Title: Two Continous Approaches to AdS/Tensor Network duality

Date: Apr 20, 2017 11:00 AM

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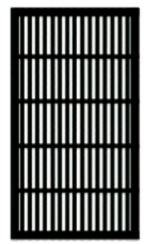
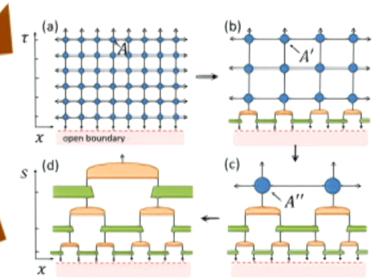
Abstract: In this talk, I would like to discuss how we can realize the correspondence between AdS/CFT and tensor network in quantum field theories (i.e. the continous limit). As the first approach I will discuss a possible connection between continuous MERA and AdS/CFT. Next I will introduce the second approach based on the optimization of Euclidean path-integral, where the strcutures of hyperbolic spaces and entanglement wedges emerge naturally. This second appraoch is closely related to the idea of tensor network renormalization.

Contents

Path-integral on flat space



Tensor Network Renormalizaton



Optimization of Path-integral

Path-integral on Hyperbolic Space

[Caputa-Kundu-Miyaji-Watanabe-TT 17]

[Evenbly-Vidal 14, 15]

[Verstraete's talk]

Tensor Networks for Quantum Field Theories II

@Perimeter Inst., Apr.18th-21th, 2017

Two Continuous Approaches to AdS/Tensor Network duality

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YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Overview of AdS/cMERA

+ 1703.00456 Caputa-Kundu-Miyaji-Watanabe-TT

Collaborators

Shinsei Ryu (Chicago)

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Masahiro Nozaki (Chicago)

Tokiro Numasawa (YITP/Osaka→McGill)

Noburo Shiba (Harvard)

Xueda Wen (UIUC)

Masamichi Miyaji (YITP, Kyoto)

Kento Watanabe (YITP, Kyoto)

References of Continuous Approach to TN I: cMERA

Original paper of cMERA:

Haegeman-Osborne-Verschelde-Verstraete [PRL 110(2013)100402]

Conformal symmetry in cMERA Hu-Vidal 1703.04798 [Guifre's talk]

Interactions in cMERA Cotler-Molina-Vilaplana-Mueller 1612.02427

Information metric and Holography for cMERA:

Nozaki-Ryu-TT 1208.3469 [JHEP 1210 (2012) 193]

Time evolution (Quantum quenches) and finite temp. in cMERA:

Mollabashi-Nozaki-Ryu-TT 1311.6095 [JHEP 1403 (2014) 098]

General Formulation (use of boundary state)

Miyaji-Ryu-Wen-TT 1412.6226 [JHEP1505 (2015) 152]

Surface/State Corr.: Miyaji-TT 1503.03542 [PTEP (2015) 073B03]

Miyaji-Numasawa-Shiba-Watanabe-TT 1506.01353 [PRL 115 (2015) 171602]

References of Continuous Approach to TN II: Optimization of Path-integral

Qualitative Argument:

1609.04645 [PRD 95(2017)066004] Miyaji-Watanabe-TT

AdS from Optimization of path-integral:

1703.00456 Caputa-Kundu-Miyaji-Watanabe-TT

170?.????? Work in progress

Refer also to Czech's talk

① Introduction

AdS/CFT (or more generally holography) [Maldacena 20 yrs ago]

⇒ ``Geometrization'' of Dynamics in QFTs

One important fact behind holography is

Quantum entanglement

⇒ ``Geometry'' of Quantum States in many-body system

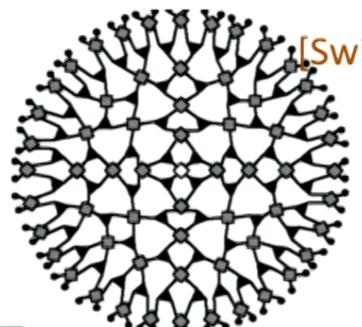
Emergent spacetime from Entanglement

Quantum States

$$|\Psi(t)\rangle = \sum_{\{i_k\}} c_{\{i_k\}}(t)\, |i_1\rangle \otimes |i_2\rangle \ldots \otimes |i_N\rangle$$

Algebraically complicated,
but geometrically look nicer !

⇔



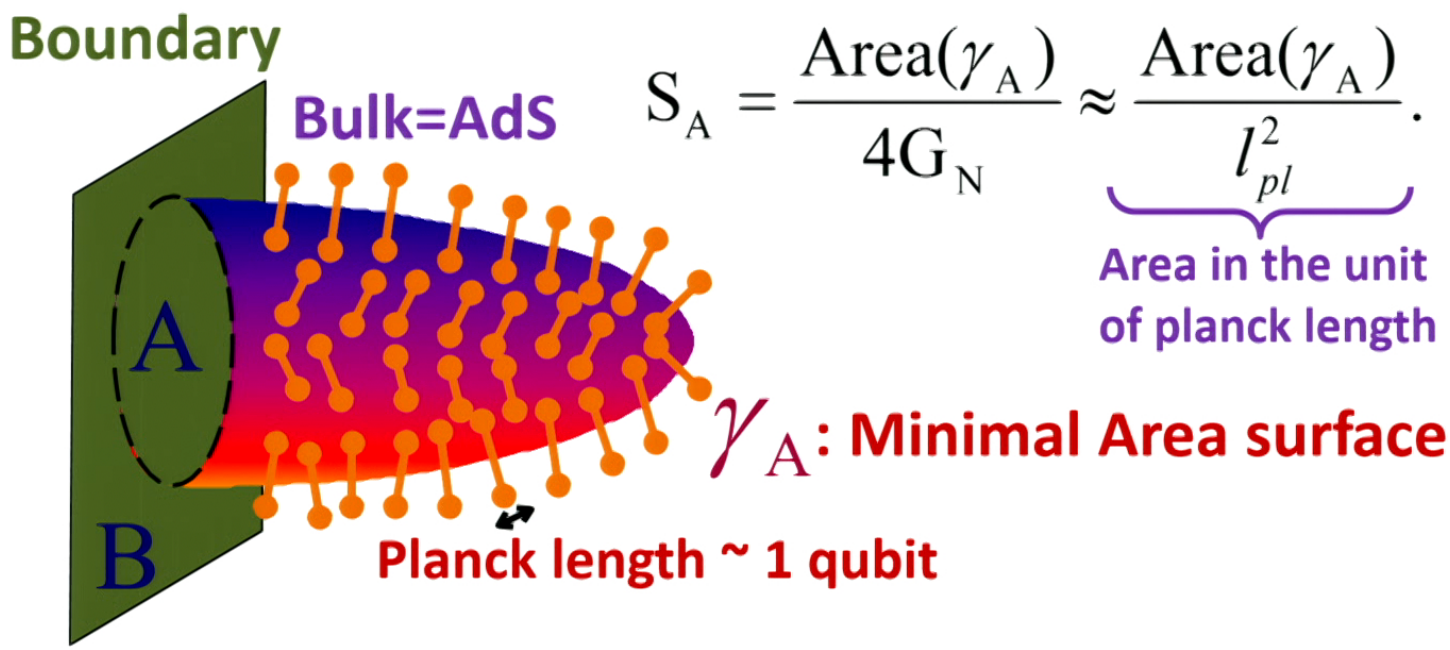
[Swingle,.]

Tensor Network
[Guifre,..]

= ?

AdS

In holography, the entanglement is computed as the area of minimal surface [Ryu-TT 06, Hybeny-Rangamani-TT 07]



$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N} \approx \underbrace{\frac{\text{Area}(\gamma_A)}{l_{pl}^2}}_{\text{Area in the unit of planck length}}.$$

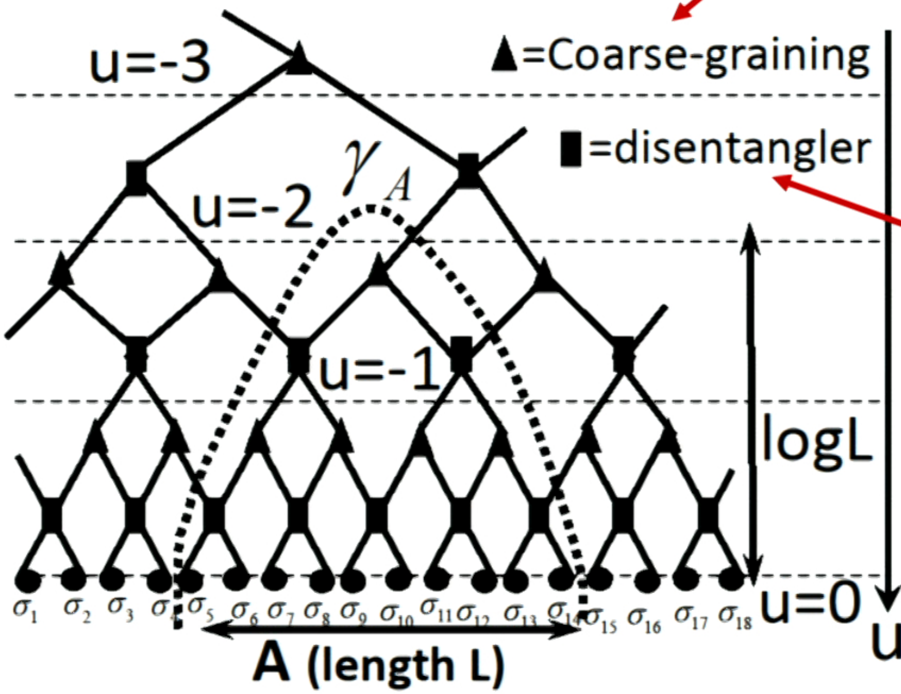
Spacetime in gravity = Collections of bits of entanglement

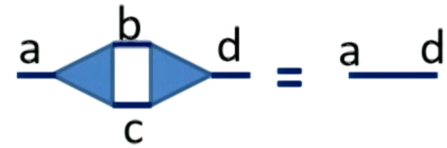
⇒ Emergent space via tensor network ?

