Title: Dynamics for holographic codes

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Abstract: In this talk I discuss the problem of introducing dynamics for holographic codes. To do this it is necessary to take a continuum limit of the holographic code. As I argue, a convenient kinematical continuum limit space is given by Jonesâ€™ semicontinuous limit. Dynamics are then furnished by a unitary representation of a discrete analogue of the conformal group known as Thompsonâ€™s group T. I will describe these representations in detail in the simplest case of a discrete AdS geometry modelled by trees. Consequences such as the ER=EPR argument are then realised in this setup. Extensions to more general tessellations with a MERA structure are possible, and will be (very) briefly sketched.

Dynamics for holographic codes

Tobias J. Osborne
Deniz Stiegemann

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PREPARED FOR SUBMISSION TO JHEP

Holographic quantum error-correcting codes: Toy models for the bulk/boundary correspondence

Fernando Pastawski,*[a] Beni Yoshida*[a] Daniel Harlow,[b] John Preskill,[a]

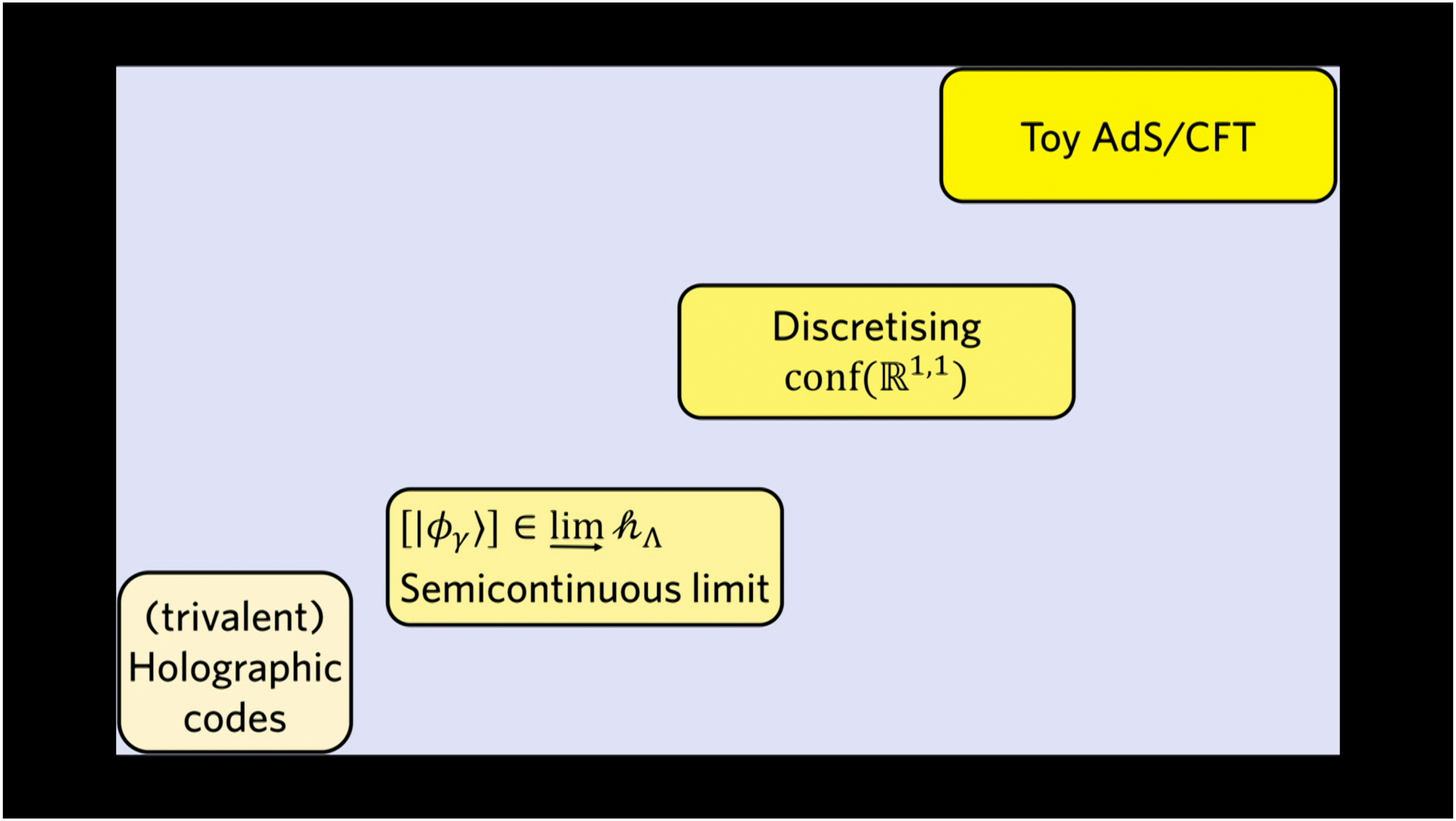
[a]*Institute for Quantum Information & Matter and Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA*

[b]*Princeton Center for Theoretical Science, Princeton University, Princeton NJ 08540 USA*

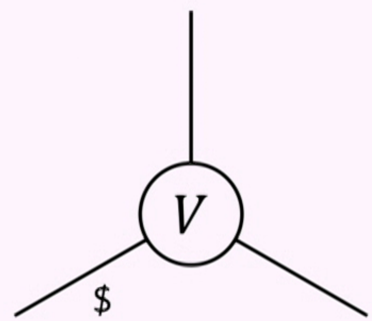
*These authors contributed equally to this work.

E-mail: fernando.pastawski@gmail.com, rouge@caltech.edu, dharlow@princeton.edu, preskill@caltech.edu

ABSTRACT: We propose a family of exactly solvable toy models for the AdS/CFT correspondence based on a novel construction of quantum error-correcting codes with a tensor network structure. Our building block is a special type of tensor with maximal

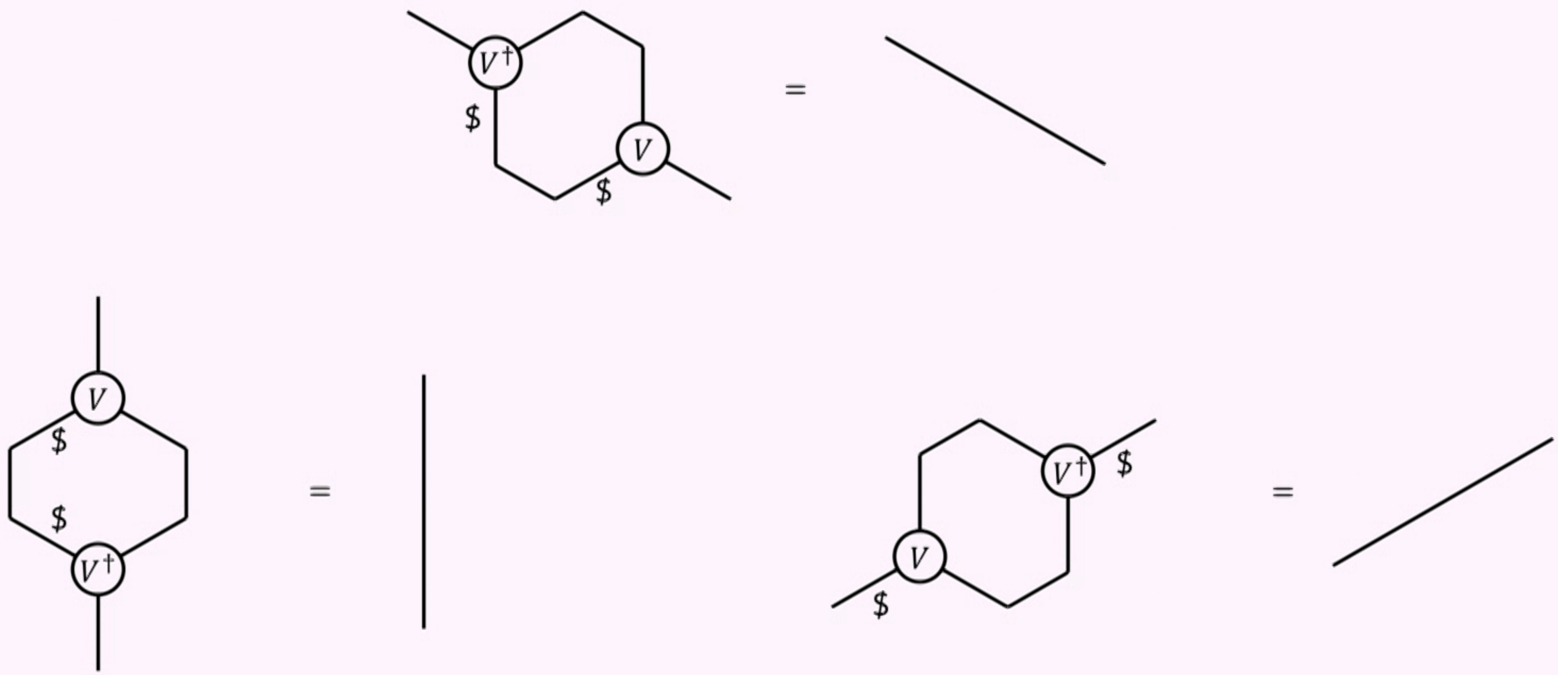

Toy AdS/CFT
Discretising
$\mathrm{conf}(\mathbb{R}^{1,1})$
$[|\phi_\gamma\rangle] \in \varinjlim \mathcal{h}_\Lambda$
Semicontinuous limit
(trivalent)
Holographic
codes

3-leg perfect tensor

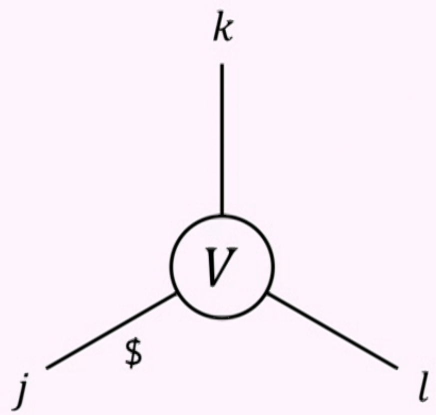


$$V: \mathbb{C}^d \otimes \mathbb{C}^d \to \mathbb{C}^d$$

3-leg perfect tensor



Example: $d = 3$

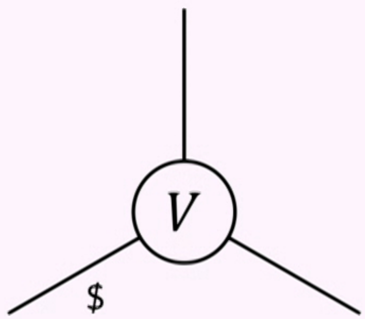


$$\langle j|V|kl\rangle \propto \begin{cases} 0 & \text{if } j = k \text{ or } k = l \text{ or } j = l \\ 1 & \text{otherwise} \end{cases}$$

Example: $d = 3$

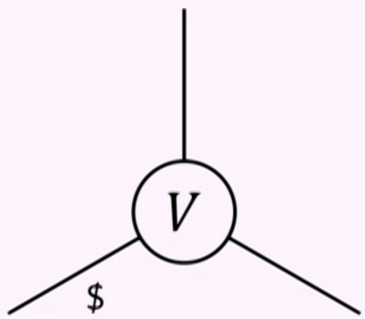
Example: $d = 3$

More examples:



(i) Frobenius algebras (finite dimensional)
(ii) OPE coefficients (infinite dimensional)
(iii) Tensor categories, e.g., $SO(3)_q$; Temperley Lieb
(iv) Planar algebras

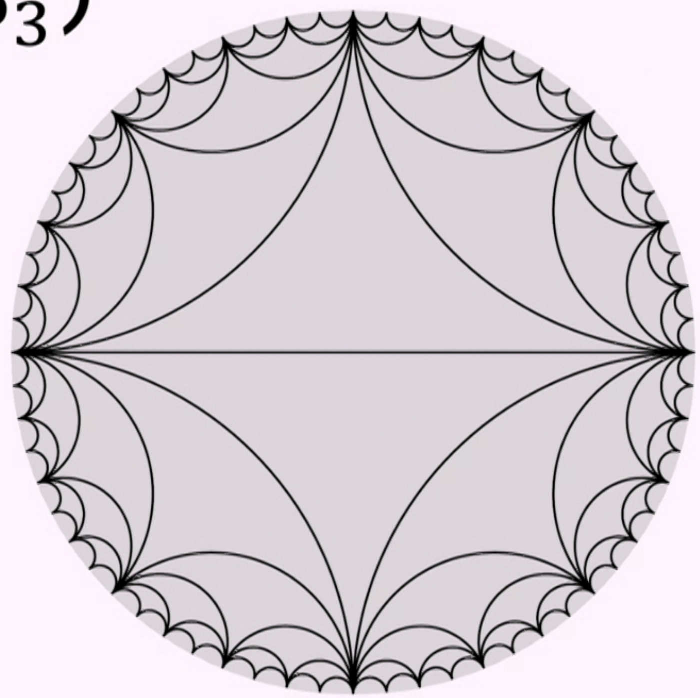
More examples:



(i) Frobenius algebras (finite dimensional)
(ii) OPE coefficients (infinite dimensional)
(iii) Tensor categories, e.g., $SO(3)_q$; Temperley Lieb
(iv) Planar algebras

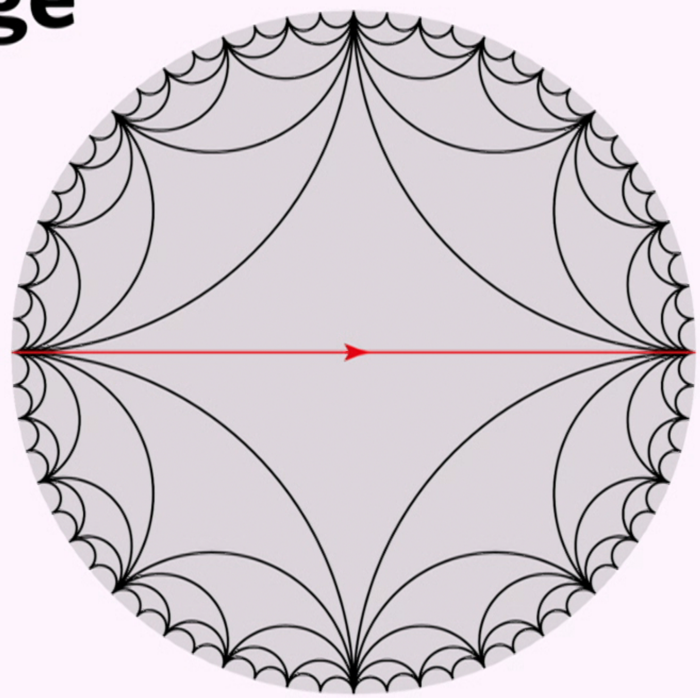
Hyperbolic tessellations τ

(slice of AdS_3)



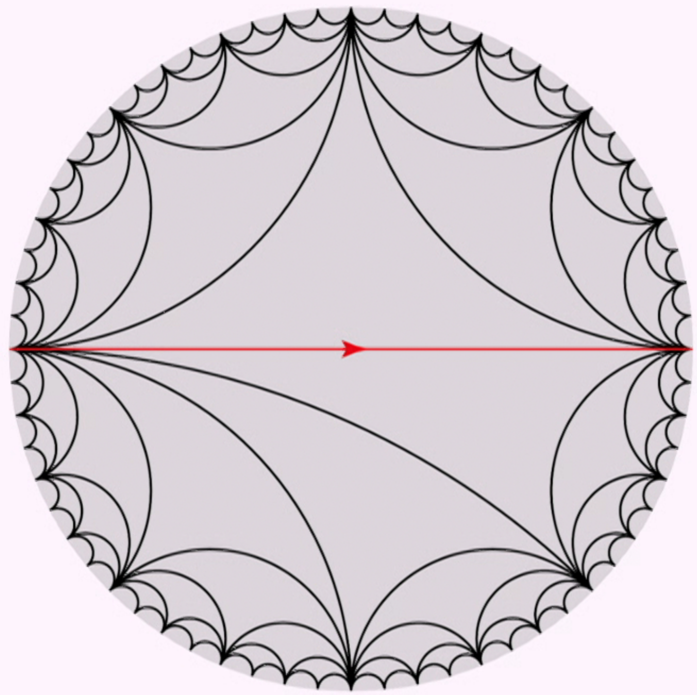
Tessellation with **distinguished oriented edge**

(τ, e)

