Title: The continuous multi-scale entanglement renormalization ansatz (cMERA)

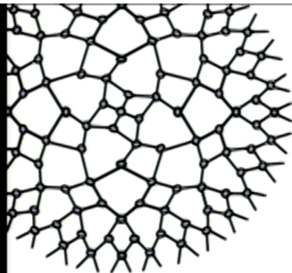
Date: Apr 19, 2017 09:30 AM

URL: http://pirsa.org/17040038

Abstract: The first half of the talk will introduce the cMERA, as proposed by Haegeman, Osborne, Verschelde and Verstratete in 2011 [1], as an extension to quantum field theories (QFTs) in the continuum of the MERA tensor network for lattice systems. The second half of the talk will review recent results [2] that show how a cMERA optimized to approximate the ground state of a conformal field theory (CFT) retains all of its spacetime symmetries, although these symmetries are realized quasi-locally. In particular, the conformal data of the original CFT can be extracted from the optimized cMERA.

[1] J. Haegeman, T. J. Osborne, H. Verschelde, F. Verstraete, Entanglement renormalization for quantum fields, Phys. Rev. Lett, 110, 100402 (2013), arXiv:1102.5524

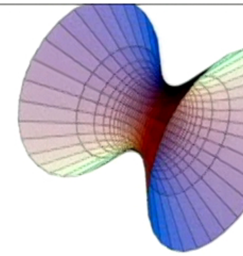
[2] Q. Hu, G. Vidal, Spacetime symmetries and conformal data in the continuous multi-scale entanglement renormalization ansatz, arXiv:1703.04798



TENSOR NETWORKS FOR QUANTUM FIELD THEORIES II

PERIMETER INSTITUTE

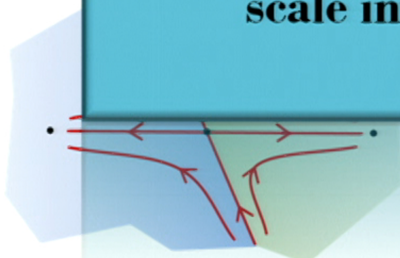
April 19th, 2017



continuous Multiscale Entanglement Renormalization Ansatz

cMERA

scale invariance and conformal symmetry



Guifre Vidal

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

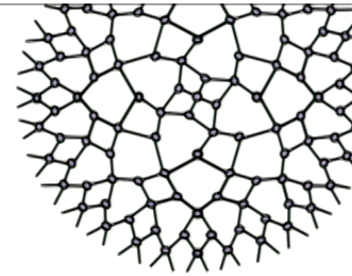
compute | calcul canada | canada

TENSOR NETWORKS INITIATIVE

SIMONS FOUNDATION

NSERC

Outline:



- What is cMERA? (1+1 free boson CFT)

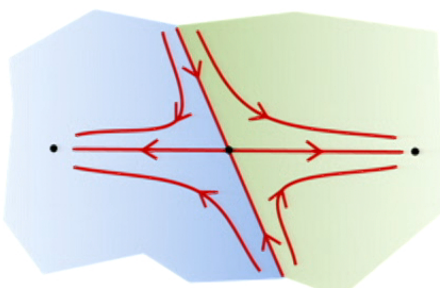
Haegeman, Osborne, Verschelde, Verstraete, PRL 2013

- Entangling evolution in scale
- Scale invariance

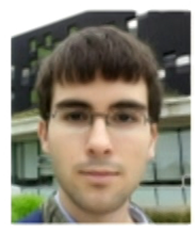
Qi Hu, GV, arxiv:1703.04798

- Conformal symmetry

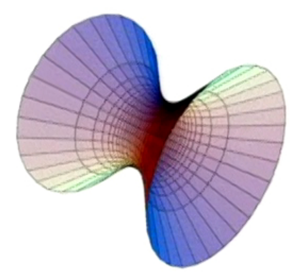
see posters by



Qi Hu
(Perimeter)



Adrian Franco-Rubio
(Perimeter)

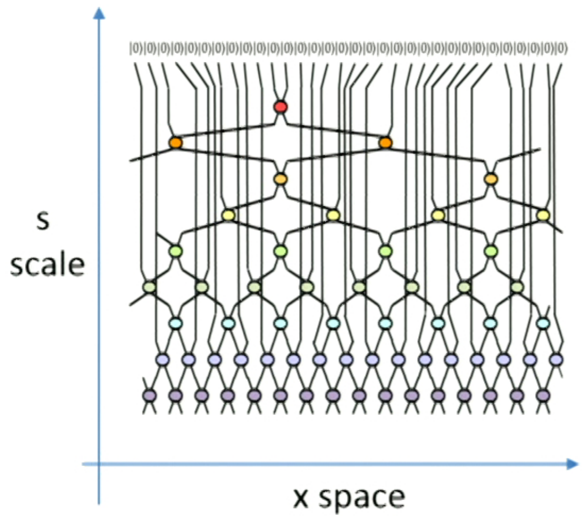


PRL 2007,2008

J. Haegeman, T. Osborne, H. Verschelde, F. Verstraete
PRL 2013

MERA

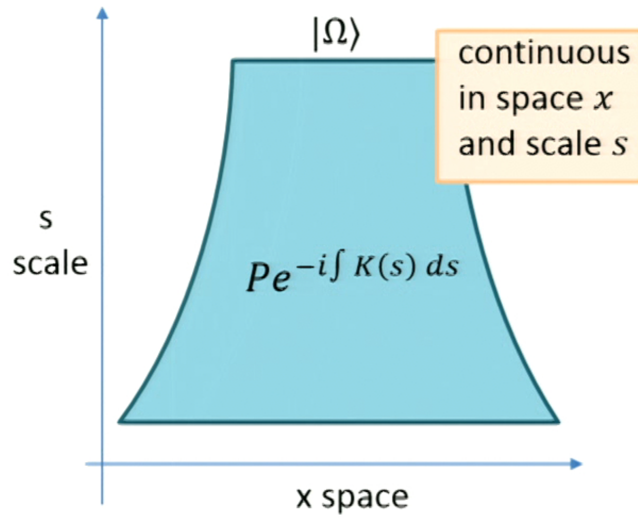
V **quantum circuit**



$$|\Psi^{\Lambda}\rangle = U|0\rangle^{\otimes N}$$

cMERA

V **unitary evolution**



$$|\Psi^{\Lambda}\rangle = U|\Omega\rangle$$

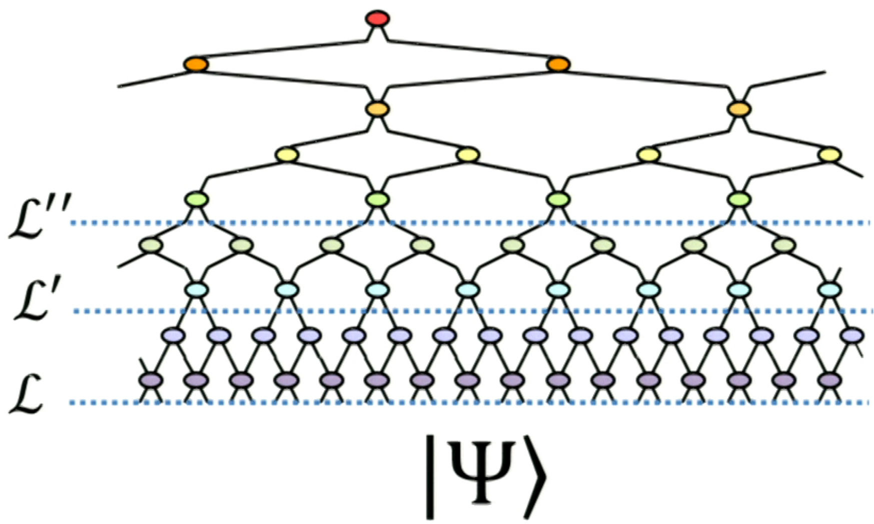
Why MERA?

1- Numerical evidence that it can approximate ground states

2- It implements Wilson's **RG** on wavefunctions

3- It suggests "ground state = **entangling evolution in scale**" (circuit complexity!)

