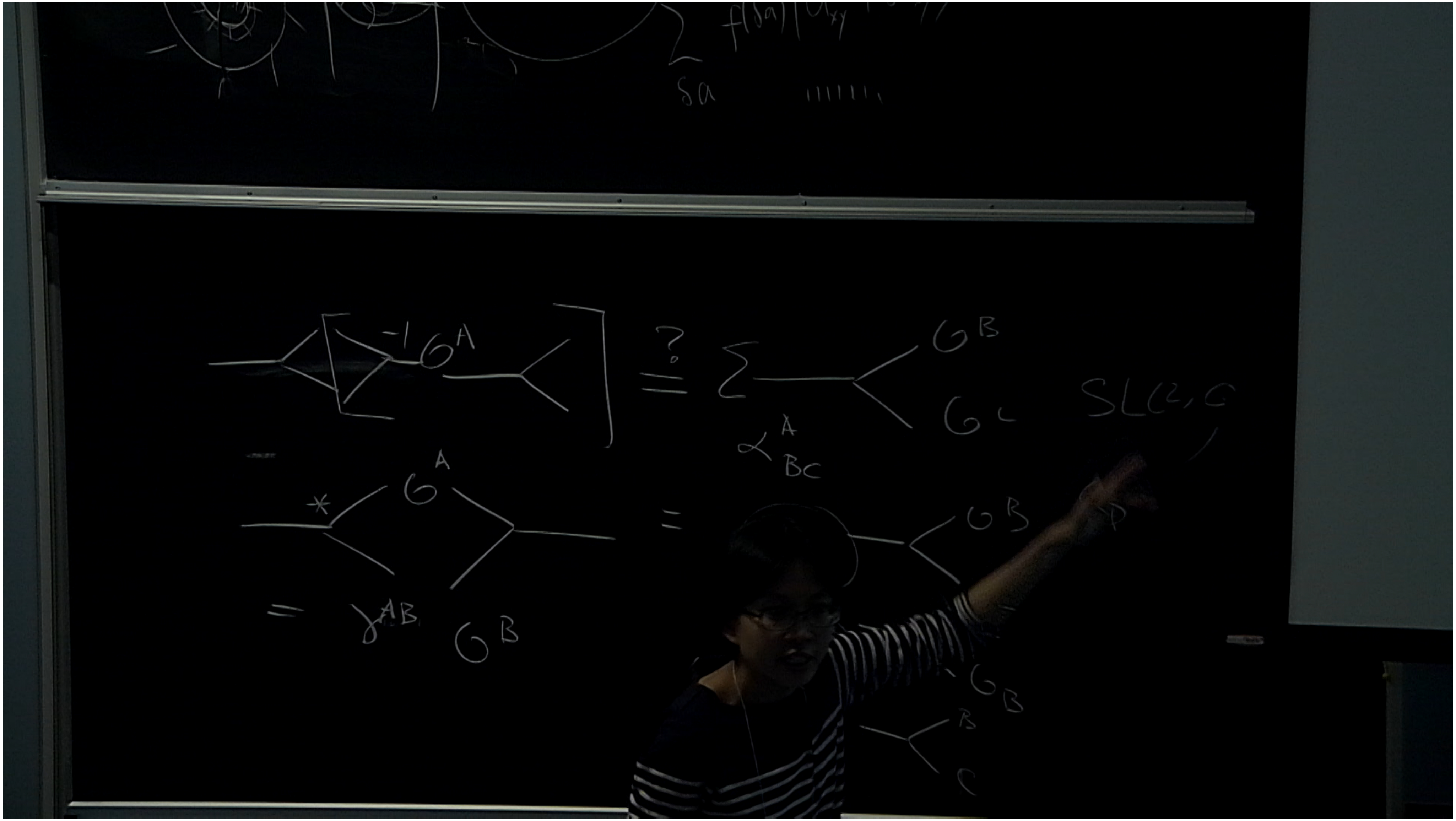
$$p^n \alpha_{AB} \gamma^{BC} \approx \delta_A^C$$