MERA [Vidal 05] [Holographic interpretations: Swingle 09, ...]

Coarse-graining = Isometry

$$[T]^{\dagger}_{abc}[T]_{bcd} = \delta_{ad}$$





Disentangler = Unitary trf.

$S_A \le \mathrm{Min}[\#\mathrm{links}]$
$\propto \log L$
$\Rightarrow$ agrees with results in 2d CFT !

The original idea: Tensor Network of MERA (∃ scale inv.)
= a time slice of AdS space

Questions [see e.g. Beny 2011, Bao et.al. 2015, Czech et.al. 2015]

(a) Special Conformal invariance ?

(b) Non-isotropic tensor → ∃ causal structure in MERA

(c) Why the EE bound is saturated ?

(d) Sub AdS scale Locality ?

Recent developments in lattice models [Sully's talk]

・Improved TN models: [Perfect TN: Pastawski-Yoshida-Harlow-Preskill 15]
⇒(a),(b),(c) [Random TN: Hayden-Nezami-Qi-Thomas-Walter-Yang 16]
[Hyper inv. TN: Evenbly 17]

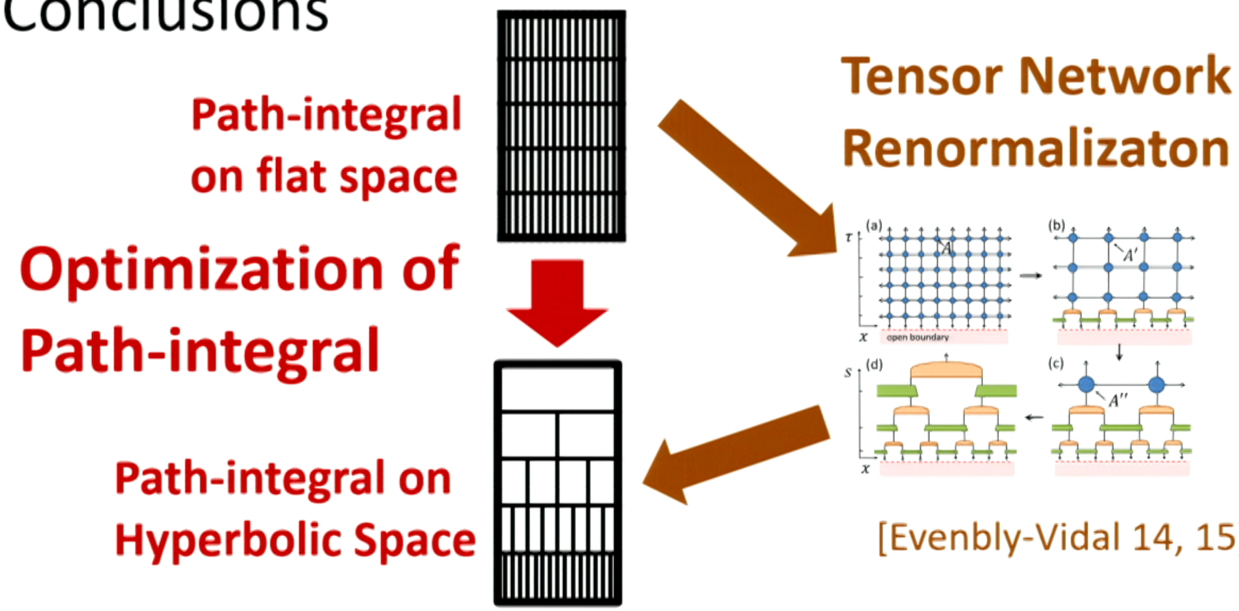
・Another Interpretation :
⇒(a),(b) [MERA as Kinematic Space (dS): Czech, Lamprou, McCandlish, Sully 15]

We expect some of these problems are due to lattice artifacts. Moreover, we would like to eventually understand the genuine AdS/CFT based on QFTs.

Therefore, in this talk we would like to focus on the continuous approaches to TNs.

Below we will discuss two different approaches.

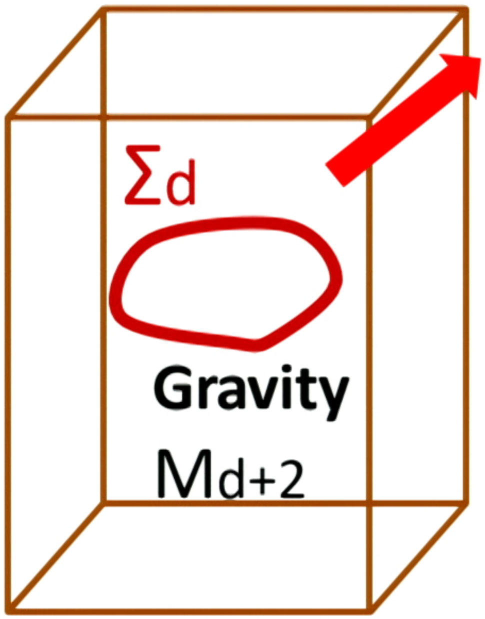
Contents



Path-integral
on flat space
Optimization of
Path-integral
Path-integral on
Hyperbolic Space
Tensor Network
Renormalizaton
[Evenbly-Vidal 14, 15]

Comment: irrespective of details, we can find a basic principle expected to be satisfied in AdS/TN:

Surface/State correspondence [Miyaji-TT 15]



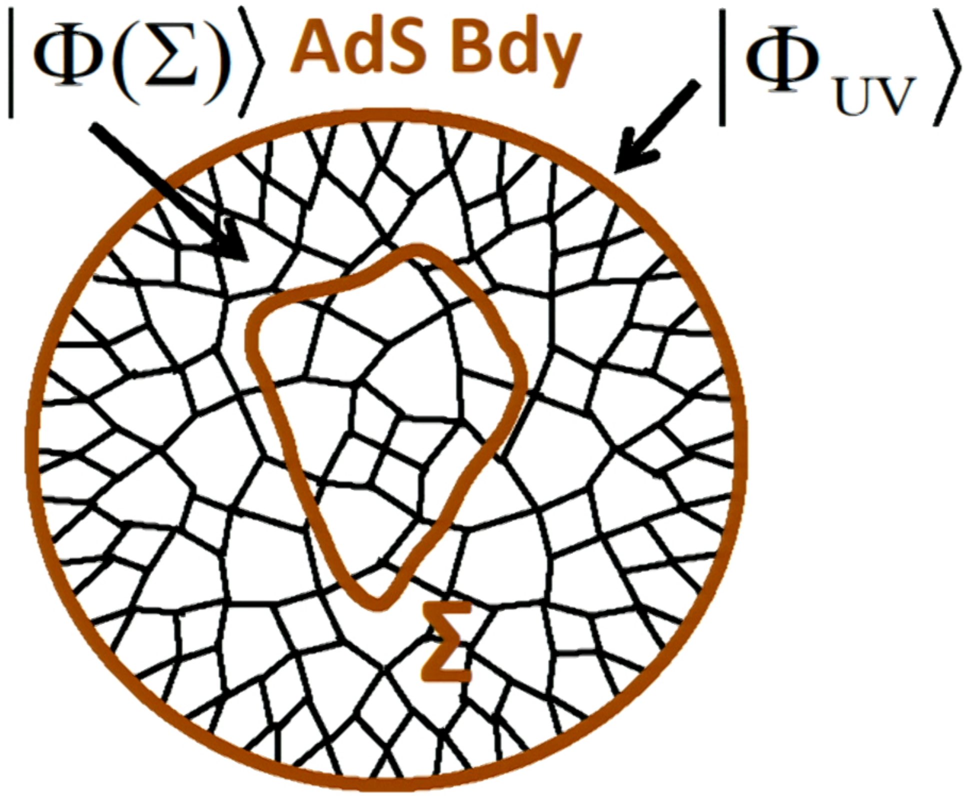
a d dim. convex space-like surface in M (closed and homologically trivial)

↕ Surface/State Correspondence

$$\left|\Phi(\Sigma)\right\rangle \in H_M$$

If Σ is open or topologically non-trivial, it corresponds to a mixed state.