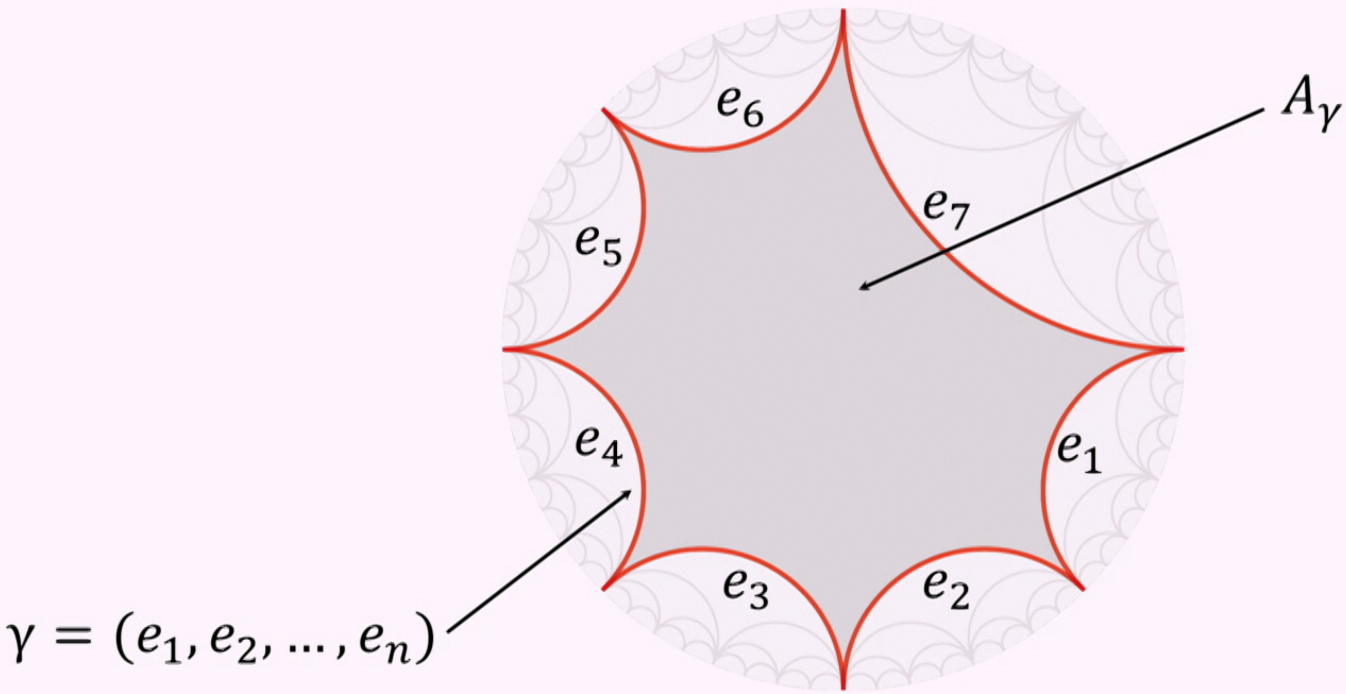


Tessellation (with flips/local moves)

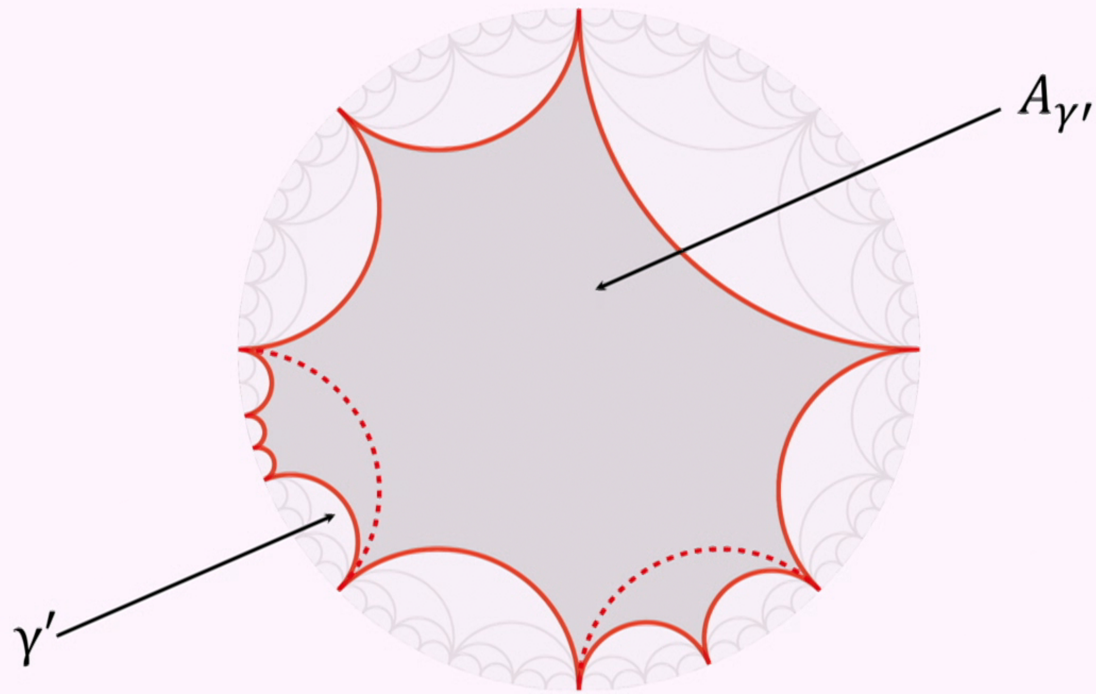
(τ', e')



Cutoffs



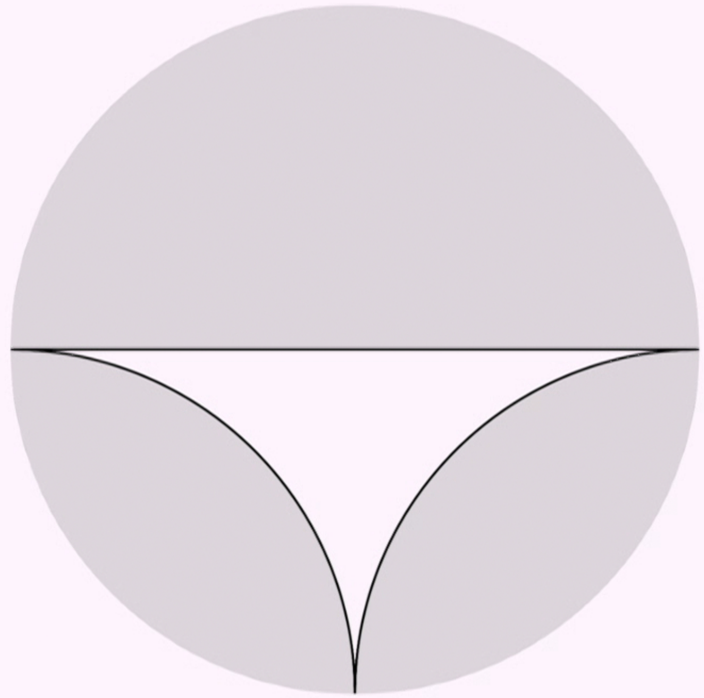
Bigger cutoff: $\gamma' \geqslant \gamma$



Directed set of cutoffs $(\mathcal{P}, \preccurlyeq)$:

1. The relation "A_γ is **contained** in $A_{\gamma'}$," written $\gamma \preccurlyeq \gamma'$ is a **partial order**
2. For every pair γ_1 and γ_2 there is a **bigger cutoff** γ: $\gamma_1 \preccurlyeq \gamma$ and $\gamma_2 \preccurlyeq \gamma$

Holographic state (ideal triangle)



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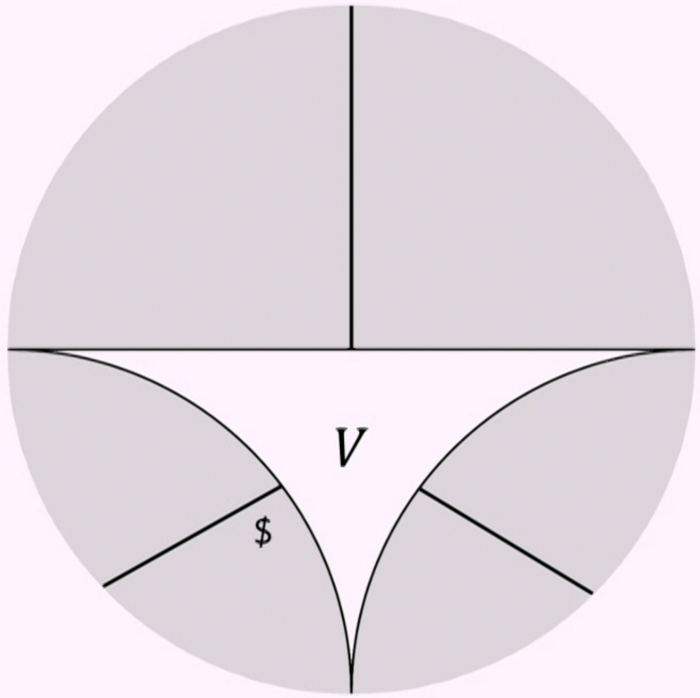
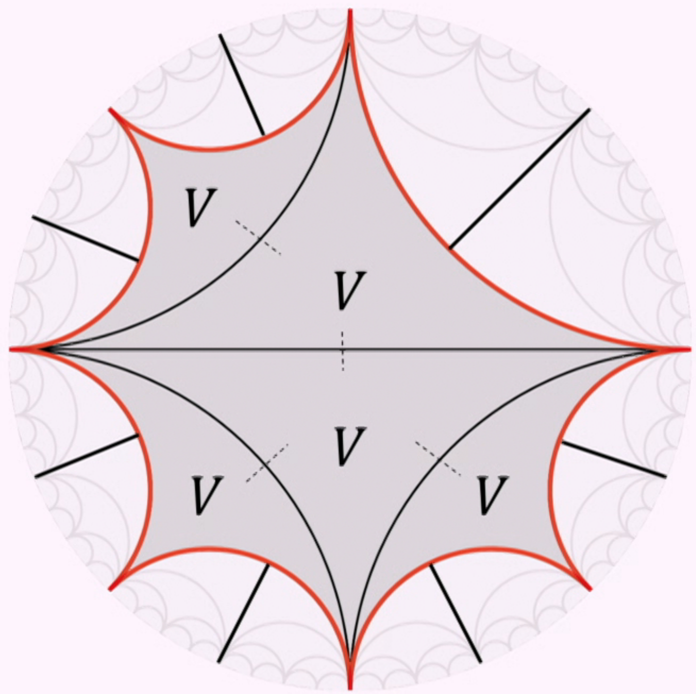
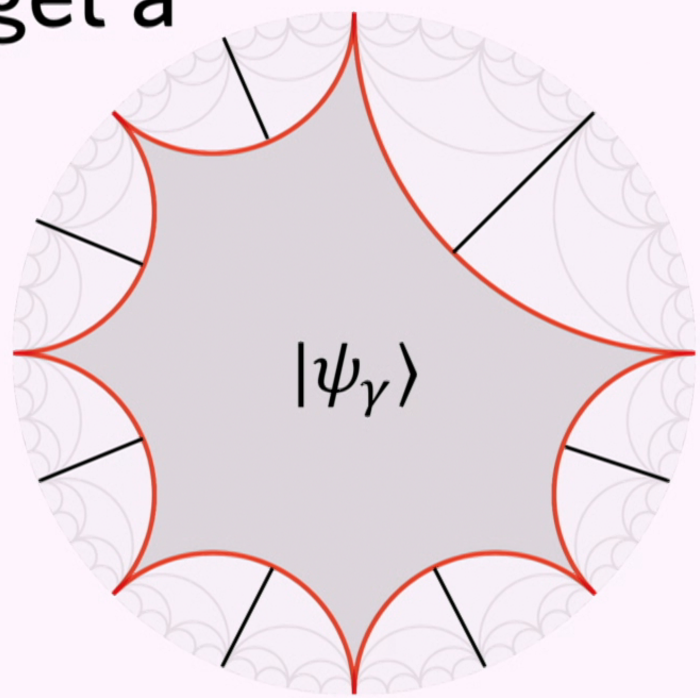


Holographic state (cutoff γ)

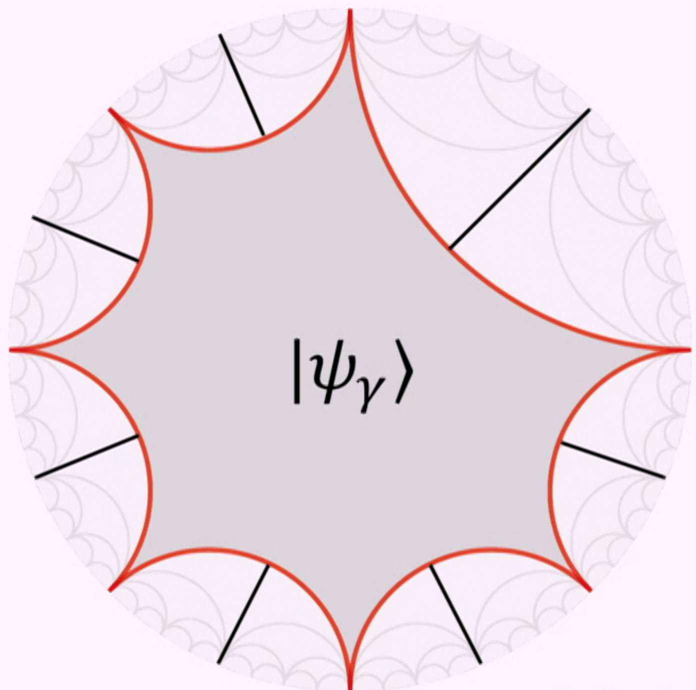


Holographic state: for every cutoff γ we get a state $|\psi_\gamma\rangle$:



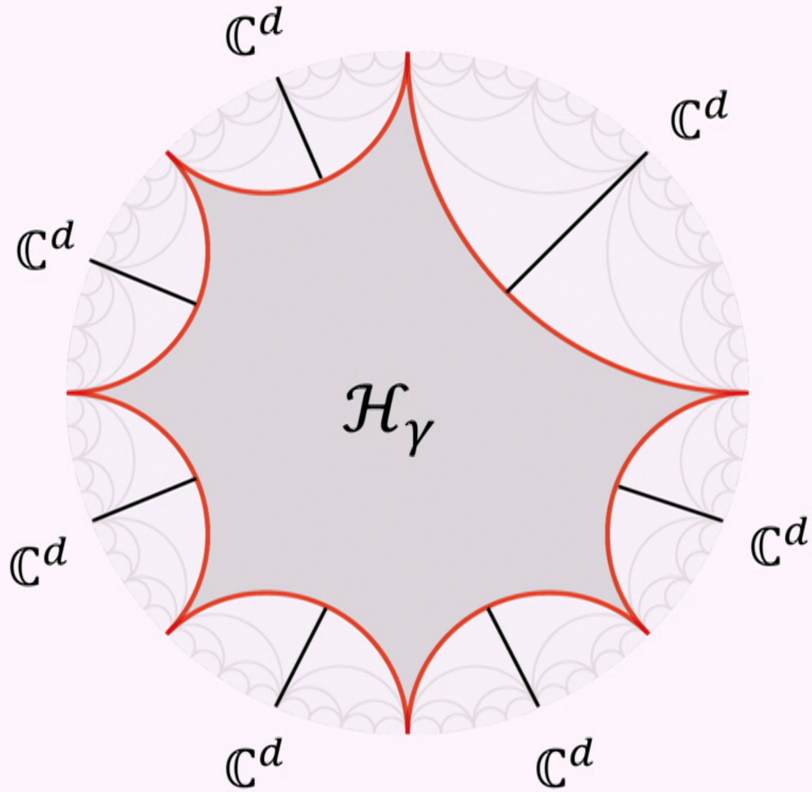
Where do holographic states live?

