Title: Emergence of conformal symmetry in critical spin chains

Date: Apr 18, 2017 11:00 AM

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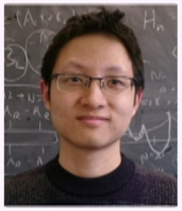
Abstract: We demonstrate that 1+1D conformal symmetry emerges in critical spin chains by constructing a lattice ansatz Hn for (certain combinations of) the Virasoro generators Ln. The generators Hn offer a new way of extracting conformal data from the low energy eigenstates of the lattice Hamiltonian on a finite circle. In particular, for each energy eigenstate, we can now identify which Virasoro tower it belongs to, as well as determine whether it is a Virasoro primary or a descendant (and similarly for global conformal towers and global conformal primaries/descendants). The central charge is obtained from a simple ground-state expectation value. Non-universal, finite-size corrections are the main source of error. We propose and demonstrate the use of periodic Matrix Product States, together with an improved ground state solver, to reach larger system sizes. We uncover that, importantly, the MPS single-particle excitation ansatz accurately describes all low energy excited states.

Emergence of conformal symmetry in critical spin chains and Matrix Product States

Ashley Milsted, Yijian Zou, Guifré Vidal



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TENSOR NETWORKS INITIATIVE

Simons Collaboration on the Many Electron Problem


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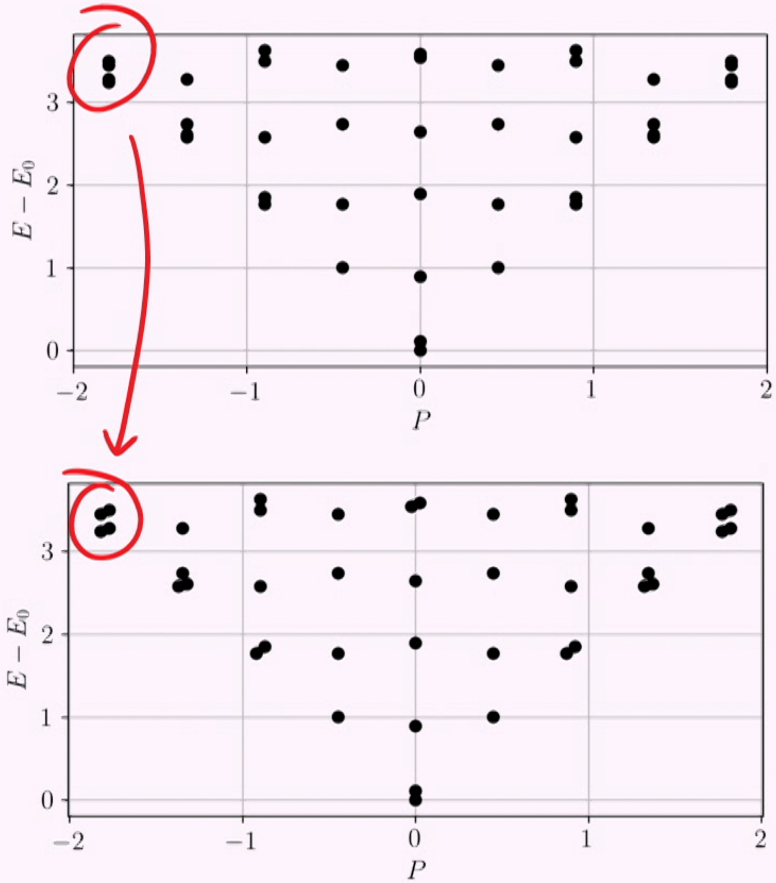
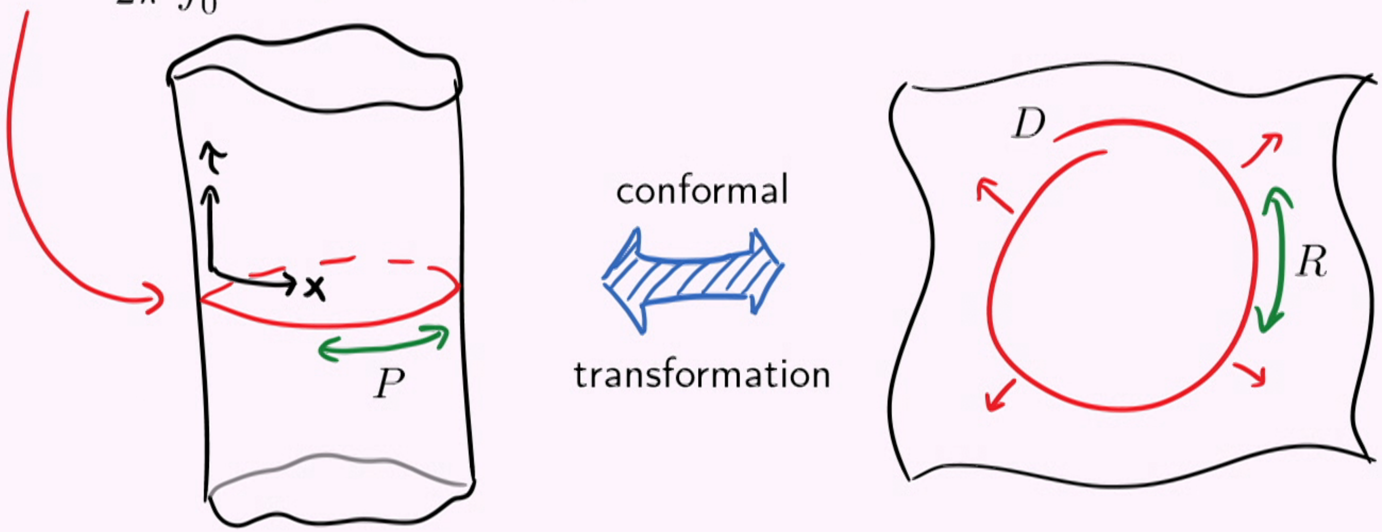


DANGER:

WE ABUSE THE X-AXIS THROUGHOUT THIS TALK!

state-operator correspondence

$$H = \frac{1}{2\pi} \int_0^L \mathrm{d}x \left(T(x) + \overline{T}(x)\right)$$



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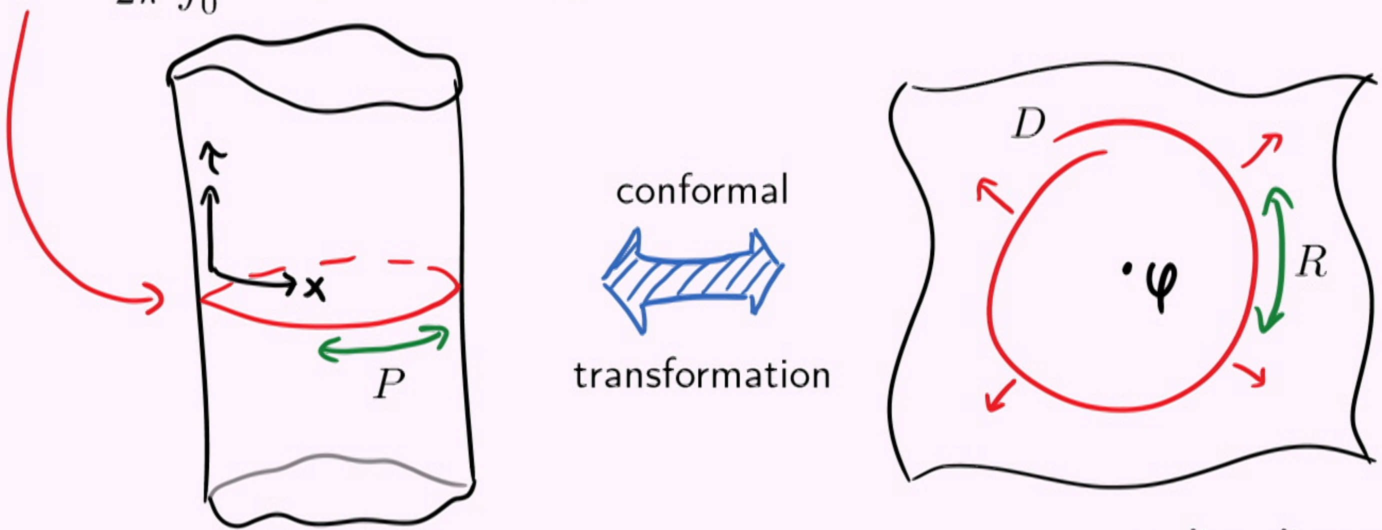


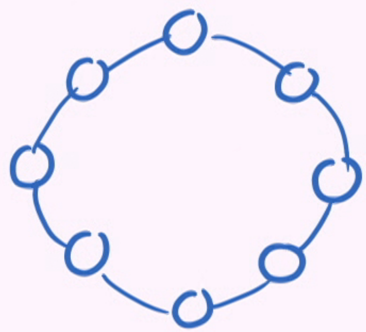
scaling dimension

$$D\varphi(0) = \Delta_\varphi \varphi(0)$$
$$R\varphi(0) = s_\varphi \varphi(0)$$

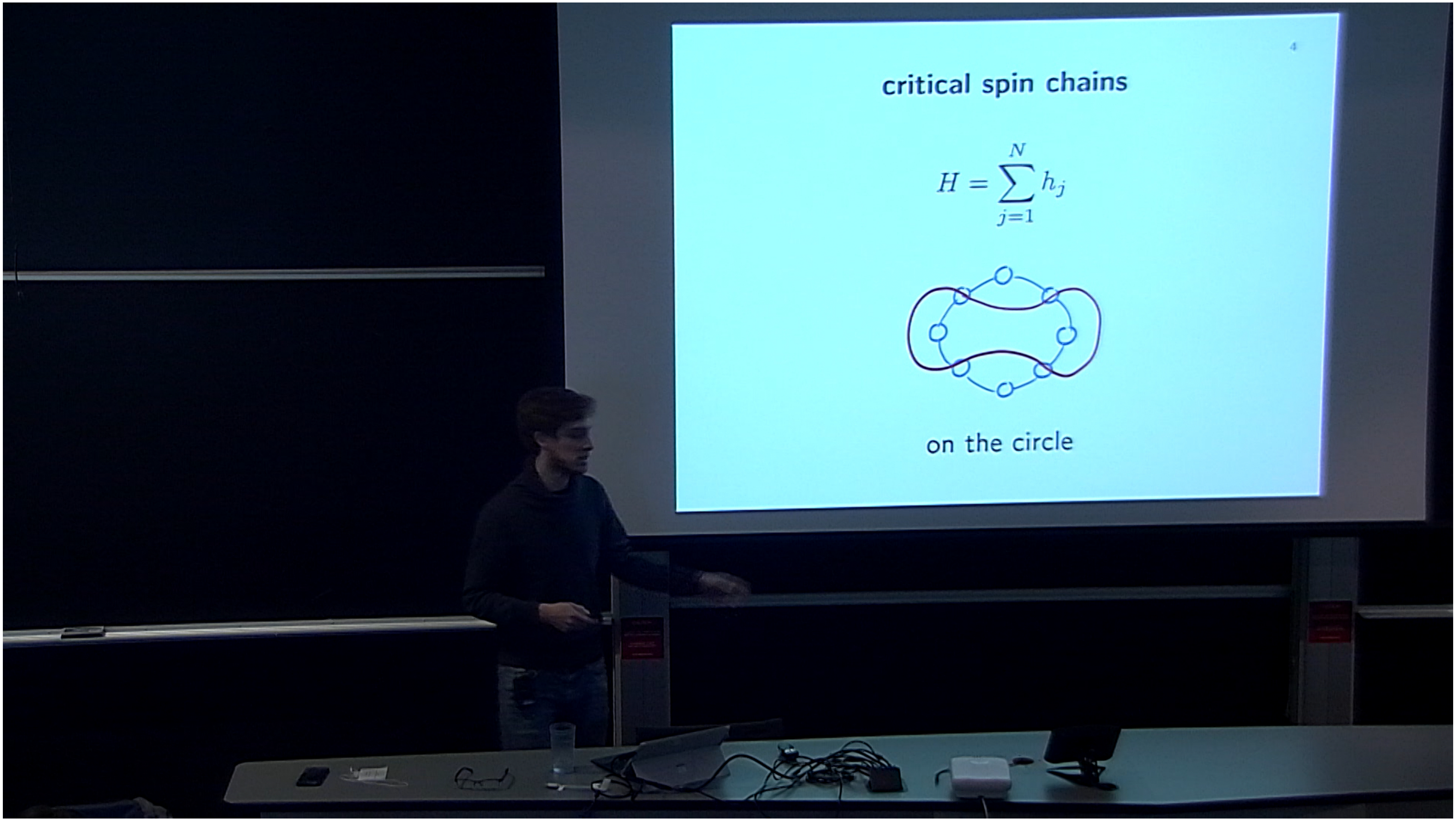
conformal spin

critical spin chains

$$H = \sum_{j=1}^{N} h_j$$



on the circle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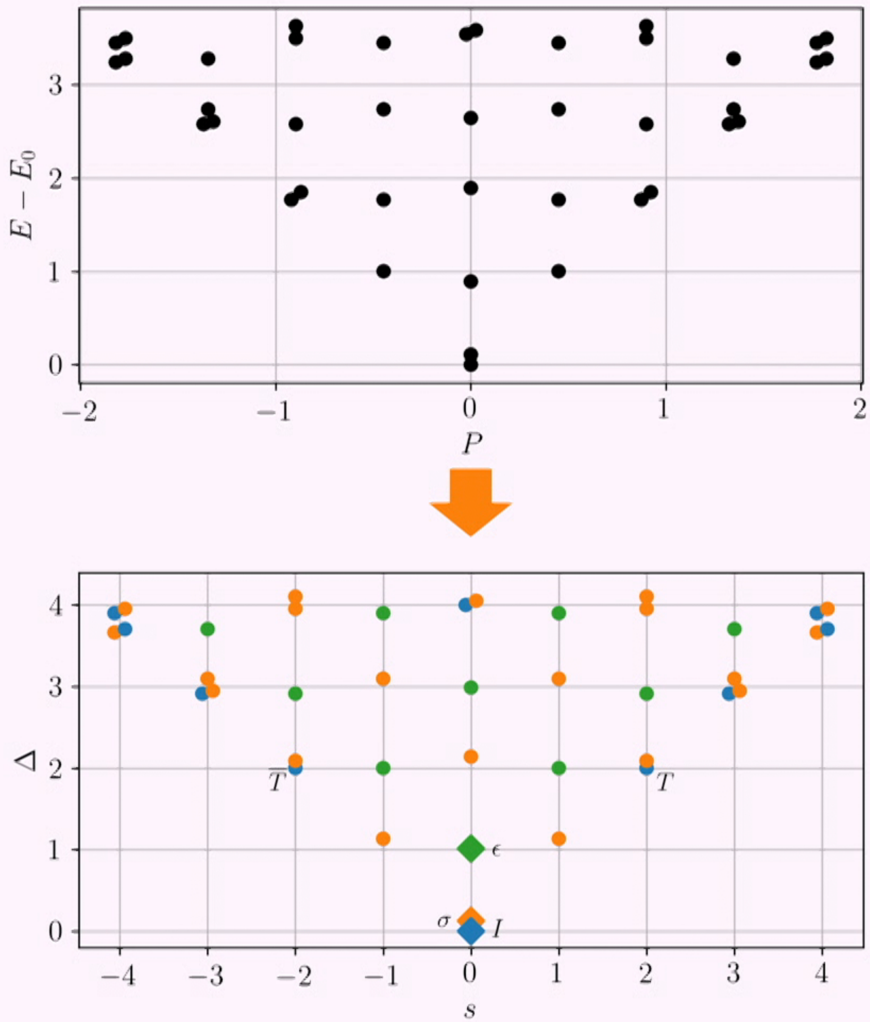
critical spin chains
$H = \sum_{j=1}^{N} h_j$
on the circle



method to identify **spin chain** energy eigenstates with **CFT operators**

...

for a generic CFT

mode expansion of hamiltonian density

CFT

$$H = \frac{1}{2\pi}\int_0^L \mathrm{d}x \left(T(x) + \overline{T}(x)\right)$$

spin chain

mode expansion of hamiltonian density

CFT

$$H = \frac{1}{2\pi}\int_0^L \mathrm{d}x\,\left(T(x) + \overline{T}(x)\right)$$

$$T(x) + \overline{T}(x) = 2\pi \sum_n e^{inx\frac{2\pi}{L}} H_n$$

spin chain

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$$H_n = \frac{1}{2\pi} \int_0^L \mathrm{d}x \; e^{-inx \frac{2\pi}{L}} \left(T(x) + \overline{T}(x) \right)$$

spin chain

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spin chain

$$H = \sum_{j=1}^{N} h_j$$

$$h_j = \frac{1}{N\eta}\sum_n e^{inj\frac{2\pi}{N}} H_n$$

mode expansion of hamiltonian density

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spin chain

$$H = \sum_{j=1}^{N} h_j$$

$$h_j = \frac{1}{N\eta}\sum_n e^{inj\frac{2\pi}{N}} H_n$$

$$H_n \equiv \eta\sum_{j=1}^{N} e^{-inj\frac{2\pi}{N}} h_j$$

mode expansion of hamiltonian density

CFT

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our proposal

$$H_n \equiv \eta \sum_{j=1}^{N} e^{-inj\frac{2\pi}{N}} h_j$$

mode expansion of hamiltonian density

6

CFT

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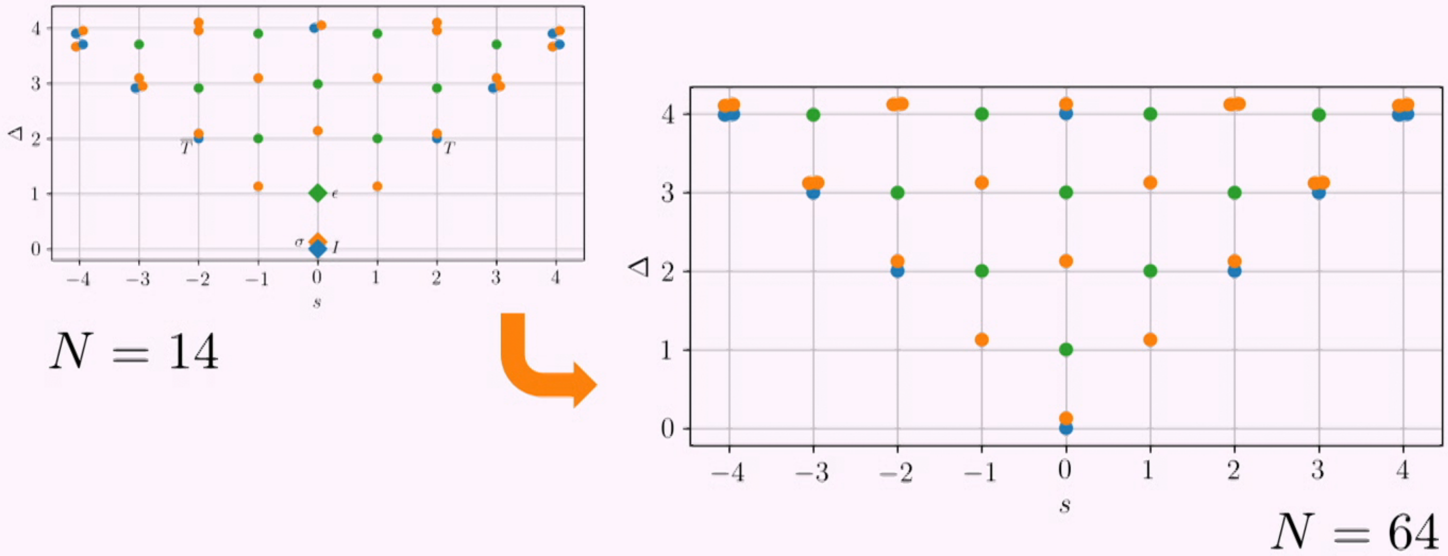
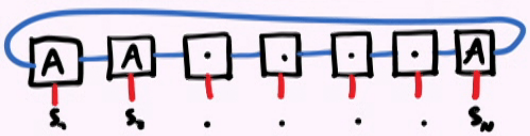
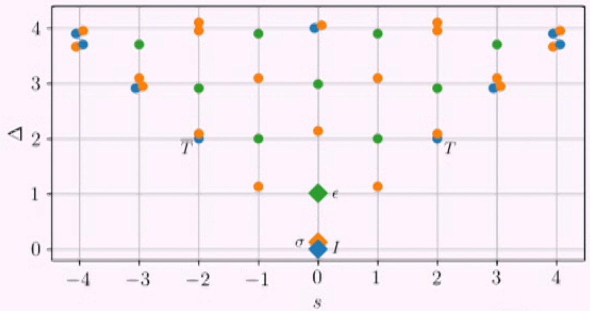
$$H_n \equiv \eta \sum_{j=1}^{N} e^{-inj\frac{2\pi}{N}} h_j$$

