Title: Unitary Networks from the Exact Renormalization of Wavefunctionals

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Abstract: The exact renormalization group (ERG) for O(N) vector models at large N on flat Euclidean space admits an interpretation as the bulk dynamics of a holographically dual higher spin gauge theory on AdS_{d+1}. The generating functional of correlation functions of single trace operators is reproduced by the on-shell action of this bulk higher spin theory, which is most simply presented in a first-order (phase space) formalism. This structure arises because of an enormous non-local symmetry of free fixed point theories. In this talk, I will review the ERG construction and describe its extension to the RG flow of the wave functionals of arbitrary states of the O(N) vector model at the free fixed point. One finds that the ERG flow of the ground state and a specific class of excited states is implemented by the action of unitary operators which can be chosen to be local. Thus the ERG equations provide a continuum notion of a tensor network. We compare this tensor network with the entanglement renormalization networks, MERA, and cMERA. The ERG tensor network appears to share the general structure of cMERA but differs in important ways.

Unitary Networks from the Exact Renormalization of Wavefunctionals

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based on a series of papers, past and future
with Onkar Parrikar, Jackson Fliss, …

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PI Workshop
"Tensor Networks for QFT II"

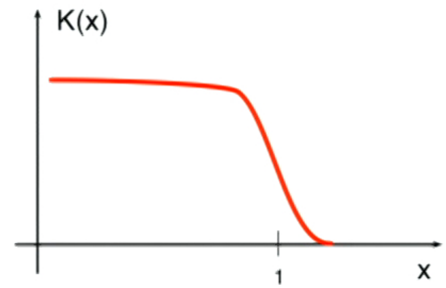
Outline

1. the Exact Renormalization Group (ERG)
2. ERG for *partition function* of free field theories
   - higher spin gauge theory holography
   - comes about through identification of an enormous non-local symmetry of free field theories
   - holographic fields described by *Cartan connection*
     - spin 2 part = graviton
3. ERG for *wave-functionals* of arbitrary states of free field theories
   - derive explicit flow through space of states
   - ERG is well-designed for *unitary* flow
     - contains rescaling and disentangler — a *continuous tensor network*
     - disentangling is in momentum space
4. Outlook

ERG

$$Z = \int [d\phi] e^{-S_o[M,\phi] - S_{int}[\phi]}$$

$$S_o[M,\phi] = \int \phi \; K^{-1}(-\Box/M^2) \; \Box\phi$$



- path integral is over all modes of field
  - regulator function gives zero weight to high momentum modes
- the RG principle is that the choice of K is immaterial

$$M \frac{d}{dM} Z = 0$$

ERG

- this comes about through the couplings of the theory becoming scale dependent
- Polchinski showed that this gives an *exact* equation

$$M\frac{\partial S_{int}}{\partial M} = \frac{1}{2}\Delta_B(x,y)\int[d\phi]\left[\frac{\delta S_{int}}{\delta\phi(x)}\frac{\delta S_{int}}{\delta\phi(y)} - \frac{\delta^2 S_{int}}{\delta\phi(x)\delta\phi(y)}\right]$$

  - employed a trick: discard a total functional derivative in the path integral

- this single equation can be expanded to extract the scale dependence of each coupling
  - in fact, we will improve on this
    - introduce separate cutoff and renormalization scale
    - complete exact system of equations for sources and correlation functions

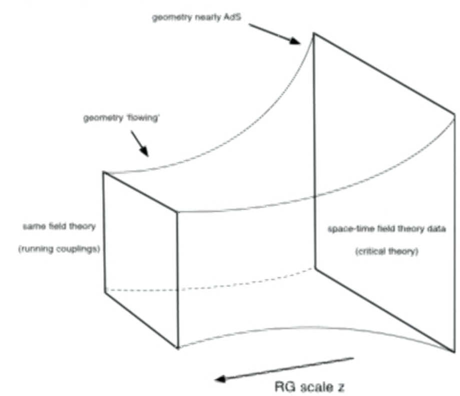
- **will apply this to a special scenario**
  - initially won't turn on explicit interactions, but instead will source 'single trace' operators
  - thus, will first study the RG properties of the generating functional of correlation functions of single trace operators
  - by single trace, we mean local operators of the form

$$\phi_a^* \partial_{\mu_1} \ldots \partial_{\mu_s} \phi^a$$

  - that is, organize elementary fields into an N-vector, and consider only U(N) (or O(N)) singlets
    - **these are bilinears, and so path integral for generating functional is Gaussian**
  - the ERG equations are a system of first order equations for the scale dependence of the sources and the corresponding expectation values
    - **this is the same data tracked by a holographic system**