4- Conjectured relation to the AdS/CFT correspondence



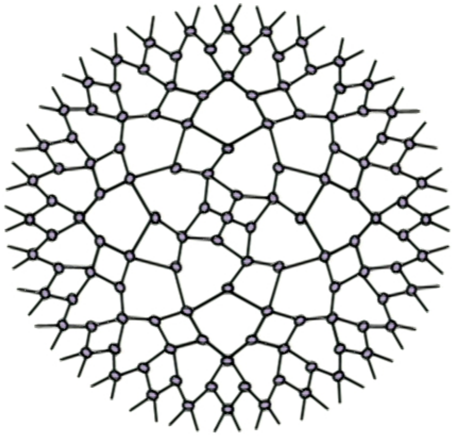
$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots$$

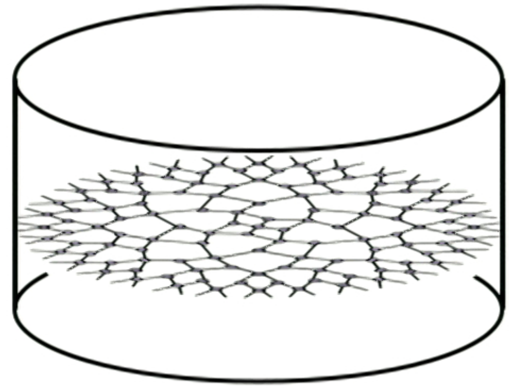
(conjectured) lattice realization of
the holographic principle

AdS_3/CFT_2

Swingle 2009, 2012

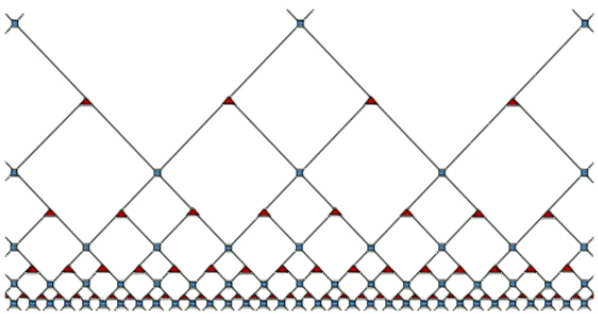
MERA = time slice of AdS_3 (hyperbolic plane H_2)

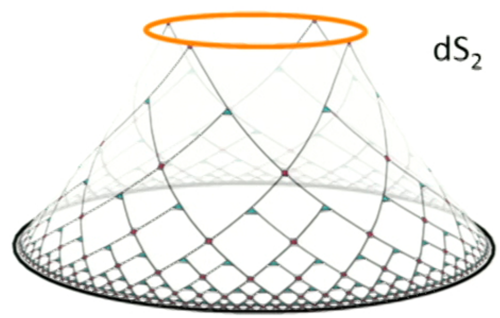




Czech, Lamprou, McCandlish, Sully, 2015-2016

MERA = kinematic space (integral transform of H_2)





Why cMERA?

We hope that cMERA…

wish list

1- will approximate ground states of **interacting** QFTs

2- will implement Wilson's **RG** on QFT wavefunctionals

3- will confirm "ground state = **entangling evolution in scale**"

4- will elucidate relation to the **AdS/CFT** correspondence

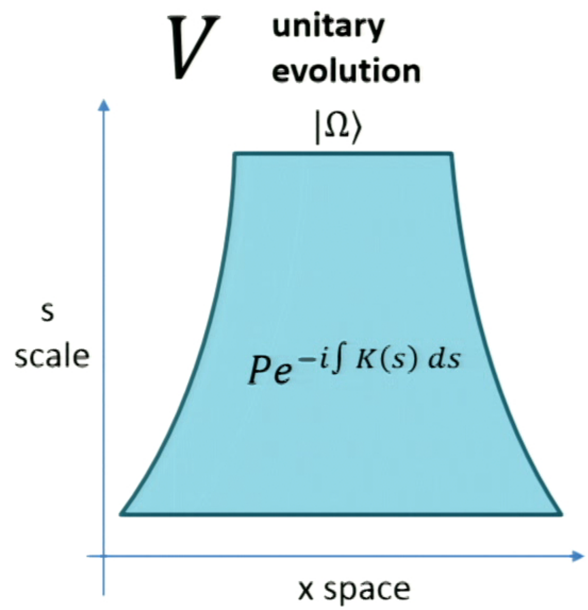
Gaussian cMERA already works for free QFTs!
Haegeman et al. PRL 2013

see poster by

Hugo Marrochio
(Perimeter)

and related talks by

Rob Leigh
Tadashi Takayanagi
Rob Myers
Bartek Czech



$$|\Psi^{\Lambda}\rangle = U|\Omega\rangle$$

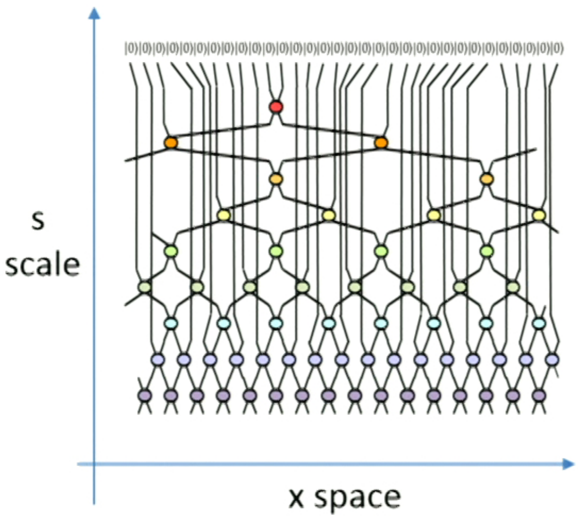
see poster by

Jordan Cotler
(Stanford)

PRL 2007,2008

MERA

V **quantum circuit**

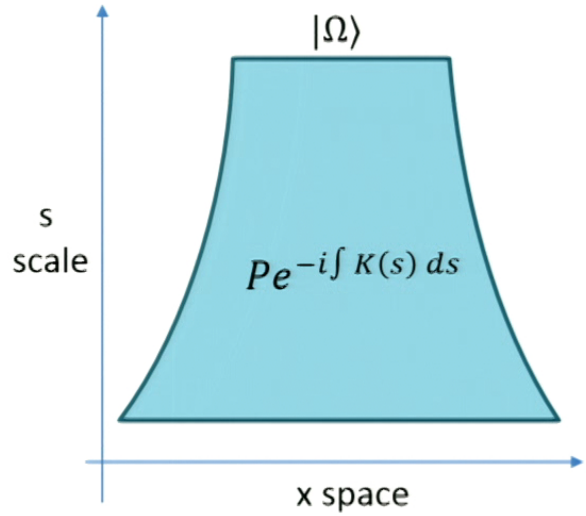


$$|\Psi^\Lambda\rangle = U|0\rangle^{\otimes N}$$

J. Haegeman, T. Osborne, H. Verschelde, F. Verstraete
PRL 2013

cMERA

V **unitary evolution**



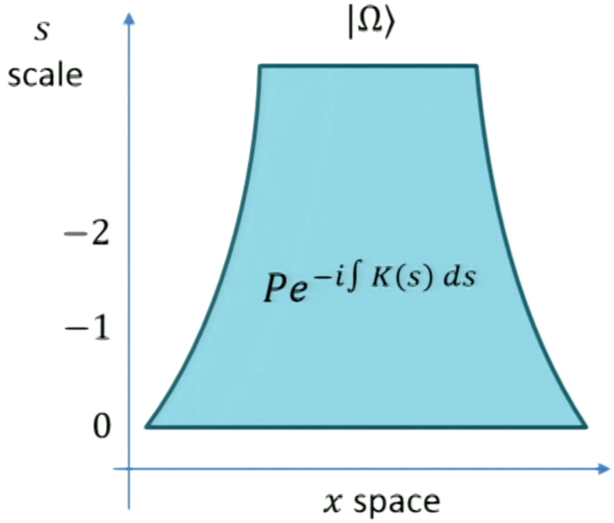
$$|\Psi^\Lambda\rangle = U|\Omega\rangle$$