$$H_0 = \eta H$$

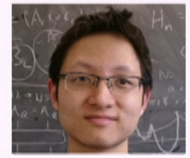


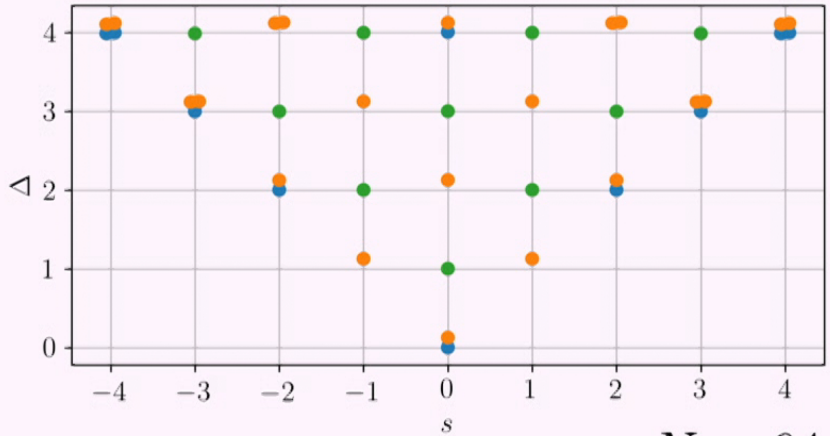


Matrix Product States **on the circle** to reach larger systems

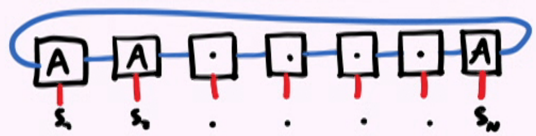


$N = 14$





$N = 64$



Matrix Product States **on the circle**
to reach larger systems

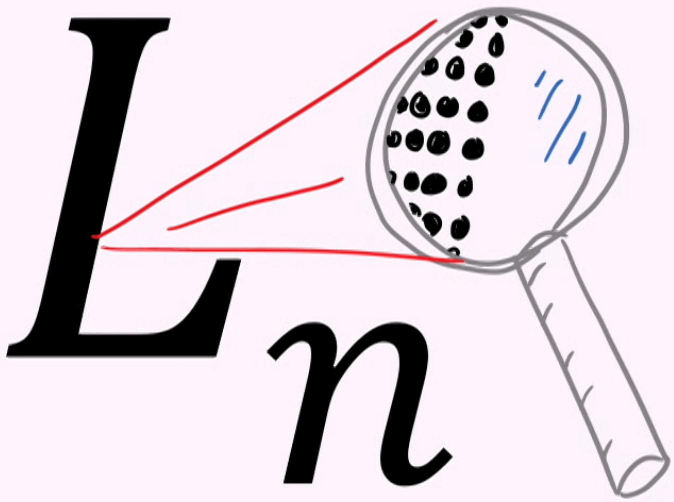
outline

emergence of 2D conformal symmetry

outline

emergence of 2D conformal symmetry

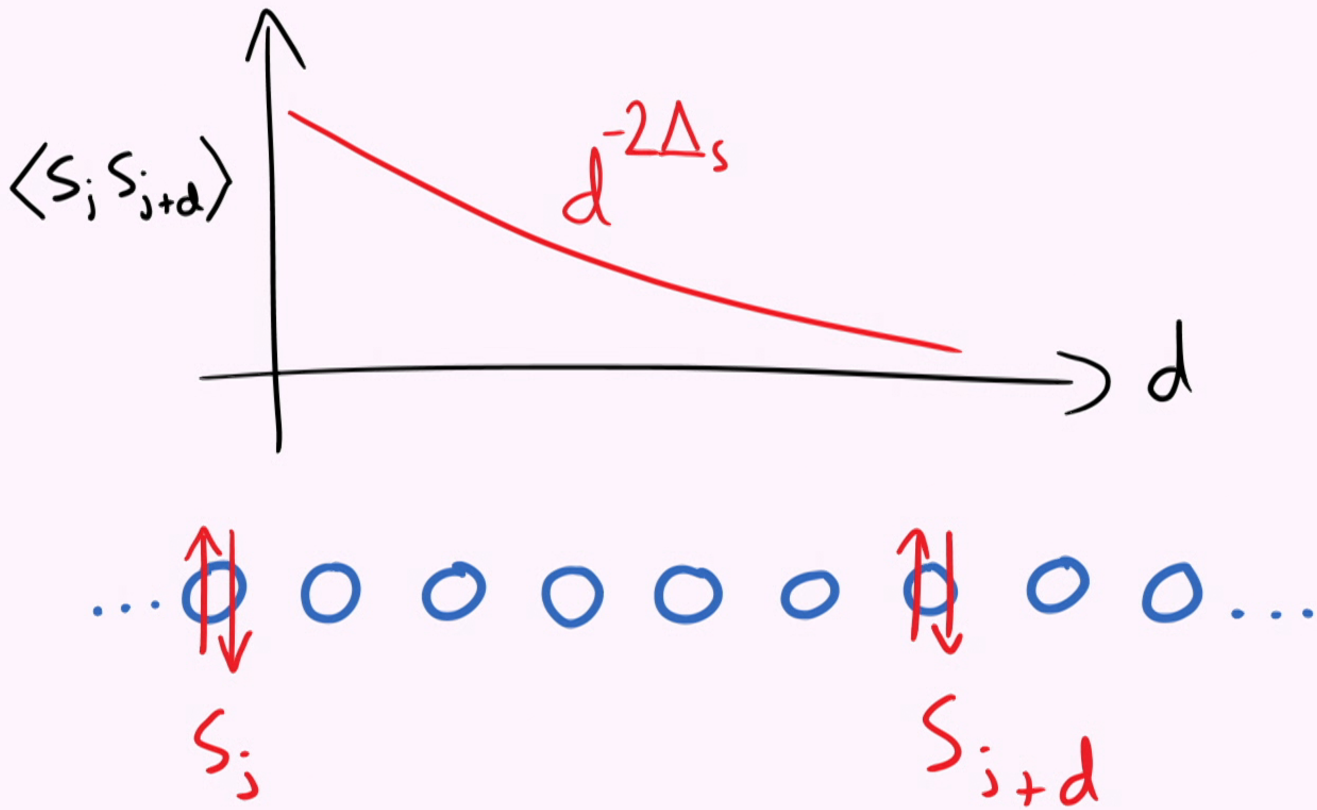
extracting **conformal data** using H_n



conformal symmetry in 1+1D

emergence on the lattice

critical many-body systems



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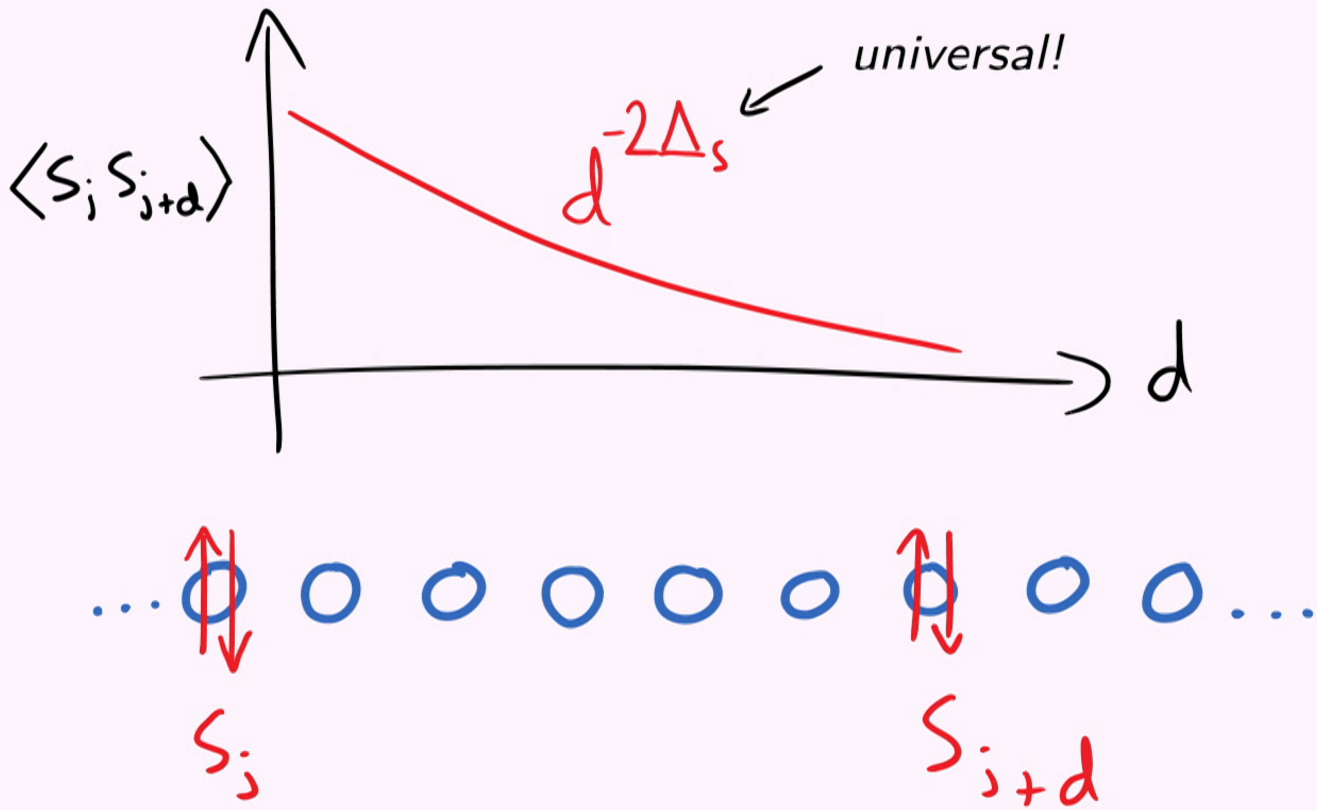
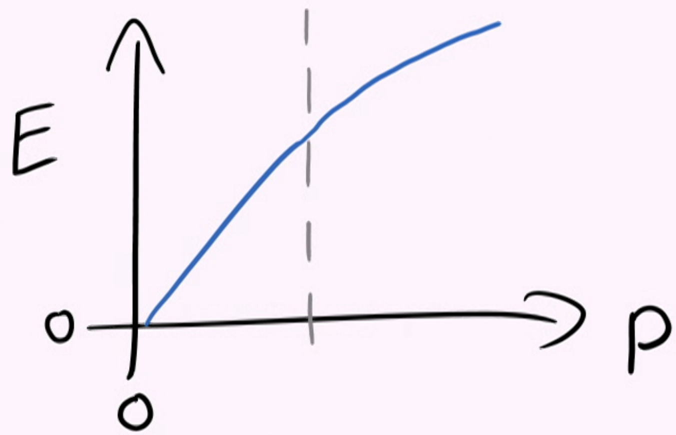
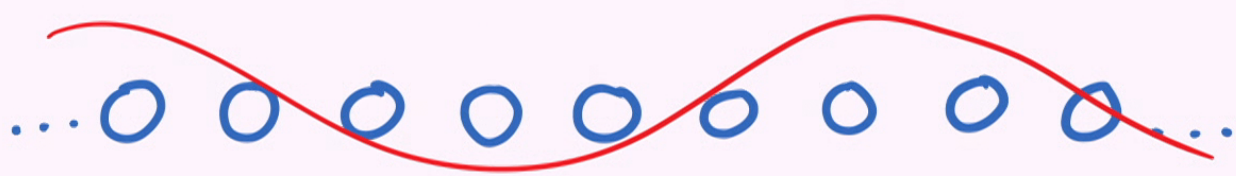


emergent symmetry



Lorentz

$$E(p) \propto |p|$$



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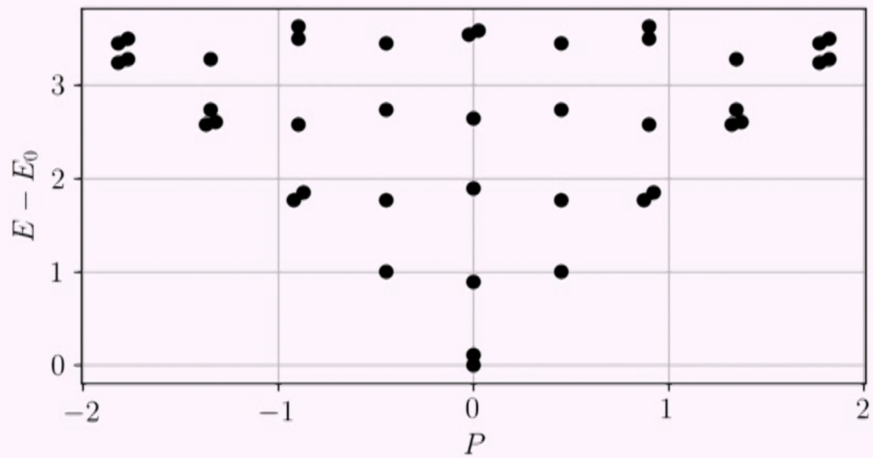


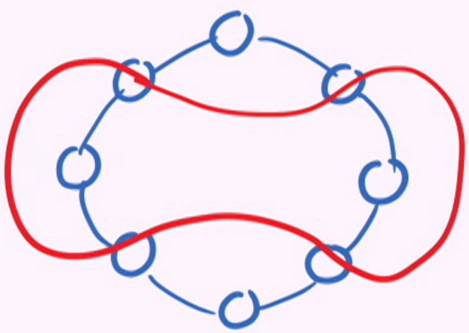
Lorentz

+

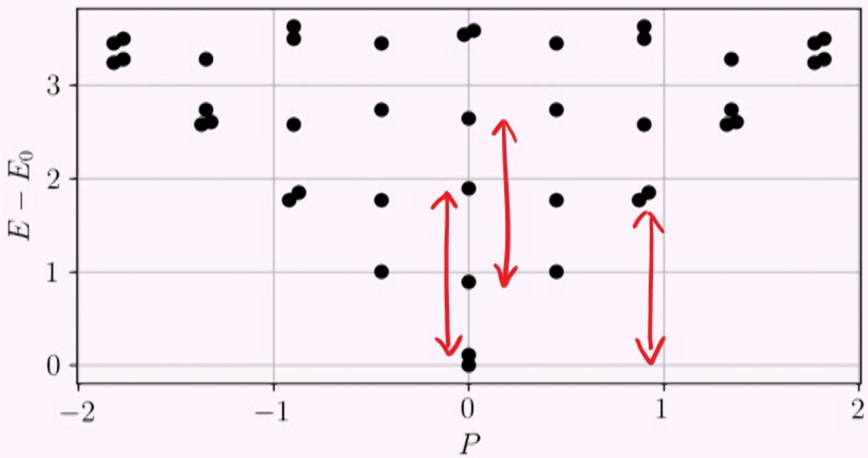
scale invariance

=

1+1D conformal



emergent symmetry



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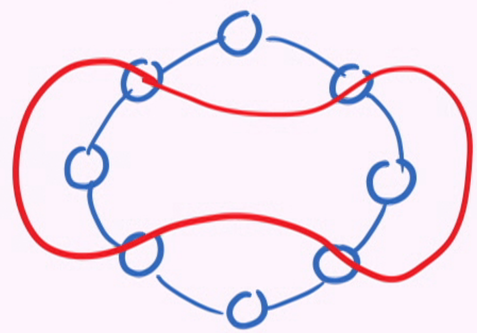
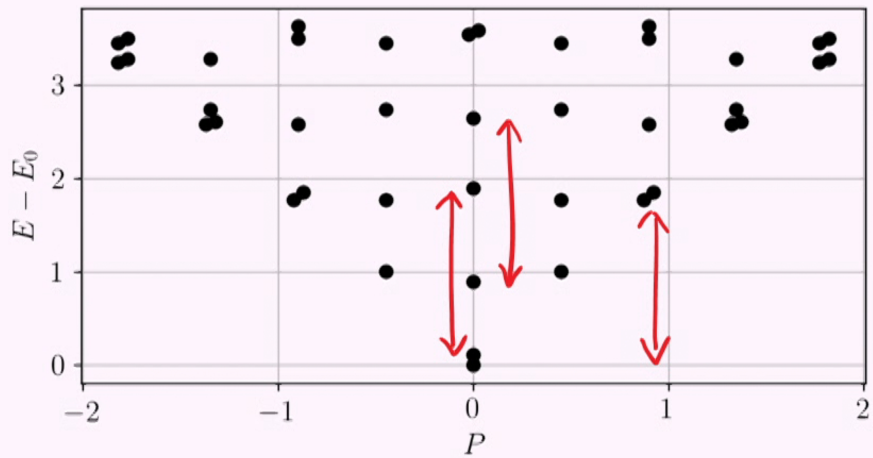


emergent symmetry



Lorentz

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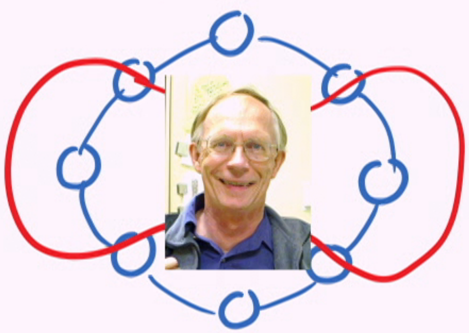
scale invariance

=

1+1D conformal

$$E_\alpha = A + \frac{B}{N}(\Delta_\alpha - \frac{c}{12}) + \mathcal{O}(N^{-x})$$

$$P_\alpha = \frac{2\pi}{N} s_\alpha$$



Cardy, Nucl. Phys. B **270** 186 (1986)

conformal data in 1+1D

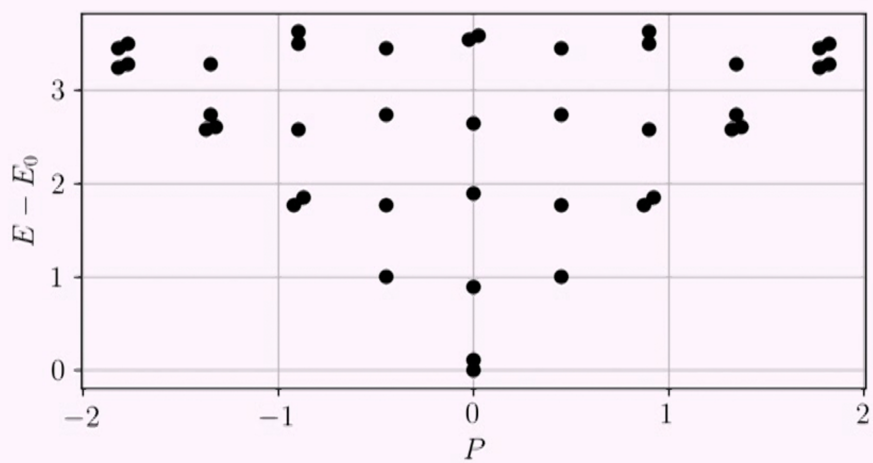
central charge: c

primary fields: $(\phi, \Delta_\phi, s_\phi)$

OPE coefficients: $C^{\phi_1}_{\phi_2\phi_3}$

(3-point correlators)

extracting conformal data: the spectrum



$$E_\alpha = A + \frac{B}{N}(\Delta_\alpha - \frac{c}{12}) + \mathcal{O}(N^{-x})$$

$$P_\alpha = \frac{2\pi}{N} s_\alpha$$

can extract:

central charge

and

scaling dimensions,

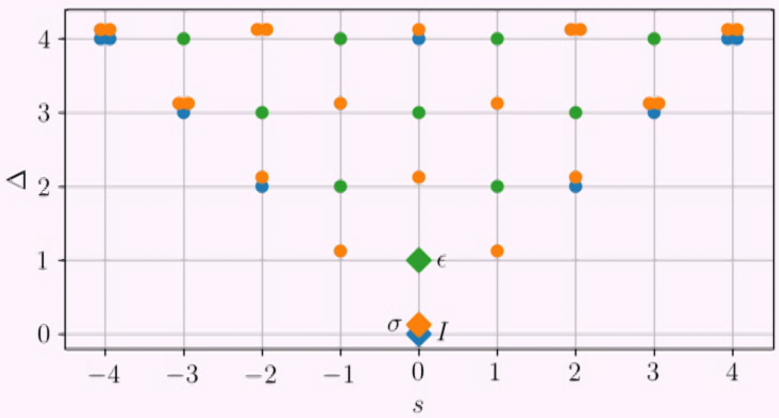
conformal spins

of CFT operators

...

can we get more?