Holography in First-Order

- given a *conserved* current $\hat{j}^{\mu_1 \ldots \mu_s}$ there is a corresponding *massless* gauge field $A_{\mu_1 \ldots \mu_s}$ in the bulk (obtained by gauge-fixing bulk tensor)
  - at linearized level, satisfies a second order PDE
  - packages together info about CFT: source and vev of $\hat{j}^{\mu_1 \ldots \mu_s}$
- we will be led to package this info together in a sort of Hamiltonian formalism, in which RG scale plays the role of time, and the bulk gauge field appears as a canonical pair, satisfying a pair of first order PDEs — the ERG equations



Exact RG and Unitary Networks

The Exactness of ERG

- **usually, we think of RG flows as irreversible, associated with coarse graining**
    - this is a practicality, rather than a necessity
    - it comes about because we throw away information
- **if we could track everything, in principle we could have unitarity**
    - typical this is impractical
    - we must track an infinite number of operators, not just the relevant subset
- **there is one case where we *must* do so**
    - in free field theories, such truncations correspond to explicit breaking of gauge symmetries (from a holographic point of view)

Symmetries of Free Fixed Points

- this symmetry acts linearly, but non-locally $|\phi^a\rangle \mapsto \mathcal{L}|\phi^a\rangle$
- or, in the space-time basis

$$\phi^a(x) \mapsto \int d^d y\ \mathcal{L}(x,y)\phi^a(y) \qquad (*)$$

- this encodes diffeomorphisms as well as higher-spin analogues

$$\mathcal{L}(x,y) = \delta^{(d)}(x-y) + \zeta^\mu(x)\partial_\mu^{(x)}\delta^{(d)}(x-y) + \zeta^{\mu\nu}(x)\partial_\mu^{(x)}\partial_\nu^{(x)}\delta^{(d)}(x-y) + \ldots$$

- we implement (*) this as a change of variables in the path integral
  - this generates an exact Ward identity — in the *background* sense
  - will be an important ingredient in RG, and is the *origin of higher spin symmetry* in the holographic bulk
  - geometry of the bulk is associated with symmetry of the fixed point

Exact RG and Unitary Networks

Generating Functionals and Ward Identities

- a standard tool is a generating functional

$$Z[A_\mu(x)] = \langle e^{i \int d^d x A_\mu(x) \hat{j}^\mu(x)} \rangle$$

- if we can compute it, it encodes all of the correlation functions of the operator $\hat{j}^\mu(x)$ that is sourced

- if the quantum theory is such that the current is conserved, we have an exact Ward identity

$$Z[A_\mu^g] = Z[A_\mu]$$

- given a path integral rep'n of Z, derive by the Fujikawa method
  - make a change of integration variables $\phi \mapsto \phi^g$
    - measure invariant (or anomalous), action transforms

$$S[\phi] + \int A_\mu j^\mu \mapsto S[\phi^g] + \int A_\mu^g j^\mu$$ (Noether)

Generating Functionals for Free QFTs

- we have local operators $\{1, \phi^2(x), j^\mu(x), T^{\mu\nu}(x), \ldots\}$
- would introduce sources ('couplings') $\{U, b(x), a_\mu(x), h_{\mu\nu}(x), \ldots\}$
- in the case of free field theory, all of these operators are bilinear in the elementary fields, and they can be collected together into a bi-local expression

$$\int d^d x \int d^d y \ \langle \phi_a | x \rangle \langle x | B | y \rangle \langle y | \phi^a \rangle = \int d^d x \int d^d y \ \phi_a^*(x) B(x,y) \phi^a(y)$$

- we can think of expanding the bi-local source quasi-locally

$$B(x,y) = b_0(x)\delta^{(d)}(x,y) + b_1^\mu(x)\partial_\mu \delta^{(d)}(x,y) + \ldots$$

- this then gives local sources for the infinite collection of spin-s currents that are conserved at the free fixed point

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Exact RG and Unitary Networks