lattice spacing
UV cut-off

$a = 1/\Lambda$

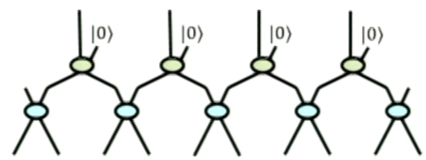
cMERA

J. Haegeman, T. Osborne, H. Verschelde, F. Verstraete
PRL 2013 (arXiv 2011)



$$|\Psi^{\Lambda}\rangle = Pe^{-i\int K(s)\,ds}|\Omega\rangle$$

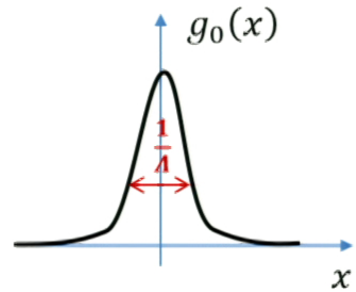
entangler $K(s) = \int k(s,x)\,dx$



- $k(s,x)$ is quasi-local

  for instance, for 1+1 boson CFT, we will choose

$$k(s,x) = -\frac{1}{2}\phi(x)\left[\int dy\; g_s(y-x)\pi(y)\right] + h.c.$$



$$g_s(y-x) = \frac{1}{2\pi}\,e^{-\frac{(e^s\Lambda)^2(x-y)^2}{4}}$$

cMERA for free boson CFT in 1+1 dimensions

(massless Klein-Gordon field)

Haegeman et al PRL 2013

Qi Hu, GV, arXiv:1703.04798

bosonic field operator $\phi(x)$

conjugate momentum operator $\pi(x)$

$$[\phi(x), \pi(y)] = i\delta(x-y)$$

momentum-space operators

$$\phi(k) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\, e^{-ikx} \phi(x) \qquad \pi(k) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\, e^{-ikx} \pi(x)$$

$$[\phi(k), \pi(q)] = i\delta(k+q)$$

A tale of three (Gaussian) states:

product state	cMERA (optimized)	CFT ground state
$\lvert\Lambda\rangle$	$\lvert\Psi^{\Lambda}\rangle$	$\lvert\Psi\rangle$

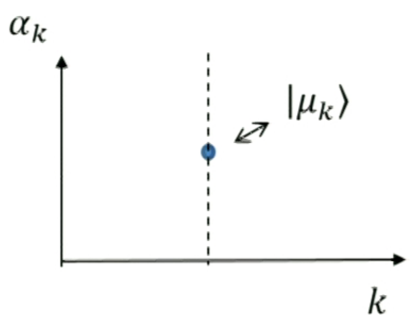
Note on Gaussian states

We consider translation invariant Gaussian states of the form

$$|\mu\rangle = \bigotimes_{k\in\mathbb{R}} |\mu_k\rangle$$

where $|\mu_k\rangle$ is a Gaussian state of the momentum k mode, characterized by the linear constraint

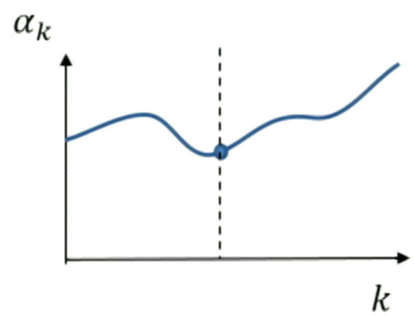
$$\left[\sqrt{\frac{\alpha_k}{2}}\,\phi(k) + i\sqrt{\frac{1}{2\alpha_k}}\,\pi(k)\right]|\mu_k\rangle = 0$$



Therefore $|\mu\rangle$ is completely characterized by the set of linear constraints

$$\left[\sqrt{\frac{\alpha_k}{2}}\,\phi(k) + i\sqrt{\frac{1}{2\alpha_k}}\,\pi(k)\right]|\mu\rangle = 0$$

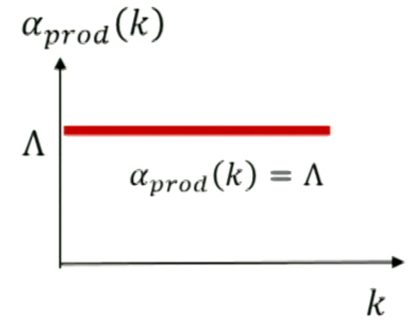
$$\forall k \in \mathbb{R}$$



complete set of linear constraints

product state

$$|\Lambda\rangle$$

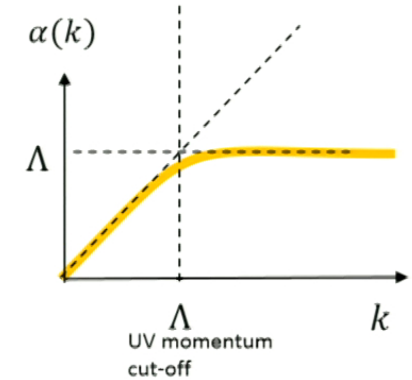


$$\left[\sqrt{\frac{\Lambda}{2}}\,\phi(k) + i\sqrt{\frac{1}{2\Lambda}}\,\pi(k)\right]|\Lambda\rangle = 0$$

$$\forall k \in \mathbb{R}$$

cMERA
(optimized)

$$|\Psi^{\Lambda}\rangle$$

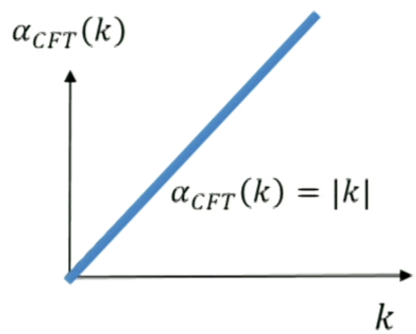


$$\left[\sqrt{\frac{\alpha(k)}{2}}\,\phi(k) + i\sqrt{\frac{1}{2\alpha(k)}}\,\pi(k)\right]|\Psi^{\Lambda}\rangle = 0$$

$$\forall k \in \mathbb{R}$$

CFT ground state

$$|\Psi\rangle$$



$$\left[\sqrt{\frac{|k|}{2}}\,\phi(k) + i\sqrt{\frac{1}{2|k|}}\,\pi(k)\right]|\Psi\rangle = 0$$

$$\forall k \in \mathbb{R}$$

product state

$$|\Lambda\rangle$$

product state (in real space):
no entanglement or correlations

$$|\Lambda\rangle = \bigotimes_{x \in \mathbb{R}} |\Lambda(x)\rangle$$

complete set of
linear constraints
in real space

$$\left[\sqrt{\frac{\Lambda}{2}}\,\phi(x) + i\sqrt{\frac{1}{2\Lambda}}\,\pi(x)\right]|\Lambda\rangle = 0$$

$$\forall x \in \mathbb{R}$$

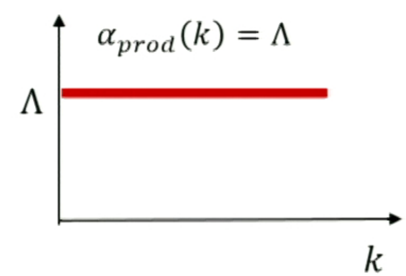
[think of a chain
of decoupled
harmonic oscillators]

Fourier
transform

complete set of
linear constraints
in momentum space

$$\left[\sqrt{\frac{\Lambda}{2}}\,\phi(k) + i\sqrt{\frac{1}{2\Lambda}}\,\pi(k)\right]|\Lambda\rangle = 0$$

$$\forall k \in \mathbb{R}$$



CFT ground state

free boson CFT
in 1+1 dimensions

$$[\phi(x), \pi(y)] = i\delta(x-y)$$

$$|\Psi\rangle$$

$$H_{CFT} = \frac{1}{2}\int_{-\infty}^{\infty} dx\, \left[\pi(x)^2 + \left(\partial_x \phi(x)\right)^2\right]$$

(massless Klein-Gordon field)