The surface/state correspondence is realized in``nice'' tensor networks description of holography (e.g. perfect TNs).



② cMERA and AdS/CFT

(2-1) cMERA formulation [Haegeman-Osborne-Verschelde-Verstraete 11] [Vidal's talk]

The continuous MERA is defined as follows:

$$\underbrace{|\Psi(u)\rangle}_{\text{State at scale} \approx \varepsilon \cdot e^{-u}} = P \cdot \exp\left(-i\int_{-\infty}^{u} ds\,(K(s)+L)\right) \cdot \underbrace{|\Omega\rangle}_{\text{IR state}}.$$

$K(u)$: (dis)entangler, $UV : u = 0$, $IR : u = -\infty$

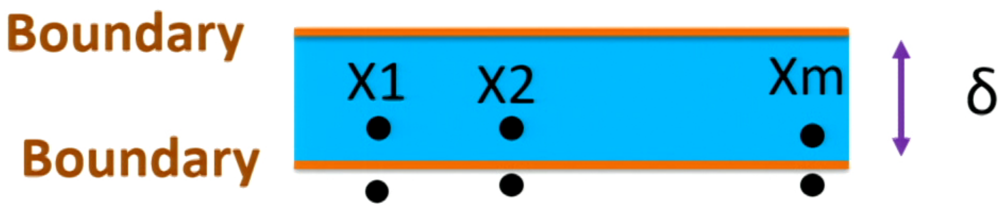
L : coarse - graining (space - like rescaling).

$|\Omega\rangle$: unentangled state in real space

$\rightarrow S_A = 0$ for any A. ➡ This is identified with so called a boundary state.

[Miyaji-Ryu-Wen-TT 14]

IR State as Boundary State



$$\frac{\langle \Omega | O(x_1) O(x_2) \cdots O(x_n) | \Omega \rangle}{\langle \Omega | \Omega \rangle} \approx \prod_{i=1}^{n} \langle O(x_i) \rangle.$$

⇒ When (xi-xj)>>δ, there are no correlations !

⇒ Disentangled !

General Structures of cMERA for 2d CFTs (below UV cutoff)

[Miyaji-Watanabe-TT 16]

Conformal Sym. (Virasoro) : $L_n, \widetilde{L}_n \quad (n \in Z)$

Space-like rescaling : L $\Rightarrow$ Defined as radius quantum quench.

IR state : $|\Omega\rangle \approx |B_0\rangle$ boundary states s.t. $(L_n - \widetilde{L}_{-n})|\Omega\rangle = 0.$

Moreover, we have $[L_n - \widetilde{L}_{-n}, \mathrm{L}] = [L_n - \widetilde{L}_{-n}, K] = 0.$

[below UV cut off]

$\Rightarrow$ Consistent with the condition $\mathrm{L}|\Omega\rangle = 0.$

Tensor Networks have Conformal Symmetry

$\mathrm{K(u)} + \mathrm{L} = \mathrm{D}$ (i.e. dilatation of CFT)

$\Rightarrow$ We can determine $K(u)$.

In this way, we can obtain K(u) and L for general 2d CFTs.

(2-2) AdS/CFT and cMERA

[Miyaji-Numasawa-Shiba-Watanabe-TT 15, Miyaji-Watanabe-TT 16]

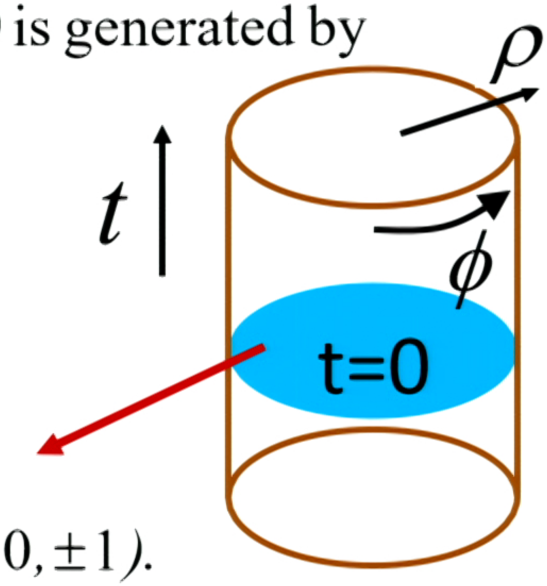
AdS3/CFT2 setup

Consider the global AdS3 space:

$$ds^2 = R^2(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\phi^2),$$

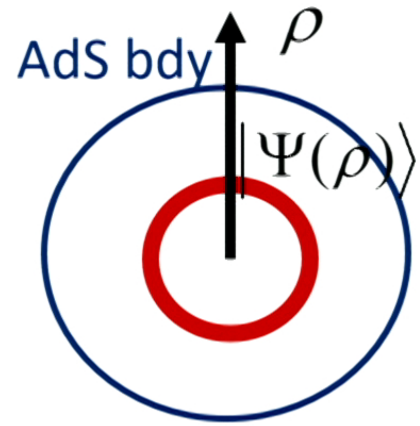
whose isometry $SL(2,R)\times SL(2,R)$ is generated by

$$L_0 = i\partial_+, \quad \tilde{L}_0 = \partial_-,$$
$$L_{\pm1} = ie^{\pm ix^+}\left[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_+ - \frac{1}{\sinh 2\rho}\partial_- \mp \frac{i}{2}\partial_\rho\right],$$
$$\tilde{L}_{\pm1} = ie^{\pm ix^-}\left[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_- - \frac{1}{\sinh 2\rho}\partial_+ \mp \frac{i}{2}\partial_\rho\right].$$



On a time slice, the isometry is generated by $l_n = L_n - \tilde{L}_{-n} \quad (n = 0, \pm1)$.

Derivation of cTN from AdS3/CFT2



$$|\Psi(\rho)\rangle = P\exp\left[-i\int_0^{\rho} M(\widetilde{\rho})d\widetilde{\rho}\right]|\Psi(0)\rangle,$$

$$M(\rho) = \int_0^{2\pi} d\phi M(\rho,\phi).$$

[⇒Surface/state correspondence: Miyaji-TT 15]

Since we have $2\partial_{\rho} = e^{i\phi}l_{-1} - e^{-i\phi}l_1$ in AdS3/CFT2, we can identify M as follows:

$$M(\rho,\phi) \approx \lim_{\delta\to 0}\frac{1}{\delta}\int_{-\delta/2}^{\delta/2} d\widetilde{\phi}\cdot\widetilde{\phi}\cdot T_{t\phi}(\phi+\widetilde{\phi}) = D(\phi).$$

In this way, M is identified with **the dilatation D**.

⇒ This reproduces the cMERA network (below UV cut off).

$$|\Psi(\rho)\rangle = |0\rangle_{R(\rho)}, \qquad R(\rho) = \frac{\sinh\rho}{\sinh\rho_{\infty}} = e^{u}.$$

(2-2) AdS/CFT and cMERA

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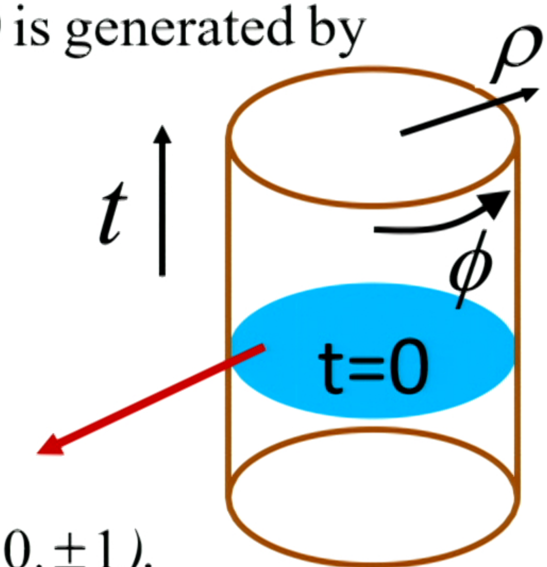

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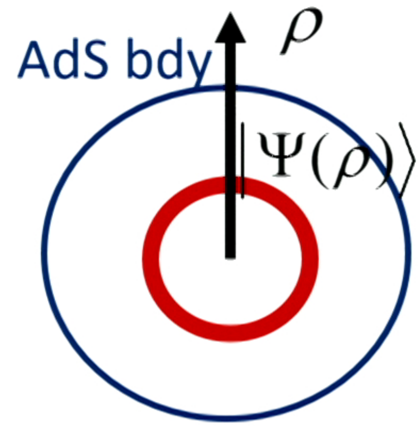

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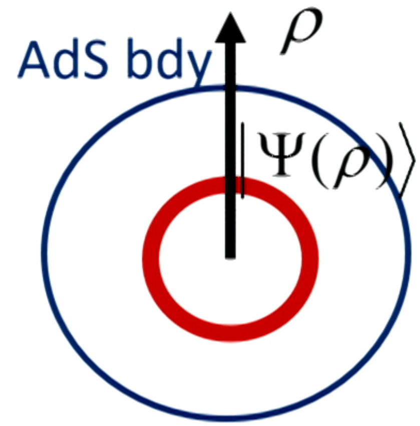

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In this way, M is identified with **the dilatation D**.