Free Majoranas

- fixed point action

$$S_0 = \int_{x,y} \psi^m(x)\gamma^\mu P_{F;\mu}(x,y)\psi^m(y) \qquad m = 1, 2, \ldots, N$$

$$P_{F;\mu}(x,y) = \partial_\mu^{(x)}\delta^{(d)}(x,y)$$

- introduce sources for *all* single-trace operators

$$S_{int} = U + \frac{1}{2}\int_{x,y} \psi^m(x)\Big(A(x,y) + \gamma^\mu W_\mu(x,y) + \gamma^{\mu\nu}A_{\mu\nu}(x,y) + \ldots\Big)\psi^m(y)$$

- the list of sources terminates, depending on space-time dimension
  - e.g., d=3: *just* $A(x,y)$ *and* $W_\mu(x,y)$

- now perform the non-local change of variables $\psi^a(x) \mapsto \int d^d y \; \mathcal{L}(x,y)\psi^a(y)$

$$S \rightarrow \psi^m \cdot \mathcal{L}^T \cdot [\gamma^\mu(P_{F;\mu} + W_\mu) + A] \cdot \mathcal{L} \cdot \psi^m$$

$$= \psi^m \cdot \gamma^\mu \mathcal{L}^T \cdot \mathcal{L} \cdot P_{F;\mu} \cdot \psi^m + \widetilde{\psi}^m \cdot \left[\gamma^\mu(\mathcal{L}^T \cdot [P_{F;\mu}, \mathcal{L}] + \mathcal{L}^T \cdot W_\mu \cdot \mathcal{L}) + \mathcal{L}^T \cdot A \cdot \mathcal{L}\right] \cdot \psi^m$$

Exact RG and Unitary Networks

- if $\mathcal{L}^T \cdot \mathcal{L} = 1$ then the fixed point action remains unchanged, while the sources transform. That is

$$Z\Big[U, A, W_\mu\Big] = Z\Big[U, \underbrace{\mathcal{L}^{-1} \cdot A \cdot \mathcal{L}}_{\text{tensor}}, \underbrace{\mathcal{L}^{-1} \cdot W_\mu \cdot \mathcal{L} + \mathcal{L}^{-1} \cdot [P_{F,\mu}, \mathcal{L}]}_{\text{connection}}.\Big]$$

- we call this group $O(L^2(\mathbb{R}^{1,d-1}))$
  - $D_\mu = P_{F,\mu} + W_\mu$ plays the role of covariant derivative
  - the fixed point theory corresponds to

$$(A, W_\mu) = (0, W_\mu^{(0)})$$

  - that is, because W_μ is a connection, the QFT is unsourced whenever A is zero and W_μ is a flat connection

Dilatations and ERG

- we extend this to RG by asking how the theory responds to a dilatation $x^\mu \mapsto \lambda x^\mu$

- one can combine $O(L^2)$ with the dilatation in a simple way, by simply allowing $\mathcal{L}^T \cdot \mathcal{L} = \lambda^{2\Delta_\psi} 1$ (we refer to this as $CO(L^2)$)

- this has the effect

$$Z[M, g; U, A, W_\mu] = Z[\lambda^{-1} M, \lambda^2 g, U^{\mathcal{L}}, A^{\mathcal{L}}, W_\mu^{\mathcal{L}}]$$

$$K = K[-z^2 D^{(0)2}/M^2]$$

metric seen by field theory

- if we parameterize $g_{\mu\nu} = z^{-2}\eta_{\mu\nu}$, we can write this equivalently as

$$Z[M, z; U, A, W_\mu] = Z[\lambda^{-1} M, \lambda^{-1} z, U^{\mathcal{L}}, A^{\mathcal{L}}, W_\mu^{\mathcal{L}}]$$

- we regard $z \in [\epsilon, \infty)$ as the renormalization scale

The Exact RG

- **we perform the ERG in two steps**
  - 1. lower the cutoff $M \mapsto \lambda M$

$$Z[M, z; U, A, W_\mu] = Z[\lambda M, z, \tilde{U}, \tilde{A}, \tilde{W}_\mu]$$

(à la Polchinski)

