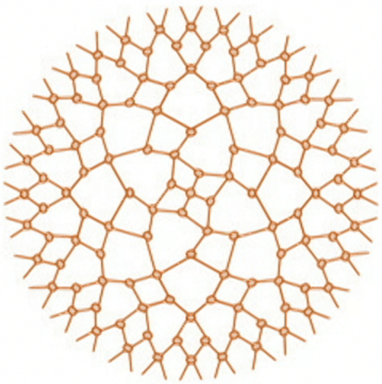
Title: Tensor Networks and Holography

Date: Apr 20, 2017 09:30 AM

URL: http://pirsa.org/17040041

Abstract:



Tensor Networks and Holography

James Sully

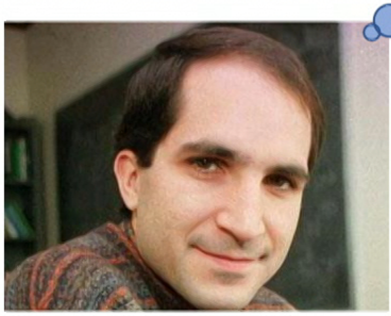
TENSOR NETWORKS FOR QUANTUM FIELD THEORIES II


WELCOME TO
DISNEYLAND!

Introduction

Like many Disney© tales, we begin with young hero, and a dream…

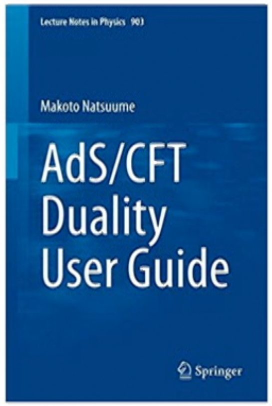
If only we understood quantum gravity…

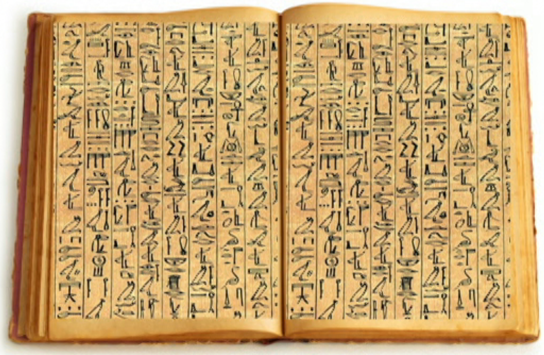


Thankfully his wish was granted:

AdS/CFT Duality

We have a **UV complete theory of quantum gravity** in terms of a **dual CFT**…





but we haven't been given the tools to completely translate between them.

How do we see classical geometry and gravitational physics emerge from CFT description?

Perhaps we need simpler toy models of holography to make progress…

Outline

1. Review of Gauge/Gravity Duality
2. MERA as Emergent Geometry
3. Gravitational Physics from Toy Tensor Networks

This Talk: pedagogical review

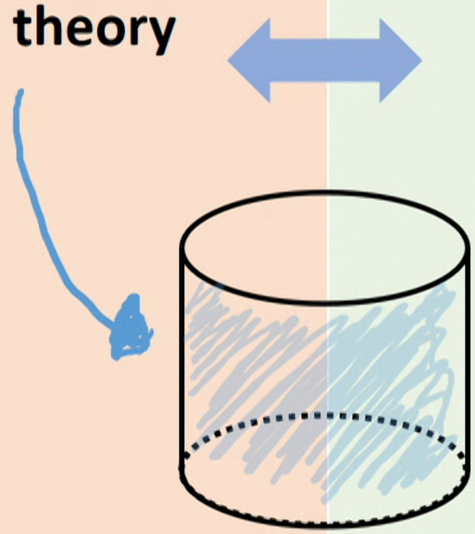
Based on: [Swingle]
[Pastawski, Yoshida, Harlow, Preskill]
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

Gauge/Gravity Duality

AdS/CFT Duality

Conformal Field theory
d-dimensions

Theory of Gravity
(d+1)-dimensions



AdS/CFT Duality

Conformal Field theory
d-dimensions

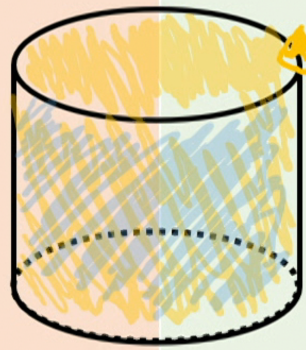
$\mathcal{O}_\Delta$

Z_{CFT}

Theory of Gravity
(d+1)-dimensions

ϕ_Δ

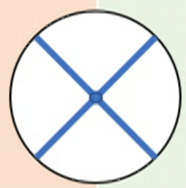
Z_{Grav}



Semiclassical Limit

Generalized Free CFT

Classical geometry, Perturbative gravity

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle = $$  $$= \lim \langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle$$

AdS/CFT Duality

[cf. Heemskerk, Penedones, Polchinski, JS]

Only particular class of CFTs expected to have a 'good' gravitational dual:

What are these CFTs?

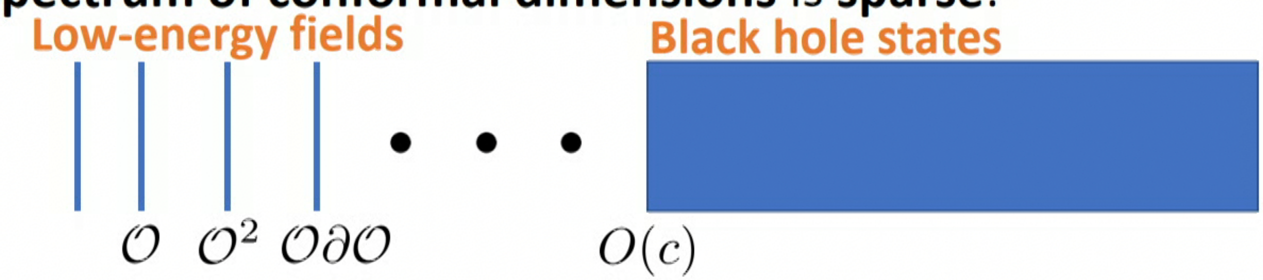
Consider a CFT that has:

1. **Large central charge**: $c >> 1$ $\left(\frac{L}{l_p}\right) \gg 1$
2. Whose **correlators factorize**:

Perturbative effective fields in bulk

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_1(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle = \langle \mathcal{O}_1(x_1)\mathcal{O}_1(x_2)\rangle \langle \mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle + O(1/c)$$

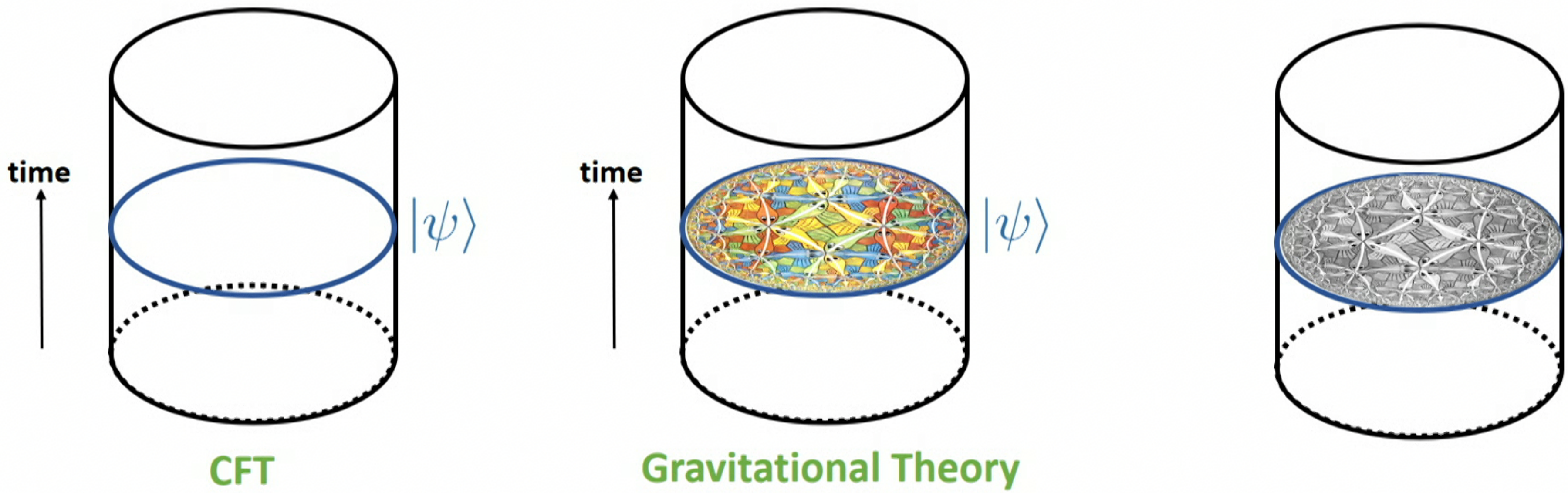
3. Whose **spectrum of conformal dimensions** is **sparse**:



Then this CFT is dual to a theory of gravity whose low-energy energy description is gravity plus effective field theory of field ϕ dual to $\mathcal{O}$.

AdS/CFT Duality

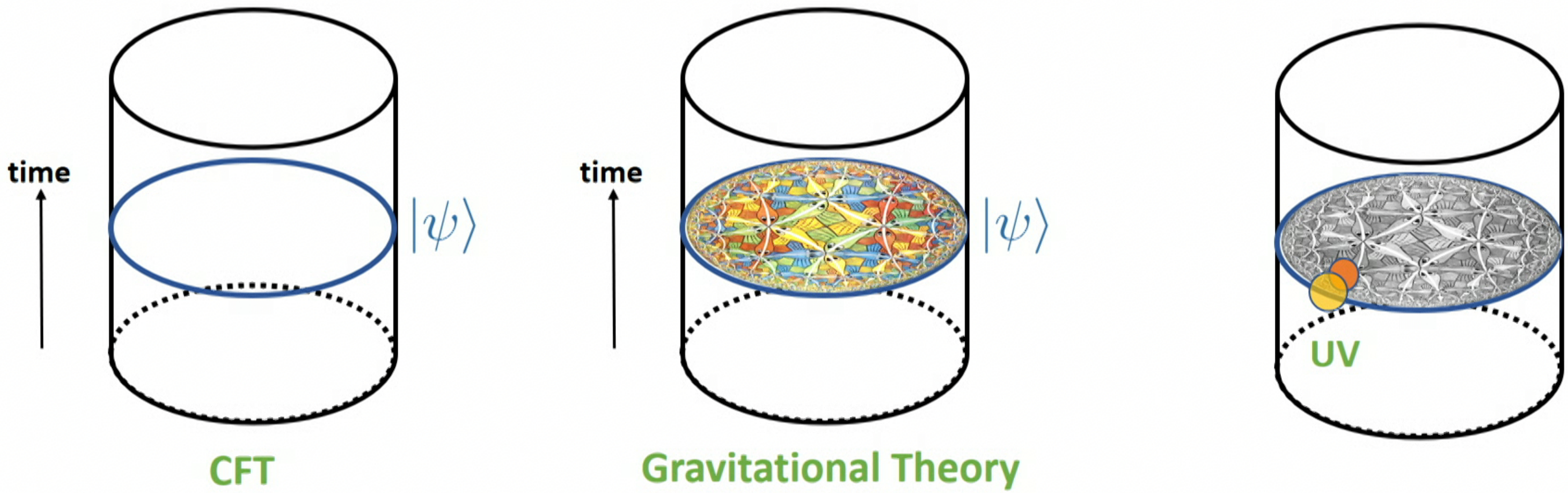
If we consider some **state, $|\psi\rangle$, in the CFT**



it also describes **some geometry in the gravitational theory**.

AdS/CFT Duality

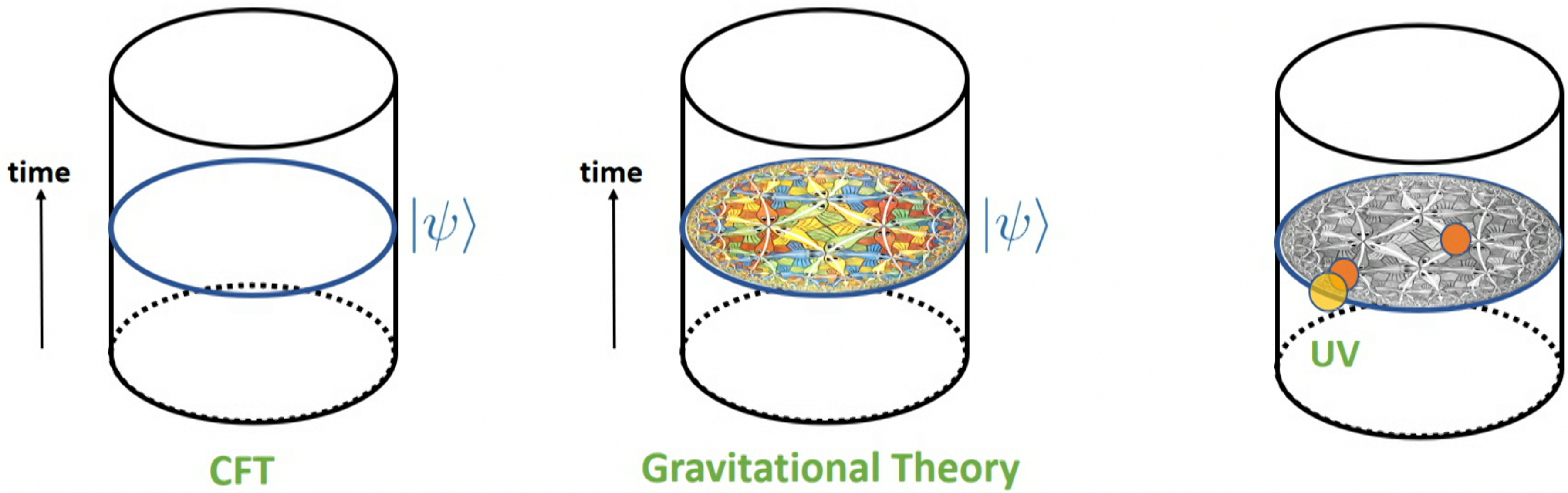
If we consider some **state, $|\psi\rangle$, in the CFT**



it also describes **some geometry in the gravitational theory**.

AdS/CFT Duality

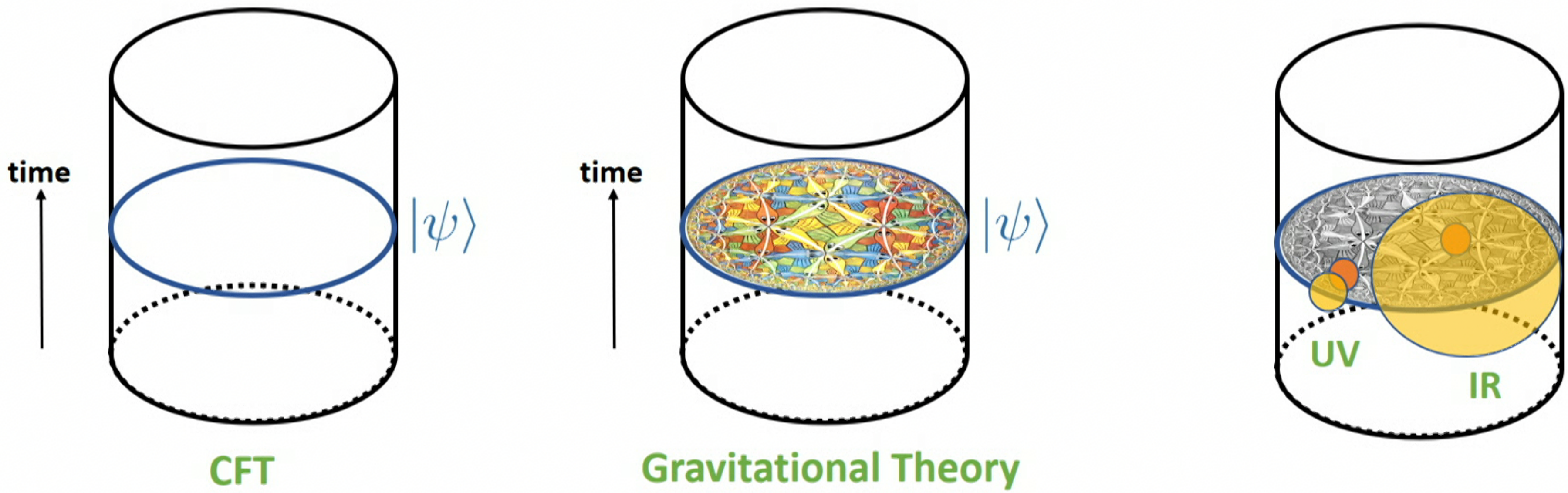
If we consider some **state, $|\psi\rangle$, in the CFT**



it also describes **some geometry in the gravitational theory**.

AdS/CFT Duality

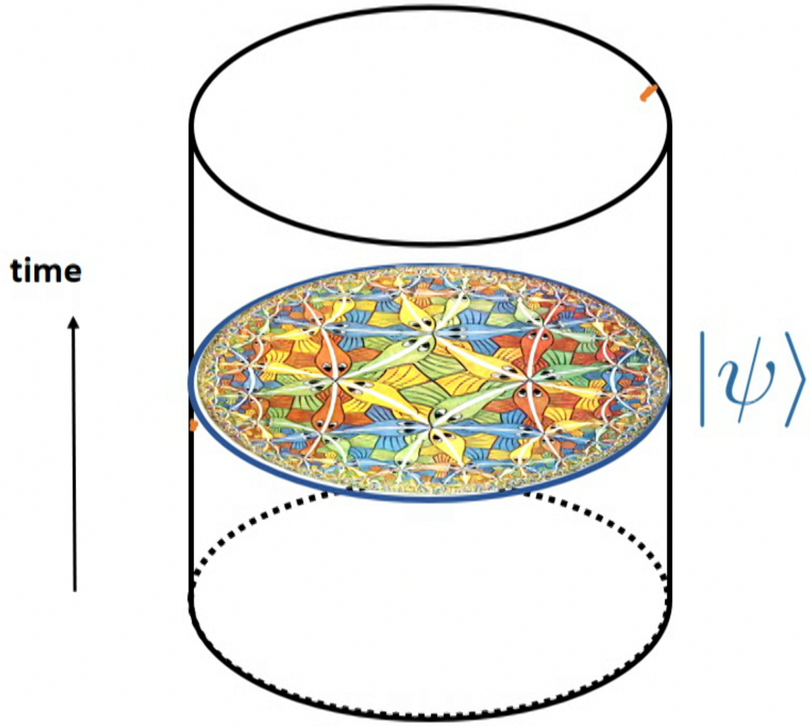
If we consider some **state, $|\psi\rangle$, in the CFT**



it also describes **some geometry in the gravitational theory**.

AdS/CFT Duality

- In the case where $|\psi\rangle$ is the **vacuum**, the dual spacetime is maximally symmetric, negative curvature **anti-de Sitter Space (AdS)**

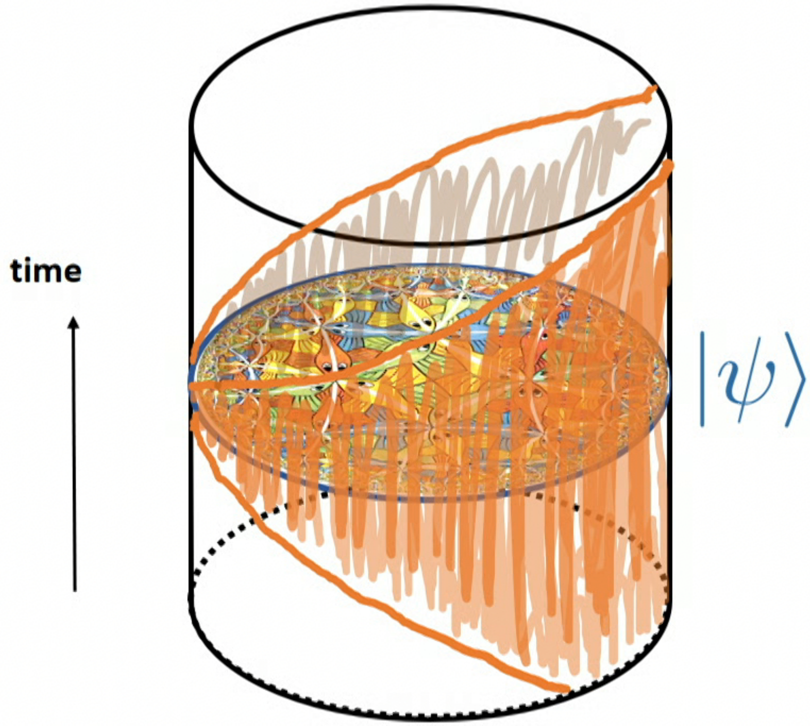


Global AdS

$$ds^2 = \frac{1}{\cos^2\rho}\left(-dt^2 + d\rho^2 + \sin^2\rho d\Omega_{d-1}^2\right)$$

AdS/CFT Duality

- In the case where $|\psi\rangle$ is the **vacuum**, the dual spacetime is maximally symmetric, negative curvature **anti-de Sitter Space (AdS)**



Global AdS

$$ds^2 = \frac{1}{\cos^2 \rho}\left(-dt^2 + d\rho^2 + \sin^2 \rho d\Omega_{d-1}^2\right)$$

Poincare Patch

$$ds^2 = \frac{1}{z^2}\left(dz^2 - dt^2 + d\vec{x}^2\right)$$

Spatial Slice: Hyperbolic Disk

$$ds^2 = \frac{1}{z^2}\left(dz^2 + d\vec{x}^2\right)$$

Reconstructing Bulk Geometry

How do we determine the bulk geometry (and the dynamics of this background) from the CFT?

Given a state $|\psi\rangle$ want to know:

1) **When is $|\psi\rangle$ dual to a classical gravitational background?**

2) **What probes can we use to most efficiently determine the classical background?**

3) **How do we describe the local dynamics in these backgrounds?**

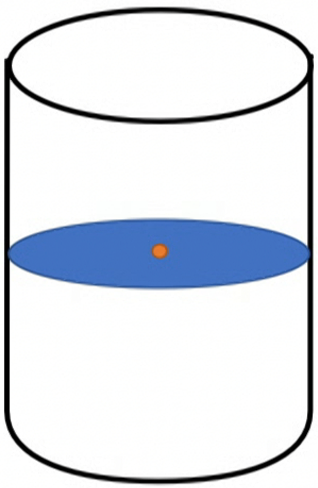
Local Bulk Operators

[Banks, Douglas, Horowitz, Martinec; Bena; Hamilton, Kabat Lifschytz, Lowe; Heemskerk, Marolf Polchinski, JS]

What does local bulk physics look like in terms of CFT operators?

- Best understood perturbatively about the AdS vacuum:
  - Near the boundary, AdS/CFT dictionary: $\lim_{z \to 0} z^{-\Delta} \phi(x, z) = O(x)$
  - Further into the bulk: **HKLL**

Global Reconstruction:



Local Bulk Operators

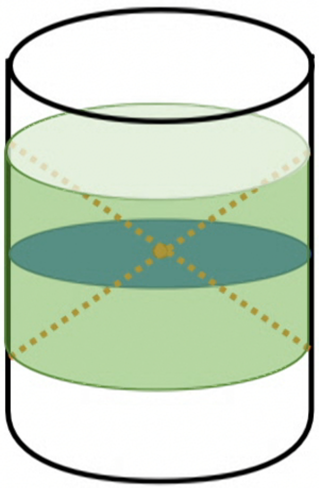
[Banks, Douglas, Horowitz, Martinec; Bena; Hamilton, Kabat Lifschytz, Lowe; Heemskerk, Marolf Polchinski, JS]

What does local bulk physics look like in terms of CFT operators?

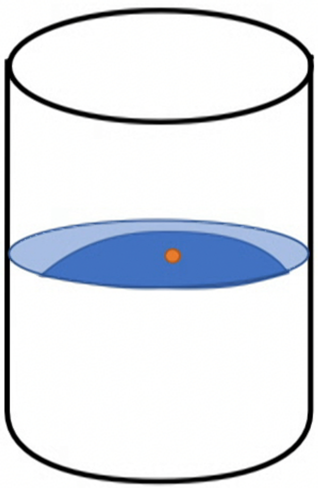
- Best understood perturbatively about the AdS vacuum:
  - Near the boundary, AdS/CFT dictionary: $\lim_{z\to 0} z^{-\Delta}\phi(x,z) = O(x)$
  - Further into the bulk: **HKLL**

Global Reconstruction:

$$\phi(x,z) = \int d^d x' K_g(x,z|x')\mathcal{O}(x')$$



Rindler Reconstruction:



Local Bulk Operators

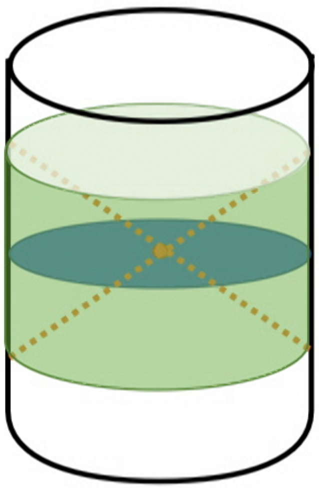
[Banks, Douglas, Horowitz, Martinec; Bena; Hamilton, Kabat Lifschytz, Lowe; Heemskerk, Marolf Polchinski, JS]

What does local bulk physics look like in terms of CFT operators?