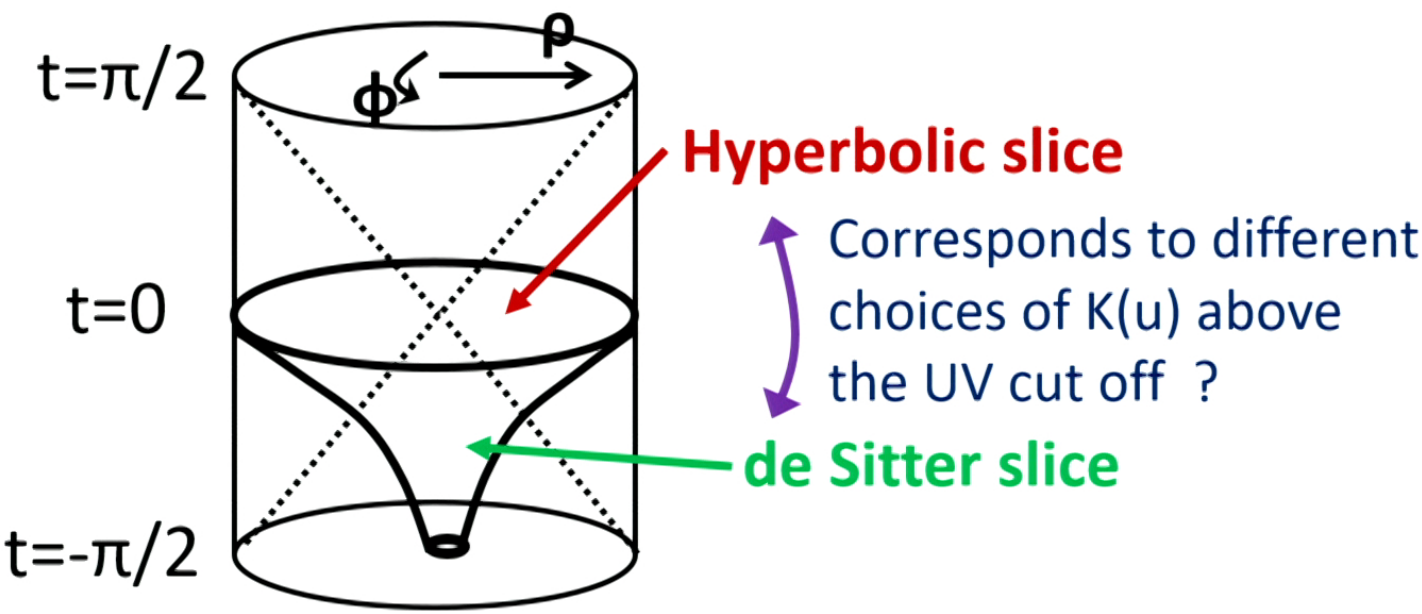
⇒ This reproduces the cMERA network (below UV cut off).

$$|\Psi(\rho)\rangle = |0\rangle_{R(\rho)}, \qquad R(\rho) = \frac{\sinh\rho}{\sinh\rho_\infty} = e^u.$$

Comments

We find that cMERA satisfies the condition for the time slice of AdS3 (=2d hyperbolic space H2) *below UV cut off*.

However, the same symmetry argument can be applied to 2d de-Sitter slices (dS2) in AdS3.



(2-3) cMERA for a Massless Free Scalar Theory

[Haegeman-Osborne-Verschelde-Verstraete 11]

Hamiltonian: $H = \frac{1}{2}\int dk^d [\pi(k)\pi(-k) + k^2\phi(k)\phi(-k)].$

Ground state $|0\rangle$: $a_k|0\rangle = 0.$

IR state: $a_x|\Omega\rangle = 0, \qquad \left(a_x \equiv \sqrt{\Lambda}\phi(x) + \frac{i}{\sqrt{\Lambda}}\pi(x) \right),$

i.e. $|\Omega\rangle = \prod_x |0\rangle_x \qquad \Rightarrow \quad S_A = 0.$

Strength of disentanglers

cMERA: $K(u) = \frac{i}{2}\int dk^d \left[\chi(u)\cdot\Gamma(k/\Lambda)a_k^+ a_{-k}^+ + (h.c.)\right],$

where $\Gamma(x)$ is a cut off function: $\Gamma(x) = \theta(1 - |x|).$

For massless scalar, $\chi(u) = \frac{1}{2}$.

(2-4) Information Metric from cMERA [Nozaki-Ryu-TT 12]

In cMERA, it is not straightforward to calculate EE analytically. Instead, there is another quantity which is more tractable: information metric.

$$1-\left|\left\langle \Psi(u)\,|\,e^{iLdu}\,|\,\Psi(u+du)\right\rangle\right|^2$$
$$=(du)^2\cdot G_{uu}\cdot\left[\int dx^d\cdot\int_0^{\Lambda e^u} dk^d\right].$$

The total volume of phase space at scale u $=V_d\cdot\Lambda^d e^{du}.$

Note: The operation e^{iLdu} removes the coarse-graining procedure to extract the strength of disentanglers.

(2-3) cMERA for a Massless Free Scalar Theory

[Haegeman-Osborne-Verschelde-Verstraete 11]

Hamiltonian: $H = \frac{1}{2}\int dk^d [\pi(k)\pi(-k) + k^2\phi(k)\phi(-k)].$

Ground state $|0\rangle$: $a_k|0\rangle = 0.$

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Heuristic and Phenomenological Interpretation of Guu

Since Guu ∝ density^2 of disentanglers , we expect

$$S_A \sim \int_{u_{IR}}^{0} du \sqrt{G_{uu}} \cdot e^{(d-1)u} \sim \text{Hol.EE}$$

⇒ **Guu ~ guu** in $ds^2_{Gravity} = g_{uu} du^2 + \frac{e^{2u}}{\varepsilon^2} \cdot d\vec{x}^2 - g_{tt} dt^2.$

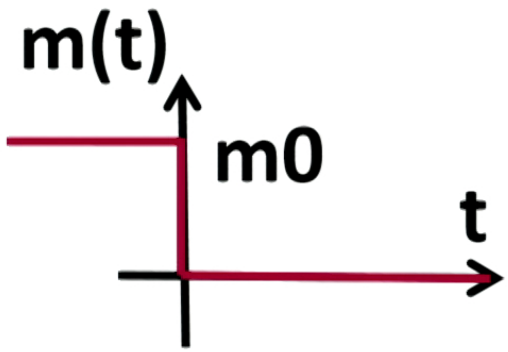
Example: Free Scalar

$$K(u) = \frac{i}{2} \int dk^d \left[\chi(u) \Gamma(k/\Lambda) a_k^{+} a_{-k}^{+} + (h.c.) \right], \quad \Rightarrow \ g_{uu} = \chi(u)^2.$$

$$\chi(u) = \frac{1}{2} \cdot \frac{e^{2u}}{e^{2u} + m^2/M^2}, \quad (\text{for } m = 0, \ \chi(\text{u}) = 1/2.)$$

(2-5) Excited States after Quantum Quenches

$$(A_k a_k + B_k a^+_{-k})|\Psi\rangle = 0, \qquad (|A_k|^2 - |B_k|^2 = 1).$$



$$A_k = \frac{1}{2}\left(\left(\frac{k^2 + m0^2}{k^2}\right)^{1/4} + \left(\frac{k^2}{k^2 + m0^2}\right)^{1/4}\right)\cdot e^{ikt},$$

$$B_k = \frac{1}{2}\left(\left(\frac{k^2 + m0^2}{k^2}\right)^{1/4} - \left(\frac{k^2}{k^2 + m0^2}\right)^{1/4}\right)\cdot e^{-ikt}.$$

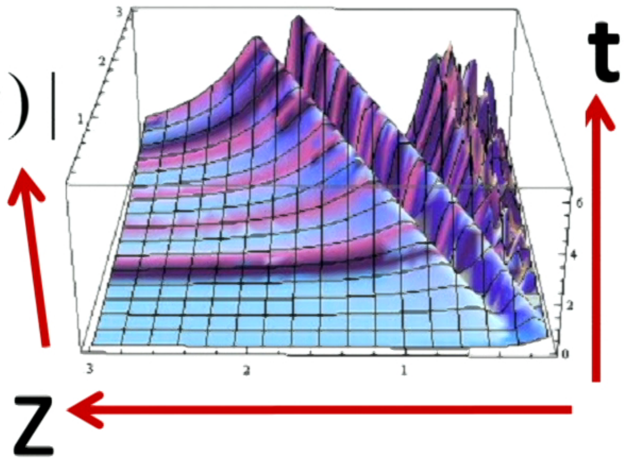
To realized these states, we extend the ansatz as

$$K(u) = \frac{i}{2}\int dk^d \Gamma(k/\Lambda)\left[\chi(u) a^+_k a^+_{-k} + \chi^*(u) a_k a_{-k}\right],$$

Time evolution (2d Scalar) under Quantum Quenches

[Mollabashi-Nozaki-Ryu-TT 13]

$$\sqrt{G_{uu}} = |\chi(u)|$$



Looks like a propagation of gravitational waves.