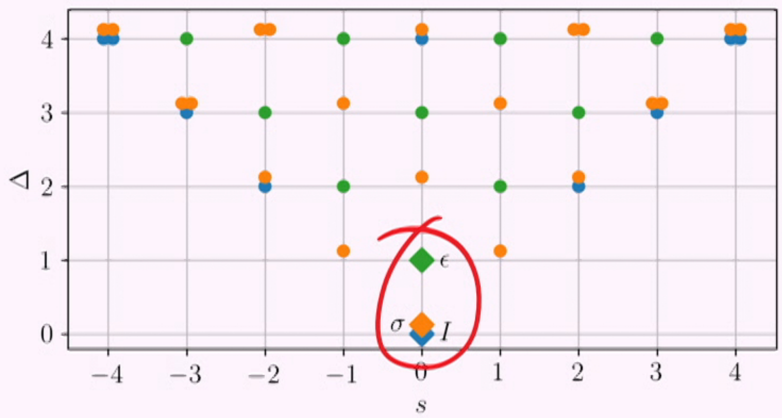
Conformal towers: Ising CFT



$$c = \frac{1}{2}$$

primary field	Δ	s	symbol
identity	0	0	I
spin	$\frac{1}{8}$	0	σ
energy density	1	0	ϵ

Conformal towers: Ising CFT



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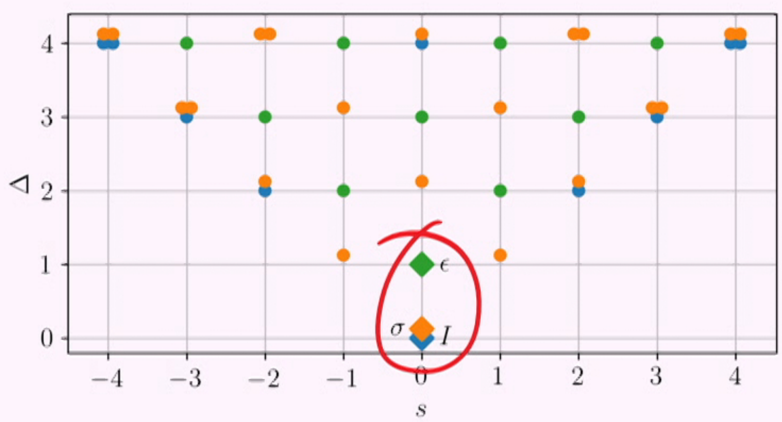

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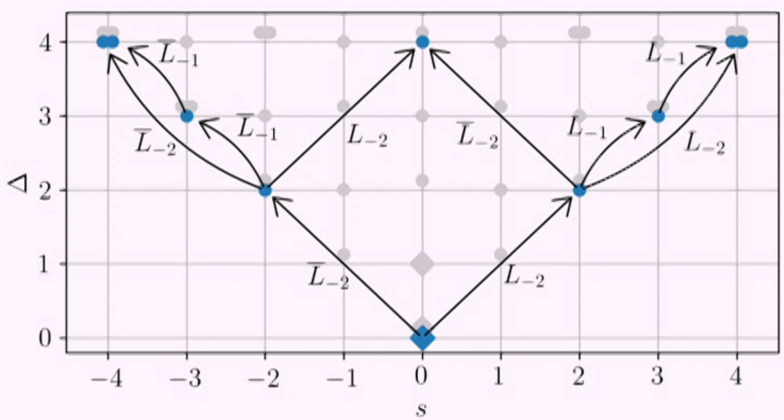
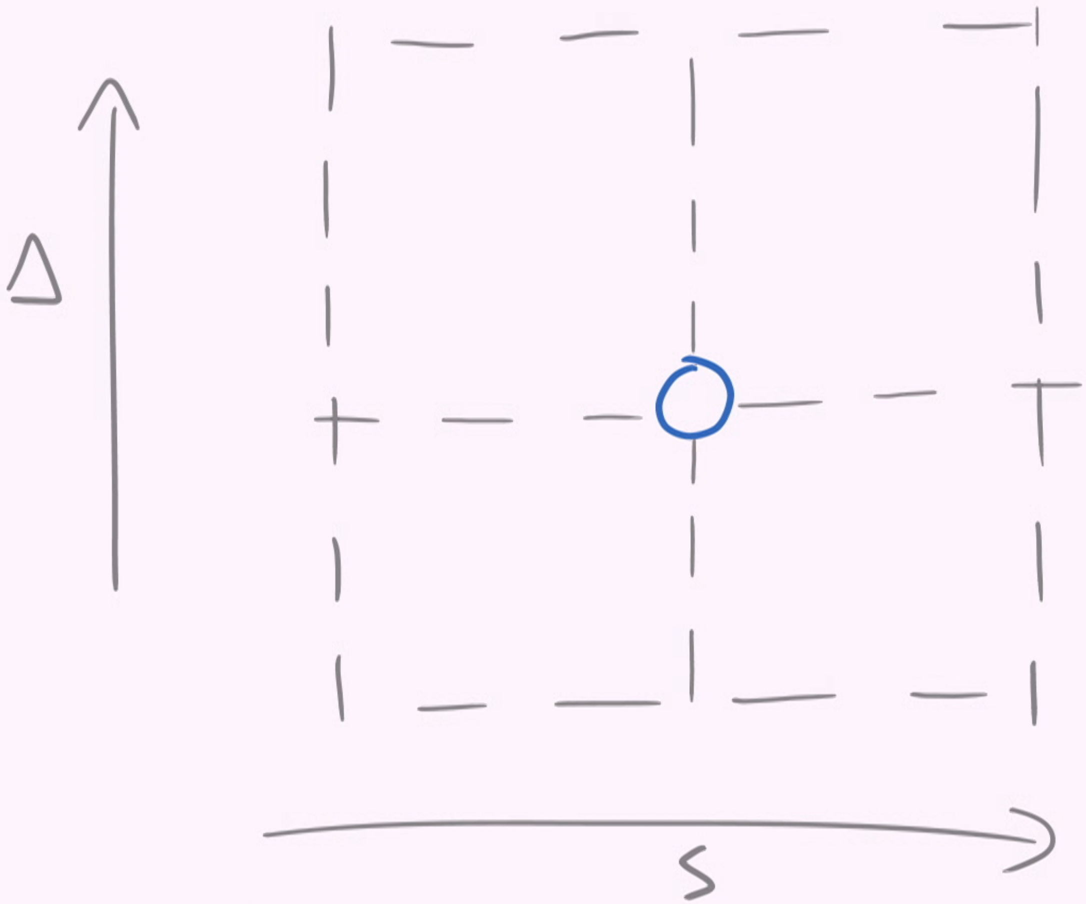


ladder operators

$$L_n, \overline{L}_n$$

conformal towers

ladder operators (Virasoro generators)



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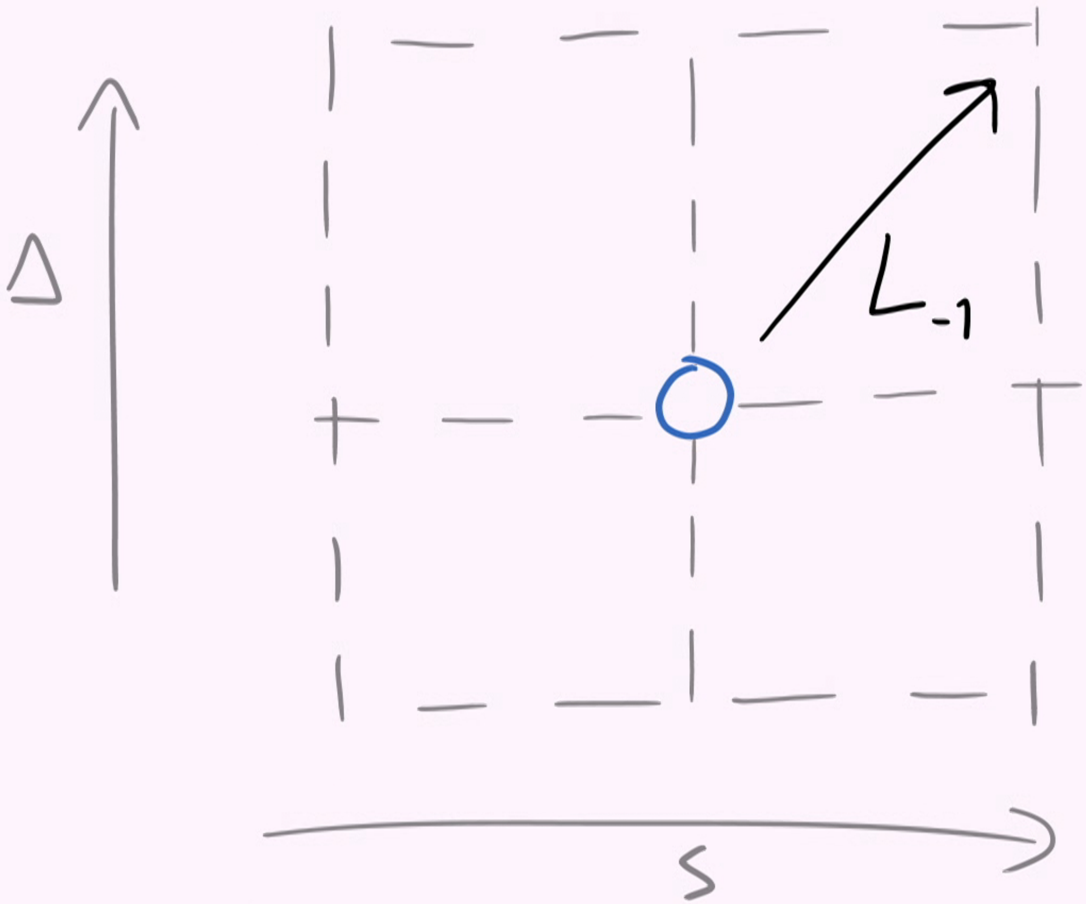

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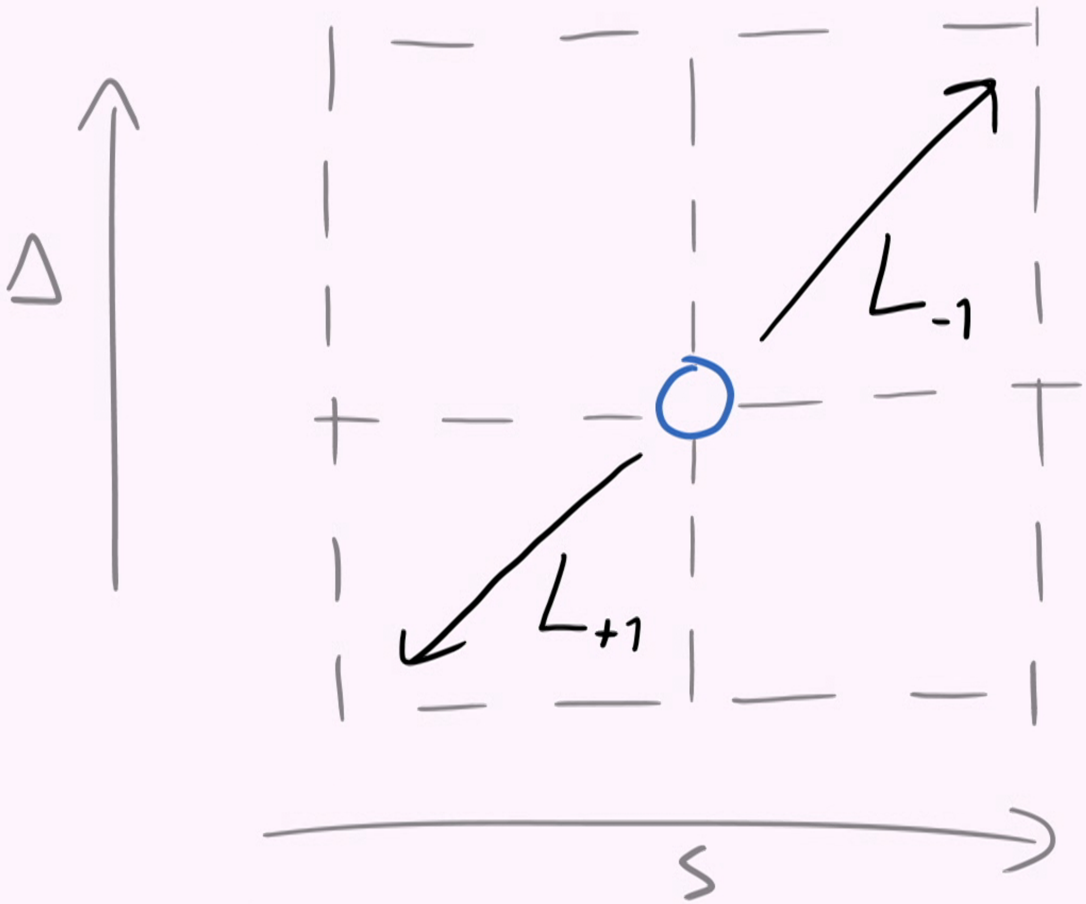

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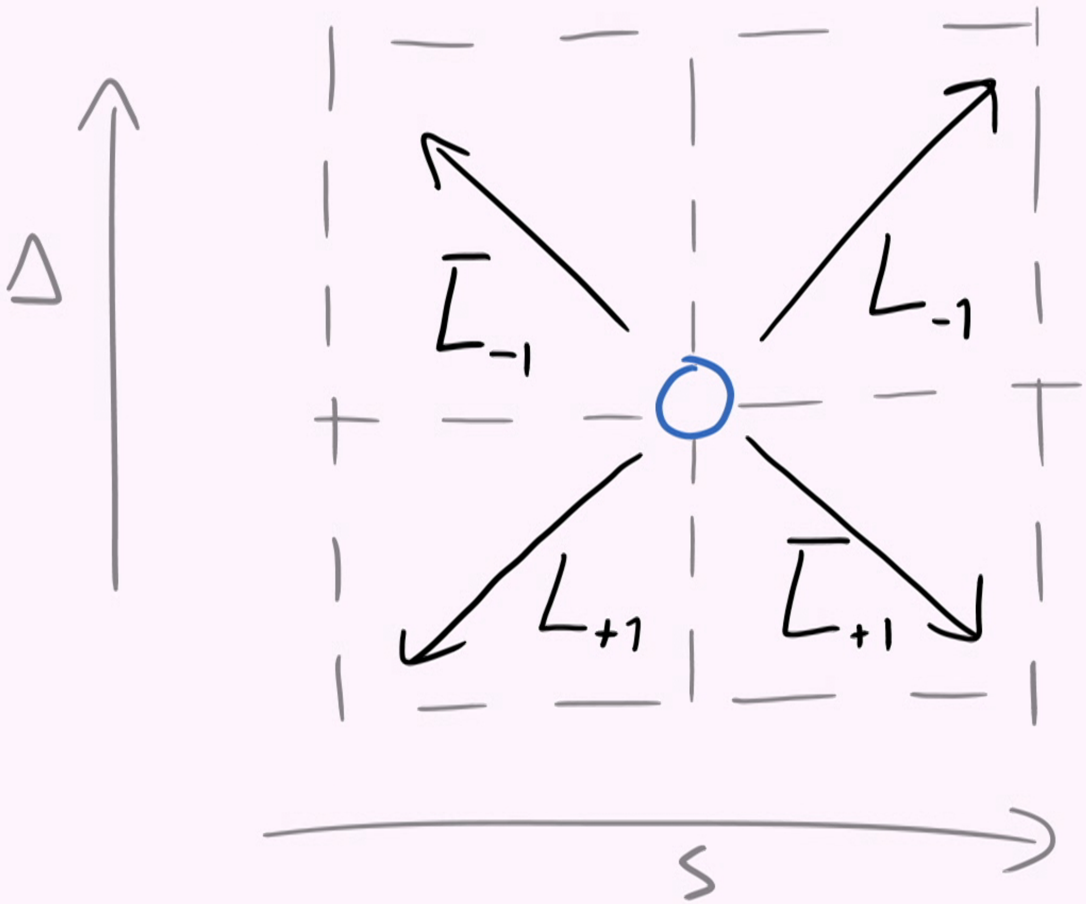
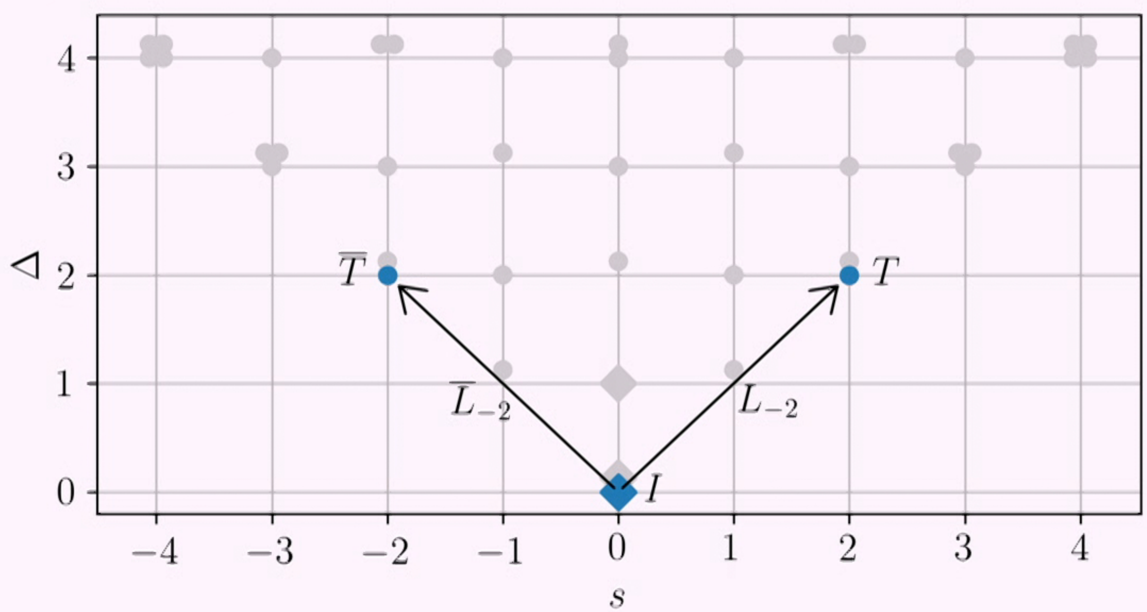
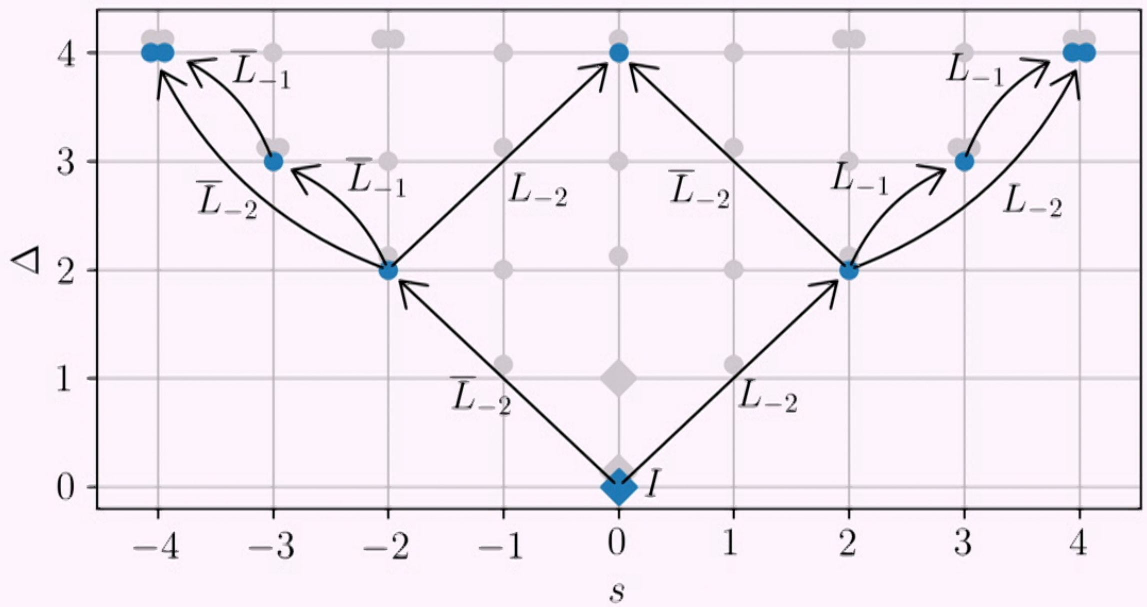


Ising CFT: energy-momentum tensor



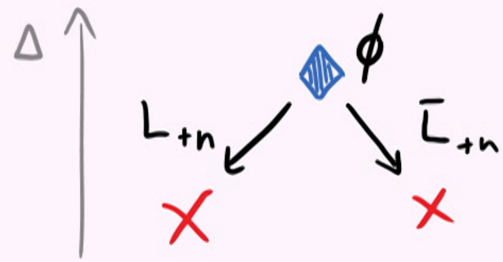
$$\sqrt{\frac{c}{2}}|T\rangle = L_{-2}|I\rangle \qquad \sqrt{\frac{c}{2}}|\overline{T}\rangle = \overline{L}_{-2}|I\rangle$$

Ising CFT: identity tower



building the tower with
ladder operators

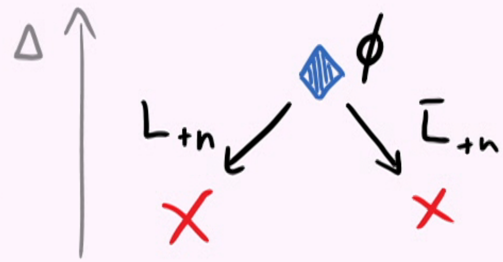
primary states



primaries are states that

cannot be lowered

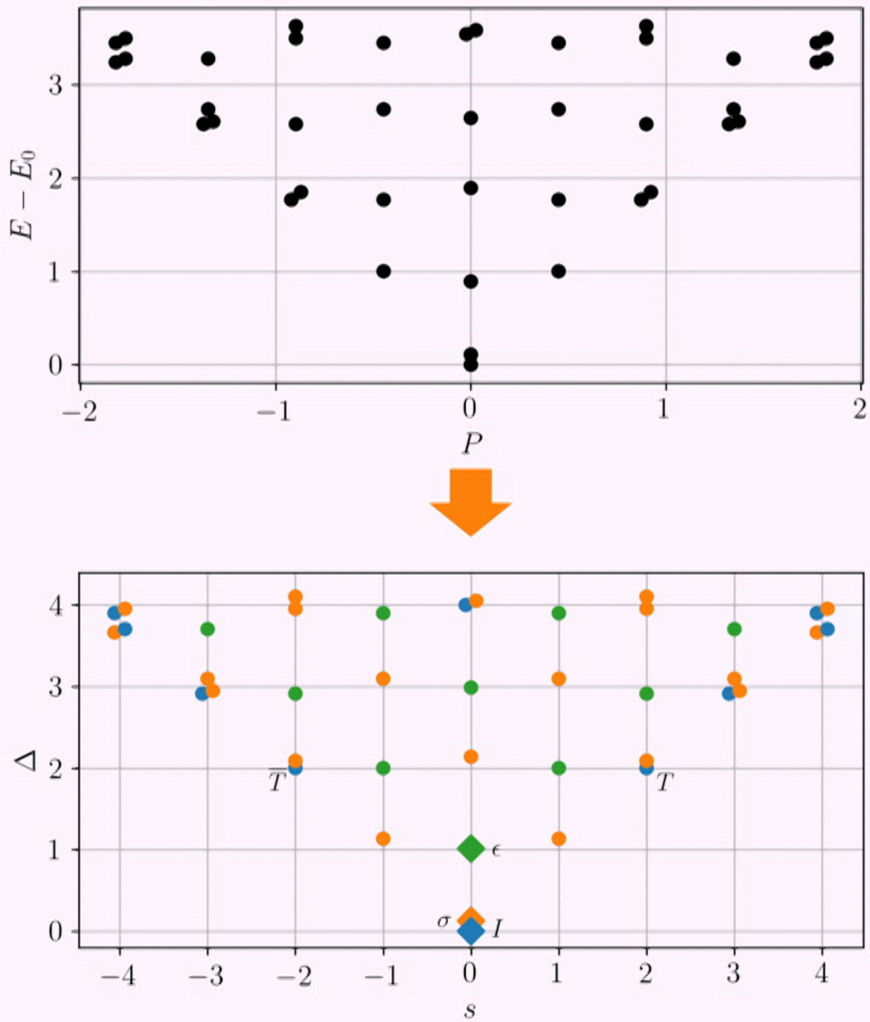
primary states



primaries are states that
cannot be lowered

$|\phi\rangle$ primary $\iff$

$$L_n|\phi\rangle = \overline{L}_n|\phi\rangle = 0 \qquad n \in [1, 2]$$



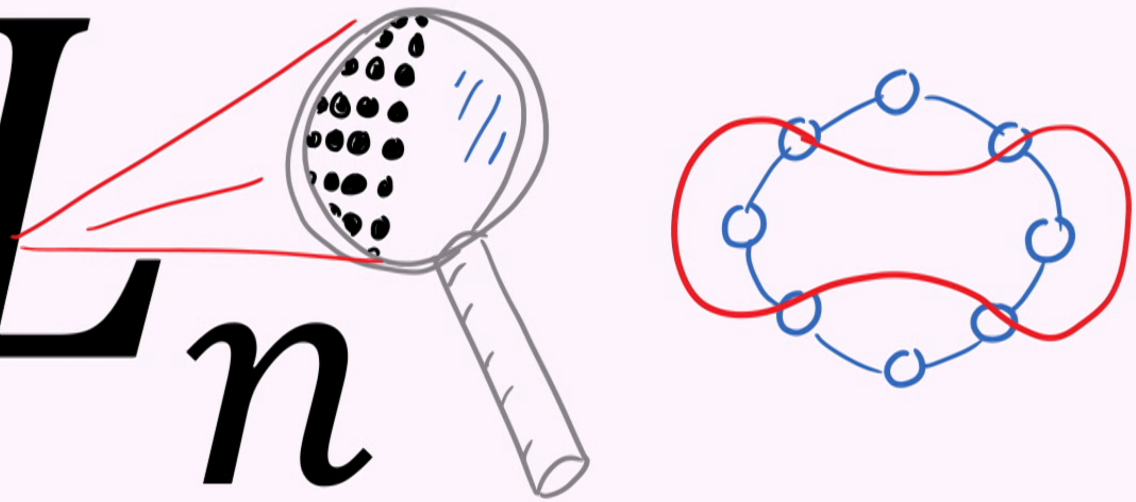
are there ladder operators on the **spin chain**?

can we use them to identify energy eigenstates with **CFT operators**

...

for any CFT?