Fourier transform

$$H_{CFT} = \frac{1}{2}\int_{-\infty}^{\infty} dk [\pi(k)\pi(-k) + k^2\phi(k)\phi(-k)]$$

annihilation operators

$$a(k) \equiv \sqrt{\frac{|k|}{2}}\,\phi(k) + i\sqrt{\frac{1}{2|k|}}\,\pi(k)$$

$$H_{CFT} = \int_{-\infty}^{\infty} dk\ \ |k|\, a(k)^\dagger a(k)$$

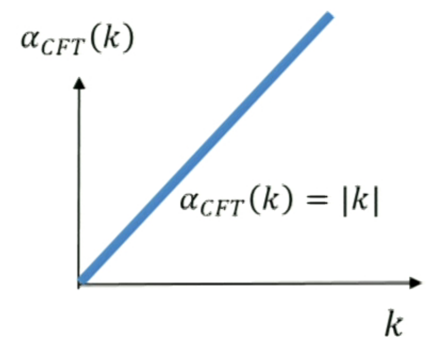
ground state

$$a(k)|\Psi\rangle = 0 \quad \forall k$$

complete set of
linear constraints
in momentum space

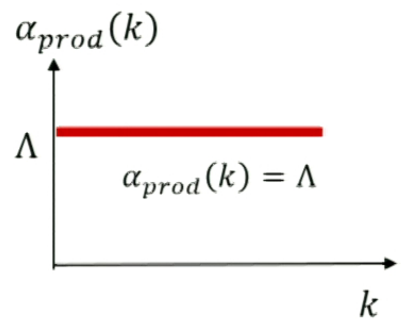
$$\left[\sqrt{\frac{|\mathrm{k}|}{2}}\,\phi(k) + i\sqrt{\frac{1}{2|\mathrm{k}|}}\,\pi(k)\right]|\Psi\rangle = 0$$

$$\forall k \in \mathbb{R}$$



product state

$|\Lambda\rangle$

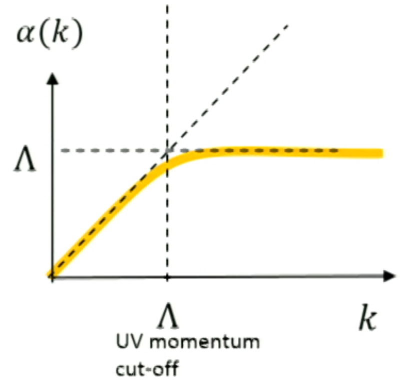


$$\left[\sqrt{\frac{\Lambda}{2}}\,\phi(k) + i\sqrt{\frac{1}{2\Lambda}}\,\pi(k)\right]|\Lambda\rangle = 0$$

$\forall k \in \mathbb{R}$

cMERA
(optimized)

$|\Psi^{\Lambda}\rangle$



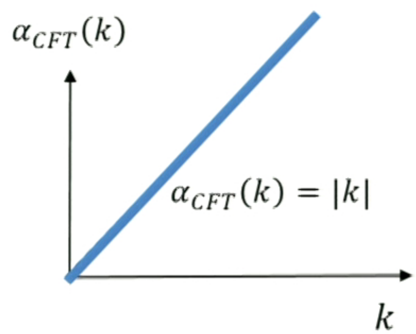
complete set of linear constraints

$$\left[\sqrt{\frac{\alpha(k)}{2}}\,\phi(k) + i\sqrt{\frac{1}{2\alpha(k)}}\,\pi(k)\right]|\Psi^{\Lambda}\rangle = 0$$

$\forall k \in \mathbb{R}$

CFT ground state

$|\Psi\rangle$

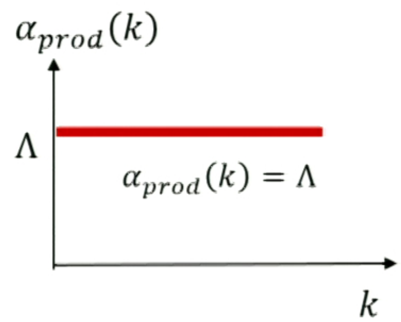


$$\left[\sqrt{\frac{|\mathrm{k}|}{2}}\,\phi(k) + i\sqrt{\frac{1}{2|\mathrm{k}|}}\,\pi(k)\right]|\Psi\rangle = 0$$

$\forall k \in \mathbb{R}$

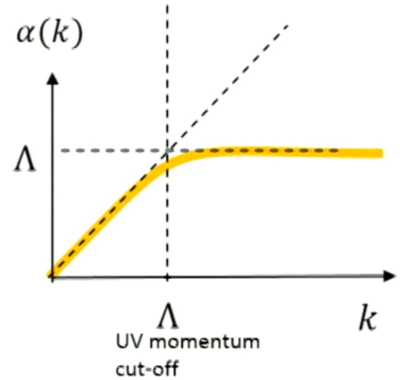
product state

$|\Lambda\rangle$



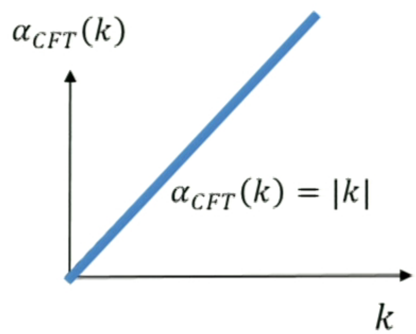
cMERA
(optimized)

$|\Psi^{\Lambda}\rangle$



CFT ground state

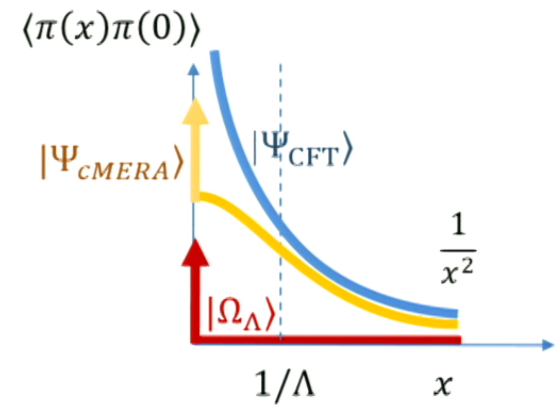
$|\Psi\rangle$



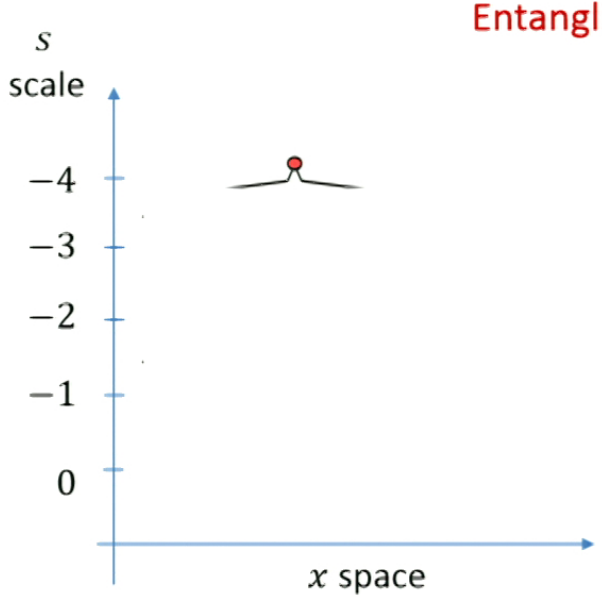
what does this mean?