- Best understood perturbatively about the AdS vacuum:
  - Near the boundary, AdS/CFT dictionary: $\lim_{z\to 0} z^{-\Delta}\phi(x,z) = \ O(x)$
  - Further into the bulk: **HKLL**

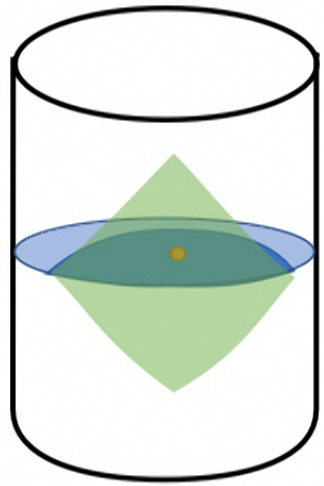
Global Reconstruction:

$$\phi(x,z) = \int d^d x' K_g(x,z|x')\mathcal{O}(x')$$



Rindler Reconstruction:

$$\phi(x,z) = \int d^d x' K_r(x,z|x')\mathcal{O}(x')$$



- **Local Bulk** Operator
  =
  Non-local boundary operator
- Supported on **causal region**

Geometry and Entropy

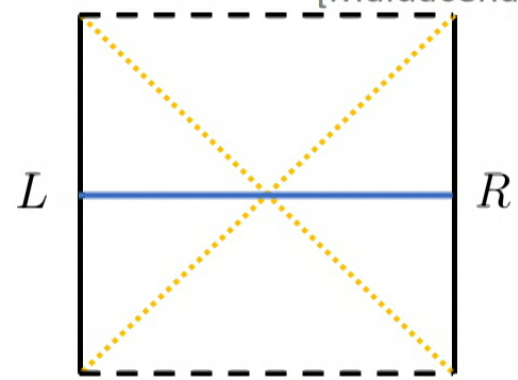
What about non-perturbative probes to study non-trivial backgrounds?

- Long history suggestive that there is a deep connection between spacetime and entropy:
  - **Black hole thermodynamics**: $\delta E = T\, dS$

$$S_{BH} = \frac{A}{4G_N}$$

  - BH thermo can be used to **derive Einstein equation** [Jacobson]
  - **Thermofield double state** of two CFTs (L and R): [Maldacena]

$$|TFD\rangle = N \sum_E e^{-\beta E/2} |E\rangle_L |E\rangle_R$$



Geometry and Entropy

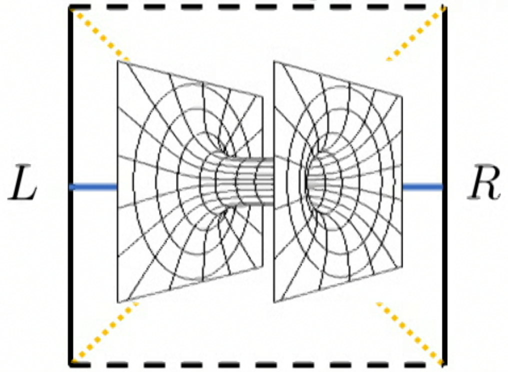
What about non-perturbative probes to study non-trivial backgrounds?

- Long history suggestive that there is a deep connection between spacetime and entropy:
  - **Black hole thermodynamics**: $\delta E = T\, dS$

$$S_{BH} = \frac{A}{4G_N}$$

  - BH thermo can be used to **derive Einstein equation** [Jacobson]
  - **Thermofield double state** of two CFTs (L and R): [Maldacena]

$$|TFD\rangle = N \sum_E e^{-\beta E/2} |E\rangle_L |E\rangle_R$$



Geometry and Entropy

What about non-perturbative probes to study non-trivial backgrounds?

- Long history suggestive that there is a deep connection between spacetime and entropy:
  - **Black hole thermodynamics**: $\delta E = T\, dS$

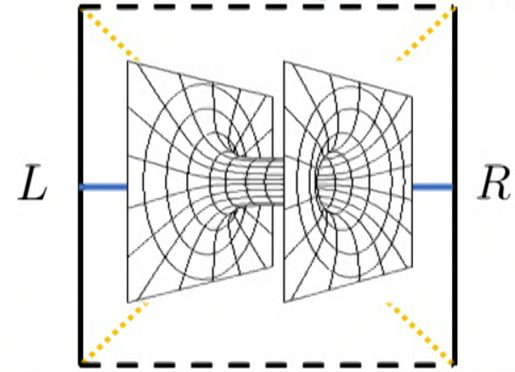
$$S_{BH} = \frac{A}{4G_N}$$

  - BH thermo can be used to **derive Einstein equation** [Jacobson]
  - **Thermofield double state** of two CFTs (L and R): [Maldacena]

$$|TFD\rangle = N \sum_E e^{-\beta E/2} |E\rangle_L |E\rangle_R$$

$$S_{BH} = -\mathrm{tr}\left[\rho_R \log \rho_R\right]$$

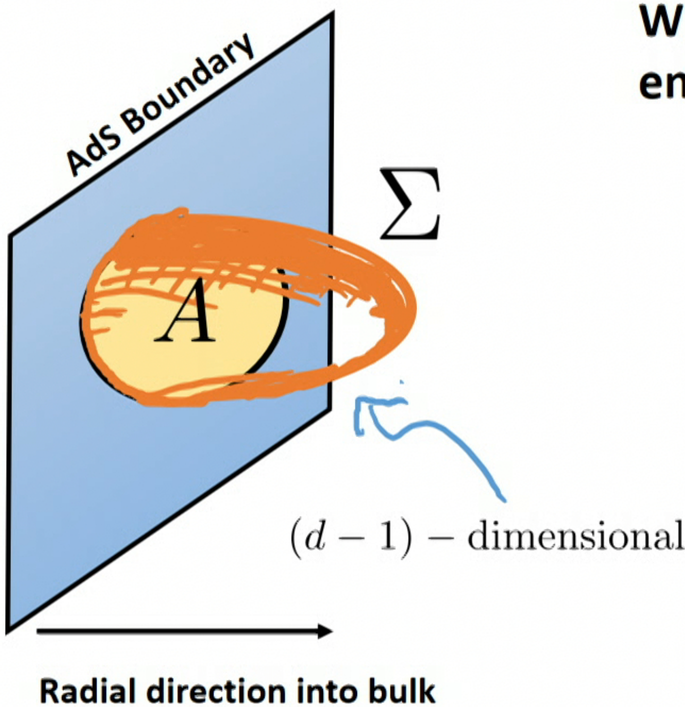
Black hole entropy is **entanglement entropy**



Ryu-Takayanagi

[Ryu, Takayanagi; Hubeny, Rangamani, Takayanagi]

- Consider a spatial region of a single CFT:



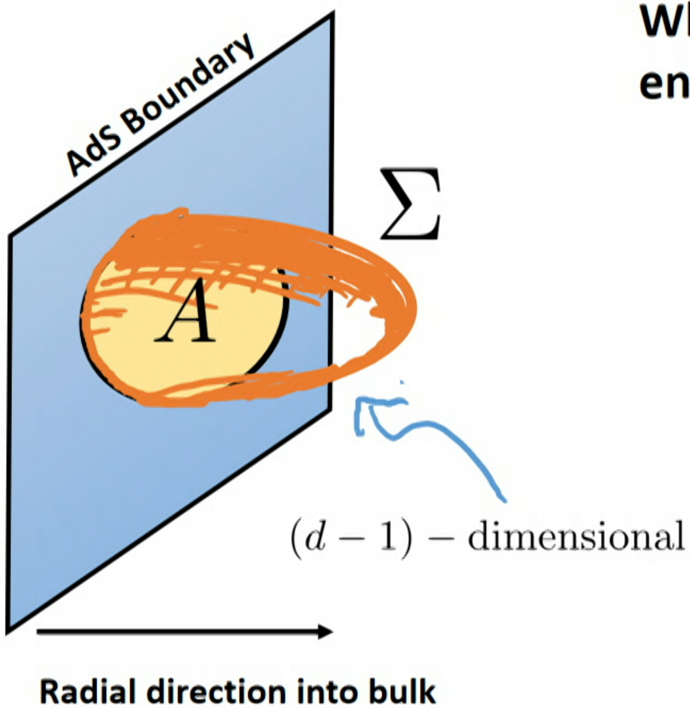
What bulk object computes the entanglement entropy of the region A?

$$S(A) = \underset{\Sigma \sim A}{\text{ext}} \frac{A_\Sigma}{4G_N}$$

Ryu-Takayanagi

[Ryu, Takayanagi; Hubeny, Rangamani, Takayanagi]

- Consider a spatial region of a single CFT:



What bulk object computes the entanglement entropy of the region A?

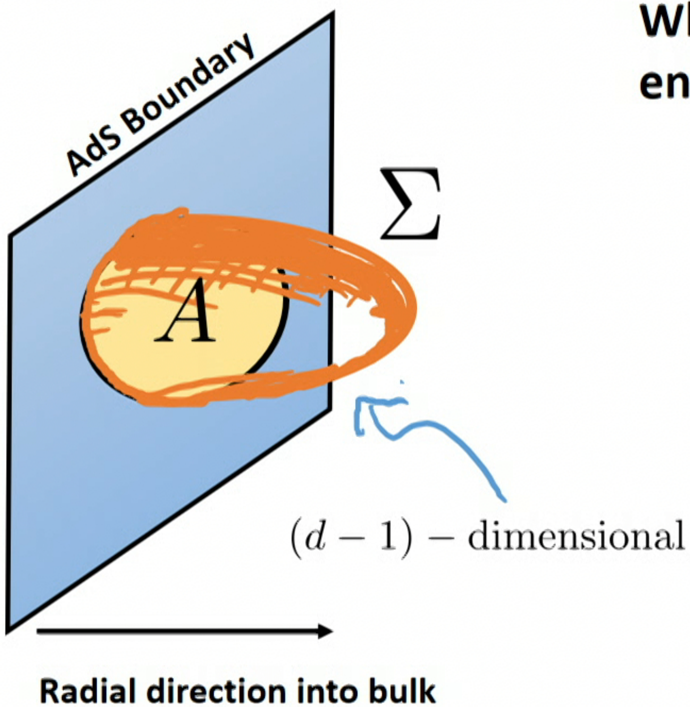
$$S(A) = \underset{\Sigma \sim A}{\text{ext}} \frac{A_\Sigma}{4G_N}$$

(This has corrections from higher-curvature and quantum contributions in the bulk.)

Ryu-Takayanagi

[Ryu, Takayanagi; Hubeny, Rangamani, Takayanagi]

- Consider a spatial region of a single CFT:



What bulk object computes the entanglement entropy of the region A?

$$S(A) = \underset{\Sigma \sim A}{\text{ext}} \frac{A_\Sigma}{4G_N}$$

(This has corrections from higher-curvature and quantum contributions in the bulk.)

Entanglement entropy gives us **non-perturbative, non-local** probes of the dual **bulk geometry**

Ryu-Takayanagi

[Lashkari, McDermott, van Raamsdonk; Faulkner, Guica, Hartman, Myers, van Raamsdonk; Swingle, van Raamsdonk; Lin, Marcollil, Ooguri, Stoica; Czech, Lamprou, McCandlish, Mosk, Sully;]

Suggestive of picture where CFT **entanglement seems to constitute bulk geometry.**

Can we reconstruct the bulk geometry from these probes?

- A lot of progress showing how to read off bulk geometry and Einstein equation from this data (**Bulk Tomography**)
- **But would like to understand how/why geometry emerges from entanglement.**

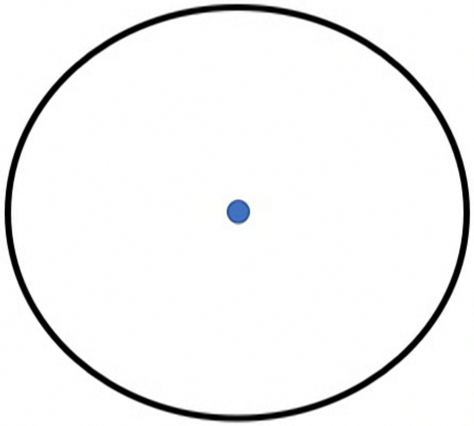
Ryu-Takayanagi

[Lashkari, McDermott, van Raamsdonk; Faulkner, Guica, Hartman, Myers, van Raamsdonk; Swingle, van Raamsdonk; Lin, Marcollil, Ooguri, Stoica; Czech, Lamprou, McCandlish, Mosk, Sully;]

Suggestive of picture where CFT **entanglement seems to constitute bulk geometry.**

Can we reconstruct the bulk geometry from these probes?

- A lot of progress showing how to read off bulk geometry and Einstein equation from this data (**Bulk Tomography**)
- **But would like to understand how/why geometry emerges from entanglement.**
- Also: entanglement can be blind to important regions of bulk geometry: '**entanglement shadows**'
  [Balasubramanian, Chowdhury, Czech, de Boer]



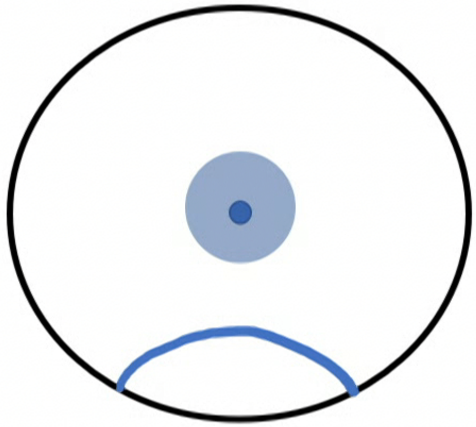
Ryu-Takayanagi

[Lashkari, McDermott, van Raamsdonk; Faulkner, Guica, Hartman, Myers, van Raamsdonk; Swingle, van Raamsdonk; Lin, Marcollil, Ooguri, Stoica; Czech, Lamprou, McCandlish, Mosk, Sully;]

Suggestive of picture where CFT **entanglement seems to constitute bulk geometry.**

Can we reconstruct the bulk geometry from these probes?

- A lot of progress showing how to read off bulk geometry and Einstein equation from this data (**Bulk Tomography**)
- **But would like to understand how/why geometry emerges from entanglement.**
- Also: entanglement can be blind to important regions of bulk geometry: '**entanglement shadows**'
  [Balasubramanian, Chowdhury, Czech, de Boer]



Ryu-Takayanagi

[Lashkari, McDermott, van Raamsdonk; Faulkner, Guica, Hartman, Myers, van Raamsdonk; Swingle, van Raamsdonk; Lin, Marcollil, Ooguri, Stoica; Czech, Lamprou, McCandlish, Mosk, Sully;]

Suggestive of picture where CFT **entanglement seems to constitute bulk geometry.**

Can we reconstruct the bulk geometry from these probes?

- A lot of progress showing how to read off bulk geometry and Einstein equation from this data (**Bulk Tomography**)
- **But would like to understand how/why geometry emerges from entanglement.**
- Also: entanglement can be blind to important regions of bulk geometry: '**entanglement shadows**'
  [Balasubramanian, Chowdhury, Czech, de Boer]

Ryu-Takayanagi

[Lashkari, McDermott, van Raamsdonk; Faulkner, Guica, Hartman, Myers, van Raamsdonk; Swingle, van Raamsdonk; Lin, Marcollil, Ooguri, Stoica; Czech, Lamprou, McCandlish, Mosk, Sully;]

Suggestive of picture where CFT **entanglement seems to constitute bulk geometry.**

Can we reconstruct the bulk geometry from these probes?

- A lot of progress showing how to read off bulk geometry and Einstein equation from this data (**Bulk Tomography**)
- **But would like to understand how/why geometry emerges from entanglement.**
- Also: entanglement can be blind to important regions of bulk geometry: '**entanglement shadows**' [Balasubramanian, Chowdhury, Czech, de Boer]

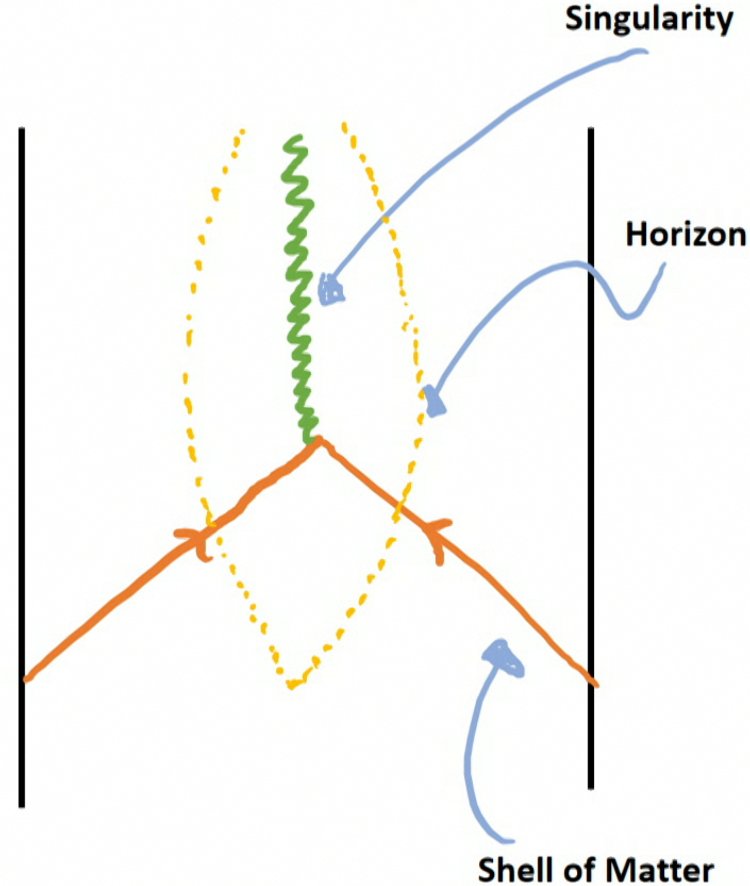
What CFT quantities are needed to see the complete bulk geometry?

Is a different type of CFT correlation responsible for generating these?

Black Hole Interior

[Almheiri, Marolf, Polchinski, JS;+Stanford; Mathur]

One of the most important examples of a region not probed by entanglement is the interior of a black hole:

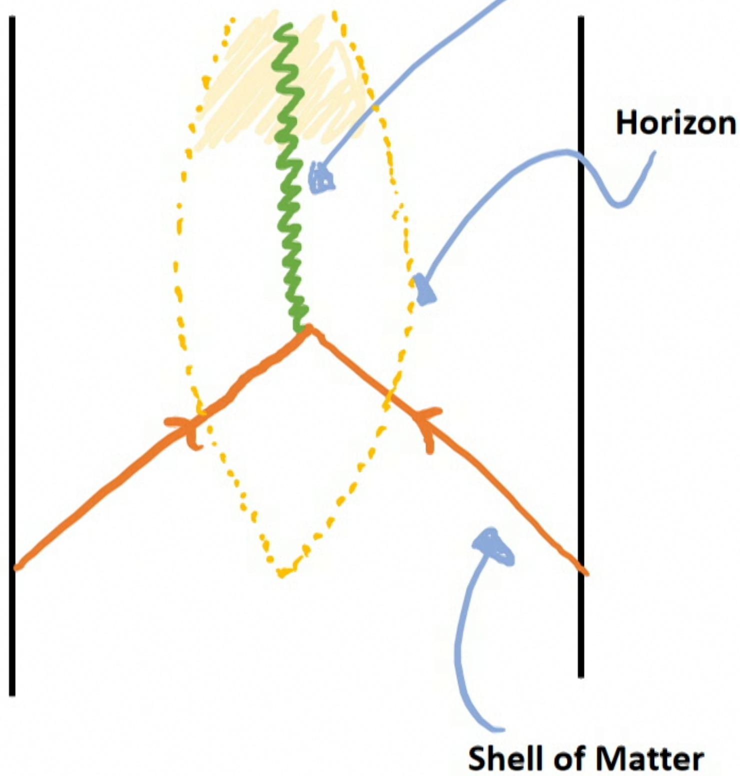


Black Hole Interior

[Almheiri, Marolf, Polchinski, JS;+Stanford; Mathur]

One of the most important examples of a region not probed by entanglement is the interior of a black hole:

- The **BH Information Paradox** gives us reason to doubt the existence of the interior of a black hole at late times (ie. Firewalls)



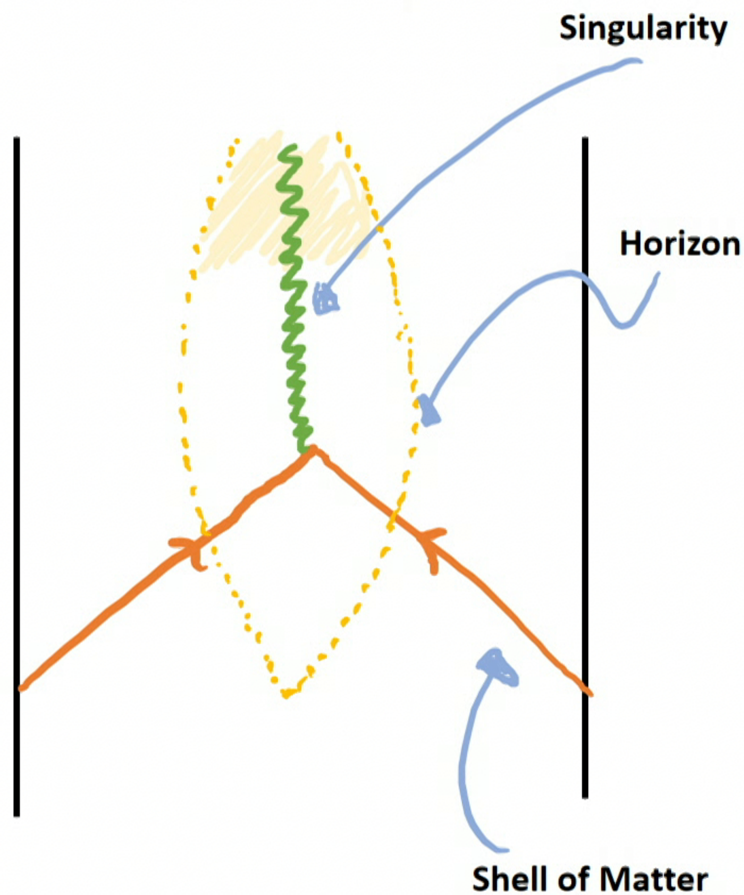
Black Hole Interior

[Almheiri, Marolf, Polchinski, JS;+Stanford; Mathur]

One of the most important examples of a region not probed by entanglement is the interior of a black hole:

- The **BH Information Paradox** gives us reason to doubt the existence of the interior of a black hole at late times (ie. Firewalls)
- The interior is in an entanglement shadow

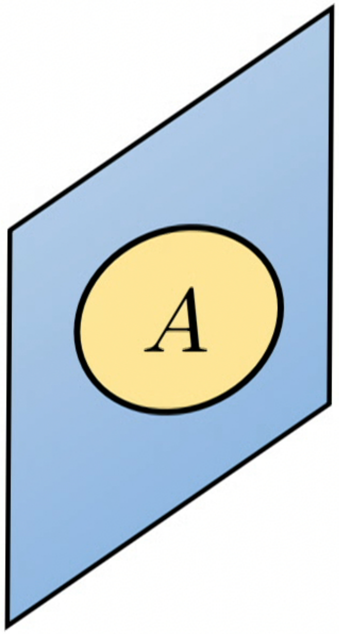
If this region of spacetime exists, how is it encoded by the CFT?



Sub-region Duality

How non-local is the mapping between the boundary and the bulk?

- Given the **density matrix** for a **spatial region** on the boundary, **how much of the bulk** can I reconstruct?

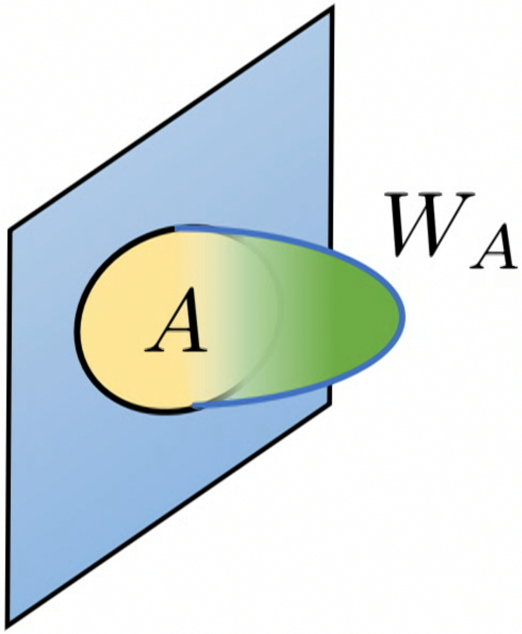


- Can compute EE for all sub-regions → **Suggests we should be able to** reconstruct out to RT surface, the **Entanglement Wedge W_A**

Sub-region Duality

How non-local is the mapping between the boundary and the bulk?

- Given the **density matrix** for a **spatial region** on the boundary, **how much of the bulk** can I reconstruct?