  - 2. bring M back to its original value via a $CO(L^2)$ transformation

$$Z[\lambda M, z; \tilde{U}, \tilde{A}, \tilde{W}_\mu] = Z[M, \lambda^{-1} z, \tilde{U}^{\mathcal{L}}, \tilde{A}^{\mathcal{L}}, \tilde{W}_\mu^{\mathcal{L}}]$$

- **there is a freedom in the choice of $\mathcal{L}$**
- **comparing the two, we arrive at a relation between the generating functionals at the same scale M, but different renormalization scale z**

$$\boxed{Z[M, z; U, A, W_\mu] = Z[M, \lambda^{-1} z, \tilde{U}^{\mathcal{L}}, \tilde{A}^{\mathcal{L}}, \tilde{W}_\mu^{\mathcal{L}}]}$$

$$Z[M, z; U, A, W_\mu] = Z[M, \lambda^{-1} z, \tilde{U}^{\mathcal{L}}, \tilde{A}^{\mathcal{L}}, \tilde{W}_\mu^{\mathcal{L}}]$$

- this is exact in general, but it is convenient to re-write it as a differential equation by expansing near

a well chosen name

$$\lambda \simeq 1 - \varepsilon, \qquad \mathcal{L} \simeq 1 + \varepsilon z W_z$$

- then we find

$$A(z + \varepsilon z) = A(z) + \varepsilon z \, [W_z, A] + \varepsilon z \beta^{(A)} + O(\varepsilon^2)$$

$$W_\mu(z + \varepsilon z) = W_\mu(z) + \varepsilon z \, [P_{F;\mu} + W_\mu, W_z] + \varepsilon z \beta_\mu^{(W)} + O(\varepsilon^2)$$

$$W_\mu^{(0)}(z + \varepsilon z) = W_\mu^{(0)}(z) + \varepsilon z \left[P_{F;\mu} + W_\mu^{(0)}, W_z^{(0)} \right] + O(\varepsilon^2)$$

output of ERG

recall:
everything bi-local in x,y

Exact RG and Unitary Networks

$$Z[M, z; U, A, W_\mu] = Z[M, \lambda^{-1} z, \tilde{U}^{\mathcal{L}}, \tilde{A}^{\mathcal{L}}, \tilde{W}_\mu^{\mathcal{L}}]$$

- or by taking $\varepsilon \to 0$

$$\partial_z \mathcal{W}_\mu^{(0)} - [P_{F;\mu}, \mathcal{W}_z^{(0)}] + [\mathcal{W}_z^{(0)}, \mathcal{W}_\mu^{(0)}] = 0$$

bi-local products

$$\partial_z \mathcal{A} + [\mathcal{W}_z, \mathcal{A}] = \beta^{(\mathcal{A})}$$

$$\partial_z \mathcal{W}_\mu - [P_{F;\mu}, \mathcal{W}_z] + [\mathcal{W}_z, \mathcal{W}_\mu] = \beta_\mu^{(\mathcal{W})}$$

- this is what is obtained from dilatations; we suppose that more generally, these are components of covariant equations

$$d\mathcal{W}^{(0)} + \mathcal{W}^{(0)} \wedge \mathcal{W}^{(0)} = 0$$

$$d\mathcal{A} + [\mathcal{W}, \mathcal{A}] = \boldsymbol{\beta}^{(\mathcal{A})}$$

$$d\mathcal{W} + \mathcal{W} \wedge \mathcal{W} = \boldsymbol{\beta}^{(\mathcal{W})}$$

other components not determined by RG

but determined by consistency 'Bianchi identities'

Exact RG and Unitary Networks

(Classical) Bulk Action

- **this is only half of the bulk system**
  - repeat analysis for the vevs (Callan-Symanzik equations)
  - give rise to bulk 'momenta'
- **the resulting system of equations is a Hamiltonian system with respect to z**
  - the ERG analysis determines the Hamiltonian
  - correspondingly, there is an action

$$Z = e^{-S_{HJ}[z;B]} \to e^{-I[\mathcal{P},\mathcal{B}]} = e^{-\int dz \ Tr(\mathcal{P}\cdot\partial_z\mathcal{B} - H(\mathcal{P},\mathcal{B}))}$$

$$I = \int dz \ Tr\left\{\mathcal{P}^I \cdot \left(\mathcal{D}_I^{(0)}\mathcal{B} - \beta_I^{(\mathcal{B})}\right) + N\Delta_B \cdot \mathcal{B}\right\}$$

$$\beta^{(\mathcal{B})} = \mathcal{B}\cdot\Delta_B\cdot\mathcal{B}$$

(here I've switched to O(N) scalar theory)