Boundary hilbert space $\mathcal{H}_\gamma$:



$$|\psi_\gamma\rangle \in \mathcal{H}_\gamma \cong \left(\mathbb{C}^d\right)^{\otimes 7}$$

Boundary hilbert space $\mathcal{H}_\gamma$:

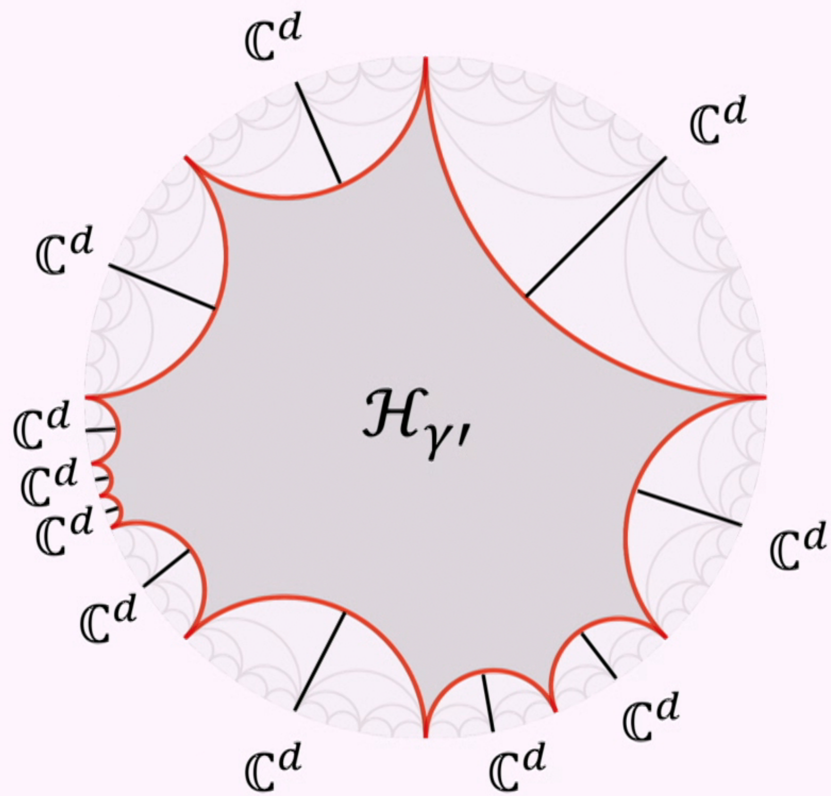


Boundary hilbert space $\mathcal{H}_\gamma$:

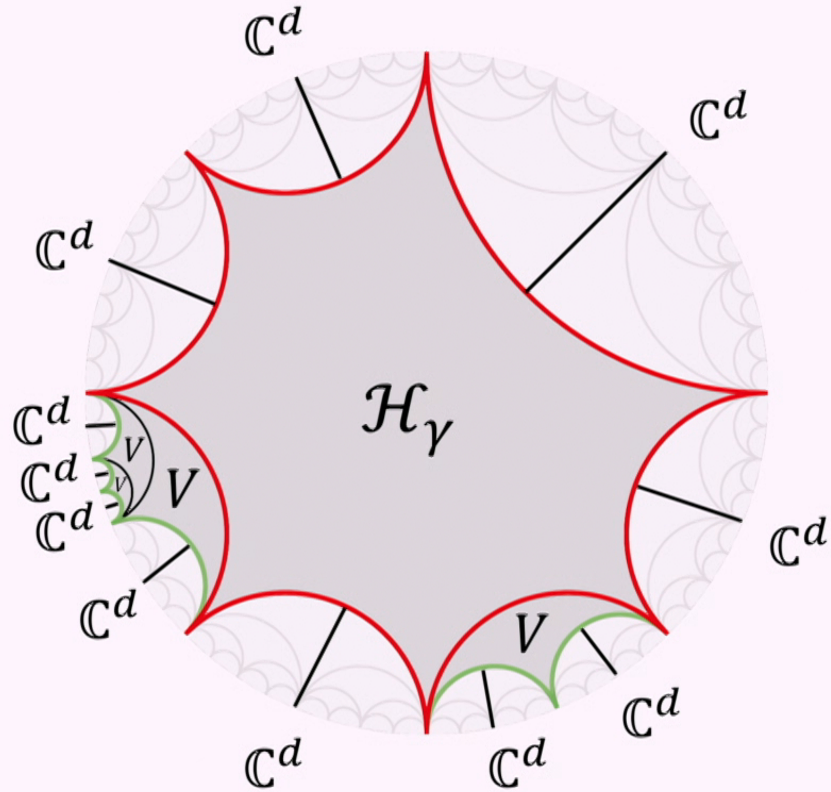
$$\mathcal{H}_\gamma \equiv \bigotimes_{e\in\gamma} \mathbb{C}^d$$

Holographic state:
sequence of states
$|\psi_\gamma\rangle$ and hilbert
spaces $\mathcal{H}_\gamma$

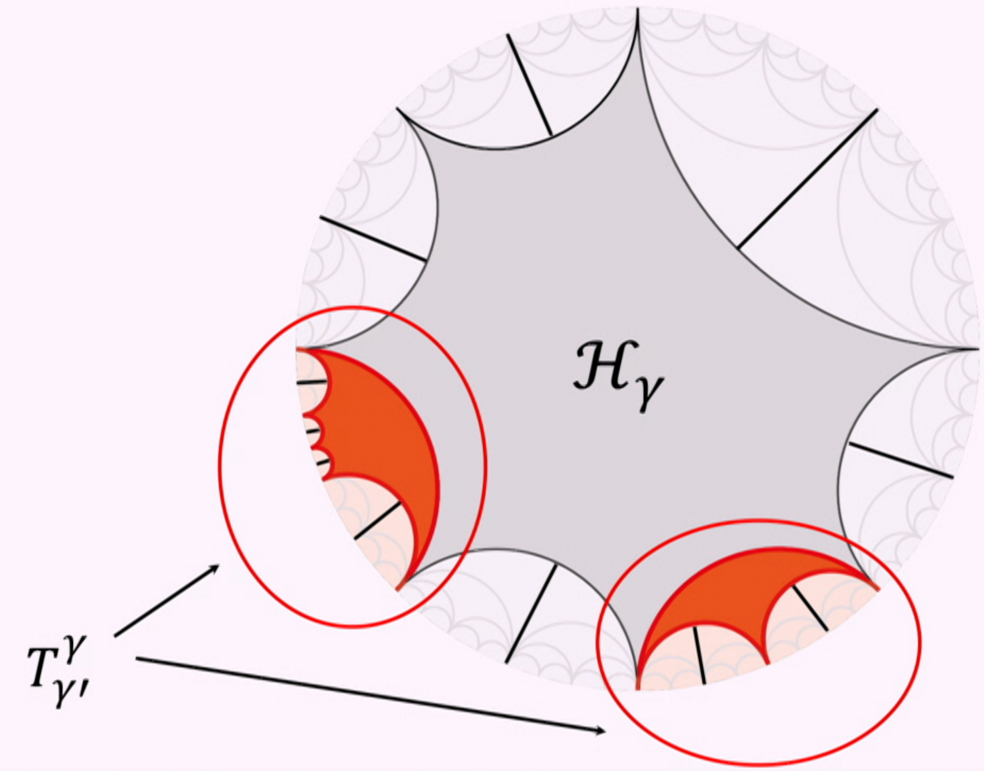
Boundary space with bigger cutoff $\mathcal{H}_{\gamma'}$:



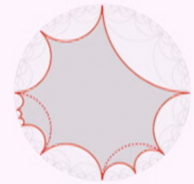
Boundary space with bigger cutoff $\mathcal{H}_{\gamma'}$:



Boundary space with bigger cutoff $\mathcal{H}_{\gamma'}$:



Embedding boundary spaces:



$$T_{\gamma'}^{\gamma} : \mathcal{H}_{\gamma} \to \mathcal{H}_{\gamma'}$$

for

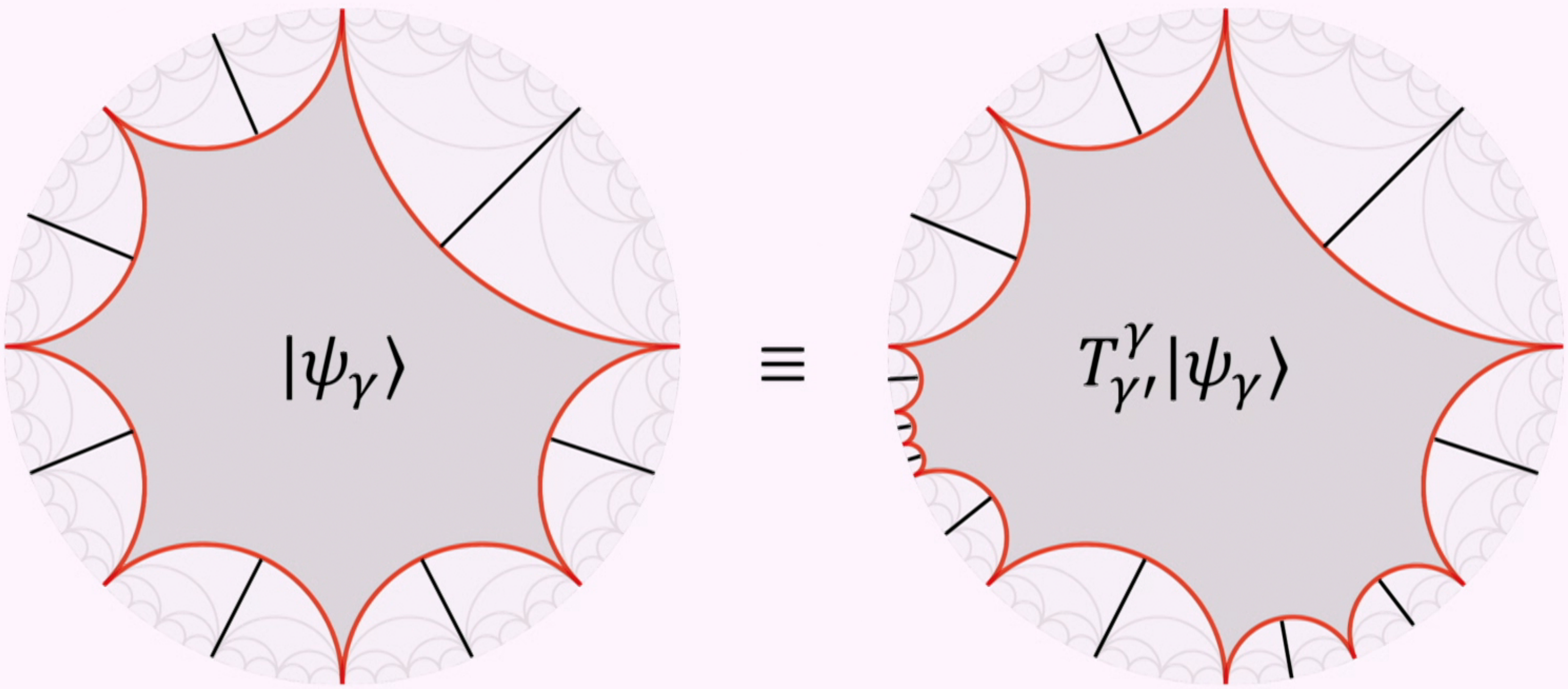
$$\gamma \preccurlyeq \gamma'$$

Definition: the **kinematical space** for holographic state is determined by the directed set $(\mathcal{P}, \preccurlyeq)$ with boundary Hilbert space $\mathcal{H}_\gamma$ for each $\gamma \in \mathcal{P}$ s.t. for all $\gamma \preccurlyeq \gamma'$ there are isometries $T^{\gamma}_{\gamma'}: \mathcal{H}_\gamma \to \mathcal{H}_{\gamma'}$, satisfying

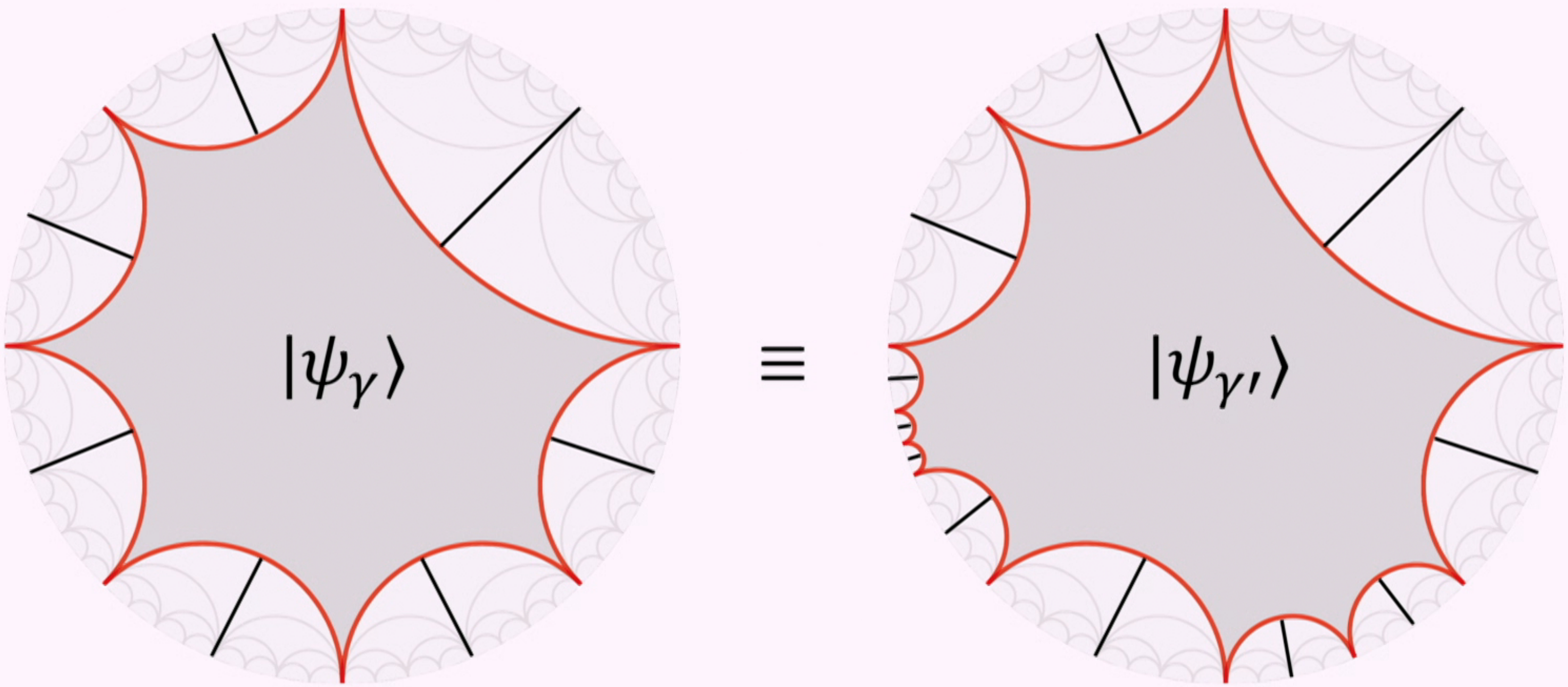
(1) $T^{\gamma}_{\gamma} = \mathbb{I}, \forall \gamma$

(2) $T^{\gamma'}_{\gamma''} T^{\gamma}_{\gamma'} = T^{\gamma}_{\gamma''}, \forall \gamma \leq \gamma' \leq \gamma''$

Equivalent holographic states:



Equivalent holographic states:



Semicontinuous limit:

$$\widehat{\mathcal{H}} = \left(\biguplus \mathcal{H}_\gamma / \sim \right)^{\|\ \|}$$

$$= \left\{ \begin{array}{l} \text{the disjoint union of } \mathcal{H}_\gamma \text{ over all } \gamma \in \mathcal{P} \\ \text{modulo the equivalence relation } |\phi\rangle_\gamma \sim |\psi\rangle_{\gamma'} \\ \text{if there is } \gamma'' \geq \gamma \text{ and } \gamma'' \geq \gamma' \text{ such that} \\ \qquad\qquad T^{\gamma}_{\gamma''} |\phi\rangle_\gamma = T^{\gamma'}_{\gamma''} |\psi\rangle_{\gamma'} \end{array} \right\}^{\|\ \|}$$

1. any book on algebra
2. R. F. Werner, unpublished (1993)
3. V. F. R. Jones, arXiv:1412.7740 (2014)

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Semicontinuous limit:

$\widehat{\mathcal{H}}$: infinite dimensional separable Hilbert space

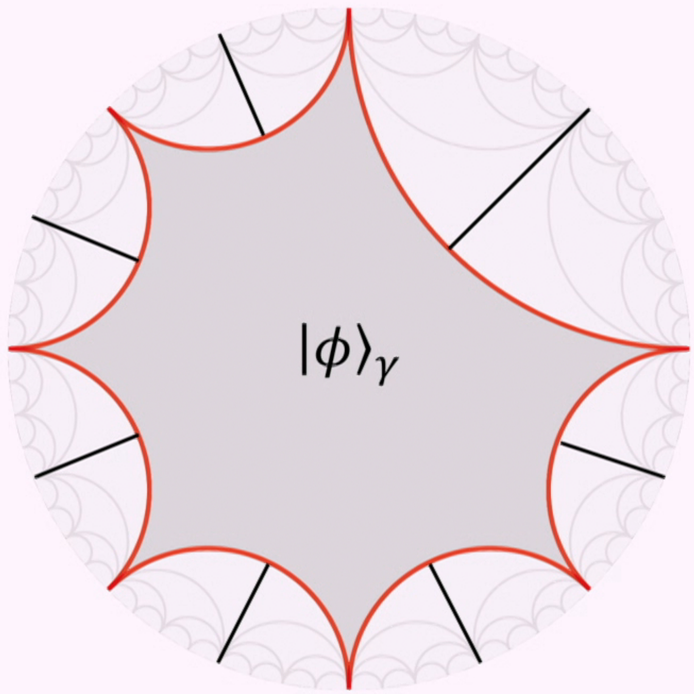
Residents of $\widehat{\mathcal{H}}$:

Sequences of boundary states $|\phi\rangle_\gamma$ for increasing cutoffs γ:

$$\big[|\psi\rangle_\gamma\big] \equiv \{|\phi\rangle_\gamma = T^{\gamma}_{\gamma'}|\psi\rangle_\gamma\}$$

$$\equiv \text{UV completion of } |\psi\rangle_\gamma$$

Residents of $\widehat{\mathcal{H}}$:



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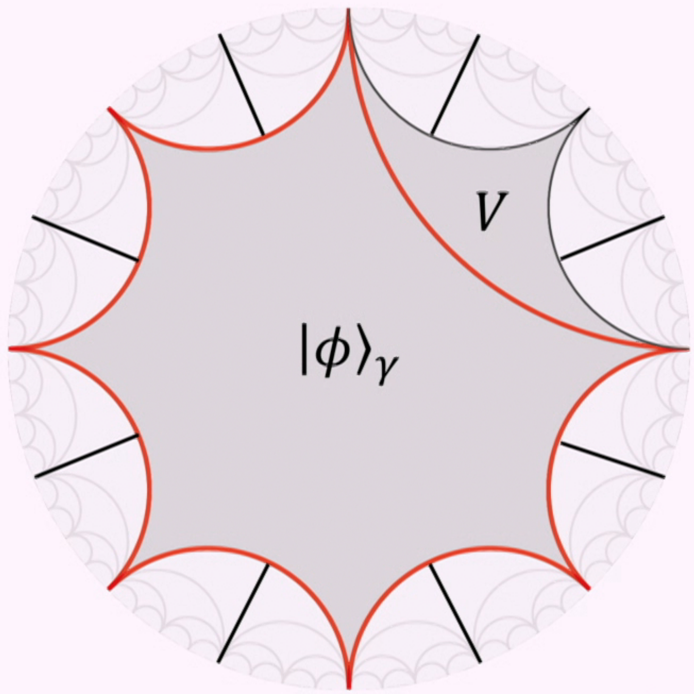

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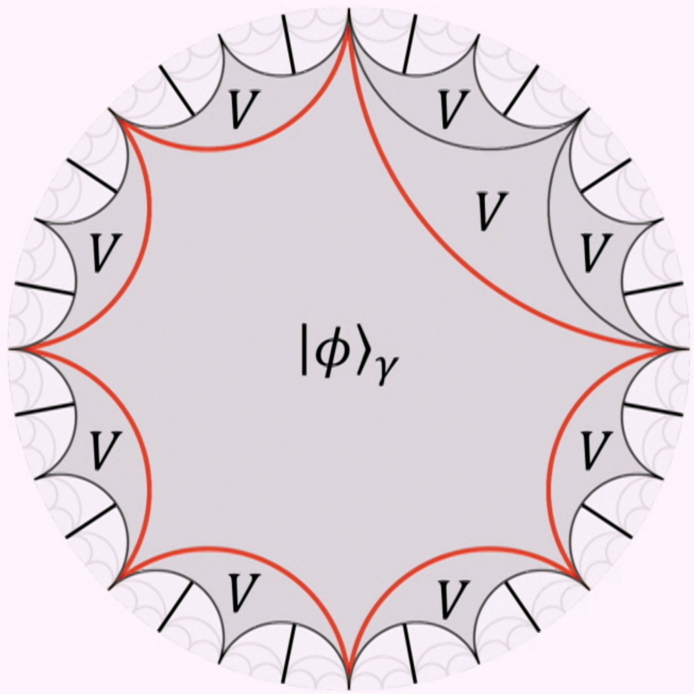

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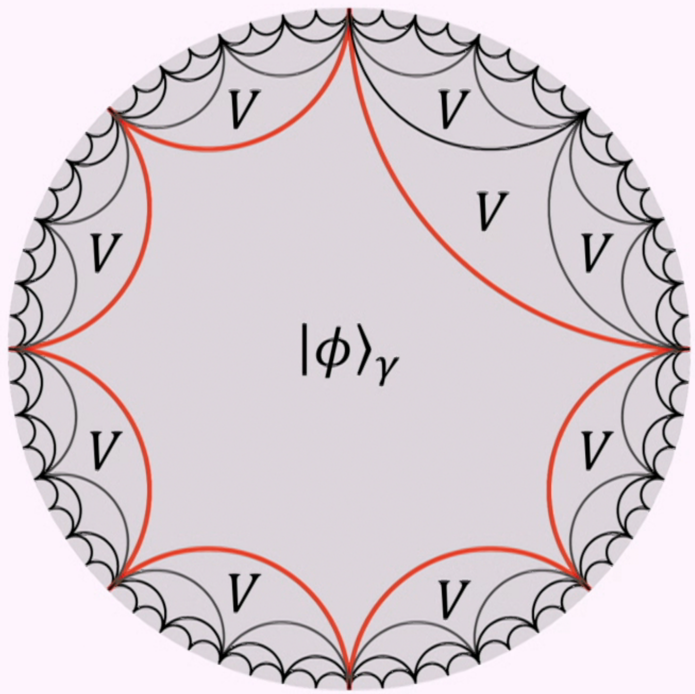
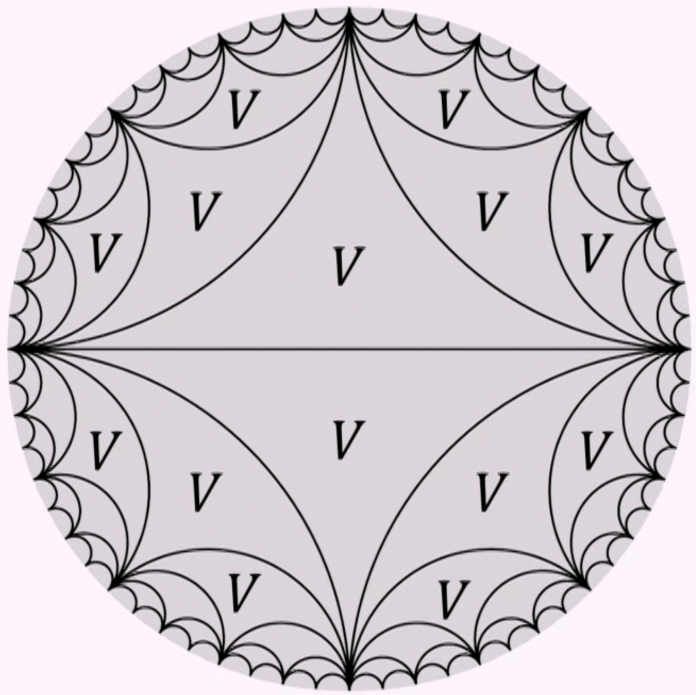
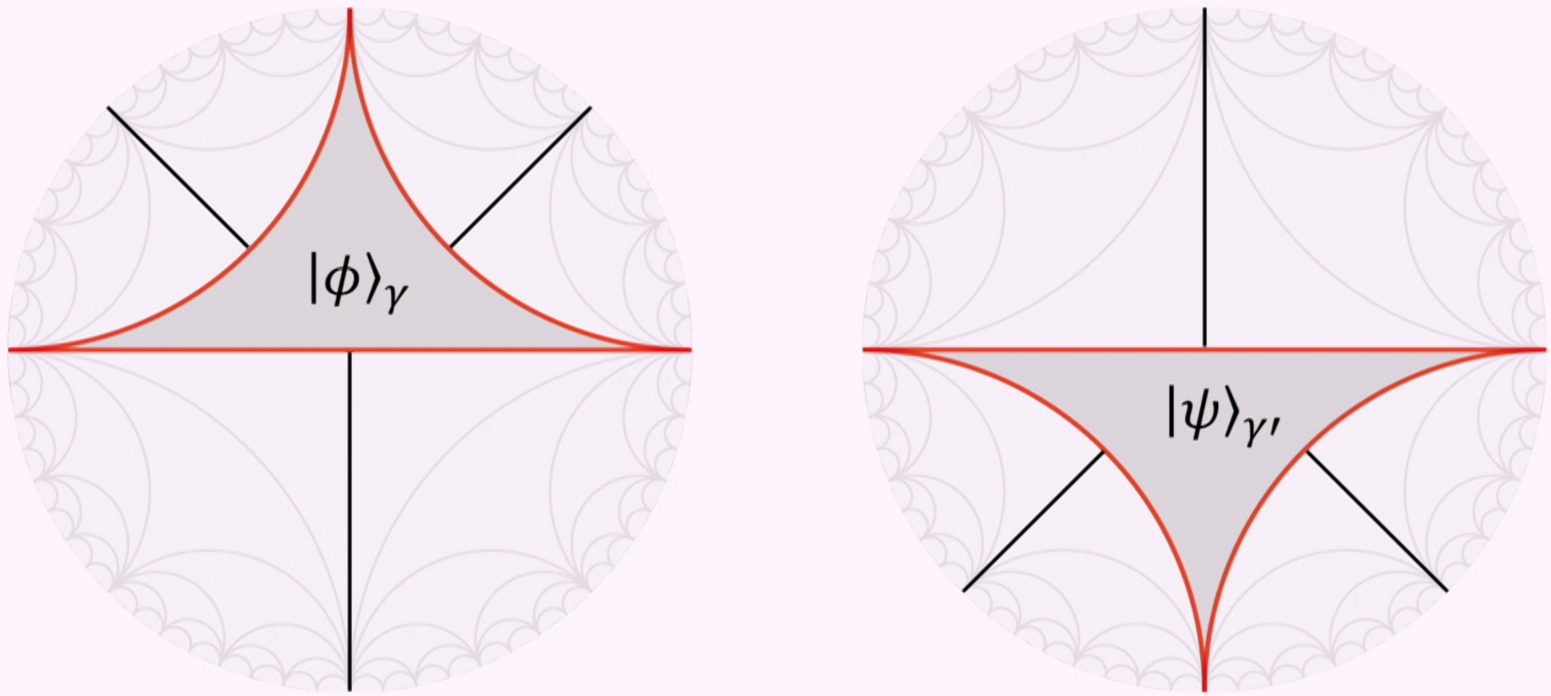


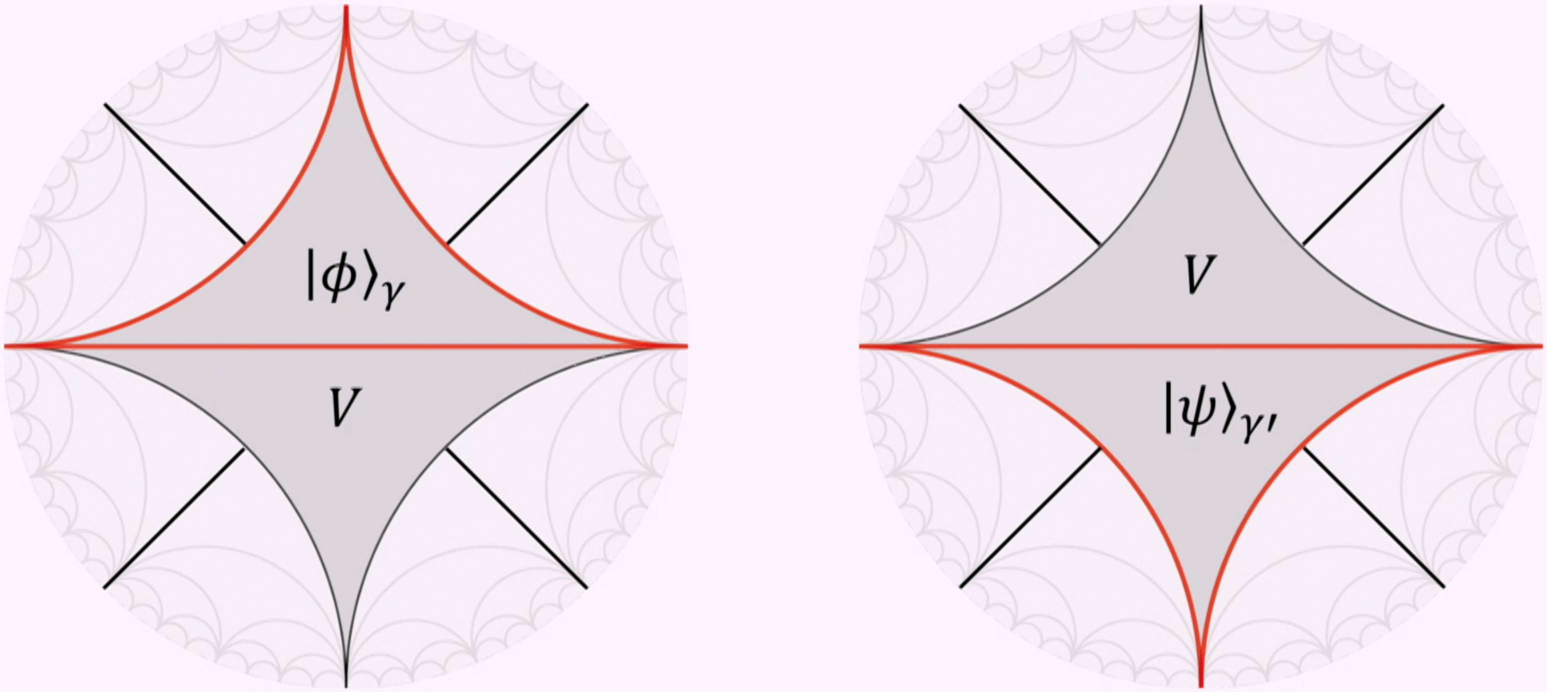
Vacuum state $\left[|\Omega\rangle_\gamma\right] \in \widehat{\mathcal{H}}$:



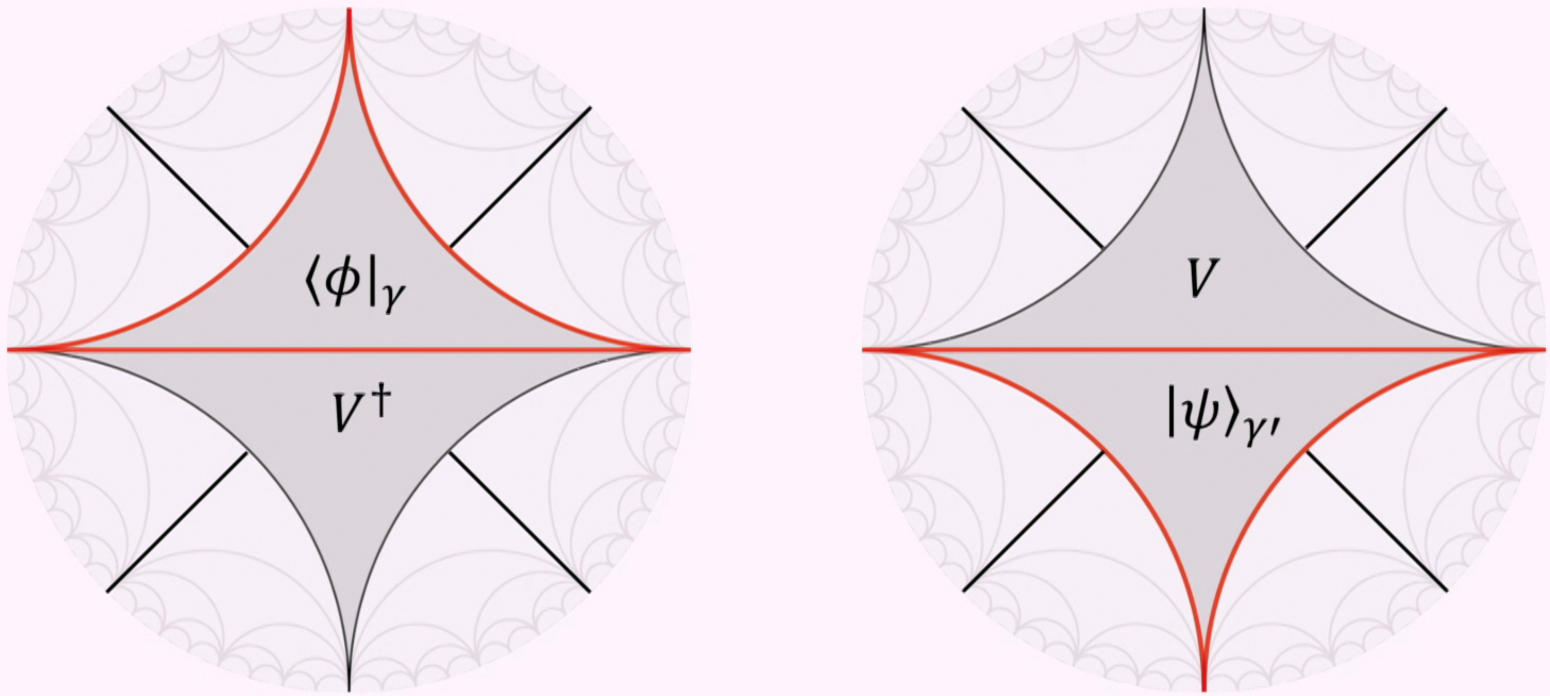
Inner product: $[|\phi\rangle_\gamma]$ and $[|\psi\rangle_{\gamma'}]$



Inner products:



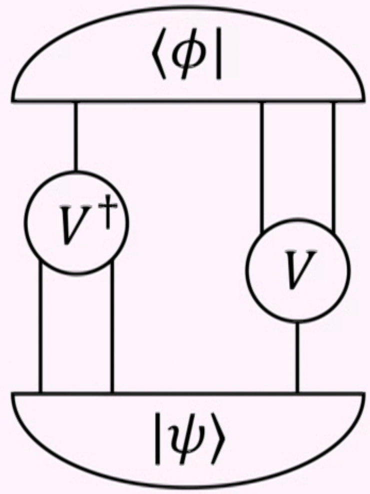
Inner products:



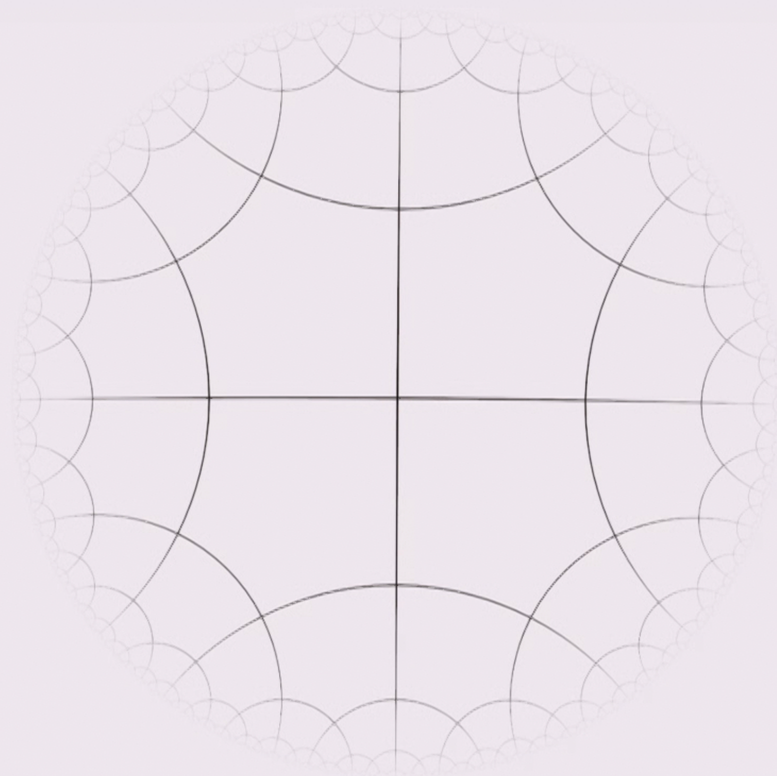
Inner products:

$$([|\phi\rangle_\gamma], [|\psi\rangle_{\gamma'}]) = \langle\phi|\big(V^\dagger \otimes \mathbb{I} \otimes \mathbb{I}\big)(\mathbb{I} \otimes \mathbb{I} \otimes V)|\psi\rangle$$

$=$



Works for all "nice" tessellations:



Hyper-invariant tensor networks and holography

Glen Evenbly[1]

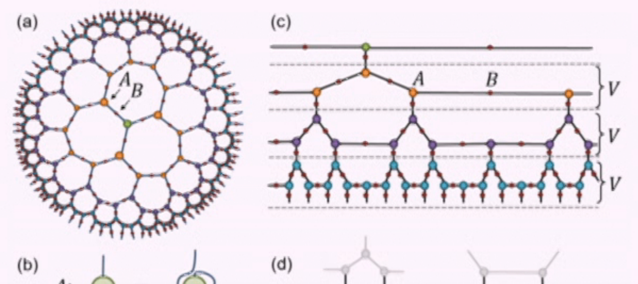
[1] *Département de Physique and Institut Quantique, Université de Sherbrooke, Québec, Canada* *

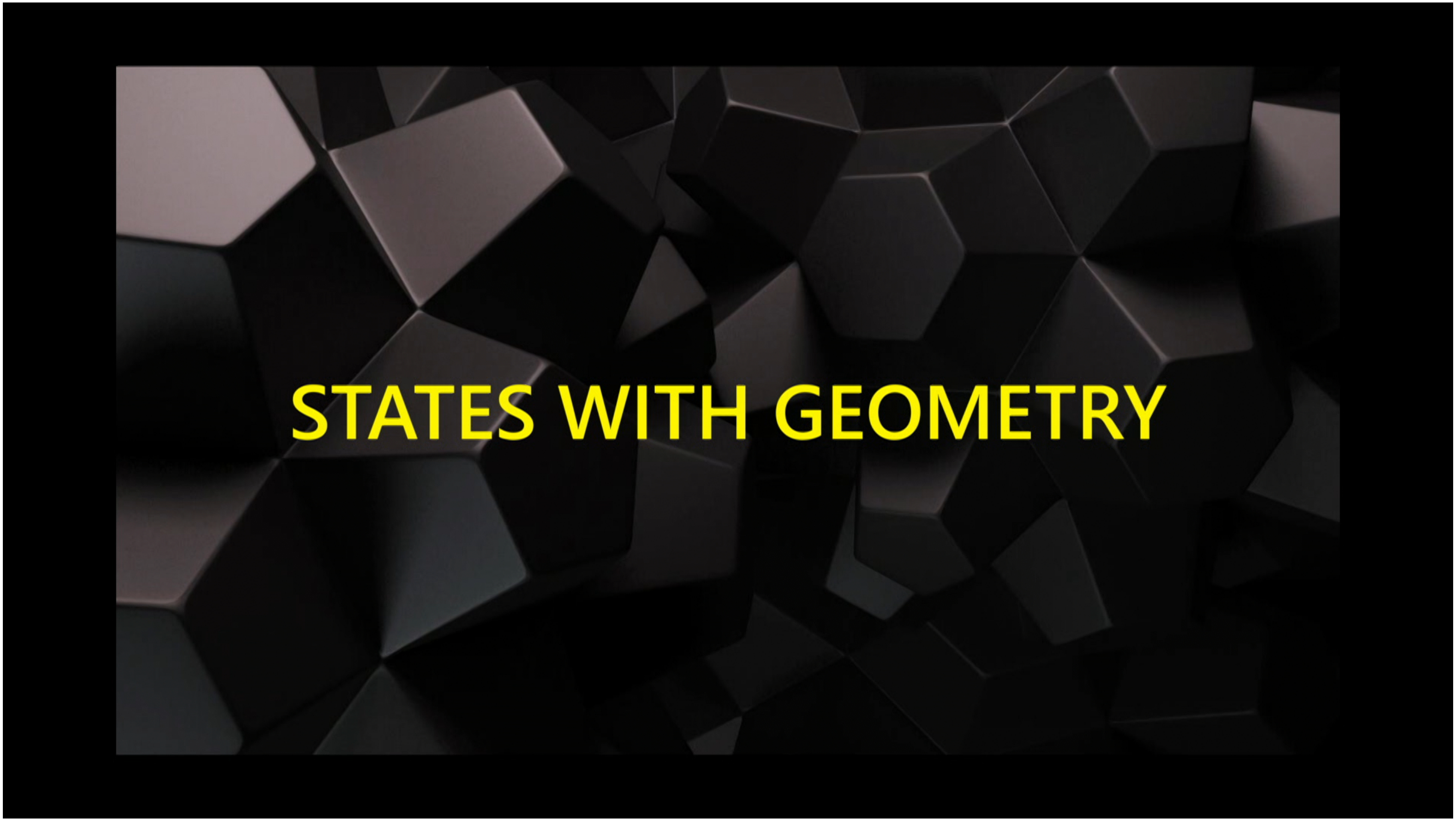
(Dated: April 14, 2017)

We propose a new class of tensor network state as a model for the AdS/CFT correspondence and holography. This class is demonstrated to retain key features of the multi-scale entanglement renormalization ansatz (MERA), in that they describe quantum states with algebraic correlation functions, have free variational parameters, and are efficiently contractible. Yet, unlike MERA, they are built according to a uniform tiling of hyperbolic space, without inherent directionality or preferred locations in the holographic bulk, and thus circumvent key arguments made against the MERA as a model for AdS/CFT. Novel holographic features of this tensor network class are examined, such as an equivalence between the causal cones $\mathcal{C}(\mathcal{R})$ and the entanglement wedges $\mathcal{E}(\mathcal{R})$ of connected boundary regions $\mathcal{R}$.

PACS numbers: 05.30.-d, 02.70.-c, 03.67.Mn, 75.10.Jm

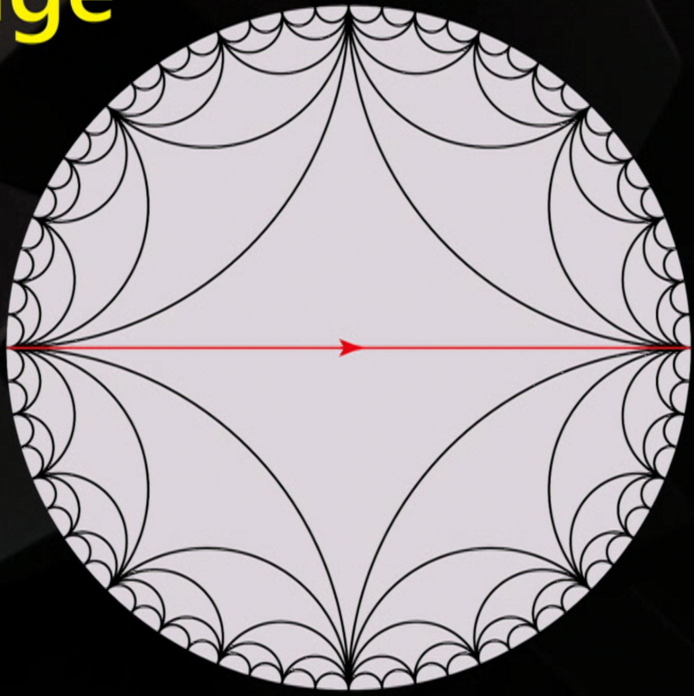
Introduction.— Tensor network methods [1, 2] have proven remarkably useful for investigating quantum many-body systems, both advancing their theoretical understanding and providing powerful tools for their numeric simulation. Introduced by Vidal, the multi-scale entanglement renormalization ansatz (MERA) [3], which describes quantum states on a D-dimensional lattice as a tensor network in $(D+1)$-dimensions, is known to be particularly well-suited for representing ground states of crit-



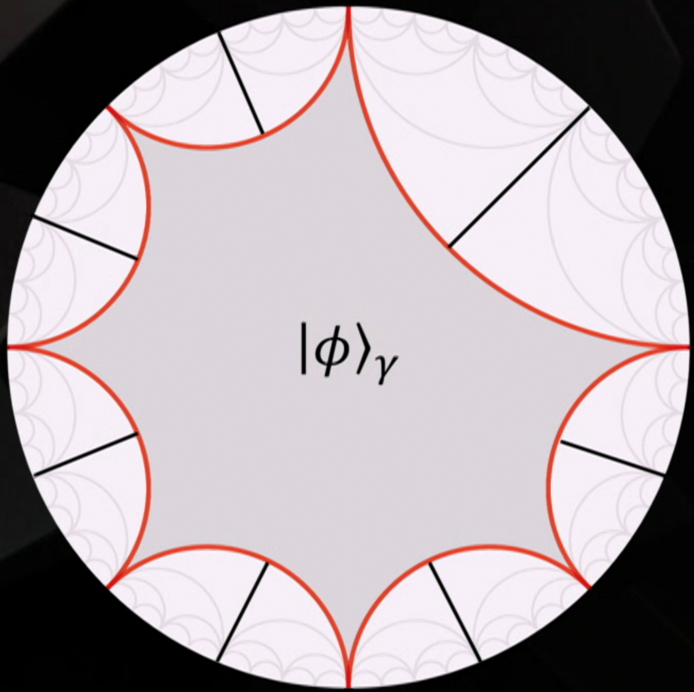

STATES WITH GEOMETRY

Tessellation with distinguished oriented edge

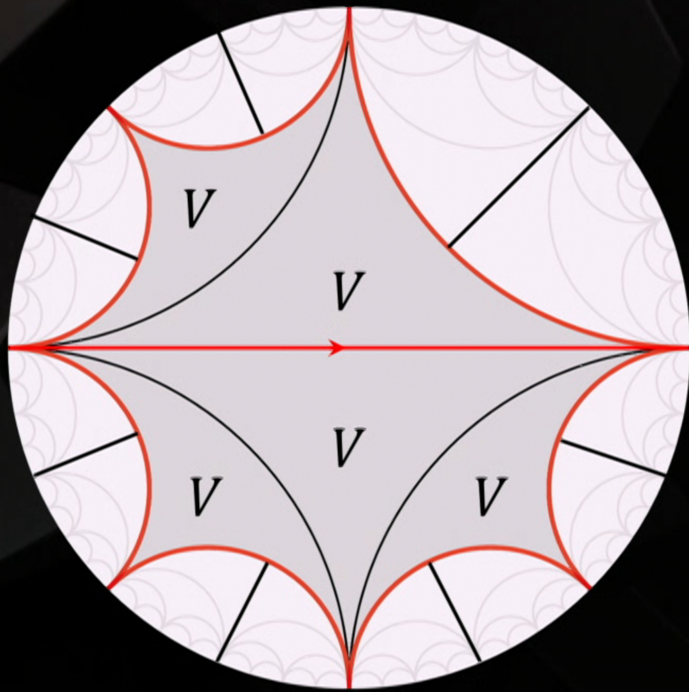
(τ, e)



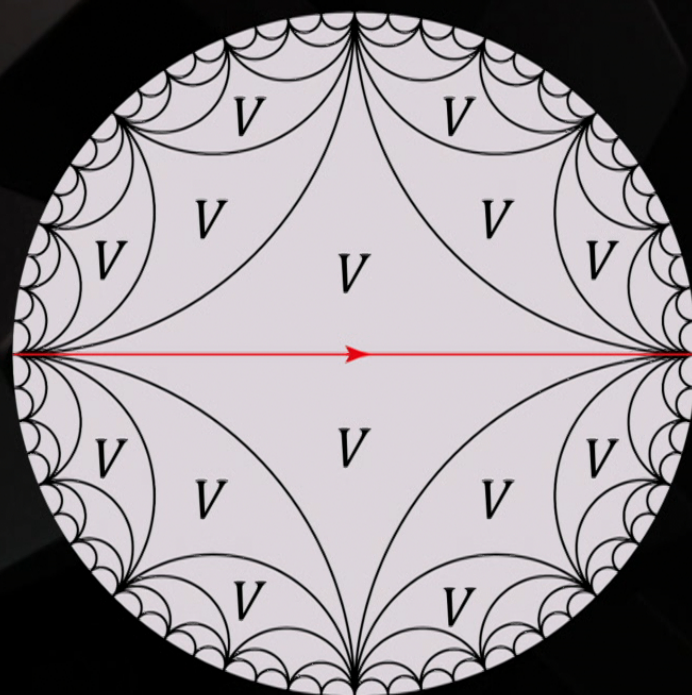
State with geometry (τ, e)



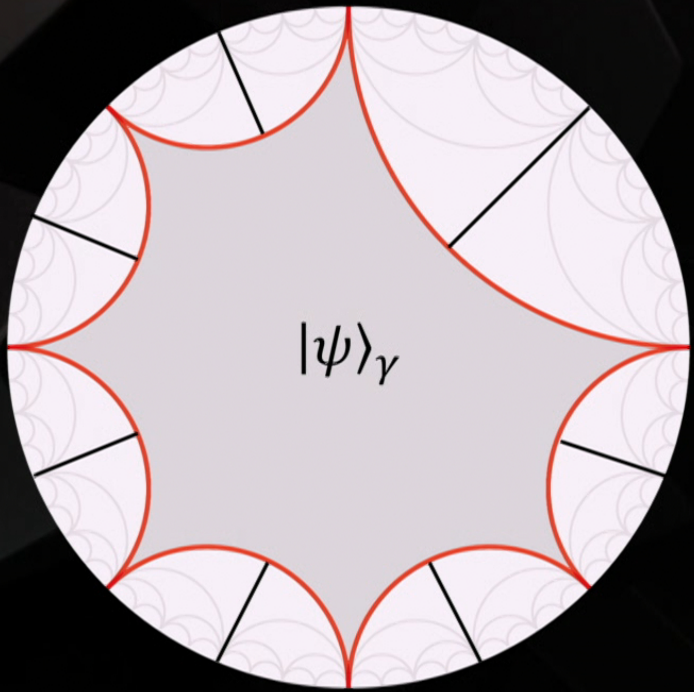
State with geometry (τ, e)



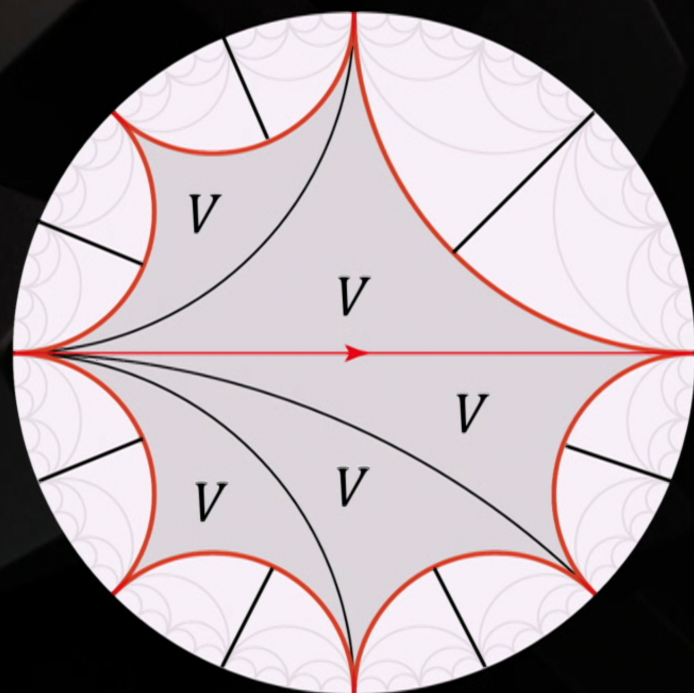
State with geometry (τ, e)