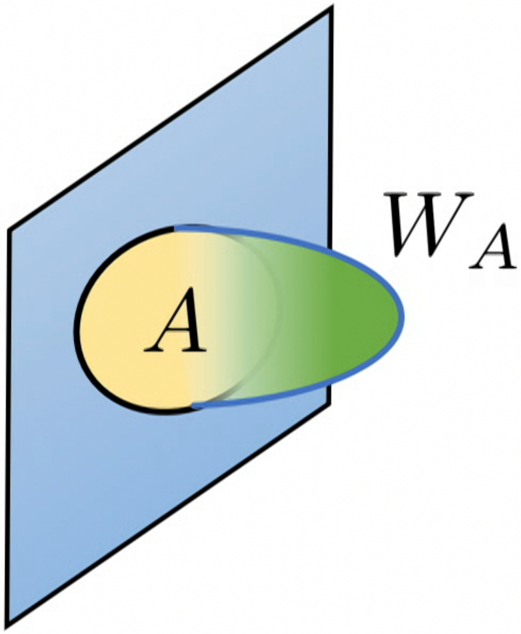


- Can compute EE for all sub-regions → **Suggests we should be able to** reconstruct out to RT surface, the **Entanglement Wedge W_A**

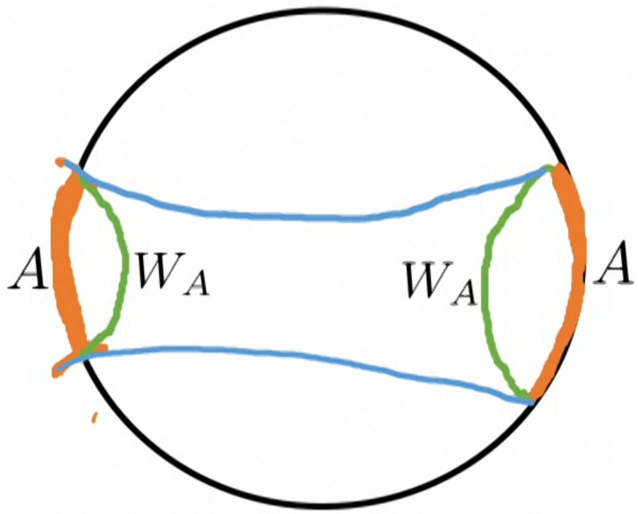
Sub-region Duality

How non-local is the mapping between the boundary and the bulk?

- Given the **density matrix** for a **spatial region** on the boundary, **how much of the bulk** can I reconstruct?



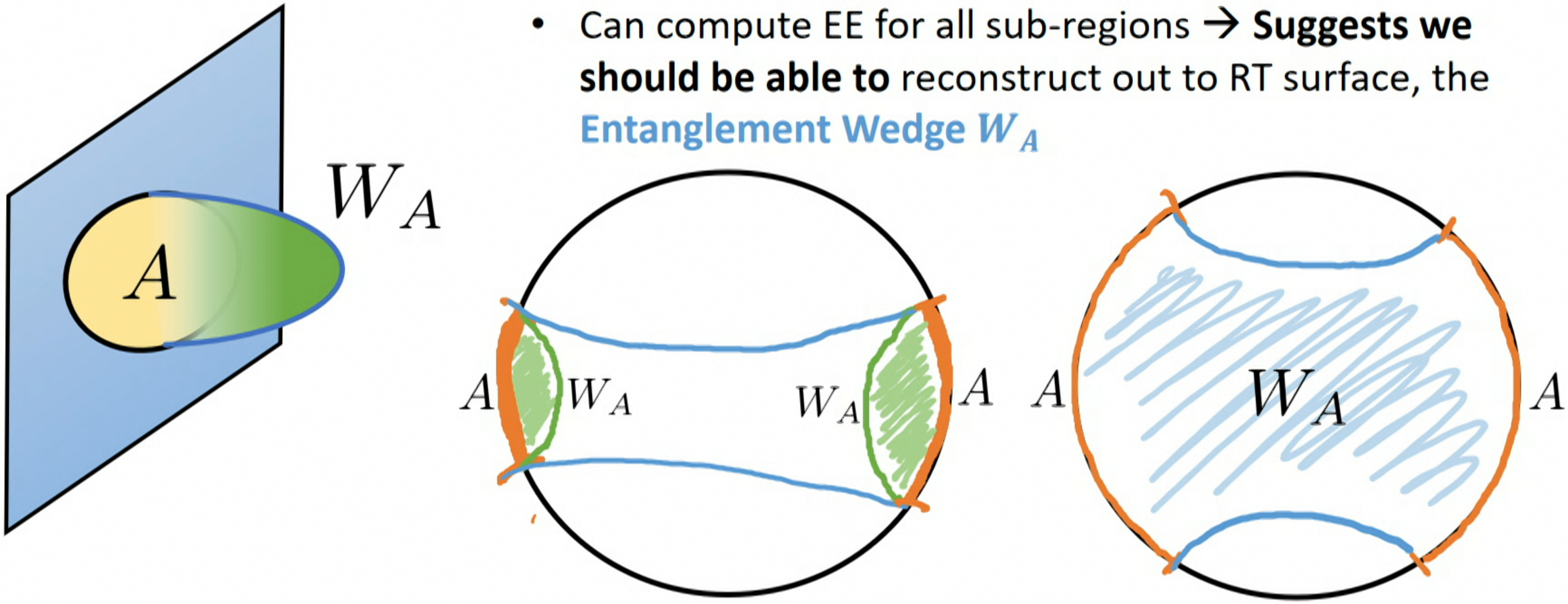
- Can compute EE for all sub-regions → **Suggests we should be able to** reconstruct out to RT surface, the **Entanglement Wedge W_A**



Sub-region Duality

How non-local is the mapping between the boundary and the bulk?

- Given the **density matrix** for a **spatial region** on the boundary, **how much of the bulk** can I reconstruct?
  - Can compute EE for all sub-regions → **Suggests we should be able to** reconstruct out to RT surface, the **Entanglement Wedge W_A**

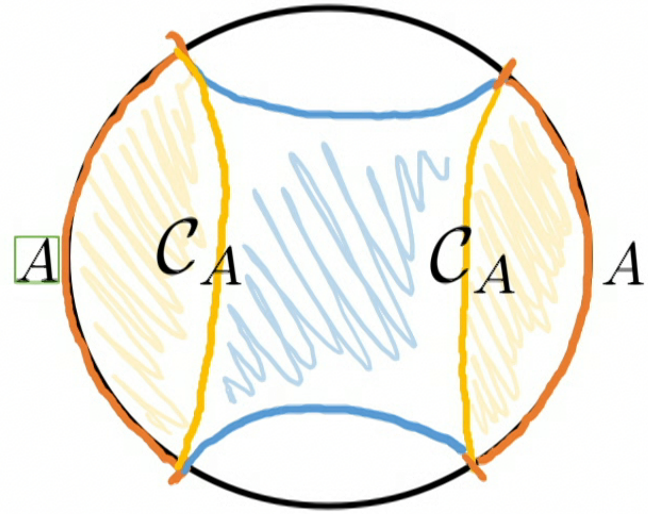


Sub-region Duality

What can we actually do?

Constructive:

- Can build bulk operators in regions causally connected to boundary **(HKLL)**

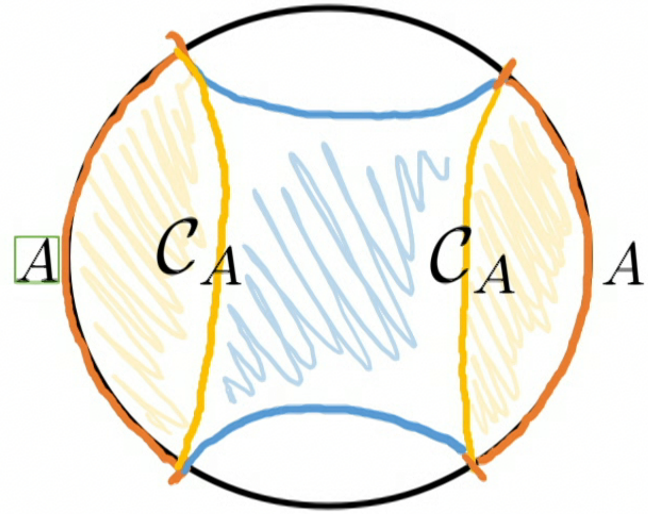


Sub-region Duality

What can we actually do?

Constructive:

- Can build bulk operators in regions causally connected to boundary **(HKLL)**



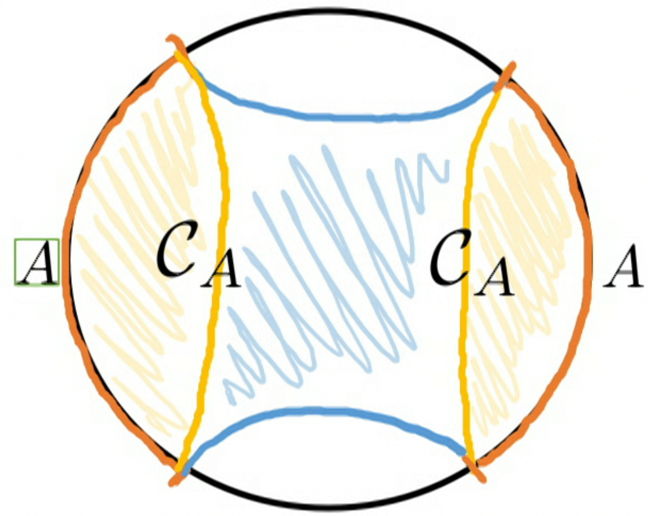
Less Constructive: [Dong, Harlow, Wall]

Sub-region Duality

What can we actually do?

Constructive:

- Can build bulk operators in regions **causally connected** to boundary **(HKLL)**



Less Constructive: [Dong, Harlow, Wall]

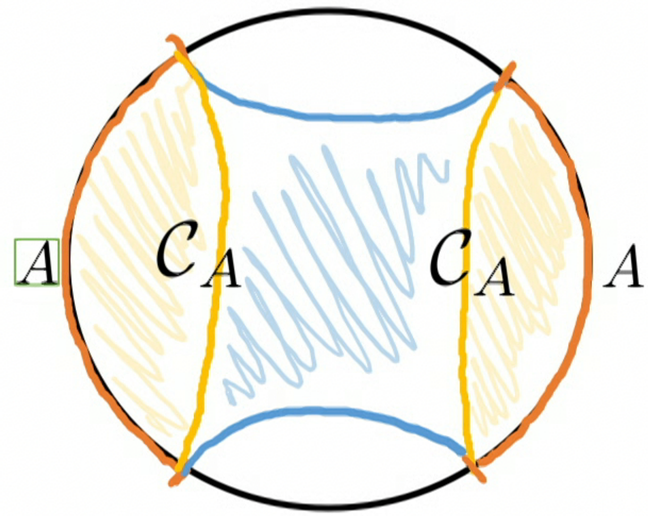
- There is a prescription to construct operators in the entanglement wedge, but requires knowledge of modular Hamiltonian for region
  → *Hard to carry out explicitly*

Sub-region Duality

What can we actually do?

Constructive:

- Can build bulk operators in regions **causally connected** to boundary **(HKLL)**



Less Constructive: [Dong, Harlow, Wall]

- There is a prescription to construct operators in the entanglement wedge, but requires knowledge of modular Hamiltonian for region
  → *Hard to carry out explicitly*

Are there concrete scenarios to explore sub-region duality?

Toy Models for Holography?

To summarize, we are interested in:

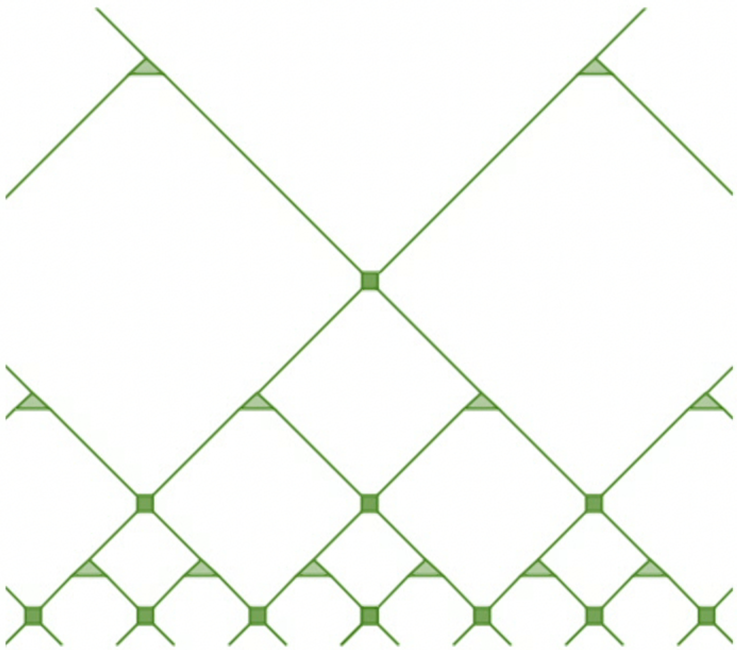
1. **How do we see the emergence of classical bulk geometry?**
2. **What are effective non-perturbative probes of this geometry?**
3. **How do see into regions shadowed from entanglement?**
4. **Are there practical approaches to understanding duality for sub-regions?**

AdS / MERA

MERA and Holography

[Swingle]

Sometime circa 2009, Brian Swingle was staring at a MERA network…

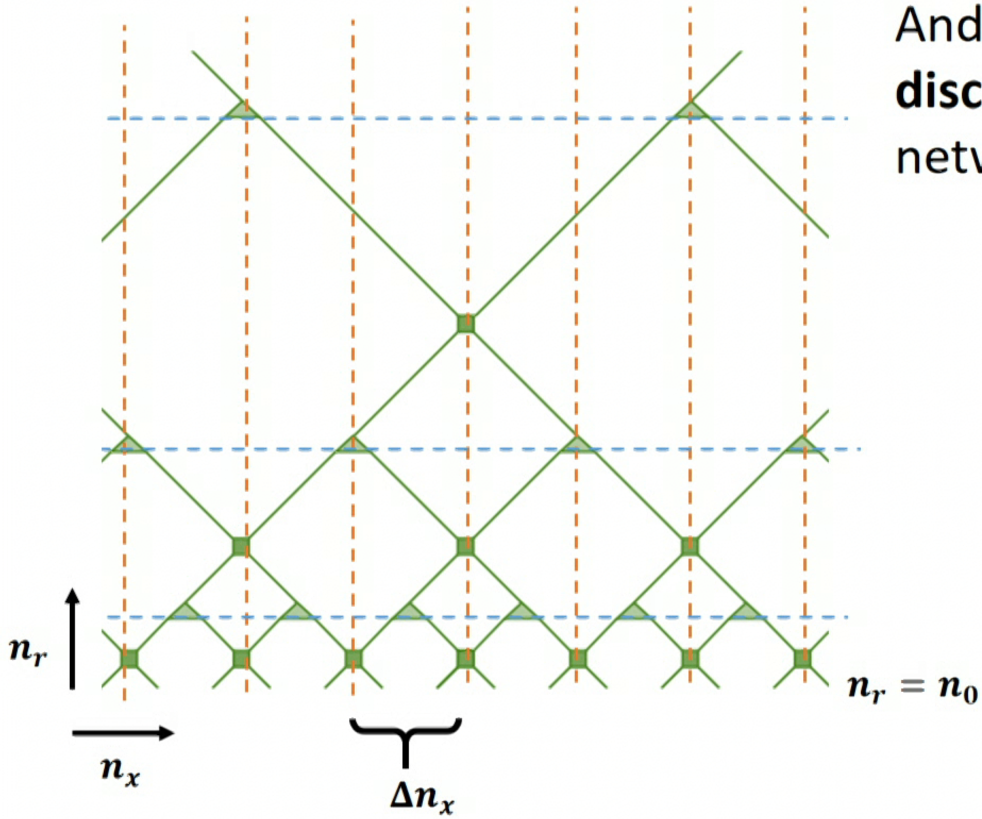


And realized that one can assign a **natural discrete Euclidean metric** to translations in the network:

MERA and Holography

[Swingle]

Sometime circa 2009, Brian Swingle was staring at a MERA network…

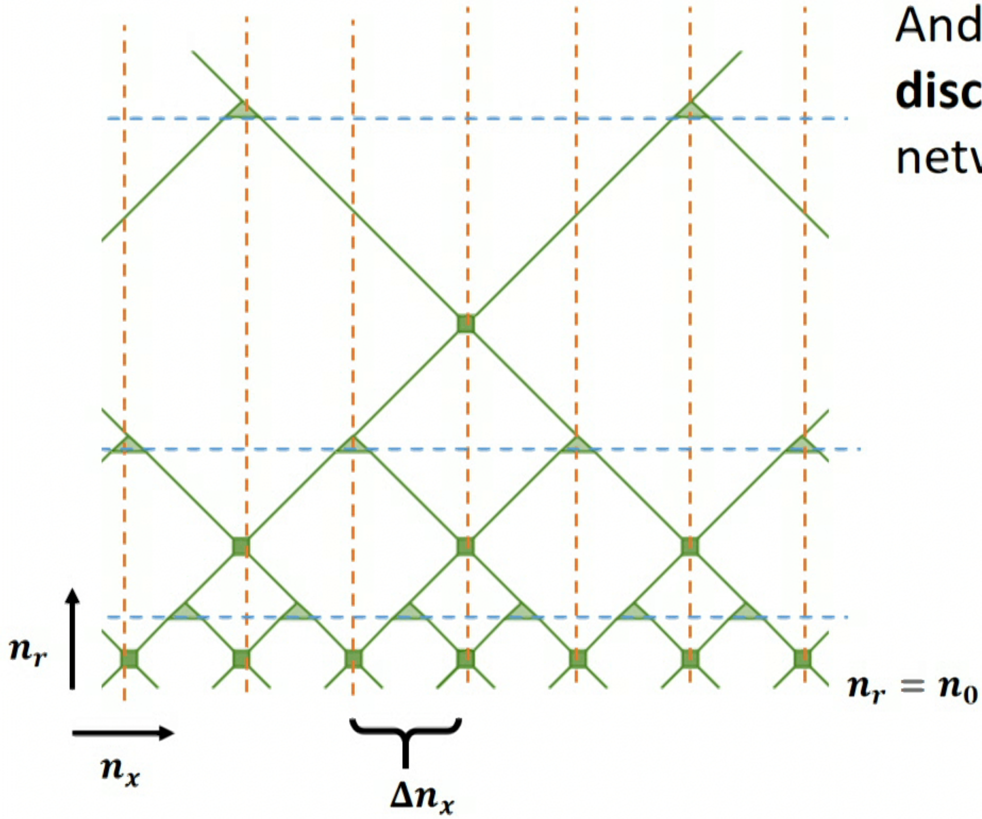


And realized that one can assign a **natural discrete Euclidean metric** to translations in the network:

MERA and Holography

[Swingle]

Sometime circa 2009, Brian Swingle was staring at a MERA network…



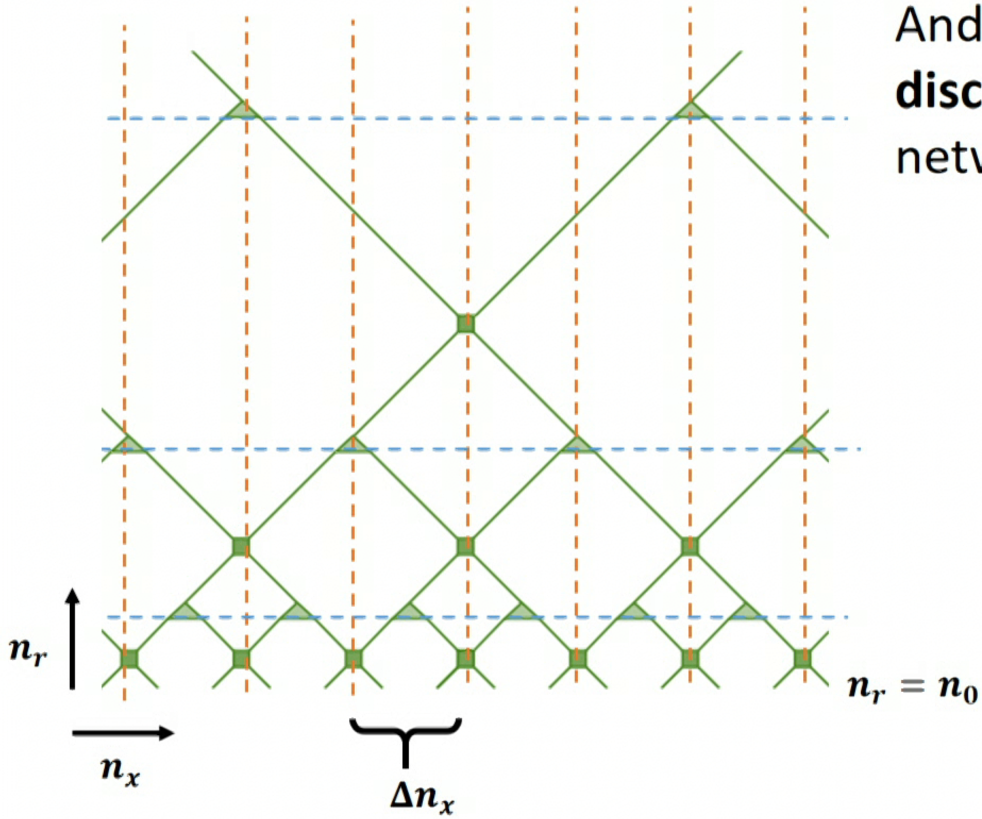
And realized that one can assign a **natural discrete Euclidean metric** to translations in the network:

$$\Delta n^2 = \Delta n_r^2 + 2^{-(n_r - n_0)} \Delta n_x^2$$

MERA and Holography

[Swingle]

Sometime circa 2009, Brian Swingle was staring at a MERA network…



And realized that one can assign a **natural discrete Euclidean metric** to translations in the network:

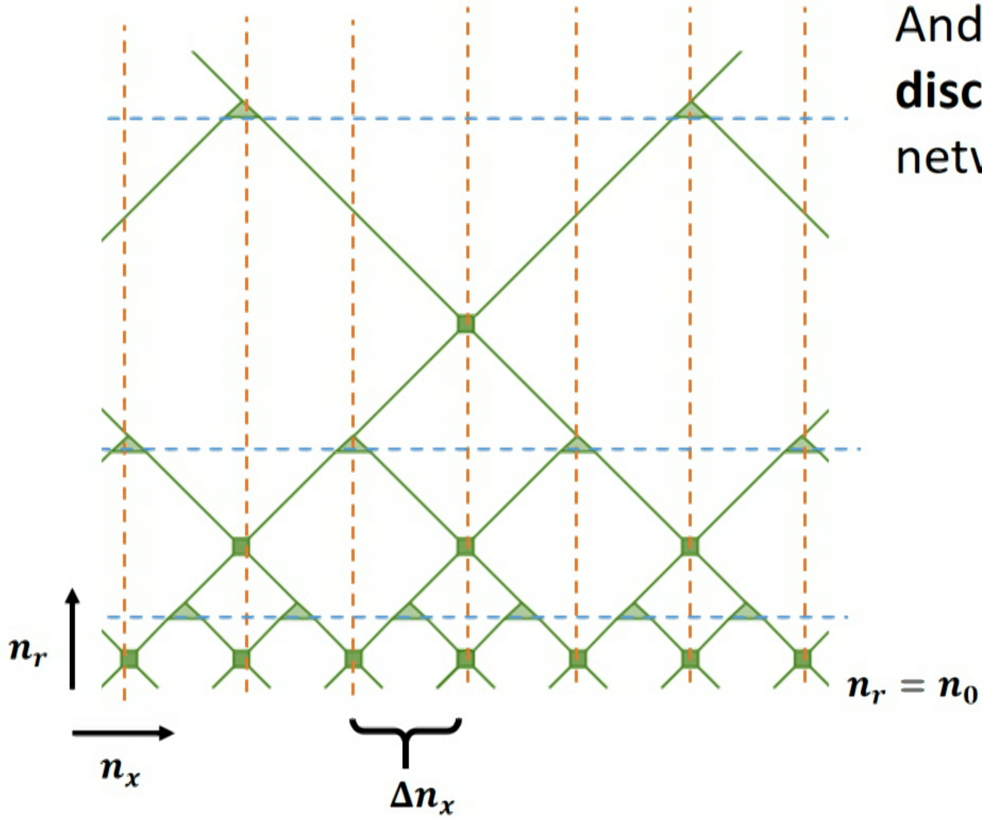
$$\Delta n^2 = \Delta n_r^2 + 2^{-(n_r - n_0)} \Delta n_x^2$$

$$ds^2 = d\rho^2 + e^{-2\rho} dx^2$$

MERA and Holography

[Swingle]