L_n



lattice ladder operators

in critical spin chains

mode expansion of hamiltonian density

CFT

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$$H_n = \frac{1}{2\pi}\int_0^L \mathrm{d}x\; e^{-inx\frac{2\pi}{L}}\left(T(x) + \overline{T}(x)\right)$$

our proposal →

spin chain

$$H = \sum_{j=1}^{N} h_j$$

$$h_j = \frac{1}{N\eta}\sum_n e^{inj\frac{2\pi}{N}} H_n$$

$$H_n \equiv \eta \sum_{j=1}^{N} e^{-inj\frac{2\pi}{N}} h_j$$

$$H_0 = \eta H$$

mode expansion of hamiltonian density

$$H_n \equiv \eta \sum_{j=1}^{N} e^{-inj\frac{2\pi}{N}} h_j$$

as an ansatz for the conformal generator

$$H_n^{CFT} \equiv L_n + \overline{L}_{-n}$$

mode expansion of hamiltonian density

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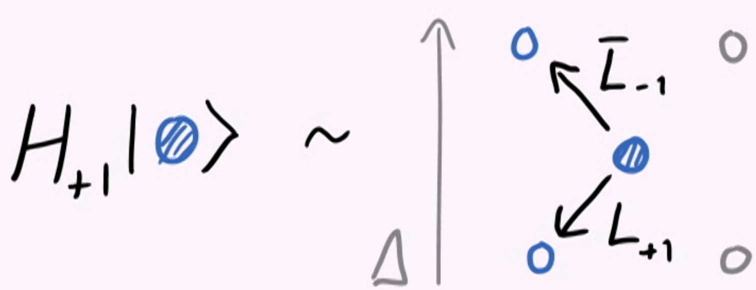
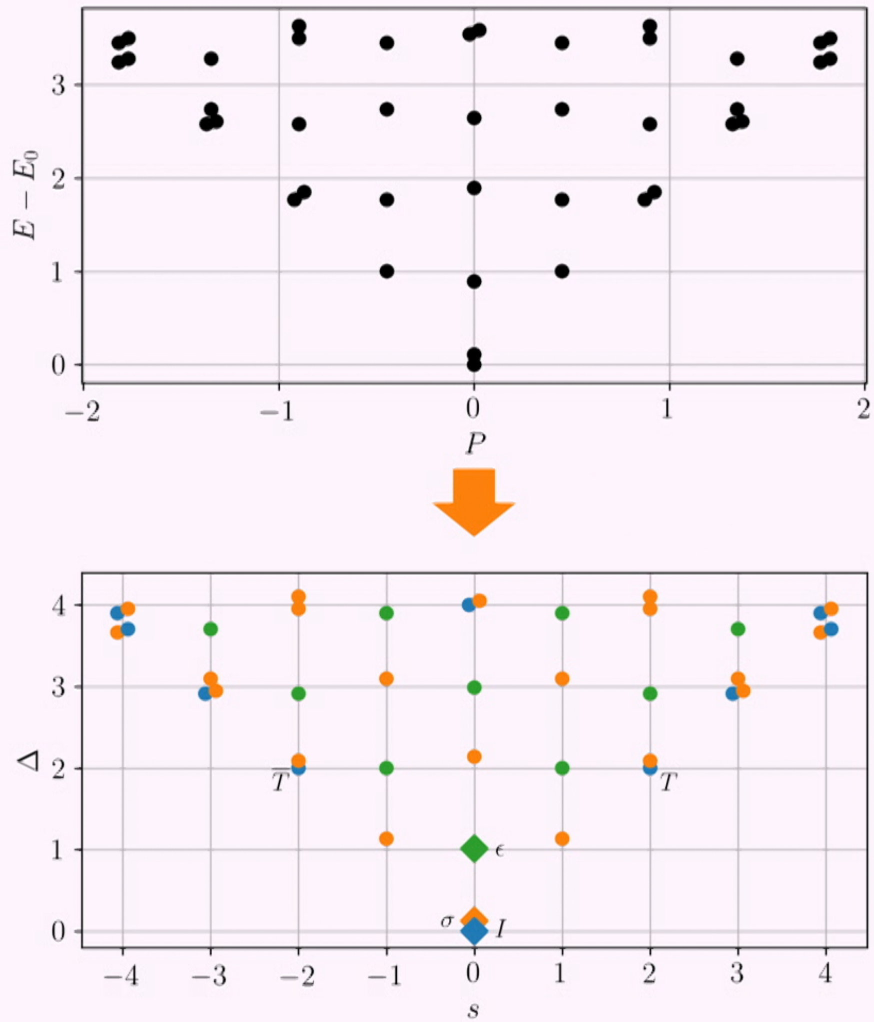




the test:

can we identify energy eigenstates with CFT operators

...

without opening the yellow book?

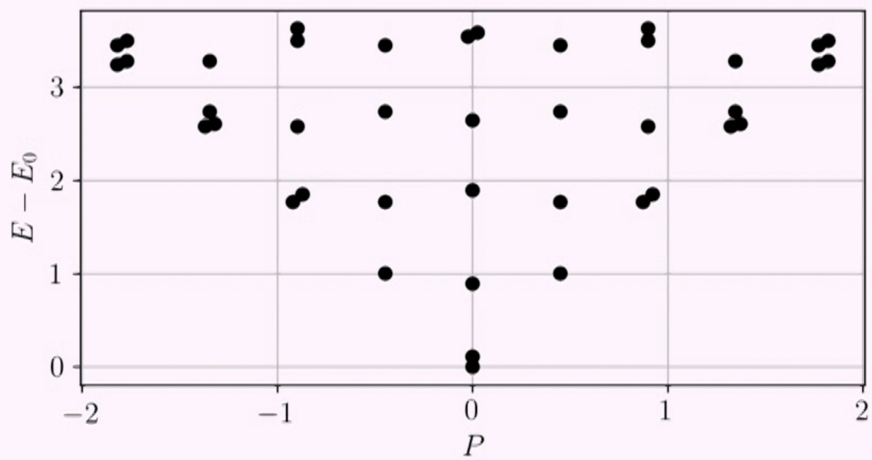
$$H \equiv - \sum_{j=1}^{N} \left[\sigma_j^X \sigma_{j+1}^X + \sigma_j^Z \right]$$

transverse field Ising model

exact diagonalization

Ising spin chain energy eigenstates

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$



$N = 14$ spins

Ising spin chain: primary states

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

primary states cannot be *lowered* (up to finite-size corrections)

$$\langle\beta|H_1 + H_{-1} + H_2 + H_{-2}|\alpha\rangle$$

Ising spin chain: primary states

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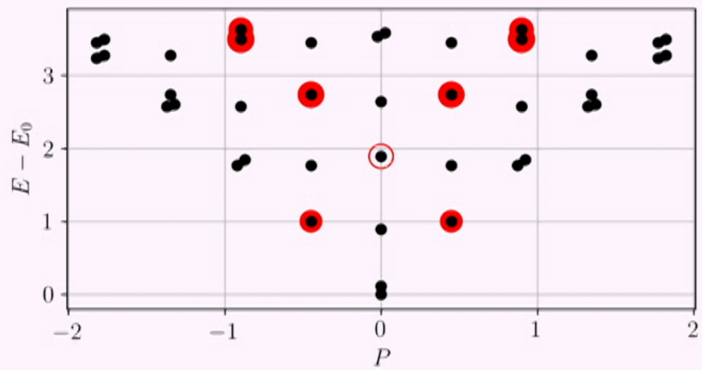
not a primary



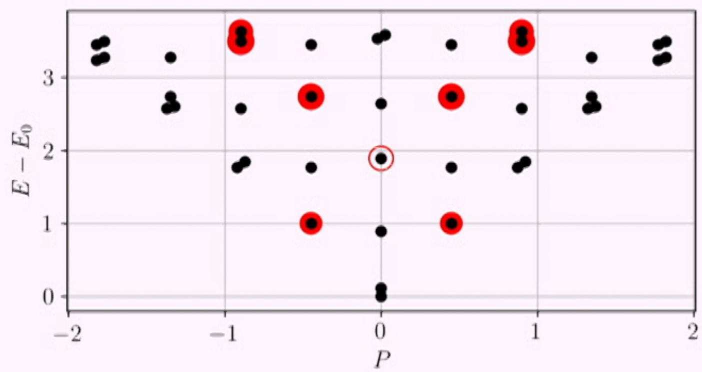
Ising spin chain: primary states

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

primary states cannot be *lowered* (up to finite-size corrections)

$$\langle\beta|H_1 + H_{-1} + H_2 + H_{-2}|\alpha\rangle$$

not a primary



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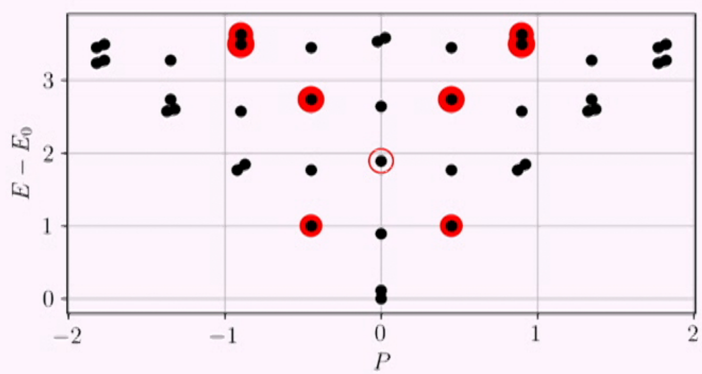
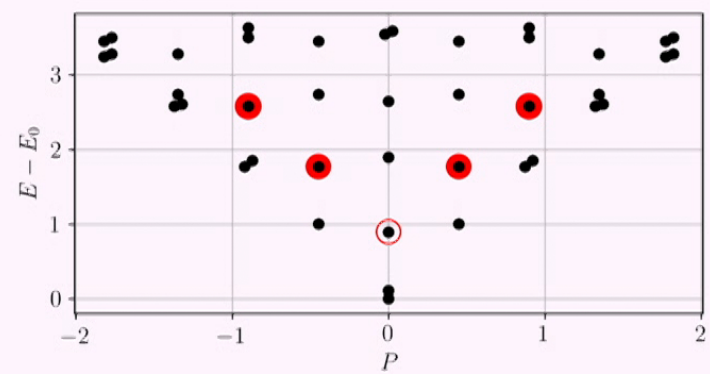


yes a primary!@1!!!



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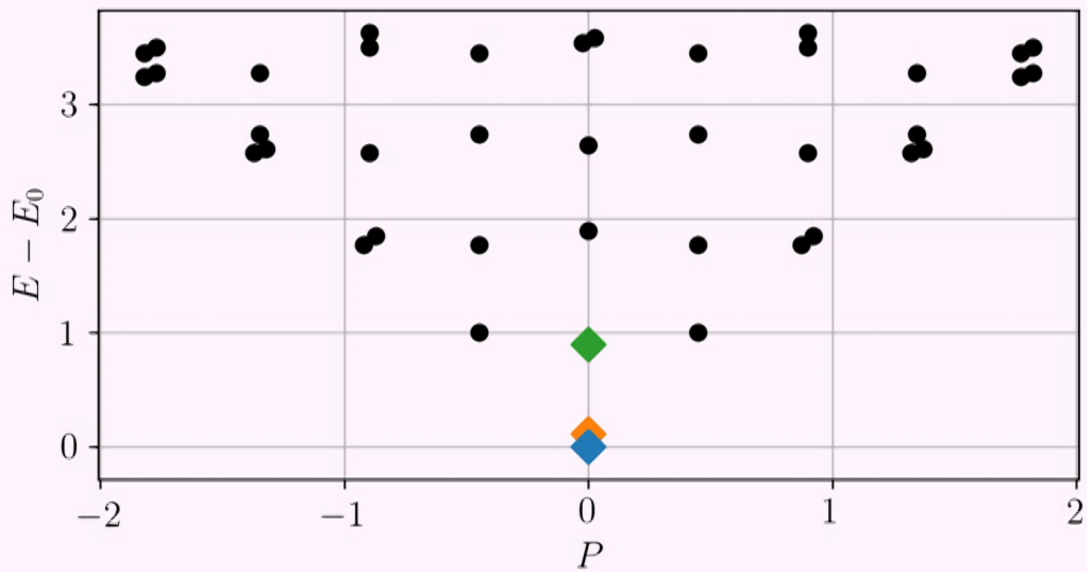

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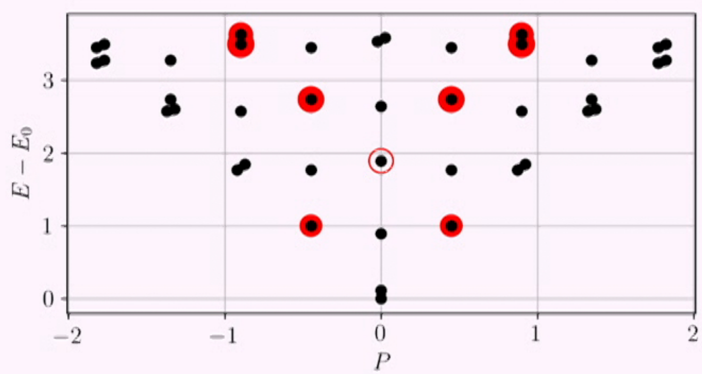

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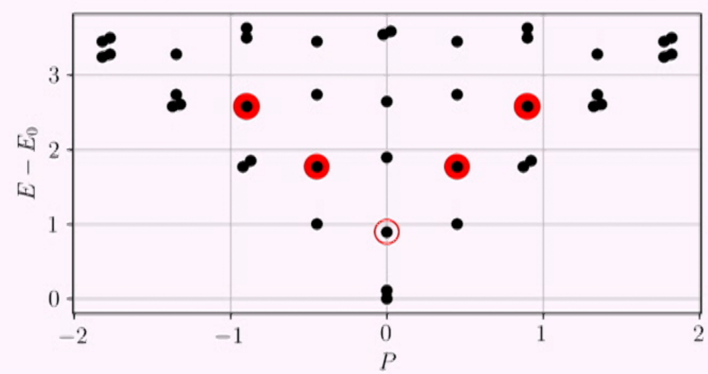

Ising spin chain: conformal towers

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

primaries $|\phi_j\rangle$

$$\langle\phi_j|e^{i(H_1+H_{-1}+H_2+H_{-2})}|\alpha\rangle$$

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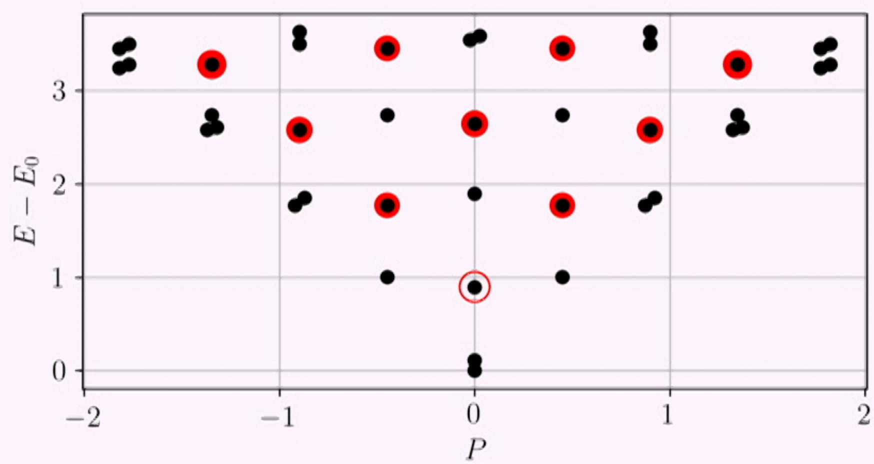

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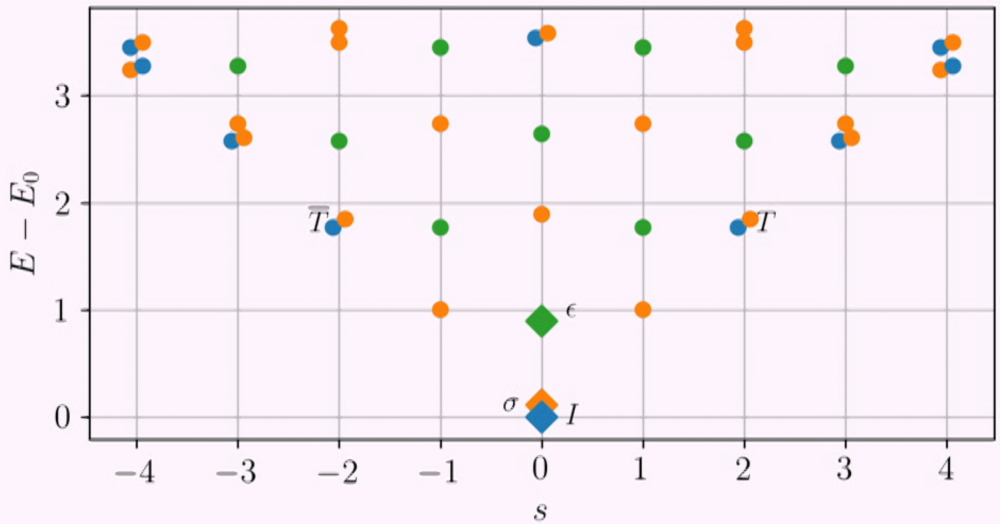

Ising spin chain: normalization

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

primaries $|\phi_j\rangle$

towers

$$H_n \equiv \eta \sum_{j=1}^{N} e^{-inj\frac{2\pi}{N}} h_j$$

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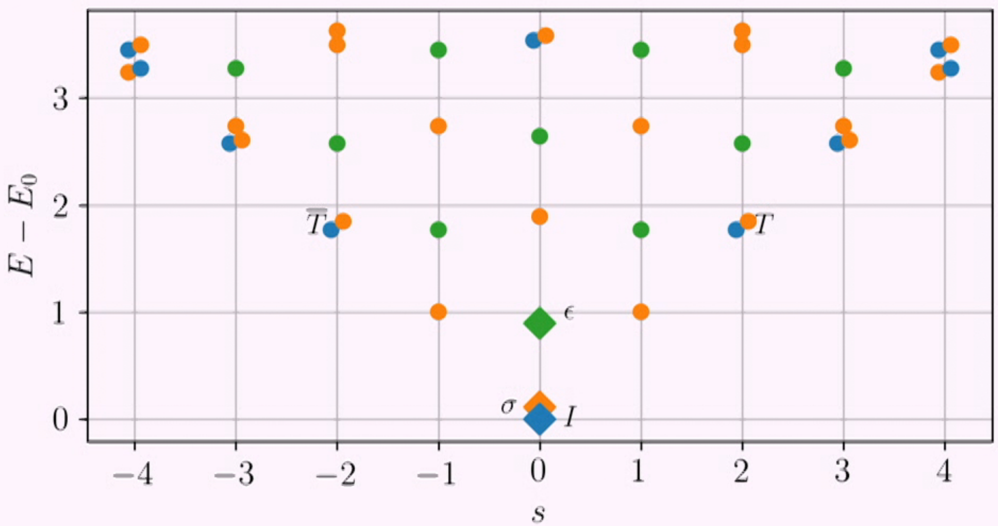