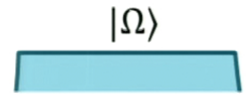
let us look at correlators:

$$\langle\pi(x)\pi(0)\rangle = \frac{1}{4\pi}\int dk\, e^{ik(x-y)}\alpha(k)$$

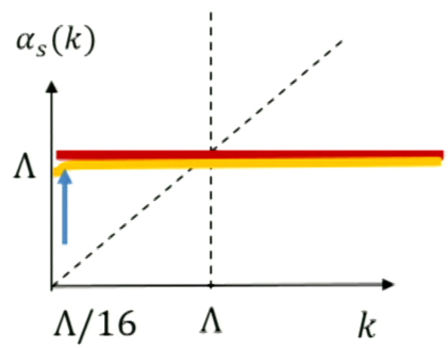


Entangling evolution in scale

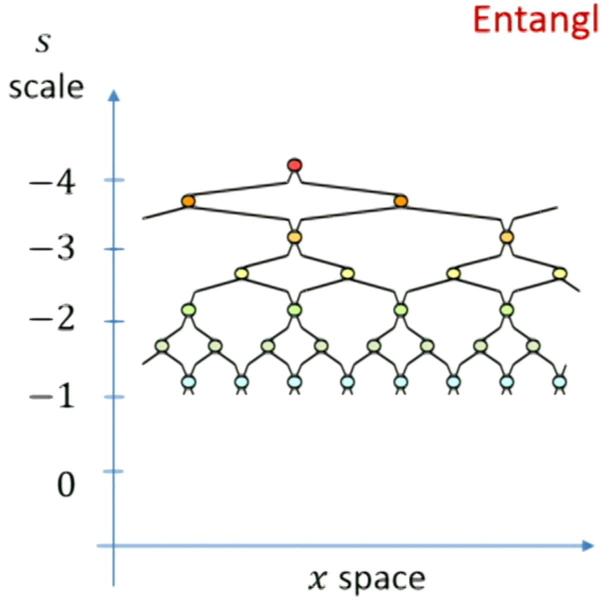


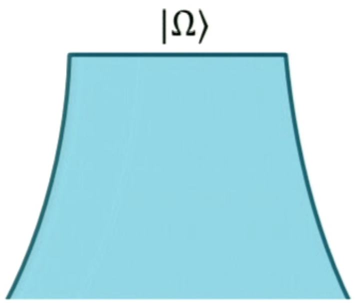


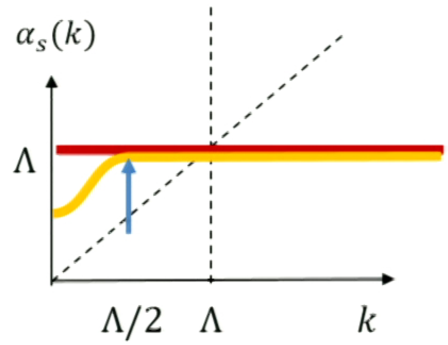
$Pe^{-i\int_{s_1}^{s_2} ds\, K(s)}$



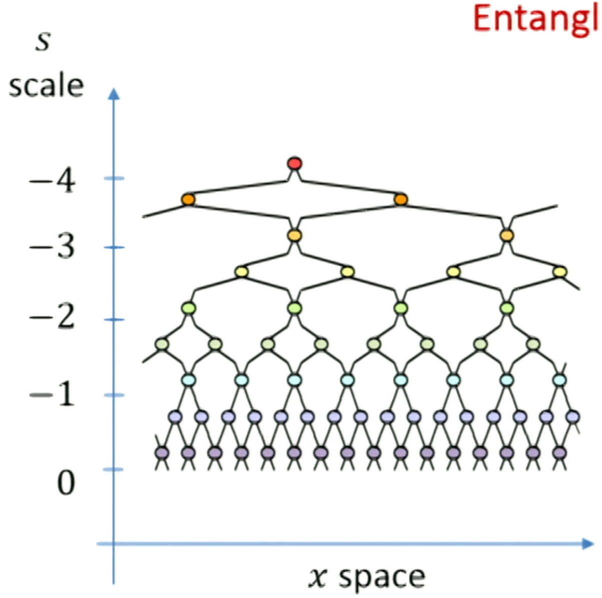
Entangling evolution in scale



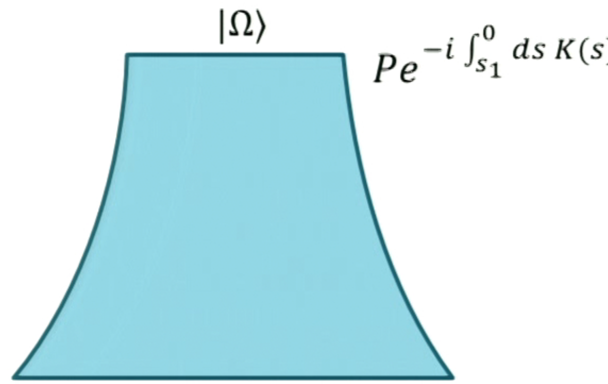




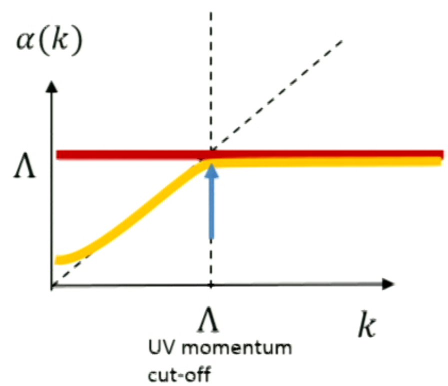
Entangling evolution in scale



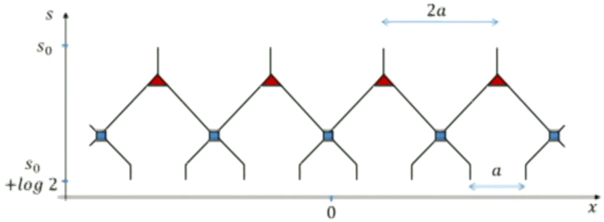
$$|\Psi^\Lambda\rangle = U|0\rangle^{\otimes N}$$

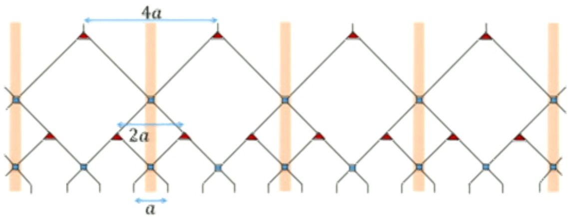


$$|\Psi^\Lambda\rangle = Pe^{-i\int_{s_1}^{0} ds\, K(s)}|\Omega\rangle$$



without rescaling of space



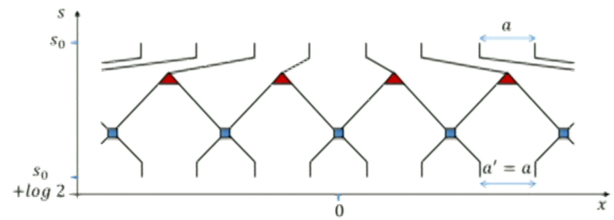


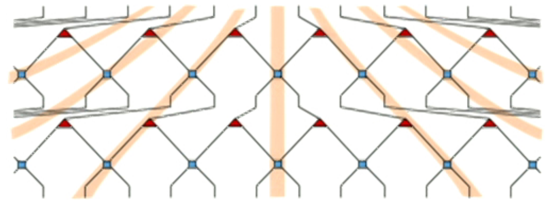
scale-dependent entangler $K(s)$

$K(s) = e^{-isL} K e^{isL}$
(interaction picture)

$Pe^{-i \int ds\, K(s)}$

with rescaling of space

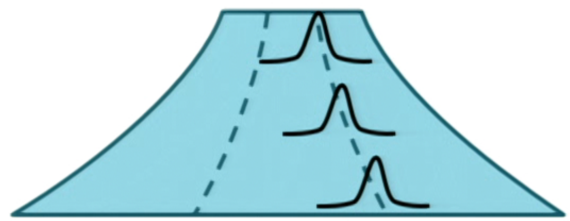




scale-independent entangler $K = K(0)$

entangler always acts at same length scale $1/\Lambda$

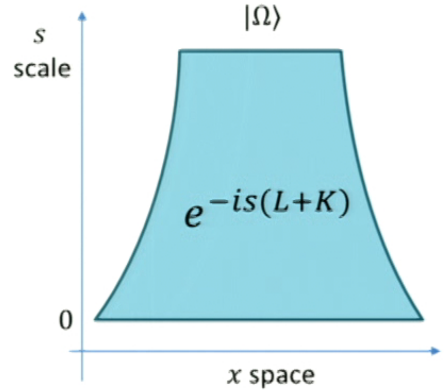
$e^{-i\Delta s\,(L+K)}$



expanding universe (deSitter)

cMERA

J. Haegeman, T. Osborne, H. Verschelde, F. Verstraete
PRL 2013 (arXiv 2011)



non-relativistic scaling operator
(rescaling of space and fields)