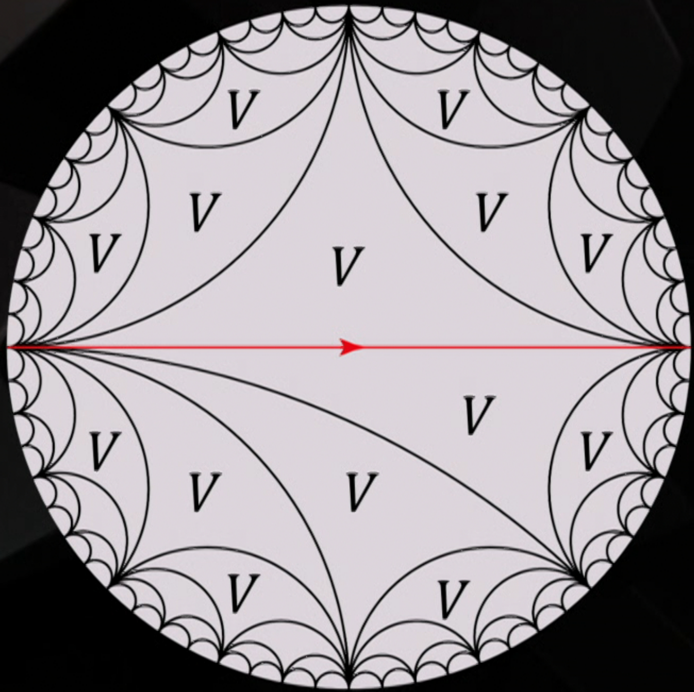
State with geometry (τ', e')



State with geometry (τ', e')



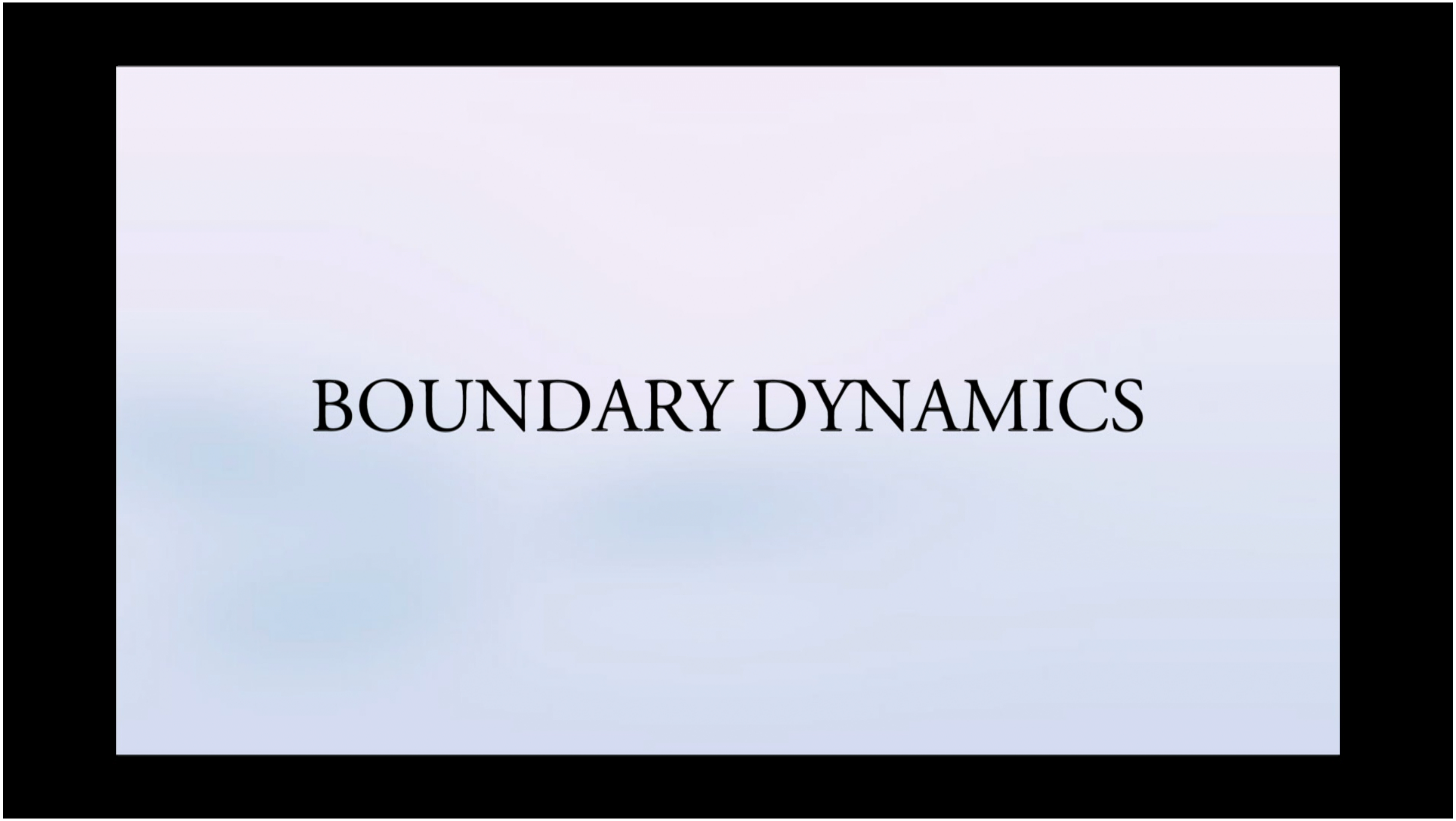
State with geometry (τ', e')



Definition: State $[|\psi_\gamma\rangle] \in \widehat{\mathcal{H}}$ has geometry (τ, e) if it is contraction of holographic tensor network for tessellation (τ, e)

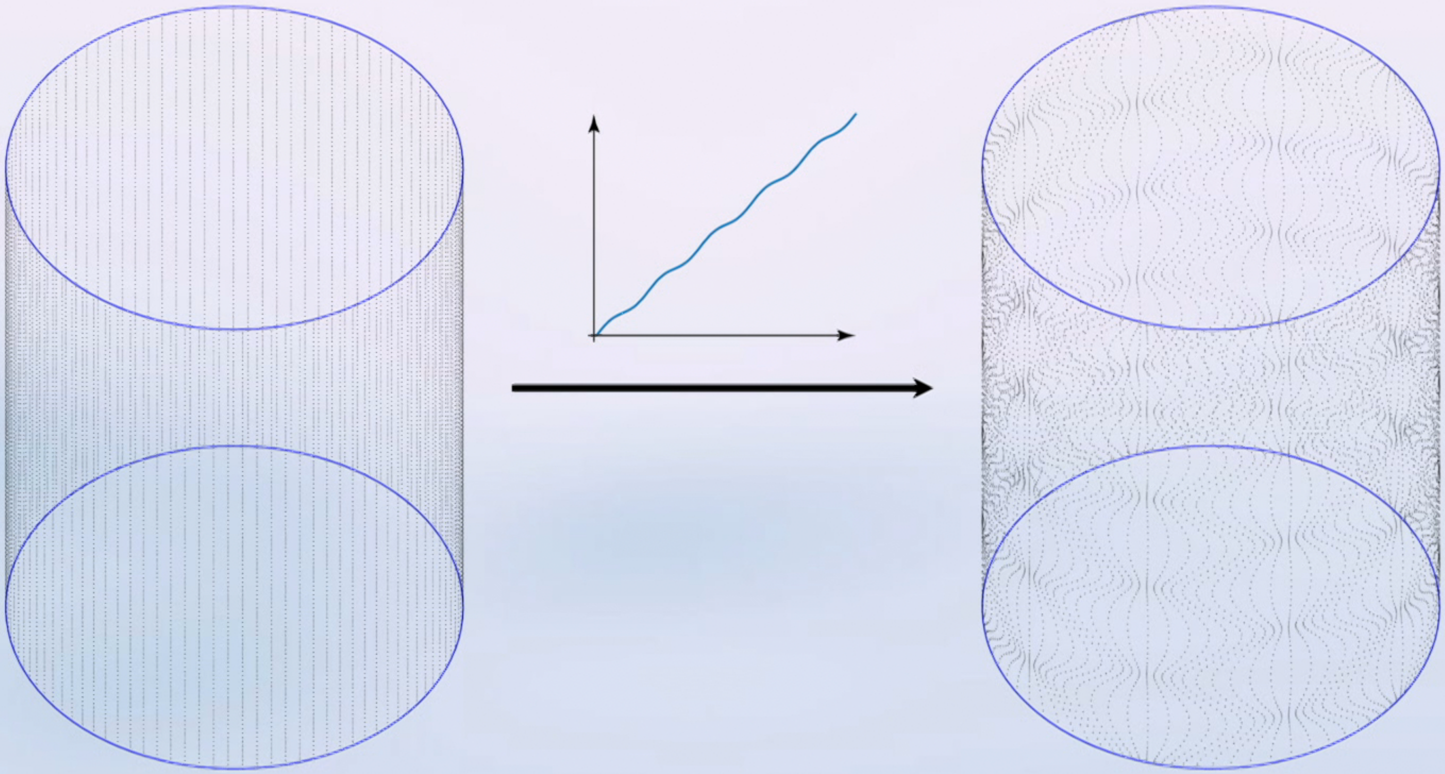
Subset $\mathcal{G}$ of $\widehat{\mathcal{H}}$ of states with a geometry (τ, e) are **semiclassical states**

Warning: subset $\mathcal{G}$ of $\widehat{\mathcal{H}}$ of states with some geometry (τ, e) is **not** a linear space


BOUNDARY DYNAMICS

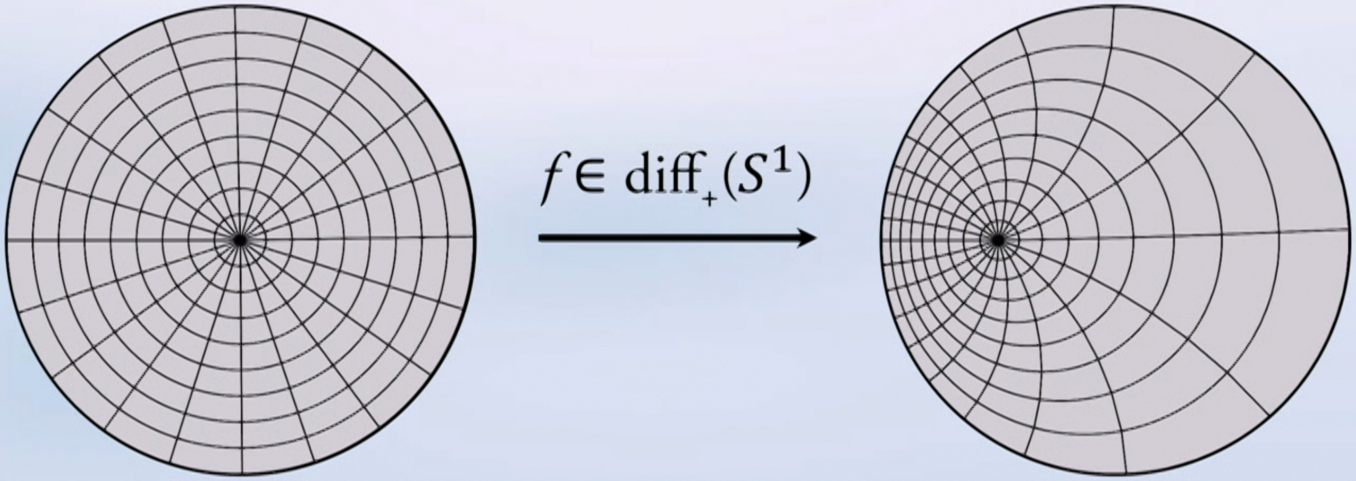
Dynamics: unitary representation of Poincaré/conformal group

$$\mathrm{conf}(\mathbb{R}^{1,1}) \cong \mathrm{diff}_+(S^1) \times \mathrm{diff}_+(S^1)$$

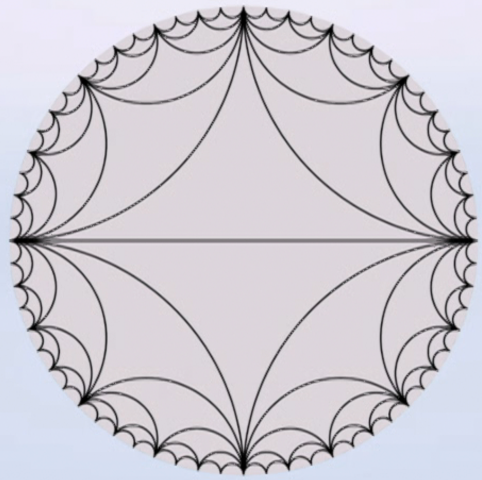


CFT Dream: find a unitary action of $\mathrm{conf}(\mathbb{R}^{1,1})$ on $\widehat{\mathcal{H}}$

$\mathrm{diff}_+(S^1)$ acts on boundary $S^1 = \partial\mathbb{D}$ of Poincaré disc $\mathbb{D}$:

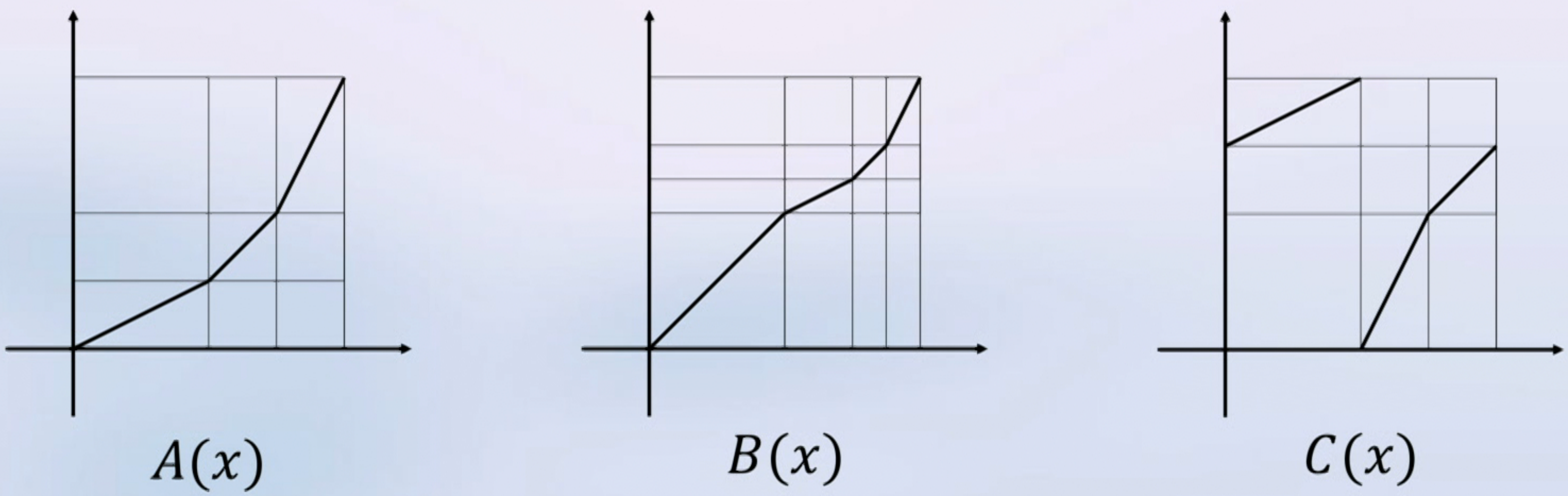


Problem: $\mathrm{diff}_+(S^1)$ is incompatible with (dyadic) tessellation



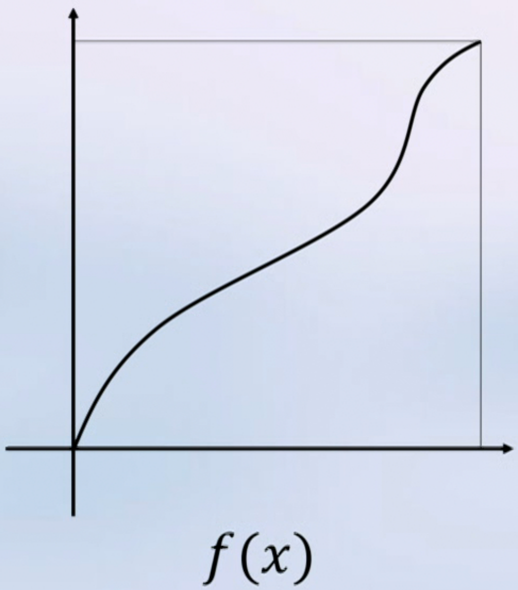
Strategy: study "discrete" version of conformal group; **Thompson's group** T