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$$H_n \equiv \eta \sum_{j=1}^{N} e^{-inj\frac{2\pi}{N}} h_j$$

$$\Delta_{\alpha,N} = \langle\alpha|H_0|\alpha\rangle - \langle I|H_0|I\rangle$$

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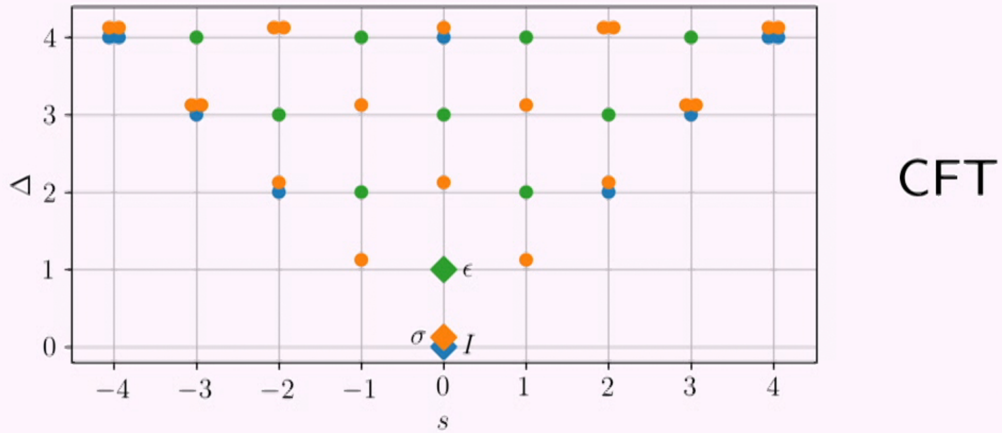
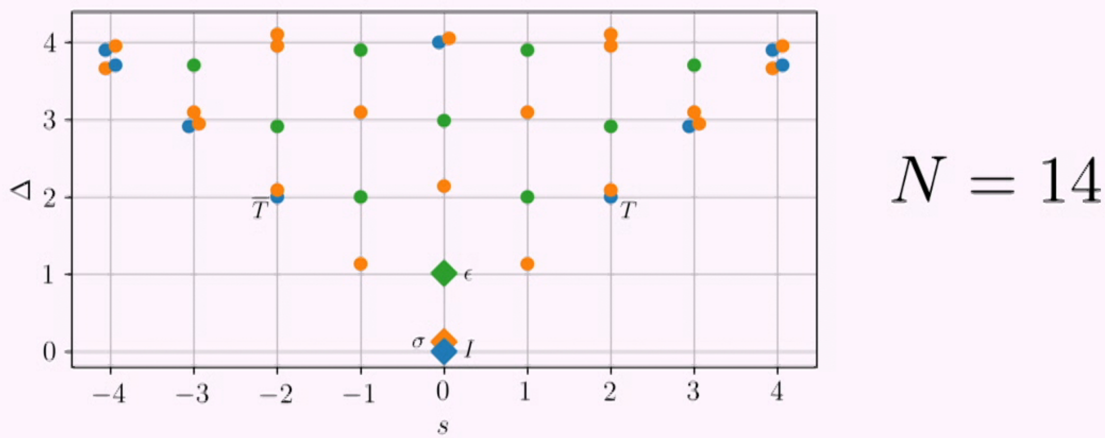
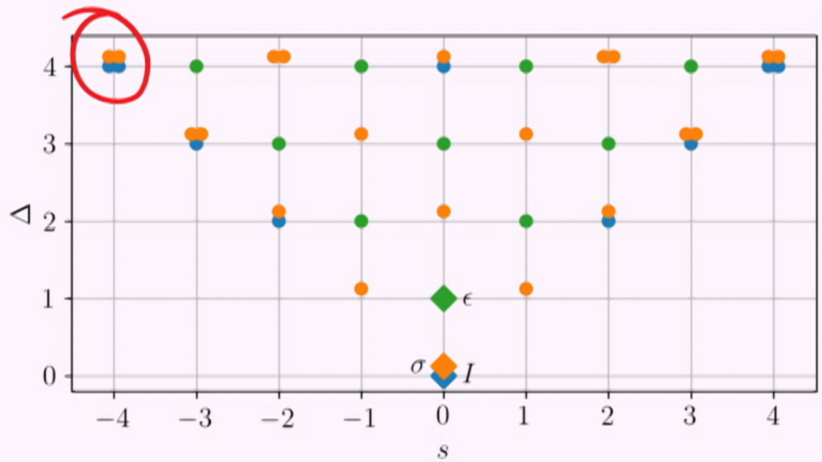
$$\eta : \Delta_{T,N} = \Delta_T = 2$$

Ising spin chain: conformal towers

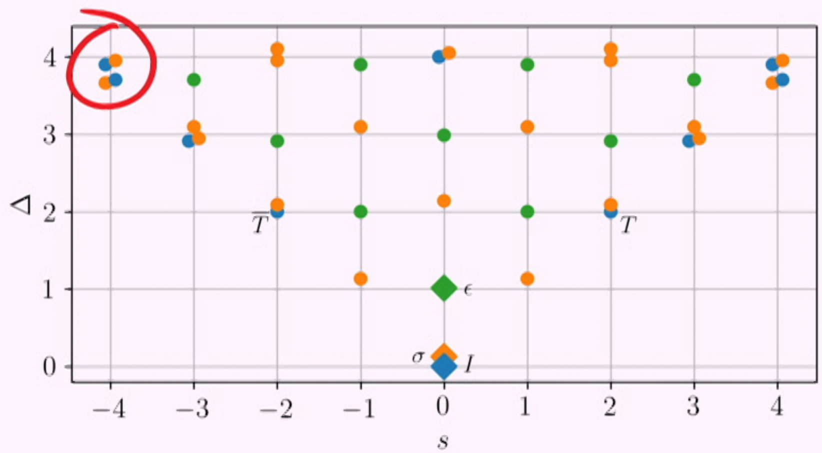




Ising spin chain: conformal towers



CFT



$N = 14$

Ising spin chain: central charge

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

primaries $|\phi_j\rangle$

towers

$$\sqrt{\frac{c}{2}}|T\rangle = L_{-2}|I\rangle$$

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$$\sqrt{\frac{c}{2}}|T\rangle = L_{-2}|I\rangle \qquad H_{-2}|I\rangle \xrightarrow{N\to\infty} \sqrt{\frac{c}{2}}|T\rangle$$

Ising spin chain: central charge

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

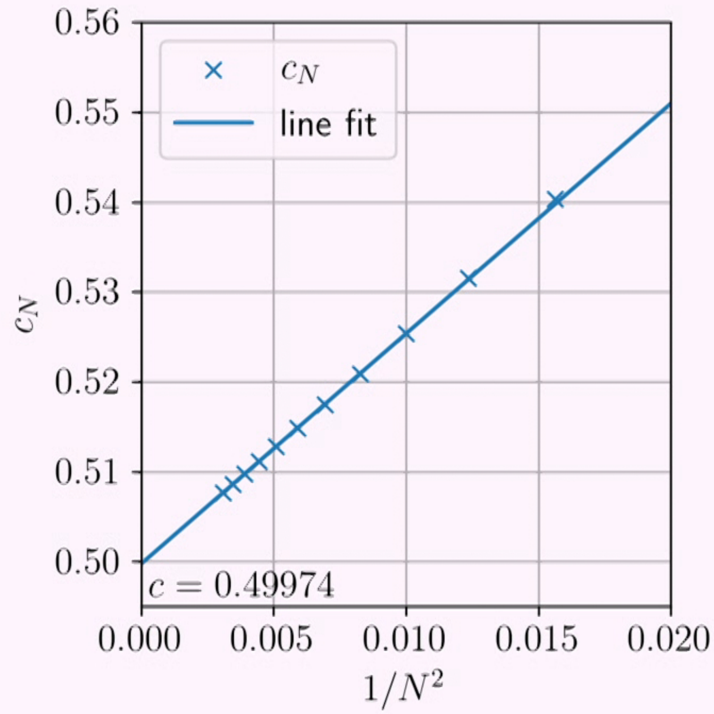
primaries $|\phi_j\rangle$

towers

$$\sqrt{\frac{c}{2}}|T\rangle = L_{-2}|I\rangle \qquad H_{-2}|I\rangle \xrightarrow{N\to\infty} \sqrt{\frac{c}{2}}|T\rangle$$

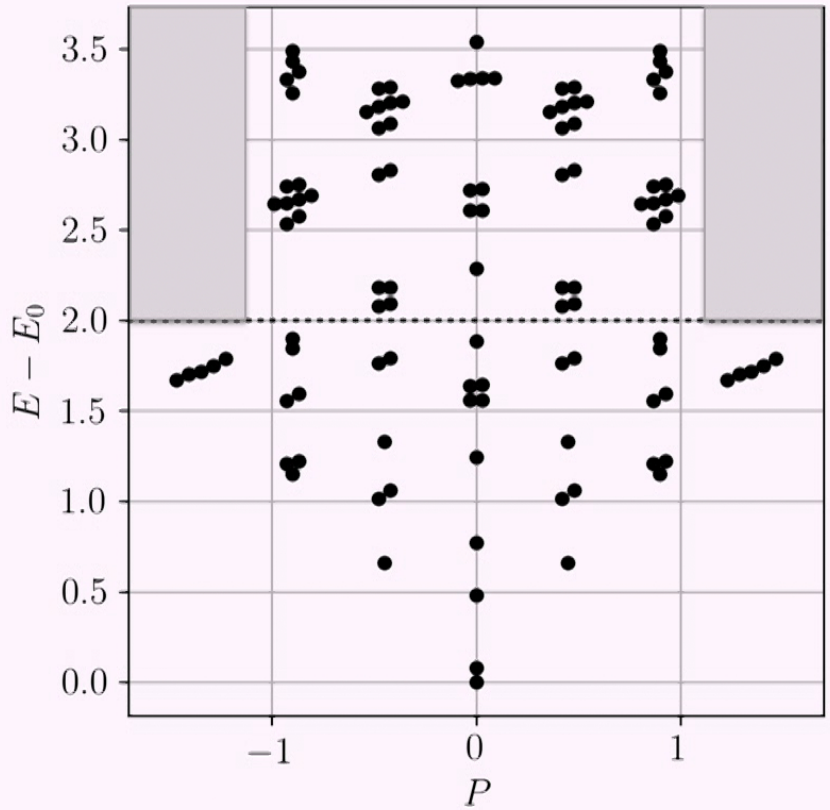
$$c_N \equiv 2\langle I|H^\dagger_{-2}H_{-2}|I\rangle$$

Ising spin chain: central charge



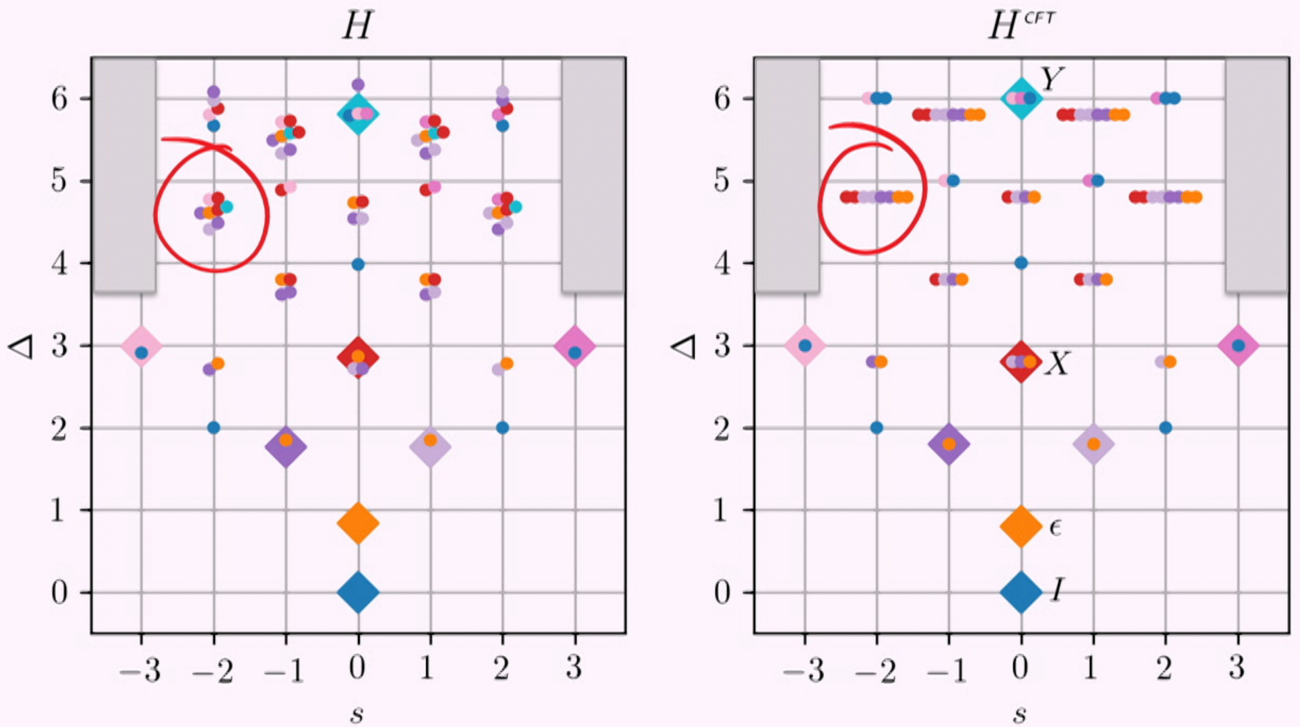
$$N \leq 18$$

3-state Potts spin chain: spectrum



$N = 14$

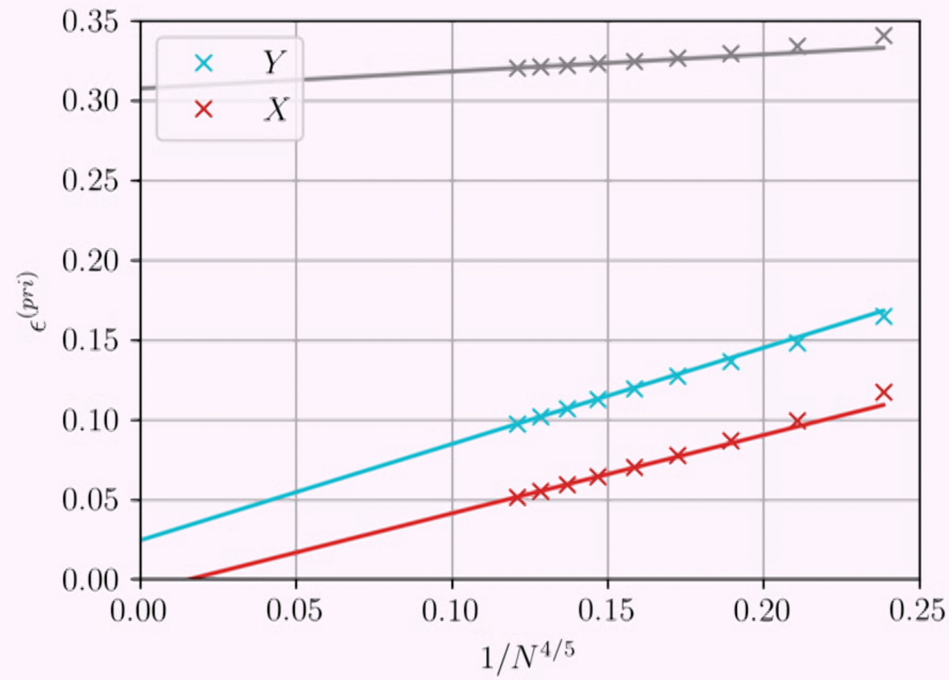
3-state Potts spin chain: towers



$N = 14$

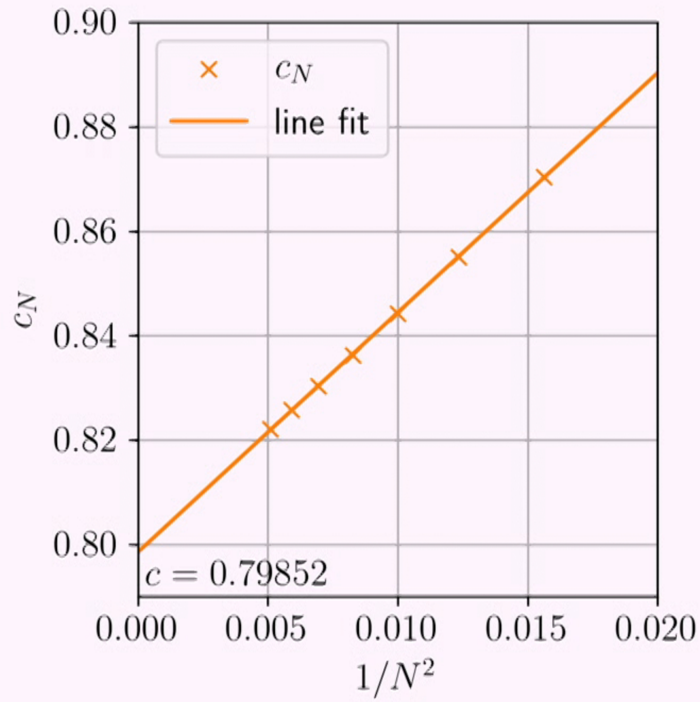
CFT

3-state Potts spin chain: finite-size scaling



size of matrix elements which lower primary candidate states

3-state Potts spin chain: central charge



$$N \leq 14$$

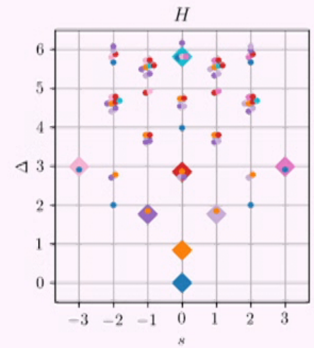
checkpoint!

Hamiltonian mode expansion acts as conformal generator **on finite spin chains**

checkpoint!

Hamiltonian mode expansion acts as conformal generator **on finite spin chains**

we can **identify** energy eigenstates with **conformal fields** *without any assumptions about the CFT*



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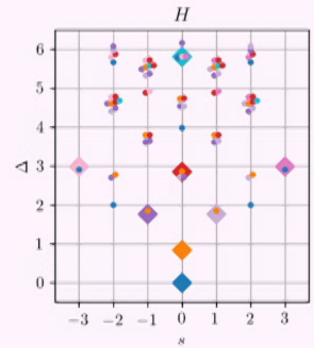

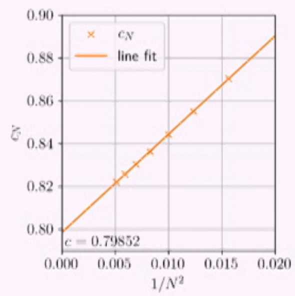

we can estimate the **central charge** from ground state expectation values

checkpoint!