Sometime circa 2009, Brian Swingle was staring at a MERA network…



And realized that one can assign a **natural discrete Euclidean metric** to translations in the network:

$$\Delta n^2 = \Delta n_r^2 + 2^{-(n_r - n_0)} \Delta n_x^2$$

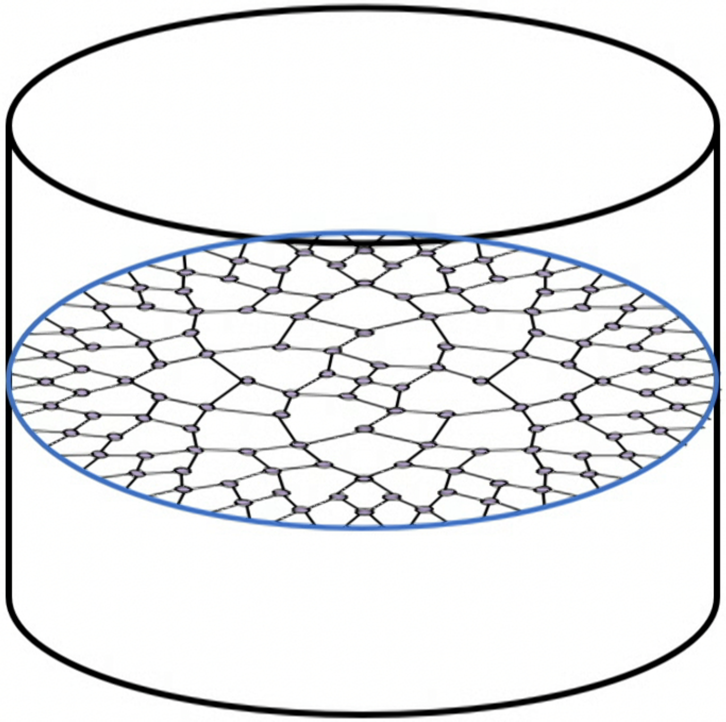
$$ds^2 = d\rho^2 + e^{-2\rho} dx^2$$

$$ds^2 = \frac{1}{z^2}(dz^2 + dx^2)$$

Hyperbolic Metric

MERA and Holography

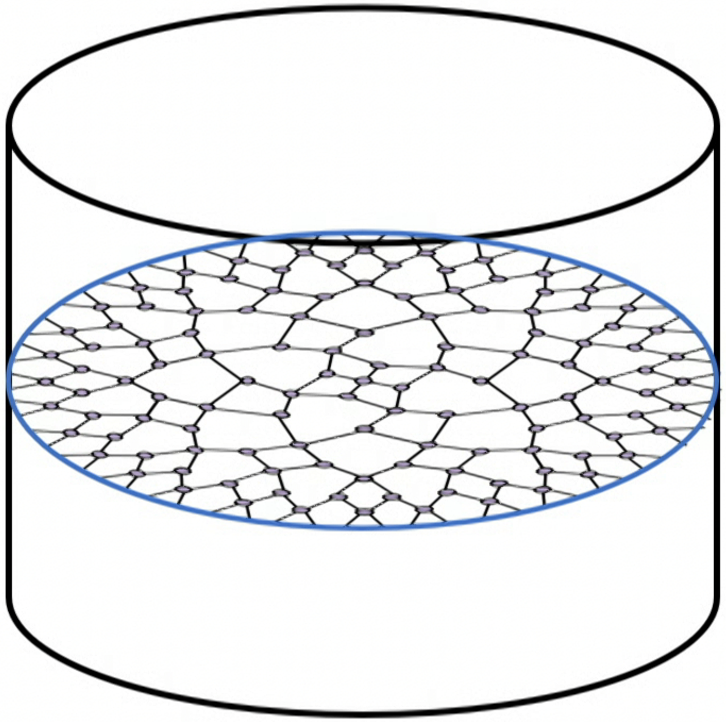
It looks as though MERA generates a spatial slice of the AdS geometry dual to the CFT ground state:



MERA and Holography

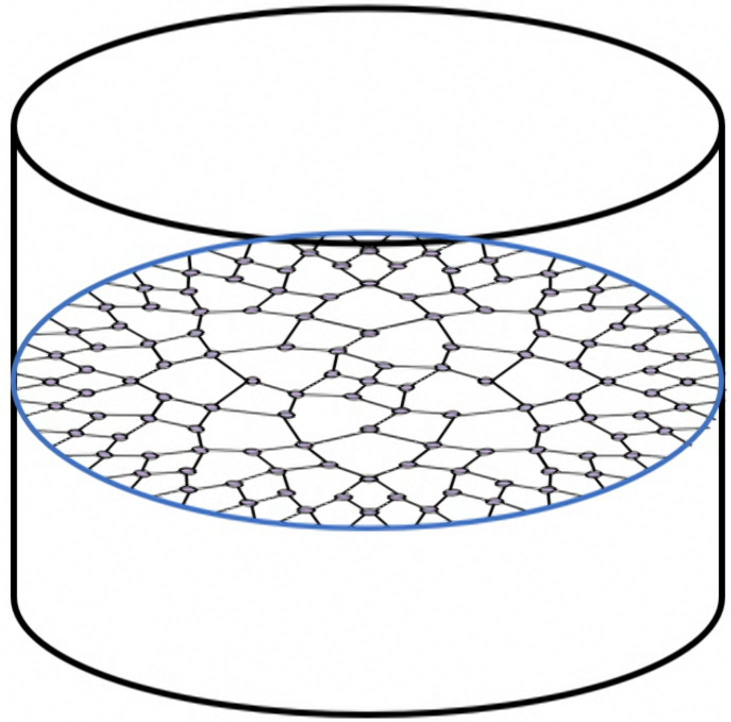
It looks as though MERA generates a spatial slice of the AdS geometry dual to the CFT ground state:

- Both have **natural hyperbolic metrics**
- Both have an **extra-dimension** that emerges



MERA and Holography

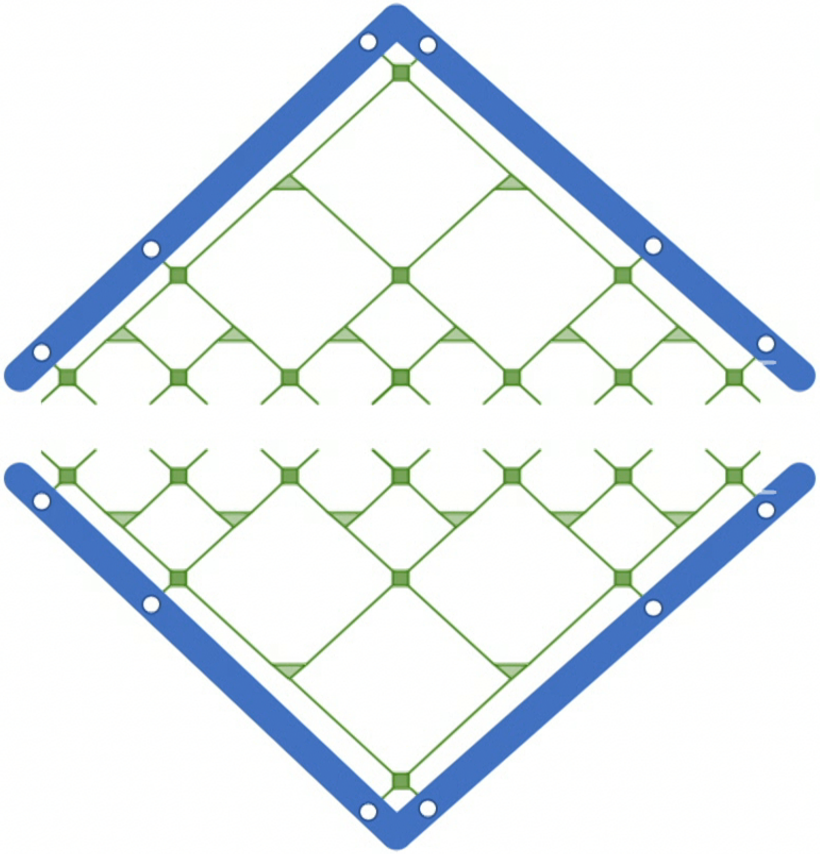
It looks as though MERA generates a spatial slice of the AdS geometry dual to the CFT ground state:



- Both have **natural hyperbolic metrics**
- Both have an **extra-dimension** that emerges
- The **extra 'radial' dimension** is associated to the **RG** in both schemes

MERA and Holography

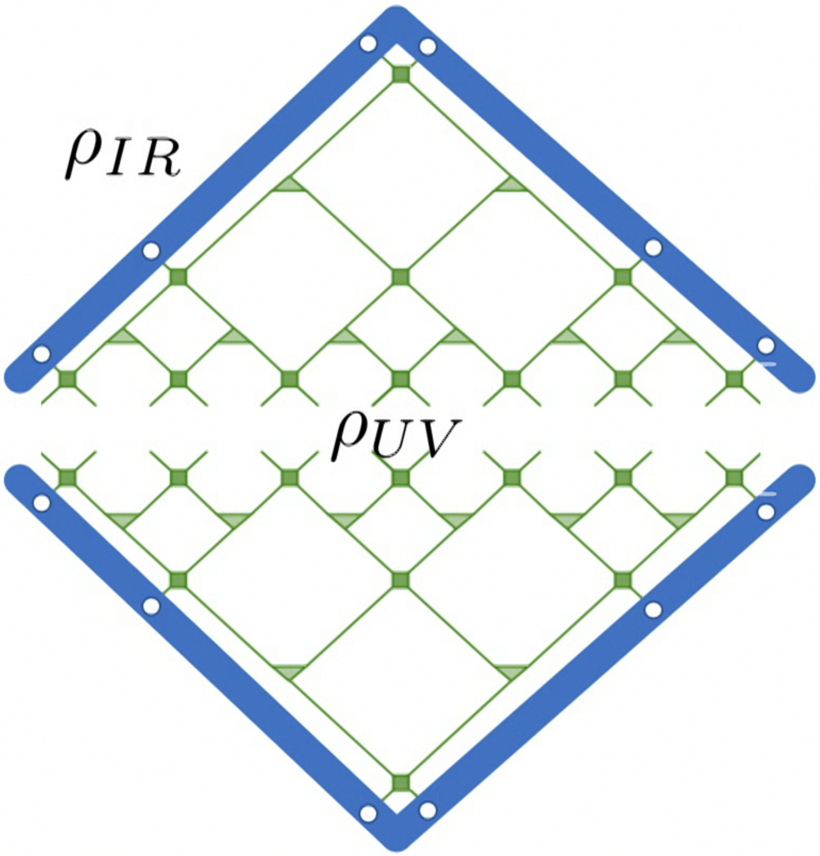
This very simple model also realize more features of the duality:



Consider the region of the MERA network responsible for computing the density matrix of a region:

MERA and Holography

This very simple model also realize more features of the duality:



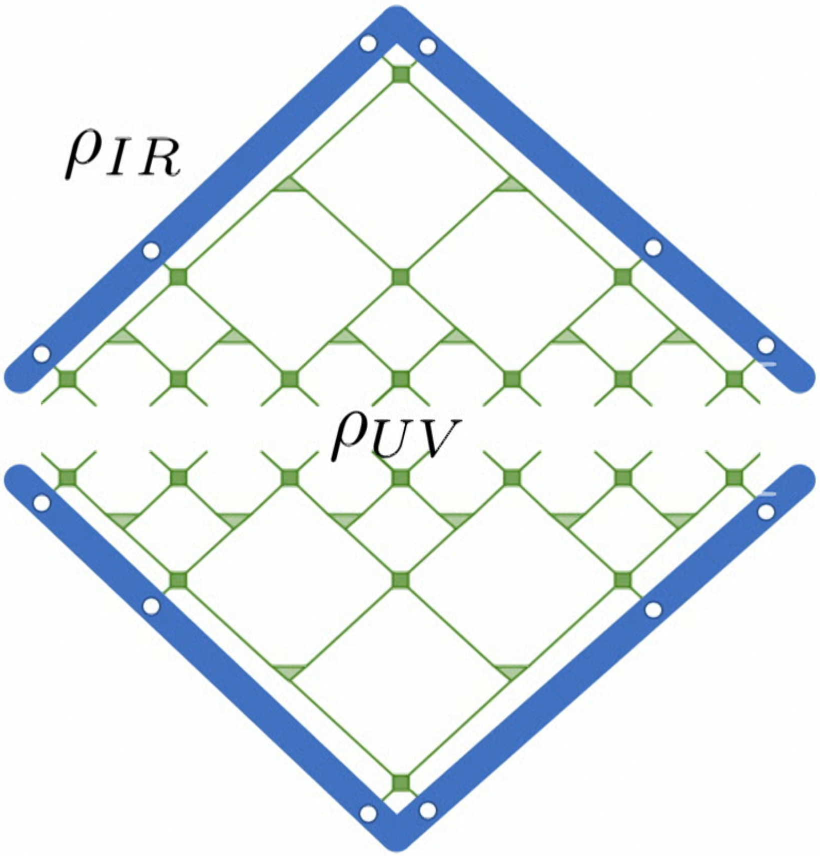
Consider the region of the MERA network responsible for computing the density matrix of a region:

- Implements unitary transformation between IR and UV density matrix
- Entanglement entropy is bounded by dimension of IR Hilbert space:

$$S(A) \leq n_{cut} \log \chi$$

MERA and Holography

This very simple model also realize more features of the duality:



Consider the region of the MERA network responsible for computing the density matrix of a region:

- Implements unitary transformation between IR and UV density matrix
- Entanglement entropy is bounded by dimension of IR Hilbert space:

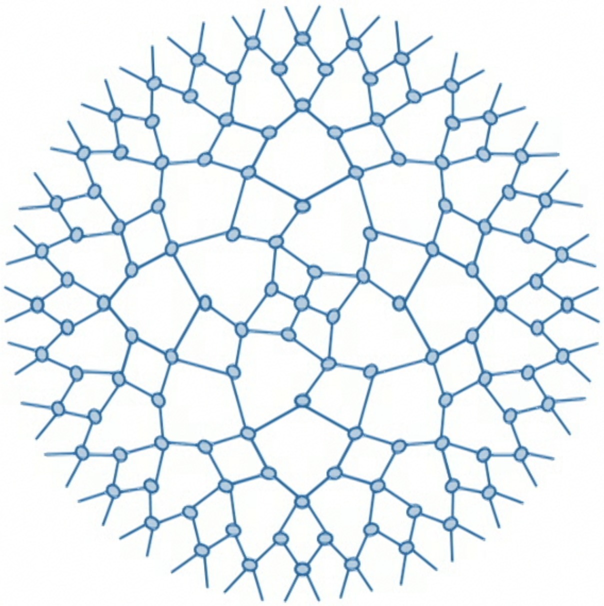
$$S(A) \leq n_{cut} \log \chi$$

- Better yet, it has been shown that each scale contributes approximately equally to the entropy so that

$$S(A) = n_{cut} \log \chi_{\text{eff}}$$

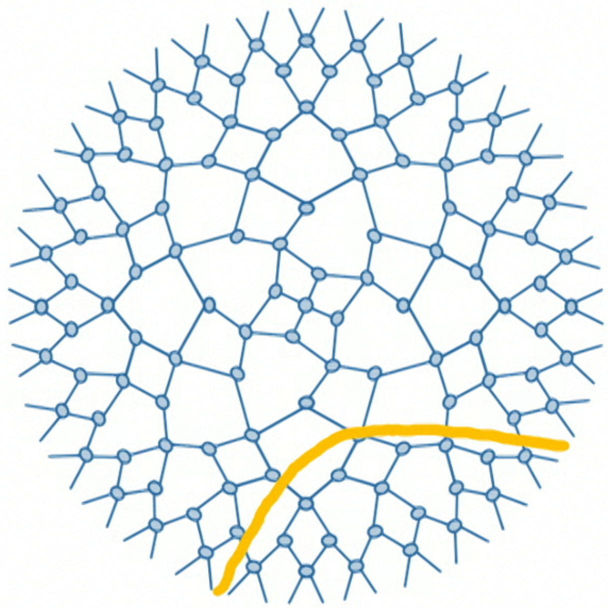
MERA and Holography

Entanglement entropy or a region in MERA: is given by length of cut:



MERA and Holography

Entanglement entropy or a region in MERA: is given by length of cut:

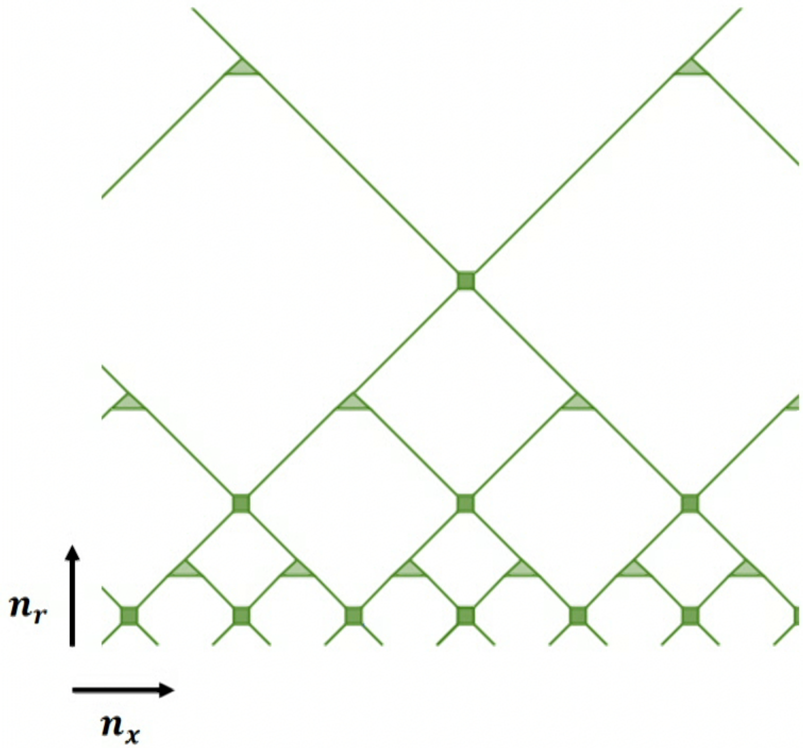


Beautiful correspondence to length of RT surface in spatial slice of AdS!

MERA and Holography

[Czech, Lamprou, McCandlish, JS; Beny]

A short detour:

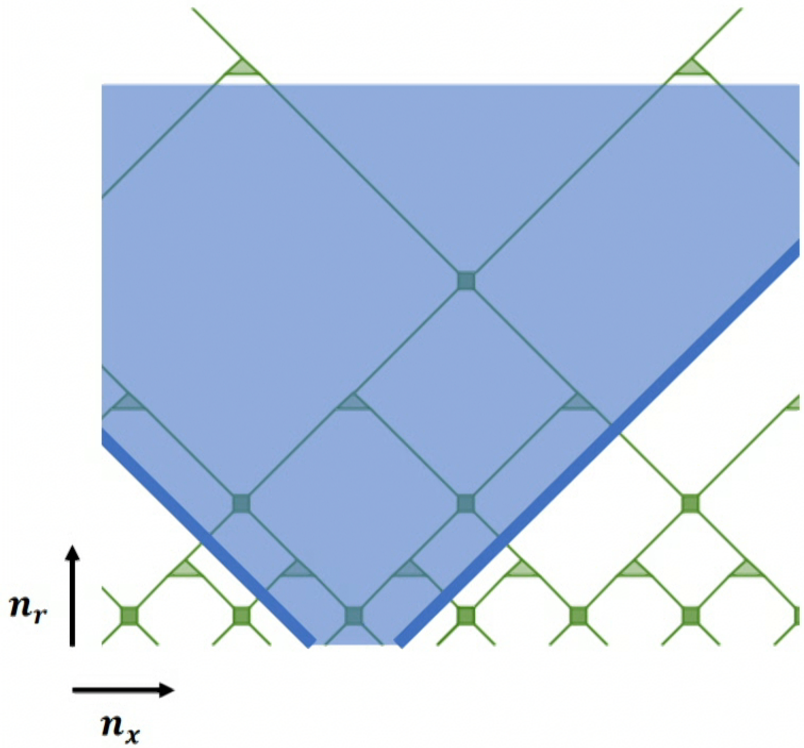


One feature of MERA not particularly well-captured by this equivalence is its **causal structure:**

MERA and Holography

[Czech, Lamprou, McCandlish, JS; Beny]

A short detour:

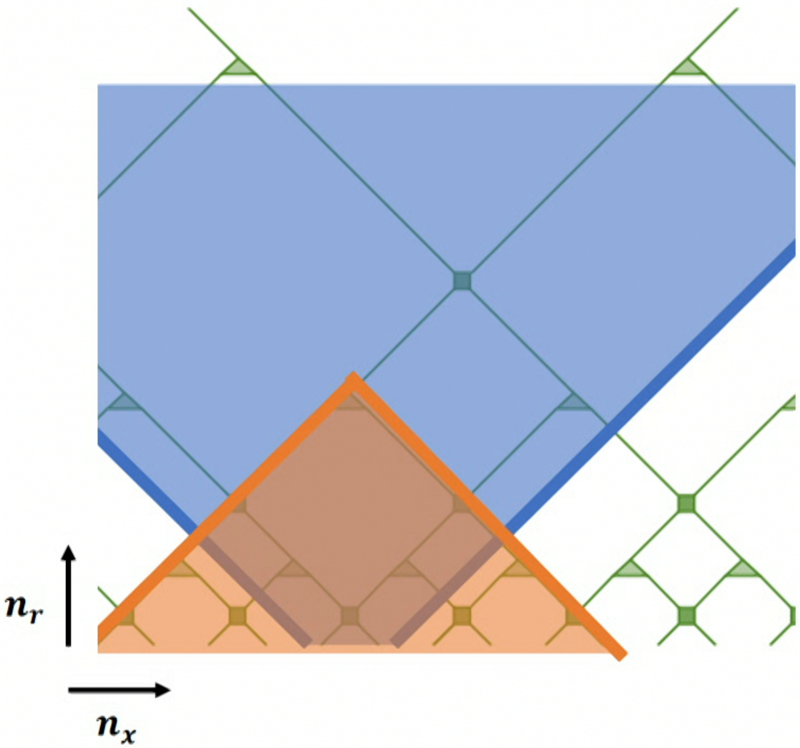


One feature of MERA not particularly well-captured by this equivalence is its **causal structure:**

MERA and Holography

[Czech, Lamprou, McCandlish, JS; Beny]

A short detour:

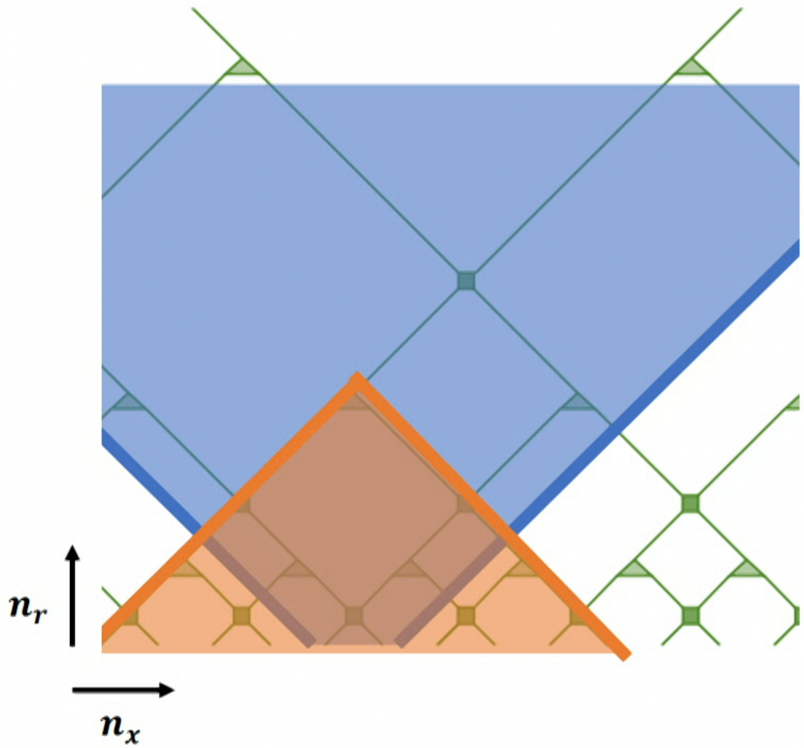


One feature of MERA not particularly well-captured by this equivalence is its **causal structure:**

MERA and Holography

[Czech, Lamprou, McCandlish, JS; Beny]

A short detour:



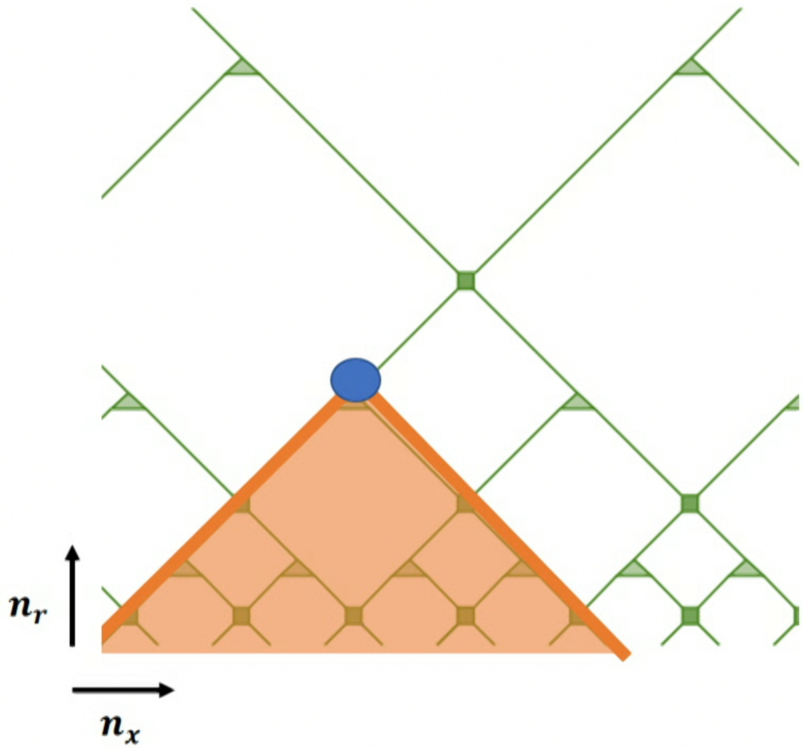
One feature of MERA not particularly well-captured by this equivalence is its **causal structure:**

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^2)$$

de Sitter Metric

Kinematic Space

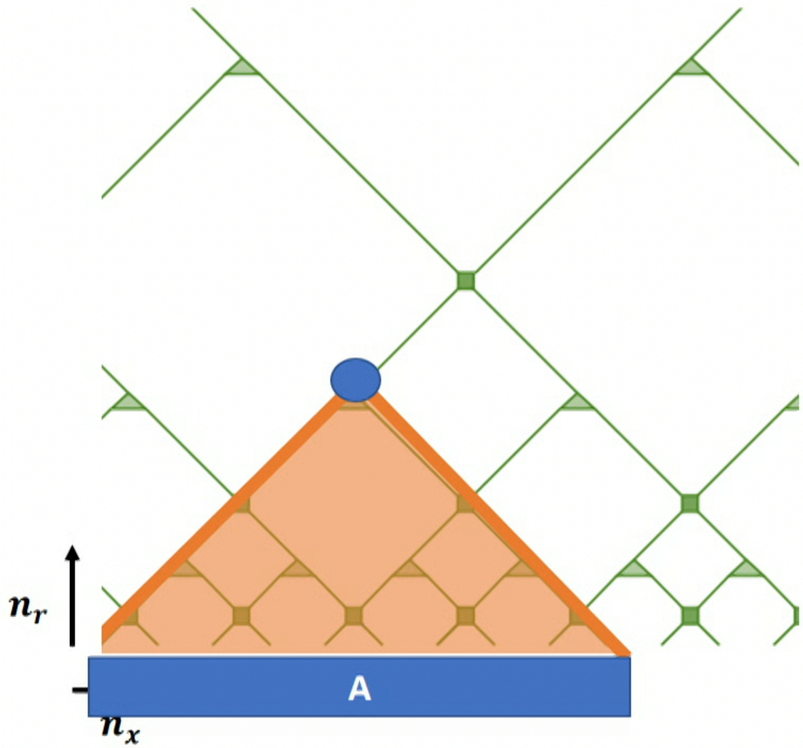
[Czech, Lamprou, McCandlish, JS; Beny]



What is this de Sitter space we have associated to MERA?