Exact RG and Unitary Networks

$$Z[M, z; U, A, W_\mu] = Z[M, \lambda^{-1} z, \tilde{U}^{\mathcal{L}}, \tilde{A}^{\mathcal{L}}, \tilde{W}_\mu^{\mathcal{L}}]$$

- or by taking $\varepsilon \to 0$

$$\partial_z \mathcal{W}_\mu^{(0)} - [P_{F;\mu}, \mathcal{W}_z^{(0)}] + [\mathcal{W}_z^{(0)}, \mathcal{W}_\mu^{(0)}] = 0$$

bi-local products

$$\partial_z \mathcal{A} + [\mathcal{W}_z, \mathcal{A}] = \beta^{(\mathcal{A})}$$

$$\partial_z \mathcal{W}_\mu - [P_{F;\mu}, \mathcal{W}_z] + [\mathcal{W}_z, \mathcal{W}_\mu] = \beta_\mu^{(\mathcal{W})}$$

- this is what is obtained from dilatations; we suppose that more generally, these are components of covariant equations

$$d\mathcal{W}^{(0)} + \mathcal{W}^{(0)} \wedge \mathcal{W}^{(0)} = 0$$

$$d\mathcal{A} + [\mathcal{W}, \mathcal{A}] = \beta^{(\mathcal{A})}$$

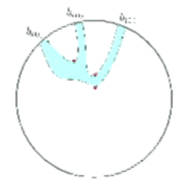
$$d\mathcal{W} + \mathcal{W} \wedge \mathcal{W} = \beta^{(\mathcal{W})}$$

other components
not determined by RG

but determined by consistency
'Bianchi identities'

Holographic Higher Spins

- all of the usual holographic machinery can be employed here
- a classical solution corresponds to an RG flow
- the trivial solution corresponds to the free fixed point
  - i.e., turn off all sources — $\mathcal{W}_A$ is flat
  - if we choose "spin-2 gauge", this connection encodes the geometry of
  - AdS_{d+1} , $\mathcal{W}^{(0)} \rightarrow \{e^a, \omega^a{}_b\}$ $\qquad \mathcal{W}^{(0)}(x,y) = -\frac{dz}{z} D(x,y) + \frac{dx^\mu}{z} P_\mu(x,y)$
    - at least when the free fixed point has relativistic symmetry
- all correlation functions can be systematically computed



  - look like "bi-local Witten diagrams"
  - these resum to the determinant — proof that no information has been lost

Conformal details

- the usual conformal group $SO(2,d) \subset CO(L^2)$
- each local operator transforms in a short conformal module $U(\Delta, s)$
- the corresponding sources transform in the dual module $U(d-\Delta, s)$
- the bulk degrees of freedom transform in

$$\oplus_s \Big(U(d-\Delta, s) \oplus U(\Delta, s) \Big)$$

- linearizing around AdS_{d+1}, one can write the equations of motion as decoupled second order PDEs
  - these are nothing but $Casimir = s(s+d-2)$
  - "Fronsdal equations" of higher spin theory

Exact RG and Unitary Networks

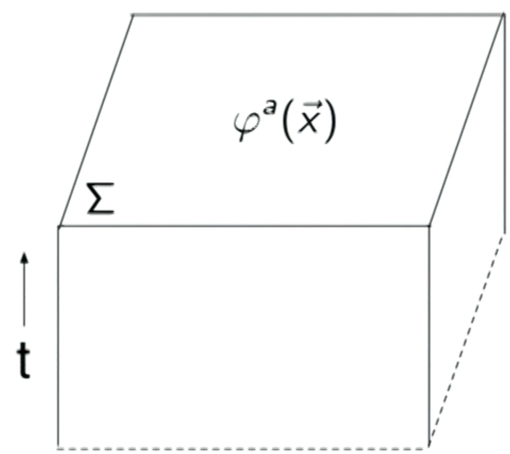
3

ERG and Wave-functionals

- in continuum QFT, we do not typically consider explicit wave functionals for states
- however, the ground state wave-functional of a free field theory is well-known, being Gaussian
  - much less understood is how the renormalization group acts on the ground state, as well as all other states
- we can in fact use ERG methods to study this
  - builds on similar concepts to those introduced previously
  - the ERG construction naturally retains 'ancillary' degrees of freedom
  - generates a flow in the space of states
    - equivalent to a continuous tensor network whose properties we can study