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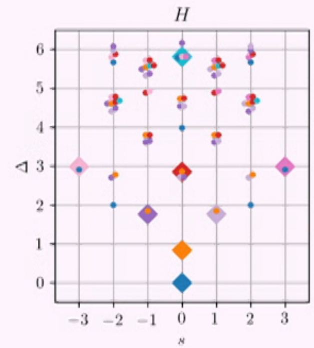
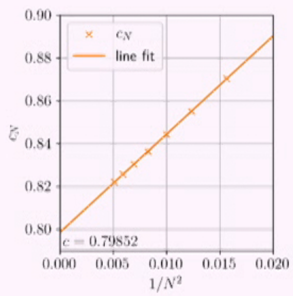
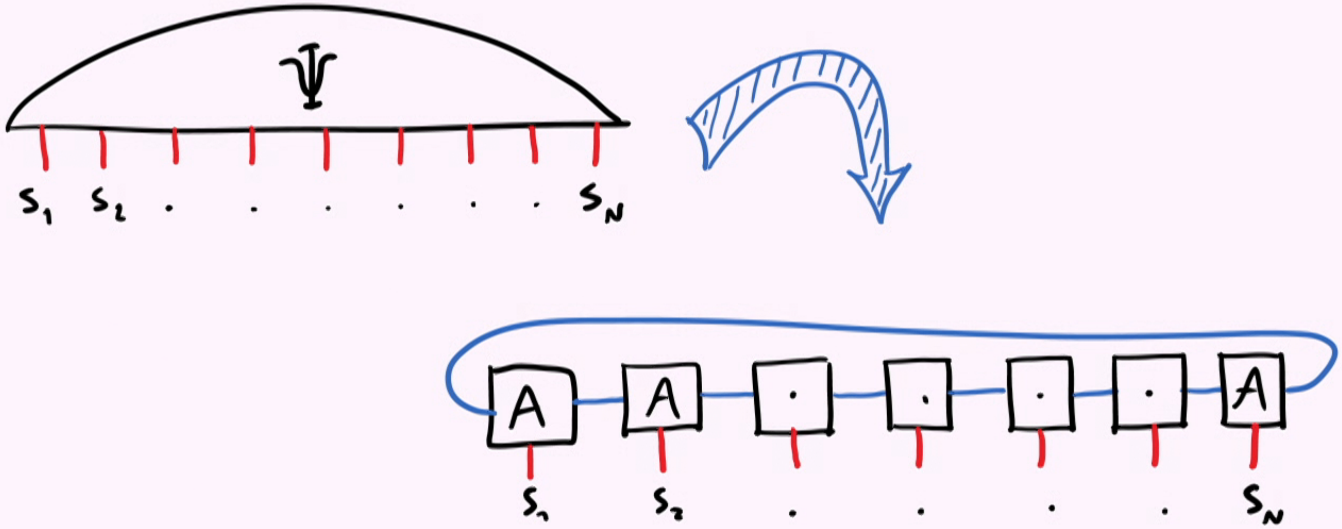




we can estimate the **central charge** from ground state expectation values

our identification method is **independent** of diagonalization technique

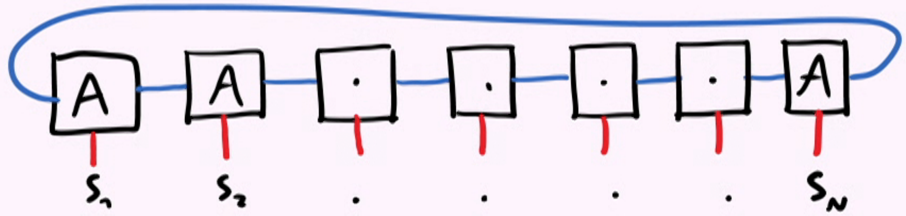


Matrix Product States on the circle

emergent conformal symmetry

in larger systems

MPS on the circle



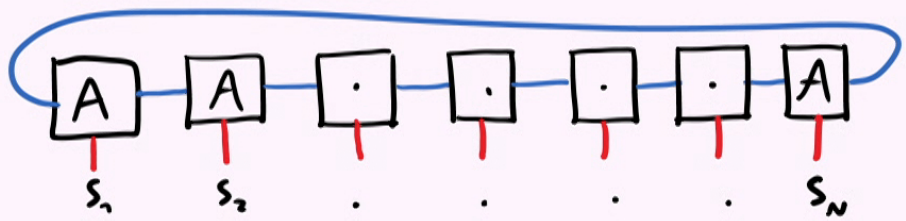
ground states

White, Phys. Rev. Lett. **69** 2863 (1992)
Verstraete, Porras, and Cirac, Phys. Rev. Lett. **93** 227205 (2004)
Pippan, White, and Evertz, Phys. Rev. B **81**, 081103 (2010)
Pirvu, Verstraete, and Vidal, Phys. Rev. B. **83** 125104 (2011)

excited states

Rommer and Östlund, Phys. Rev. B **55**, 2164 (1997)
Pirvu, Haegeman, and Verstraete, Phys. Rev. B **85** 035130 (2012)

MPS on the circle



ground states Time cost: $\mathcal{O}(N\chi^5)$

White, Phys. Rev. Lett. **69** 2863 (1992)
Verstraete, Porras, and Cirac, Phys. Rev. Lett. **93** 227205 (2004)
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Rommer and Östlund, Phys. Rev. B **55**, 2164 (1997)
Pirvu, Haegeman, and Verstraete, Phys. Rev. B **85** 035130 (2012)

MPS

extracting conformal data

finite MPS with OBC

usual finite-size-scaling techniques apply

infinite MPS *(finite entanglement scaling)*

Tagliacozzo, de Oliveira, Iblisdir, and Latorre, Phys. Rev. B **78** 024410 (2008)

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Stojevic, Haegeman, McCulloch, Tagliacozzo and Verstraete,
Phys. Rev. B **91** 035120 (2015)

MPS

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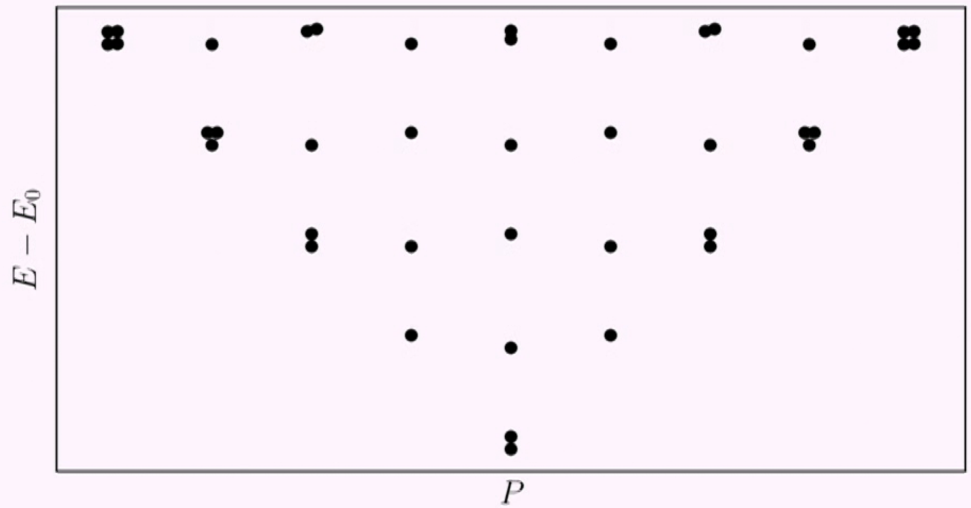
Time cost: $\mathcal{O}(\chi^3)$

MPS on the circle

scaling dimensions of the Ising model

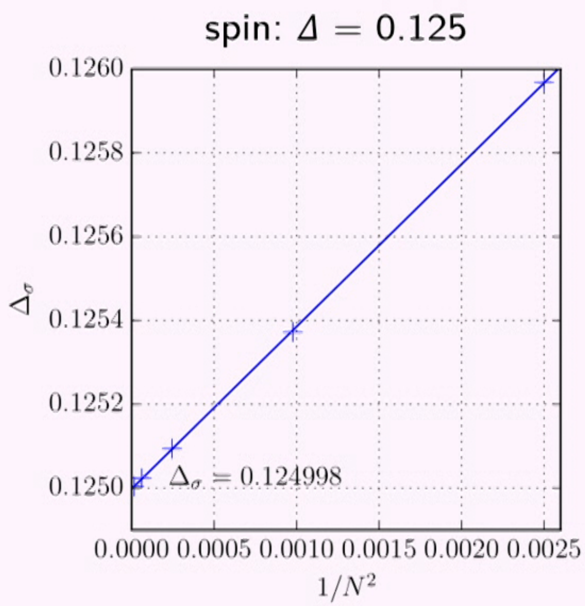


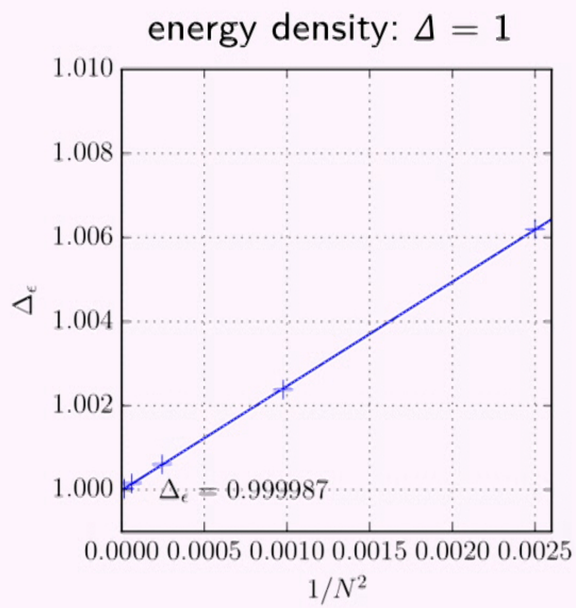
$$E_\alpha = A + \frac{B}{N}(\Delta_\alpha - \frac{c}{12}) + \mathcal{O}(N^{-x})$$

$$P_\alpha = \frac{2\pi}{N} s_\alpha$$

MPS on the circle

scaling dimensions of the Ising model





$$\Delta_\sigma = 0.124998$$

$$\Delta_\sigma = 0.999987$$

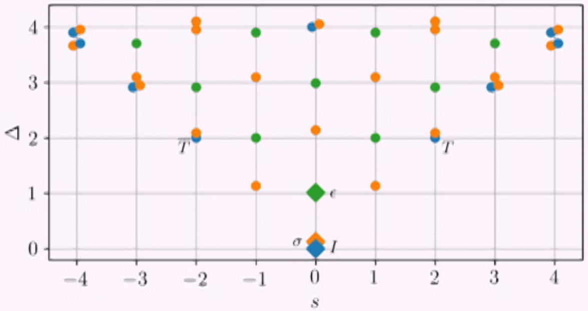
MPS on the circle

scaling dimensions of the Ising model

$\Delta_\sigma = 0.125$	$\Delta_\sigma = 1$	CFT result
$\Delta_\sigma = 0.124998$	$\Delta_\epsilon = 0.999987$	MPS on the circle (FSS): $\chi \leq 36$, $N \leq 256$
$\Delta_\sigma = 0.1246$	$\Delta_\epsilon = 0.998$	from infinite MPS: $\chi \leq 64$ Stojevic et al., Phys. Rev. B **91** 035120 (2015)

MPS on the circle

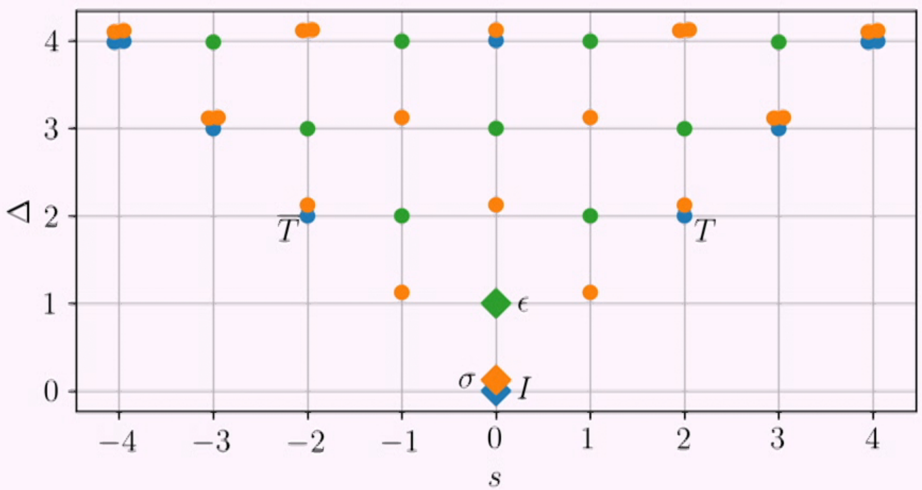
conformal data with H_n



ED

$N = 14$

Time cost: $\mathcal{O}(d^N)$



MPS

$N = 64$

$\chi = 24$

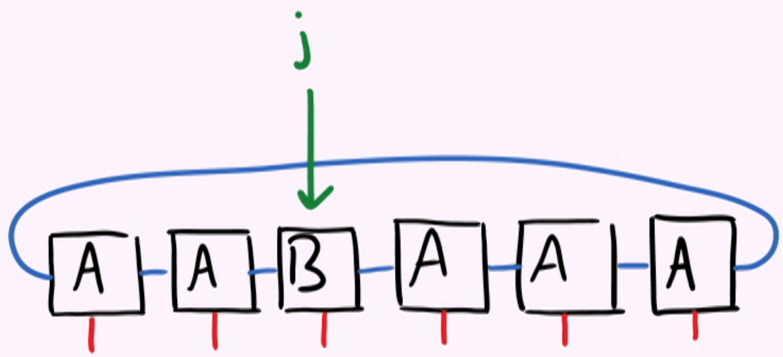
Max. time cost:

$\mathcal{O}(N\chi^6)$

MPS on the circle

"single-sum" excitations

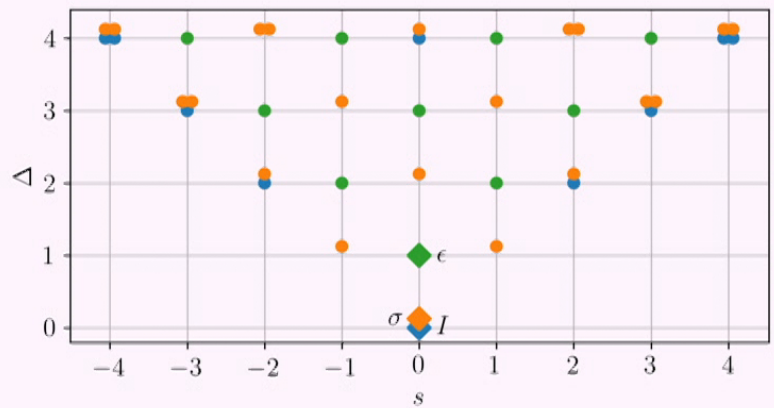
$$\sum_{j=1}^{N} e^{ipj\frac{2\pi}{N}} T^j$$



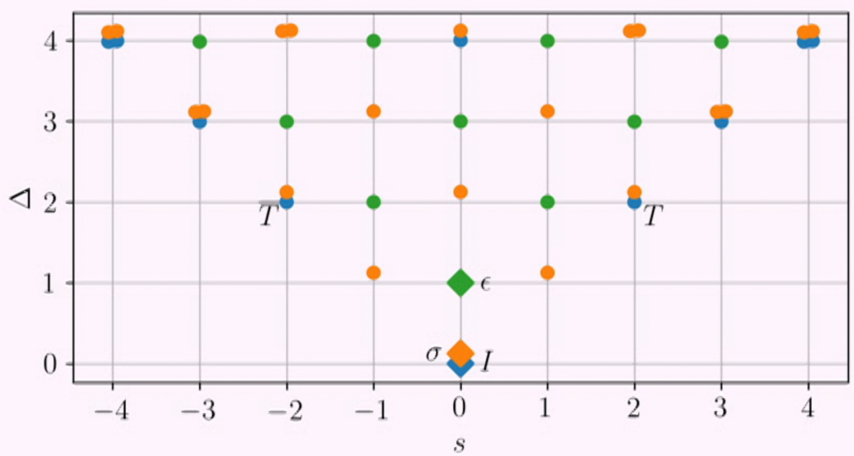
Rommer and Östlund, Phys. Rev. B **55**, 2164 (1997)
Pirvu, Haegeman, Verstraete, Phys. Rev. B **85** 035130 (2012)