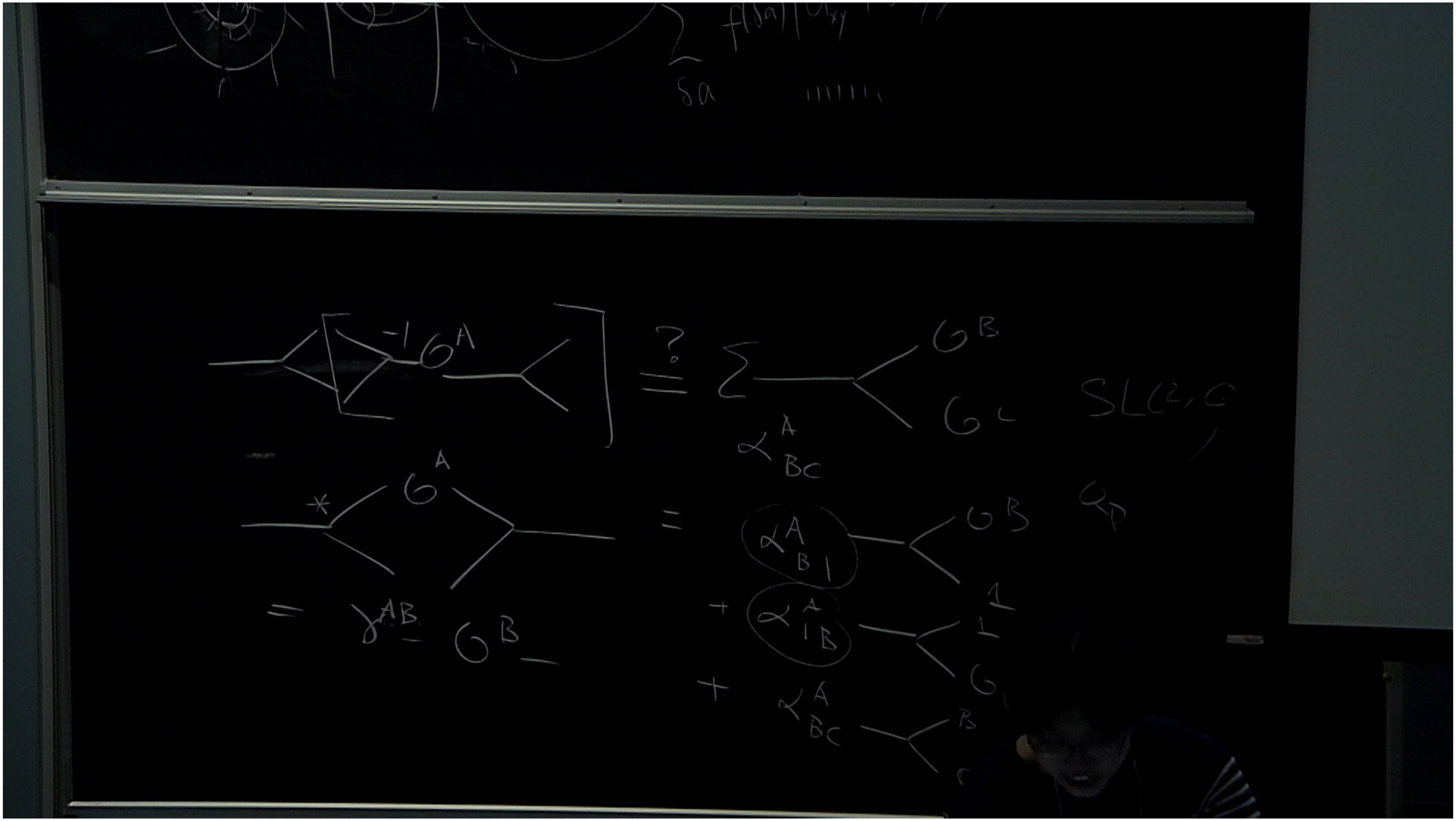
Scaling transformation—
effected by global isometry
of the tree