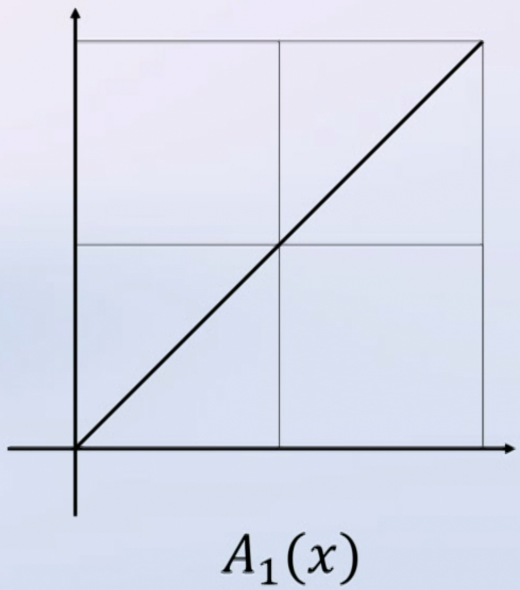
Thompson's group T: generated by $A(x)$, $B(x)$, and $C(x)$ under composition



J. W. Cannon, W. J. Floyd, and W. R. Parry, *Enseign. Math.*, vol. 42, no. 3–4, pp. 215 – 256, 1996

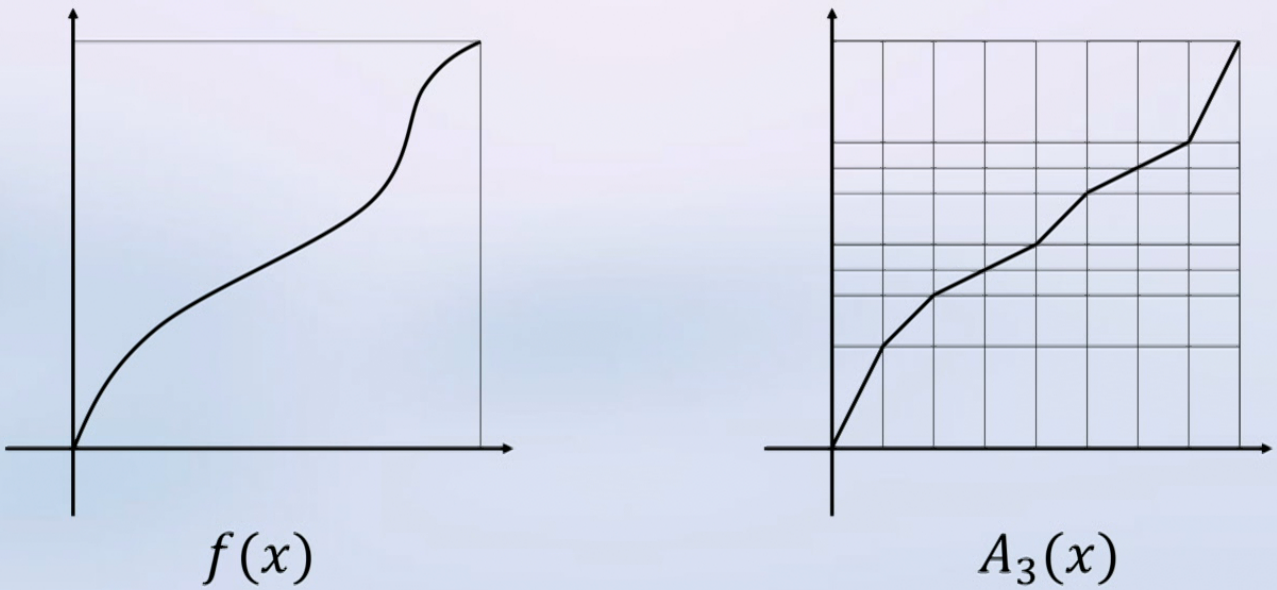
Proposition ("well known"): let $f \in \text{diff}_+(S^1)$. Then $\exists$ sequence $A_n(x) \in T$ s.t. $\|A_n - f\|_\infty \to 0$.



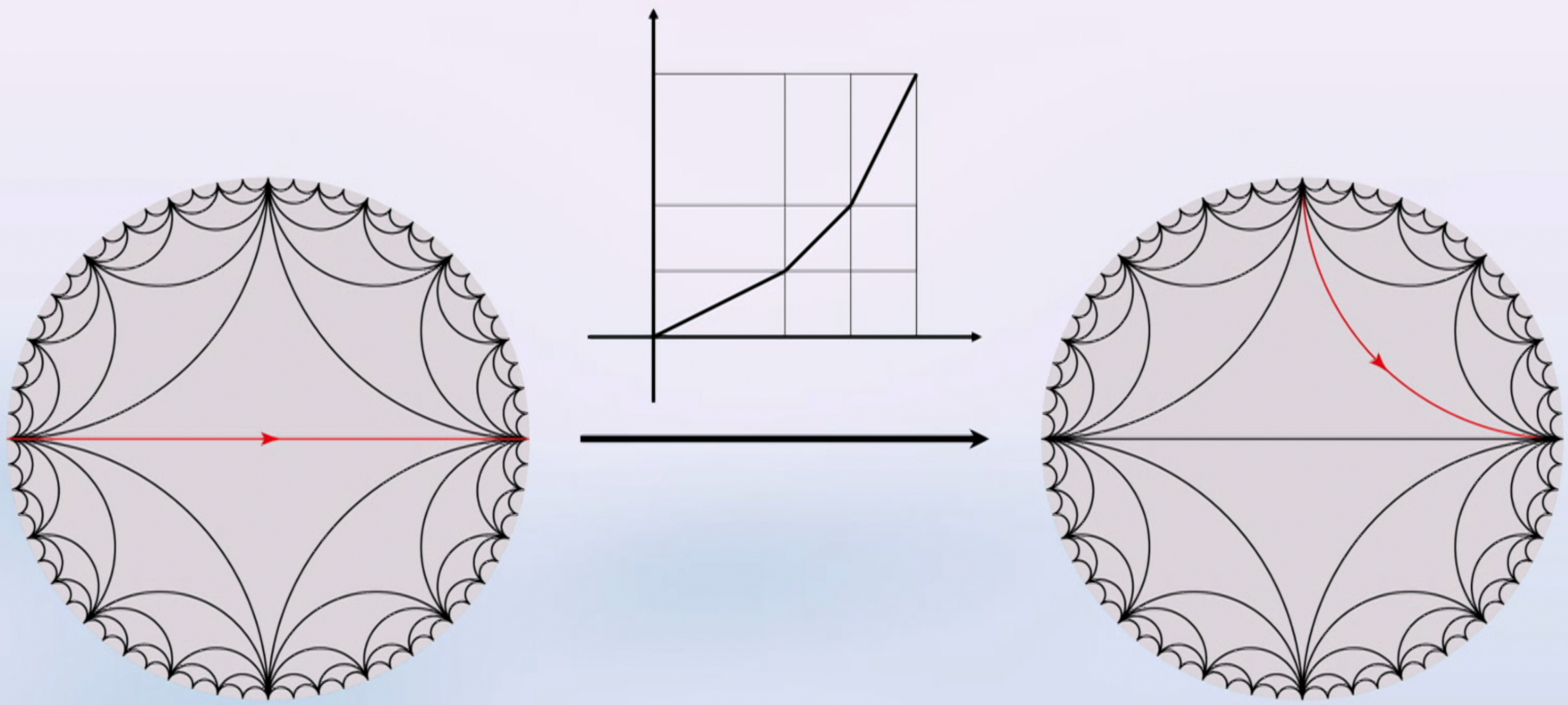


see e.g., A. Akhmedov and M. P. Cohen, arXiv:1508.04604

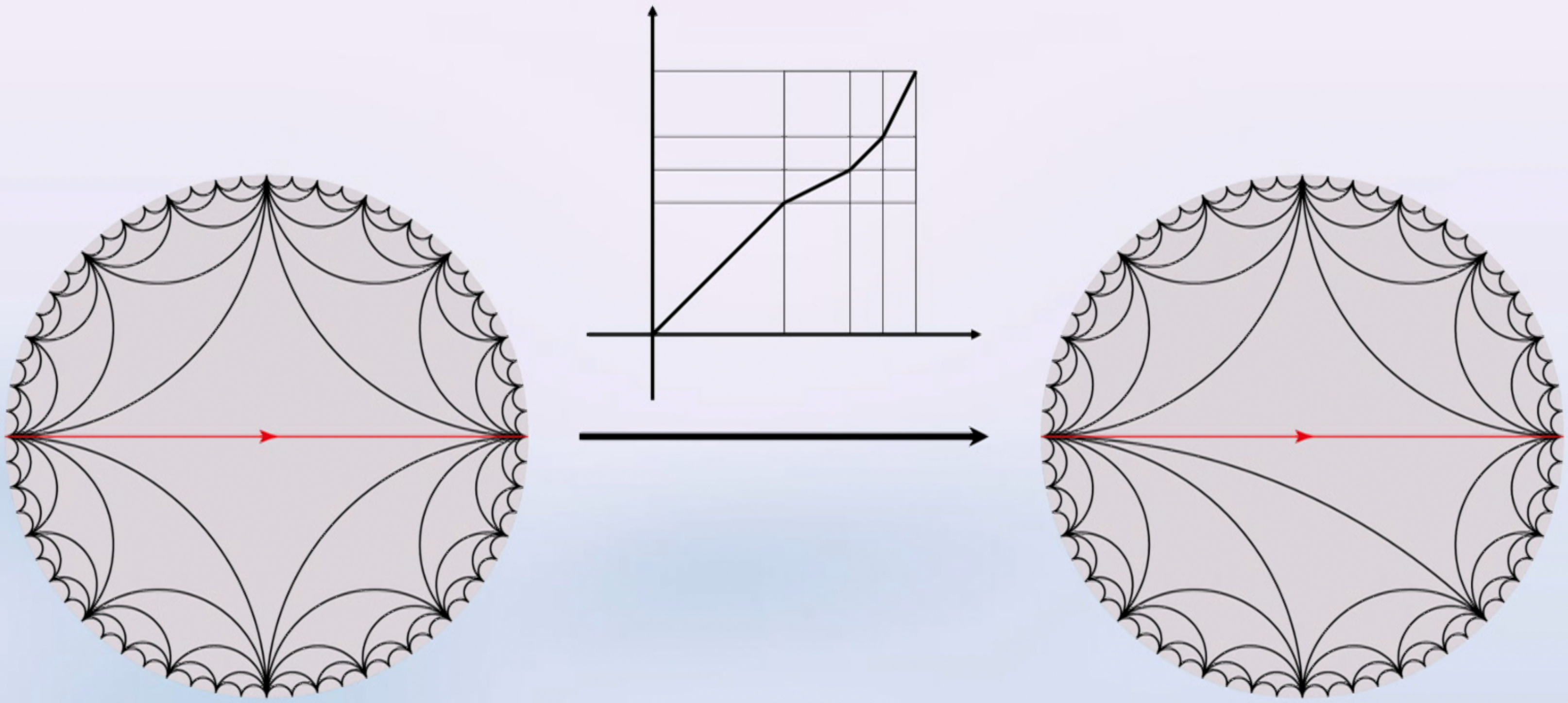
Proposition ("well known"): let $f \in \mathrm{diff}_+(S^1)$. Then $\exists$ sequence $A_n(x) \in T$ s.t. $\|A_n - f\|_\infty \to 0$.



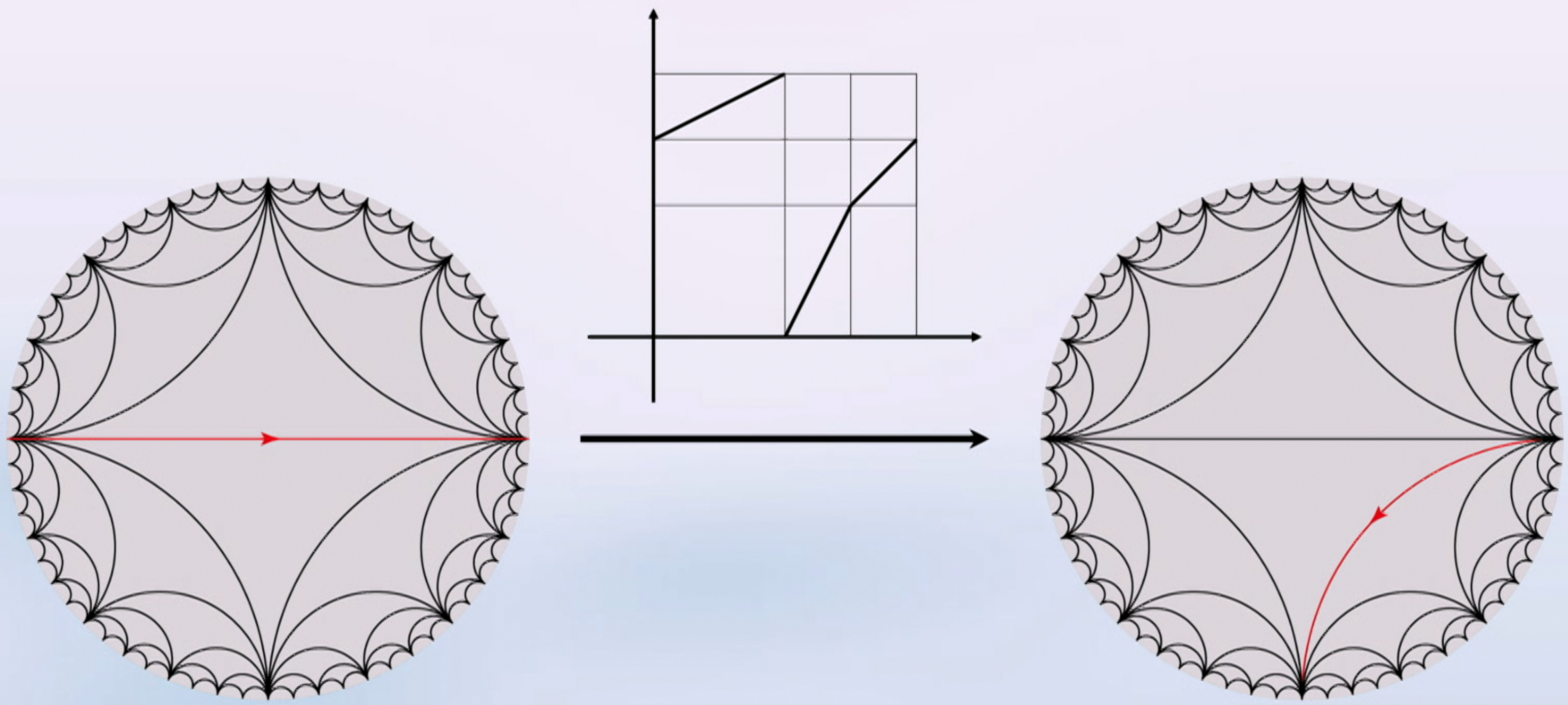
see e.g., A. Akhmedov and M. P. Cohen, arXiv:1508.04604



R. C. Penner, Adv. Math. 98, 143–215 (1993)
Geometric Galois Actions: Volume 2, (Cambridge University Press, 1997)



R. C. Penner, Adv. Math. 98, 143–215 (1993)
Geometric Galois Actions: Volume 2, (Cambridge University Press, 1997)

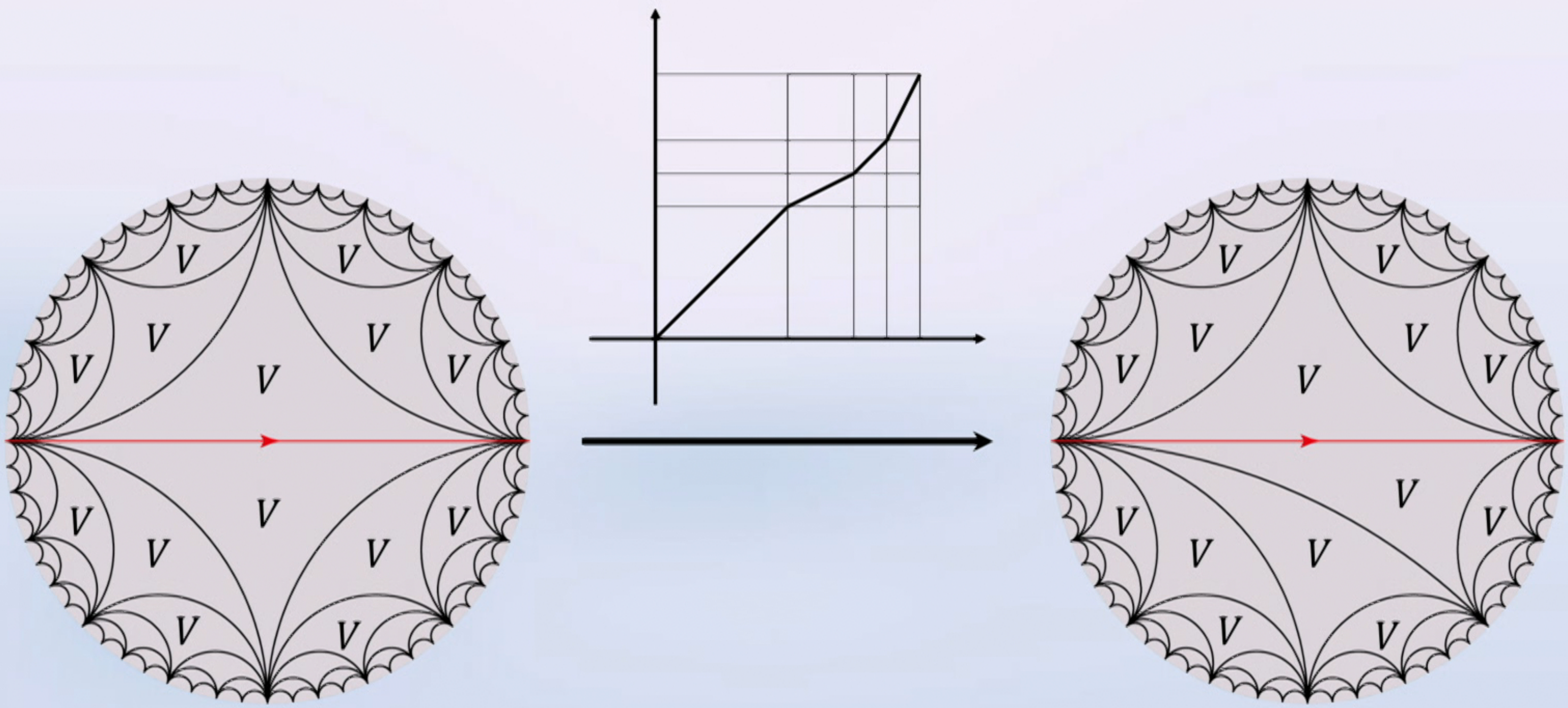


R. C. Penner, Adv. Math. 98, 143–215 (1993)
Geometric Galois Actions: Volume 2, (Cambridge University Press, 1997)

Theorem (Imbert, Lochak & Scheps, Penner): Thompson's group T is isomorphic to group of Pachner flips on tessellation (τ, e) with distinguished oriented edge.