ERG for Wave-functionals

- to employ these exact methods for wave-functionals, we need to carefully construct familiar concepts
- wave-functional in 'position basis' $\langle \varphi^a(\vec{x}) | \Psi \rangle$ obtained by path integral over half space-time in Euclidean time, with specified boundary condition on a space-like hypersurface



$$\langle \varphi^a(\vec{x}) | \Psi \rangle \equiv Z[M_-; \varphi^a]$$

- extract ground state by usual limiting procedure

- generate large class of states by operator insertions in Euclidean time

ERG for Wave-functionals

- various technical points to manage
  - convergence
  - normalizability
  - well-defined canonical structure (boundary conditions)
- here we are interested in states corresponding to insertions of O(N)-invariant operators

$$|\psi\rangle = e^{-\delta \hat{H}} \hat{\mathcal{O}}_1(0,\vec{x}_1)\hat{\mathcal{O}}_2(0,\vec{x}_2)\cdots\hat{\mathcal{O}}_n(0,\vec{x}_n)|\Omega\rangle$$

- it is useful to introduce the *generating functional of states*

$$|\psi[b]\rangle = \mathcal{T}_E e^{-\frac{1}{2}\int_{M_-} d^d x \int_{M_-} d^d y\, \hat{\phi}^a(x) b(x,y) \hat{\phi}^a(y)} |\Omega\rangle$$

- (can generalize to contour ordering in complex time)
- *b(x,y)* plays a similar role to the sources considered earlier

ERG for Wave-functionals

- **again, there is a large non-local symmetry present**
  - restrict $\mathcal{L}(x, y)$ to preserve Σ
  - need to regulate appropriately

$$S_\phi = \frac{1}{2}\int_M \phi(x) \circ K^{-1}\left(-\vec{D}^2/M^2\right) \circ D^2 \circ \phi(x) + \frac{1}{2z^{d-2}}\int_\Sigma \varphi(\vec{x}) \cdot K^{-1}\left(-\vec{D}^2/M^2\right) \cdot D_t \cdot \phi|_\Sigma(\vec{x})$$

  - cannot introduce arbitrary number of time derivatives
    - sufficient to introduce 'spatial' regulator $K\left(-\vec{D}^2/M^2\right)$

- **recall for partition function, we implemented ERG as a 2-step process**
  - lower cutoff $M \mapsto \lambda M$
  - use CO(L[2]) to take $\lambda M \mapsto M, \quad z \mapsto \lambda^{-1} z$

Exact RG and Unitary Networks

ERG for Wave-functionals

- **for the generator of states, we employ a similar process**
  - the novelty is that we have to take care with boundary terms
  - there is dependence on a bulk kernel Δ_B but now also a boundary kernel Δ_Σ , and a CO(L^2) transformation given by W_z and w_z
    - these know about the details of the regulator
- **for the partition function, we required M-independence as the basic RG requirement**
  - for wave-functionals, this would be too strong
  - instead we just eliminate M-derivatives from the ERG equations

$$z\frac{\partial}{\partial z}\Psi = \left(z Tr_{\Sigma\times C}\left(([W_z, b]_\circ + b \circ \Delta_B \circ b) \circ \frac{\delta}{\delta b}\right) + z\frac{N}{2} Tr_\Sigma g + z\int_\Sigma \varphi \cdot g^T \cdot \frac{\delta}{\delta\varphi}\right)\Psi$$

$$g(z;\vec{x},\vec{y}) := \left(\tfrac{1}{2}\Delta_\Sigma + w_z\right)(\vec{x},\vec{y})$$