Kinematic Space

[Czech, Lamprou, McCandlish, JS; Beny]

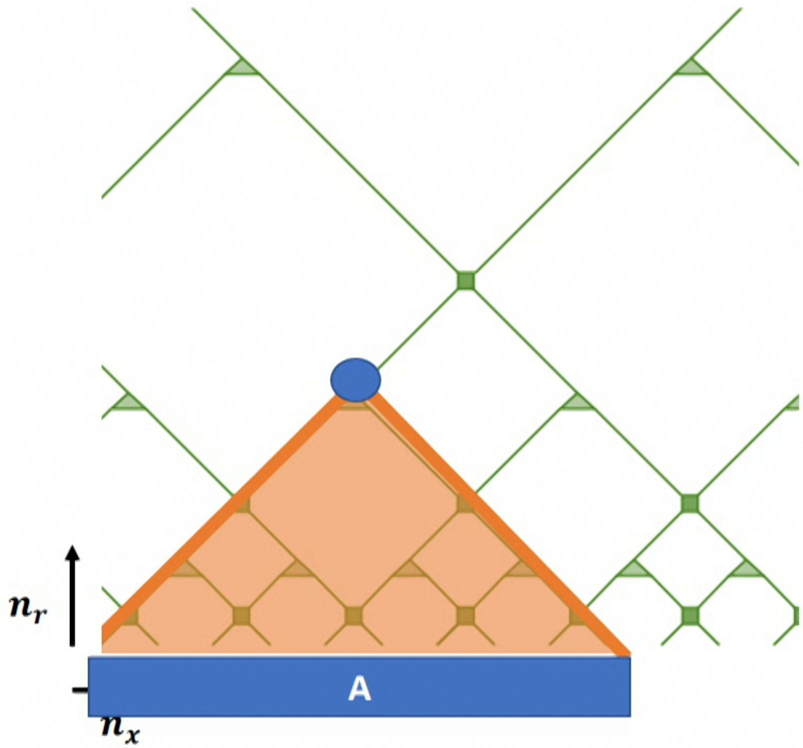


What is this de Sitter space we have associated to MERA?

Natural to associate **point** with **interval** on boundary in its **causal past**.

Kinematic Space

[Czech, Lamprou, McCandlish, JS; Beny]



What is this de Sitter space we have associated to MERA?

Natural to associate **point** with **interval** on boundary in its **causal past**.

de Sitter is the space of intervals of the CFT:

Kinematic Space

MERA: dS vs AdS

Is there a correct answer for the right gravitational analog of MERA?

Probably not: real space and kinematic space are really **dual** to each other

MERA: dS vs AdS

Is there a correct answer for the right gravitational analog of MERA?

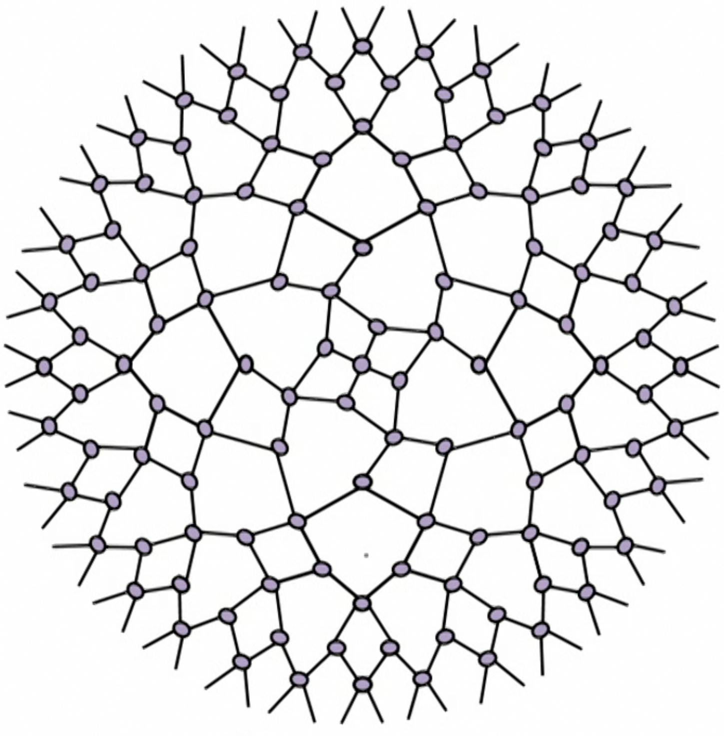
Probably not: real space and kinematic space are really **dual** to each other

1) **Forget that MERA us composed of unitaries with a direction and consider it as an undirected graph. This looks like hyperbolic space.**

MERA: dS vs AdS

Is there a correct answer for the right gravitational analog of MERA?

Probably not: real space and kinematic space are really **dual** to each other

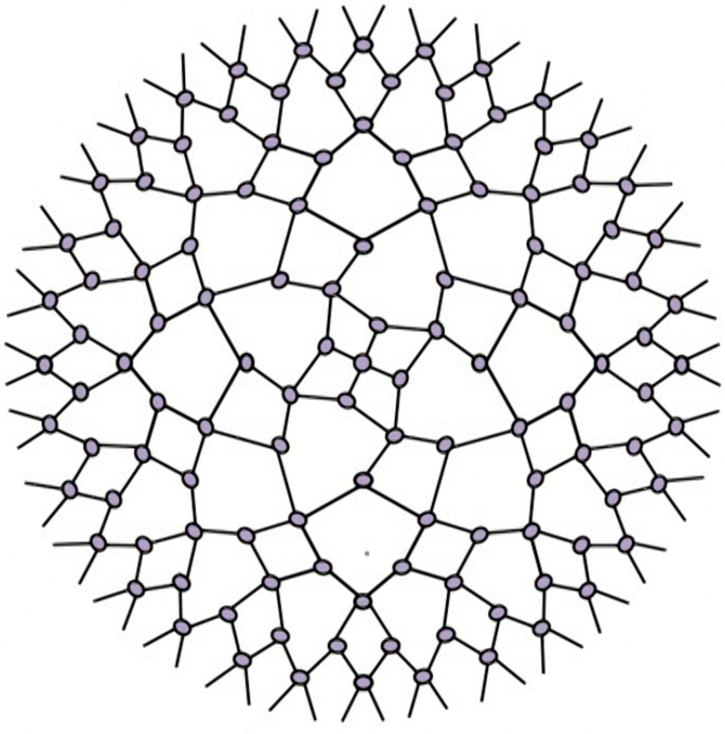


1) **Forget that MERA us composed of unitaries with a direction and consider it as an undirected graph. This looks like hyperbolic space.**

MERA: dS vs AdS

Is there a correct answer for the right gravitational analog of MERA?

Probably not: real space and kinematic space are really **dual** to each other



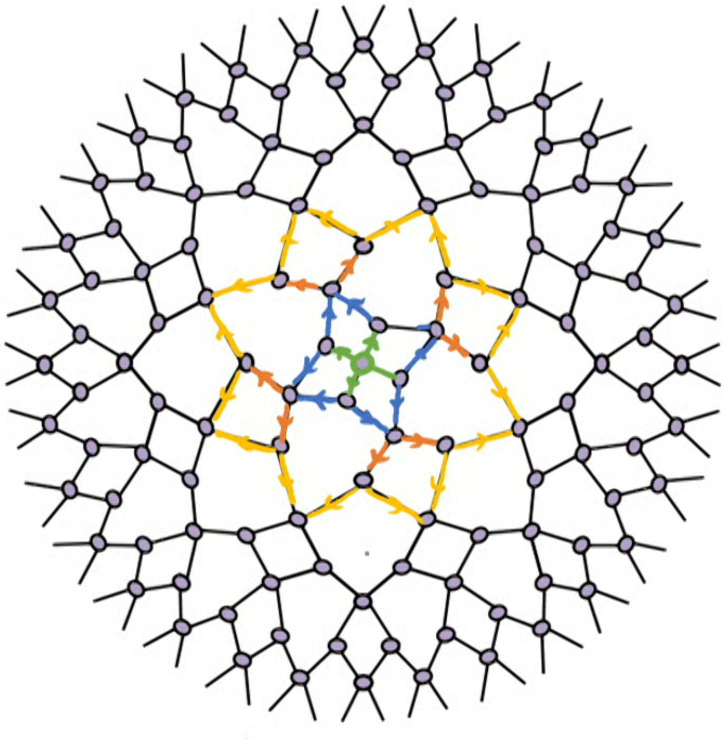
1) **Forget that MERA us composed of unitaries with a direction and consider it as an undirected graph. This looks like hyperbolic space.**

2) **From the center, add arrows to indicate the direction/action of the isometries and unitaries. (Now we have MERA with a causal structure.) This looks like de Sitter.**

MERA: dS vs AdS

Is there a correct answer for the right gravitational analog of MERA?

Probably not: real space and kinematic space are really **dual** to each other



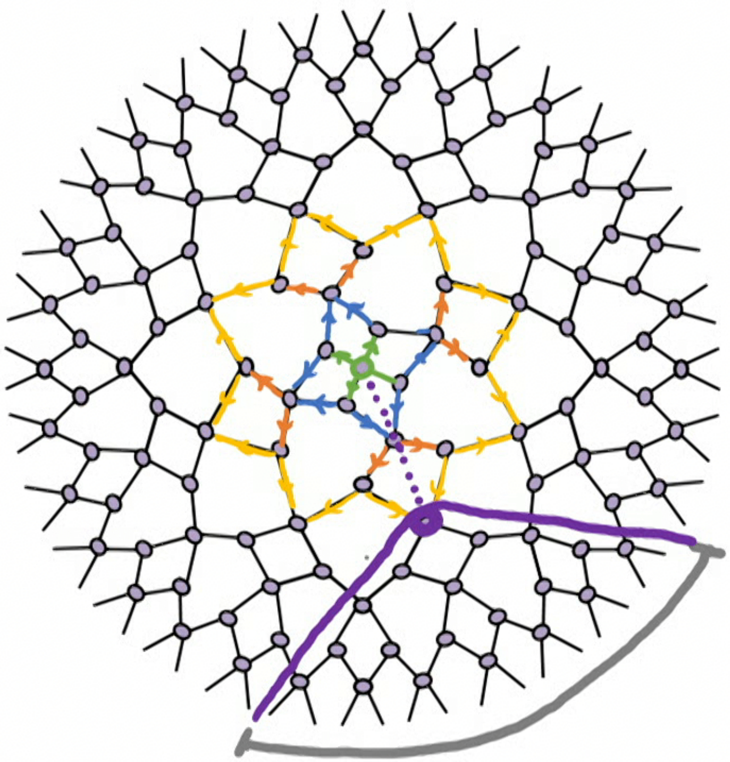
1) **Forget that MERA us composed of unitaries with a direction and consider it as an undirected graph. This looks like hyperbolic space.**

2) **From the center, add arrows to indicate the direction/action of the isometries and unitaries. (Now we have MERA with a causal structure.) This looks like de Sitter.**

MERA: dS vs AdS

Is there a correct answer for the right gravitational analog of MERA?

Probably not: real space and kinematic space are really **dual** to each other



1) **Forget that MERA us composed of unitaries with a direction and consider it as an undirected graph. This looks like hyperbolic space.**

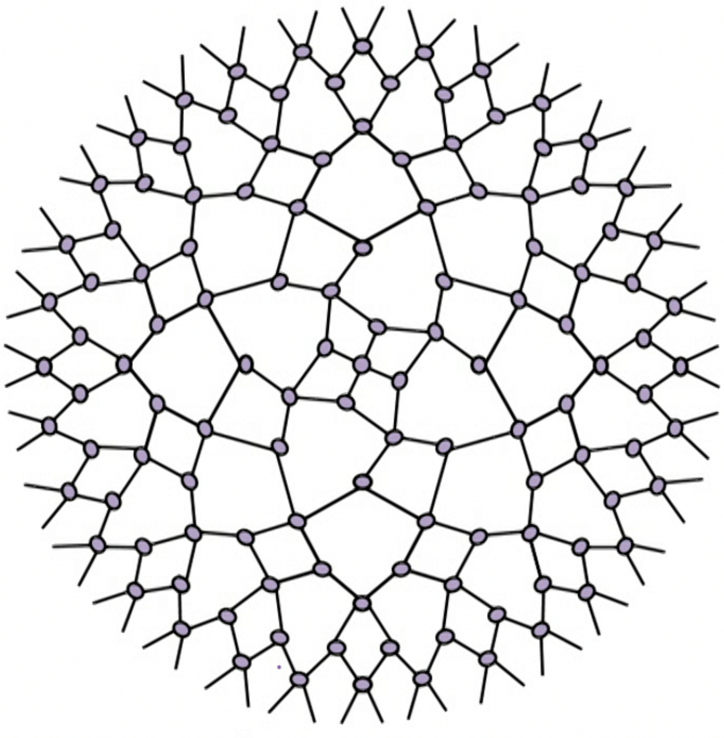
2) **From the center, add arrows to indicate the direction/action of the isometries and unitaries. (Now we have MERA with a causal structure.) This looks like de Sitter.**

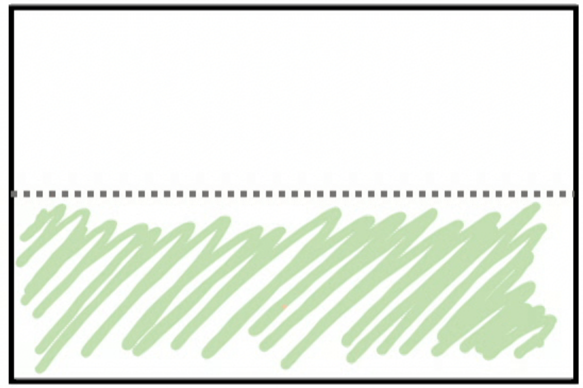
3) **This duality assigns an interval on the boundary to the point on the RT surface closest to the center.**

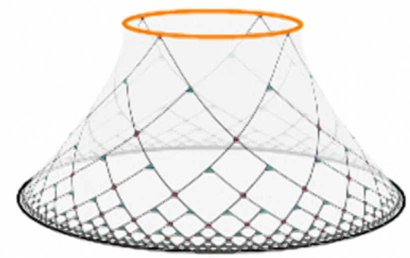
MERA: dS vs AdS

Is there a correct answer for the right gravitational analog of MERA?

Probably not: real space and kinematic space are really **dual** to each other



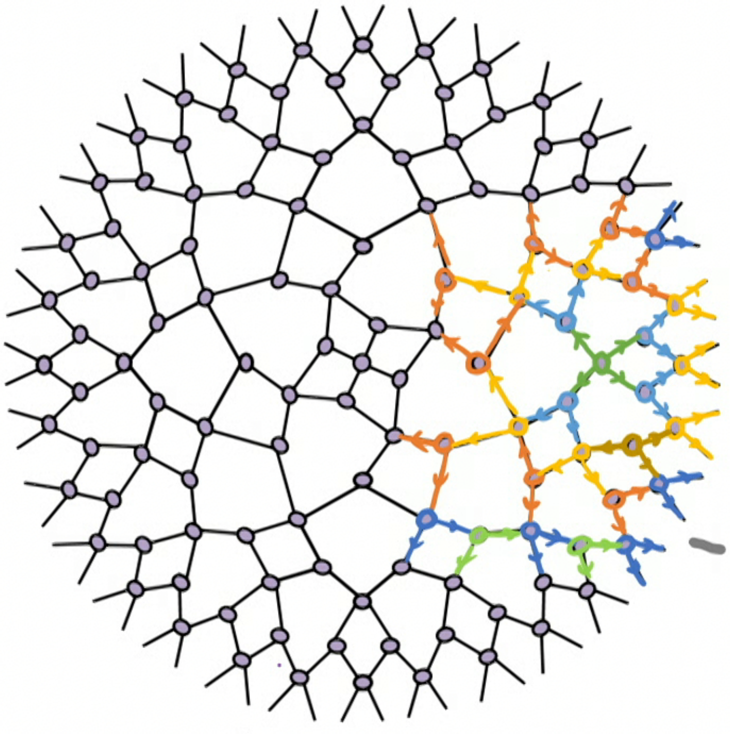


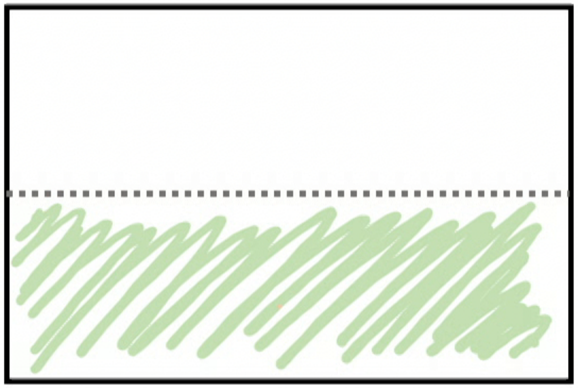


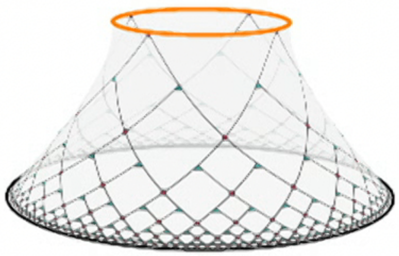
MERA: dS vs AdS

Is there a correct answer for the right gravitational analog of MERA?

Probably not: real space and kinematic space are really **dual** to each other



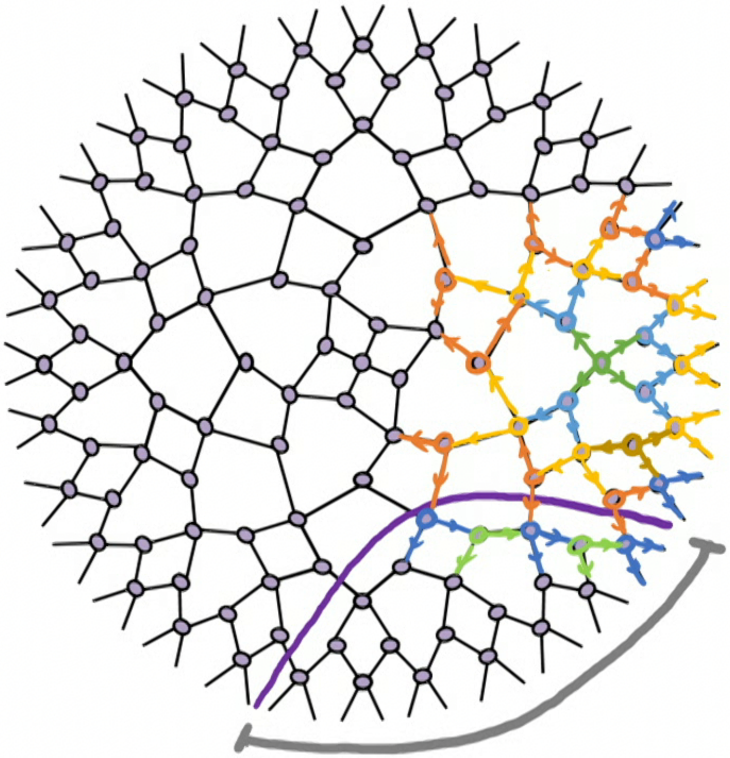


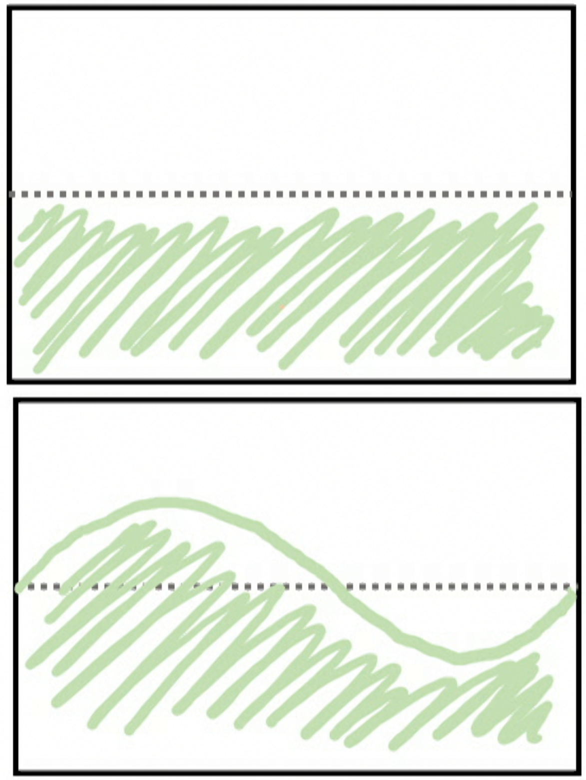


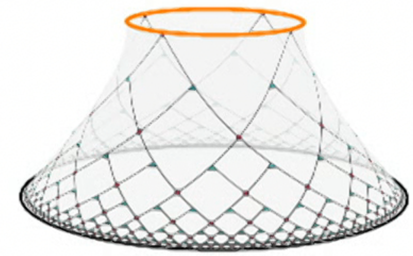
MERA: dS vs AdS

Is there a correct answer for the right gravitational analog of MERA?

Probably not: real space and kinematic space are really **dual** to each other



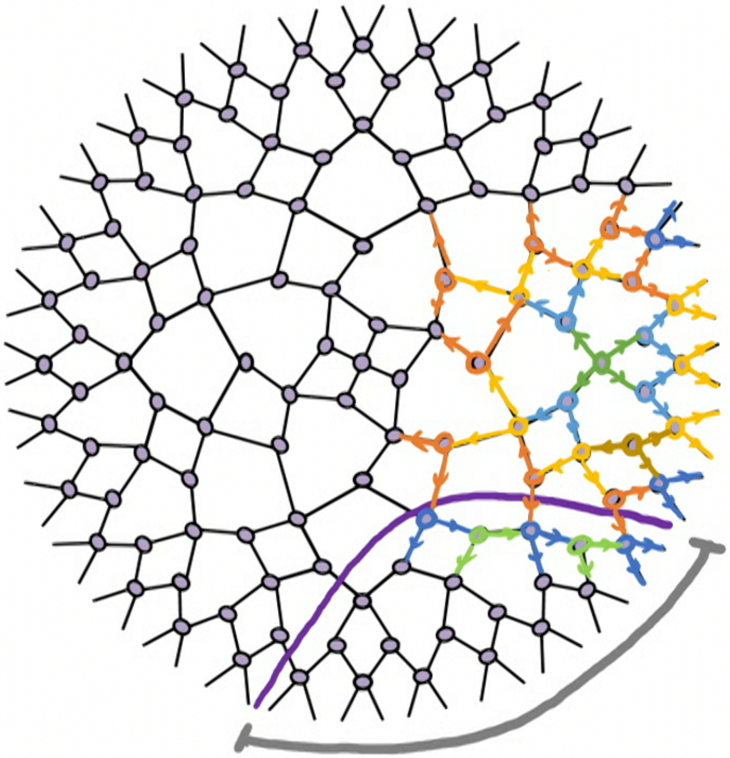


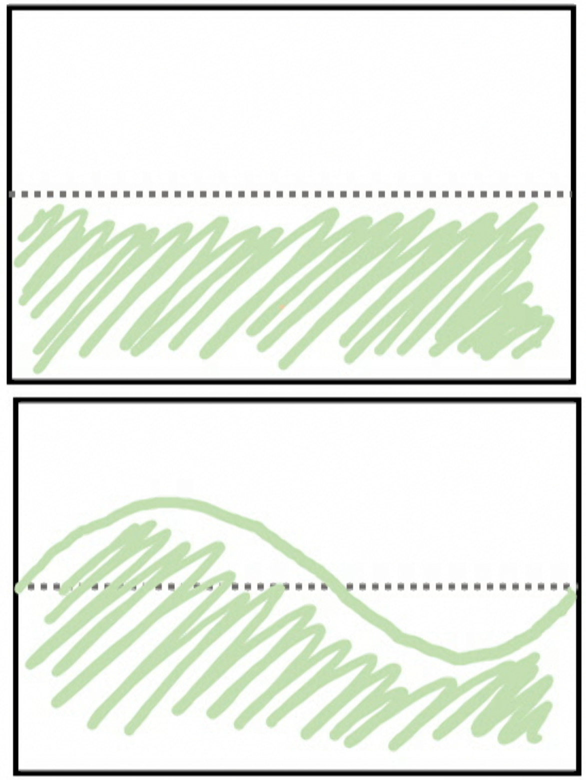


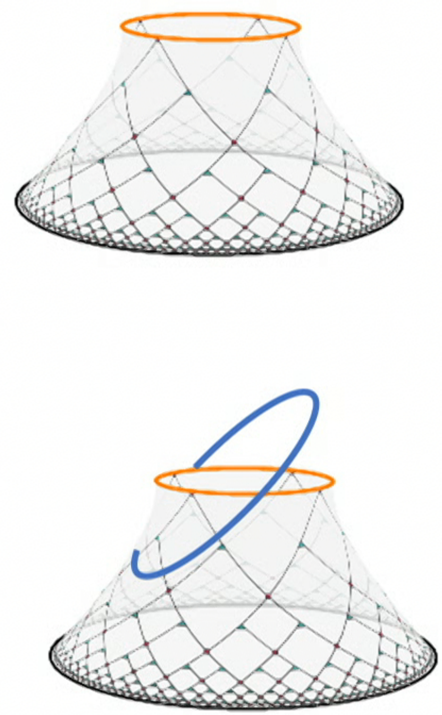
MERA: dS vs AdS

Is there a correct answer for the right gravitational analog of MERA?