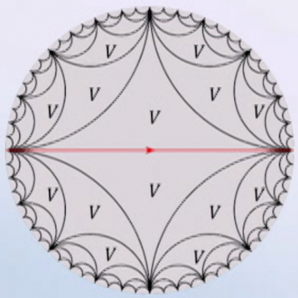
R. C. Penner, Adv. Math. 98, 143–215 (1993)
Geometric Galois Actions: Volume 2, (Cambridge University Press, 1997)

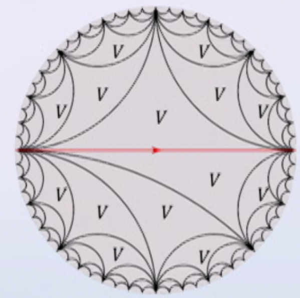
Unitary representation of T on $\widehat{\mathcal{H}}$:



V. F. R. Jones, arXiv:1412.7740 (2014)

Unitary representation of T on $\widehat{\mathcal{H}}$:





$$\left[|\Omega\rangle_\gamma\right] \longrightarrow |f\rangle \equiv \pi(f)\left[|\Omega\rangle_\gamma\right]$$

V. F. R. Jones, arXiv:1412.7740 (2014)

Unitary representation of T on $\widehat{\mathcal{H}}$:

Corollary: let $f \in T$, then every state

$$|f\rangle \equiv \pi(f)\big[|\Omega\rangle_\gamma\big]$$

is a state with *geometry* $(f(\tau), f(e))$

V. F. R. Jones, arXiv:1412.7740 (2014)

Unitary representation of T on $\widehat{\mathcal{H}}$:

Definition: let $\mathcal{V} \subset \widehat{\mathcal{H}}$ be space **spanned** by states in $\mathcal{G}$, for $f \in T$:

$$|f\rangle \equiv \pi(f)\big[|\Omega\rangle_\gamma\big]$$

V. F. R. Jones, arXiv:1412.7740 (2014)

Unitary representation of T on $\mathcal{V}$:

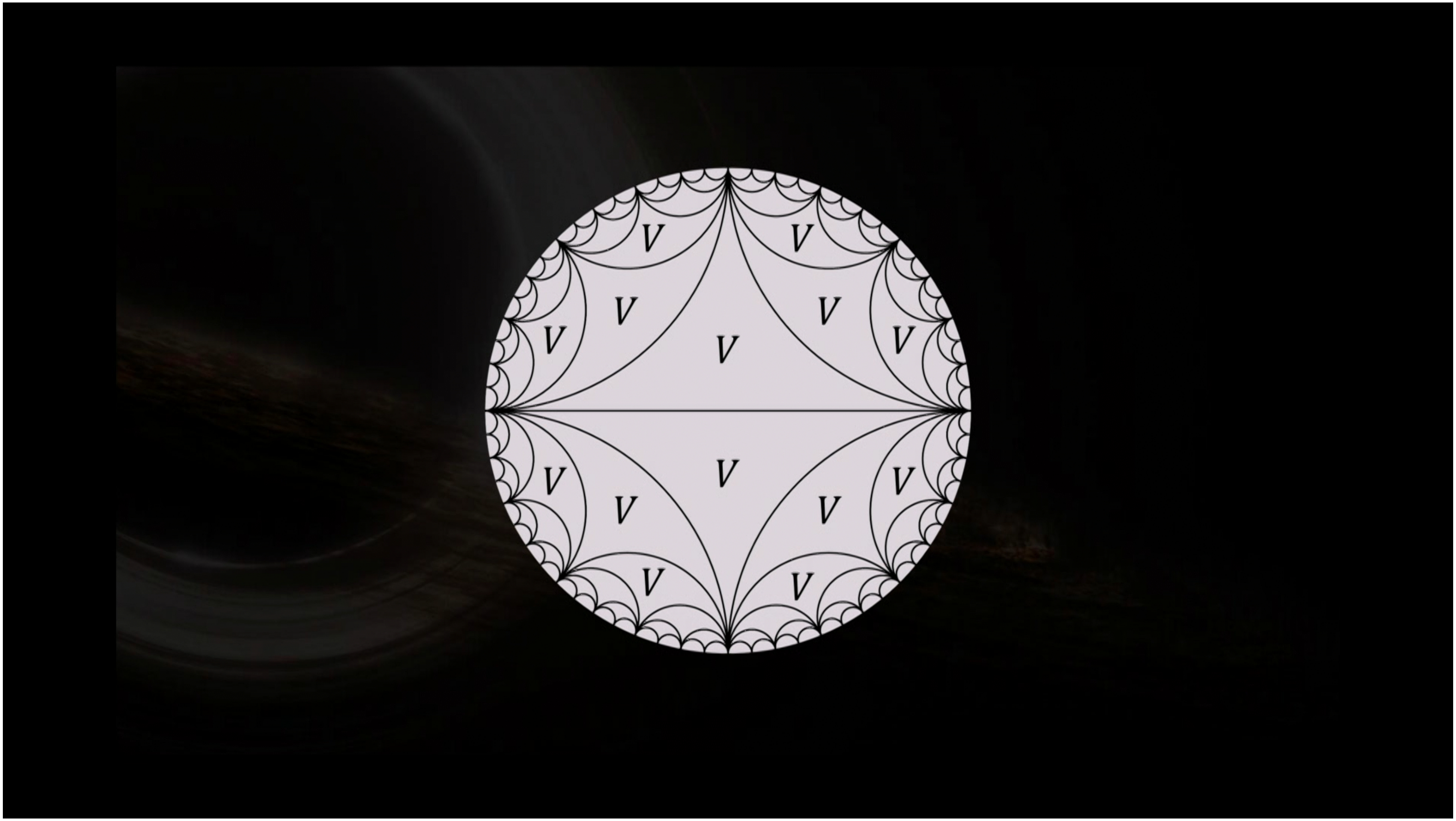
Theorem (Jones): the action

$$\pi(f)|g\rangle \equiv |fg\rangle$$

furnishes a **unitary** representation of T on $\mathcal{V}$

V. F. R. Jones, arXiv:1412.7740 (2014)
V. F. R. Jones, arXiv:1607.08769 (2016)


BTZ BLACK
HOLES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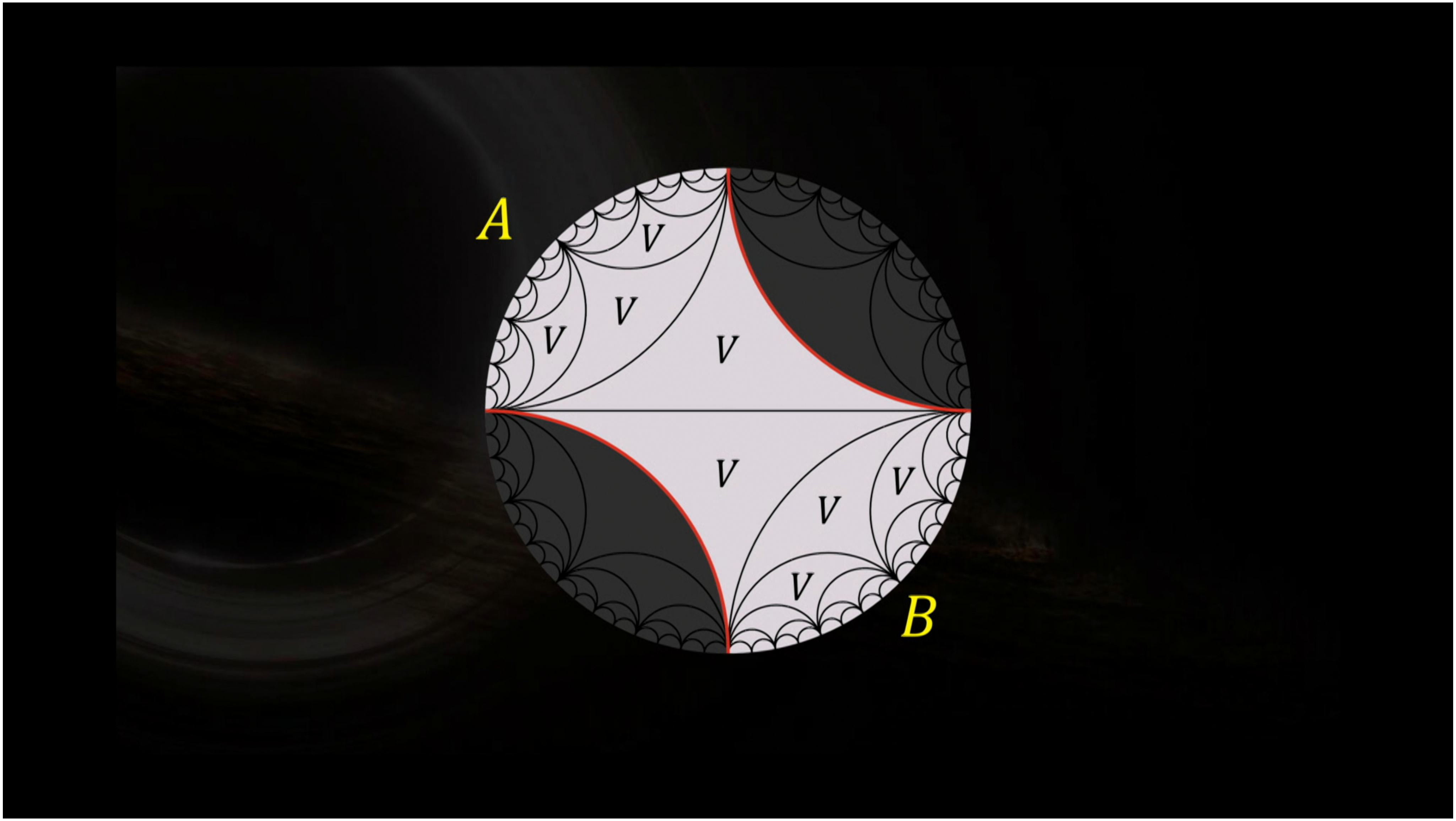
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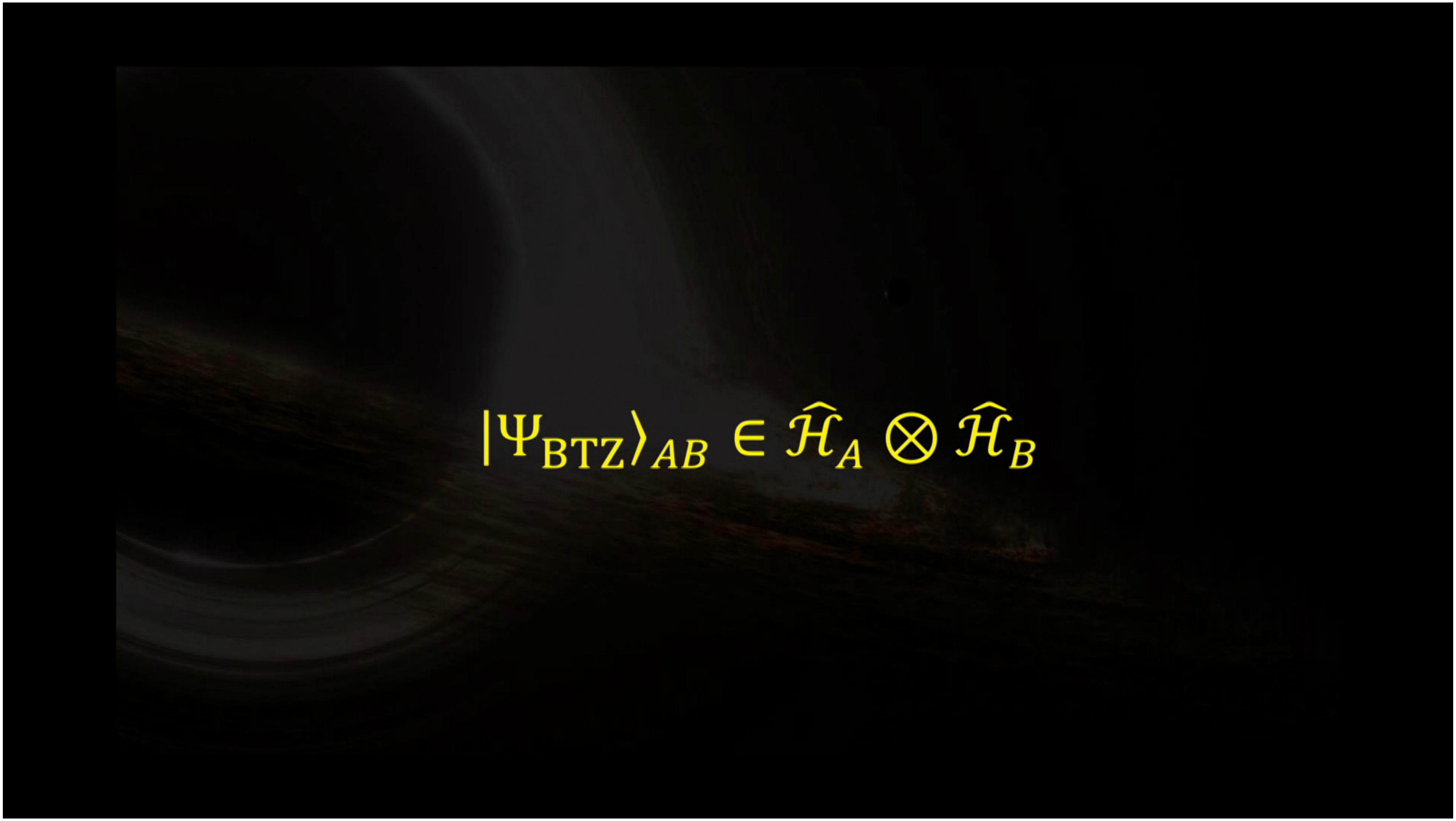

$$|\Psi_{\mathrm{BTZ}}\rangle_{AB} \in \widehat{\mathcal{H}}_A \otimes \widehat{\mathcal{H}}_B$$

$|\Psi_{\mathrm{BTZ}}\rangle_{AB}$ is entangled state
(no geometry) of Thompson CFTs *A* & *B*
AND
$|\Psi_{\mathrm{BTZ}}\rangle_{AB}$ is a state with geometry of
tessellation of BTZ

M. Van Raamsdonk, Gen. Relativ. Grav. 42, 2323–2329 (2010)
J. Maldacena and L. Susskind, Fortsch. Phys. 61, 781–811 (2013)

Unitary representation of groupoid tensor category

Objects: tessellations of Riemann surfaces
Morphisms: cobordisms

Continuum	Discretuum
$\mathbb{D}$	(τ, e)
$\mathrm{conf}(\mathbb{R}^{1,1})$	$T \times T$
CFT hilbert space $\mathcal{H}$	$\widehat{\mathcal{H}}$
$\mathcal{H}_{\mathrm{AdS}} \subset \mathcal{H}_{\mathrm{CFT}}$	$\mathcal{V} \subset \widehat{\mathcal{H}}$
(Large) bulk diffeomorphism	Pachner flip
Primary field	ϕ_α
Fusion rules	$\mathcal{E}(X) \equiv V(X \otimes \mathbb{I})V^\dagger$