ERG for Ground State

- the ground state is obtained by setting b(x,y)=0
  - then, the $\delta/\delta b$ terms disappear
    - the ground state does not mix with other states, and satisfies

$$z\frac{d}{dz}\left|\Omega(z)\right\rangle = i\Big(\mathbf{K}(z)+\mathbf{L}(z)\Big)\left|\Omega(z)\right\rangle$$

$$\mathbf{K}(z) = \frac{z}{2}\left(\hat{\pi}\cdot\Delta_{\Sigma}(z)\cdot\hat{\phi}+\hat{\phi}\cdot\Delta_{\Sigma}^{\mathsf{T}}(z)\cdot\hat{\pi}\right) \quad \text{“disentangler”}$$

$$\mathbf{L}(z) = \frac{z}{2}\left(\hat{\pi}\cdot w_z(z)\cdot\hat{\phi}+\hat{\phi}\cdot w_z^{\mathsf{T}}(z)\cdot\hat{\pi}\right) \quad \text{scale transformation}$$

- **K** and **L** are both Hermitian
- can be solved in terms of path-ordered exponential

$$\Psi_{\Omega}[z_*,\varphi] = \left\langle\varphi\middle|\Omega(z_*)\right\rangle = \left\langle\varphi\middle|\mathcal{P}e^{\frac{i}{2}\int_{\epsilon}^{z_*}dz\int_{\Sigma}\left(\hat{\pi}\cdot g(z)\cdot\hat{\phi}+\hat{\phi}\cdot g^{\mathsf{t}}(z)\cdot\hat{\pi}\right)}\middle|\Omega(\epsilon)\right\rangle.$$

ERG for States

- for any other state, we have

$$z\partial_z \,|\Psi_C[b]\rangle = \left(-Tr\,\boldsymbol{\beta}\circ\frac{\delta}{\delta b} + i\mathbf{K} + i\mathbf{L}\right)|\Psi_C[b]\rangle$$

- **$\mathbf{K}$ and $\mathbf{L}$ are state independent, β causes mixing of states along the flow**
- **if we think of $|\Psi_C[b]\rangle$ as a family of states in the space of b, we can regard this equation as a flow along the integral curves of β**
  - that is, we introduce a "running" source $\mathcal{B}(z;x,y)$ satisfying

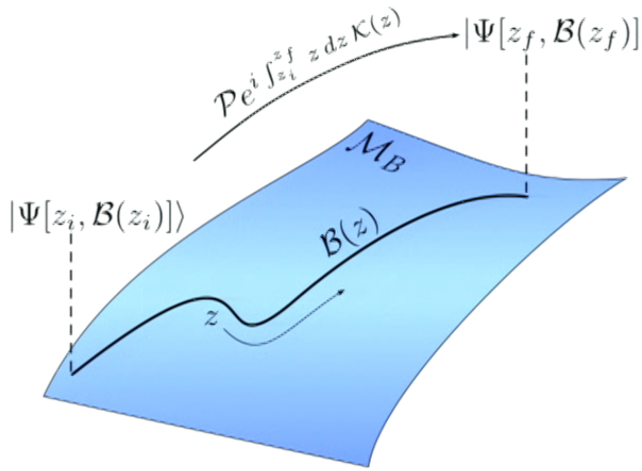
$$z\partial_z\mathcal{B} = \beta[\mathcal{B}]$$

$$z\frac{d}{dz}\,|\Psi_C[z,\mathcal{B}(z)]\rangle = i\,(\mathbf{K}+\mathbf{L})\,|\Psi_C[z,\mathcal{B}(z)]\rangle$$

  - (same equation as ground state)

ERG for States

- claim: RG principle for states should be that the flow is along integral curves of β
  - then, state changes by a unitary operator
  - consistent with RG-invariance of norm (~ partition function)



MERA?

- **of course such unitary evolution is present in MERA**
  - in MERA though, the precept is that unitaries be chosen to disentangle spatially, culminating in a specific IR state
- **our result should be interpreted as a continuous tensor network**
- **the disentangling is happening in *momentum space***
  - ground state is a product state in momentum space
    - excited states are typically not
  - by looking at non-trivial states, can show that **K** disentangles states above and below RG scale
- **K is given by the choice of regulator**
  - optimization, as in MERA, then explores different choices of regulator

MERA?

- **thus, conceptually very similar, but details differ for entanglement**
  - this essentially follows from locality in position space of the free fixed point
  - it is not clear that there is a suitable **K** that would give rise to a position space product state in IR
- **does not preclude the study of real space entanglement using ERG**
  - given that we understand wave-functionals, we also understand how to do ERG for density matrices
  - the flow of reduced density matrices, entanglement entropy, etc., apparently requires a sophisticated regulator, and is currently under study