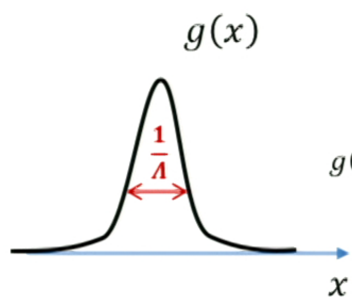
$$L \equiv -\frac{1}{2}\int dx \left[\pi(x)x\partial_x\phi(x) + \frac{1}{2}\,\phi(x)\pi(x) + h.c.\right]$$

$$x \to e^{s}x$$
$$\phi(x) \to e^{s/2}\phi(e^{s}x)$$
$$\pi(x) \to e^{s/2}\pi(e^{s}x)$$

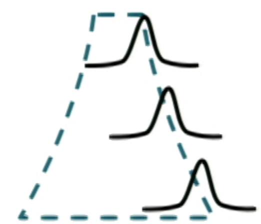
entangler

$$K \equiv -\frac{1}{2}\int dx\,\left(\phi(x)\left[\int dy\, g(y-x)\pi(y)\right] + h.c.\right)$$

for a CFT, K can be chosen to be independent of the scale parameter s



$$g(x) = \frac{1}{2\pi}\,e^{-\frac{\Lambda^2x^2}{4}}$$



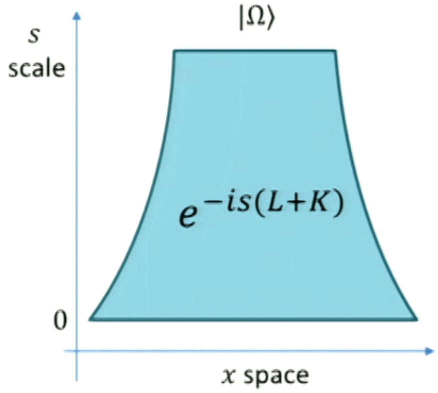
entangling evolution in scale, with rescaling

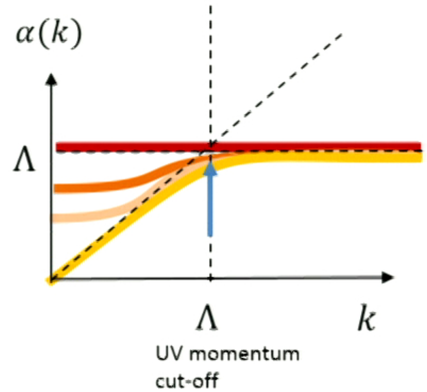
$$|\Psi^\Lambda(s)\rangle = e^{-is(L+K)}|\Omega\rangle$$

product state

$|\Omega\rangle = |\Lambda\rangle$

specific choice of product state





$|\Psi^\Lambda(0)\rangle = |\Lambda\rangle$

$|\Psi^\Lambda(s)\rangle$

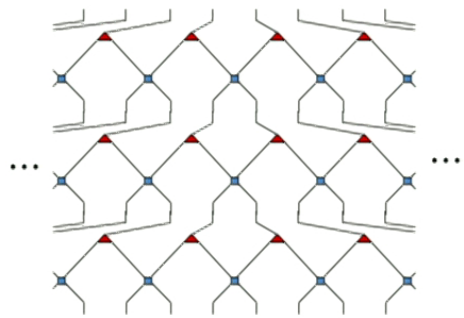
$|\Psi^\Lambda(\infty)\rangle = |\Psi^\Lambda\rangle$

$$|\Psi^\Lambda\rangle = \lim_{s\to\infty} e^{-is(L+K)}|\Omega\rangle$$

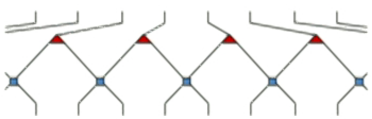
cMERA for CFT = fixed-point of entangling evolution

MERA for a critical theory

(infinite system)



$$|\Psi^{\Lambda}\rangle = \lim_{n\to\infty} W^{n}|\Omega\rangle$$

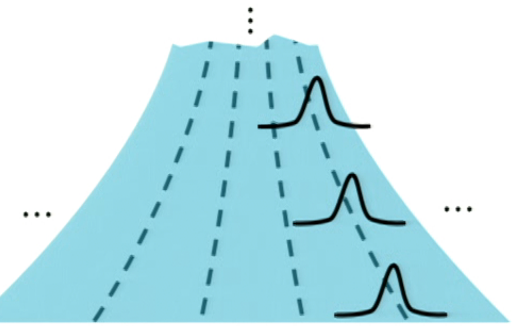
$W \equiv$ 

let us call W "scale transformation"

then $|\Psi^{\Lambda}\rangle$ is scale invariant,

$$\mathrm{W}|\Psi^{\Lambda}\rangle = \ \Psi^{\Lambda}\rangle$$

cMERA for a CFT



$$|\Psi^{\Lambda}\rangle = \lim_{s\to\infty} e^{-is(L+K)}|\Omega\rangle$$

$e^{-i\Delta s(L+K)}$ 

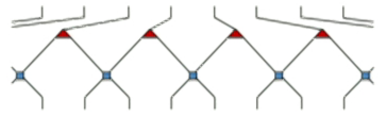
let us call $L + K$
"generator of scale transformations"

then $|\Psi^{\Lambda}\rangle$ is scale invariant,

$$(L+K)|\Psi^{\Lambda}\rangle = 0$$

useful definition of scale transformation/invariance?

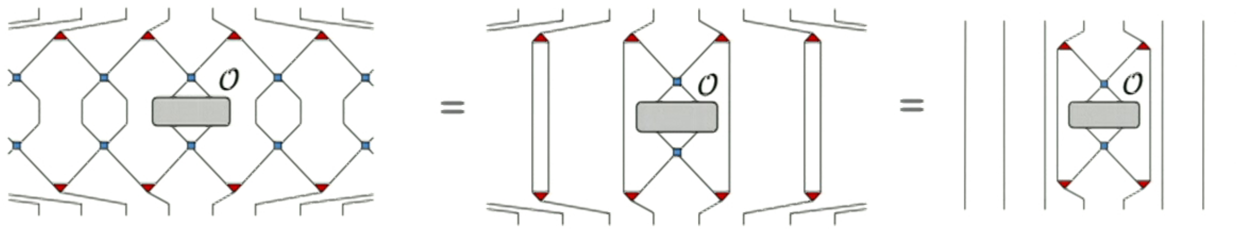
$W \equiv$



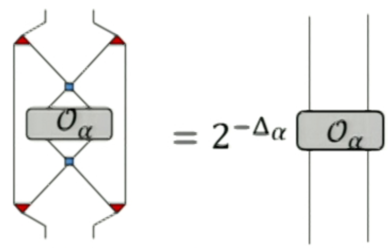
Action of W on a local operator $\mathcal{O}$ $\qquad \mathcal{O}' = W^\dagger \mathcal{O} W$

W

$W^\dagger$



linear map for
local operator $\mathcal{O}$
→ eigenvectors?



Δ_α scaling dimension
of scaling operator $\mathcal{O}_\alpha$

more generally,
we can extract
conformal data
from MERA

$\mathcal{O}_\alpha$
scaling operator

Δ_α scaling dimension

s_α conformal spin

$C_{\alpha\beta\gamma}$ operator product expansion (OPE)

useful definition of scale transformation/invariance?