MPS on the circle

"single-sum" excitations ***capture everything***



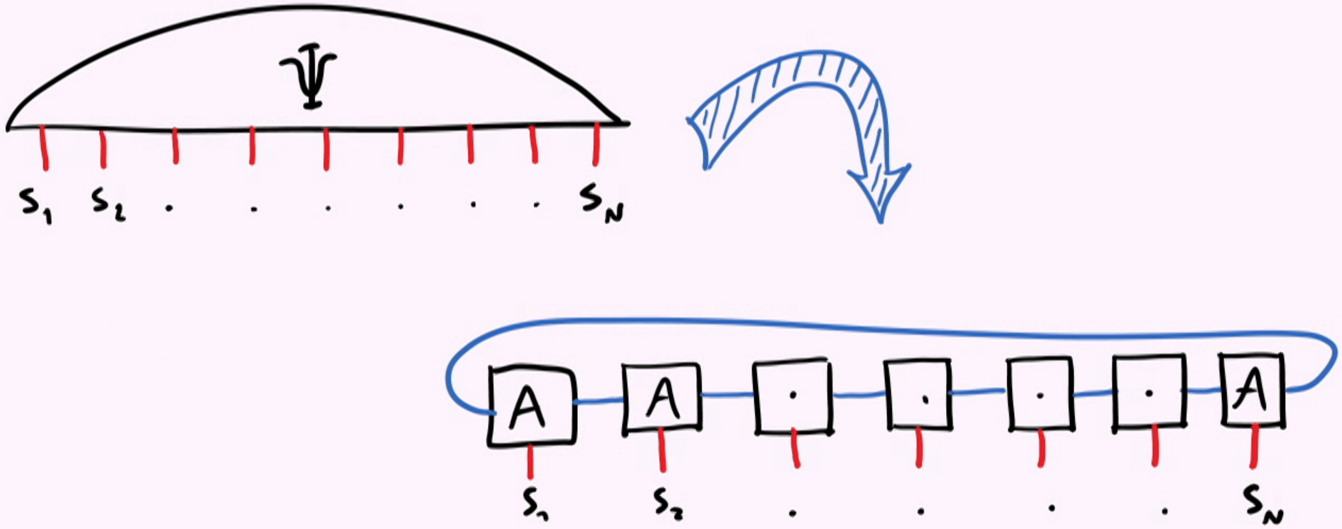
CFT



MPS

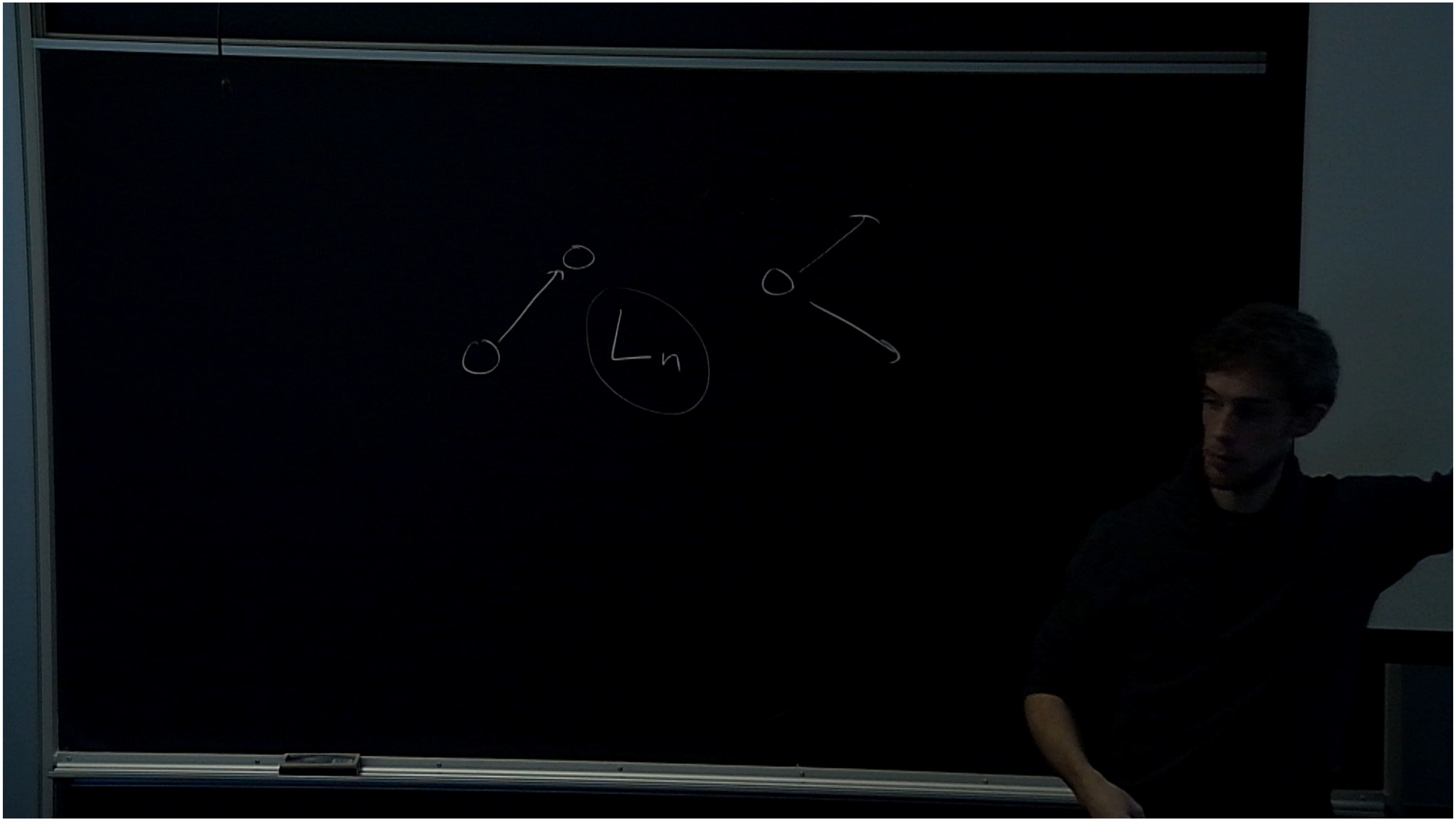
$N = 64$

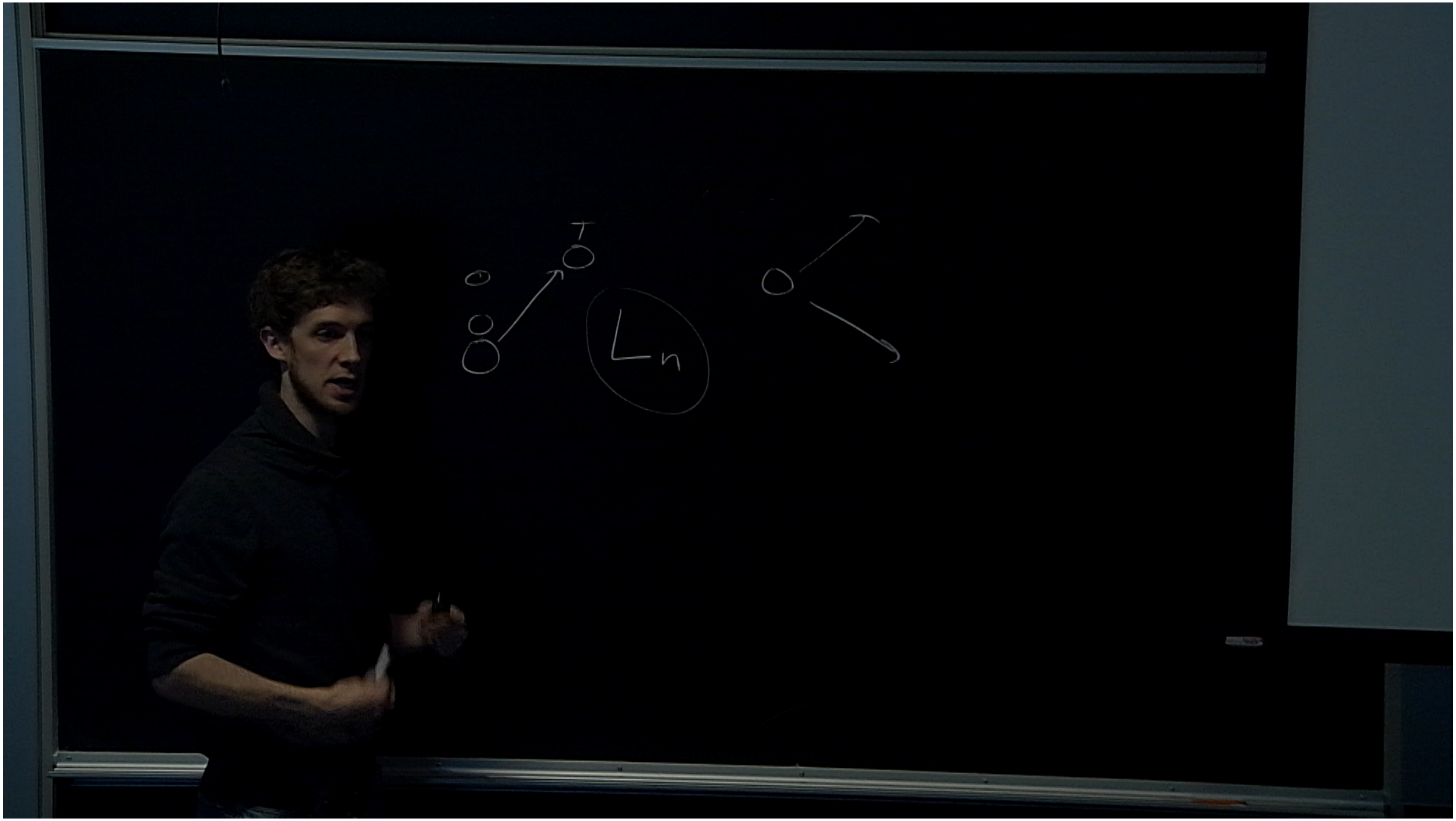
$\chi = 24$

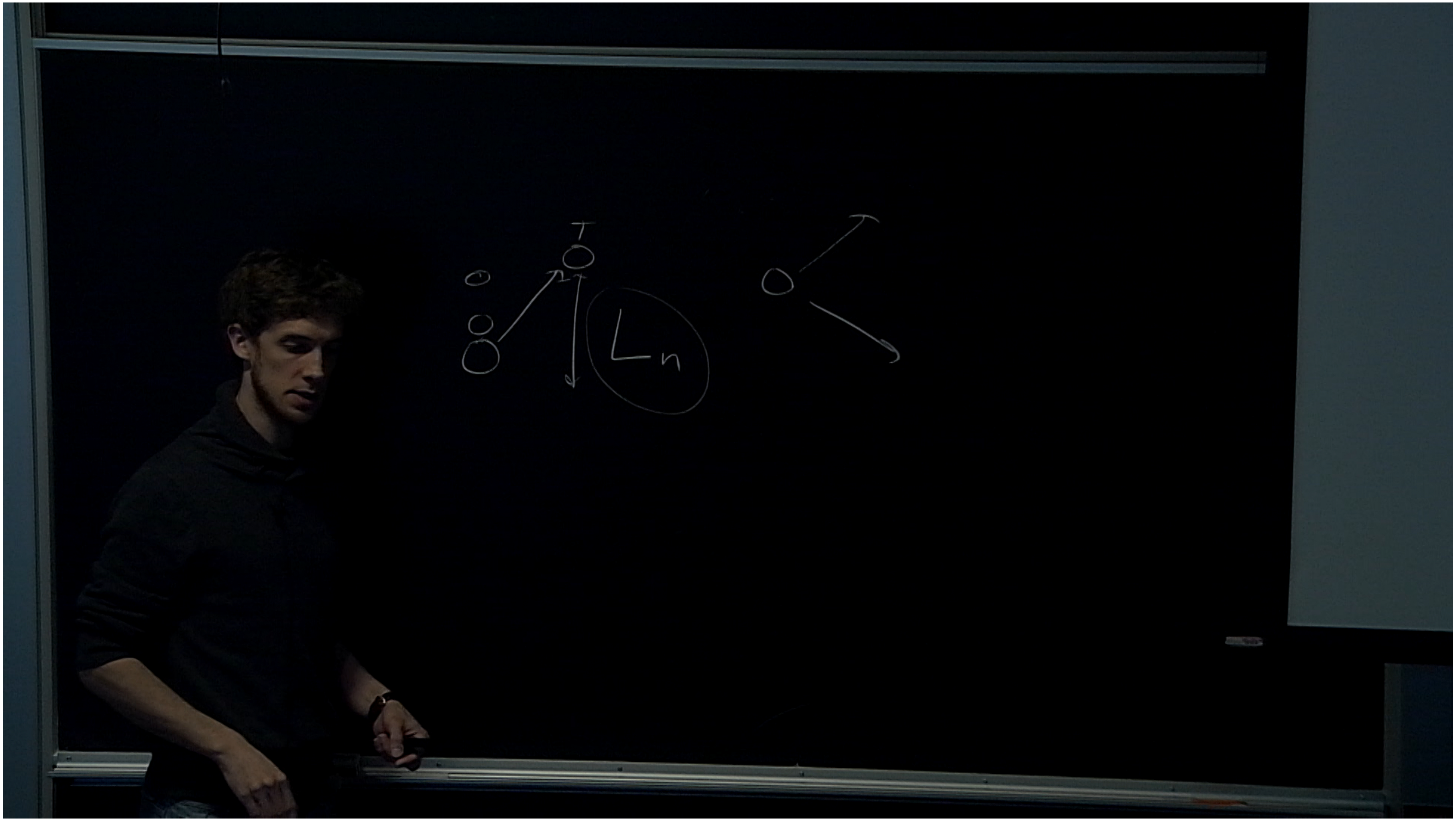


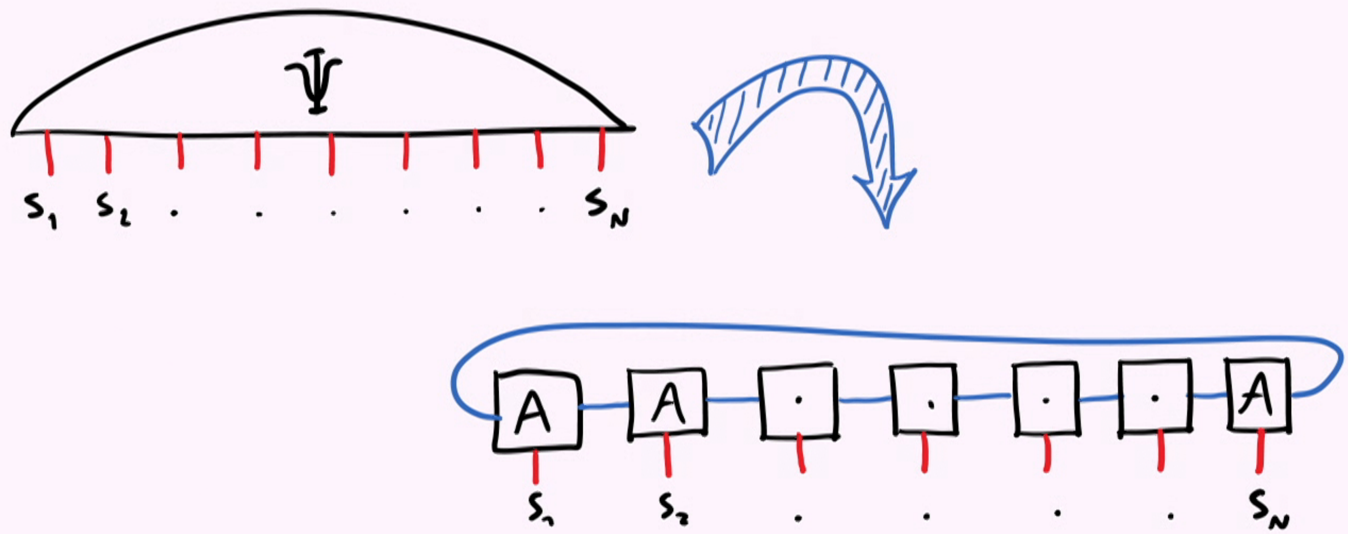
Matrix Product States on the circle

local gradient descent


L_n


L_n


Ln



Matrix Product States on the circle

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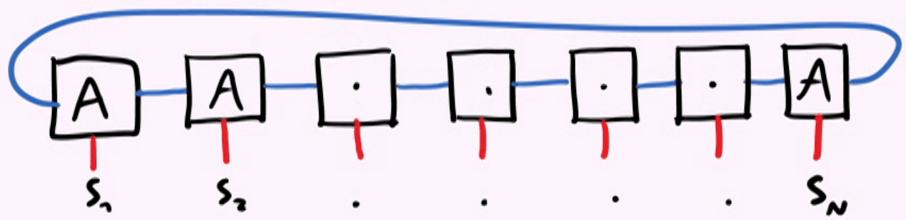
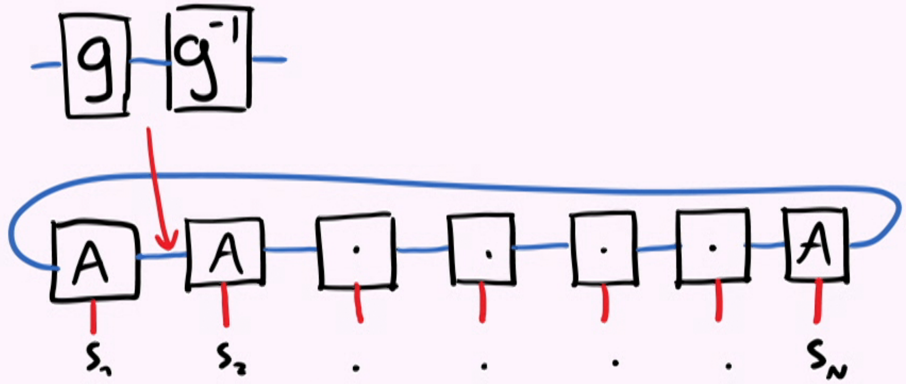


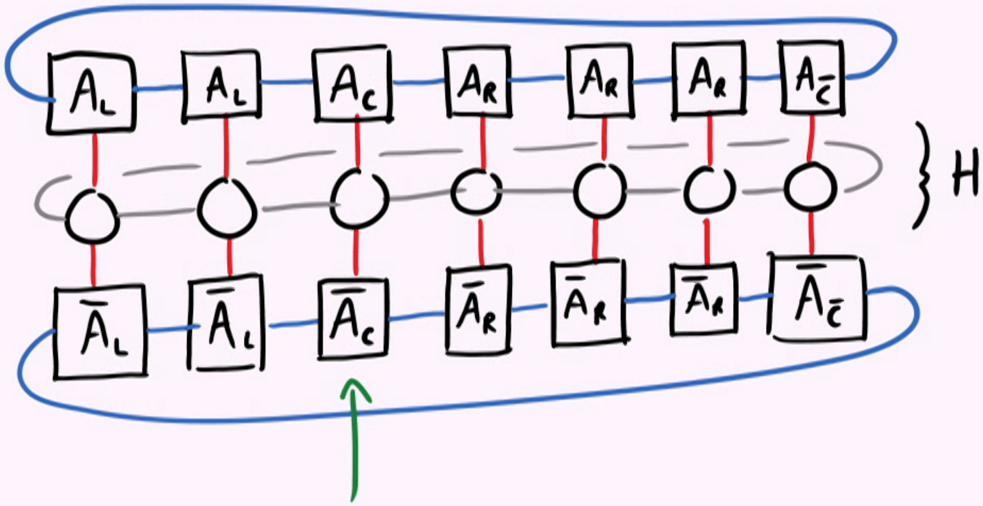
$$|\Psi\rangle = \sum_{s_1 \ldots s_N} \mathrm{tr}(A^{s_1} A^{s_2} \ldots A^{s_N}) |s_1 \ldots s_N\rangle$$



$$|\Psi\rangle = \sum_{s_1 \ldots s_N} \mathrm{tr}(A^{s_1} g g^{-1} A^{s_2} \ldots A^{s_N}) |s_1 \ldots s_N\rangle$$

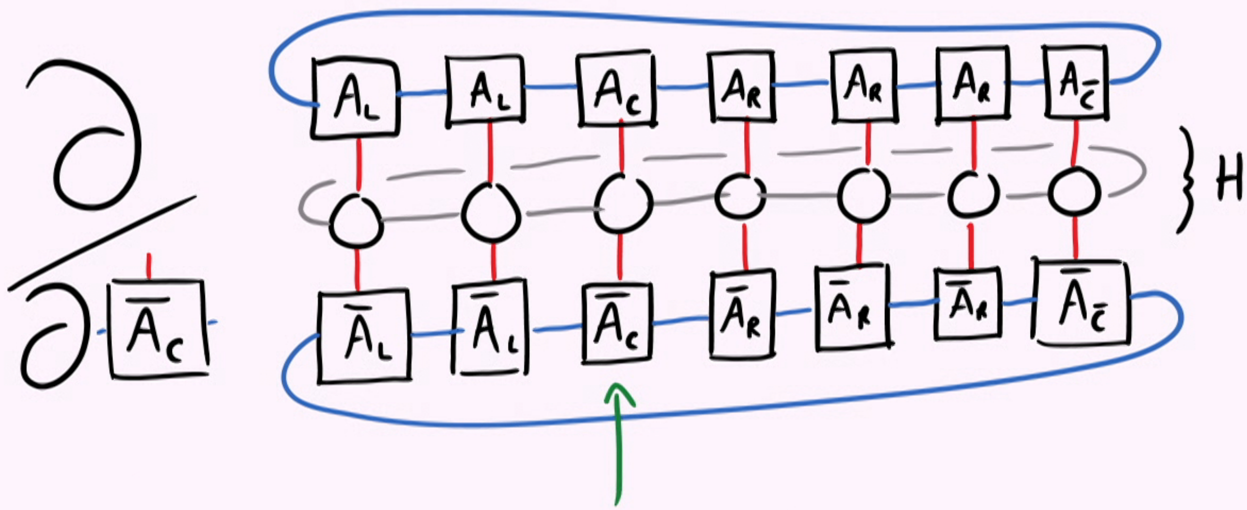
"gauge" freedom

ground states via *local* gradient descent



work in "center gauge"

ground states via *local* gradient descent



work in "center gauge"

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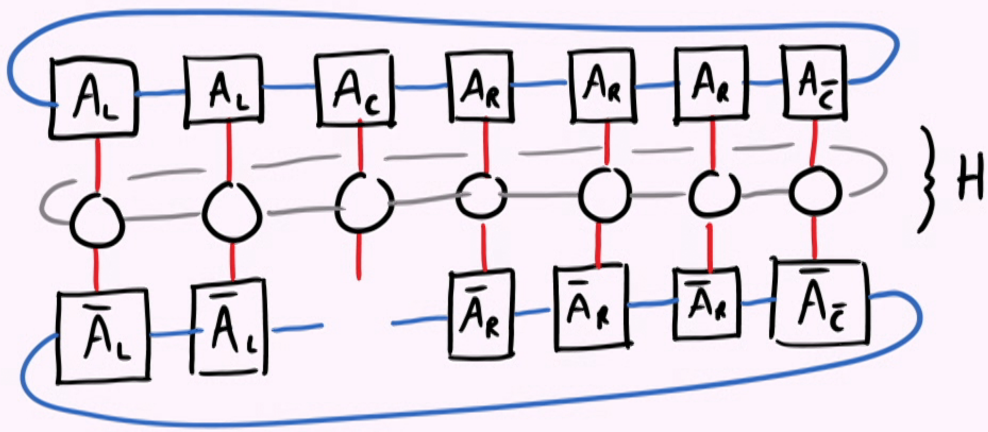
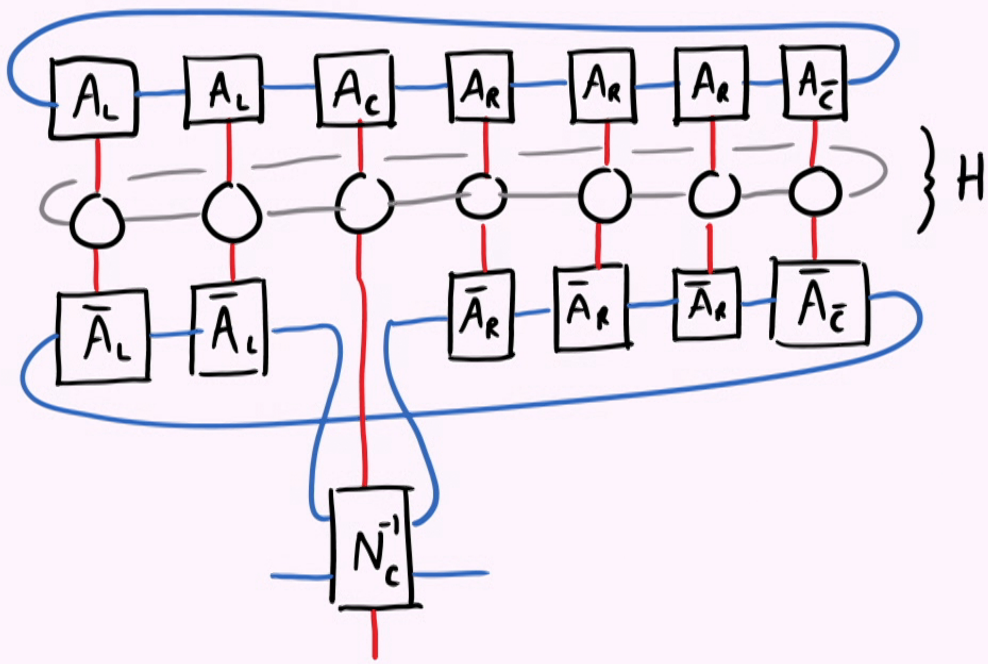
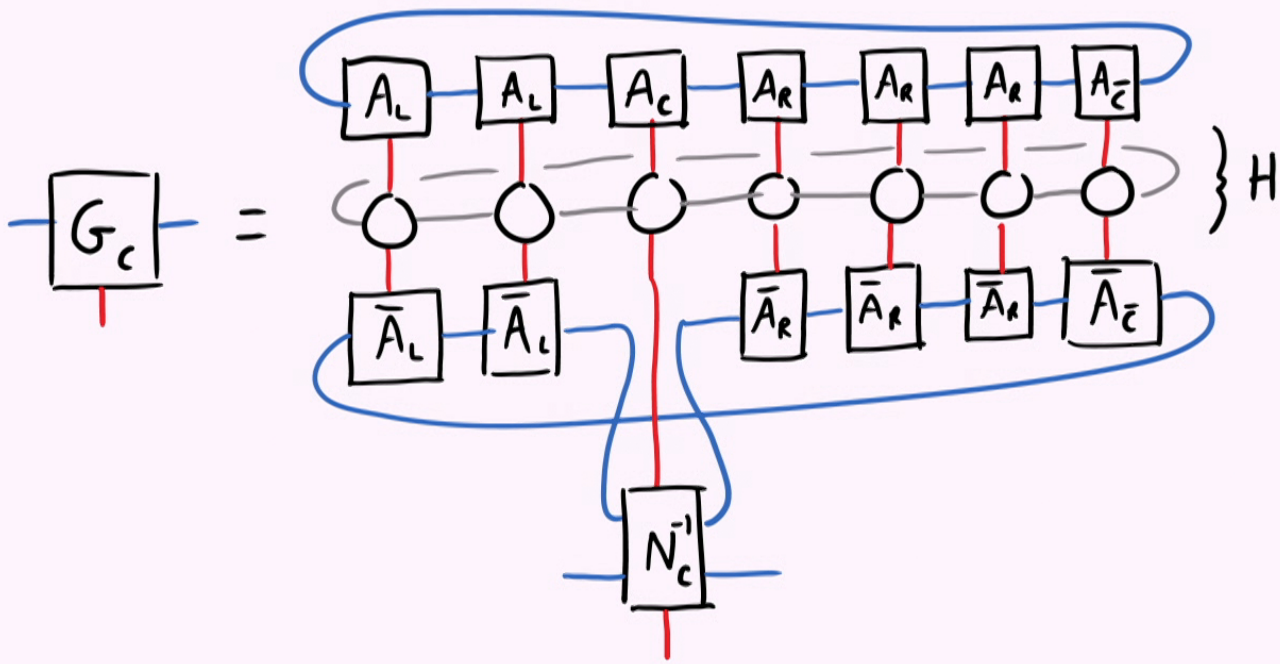


ground states via *local* gradient descent



ground states via *local* gradient descent



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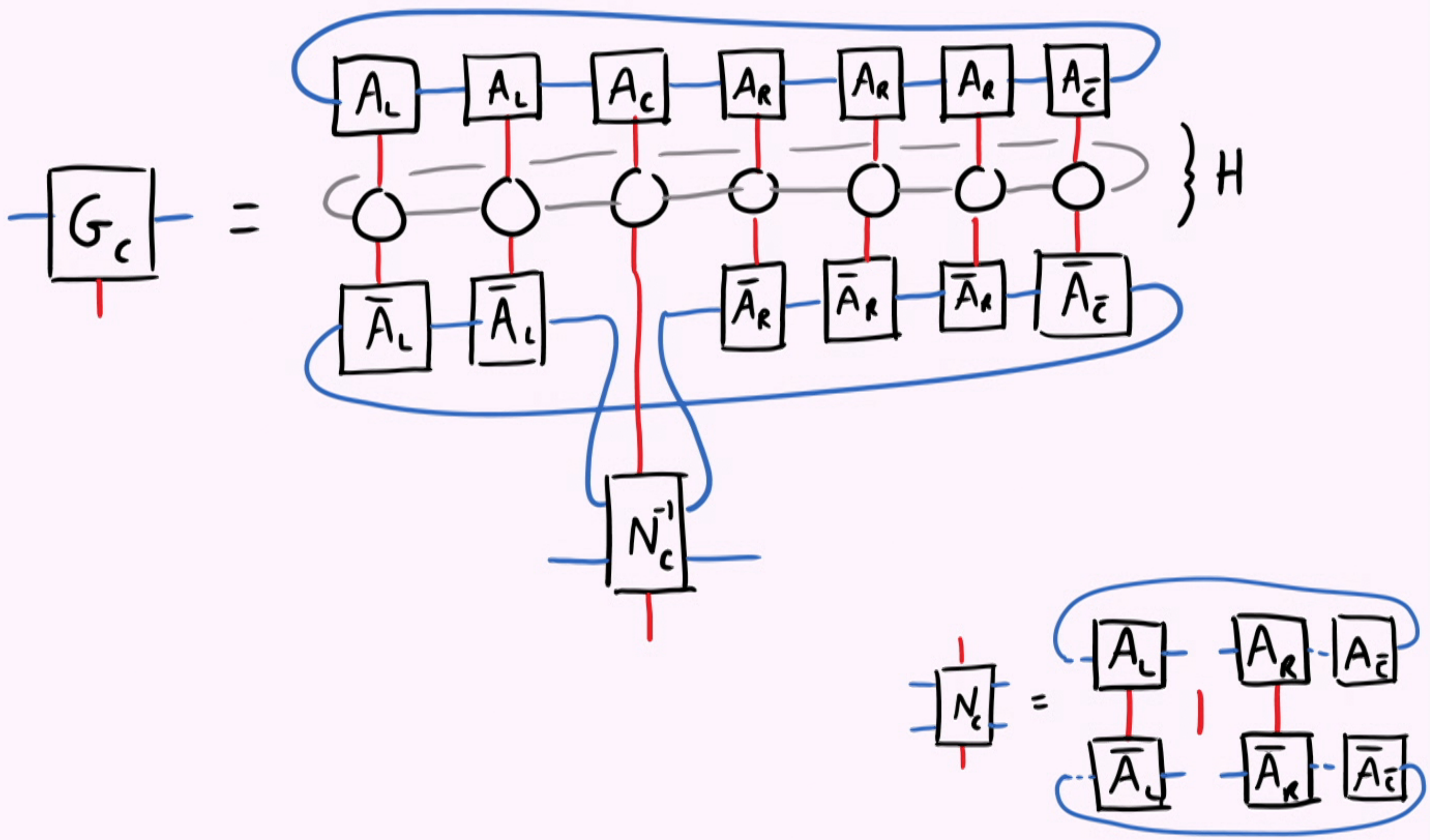