Probably not: real space and kinematic space are really **dual** to each other





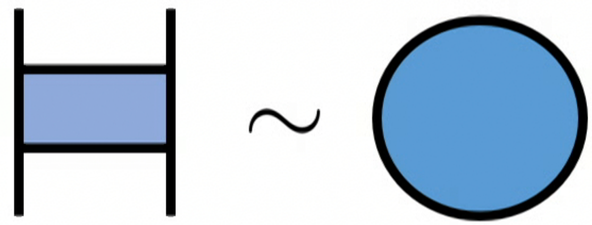


MERA and Holography

Comments

- Every tensor of MERA gives a large region of AdS size

$$\delta S = O(c)$$



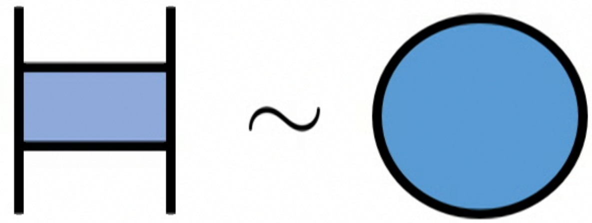
$$\sim L_{AdS}$$

MERA and Holography

Comments

- Every tensor of MERA gives a large region of AdS size

$\delta S = O(c)$



$\sim L_{AdS}$

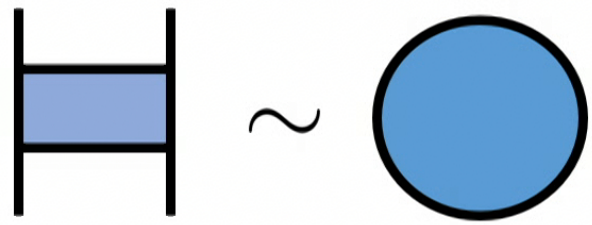
- Can we probe sub-AdS physics?

MERA and Holography

Comments

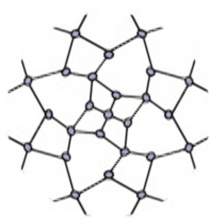
- Every tensor of MERA gives a large region of AdS size

$$\delta S = O(c)$$



$$\sim L_{AdS}$$

- Can we probe sub-AdS physics?

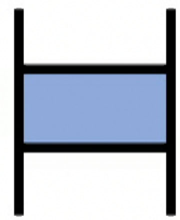


MERA and Holography

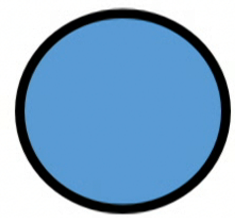
Comments

- Every tensor of MERA gives a large region of AdS size

$$\delta S = O(c)$$

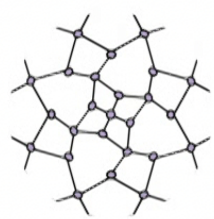


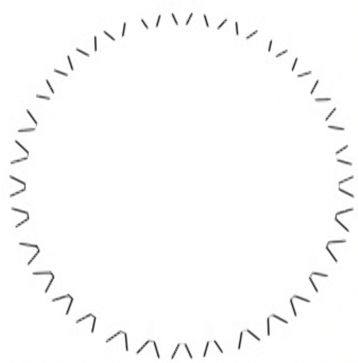
$$\sim$$



$$\sim L_{AdS}$$

- Can we probe sub-AdS physics?



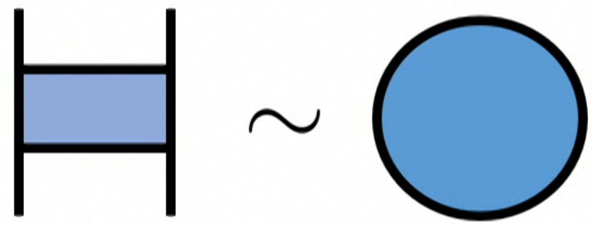


MERA and Holography

Comments

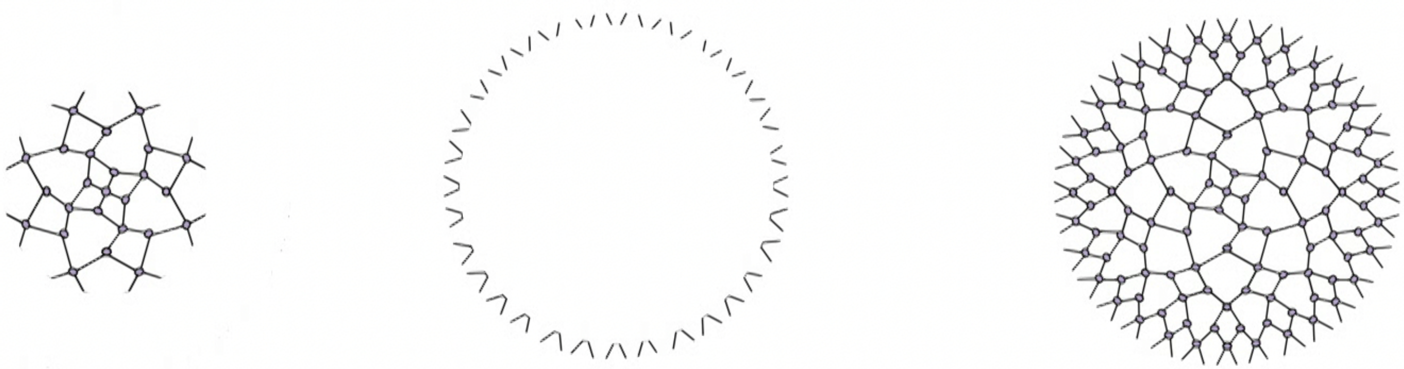
- Every tensor of MERA gives a large region of AdS size

$\delta S = O(c)$

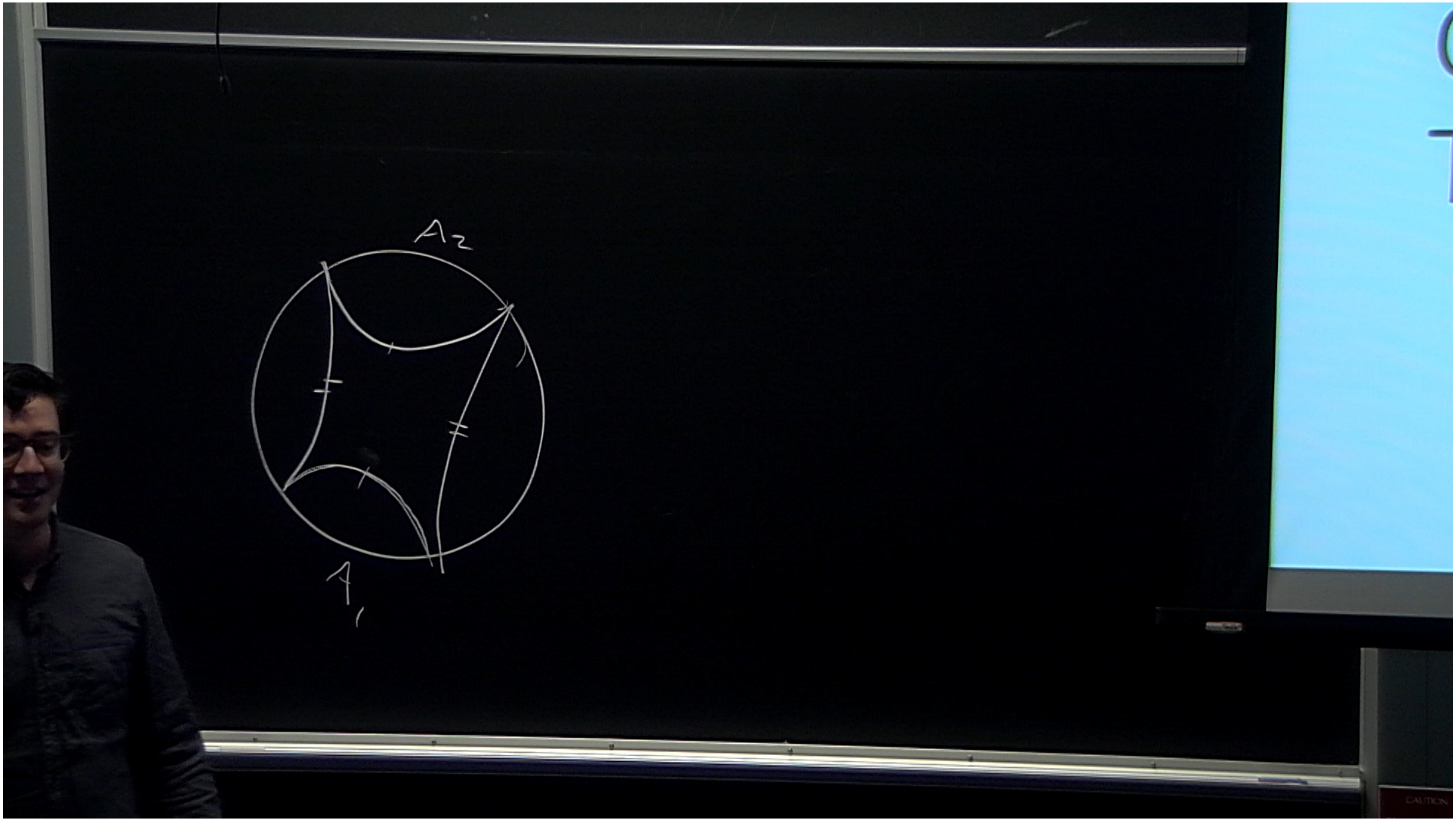


$\sim L_{AdS}$

- Can we probe sub-AdS physics?



Classical Geometry and Random Tensor Networks



A_2
A_1

Beyond the Vacuum

AdS/MERA gives a beautiful description of the ground state. How do we extend this?

Want to understand both directions:

1. **Given a state $|\psi\rangle$ far from the vacuum, how do I find an appropriate tensor network description?**
   - Tensor networks are **enormously redundant**. There may be one that looks like dual bulk geometry, but what about a generic one…

Beyond the Vacuum

AdS/MERA gives a beautiful description of the ground state. How do we extend this?

Want to understand both directions:

1. **Given a state $|\psi\rangle$ far from the vacuum, how do I find an appropriate tensor network description?**
   - Tensor networks are **enormously redundant**. There may be one that looks like dual bulk geometry, but what about a generic one…
   - What **constraints** do I need so that this tensor network represents the dual geometry?
2. **Given a bulk geometry, how can I convert it to a tensor network to write down a CFT state consistent with that geometry (ie the same EE as given by RT)?**

Beyond the Vacuum

AdS/MERA gives a beautiful description of the ground state. How do we extend this?

Want to understand both directions:

1. **Given a state $|\psi\rangle$ far from the vacuum, how do I find an appropriate tensor network description?**
   - Tensor networks are **enormously redundant**. There may be one that looks like dual bulk geometry, but what about a generic one…
   - What **constraints** do I need so that this tensor network represents the dual geometry?
2. **Given a bulk geometry, how can I convert it to a tensor network to write down a CFT state consistent with that geometry (ie the same EE as given by RT)?**
   - There are many CFT states with same classical geometry, so not unique

Geometry → TN → State

[Czech, Hayden, Lashkari, Swingle]

Let's begin with second question:

Finding geometric TN would likely be hard (impossible?) for a *generic* CFT.
But CFTs conjectured to have **good bulk** descriptions are **special**:

Geometry → TN → State

[Czech, Hayden, Lashkari, Swingle]

Let's begin with second question:

Finding geometric TN would likely be hard (impossible?) for a *generic* CFT. But CFTs conjectured to have **good bulk** descriptions are **special**:

- **Large central charge**, *c*, and a **large gap**.

One entropic consequence:

- Consider the **smooth min- and max-entropies**

Geometry → TN → State

[Czech, Hayden, Lashkari, Swingle]

Let's begin with second question:

Finding geometric TN would likely be hard (impossible?) for a *generic* CFT. But CFTs conjectured to have **good bulk** descriptions are **special**:

- **Large central charge**, *c*, and a **large gap**.

One entropic consequence:

- Consider the **smooth min- and max-entropies**

$$H^{\epsilon}_{\max}(\rho) = \min_{\|\sigma-\rho\|<\epsilon} \log(\mathrm{rank}(\sigma))$$

Geometry → TN → State

[Czech, Hayden, Lashkari, Swingle]

Let's begin with second question:

Finding geometric TN would likely be hard (impossible?) for a *generic* CFT. But CFTs conjectured to have **good bulk** descriptions are **special**:

- **Large central charge**, c, and a **large gap**.

One entropic consequence:

- Consider the **smooth min- and max-entropies**

$$H^{\epsilon}_{\max}(\rho) = \min_{\|\sigma - \rho\| < \epsilon} \log(\text{rank}(\sigma))$$

$$\begin{pmatrix} \rho_1 & & & & \\ & \ddots & & & \\ & & \rho_k & & \\ & & & \ddots & \\ & & & & \rho_M \end{pmatrix} \Rightarrow \begin{pmatrix} 1/k & & & & & \\ & \ddots & & & & \\ & & 1/k & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}$$

Geometry → TN → State

[Czech, Hayden, Lashkari, Swingle]

Let's begin with second question:

Finding geometric TN would likely be hard (impossible?) for a *generic* CFT. But CFTs conjectured to have **good bulk** descriptions are **special**:

- **Large central charge**, *c*, and a **large gap**.

One entropic consequence:

- Consider the **smooth min- and max-entropies**

$$H^{\epsilon}_{\max}(\rho) = \min_{\|\sigma-\rho\|<\epsilon} \log(\mathrm{rank}(\sigma)) \qquad H^{\epsilon}_{\min}(\rho) = \max_{\|\sigma-\rho\|<\epsilon} -\log(\rho_{\max})$$

$$\begin{pmatrix} \rho_1 & & & & \\ & \ddots & & & \\ & & \rho_k & & \\ & & & \ddots & \\ & & & & \rho_M \end{pmatrix} \Rightarrow \begin{pmatrix} 1/k & & & & \\ & \ddots & & & \\ & & 1/k & & \\ & & & 0 & \\ & & & & \ddots \\ & & & & & 0 \end{pmatrix} \qquad \begin{pmatrix} \rho_1 & & & & \\ & \ddots & & & \\ & & \rho_k & & \\ & & & \ddots & \\ & & & & \rho_M \end{pmatrix} \Rightarrow \begin{pmatrix} 1/\rho_{\max} & & & \\ & \ddots & & \\ & & 1/\rho_{\max} & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix}$$

Geometry → TN → State

[Czech, Hayden, Lashkari, Swingle]

Let's begin with second question:

Finding geometric TN would likely be hard (impossible?) for a *generic* CFT. But CFTs conjectured to have **good bulk** descriptions are **special**:

- **Large central charge**, *c*, and a **large gap**.

One entropic consequence:

- Consider the **smooth min- and max-entropies**

$$H^{\epsilon}_{\max}(\rho) = \min_{\|\sigma-\rho\|<\epsilon} \log(\mathrm{rank}(\sigma)) \qquad H^{\epsilon}_{\min}(\rho) = \max_{\|\sigma-\rho\|<\epsilon} -\log(\rho_{\max})$$

$$\begin{pmatrix} \rho_1 & & & & \\ & \ddots & & & \\ & & \rho_k & & \\ & & & \ddots & \\ & & & & \rho_M \end{pmatrix} \Rightarrow \begin{pmatrix} 1/k & & & & \\ & \ddots & & & \\ & & 1/k & & \\ & & & 0 & \\ & & & & \ddots \\ & & & & & 0 \end{pmatrix} \qquad \begin{pmatrix} \rho_1 & & & & \\ & \ddots & & & \\ & & \rho_k & & \\ & & & \ddots & \\ & & & & \rho_M \end{pmatrix} \Rightarrow \begin{pmatrix} 1/\rho_{\max} & & & \\ & \ddots & & \\ & & 1/\rho_{\max} & \\ & & & \ddots \\ & & & & 0 \end{pmatrix}$$

- **One finds:** $H^{\epsilon}_{\min\,/\,\max}(\rho) = S_{vN}(\rho) \pm O(1/c)$ $\Rightarrow$ **nearly flat spectrum**

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

Geometry → TN

The lesson we should draw:

To good approximation (leading order in 1/c), we can consider all density matrices to be **maximally mixed.**

Geometry → TN → State

[Czech, Hayden, Lashkari, Swingle]

Let's begin with second question:

Finding geometric TN would likely be hard (impossible?) for a *generic* CFT. But CFTs conjectured to have **good bulk** descriptions are **special**:

- **Large central charge**, *c*, and a **large gap**.

One entropic consequence:

- Consider the **smooth min- and max-entropies**

$$H^{\epsilon}_{\max}(\rho) = \min_{\|\sigma-\rho\|<\epsilon} \log(\mathrm{rank}(\sigma)) \qquad H^{\epsilon}_{\min}(\rho) = \max_{\|\sigma-\rho\|<\epsilon} -\log(\rho_{\max})$$

$$\begin{pmatrix} \rho_1 & & & & \\ & \ddots & & & \\ & & \rho_k & & \\ & & & \ddots & \\ & & & & \rho_M \end{pmatrix} \Rightarrow \begin{pmatrix} 1/k & & & & \\ & \ddots & & & \\ & & 1/k & & \\ & & & 0 & \\ & & & & \ddots \\ & & & & & 0 \end{pmatrix} \qquad \begin{pmatrix} \rho_1 & & & & \\ & \ddots & & & \\ & & \rho_k & & \\ & & & \ddots & \\ & & & & \rho_M \end{pmatrix} \Rightarrow \begin{pmatrix} 1/\rho_{\max} & & & \\ & \ddots & & \\ & & 1/\rho_{\max} & \\ & & & \ddots \\ & & & & 0 \end{pmatrix}$$

- **One finds:** $H^{\epsilon}_{\min\,/\,\max}(\rho) = S_{vN}(\rho) \pm O(1/c)$ $\Rightarrow$ **nearly flat spectrum**

Geometry → TN

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

The lesson we should draw:

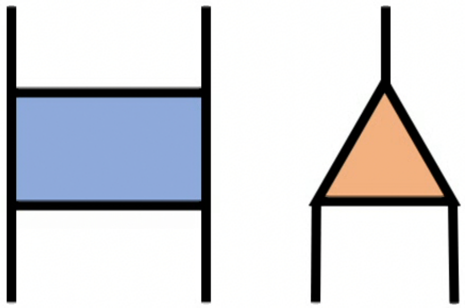
To good approximation (leading order in $1/c$), we can consider all density matrices to be **maximally mixed.**

Geometry → TN

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

The lesson we should draw:
To good approximation (leading order in 1/c), we can consider all density matrices to be **maximally mixed.**

Useful toy model:



MERA Tensors
Detailed spectrum,
OPE Coefficients

Geometry → TN

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

The lesson we should draw:
To good approximation (leading order in $1/c$), we can consider all density matrices to be **maximally mixed.**

Useful toy model:

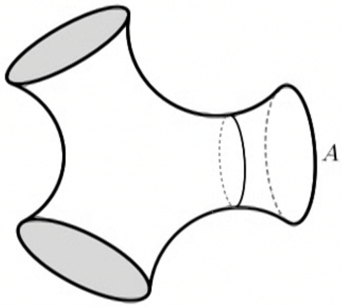
MERA Tensors
Detailed spectrum, OPE Coefficients

Random Tensor
Flat spectrum (large bond dimension)

Geometry → TN → State

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

Consider an arbitrary geometry:



Geometry → TN → State

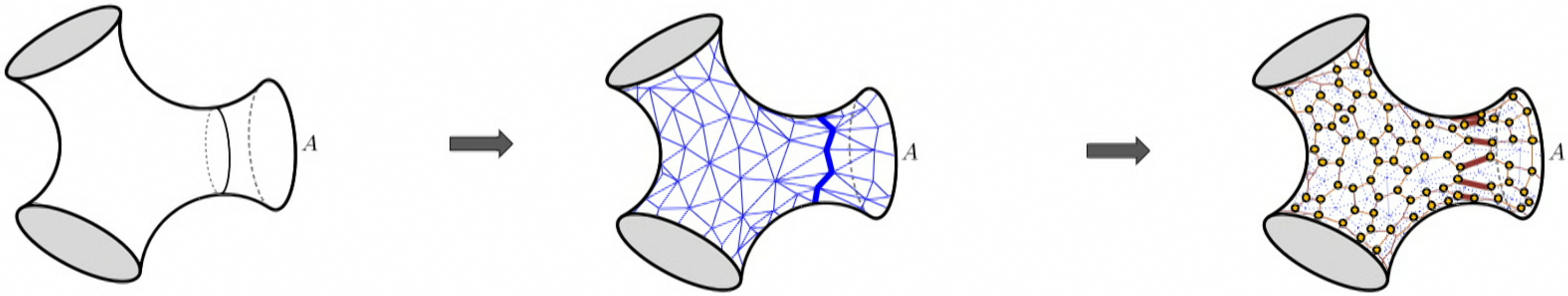
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

Consider an arbitrary geometry:

Geometry → TN → State

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

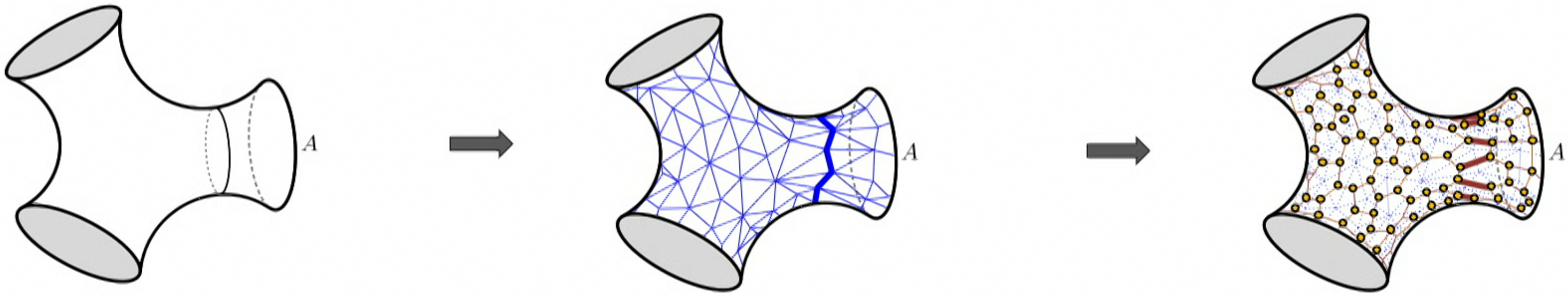
Consider an arbitrary geometry:



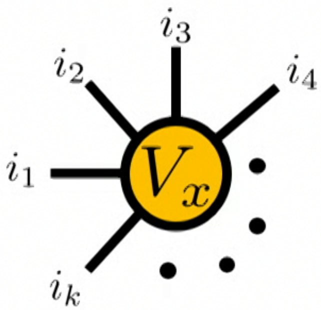
Geometry → TN → State

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

Consider an arbitrary geometry:



- **Number of edges** along cut is (approximately) the **same as bulk area**



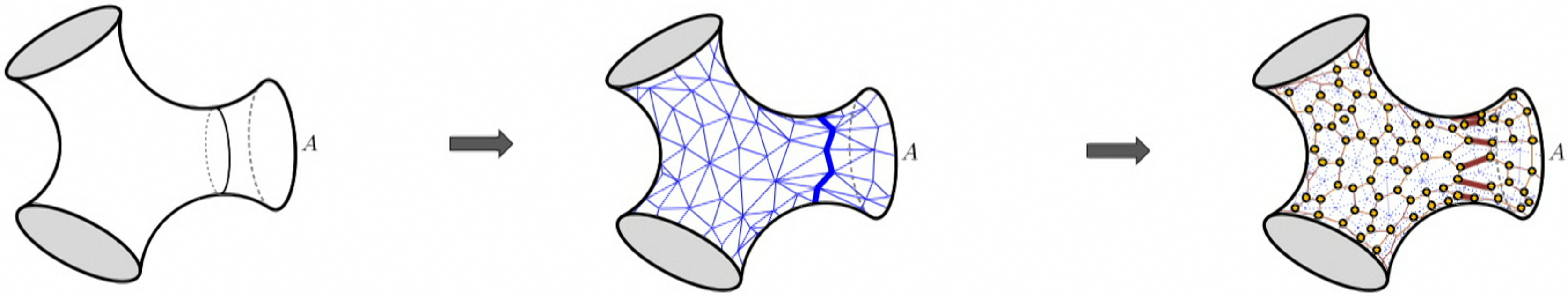
$$|V_x\rangle \in H_D^{\otimes k}$$

Random (e.g. Haar)

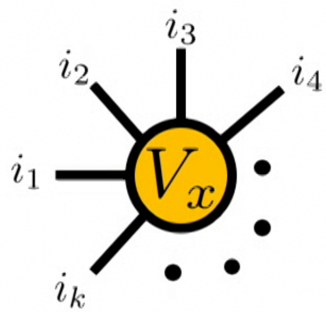
Geometry → TN → State

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

Consider an arbitrary geometry:



- **Number of edges** along cut is (approximately) the **same as bulk area**



$$|V_x\rangle \in H_D^{\otimes k}$$

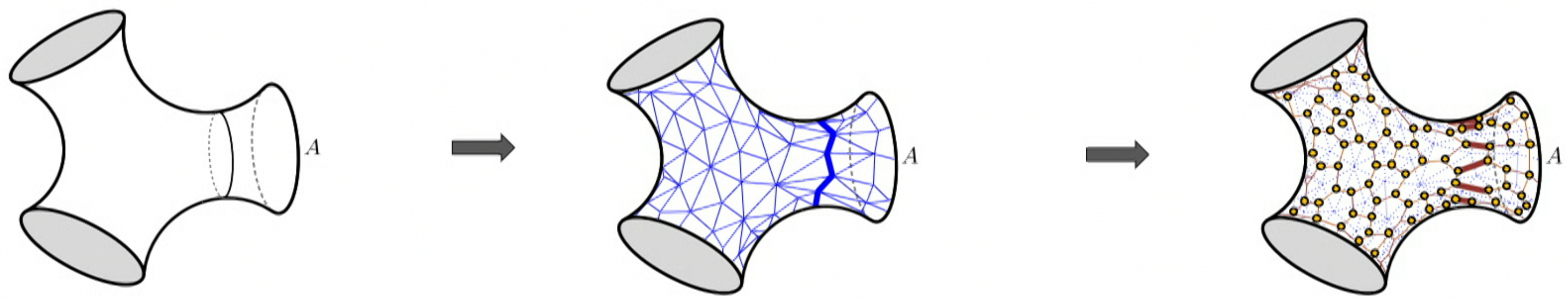
Random (e.g. Haar)

$$|xy\rangle = \sum_{i=1}^{D} |i\rangle|i\rangle$$

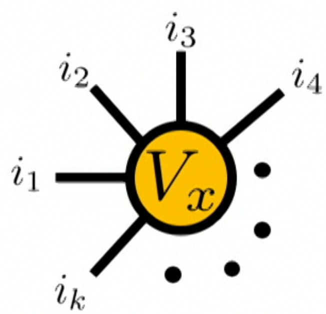
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

Geometry → TN → State

Consider an arbitrary geometry:



- **Number of edges** along cut is (approximately) the **same as bulk area**



$$|V_x\rangle \in H_D^{\otimes k}$$

Random (e.g. Haar)

$$|xy\rangle = \sum_{i=1}^{D} |i\rangle|i\rangle$$

$$|\psi\rangle = \left(\bigotimes_{\langle xy\rangle \in E} \langle xy|\right)\left(\bigotimes_{x\in V} |V_x\rangle\right)$$

Geometry → TN

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

What is the entanglement entropy?