$e^{-i\Delta s(L+K)}$ 

linear map on local operators

$$-i[L + K, \mathcal{O}_\alpha(0)] = \Delta_\alpha \mathcal{O}_\alpha(0)$$

Δ_α scaling dimension of scaling operator $\mathcal{O}_\alpha$

notice that the cut-off Λ is finite

K entangles → changes the cut-off from Λ to Λ'

L rescales space → the cut-off goes back to Λ

Scaling operators

(free boson 1+1)

dilation operator $L+K$ is quadratic and does not mix $\phi(x)$ and $\pi(x)$

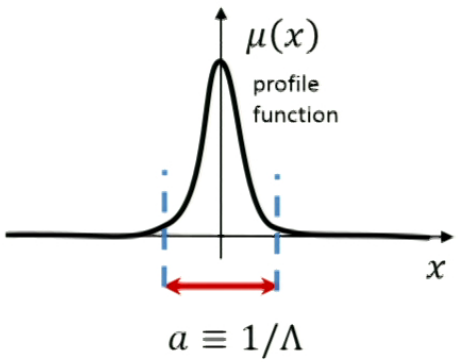
Qi Hu, G. V.
arXiv:1703.04798

ansatz for linear scaling operators:

$$\phi^{\Lambda}(x) = \int dx\, \mu_{\phi}(x-y)\phi(y)$$

$$\pi^{\Lambda}(x) = \int dx\, \mu_{\pi}(x-y)\pi(y)$$

$\mu_{\phi}(x)$, $\mu_{\pi}(x)$ local profile functions



$$i[L+K, \mathcal{O}_{\alpha}(0)] = -\Delta_{\alpha}\mathcal{O}_{\alpha}(0)$$

equation for profile $\mu_{\alpha}(x)$

exact solution with exact CFT scaling dimensions!:

ϕ^{Λ}: $\Delta_{\phi} = 0$ $\quad \mu_{\phi}(x) = \int dk\, e^{-ikx}\sqrt{\frac{\alpha(k)}{|k|}}$

π^{Λ}: $\Delta_{\pi} = 1$ $\quad \mu_{\pi}(x) = \int dk\, e^{-ikx}\sqrt{\frac{|k|}{\alpha(k)}}$

scaling operators are smeared fields

with

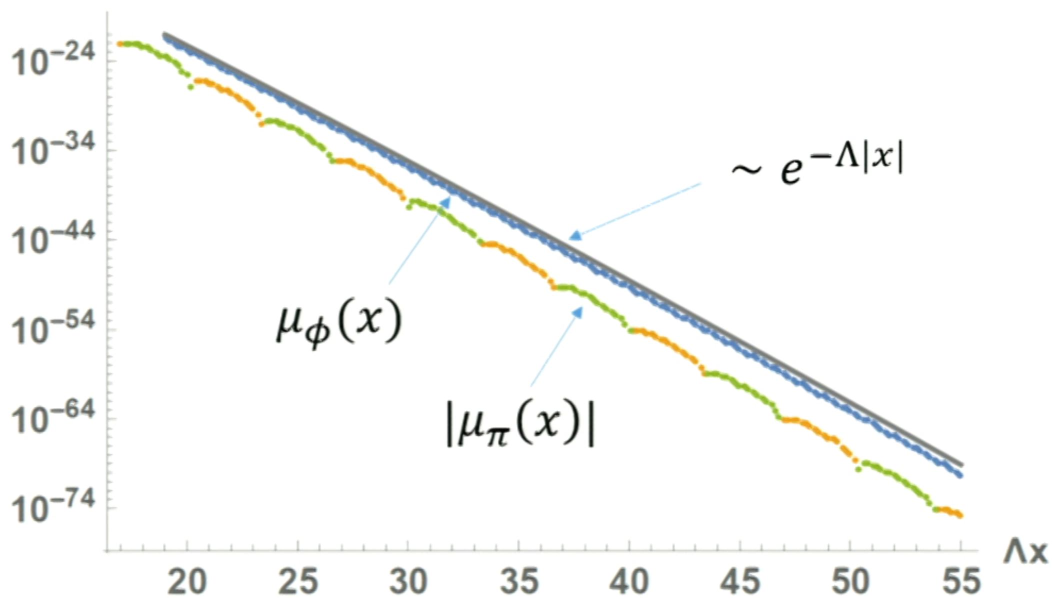
$\mu_{\phi}(x), \mu_{\pi}(x) \sim e^{-\Lambda|x|}$

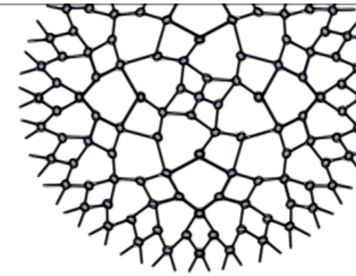
for $|x| \gg 1/\Lambda$

Smeared fields

$$\phi^\Lambda(x) = \int dx\, \mu_\phi(x-y)\phi(y)$$

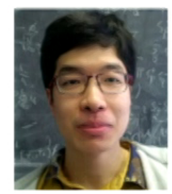
$$\pi^\Lambda(x) = \int dx\, \mu_\pi(x-y)\pi(y)$$



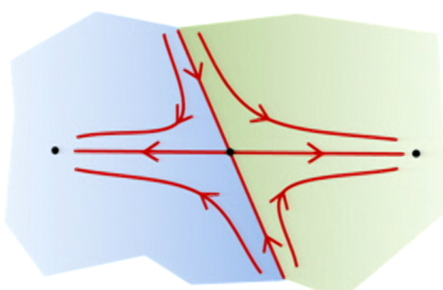


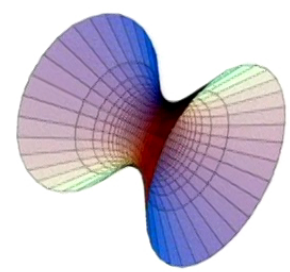
- What is cMERA? (1+1 free boson CFT)
- Entangling evolution in scale
- Scale invariance
- Conformal symmetry

Qi Hu, G. V.
"Spacetime symmetries and conformal data in cMERA"
arXiv:1703.04798



Qi Hu





Consider the symplectic map/canonical transformation

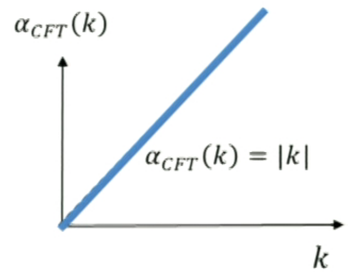
$$V\,\phi(k)\,V^{\dagger} = \sqrt{\frac{\alpha(k)}{|k|}}\,\phi(k) \qquad V\,\pi(k)\,V^{\dagger} = \sqrt{\frac{|k|}{\alpha(k)}}\,\pi(k)$$

Then $|\Psi^{\Lambda}\rangle = V|\Psi\rangle$

Indeed:

CFT ground state

$|\Psi\rangle$

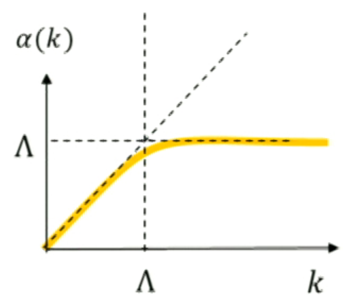


$$\left[\sqrt{\frac{|k|}{2}}\,\phi(k) + i\sqrt{\frac{1}{2|k|}}\,\pi(k)\right]|\Psi\rangle = 0 \qquad \forall k \in \mathbb{R}$$

$\downarrow V$

cMERA

$|\Psi^{\Lambda}\rangle$



$$\left[\sqrt{\frac{\alpha(k)}{2}}\,\phi(k) + i\sqrt{\frac{1}{2\alpha(k)}}\,\pi(k)\right]|\Psi^{\Lambda}\rangle = 0 \qquad \forall k \in \mathbb{R}$$

V acts non-trivially at short distances $\leq 1/\Lambda$

Remarks:

1) $\phi(x) \to V\,\phi(x)\,V^\dagger \equiv \phi^\Lambda(x)$

$\pi(x) \to V\,\pi(x)\,V^\dagger \equiv \pi^\Lambda(x)$

smeared scaling operators!