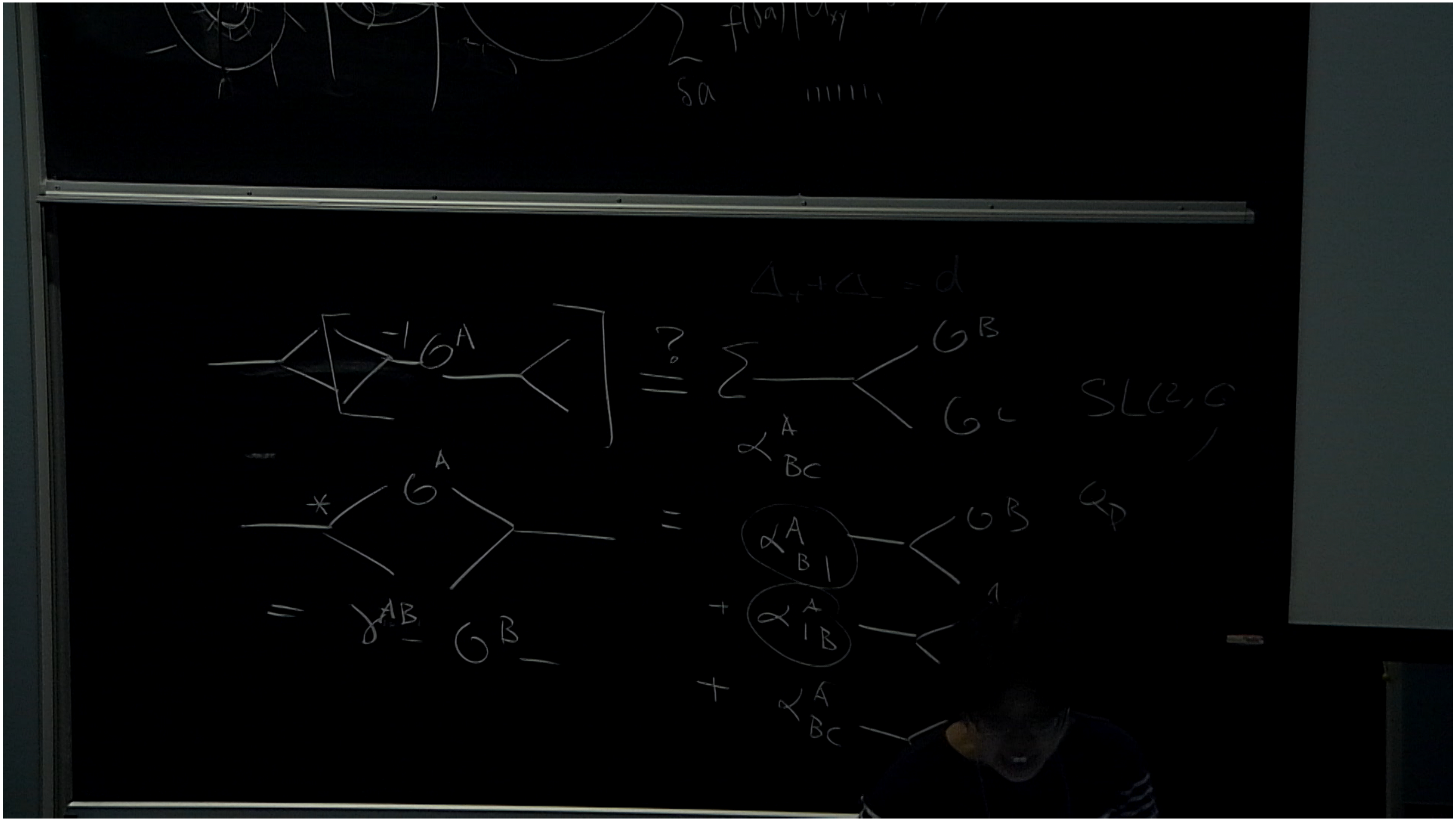
The cutoff surface is responsible
for reading off conformal dimensions



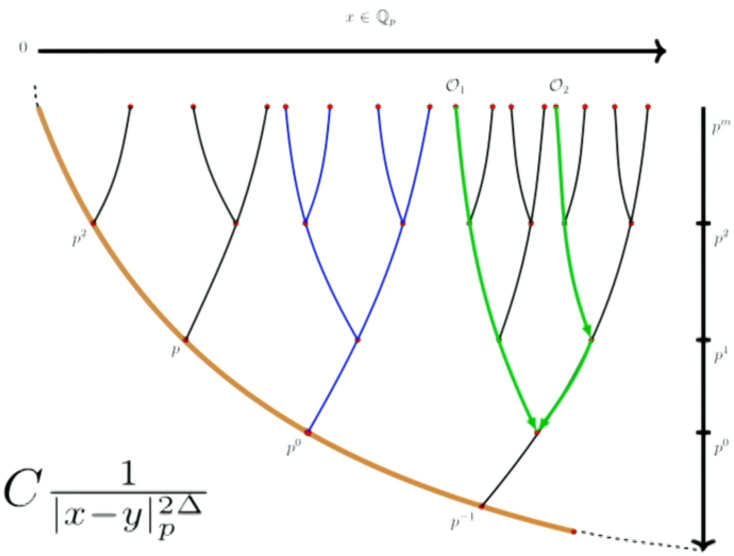



SL(2,C)