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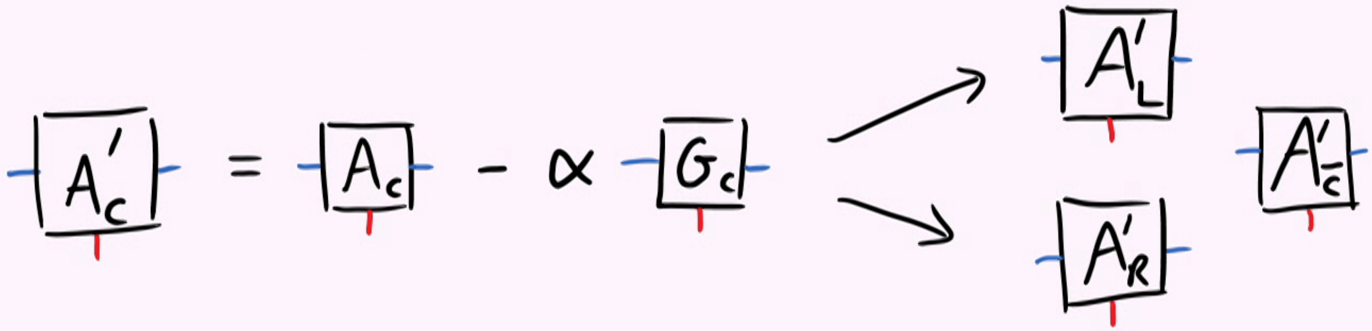
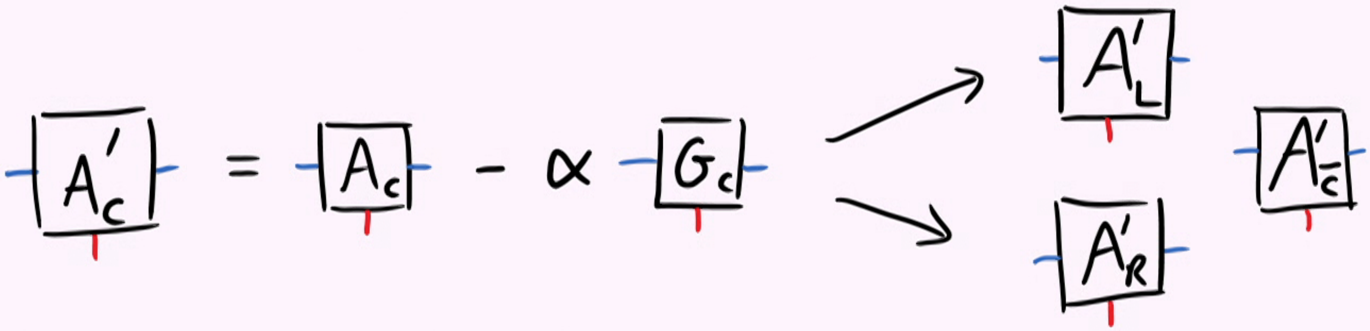
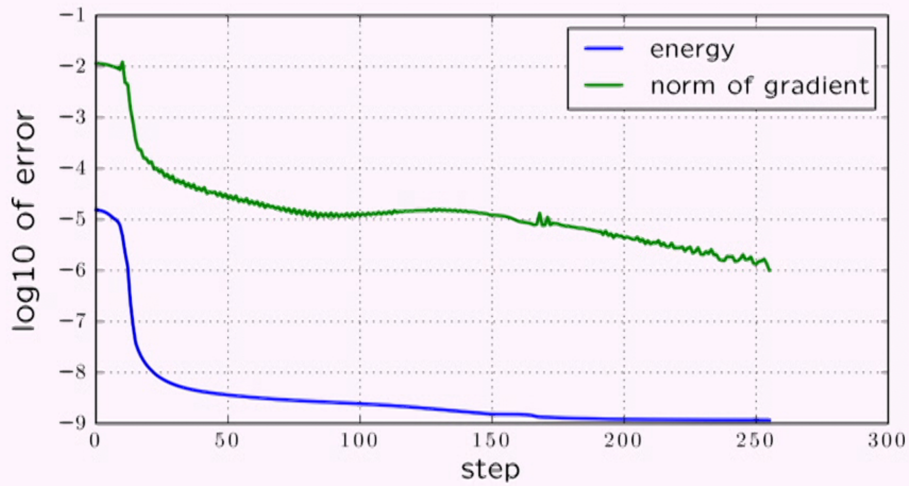


ground states via *local* gradient descent





time cost

$$\mathcal{O}(N\chi^5)$$

conclusions

Hamiltonian mode expansion acts as conformal generator **on finite spin chains**

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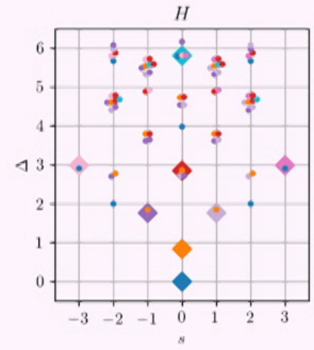
Hamiltonian mode expansion acts as conformal generator **on finite spin chains**

we can **identify** energy eigenstates with **conformal fields** *without any assumptions about the CFT*



conclusions

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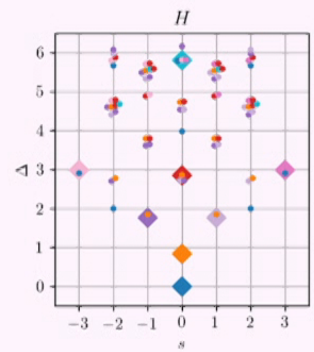

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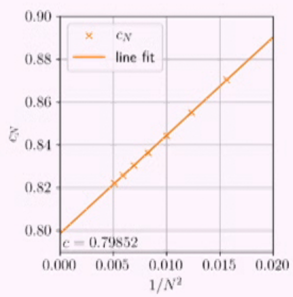

we can estimate the **central charge** from ground state expectation values

conclusions

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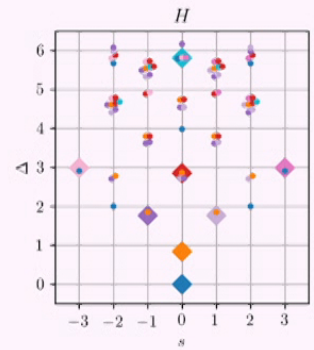
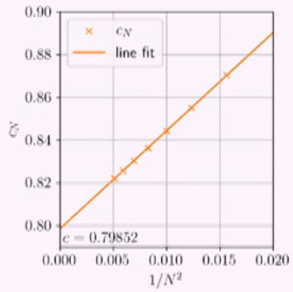


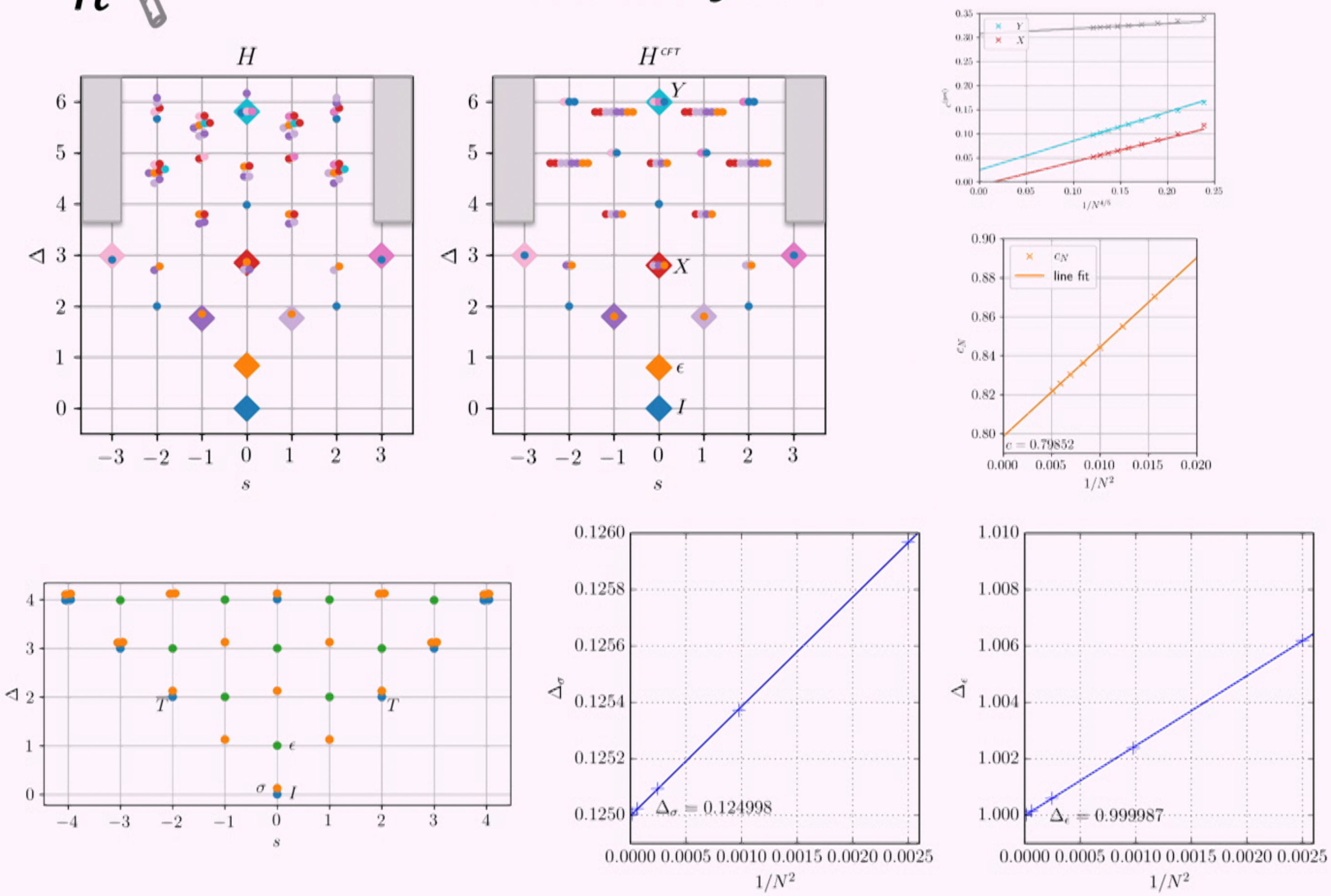


we can estimate the **central charge** from ground state expectation values

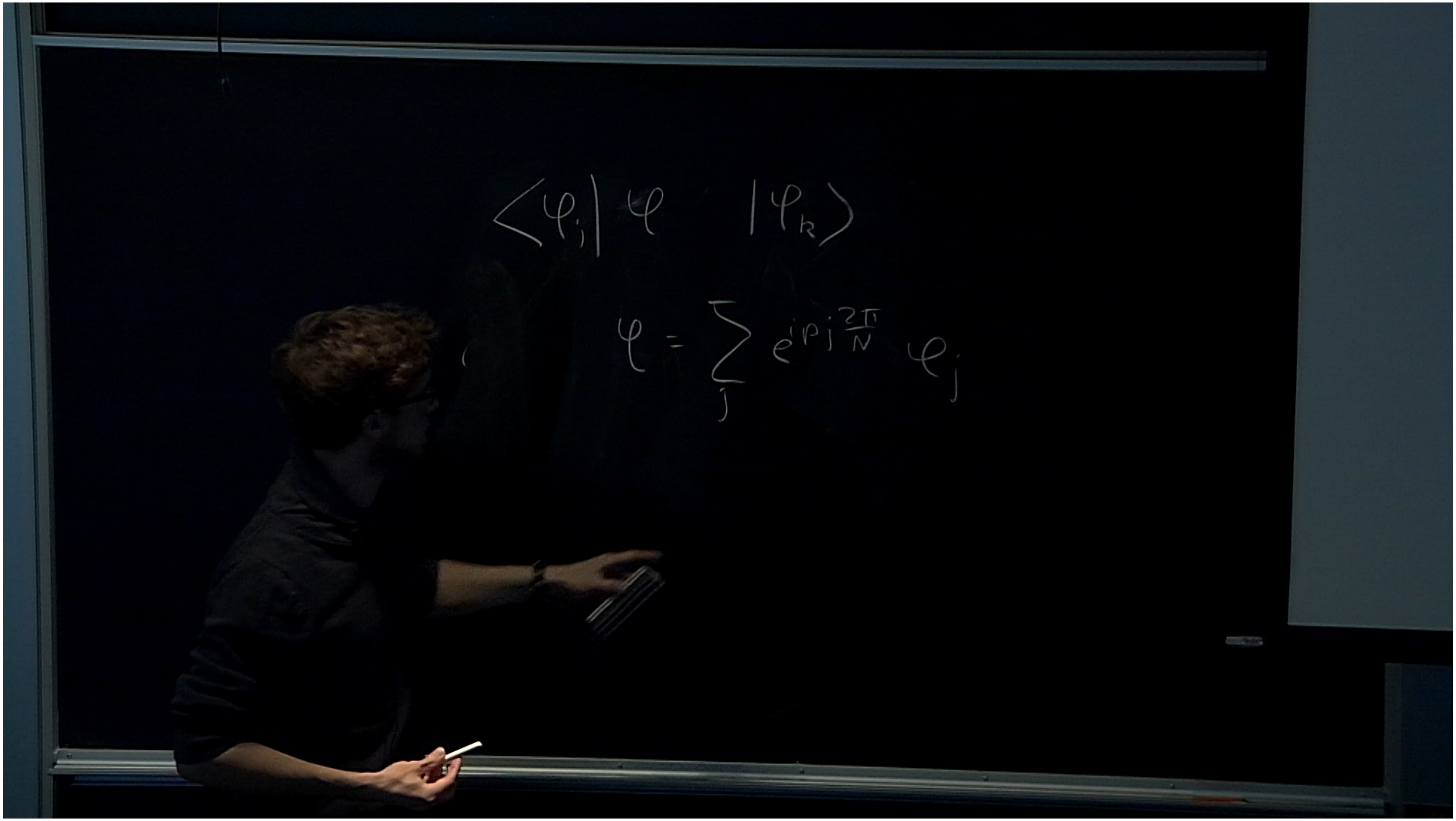
our identification method is **independent** of diagonalization technique

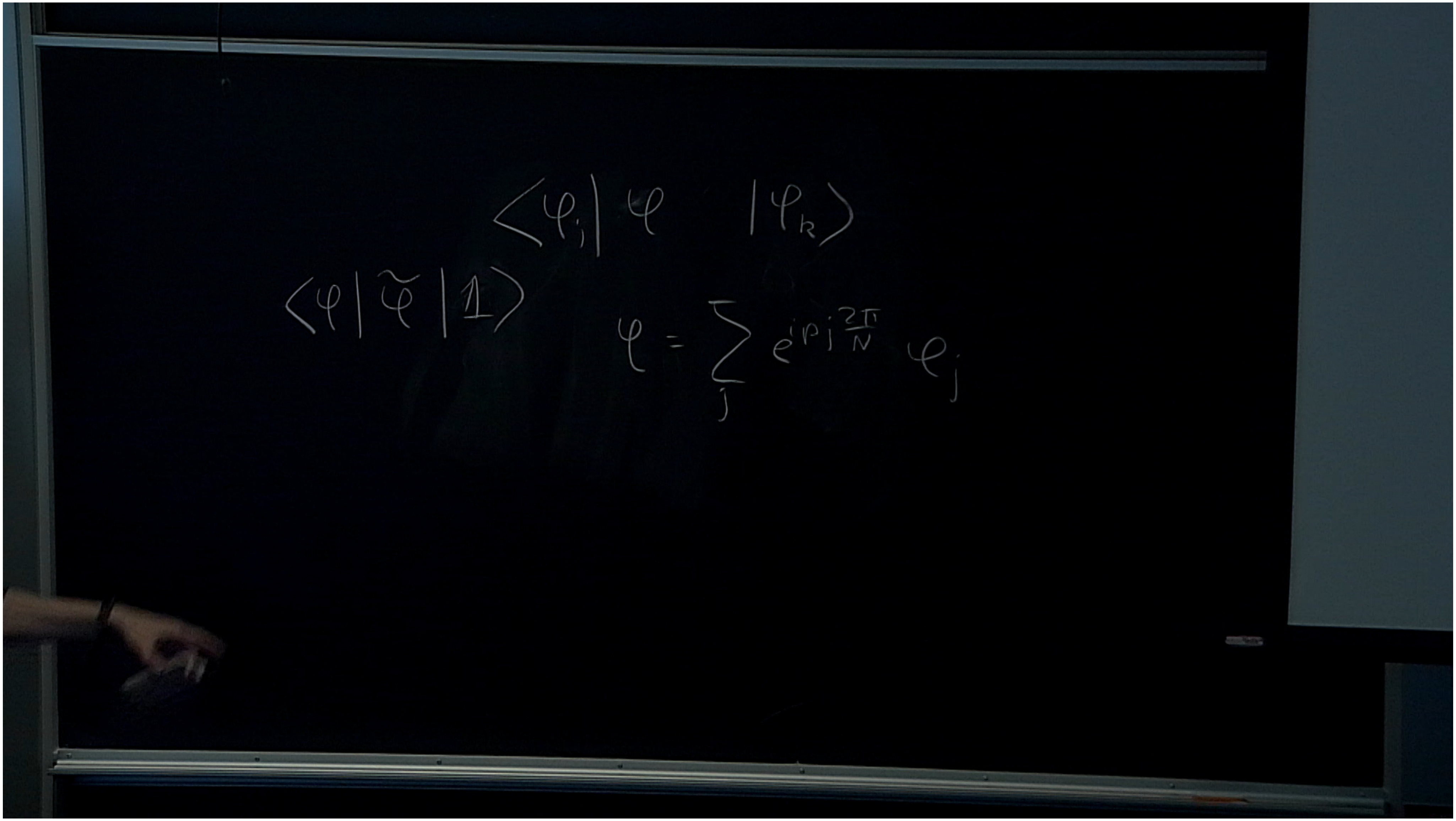
and MPS on the circle are a great choice!

thank you!



H
H^{CFT}
Y
X
ϵ
I
Δ
s
T
σ
c_N
line fit
$c = 0.79852$
$1/N^2$
$\Delta_\sigma = 0.124998$
$\Delta_\epsilon = 0.999987$


$\langle \varphi_i | \varphi \quad | \varphi_k \rangle$
$\varphi = \sum_j e^{ipj\frac{2\pi}{N}} \varphi_j$


$\langle \varphi_i | \varphi \quad | \varphi_k \rangle$
$\langle \varphi | \tilde{\varphi} | 1 \rangle$
$\varphi = \sum_j e^{ipj\frac{2\pi}{N}} \varphi_j$