2) V commutes with space derivative ∂_x

$$\partial_x\phi(x) \to V\,\partial_x\phi(x)\,V^\dagger = \partial_x\left(V\phi(x)V^\dagger\right) = \partial_x\phi^\Lambda(x)$$

Then:

CFT

left movers

$$\partial\phi(x) \equiv \frac{1}{2}(\partial_x\phi(x) - \pi(x))$$

right movers

$$\bar{\partial}\phi(x) \equiv \frac{1}{2}\left(\partial_x\phi(x) + \pi(x)\right)$$

stress tensor

$$T(x) \equiv :\partial\phi(x)\partial\phi(x):$$

$$\bar{T}(x) \equiv :\bar{\partial}\phi(x)\bar{\partial}\phi(x):$$

local density

normal order : : is with respect to CFT annihilation operators

$$a(k) \equiv \sqrt{\frac{|k|}{2}}\,\phi(k) + i\sqrt{\frac{1}{2|k|}}\,\pi(k)$$

cMERA

left movers (smeared)

$$\partial\phi^\Lambda(x) \equiv \frac{1}{2}\left(\partial_x\phi^\Lambda(x) - \pi^\Lambda(x)\right)$$

right movers (smeared)

$$\bar{\partial}\phi^\Lambda(x) \equiv \frac{1}{2}\left(\partial_x\phi^\Lambda(x) + \pi^\Lambda(x)\right)$$

stress tensor (quasi-local)

$$T^\Lambda(x) \equiv :\partial\phi^\Lambda(x)\partial\phi^\Lambda(x):$$

$$\bar{T}^\Lambda(x) \equiv :\bar{\partial}\phi^\Lambda(x)\bar{\partial}\phi^\Lambda(x):$$

local only at distances larger than $1/\Lambda$

normal order : : is with respect to cMERA annihilation operators

$$a^\Lambda(k) \equiv \sqrt{\frac{\alpha(k)}{2}}\,\phi(k) + i\sqrt{\frac{1}{2\alpha(k)}}\,\pi(k)$$

CFT

stress tensor

$$T(x) \equiv :\partial\phi(x)\partial\phi(x):$$

$$\bar{T}(x) \equiv :\bar{\partial}\phi(x)\bar{\partial}\phi(x):$$

local density

cMERA

$$T^{\Lambda}(x) = V\,T(x)\,V^{\dagger}$$

$$\bar{T}^{\Lambda}(x) = V\,\bar{T}(x)\,V^{\dagger}$$

stress tensor (quasi-local)

$$T^{\Lambda}(x) = :\partial\phi^{\Lambda}(x)\partial\phi^{\Lambda}(x):$$

$$\bar{T}^{\Lambda}(x) = :\bar{\partial}\phi^{\Lambda}(x)\bar{\partial}\phi^{\Lambda}(x):$$

local only at distances larger than $1/\Lambda$

energy density

$$h(x) = T(x) + \bar{T}(x)$$

$$= :\frac{\pi(x)^2 + (\partial_x\phi(x))^2}{2}:$$

energy density (quasi-local)

$$h^{\Lambda}(x) = Vh(x)V^{\dagger} = T^{\Lambda}(x) + \bar{T}^{\Lambda}(x)$$

$$= :\frac{\pi^{\Lambda}(x)^2 + (\partial_x\phi^{\Lambda}(x))^2}{2}:$$

momentum density

$$p(x) = T(x) - \bar{T}(x)$$

$$= :\pi(x)\partial_x\phi(x):$$

momentum density (quasi-local)

$$p^{\Lambda}(x) = Vp(x)V^{\dagger} = T^{\Lambda}(x) - \bar{T}^{\Lambda}(x)$$

$$= :\pi^{\Lambda}(x)\partial_x\phi^{\Lambda}(x):$$

Spacetime symmetries

$G^{\Lambda} \equiv V\, G^{CFT}\, V^{\dagger}$

CFT		cMERA	
Hamiltonian			
$H = \int_{-\infty}^{\infty} dx\, h(x)$ $= \int dk\, \lvert k\rvert a_k^{\dagger}\, a_k$	$H\lvert\Psi\rangle = 0$	$H^{\Lambda} = \frac{1}{2}\int_{-\infty}^{\infty} dx\, h^{\Lambda}(x)$ $= \int dk\, \lvert k\rvert {a^{\Lambda}}_k^{\dagger}\, a_k^{\Lambda}$	$H^{\Lambda}\lvert\Psi^{\Lambda}\rangle = 0$
Momentum			
$P = \int_{-\infty}^{\infty} dx\, p(x)$ $= \int dk\, k\, a_k^{\dagger} a_k$	$P\lvert\Psi\rangle = 0$	$P^{\Lambda} = \int_{-\infty}^{\infty} dx\, p^{\Lambda}(x)$ $= \int dk\, k\, {a^{\Lambda}}_k^{\dagger} a_k^{\Lambda} \quad (= \mathrm{P})$	$P^{\Lambda}\lvert\Psi^{\Lambda}\rangle = 0$
Dilation			
$D = \int_{-\infty}^{\infty} dx\, x\, p(x)$ $= \int dk\, a_k^{\dagger}\left(k\partial_k + \frac{1}{2}\right) a_k$	$D\lvert\Psi\rangle = 0$	$D^{\Lambda} = \int_{-\infty}^{\infty} dx\, x\, p^{\Lambda}(x)$ $= \int dk\, a_k^{\Lambda\dagger}\left(k\partial_k + \frac{1}{2}\right) a_k^{\Lambda}$	$D^{\Lambda} = L + K$!!!
Boost			
$B = \frac{1}{2}\int_{-\infty}^{\infty} dx\, x\, h(x)$ $= \int dk\, sgn(k)\, a_k^{\dagger}\left(k\partial_k + \frac{1}{2}\right) a_k$	$B\lvert\Psi\rangle = 0$	$B^{\Lambda} = \frac{1}{2}\int_{-\infty}^{\infty} dx\, x\, h^{\Lambda}(x)$ $= \int dk\, sgn(k)\, {a^{\Lambda}}_k^{\dagger}\left(k\partial_k + \frac{1}{2}\right) a_k^{\Lambda}$	

dilation and boost operators $D^{\Lambda}, \mathrm{B}^{\Lambda}$ → scaling operators $\mathcal{O}_{\alpha}$

$$-i\left[D^{\Lambda}, \mathcal{O}_{\alpha}(0)\right] = \Delta_{\alpha}\mathcal{O}_{\alpha}(0)$$

Δ_{α} scaling dimension

$$-i\left[B^{\Lambda}, \mathcal{O}_{\alpha}(0)\right] = \mathrm{s}_{\alpha}\mathcal{O}_{\alpha}(0)$$

s_{α} conformal spin

$$\langle\Psi^{\Lambda}|\mathcal{O}_{\alpha}\mathcal{O}_{\beta}\mathcal{O}_{\gamma}|\Psi^{\Lambda}\rangle \sim C_{\alpha\beta\gamma}$$

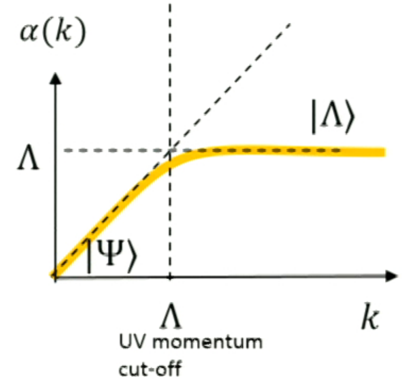
$C_{\alpha\beta\gamma}$ operator product expansion (OPE)

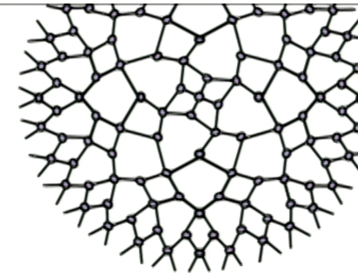
from cMERA we can extract the conformal data of the CFT

Summary:

cMERA
(optimized)

$|\Psi^{\Lambda}\rangle$





two new results: $|\Psi^{\Lambda}\rangle$ retains (quasi-local version of) the symmetries of CFT ground state $|\Psi\rangle$

Qi Hu, G. V.
“Spacetime symmetries and conformal data in cMERA”
arXiv:1703.04798

$|\Psi^{\Lambda}\rangle$ extract conformal data $\mathcal{O}_{\alpha} \rightarrow \{\Delta_{\alpha}, s_{\alpha}, C_{\alpha\beta\gamma}\}$