We can also (analytically) confirm **$\chi(u) \propto t$ and $S_A \propto t$** at late time. The same is true in higher dim.

This is consistent with the known CFT (2d) [Calabrese-Cardy 05]. and with the holographic result (any d).

[Arrastia-Aparicio-Lopez 10 , Albash-Johnson 10, Balasubramanian et.al. 10, 11, Hartman-Maldacena 13, Liu-Suh 13]

③ Optimization of Path-Integral and AdS/CFT

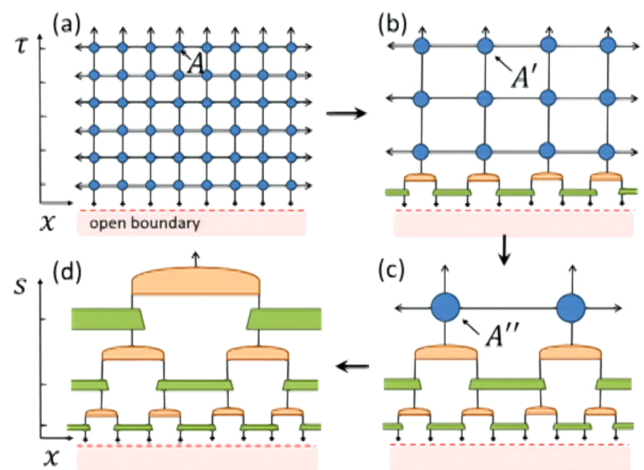
(3-1) Motivation

Remember that the MERA can be obtained from the `optimization' of tensor networks

⇒ **Tensor network renormalization** [Evenbly-Vidal 14, 15]

Euclidean Path-Integral ⇒

MERA ⇐



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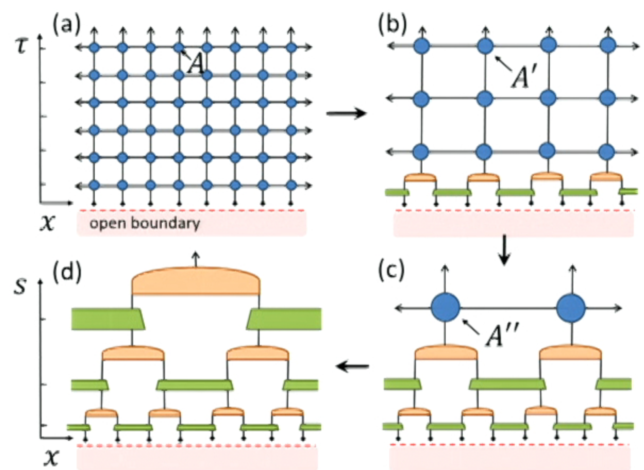
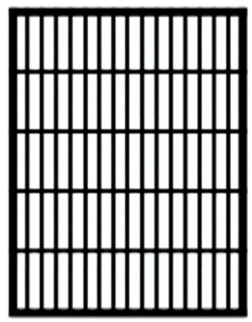


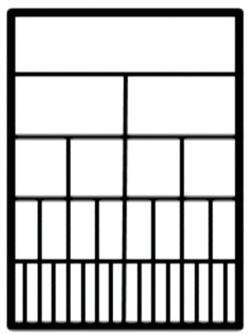
Optimization of Path-Integral

Space (x)

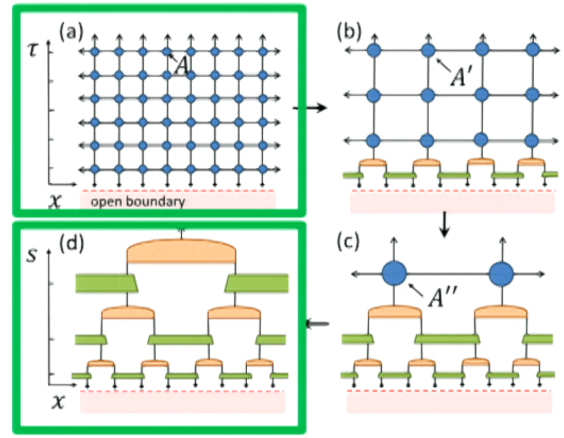
Euclidean Time (-z)



Optimization of Path-integral

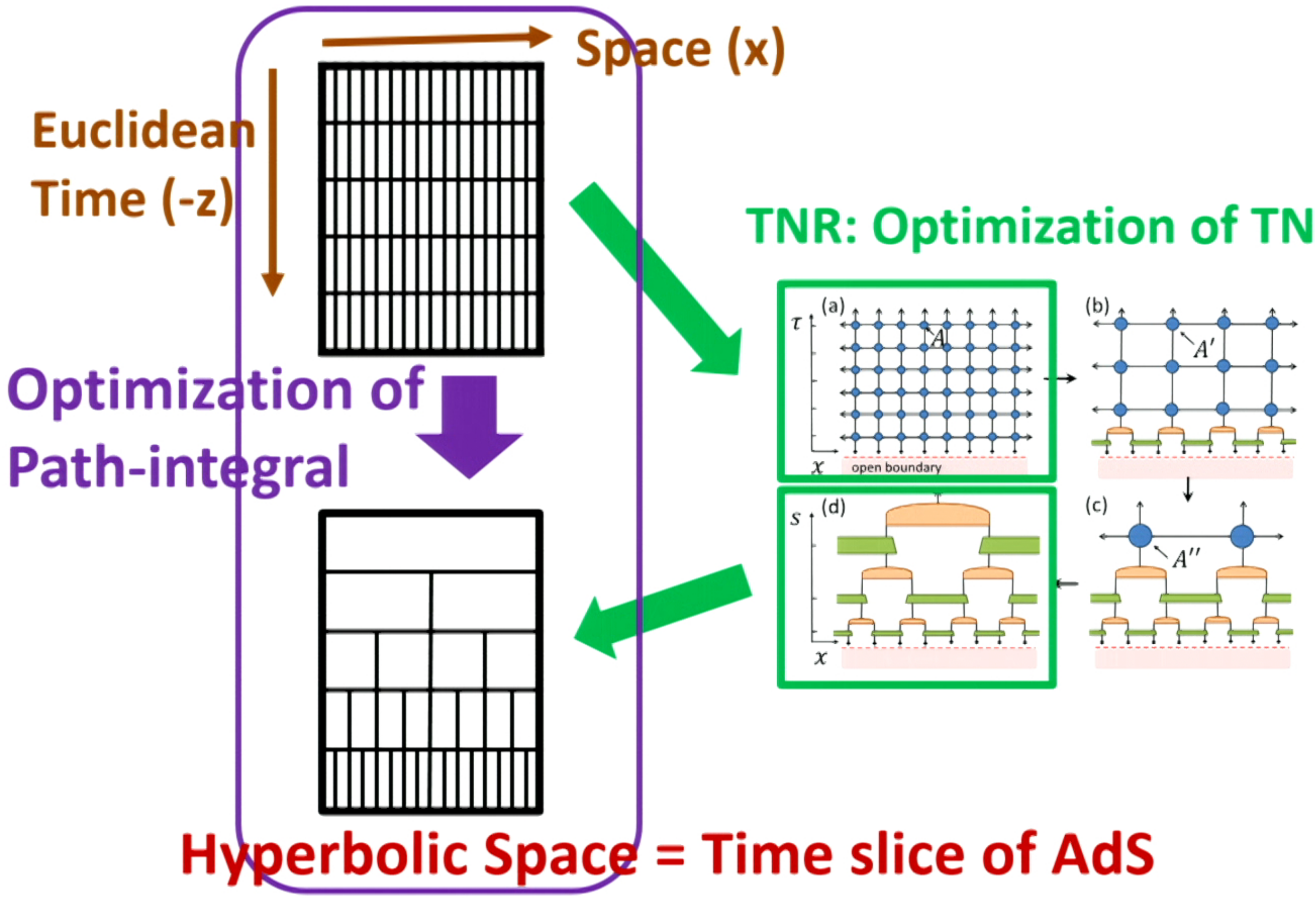


TNR: Optimization of TN



Hyperbolic Space = Time slice of AdS

Optimization of Path-Integral



(3-2) Formulating Optimization of Path-integral

A Basic Rule: Simplify the path-integral such that it produces the correct UV wave functional $\Psi_{UV}^{\mathrm{Flat}}[\Phi(x)]$.

Below we focus on 2d CFTs for simplicity.

Modification of discretizations in path-integral
= Curved space metric s.t. one cell (bit) = unit length:

$$ds^2 = e^{2\varphi(x,z)}(dx^2 + dz^2).$$

[cf. Original flat metric:

$$ds^2 = \varepsilon^{-2} \cdot (dx^2 + dz^2),$$

where ε is the UV cutoff.]