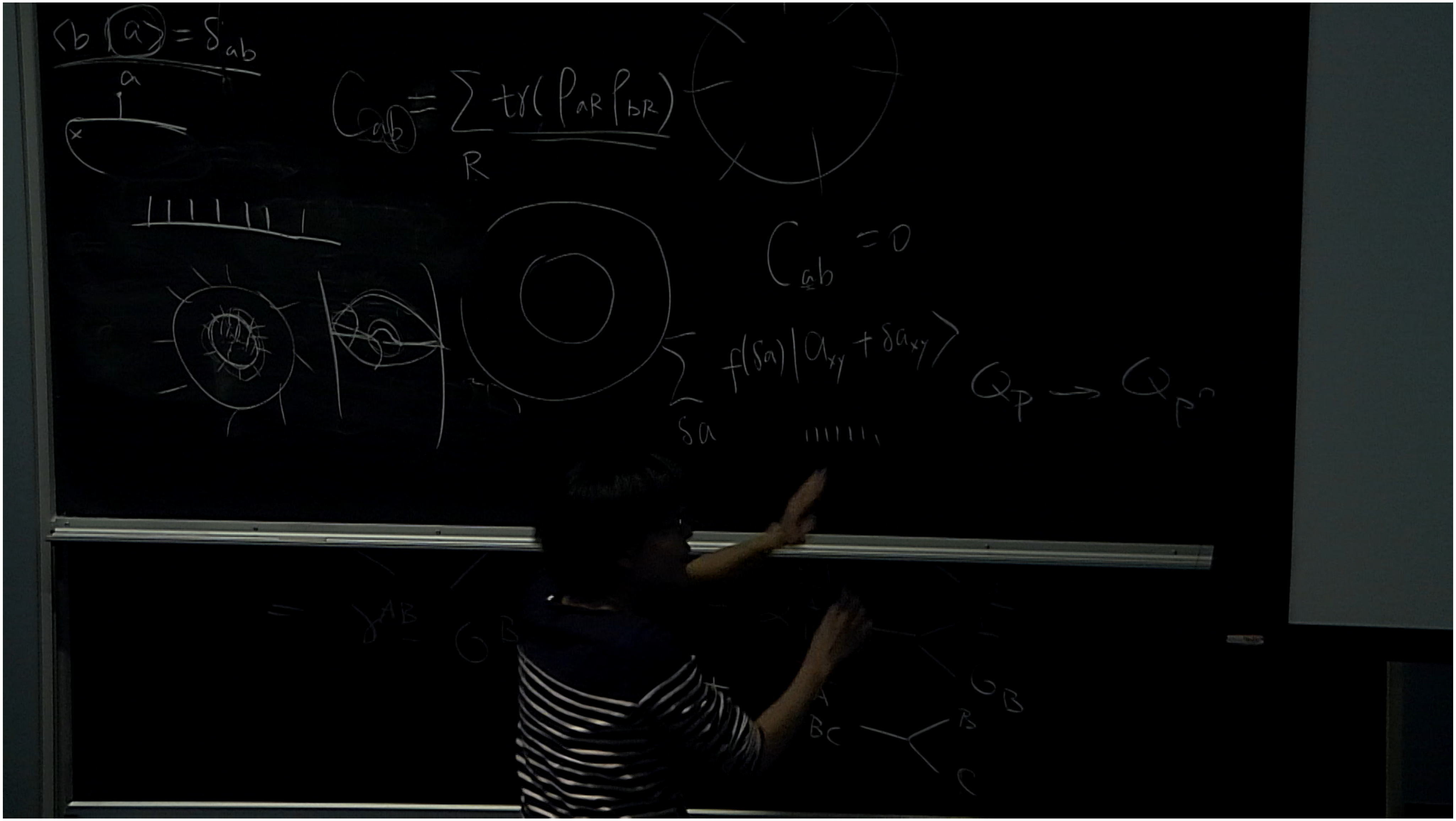


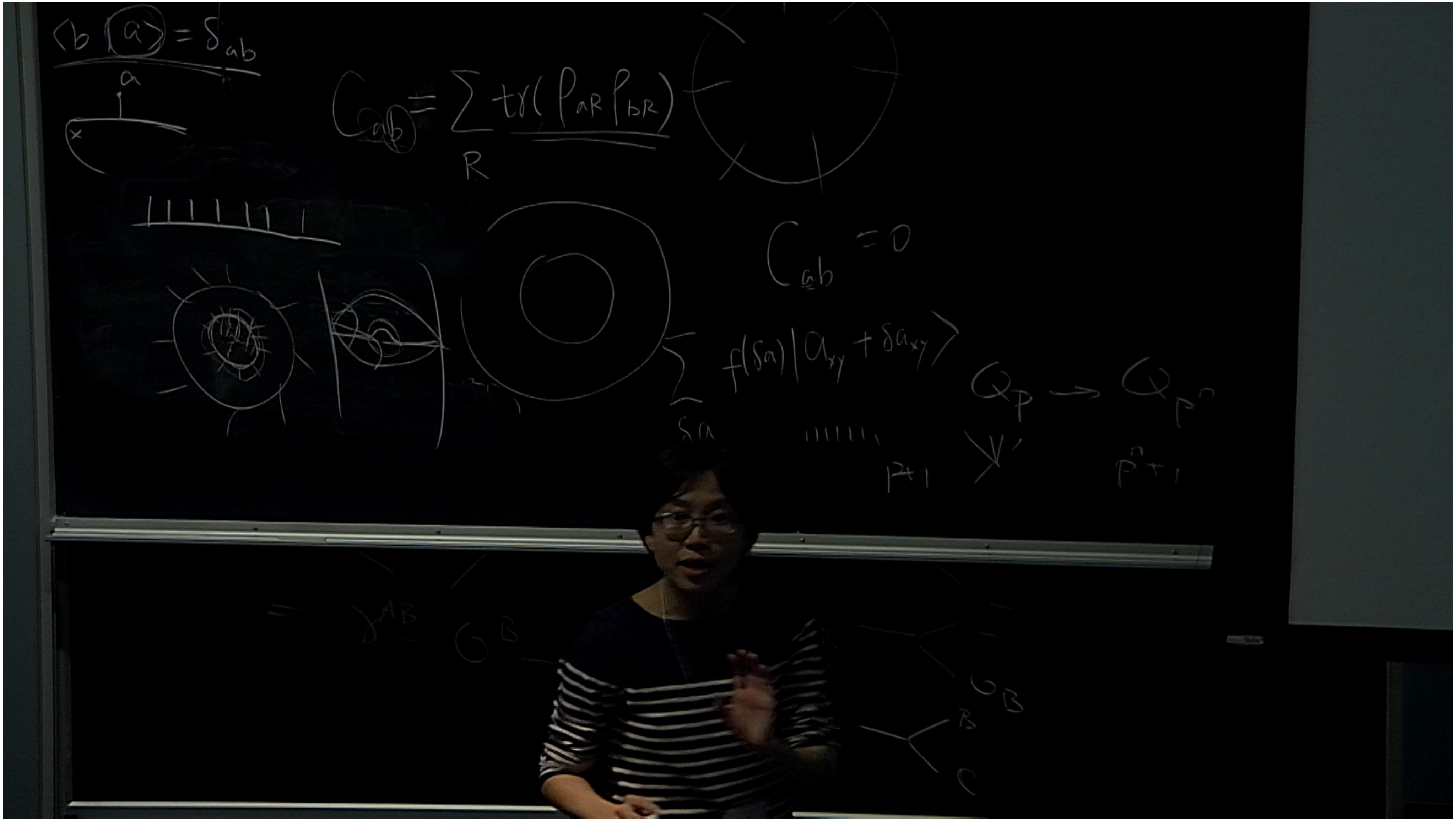
Correlation functions

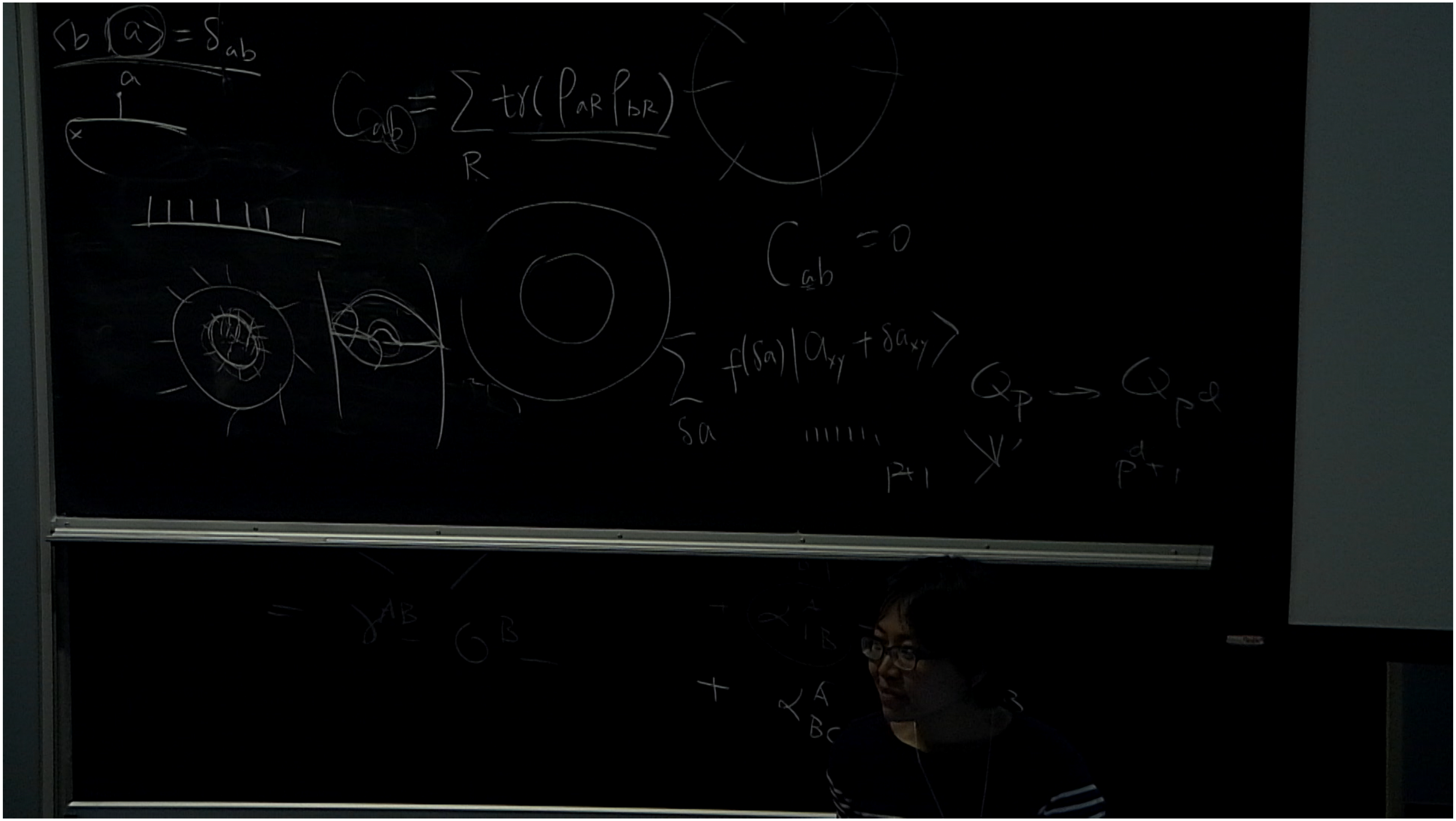


$$\langle O(x)O(y)\rangle_p = C\frac{1}{|x-y|_p^{2\Delta}}$$

Can be generalised to 3 and 4 pt function etc
(Feynman rules for Witten diagram in this prescription slightly different from standard) —here diagram organised into p-adic conformal blocks right away


⟨b|a⟩ = δab
Cab = Σ tr(ρaR ρbR)
R
Cab = 0
Σ f(δa)|axy + δaxy⟩
δa
Qp → Qpⁿ
BC
B
C


⟨b | a⟩ = δab
Cab = Σ tr(ρaR ρbR)
R
Cab = 0
Qp → Qp^n
p+1
p^n+1


⟨b|a⟩ = δab
Cab = Σ tr(ρaR ρbR)
R
Cab = 0
Σ f(δa) |axy + δaxy⟩
δa
Qp → Qpd

Summary

- We explored to some detail the tensor network AdS correspondence
- the HKLL relation built from Green's function of the graph emerges

—What is spacelike vs time like separation?— seems to be controlled by the values of the eigenvalues

- interaction vertices can also be read off
- placed on a Bruhat-Tits tree the p-adic ads/cft dictionary could emerge (up to some extra constraints to be understood)
- Generalizations? fermions, spin 1 particles, spin 2 ?!
- When is the bulk theory weakly coupled? has a large gap? are these features typical in the large D limit? (Log D ~ central charge)

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