Bounded from above by the smallest cut through the tensor network.

Bounded from below by the second Renyi entropy:

$$S_2(A) = -\log \mathrm{tr}\rho_A^2$$

One can show:

$$\mathbb{E}\left[\mathrm{tr}\rho_A^2\right] \propto \sum_{\Gamma_A} e^{-\log(D)|\partial\Gamma_A|}$$

Geometry → TN

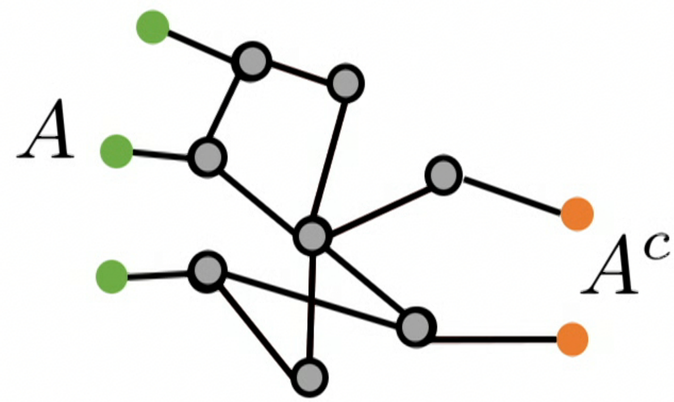
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

What is the entanglement entropy?

Bounded from above by the smallest cut through the tensor network.

Bounded from below by the second Renyi entropy:

$$S_2(A) = -\log \mathrm{tr}\rho_A^2$$



One can show:

$$\mathbb{E}\left[\mathrm{tr}\rho_A^2\right] \propto \sum_{\Gamma_A} e^{-\log(D)|\partial\Gamma_A|}$$

Geometry → TN

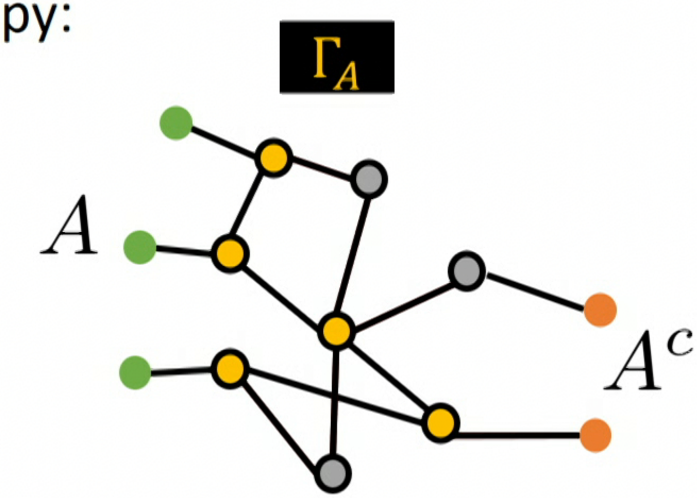
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

What is the entanglement entropy?

Bounded from above by the smallest cut through the tensor network.

Bounded from below by the second Renyi entropy:

$$S_2(A) = -\log \mathrm{tr}\rho_A^2$$



One can show:

$$\mathbb{E}\left[\mathrm{tr}\rho_A^2\right] \propto \sum_{\Gamma_A} e^{-\log(D)|\partial\Gamma_A|}$$

Geometry → TN

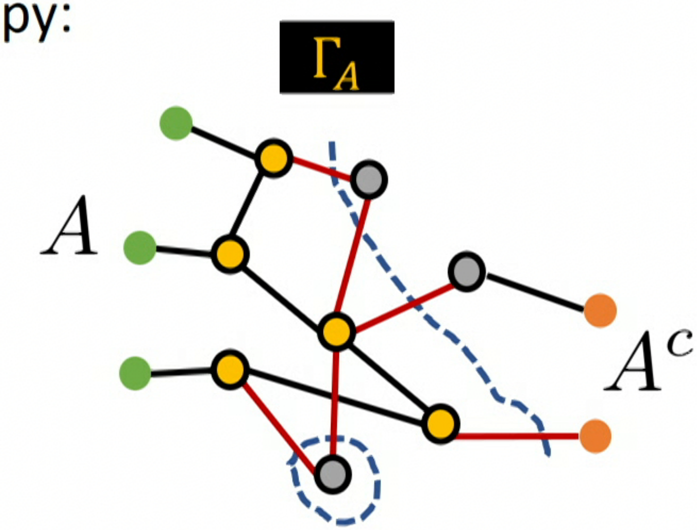
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

What is the entanglement entropy?

Bounded from above by the smallest cut through the tensor network.

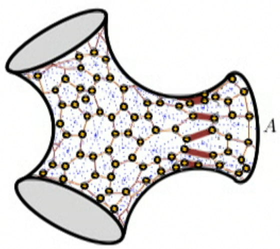
Bounded from below by the second Renyi entropy:

$$S_2(A) = -\log \mathrm{tr}\rho_A^2$$



One can show:

$$\mathbb{E}\left[\mathrm{tr}\rho_A^2\right] \propto \sum_{\Gamma_A} e^{-\log(D)|\partial\Gamma_A|}$$



Geometry → TN

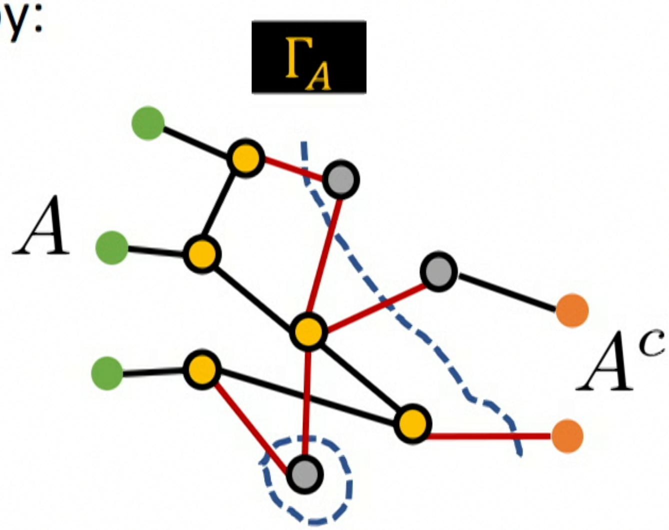
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

What is the entanglement entropy?

Bounded from above by the smallest cut through the tensor network.

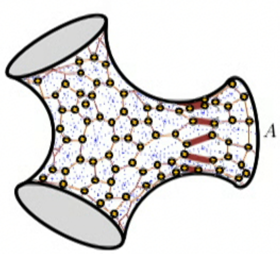
Bounded from below by the second Renyi entropy:

$$S_2(A) = -\log \mathrm{tr}\rho_A^2$$



One can show:

$$\mathbb{E}\left[\mathrm{tr}\rho_A^2\right] \propto \sum_{\Gamma_A} e^{-\log(D)|\partial\Gamma_A|}$$



$$S_2(A) = \min_{\Gamma_A} -\log(D)|\partial\Gamma_A| + \ldots$$

Geometry → TN → State

[Pastawski, Yoshida, Harlow, Preskill]

Comments

- **Too flat spectrum** in this toy model
  - Match the entanglement entropy of a CFT, but not the higher Renyi entropies

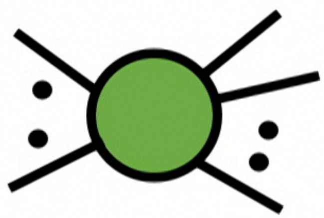
Geometry → TN → State

[Pastawski, Yoshida, Harlow, Preskill]

Comments

- **Too flat spectrum** in this toy model
  - Match the entanglement entropy of a CFT, but not the higher Renyi entropies

- Idealization of random tensors: **Perfect Tensors**

$k \leq n/2$  $n - k$

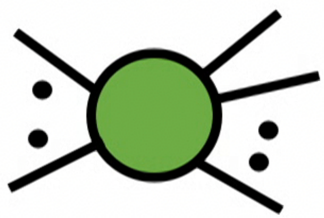
Geometry → TN → State

[Pastawski, Yoshida, Harlow, Preskill]

Comments

- **Too flat spectrum** in this toy model
  - Match the entanglement entropy of a CFT, but not the higher Renyi entropies

- Idealization of random tensors: **Perfect Tensors**

$k \leq n/2$  $n - k$

Isometry for any division (k, n-k)

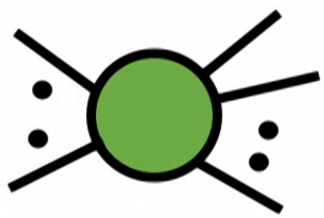
Geometry → TN → State

[Pastawski, Yoshida, Harlow, Preskill]

Comments

- **Too flat spectrum** in this toy model
  - Match the entanglement entropy of a CFT, but not the higher Renyi entropies

- Idealization of random tensors: **Perfect Tensors**

$k \leq n/2$  $n-k$

Isometry for any division (k, n-k)

  - Perfect tensor formulation actually predates work with random tensors

State → TN → Geometry

What about the other direction?

How do we take a **CFT state** and **find a tensor network** that will tell us about geometry?

- Not much progress in this direction
- This is an **even more interesting question**!

Could even ask within scope of 'maximally entangled' toy models:

- Is there an algorithm to find random tensor network description of state?
- Are there constraints needed?

Bulk Physics and Holographic Maps

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

Bulk States

So far we have been discussing correspondence between a tensor network and a bulk geometry.

But we would also like to describe the perturbative physics in this background.

Bulk States

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

So far we have been discussing correspondence between a tensor network and a bulk geometry.

But we would also like to describe the perturbative physics in this background.

- Want **not one state**, but a **Hilbert space of perturbative states** associated to each tensor network.

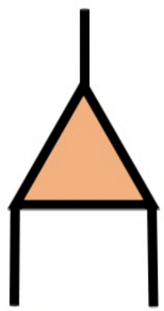
Bulk States

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

So far we have been discussing correspondence between a tensor network and a bulk geometry.

But we would also like to describe the perturbative physics in this background.

- Want **not** one state, but a Hilbert space **of perturbative states** associated to each tensor network.
  - **Already happens in MERA:**



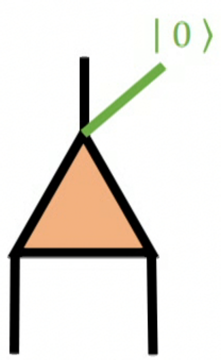
Bulk States

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

So far we have been discussing correspondence between a tensor network and a bulk geometry.

But we would also like to describe the perturbative physics in this background.

- Want **not one state**, but a **Hilbert space of perturbative states** associated to each tensor network.
  - **Already happens in MERA:**



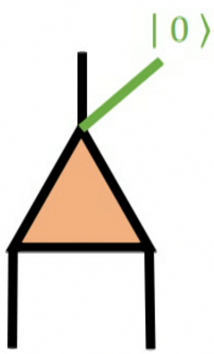
Bulk States

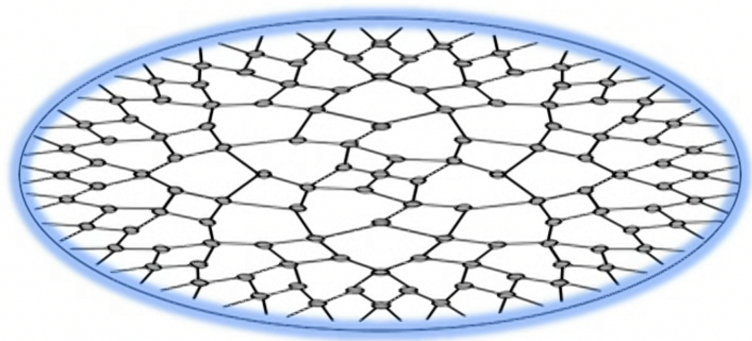
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

So far we have been discussing correspondence between a tensor network and a bulk geometry.

But we would also like to describe the perturbative physics in this background.

- Want **not one state**, but a **Hilbert space of perturbative states** associated to each tensor network.
  - **Already happens in MERA:**





[Hayden, Nezami, Qi, Thomas, Walter, Yang]

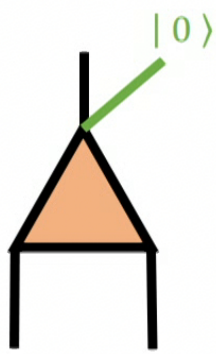
Bulk States

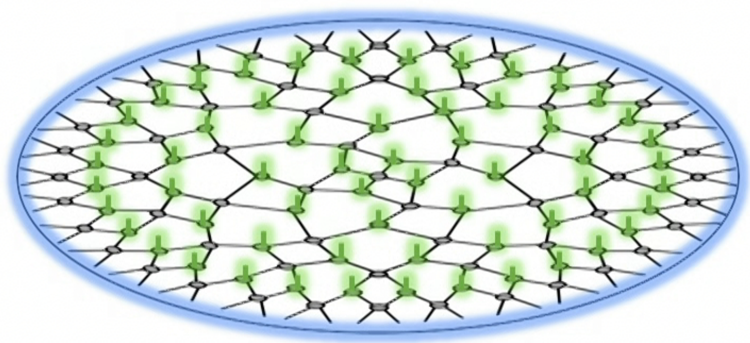
So far we have been discussing correspondence between a tensor network and a bulk geometry.

But we would also like to describe the perturbative physics in this background.

- Want **not** **one state**, but a **Hilbert space** **of perturbative states** associated to each tensor network.
  - **Already happens in MERA:**

$$\mathcal{H}_{\text{Bulk}} \times \mathcal{H}_{\text{Boundary}}$$





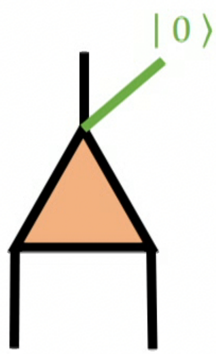
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

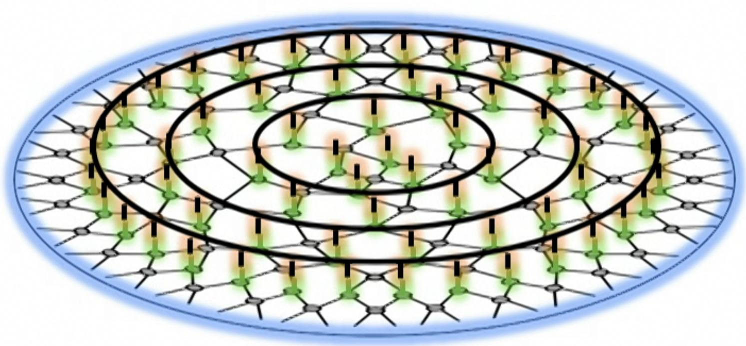
Bulk States

So far we have been discussing correspondence between a tensor network and a bulk geometry.

But we would also like to describe the perturbative physics in this background.

- Want **not** one state, but a Hilbert space **of perturbative states** associated to each tensor network.
  - **Already happens in MERA:**





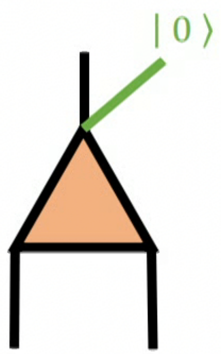
Bulk States

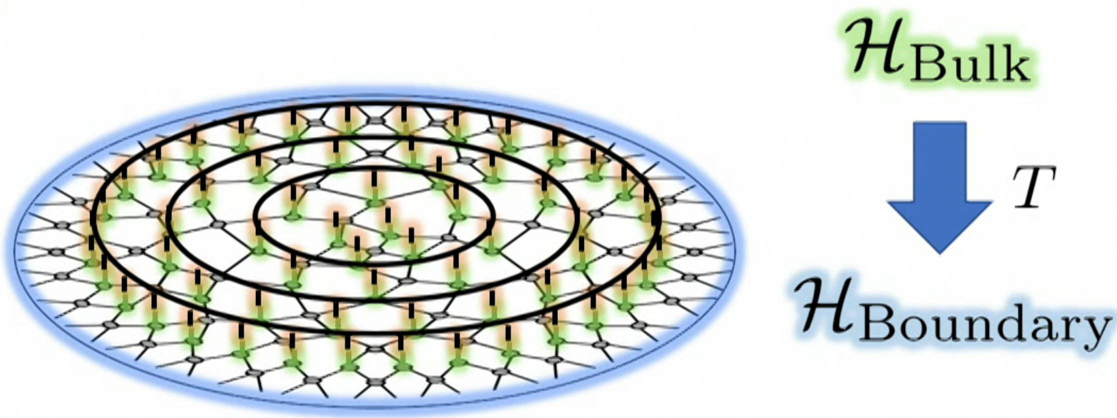
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

So far we have been discussing correspondence between a tensor network and a bulk geometry.

But we would also like to describe the perturbative physics in this background.

- Want **not one state**, but a **Hilbert space of perturbative states** associated to each tensor network.
  - **Already happens in MERA:**





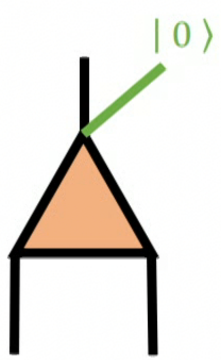
Bulk States

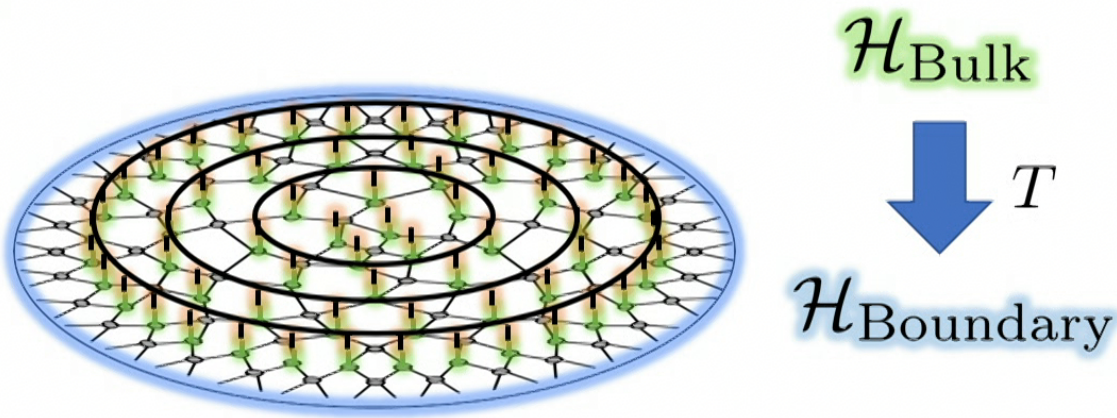
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

So far we have been discussing correspondence between a tensor network and a bulk geometry.

But we would also like to describe the perturbative physics in this background.

- Want **not one state**, but a **Hilbert space of perturbative states** associated to each tensor network.
  - **Already happens in MERA:**



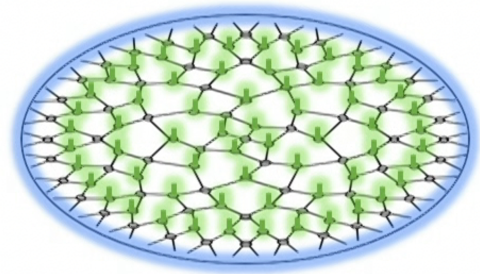


Holographic Map: **Bulk states ↔ Boundary states**

Bulk States

In the MERA example we have an **isomorphism**:

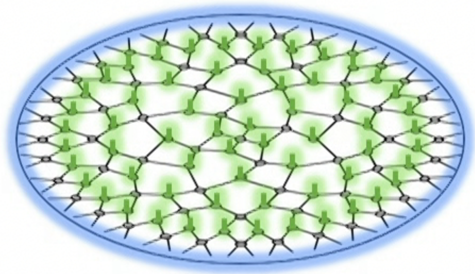
$$\mathcal{H}_{\text{Bulk}} \cong \mathcal{H}_{\text{Boundary}}$$



Bulk States

In the MERA example we have an **isomorphism**:

$$\mathcal{H}_{\mathrm{Bulk}} \cong \mathcal{H}_{\mathrm{Boundary}}$$



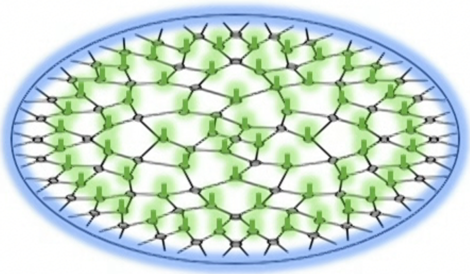
But this is **more** than we typically want:

- Typical bulk state will have **entanglement structure unrelated to the MERA network**.

Bulk States

In the MERA example we have an **isomorphism**:

$$\mathcal{H}_{\text{Bulk}} \cong \mathcal{H}_{\text{Boundary}}$$



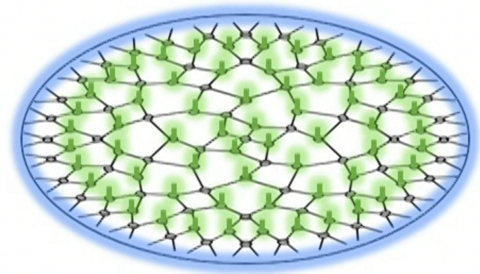
But this is **more** than we typically want:

- Typical bulk state will have **entanglement structure unrelated to the MERA network**.
- **Smaller subset of 'perturbative bulk states'** leave leading order entropies unchanged.

Bulk States

In the MERA example we have an **isomorphism**:

$$\mathcal{H}_{\text{Bulk}} \cong \mathcal{H}_{\text{Boundary}}$$



But this is **more** than we typically want:

- Typical bulk state will have **entanglement structure unrelated to the MERA network**.
- **Smaller subset of 'perturbative bulk states'** leave leading order entropies unchanged.