Ground state UV wave function in curved space

$$\Psi_{UV}^{g}[\Phi(x)] = \int \prod_{\substack{0<z<\infty \\ -\infty<x<\infty}} D\Phi(x,z)\, e^{-S_{CFT}(\Phi)} \cdot \delta\big(\Phi(x) - \Phi(x, z=0)\big)$$

In CFTs, owing to conformal sym., we have

$$\Psi_{UV}^{g_{ab}=e^{2\varphi}\delta_{ab}}\left[\Phi(x)\right]=N[\varphi(x,z)]\cdot\Psi_{UV}^{\mathrm{Flat}}\left[\Phi(x)\right].$$

Our Proposal (Optimization of Path-integral for CFTs):

Minimize $N[\varphi(x,z)]$ w.r.t $\varphi(x,z)$
with the boundary condition $e^{2\varphi}\,|_{z=\varepsilon}=\varepsilon^{-2}$.

Motivation

The normalization N estimates repetitions of same operations of path-integration. → Minimize this !

⇒ Our conjecture: $N[\varphi(x,z)]\approx\exp[C[\varphi]]$

$C[\varphi]\equiv$ complexity of $\mathrm{TN}[\varphi]$

[Refer to a nice explanation based on TN : Czech's talk afternoon]

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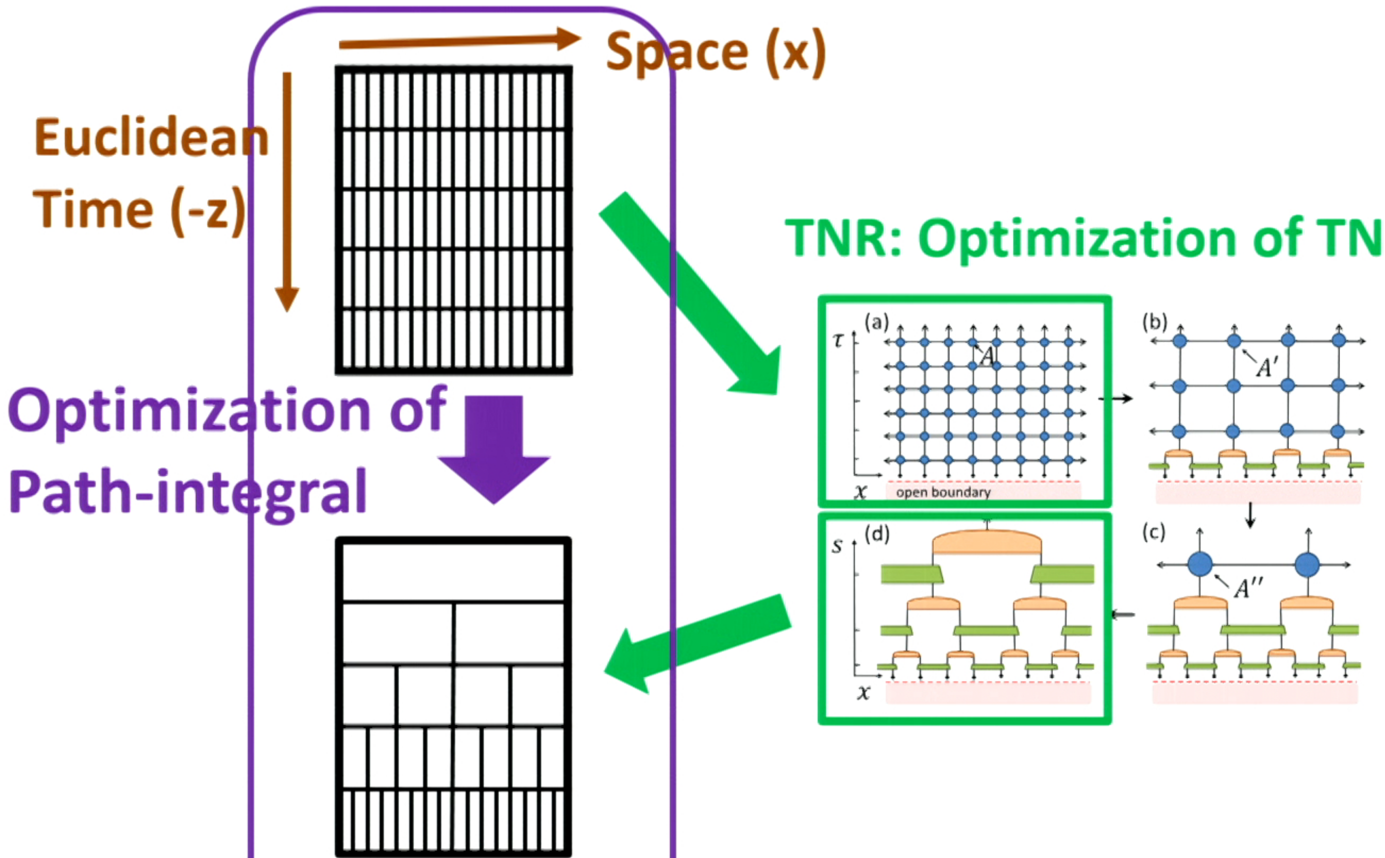
Optimization of Path-Integral



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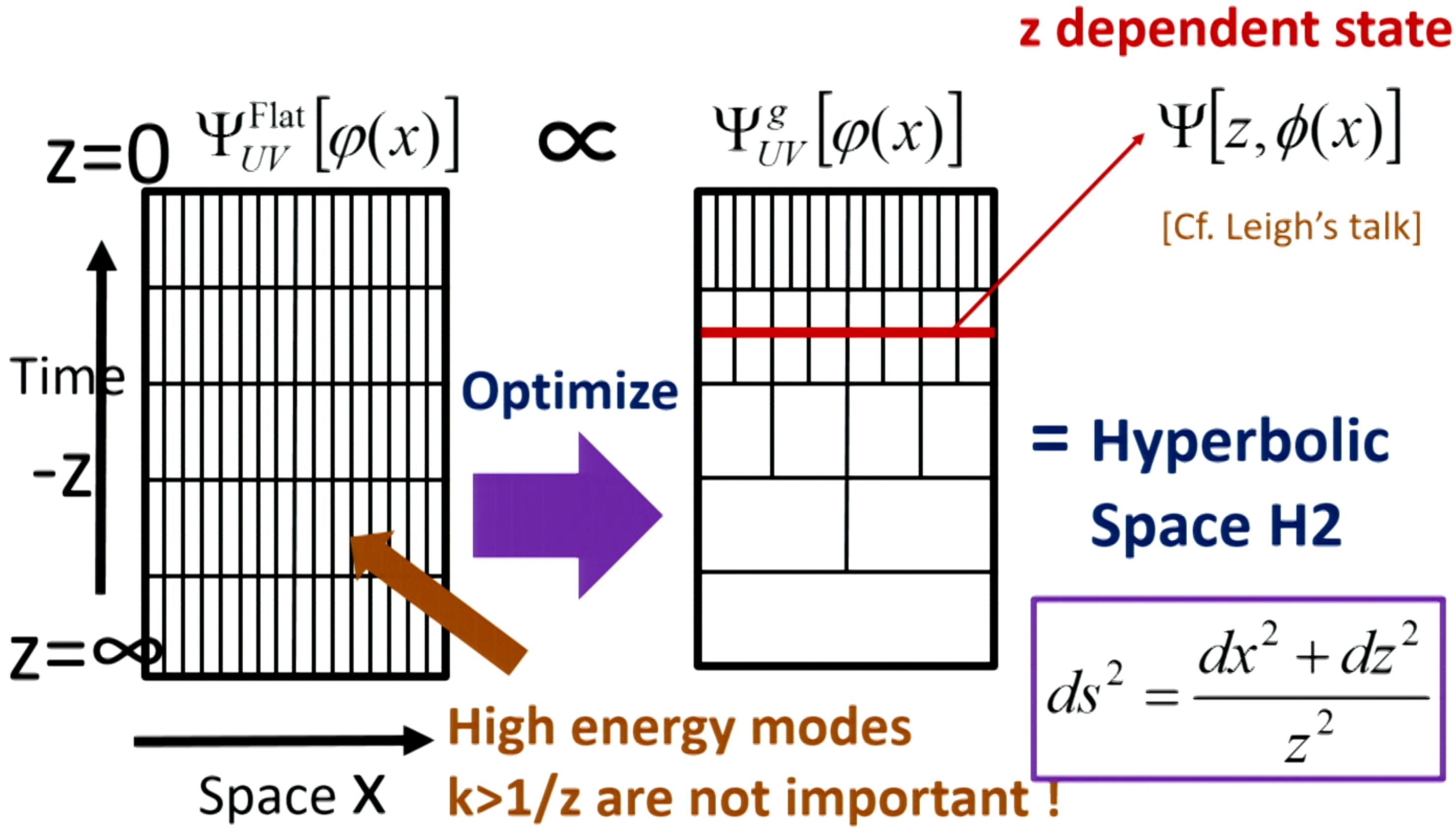
A Sketch: Optimization of Path-Integral



(3-3) 2d CFT Vacuum

$\Psi_g[\Phi(x)]$ **depends on metric due to Weyl anomaly and UV regularization.**

$$\frac{\Psi_\varphi}{\Psi_0} \propto e^{\frac{c}{24\pi}S_L},$$

Conformal Anomaly

UV div. $\sim\varepsilon^{-2}$

$$S_L = \int dxdz\left[(\partial_x\varphi)^2 + (\partial_z\varphi)^2 + \frac{\mu}{4}e^{2\varphi}\right]$$

Liouville Theory

[Polyakov 1981,...]

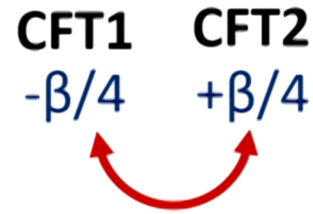
$$= \int dxdz\left[(\partial_x\phi)^2 + \left(\partial_z\varphi + \sqrt{\mu}\cdot e^{\varphi}/2\right)^2\right]$$

$$\Rightarrow \text{Minimum: } e^{2\varphi} = \frac{4}{\mu}\cdot z^{-2}.$$

Reproduces H_2 !

(we set $\mu = 4$)

(3-4) BTZ and Thermo Field Double

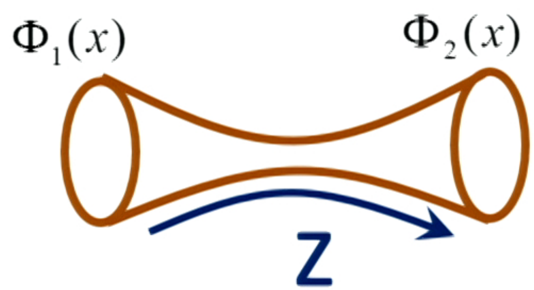


TFD state is described as

$$\Psi_g\left[\Phi_1(x),\Phi_2(x)\right]$$
$$=\int \prod_{\substack{-\beta/4<z<\beta/4 \\ -\infty<x<\infty}} D\Phi(x,z)\, e^{-S_{CFT}(\Phi)} \cdot \delta\left(\Phi_1(x)-\Phi(x,z=\beta/4)\right)\delta\left(\Phi_2(x)-\Phi(x,z=-\beta/4)\right)$$

$$\underset{Opt}{\Longrightarrow} \quad e^{2\varphi} = \frac{4\pi^2}{\beta^2}\cdot\frac{1}{\cos^2\left(2\pi z/\beta\right)}.$$

This agrees with the time slice of BTZ black hole !
(i.e. Einstein-Rosen Bridge).



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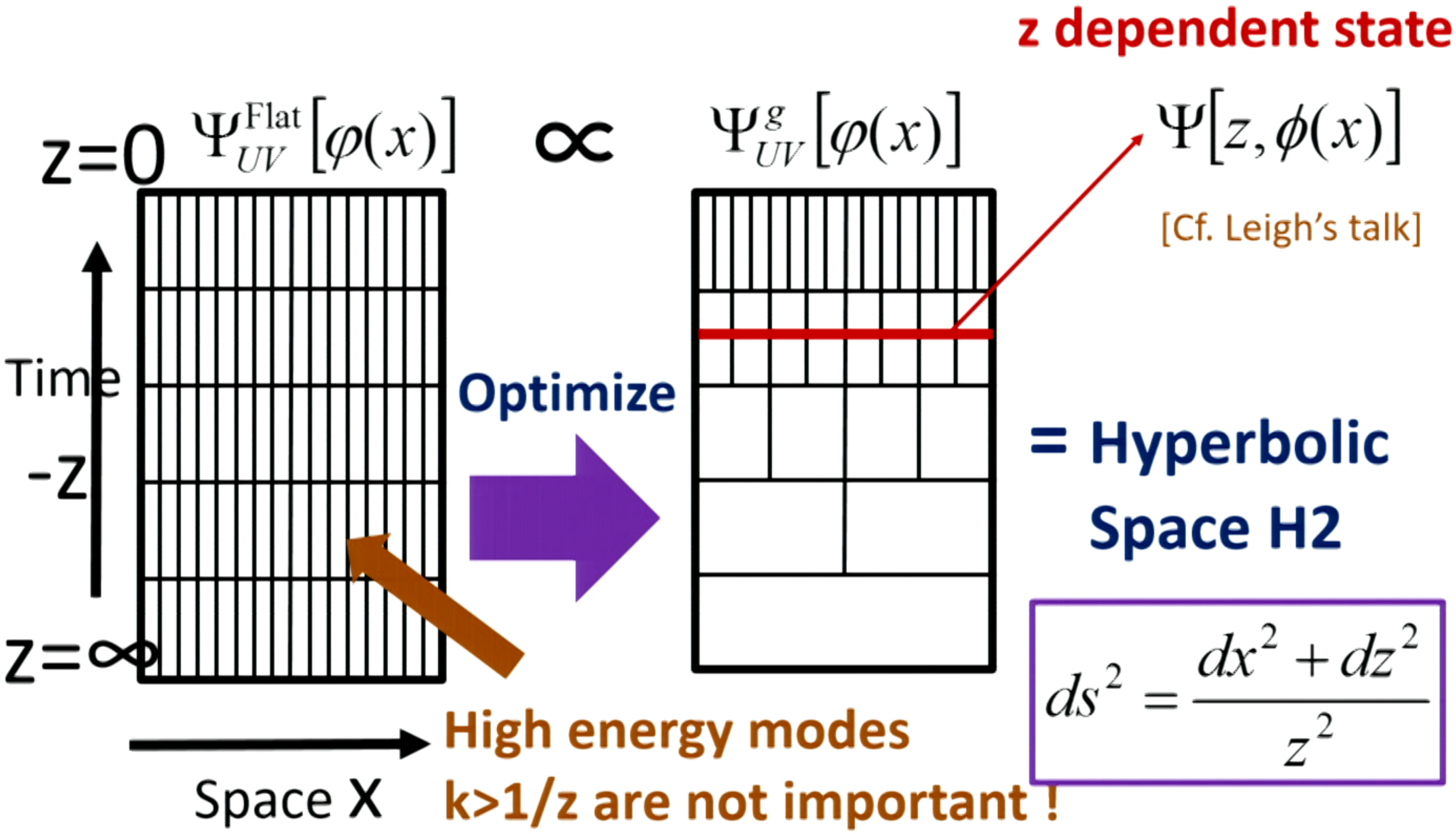
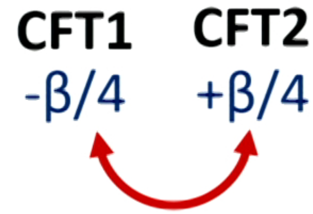


= Hyperbolic Space H2

$$ds^2 = \frac{dx^2 + dz^2}{z^2}$$

(3-4) BTZ and Thermo Field Double



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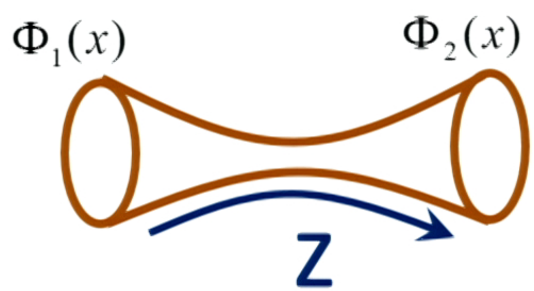
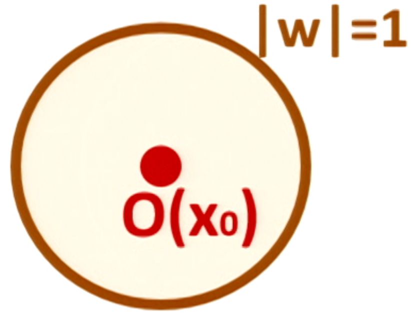


(3-5) Global AdS3 and Excitation



We insert an operator O(x) in the center of disk.

O(x): conformal dim. = (h,h) $\Rightarrow O(x) \sim e^{-2h\cdot\varphi}$.

Thus we minimize $\frac{\Psi_\varphi}{\Psi_0} \propto e^{\frac{c}{24\pi}S_L} \cdot e^{-2h\varphi(x_0)}$.

$$\partial_w \partial_{\bar{w}} \varphi - \frac{\mu}{16} e^{2\varphi} + \frac{6\pi h}{c} \delta^2(w) = 0.$$

Solution: $A(w) = w^a, \quad B(\overline{w}) = \overline{w}^a, \qquad (a \equiv 1 - 12h/c).$

Metric: $$ds^2 = \frac{4 d\zeta d\overline{\zeta}}{(1 - |\zeta|^2)^2}, \qquad \zeta \equiv w^a = r e^{i\theta}$$

⇒ Deficit angle geometry $\theta \sim \theta + 2\pi a.$

This agrees with the gravity dual if h/c<<1.

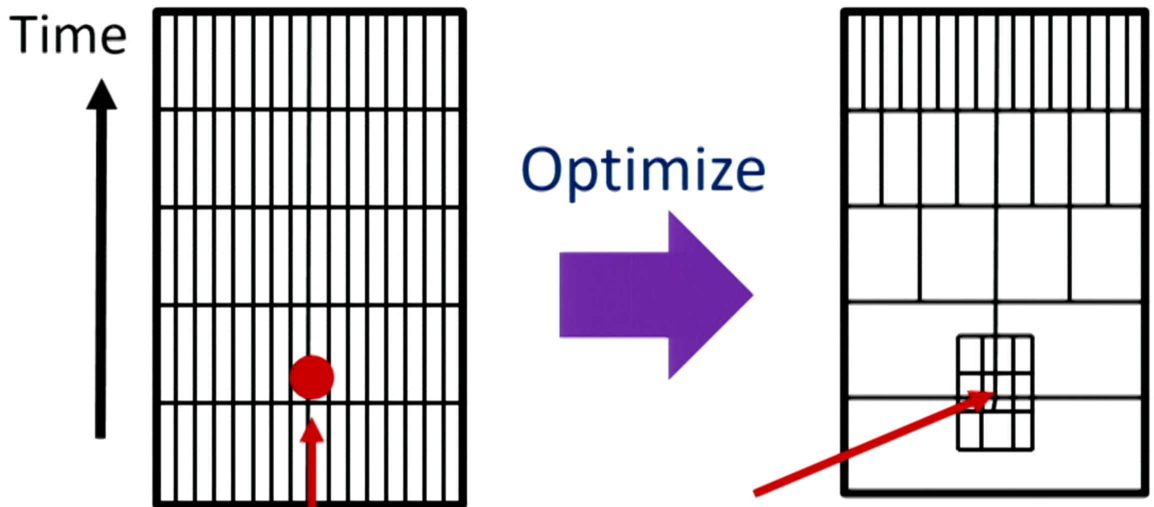
Note: the AdS/CFT predicts $a = \sqrt{1 - 24h/c}.$

Interestingly, if we consider the quantum Liouville CFT,

then $h = \frac{\gamma\alpha}{4}(Q - \alpha\gamma/2.), \quad c = 1 + 3Q^2, \quad (Q \equiv 2/\gamma + \gamma).$

⇒ We get $a = \sqrt{1 - 24h/c}.$

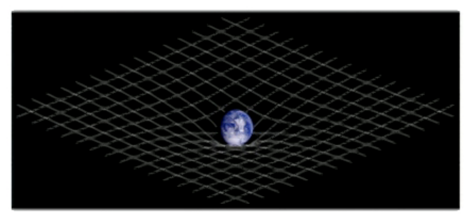
Heuristic Summary



Local excitation
(energy source)

We locally need a fine graining
⇒ The metric gets larger !

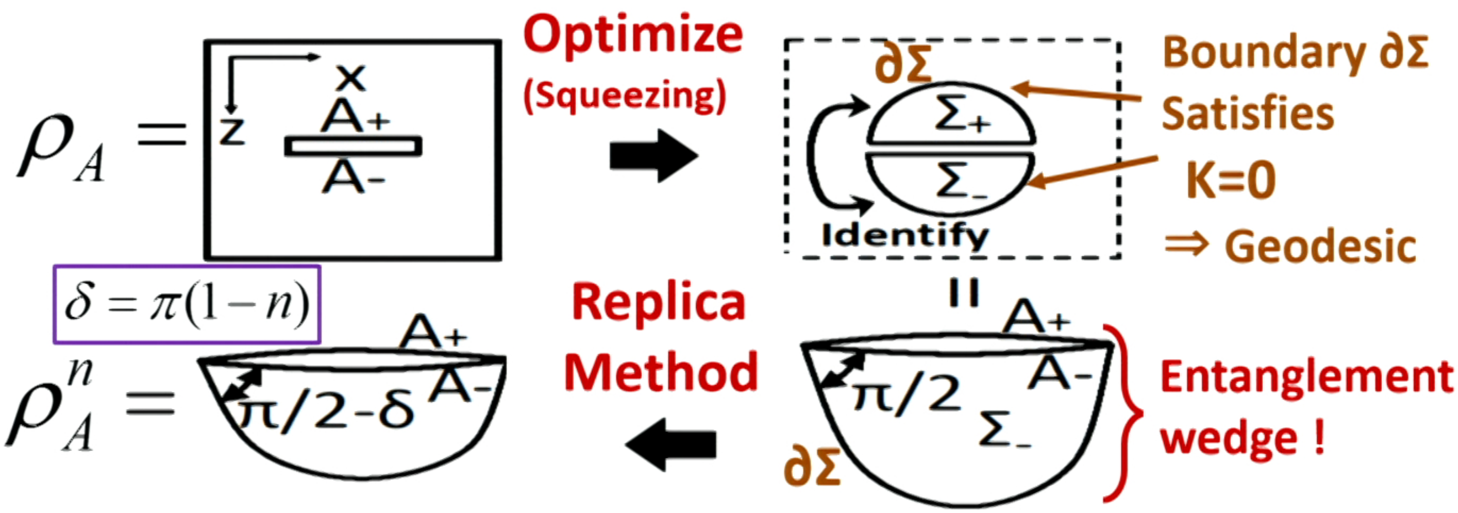
This agrees with
general relativity !



(3-6) Entanglement Wedge and Entropy

We optimize the reduced density matrix ρ_A

⇒Geometry obtained by pasting two half disks :



$$S_L^{(n)}[\varphi] = 2\int_\Sigma dxdz\left[(\partial_x\varphi)^2 + (\partial_z\varphi)^2 + e^{2\varphi}\right] + 2\int_{\partial\Sigma} ds\left[K\varphi + \mu_B e^{\varphi}\right]$$

➡ **Reproduce the correct HEE !**

$$S_A = \frac{c}{6}\int_{\partial\Sigma} ds\, e^{\varphi}$$

$$\mu_B = \pi(1-n)$$

(3-7) Higher dimensional Generalization

For simplicity, we focus on the optimization of the Weyl rescaling mode:

$$ds^2 = e^{2\varphi(x,z)}\left(dz^2 + d\vec{x}^2\right)$$

We argue that the function which we need to minimize for our optimization is given by

$$S_d = N\int dz dx^d \left[e^{(d+1)\varphi} + e^{(d-1)\varphi}\left(\partial_z\varphi\right)^2 + e^{(d-1)\varphi}\left(\partial_x\varphi\right)^2\right]$$

$$+2N\int_{bdy} dx^{d-1}\left[\frac{e^{(d-1)\varphi}\cdot K_0}{d(d-1)} + \frac{\mu_B}{d}e^{(d-1)\varphi}\right].$$

This reproduces correct metrics, ent. wedge/entropy etc.

(3-8) Comments on Complexity

Recent proposals of holographic complexity:

(1) C=Vol [Susskind 2014,..]

(2) C=Action [Brown-Roberts-Susskind-Swingle-Zhao 2015, Lehner-Myers-Poisson-Sorkin 2016, Chapman-Marrochio-Myers 2016,...]

A Field theory definition of complexity is still missing.

[other approach: Myers's talk]

One natural proposal based on our argument:

Complexity of 2d CFT = $\mathrm{Min}\left[S_L[\varphi]\right]$

[Refer to a nice explanation based on TN : Czech's talk afternoon]

Evaluation of complexity (tentative results)

2d CFT (1) Poincare AdS3: $$S_L = \frac{c}{12\pi} \cdot \frac{L}{\varepsilon}.$$

(2) global AdS3: $$S_L = \frac{c}{6} \cdot \left[\frac{1}{\varepsilon} - 1 \right].$$

(3) BTZ(TFD): $$S_L = \frac{c}{3\varepsilon}.$$

3d CFT global AdS4: $$S_L = 4\pi N \left[\frac{1}{\varepsilon^2} + \frac{1}{2} + \log\left(\frac{2}{\varepsilon} \right) \right].$$

4d CFT global AdS5: $$S_L = 2\pi^2 N \left[\frac{2}{3\varepsilon^3} + \frac{1}{\varepsilon} - \frac{5}{12} \right].$$

④ Conclusions

In this talk we discussed two different approaches to AdS/TN duality.

cMERA

Our general symmetry argument supports the interpretation of a cMERA network as a slice of AdS.

Problems: (1) Explicit analysis can only be done for free CFTs.
How about the holographic CFTs ?
(2) cMERA = H2 slice or dS2 slice of AdS ?
(3) AdS/cMERA above the UV cut off scale ?

Optimization of Path-integral

We proposed :

Optimization of Euclidean path-integral of a CFT state
⇔ Time slice of its gravity dual in AdS/CFT

We explicitly studies this for 2d CFTs where the optimization can be done by minimizing the Liouville action.
⇒ Correct gravity dual metrics and entanglement wedge/entropy.

The Liouville action ~ a field theoretic counterpart of complexity.
The minimizing complexity of TNs ⇔ TN renormalization

Problems: Time-dependent dynamics ?
Correlation functions ?
Sub AdS locality ?