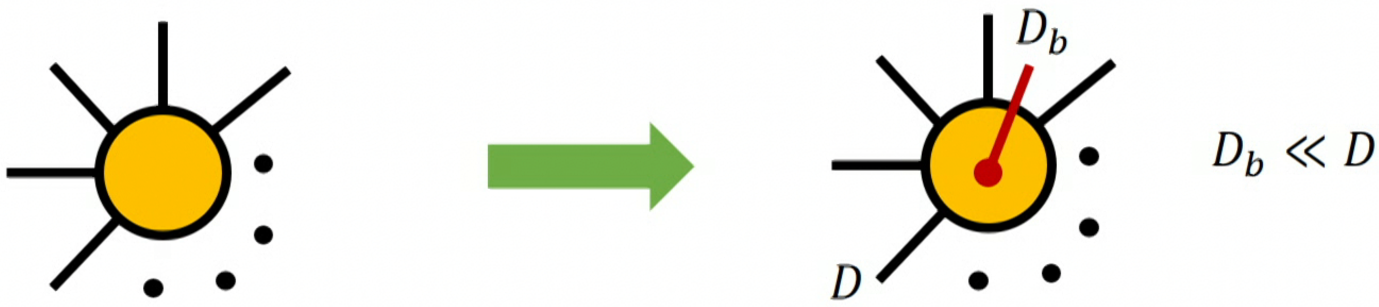
A better requirement is to have an **isometry**

$$\mathcal{H}_{\text{Bulk}} \hookrightarrow \mathcal{H}_{\text{Boundary}} \qquad \dim \mathcal{H}_{\text{Bulk}} \ll \dim \mathcal{H}_{\text{Boundary}}$$

Bulk States in Random TN

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

In the random TN model, we can also create bulk Hilbert space:



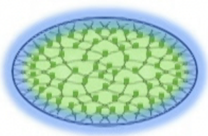
Bulk States in Random TN

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

In the random TN model, we can also create bulk Hilbert space:

This is sufficient to generate an isometry $\mathcal{H}_{\text{Bulk}} \hookrightarrow \mathcal{H}_{\text{Boundary}}$:

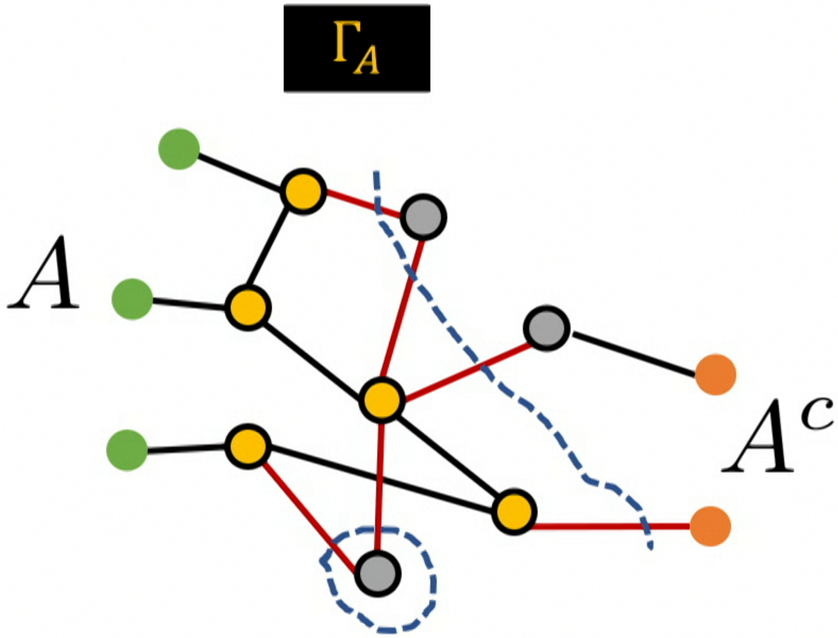
T



$$|\psi_{\text{B}}\rangle \to |\psi_{\partial}\rangle = T|\psi_{\text{B}}\rangle$$

Bulk States in Random TN

Moreover, **reproduce quantum corrections** to the RT formula:



$$S_2(A) = -\log(D)|\partial\Gamma_A| + S_2(E_A, \rho_B)$$

Bulk States in Random TN

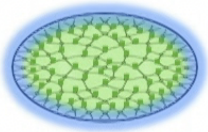
[Hayden, Nezami, Qi, Thomas, Walter, Yang]

In the random TN model, we can also create bulk Hilbert space:

$D_b \ll D$

This is sufficient to generate an isometry $\mathcal{H}_{\text{Bulk}} \hookrightarrow \mathcal{H}_{\text{Boundary}}$:

T

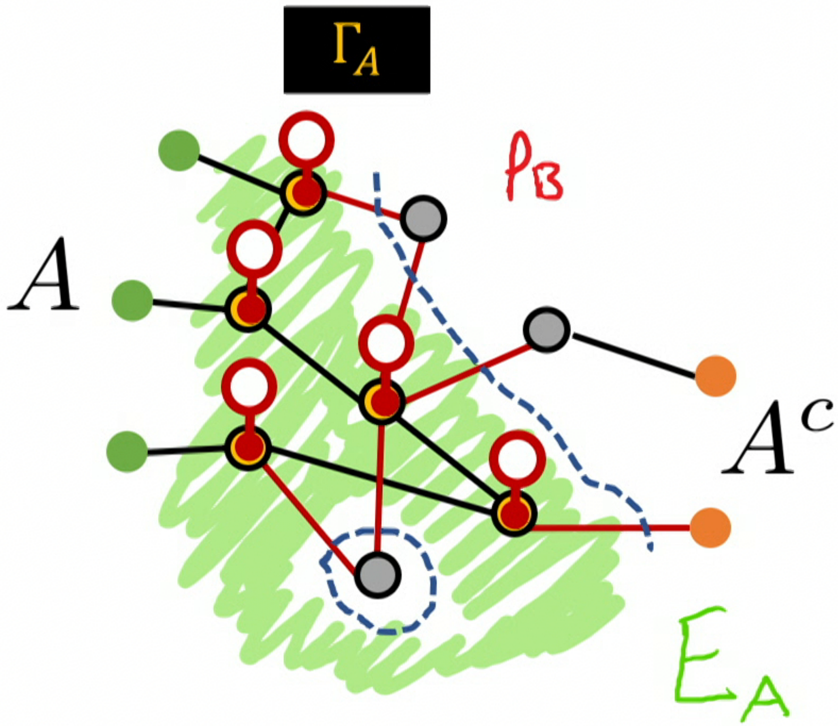


$$|\psi_{\text{B}}\rangle \to |\psi_\partial\rangle = T|\psi_{\text{B}}\rangle$$ **(Bulk EFT)**

$$\phi_B \to \mathcal{O}_\partial = T\phi_B T^\dagger$$ **(HKLL for random TN)**

Bulk States in Random TN

Moreover**, reproduce quantum corrections** to the RT formula:

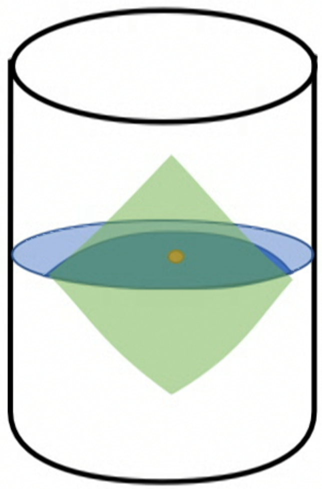


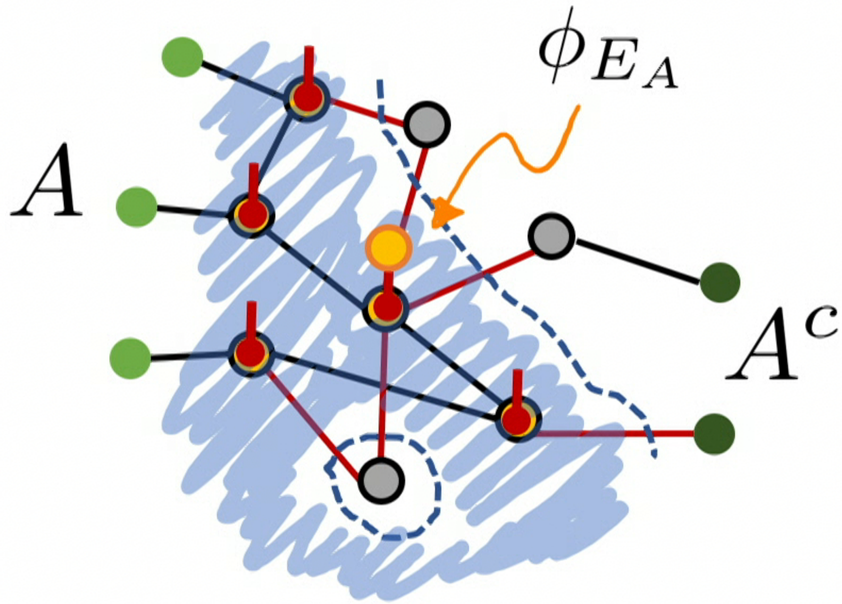
$$S_2(A) = -\log(D)|\partial\Gamma_A| + S_2(E_A, \rho_B)$$

Subregion Duality and Error-Correcting Codes

Subregion Duality

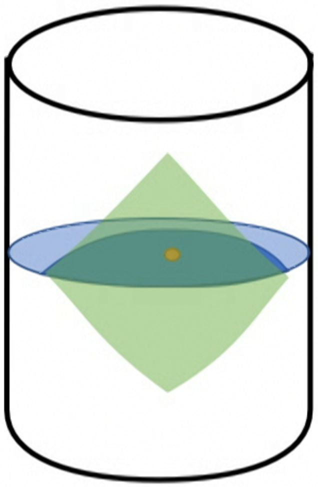
We would also like to understand how to reconstruct bulk operators using only subregion of the boundary.

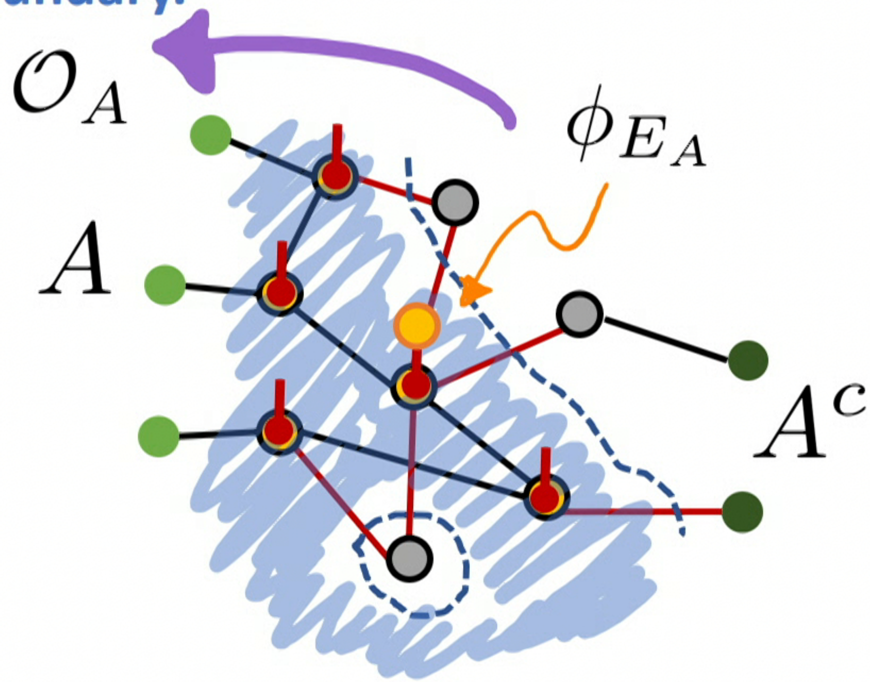




Subregion Duality

We would also like to understand how to reconstruct bulk operators using only subregion of the boundary.

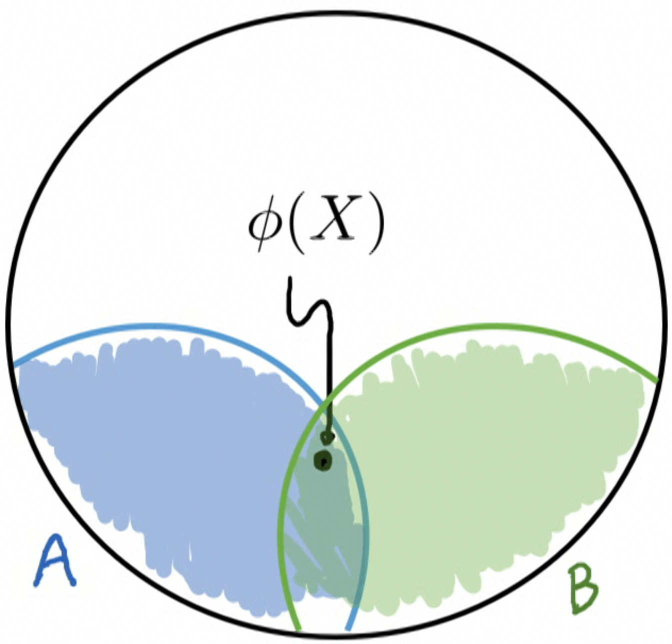




Error-Correction

[Almheiri, Dong, Harlow]

To understand construction, it's useful to consider a puzzle:



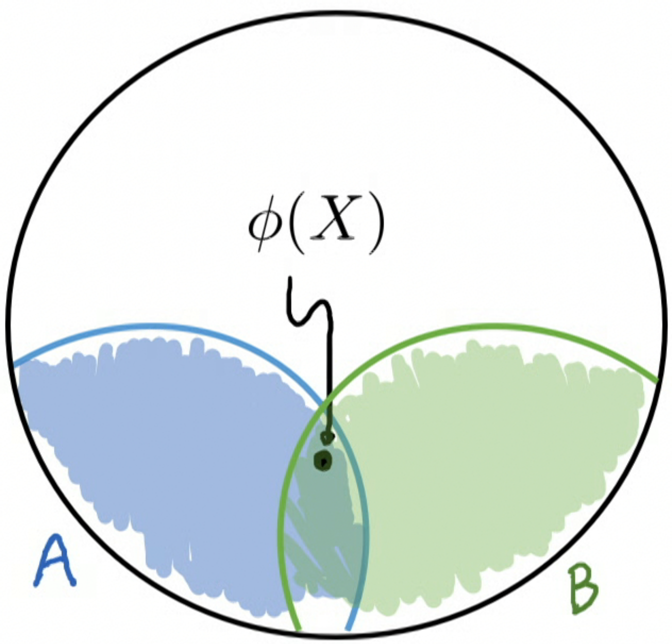
Puzzle:

$\phi(X) \in \ E_A$: Dual to operator $O_{\phi(X)} \in H_A$

Error-Correction

[Almheiri, Dong, Harlow]

To understand construction, it's useful to consider a puzzle:



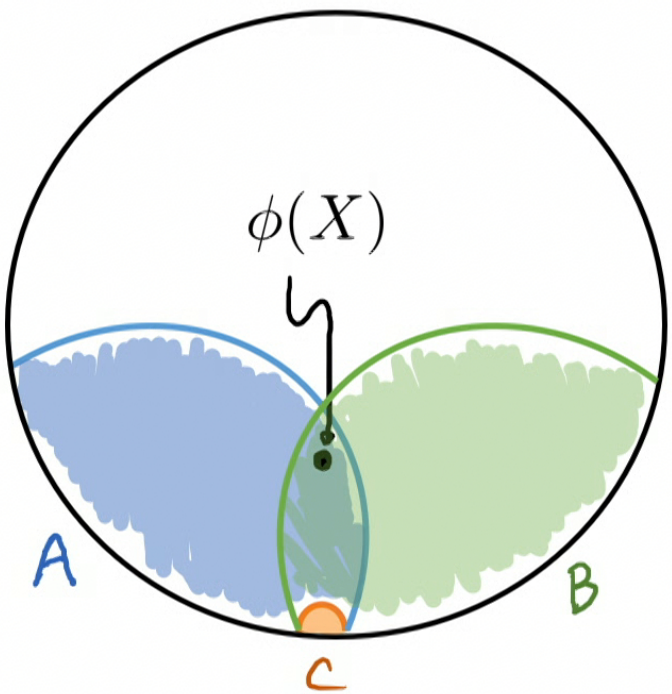
Puzzle:

$\phi(X) \in \ E_A$: Dual to operator $O_{\phi(X)} \in H_A$

Error-Correction

[Almheiri, Dong, Harlow]

To understand construction, it's useful to consider a puzzle:



Puzzle:

$\phi(X) \in \ E_A$: Dual to operator $O_{\phi(X)} \in H_A$

$\phi(X) \in \ E_B$: Dual to operator $O_{\phi(X)} \in H_B$

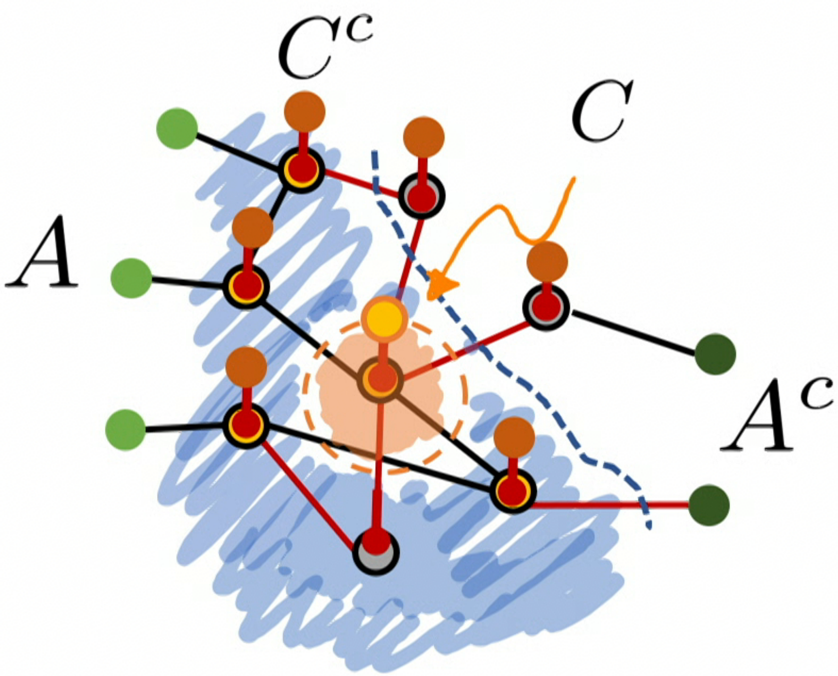
If $\boldsymbol{\phi(X)}$ is a fixed boundary operator

$\Rightarrow O_{\phi(X)} \in H_A \cap H_B = H_C$

Error-Correction

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

When can a logical bulk operator be encoded by a physical operator in a region A?



Need there to be no mutual information between C and C^cA^c:

$$S(C) + S(C^cA^c) = S(CC^cA^c)$$

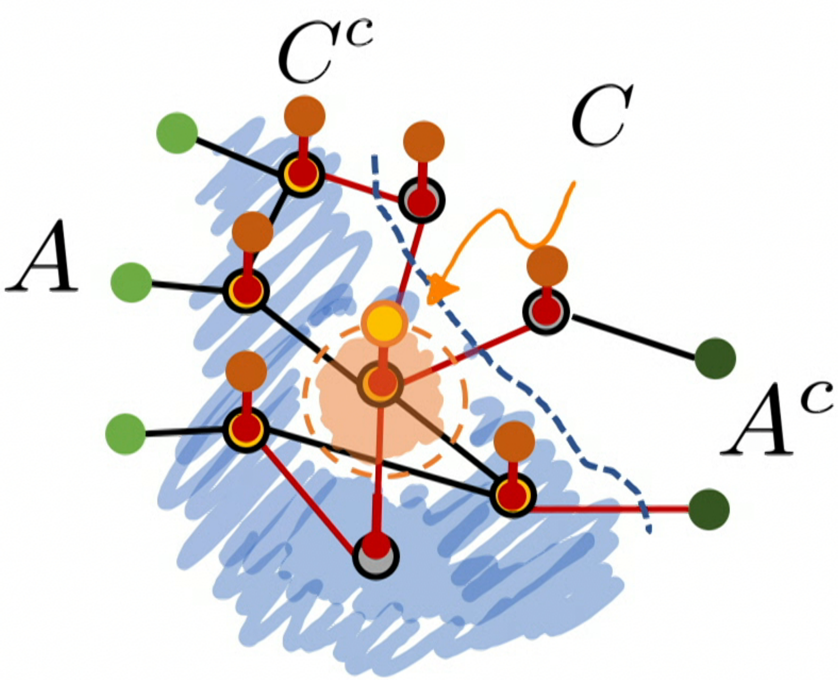
Can calculate by treating bulk legs on equal footing with boundary legs

- Just find min cuts

Error-Correction

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

When can a logical bulk operator be encoded by a physical operator in a region A?



Need there to be no mutual information between C and $C^c A^c$:

$$S(C) + S(C^c A^c) = S(C C^c A^c)$$

Can calculate by treating bulk legs on equal footing with boundary legs

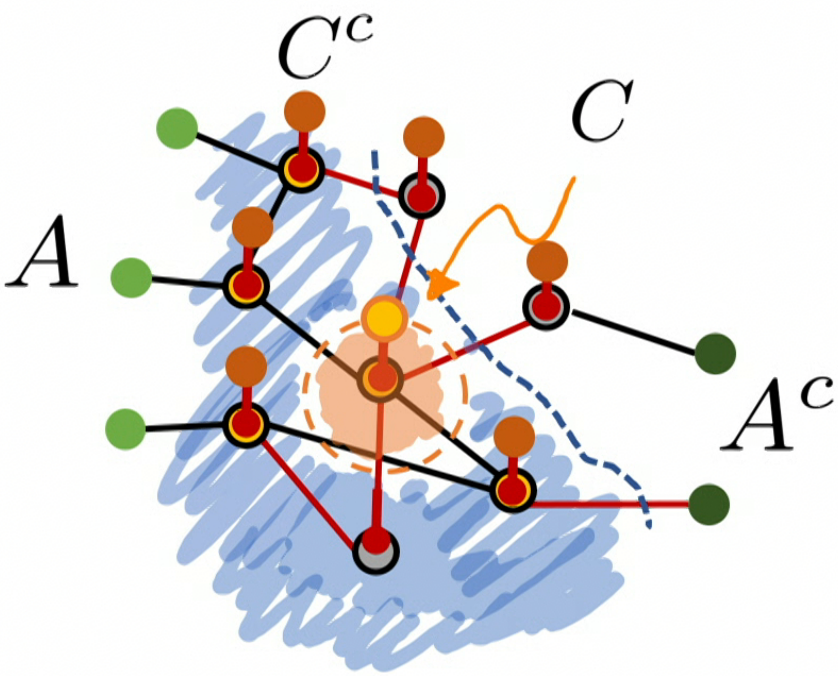
- Just find min cuts

When is mutual information vanishing?

Error-Correction

[Hayden, Nezami, Qi, Thomas, Walter, Yang]

When can a logical bulk operator be encoded by a physical operator in a region A?



Need there to be no mutual information between C and $C^c A^c$:

$$S(C) + S(C^c A^c) = S(C C^c A^c)$$

Can calculate by treating bulk legs on equal footing with boundary legs

- Just find min cuts

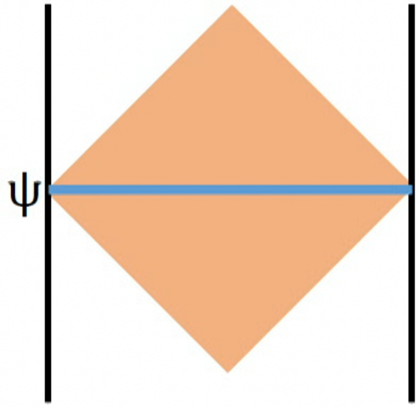
When is mutual information vanishing?

$$C \subset E_A$$

Reconstruct entanglement wedge using QECC

Space < Spacetime

So far we have used tensor networks to understand spatial geometry in an emergent extra dimension. What about spacetime?

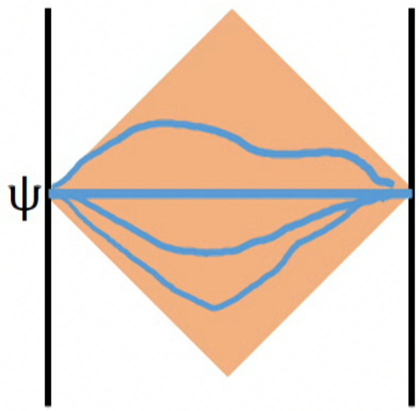


A state in the CFT describes an entire causal diamond in the bulk geometry.

- What is the tensor network description of an arbitrary slice of this diamond?

Space < Spacetime

So far we have used tensor networks to understand spatial geometry in an emergent extra dimension. What about spacetime?



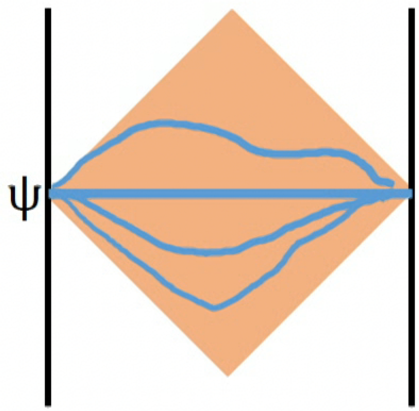
A state in the CFT describes an entire causal diamond in the bulk geometry.

- What is the tensor network description of an arbitrary slice of this diamond?

In a non-static geometry, entropy given by extremal (*not minimal*) surface.

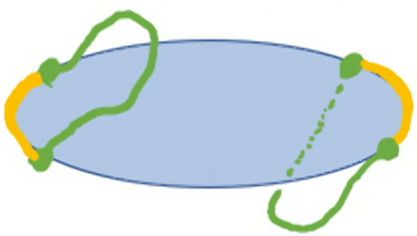
Space < Spacetime

So far we have used tensor networks to understand spatial geometry in an emergent extra dimension. What about spacetime?



A state in the CFT describes an entire causal diamond in the bulk geometry.

- What is the tensor network description of an arbitrary slice of this diamond?



In a non-static geometry, entropy given by extremal (*not minimal*) surface.

- Different HRT slices may not lie on same slice

Real-time Evolution

We would also like to understand real-time evolution: