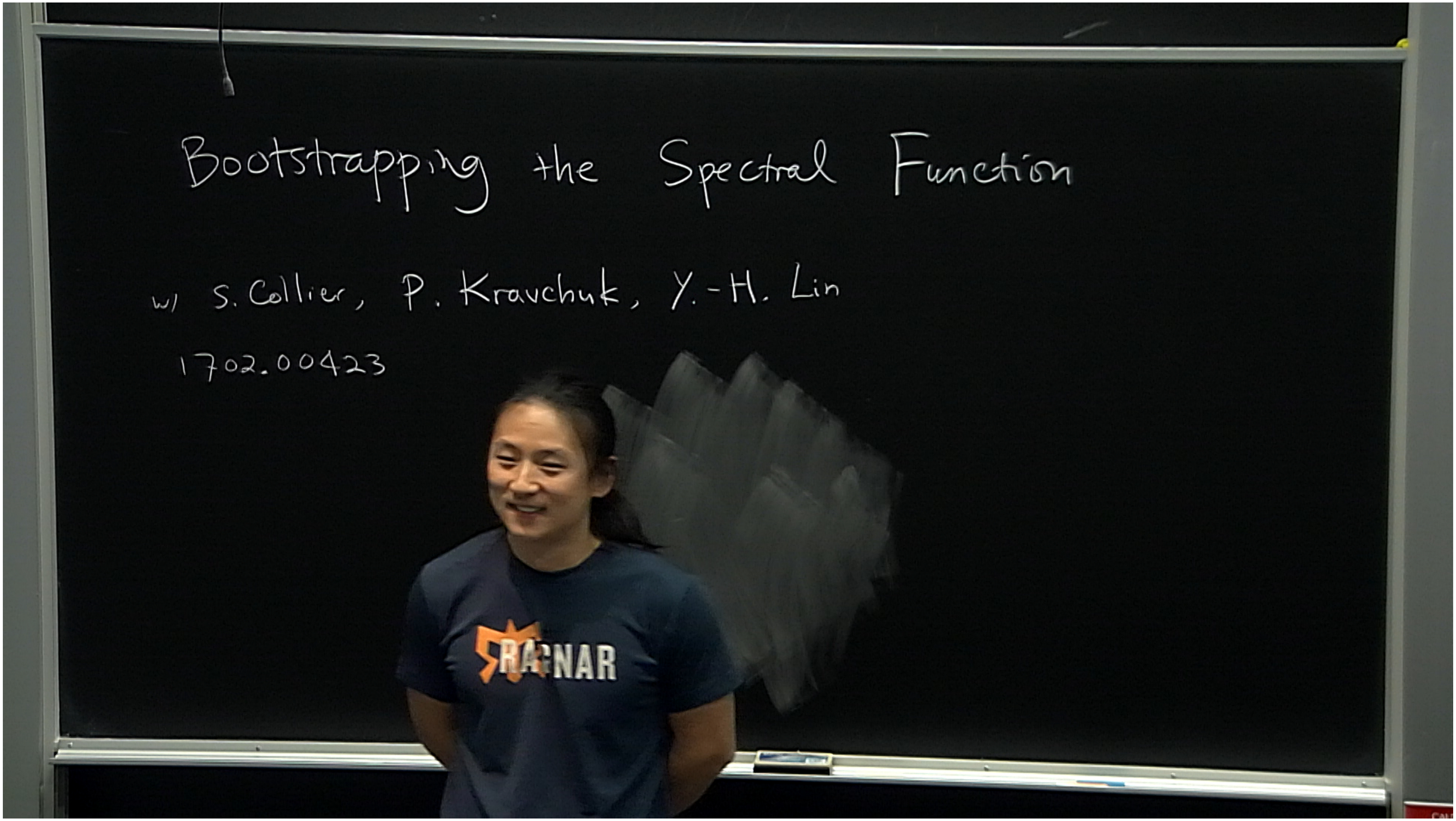
Title: Bootstrapping the spectral function

Date: Apr 04, 2017 02:30 PM

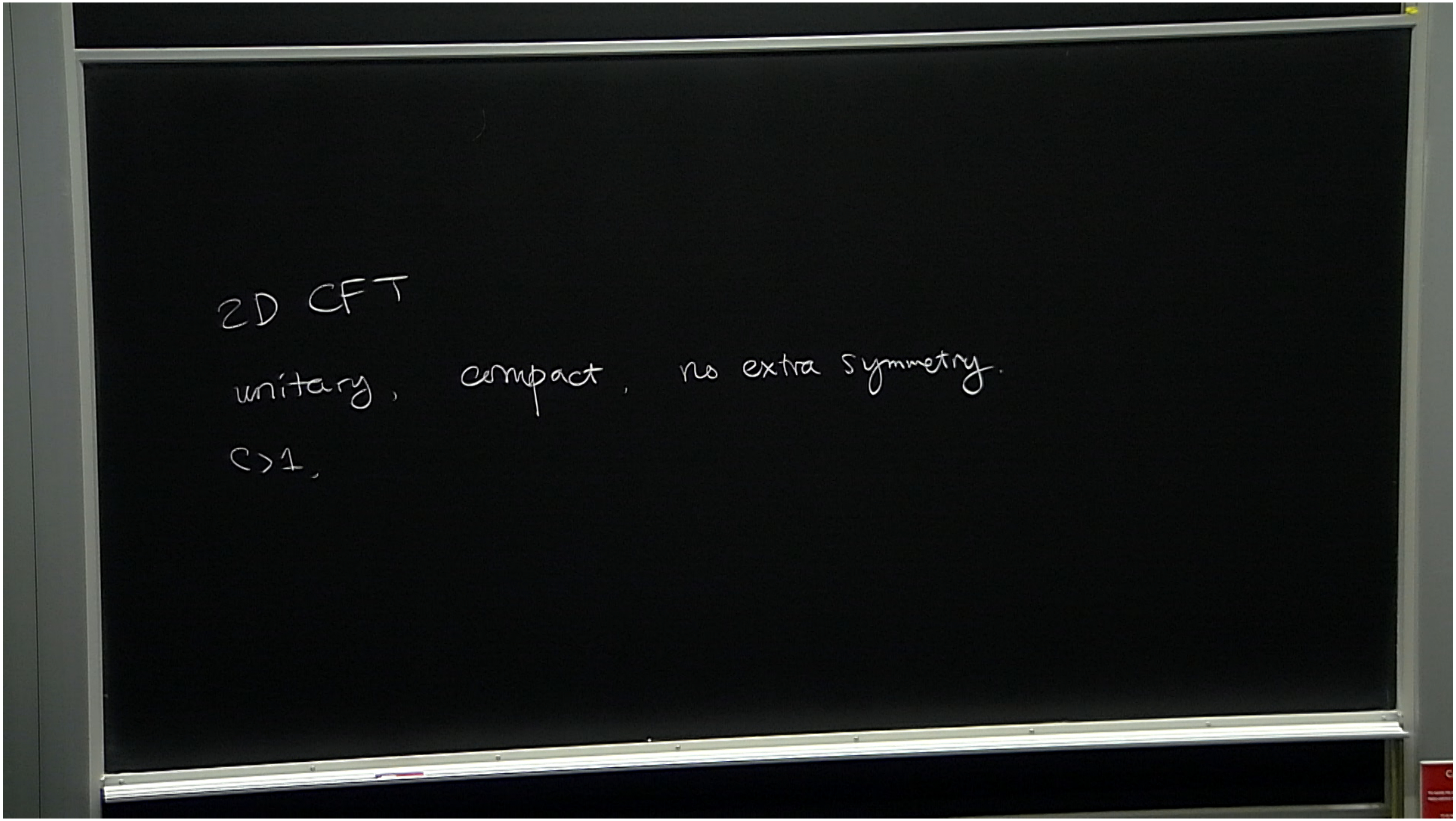
URL: http://pirsa.org/17040028

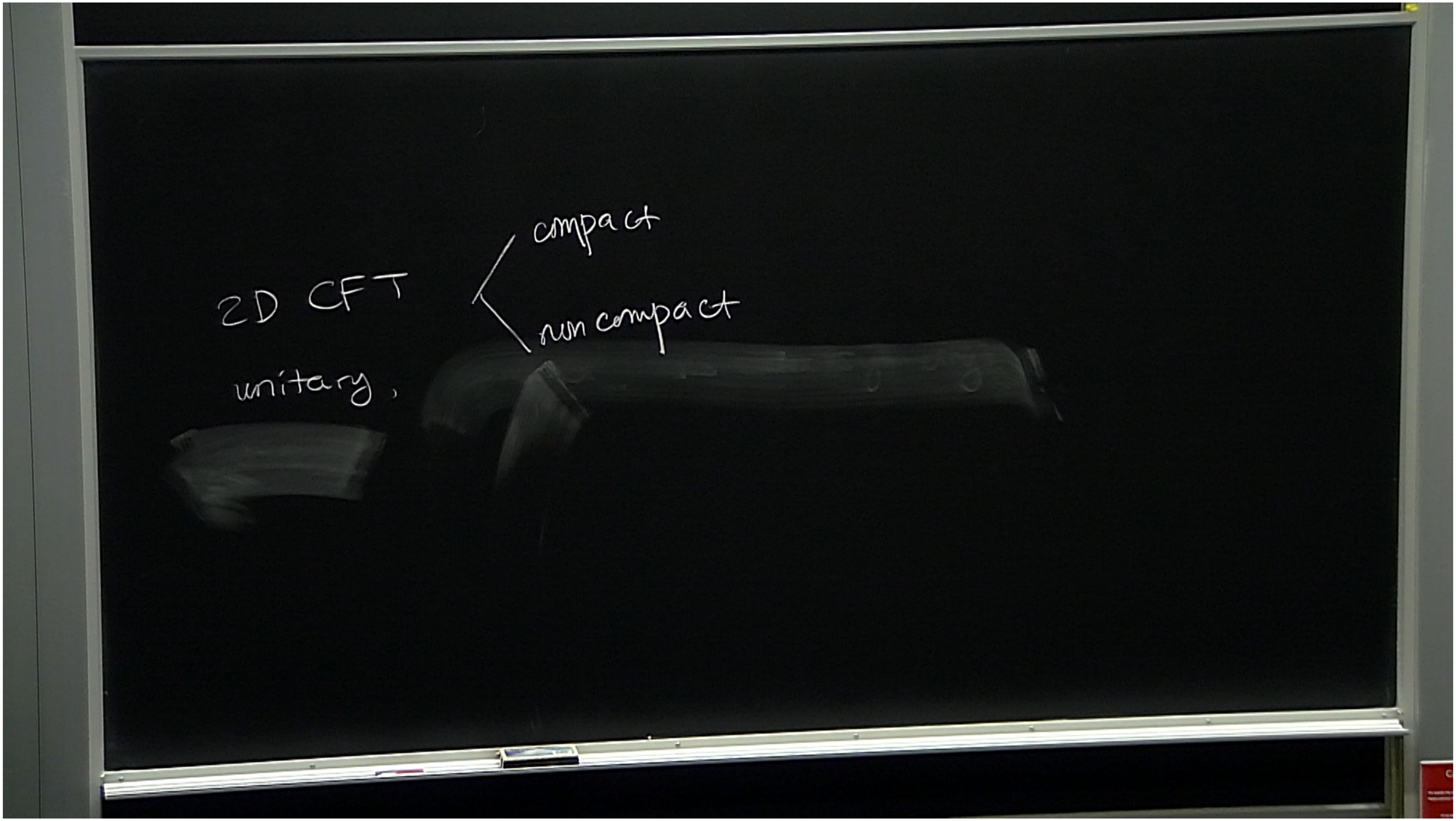
Abstract: <p>I will introduce the spectral function method in the context of conformal bootstrap. I will discuss some applications of this method in two dimensions: (1) substantial evidence for the conjecture that the only unitary c>1 CFT with Virasoro primaries of bounded spin is Liouville theory, (2) detailed modular constraints on the spectrum of small c CFTs, (3) spectral density of large c CFTs with large gap, in connection to the (non-)universality of BTZ black hole entropy.</p>

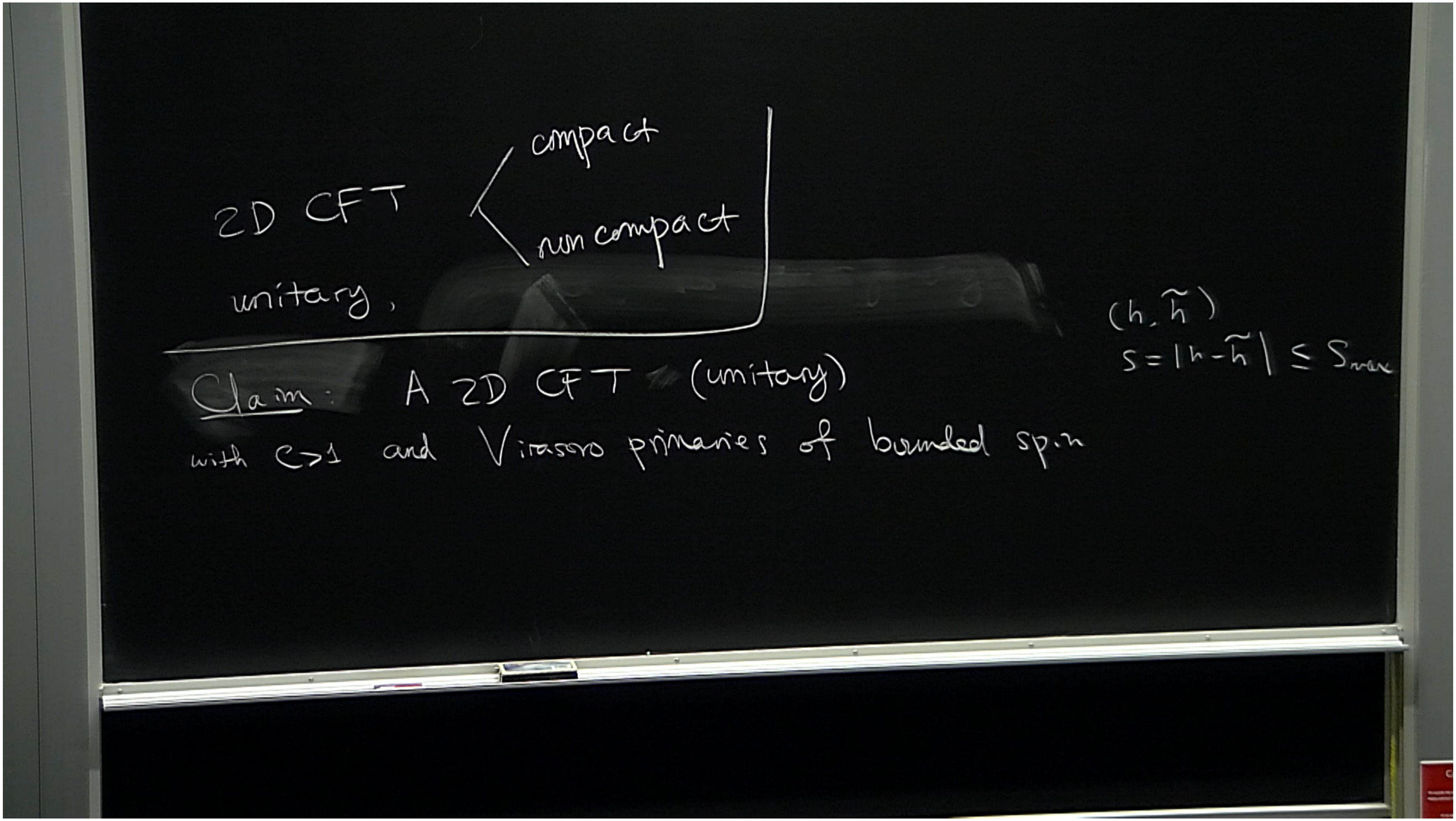

Bootstrapping the Spectral Function
w/ S. Collier, P. Kravchuk, Y.-H. Lin
1702.00423

2D CFT

unitary, compact,


2D CFT
unitary, compact, no extra symmetry.
c>1.


2D CFT
unitary,
compact
non compact


2D CFT
compact
non compact
unitary,
$(h, \tilde{h})$
$s = |h - \tilde{h}| \leq s_{max}$
Claim: A 2D CFT (unitary)
with $c > 1$ and Virasoro primaries of bounded spin

$$\text{2D CFT} \begin{cases} \text{compact} \\ \text{non compact} \end{cases}$$

unitary,

Claim: A 2D CFT (unitary) with $c>1$ and Virasoro primaries of bounded spin must be Liouville CFT.

$(h, \tilde{h})$

$s = |h - \tilde{h}| \leq s_{max}$

2D CFT — compact / non compact

unitary,

$(h, \tilde{h})$

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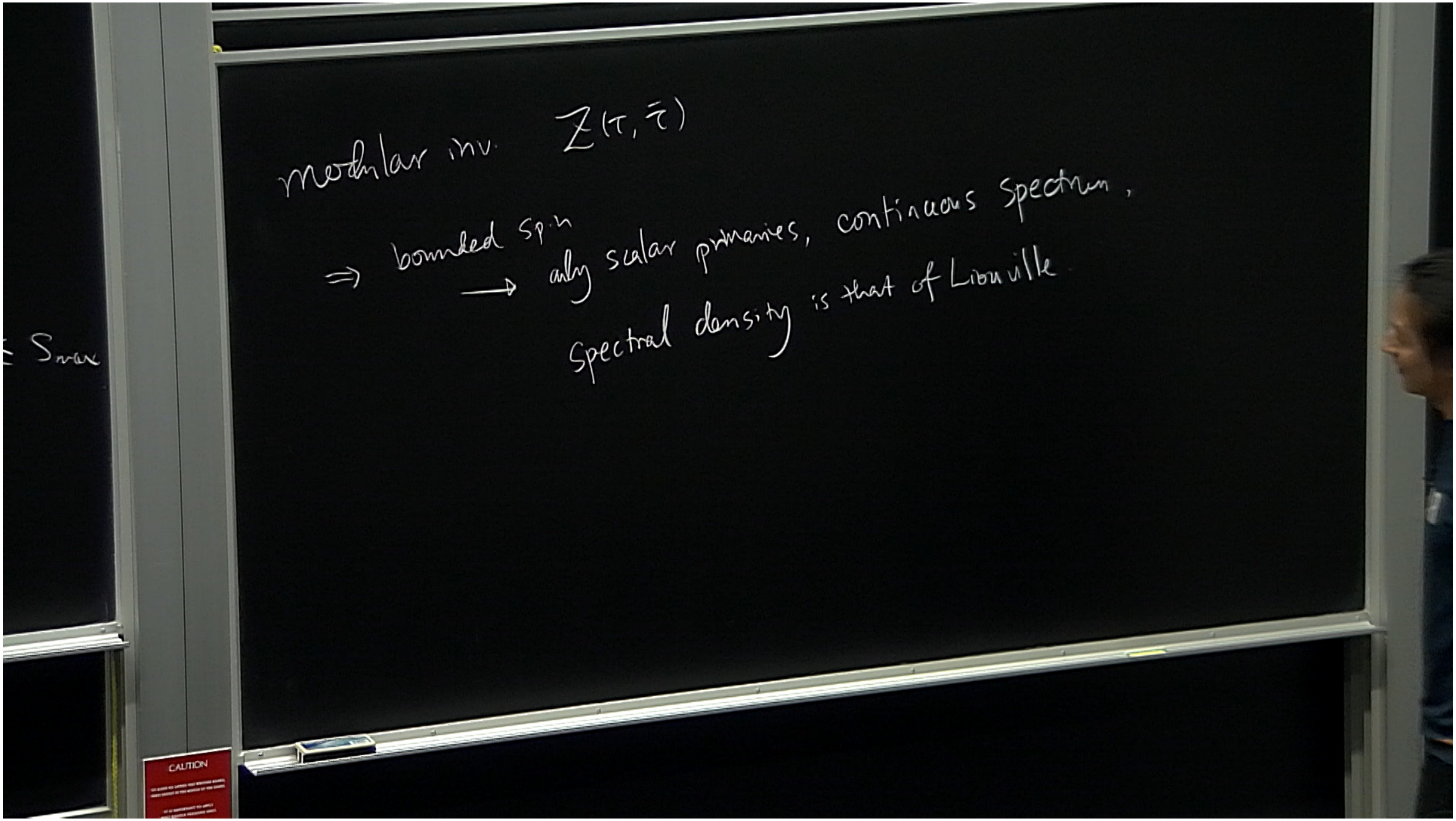
Claim: A 2D CFT (unitary) with $c > 1$ and Virasoro primaries of bounded spin must be Liouville CFT.


modular inv. $Z(\tau, \bar{\tau})$
$\Rightarrow$ bounded spin
$\longrightarrow$ only scalar primaries, continuous spectrum,
spectral density is that of Liouville
$\leq S_{max}$

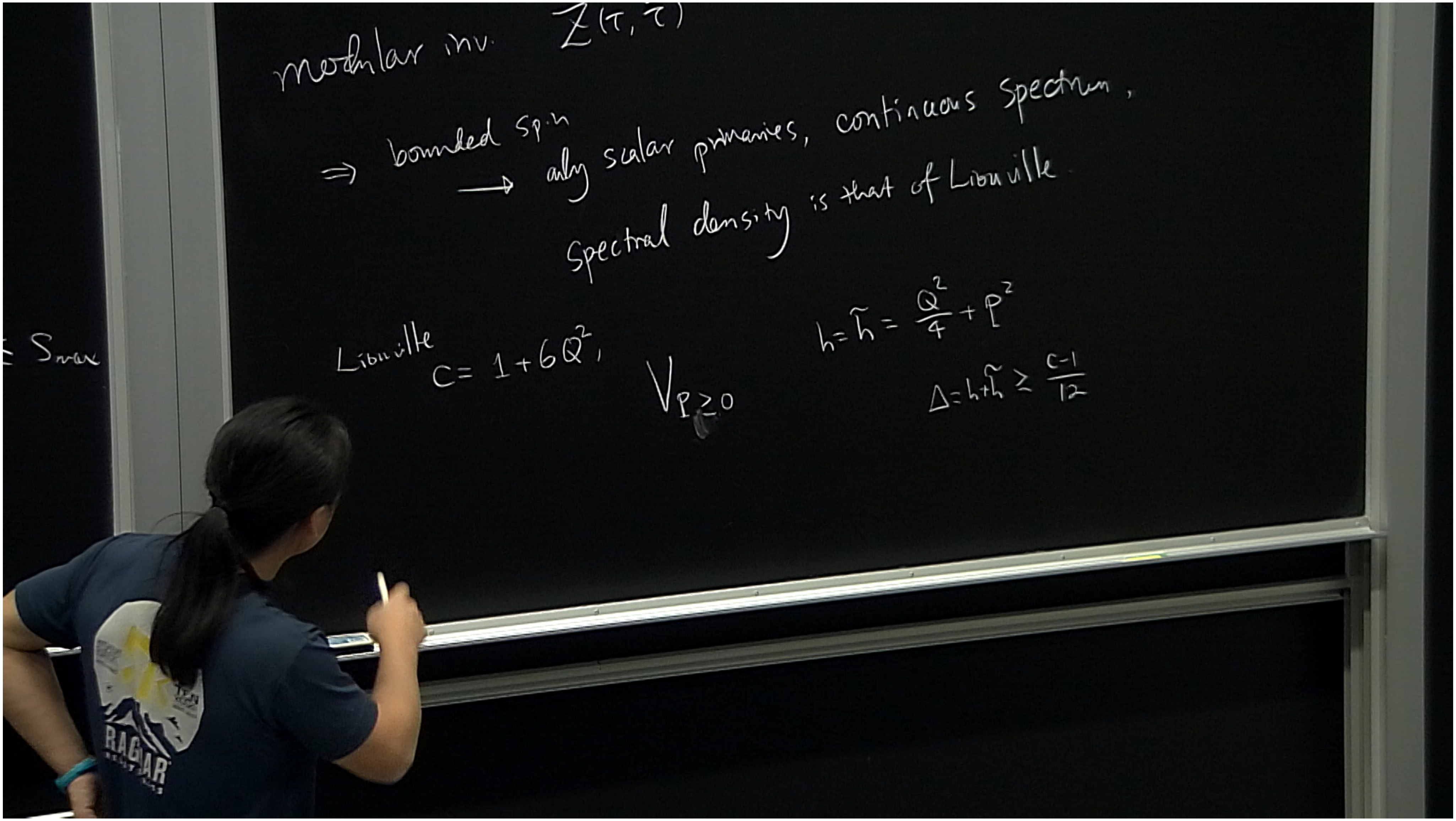
modular inv. $Z(\tau, \bar{\tau})$

$\Rightarrow$ bounded spin $\longrightarrow$ only scalar primaries, continuous spectrum, spectral density is that of Liouville.

$\leq S_{max}$

Liouville $c = 1 + 6Q^2$, $h = \tilde{h} = \frac{Q^2}{4} + P^2$

$V_{P \geq 0}$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$\Rightarrow$ bounded spin
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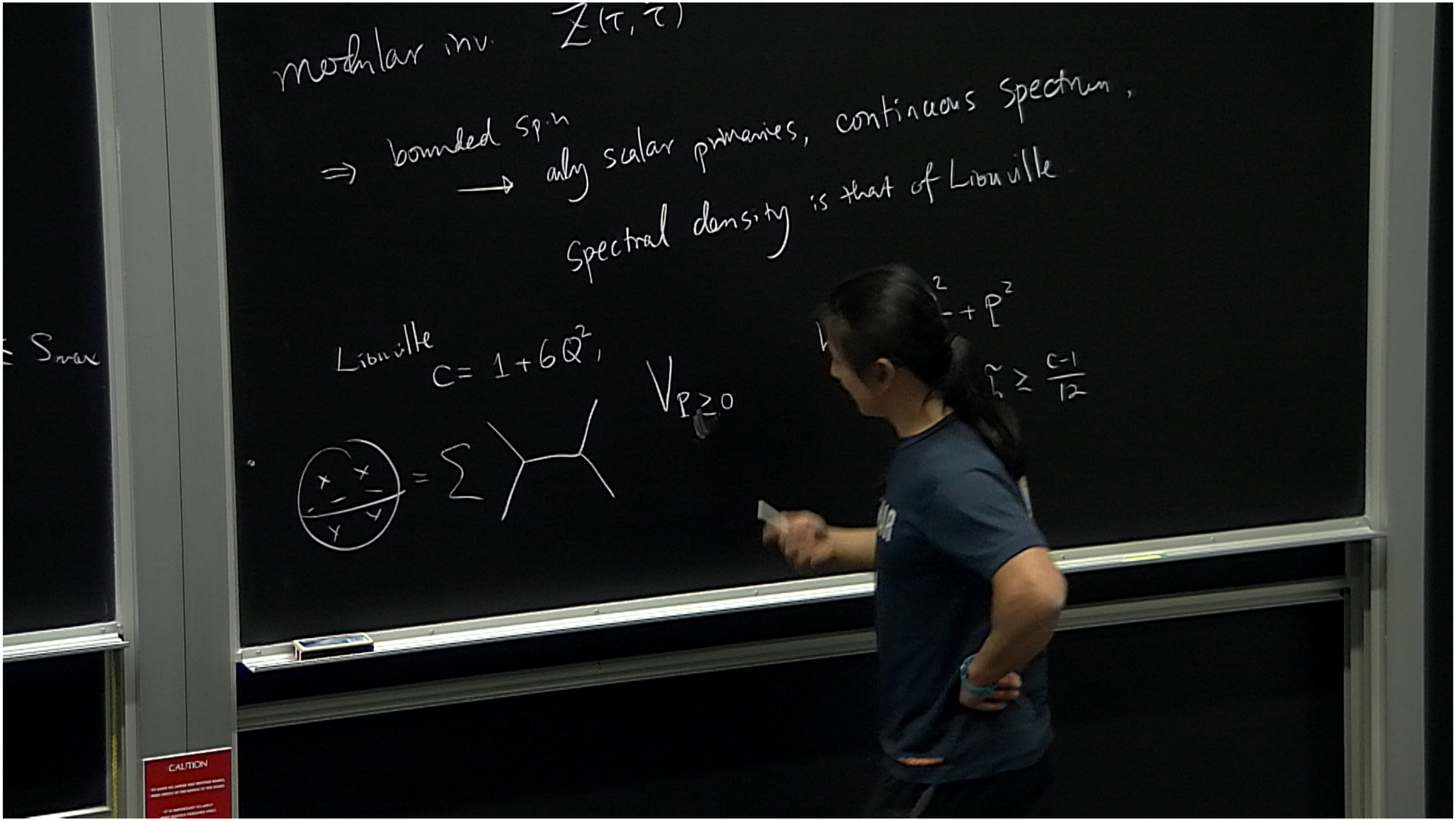
spectral density is that of Liouville.
$\leq S_{max}$
Liouville
$c = 1 + 6Q^2$,
$V_{P \geq 0}$
$h = \tilde{h} = \frac{Q^2}{4} + P^2$
$\Delta = h + \tilde{h} \geq \frac{c-1}{12}$


modular inv. $Z(\tau, \bar{\tau})$
$\Rightarrow$ bounded spin
$\longrightarrow$ only scalar primaries, continuous spectrum,
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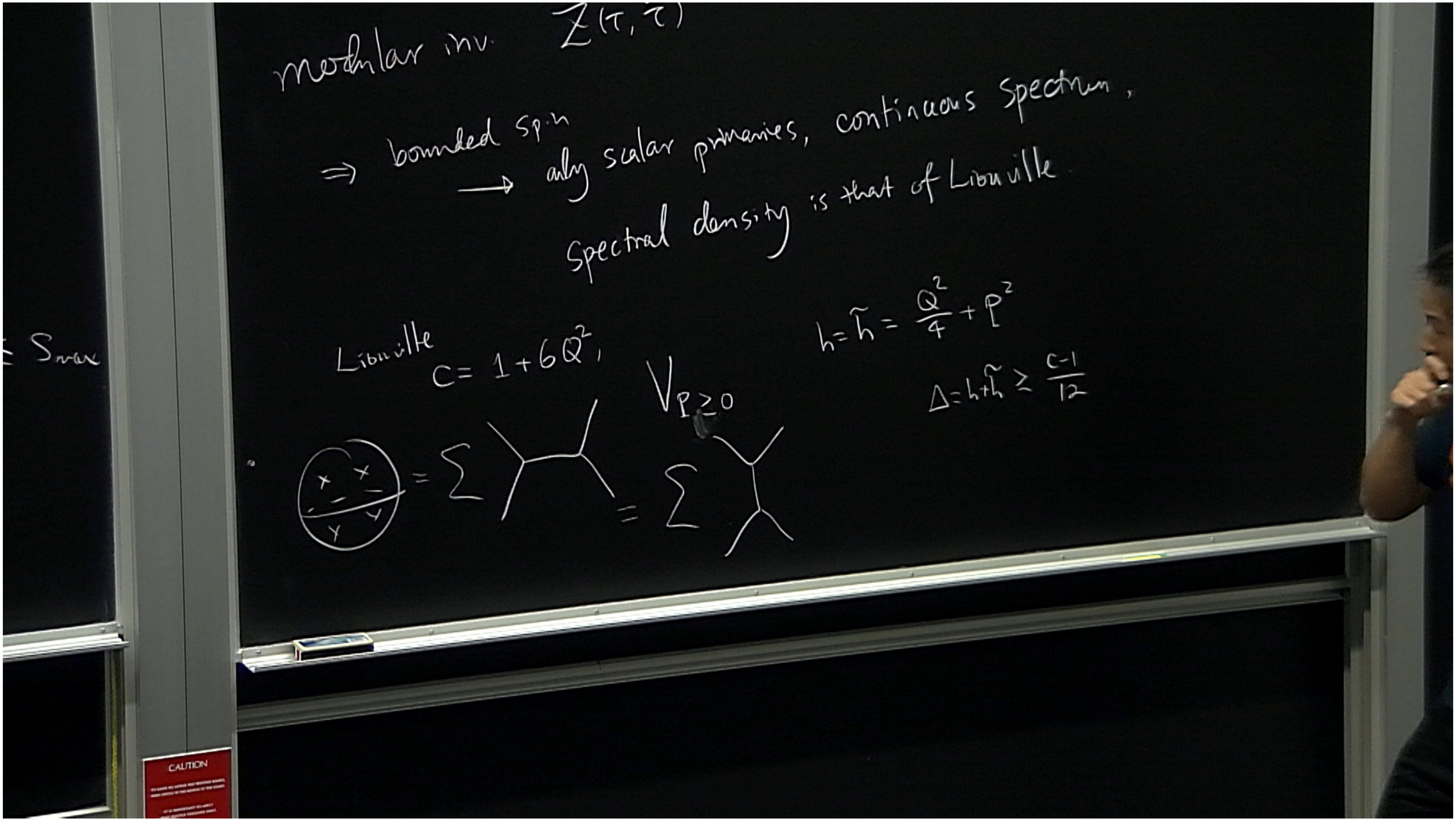
Liouville
$c = 1 + 6Q^2$,
$V_{P \geq 0}$
$+ P^2$
$\tilde{h} \geq \frac{c-1}{12}$
S_{max}
CAUTION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Liouville
$c = 1 + 6Q^2$,
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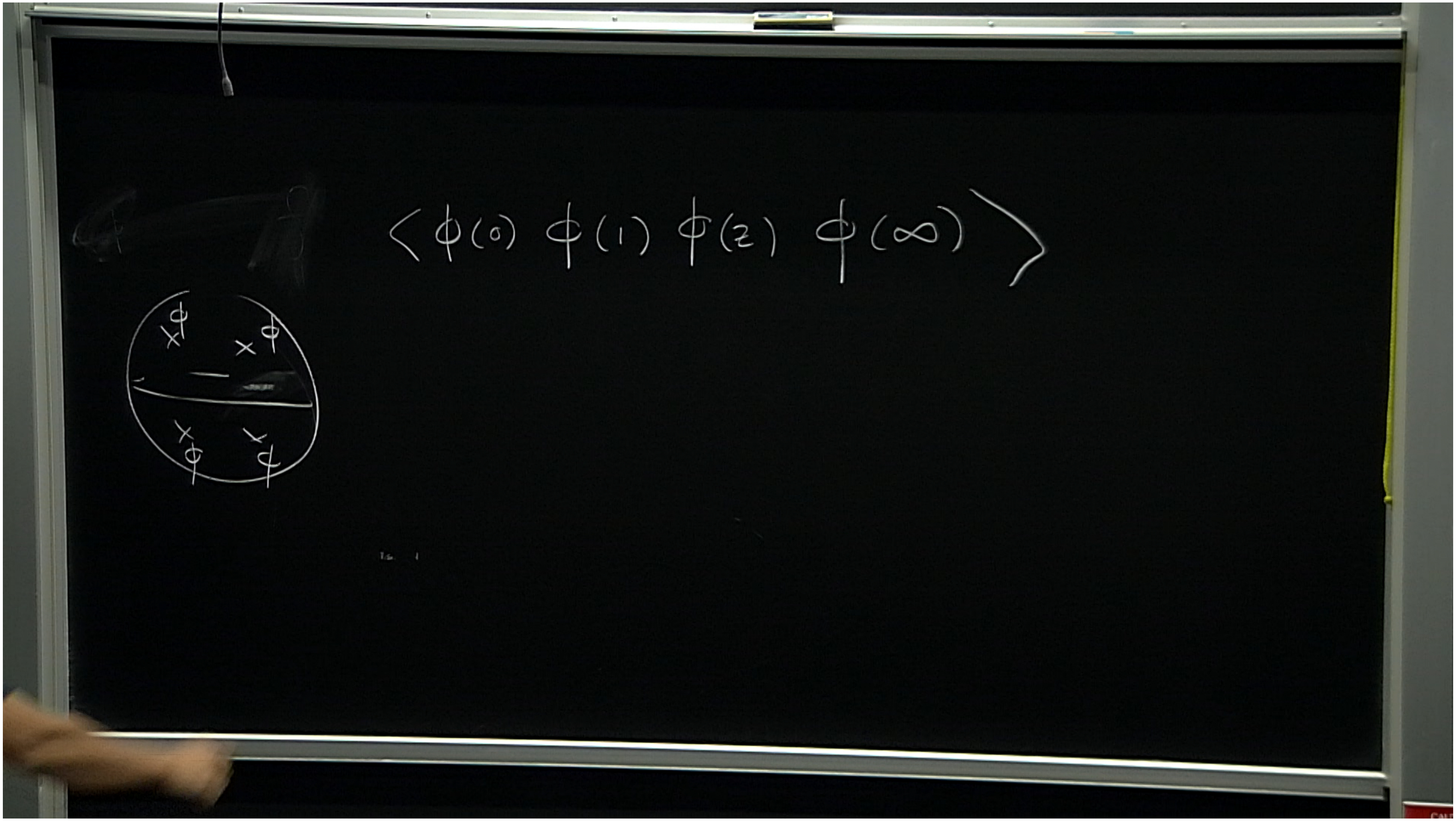
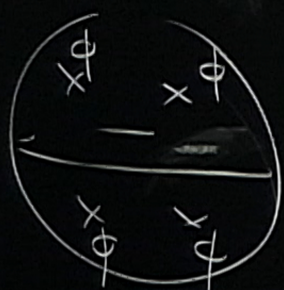
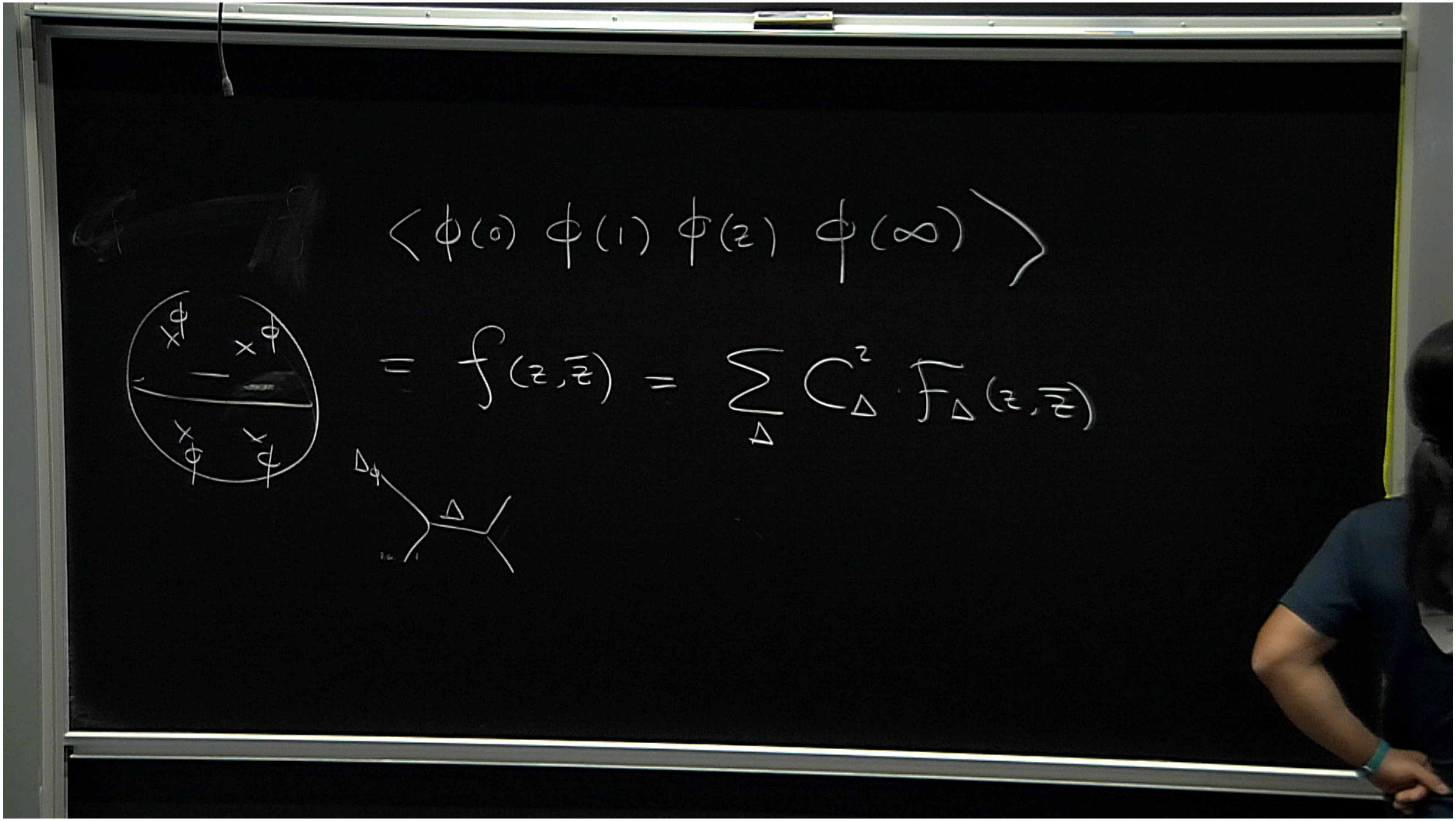
S_{max}

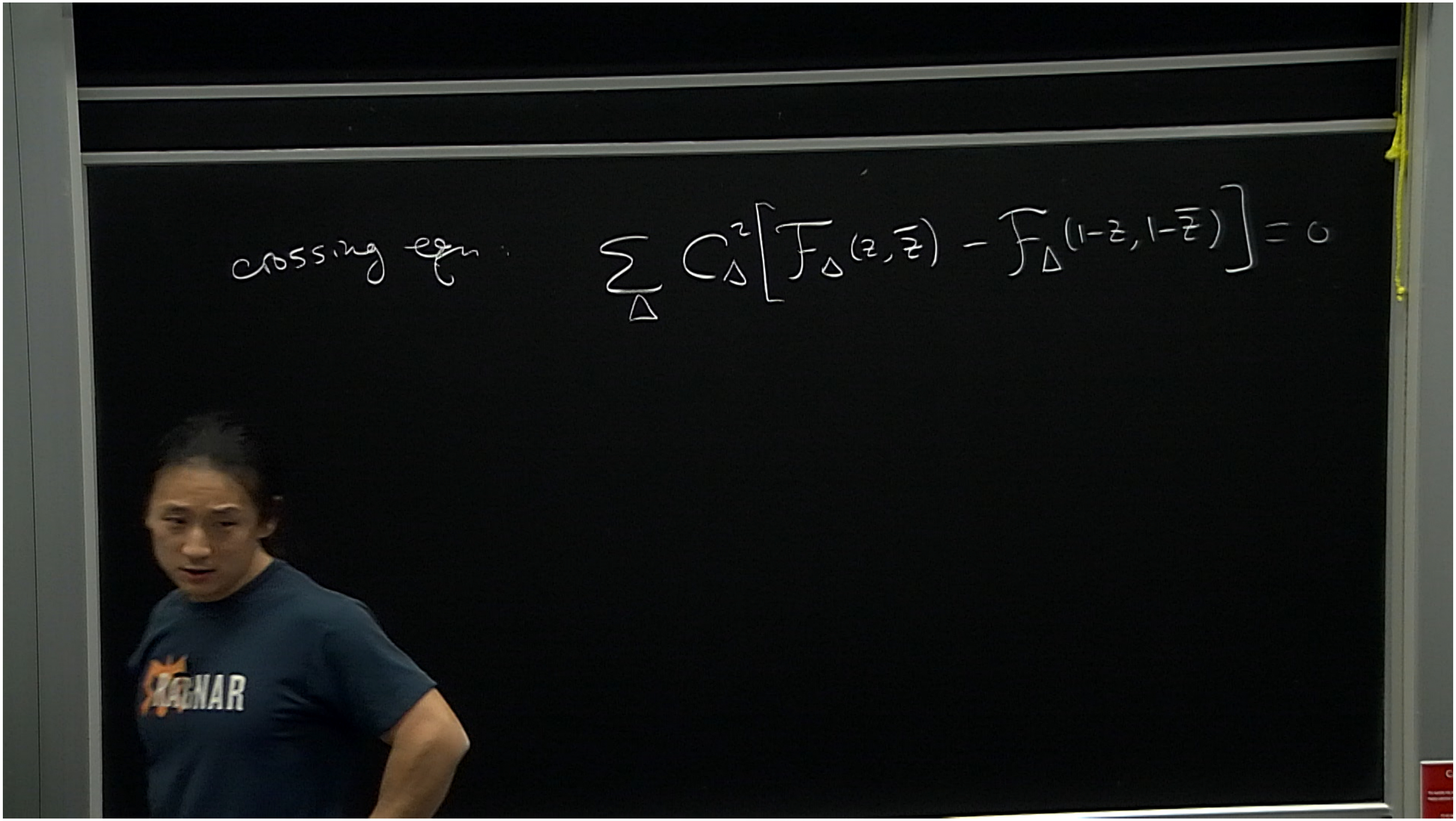

$\langle \phi(0)\, \phi(1)\, \phi(z)\, \phi(\infty) \rangle$

$$\langle \phi(0)\,\phi(1)\,\phi(z)\,\phi(\infty) \rangle$$



$$= f(z,\bar{z})$$


$\langle \phi(0)\, \phi(1)\, \phi(z)\, \phi(\infty) \rangle$
$= f(z, \bar{z}) = \sum_{\Delta} C_{\Delta}^{2} \cdot F_{\Delta}(z, \bar{z})$
Δ_ϕ
Δ


crossing eqn:
$\sum_{\Delta} C_{\Delta}^{2}\left[\mathcal{F}_{\Delta}(z,\bar{z}) - \mathcal{F}_{\Delta}(1-z,1-\bar{z})\right] = 0$

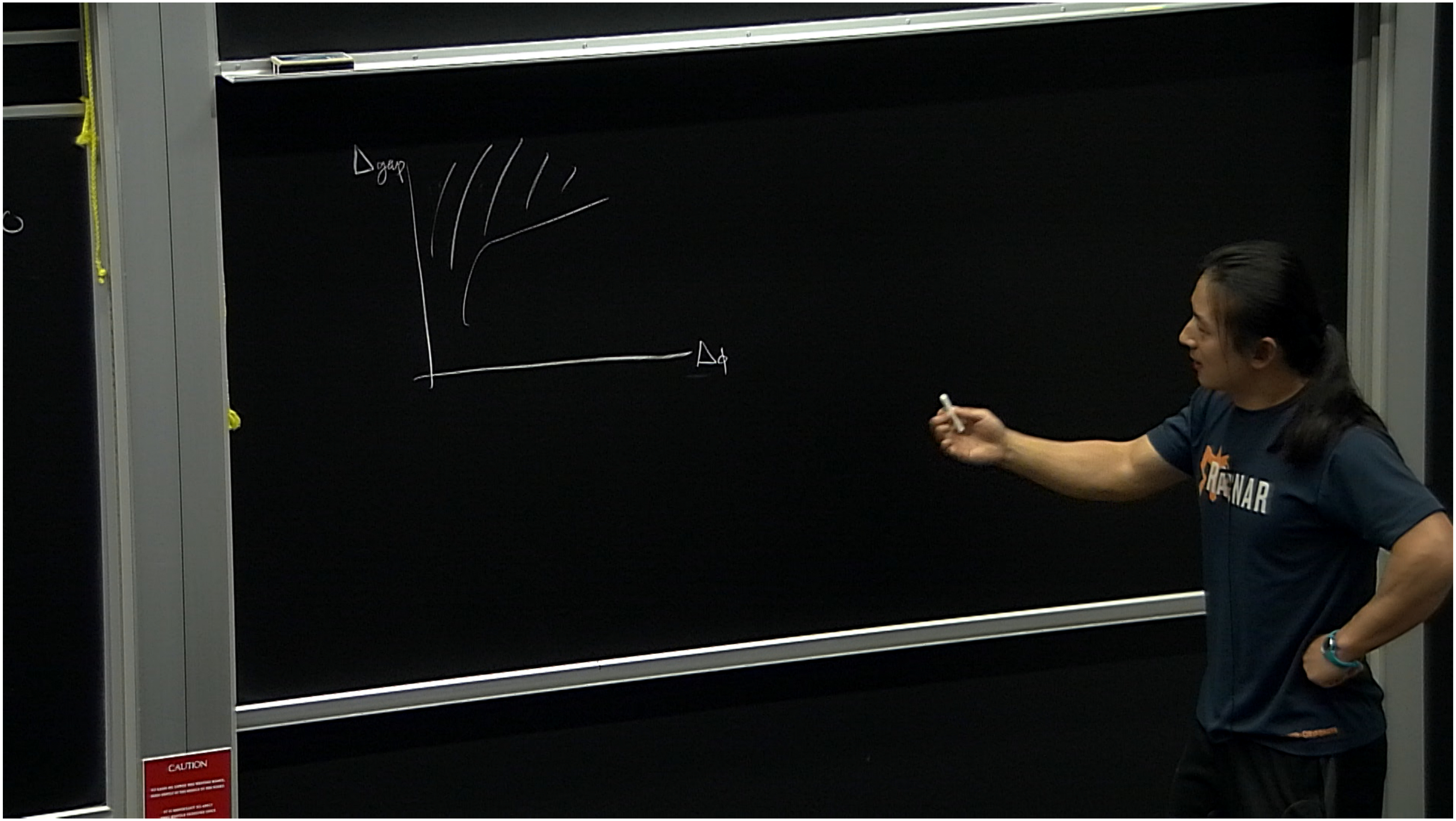
crossing eqn: $$\sum_{\Delta} C_\Delta^2 \left[F_\Delta(z,\bar{z}) - F_\Delta(1-z, 1-\bar{z}) \right] = 0$$

$$\Updownarrow$$

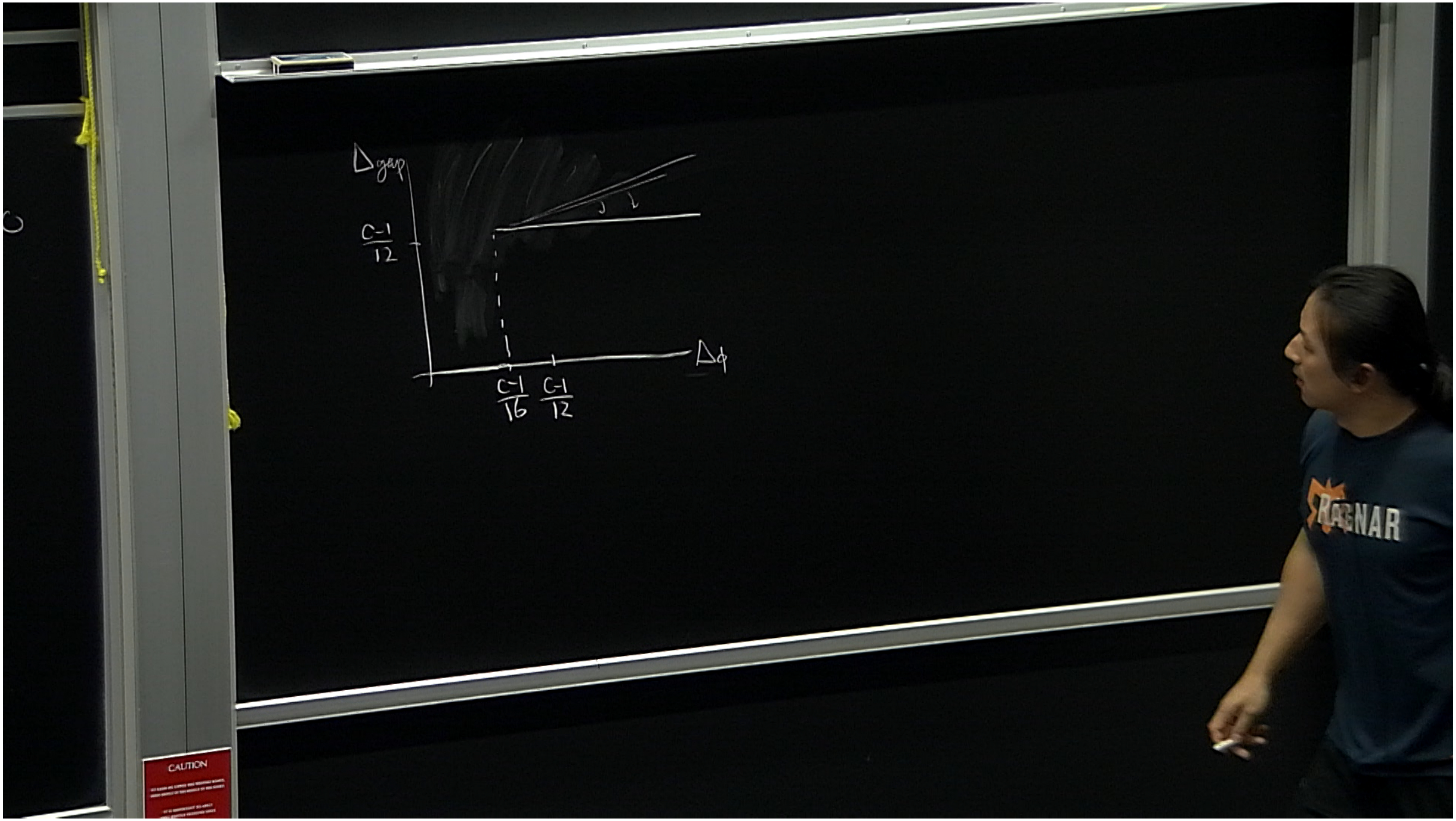
$$\sum_{\Delta} C_\Delta^2 \, F_{m,n}(\Delta) = 0, \qquad z = \bar{z} = \tfrac{1}{2}$$

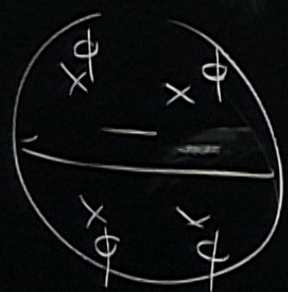
$m+n$ odd

$$F_{m,n}(\Delta) \equiv \partial_z^m \partial_{\bar{z}}^n \Big|_{z=\bar{z}=\frac{1}{2}} F_\Delta(z,\bar{z})$$



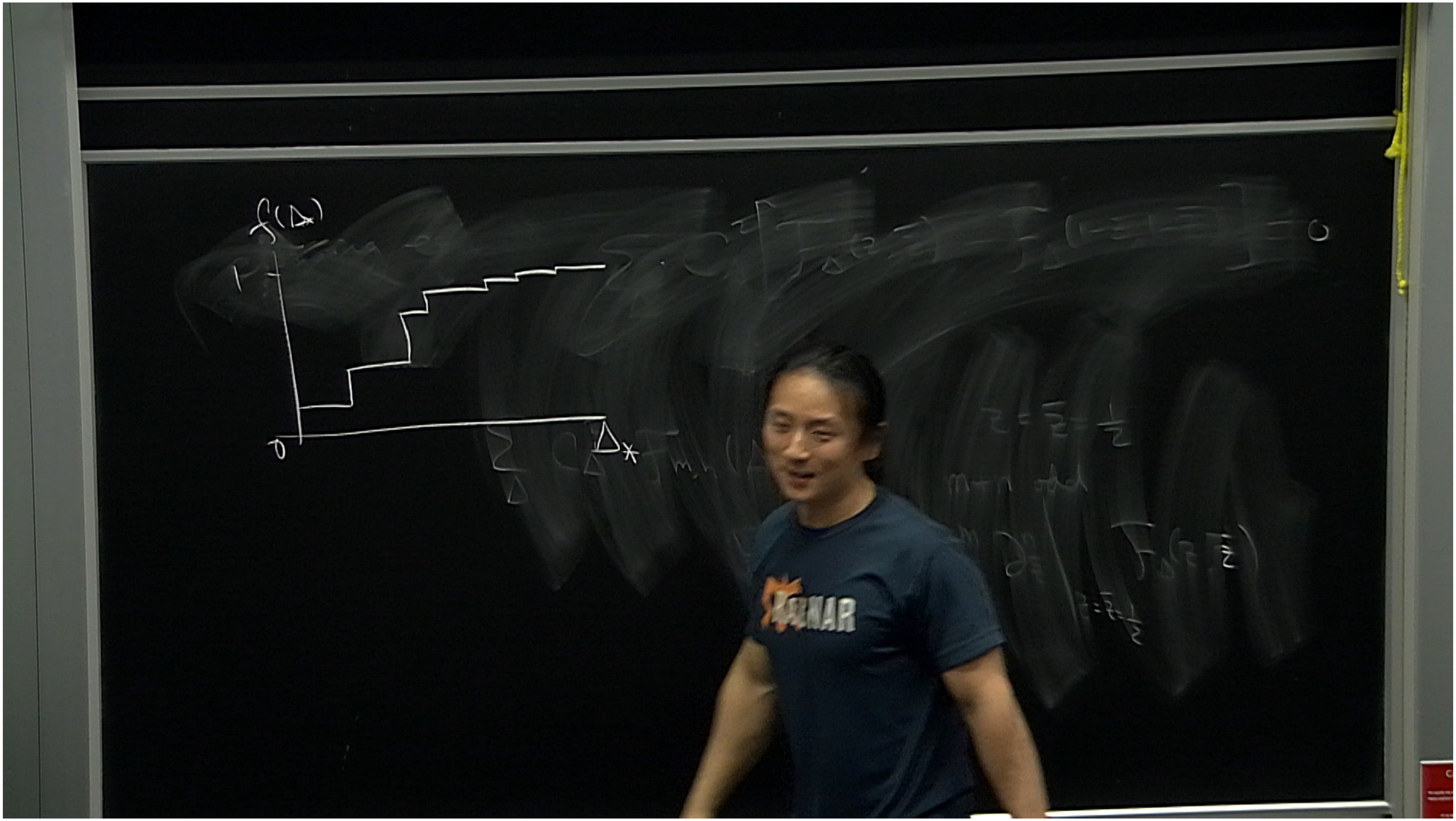
Δ_{gap}
Δ_ϕ
CAUTION

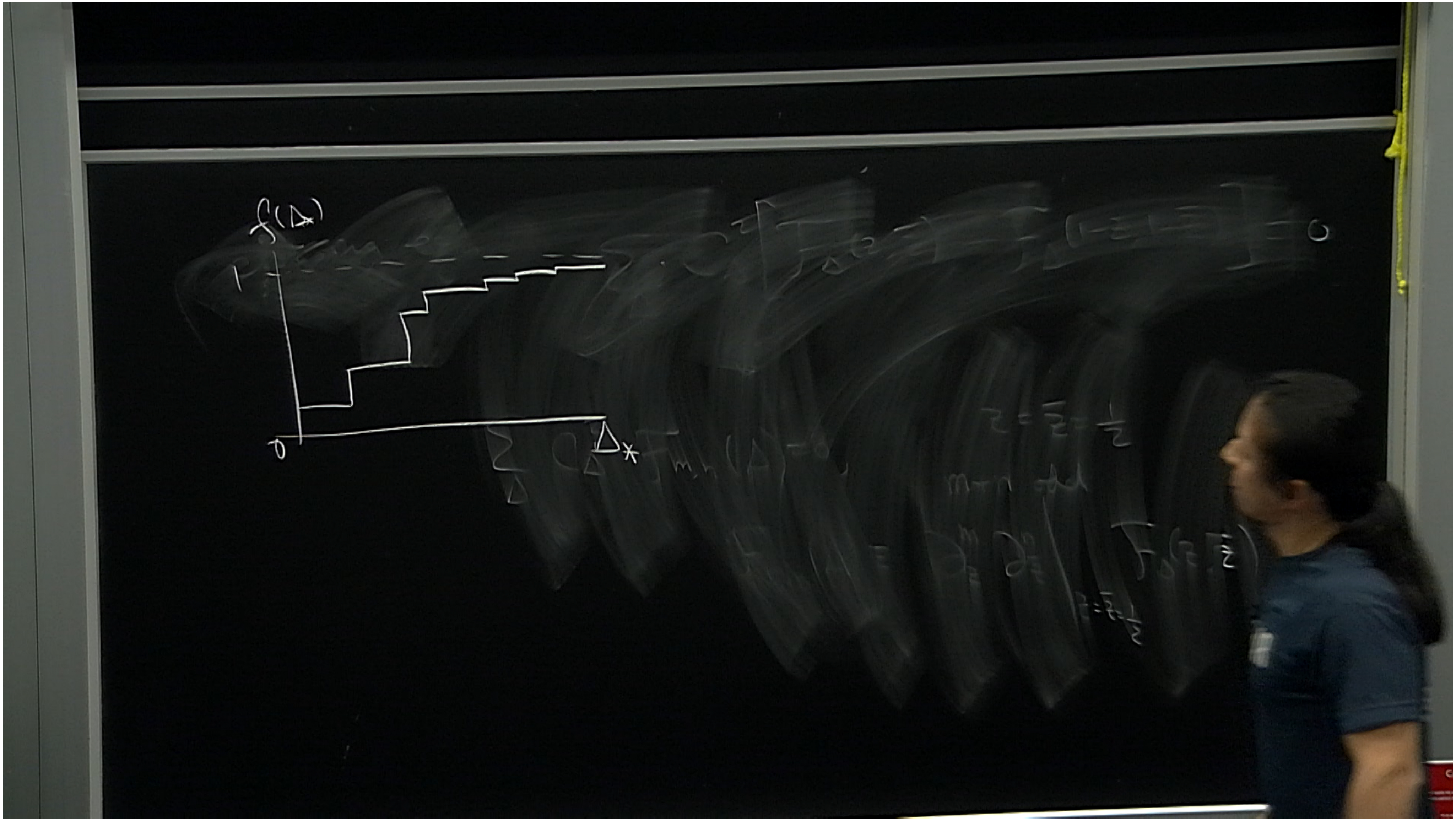

Δgap
(c-1)/12
(c-1)/16
(c-1)/12
Δφ
CAUTION

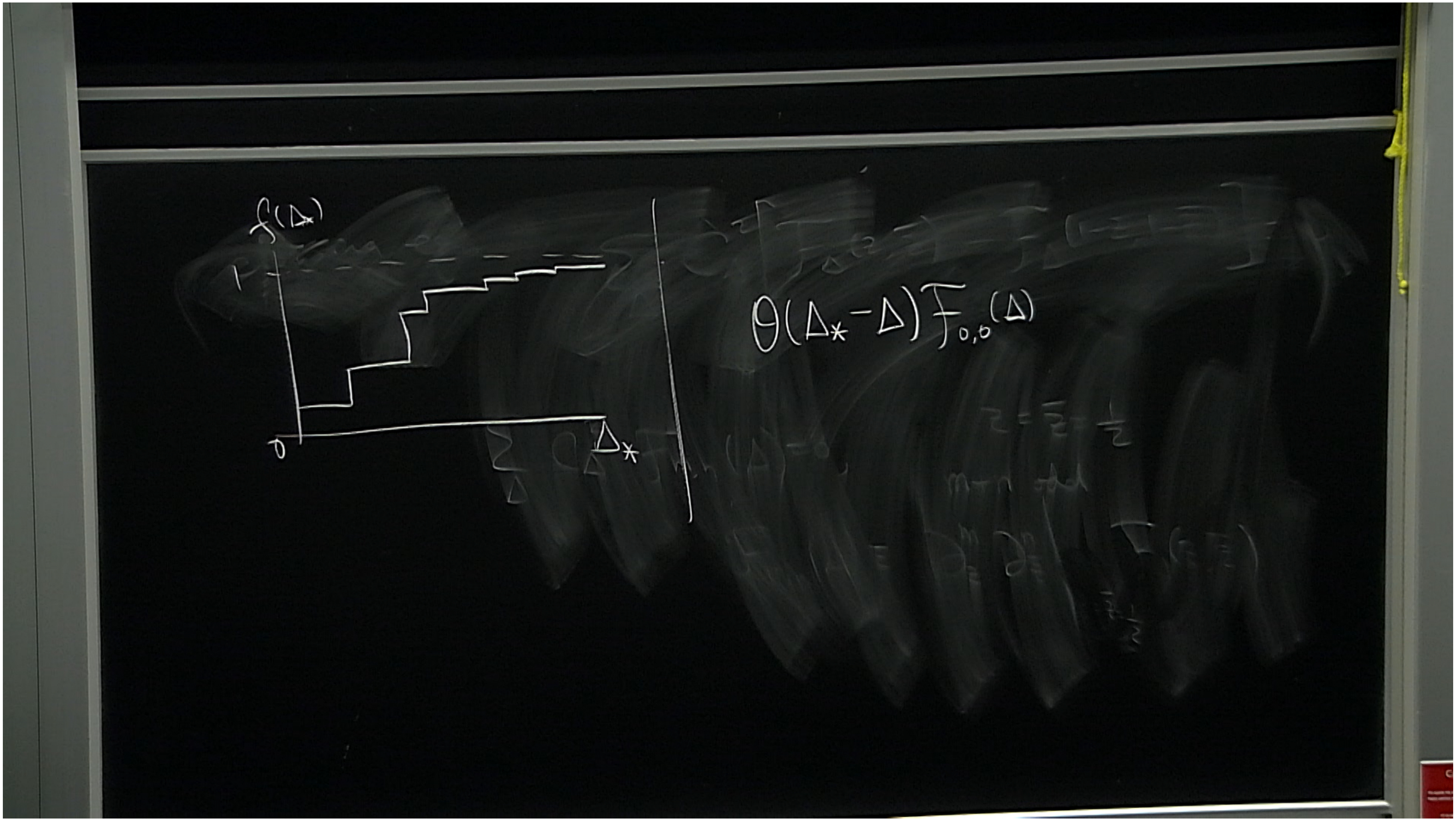


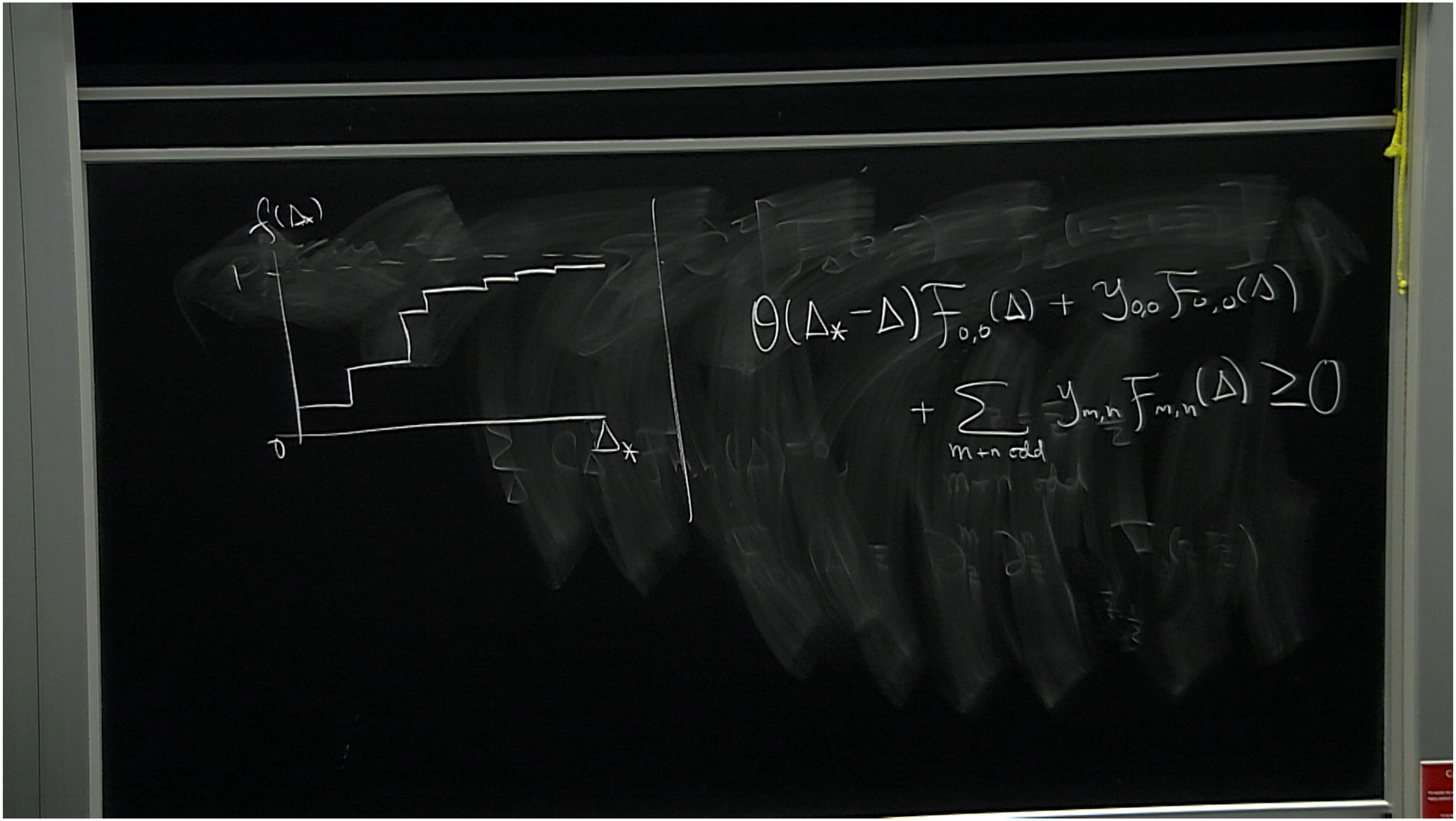
Spectral function

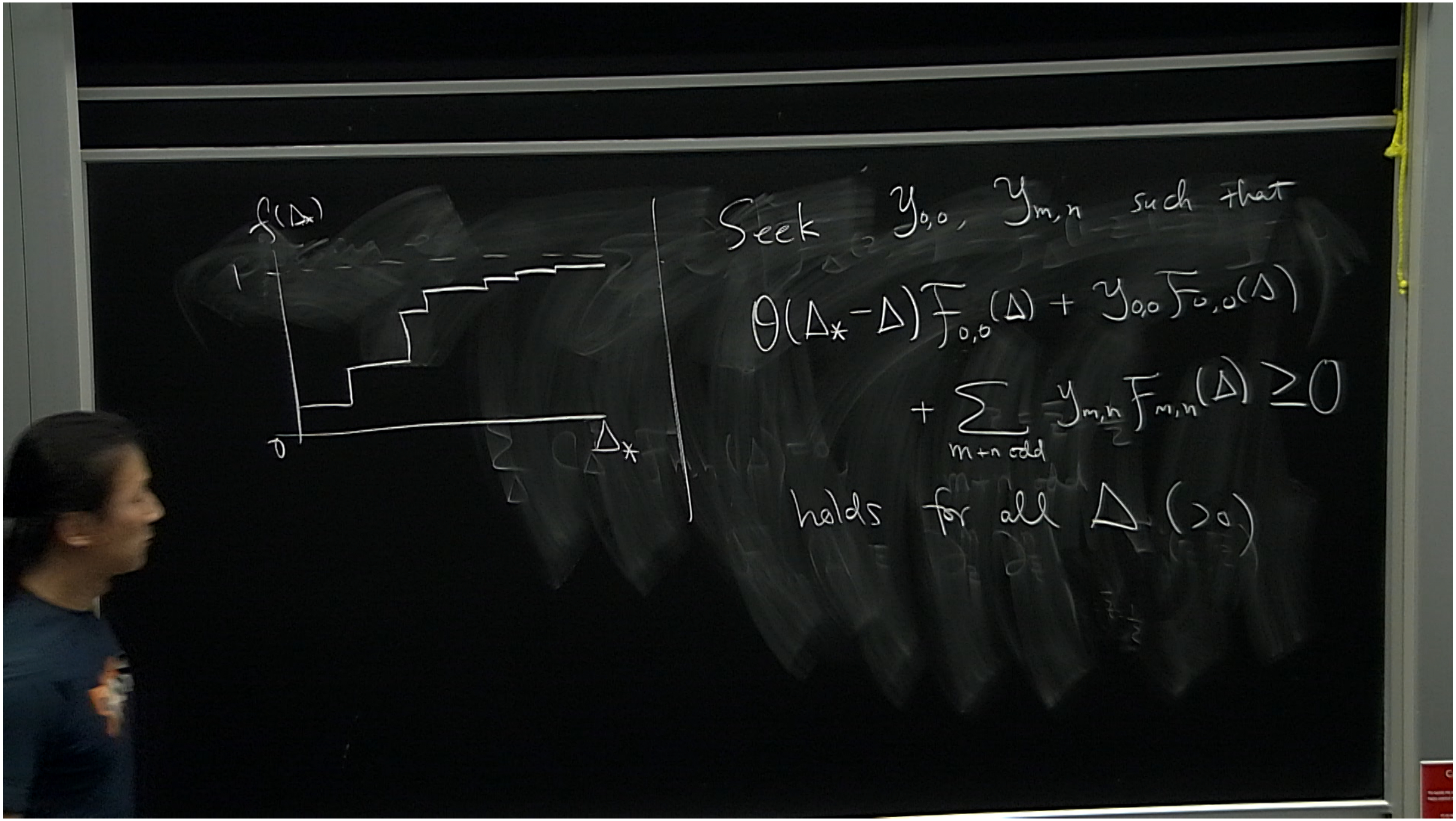
$$f(\Delta_*) := \frac{\sum_{\Delta \leq \Delta_*} C_\Delta^2 \, F_\Delta\left(z = \bar{z} = \frac{1}{2}\right)}{\sum_{\Delta'} C_{\Delta'}^2 \, F_{\Delta'}\left(z = \bar{z} = \frac{1}{2}\right)}$$

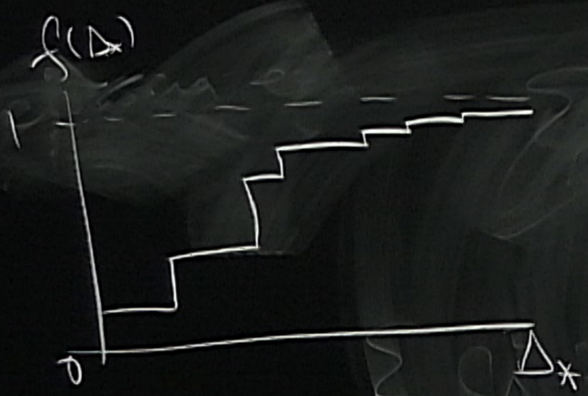

f(Δ*)
1
0
Δ*




f(Δ)
1
0
Δ*
θ(Δ* − Δ) F0,0(Δ)


$f(\Delta_*)$
1
0
Δ_*
$\theta(\Delta_* - \Delta) F_{0,0}(\Delta) + y_{0,0} F_{0,0}(\Delta)$
$+ \sum_{m+n \text{ odd}} y_{m,n} F_{m,n}(\Delta) \geq 0$

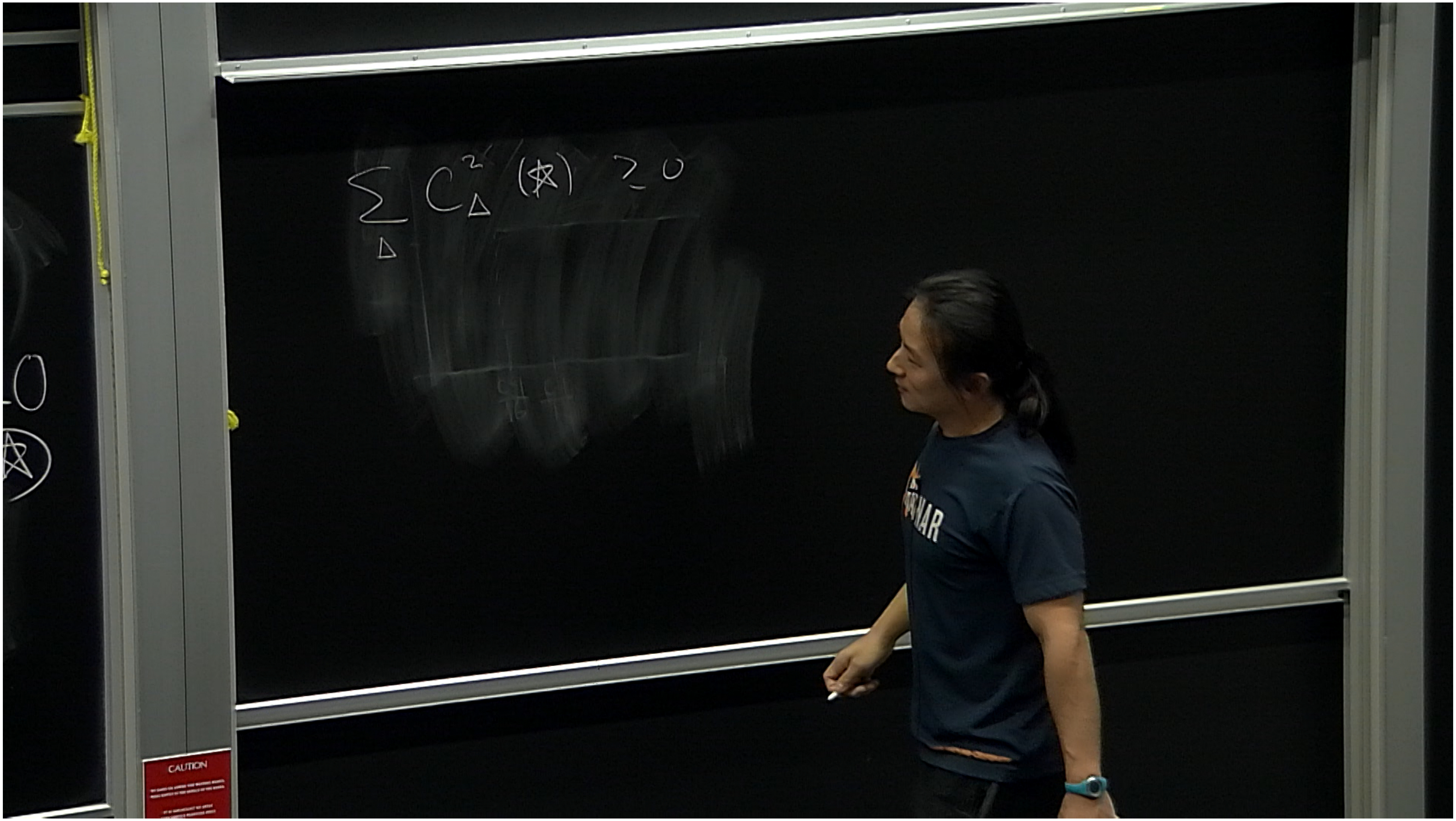

$f(\Delta_*)$
1
0
Δ_*
Seek $y_{0,0}$, $y_{m,n}$ such that
$\theta(\Delta_*-\Delta)F_{0,0}(\Delta) + y_{0,0}F_{0,0}(\Delta)$
$+ \sum_{m+n \text{ odd}} y_{m,n}F_{m,n}(\Delta) \geq 0$
holds for all Δ (>0)

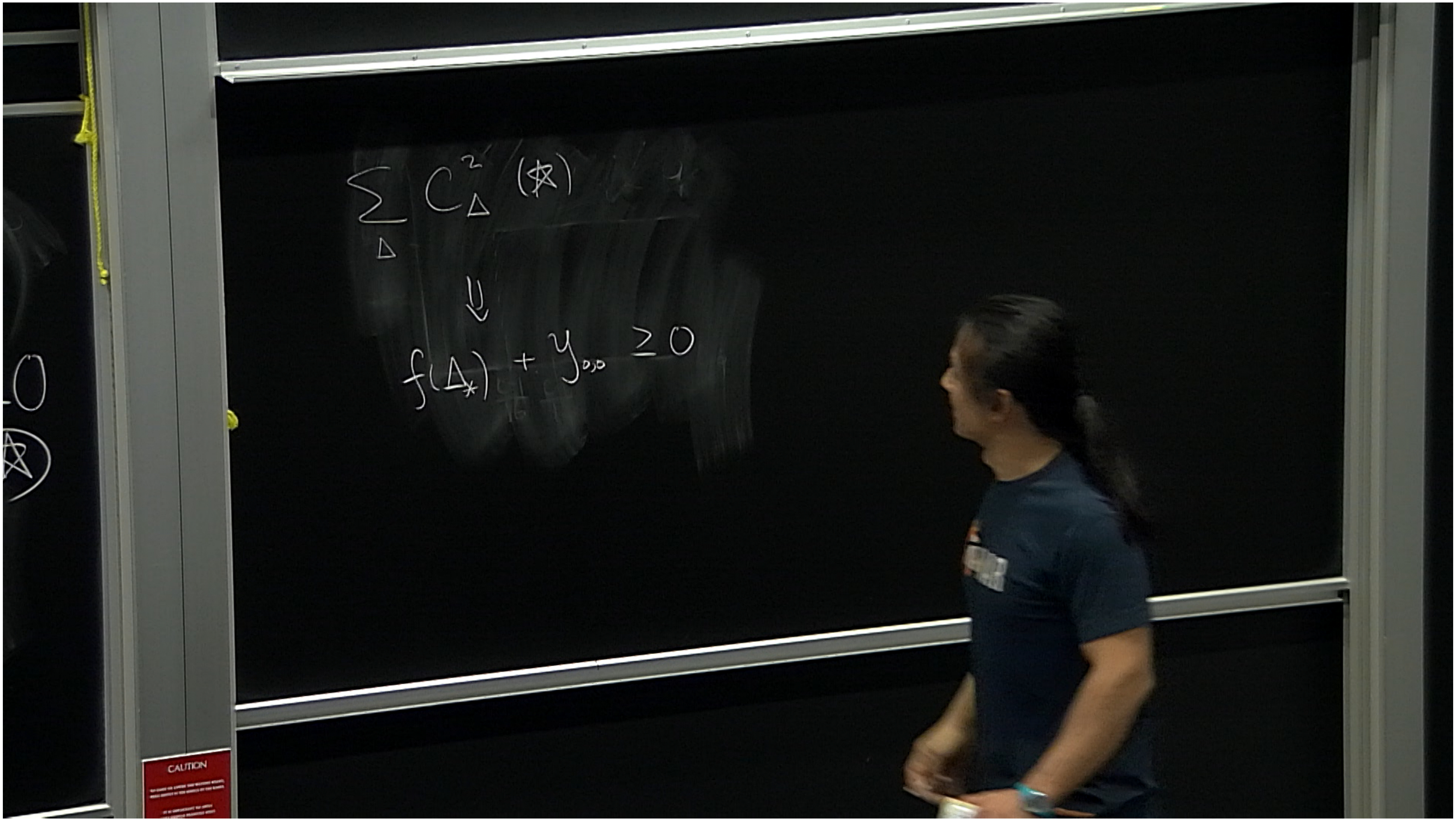


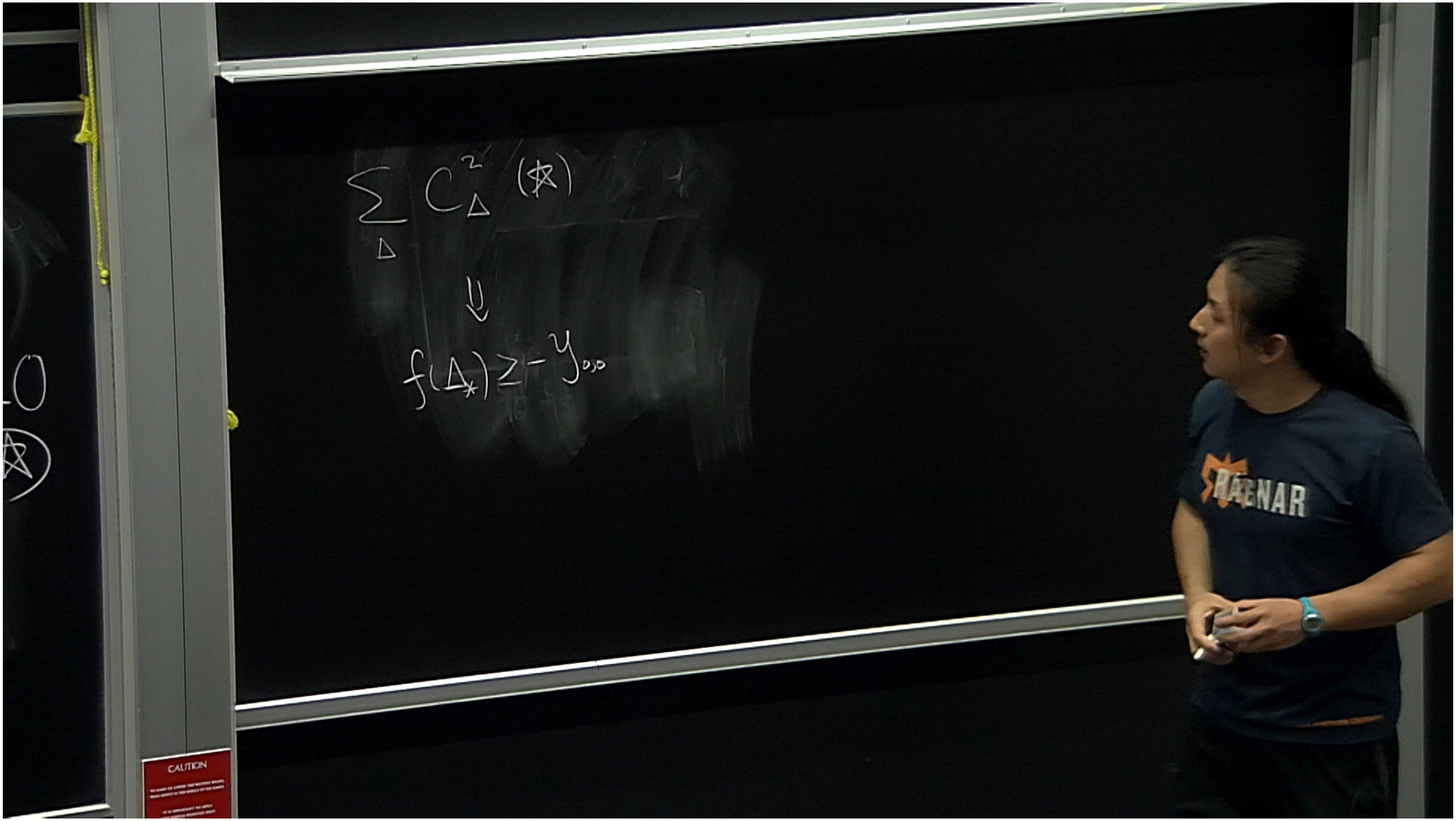
Seek $y_{0,0}$, $y_{m,n}$ such that

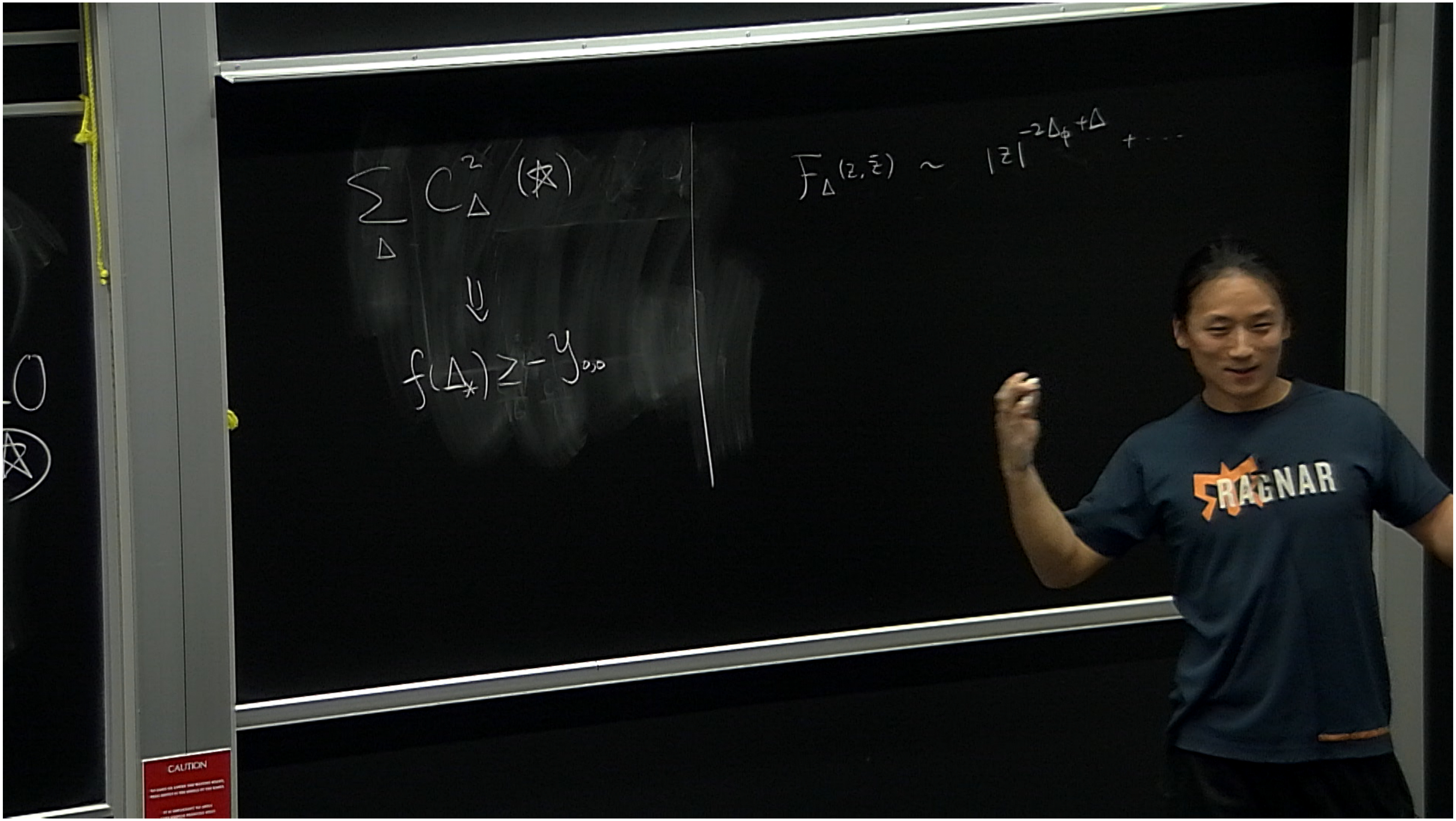
$$\Theta(\Delta_* - \Delta) F_{0,0}(\Delta) + y_{0,0} F_{0,0}(\Delta) + \sum_{m+n \text{ odd}} y_{m,n} F_{m,n}(\Delta) \geq 0 \qquad (\star)$$

holds for all Δ (>0)

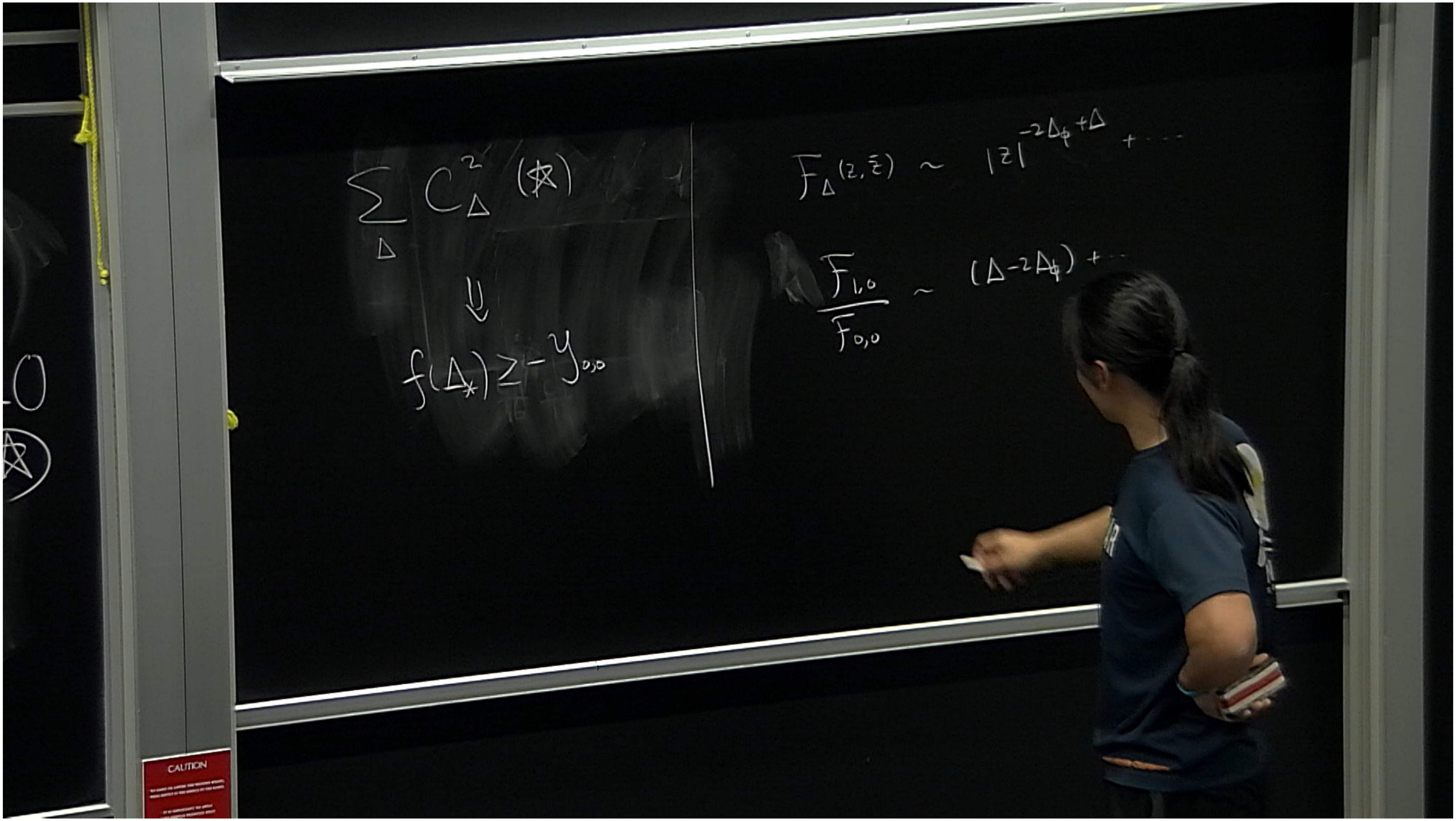

$\sum_{\Delta} C_{\Delta}^{2} (\star) \geq 0$
CAUTION


$\sum_{\Delta} C_{\Delta}^{2} (\star)$
$\Downarrow$
$f(\Delta_*) + y_{0,0} \geq 0$
CAUTION

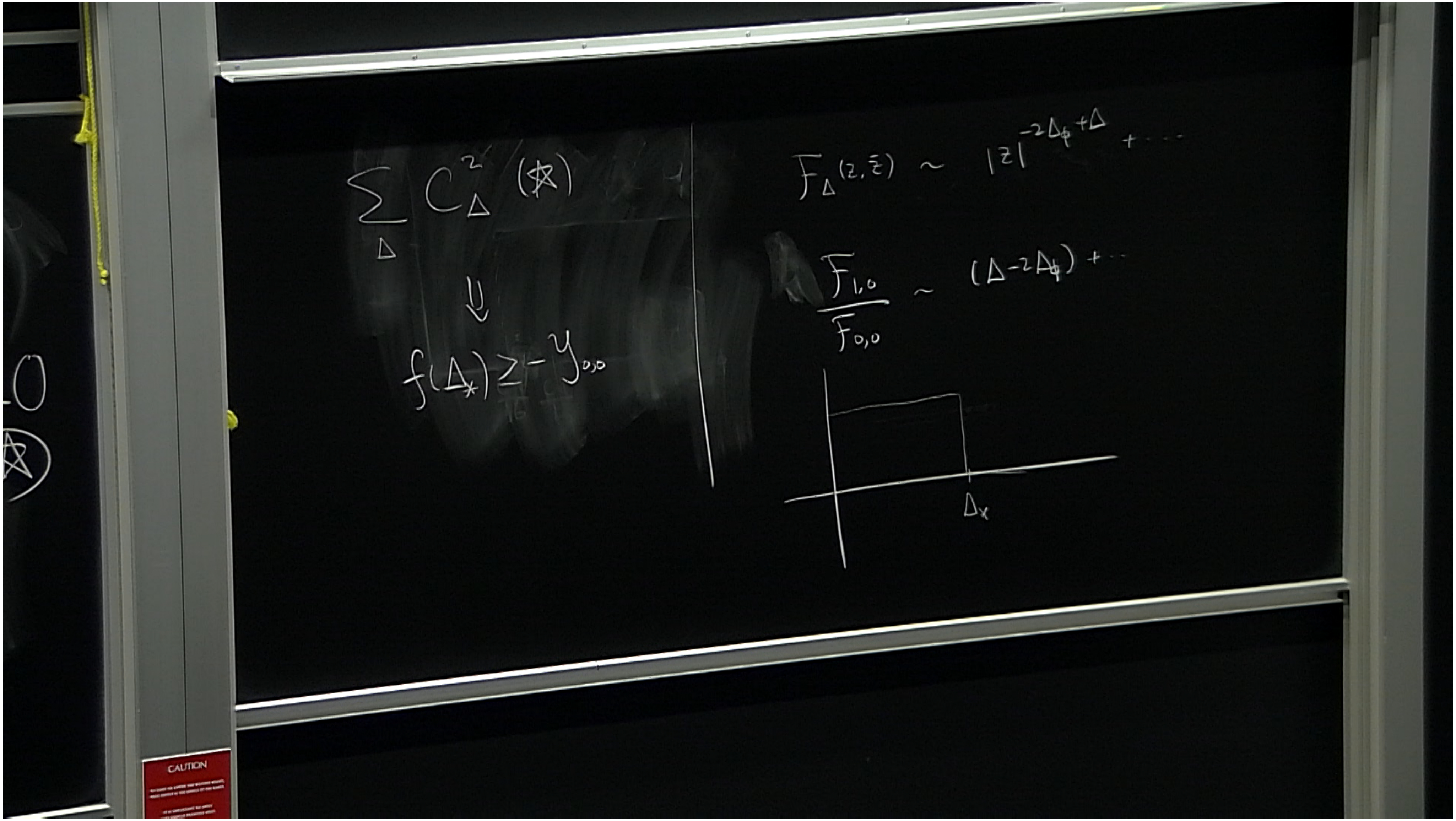

$\sum_{\Delta} C_{\Delta}^2 \ (\star)$
$\Downarrow$
$f(\Delta_*) \geq -y_{0,0}$
CAUTION



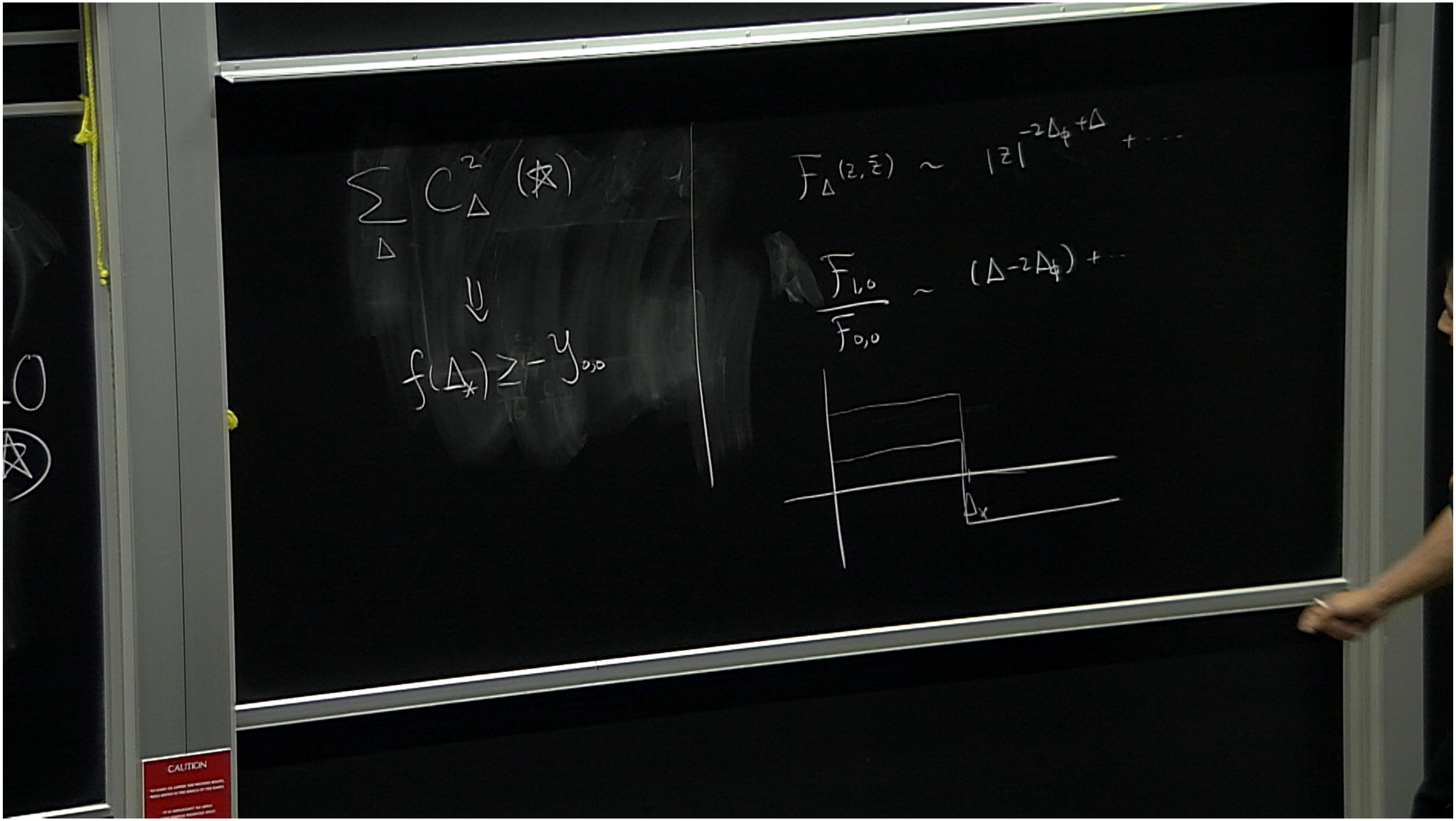
$\sum_{\Delta} C_{\Delta}^{2}$ (☆)
$\Downarrow$
$f(\Delta_*) \geq -y_{0,0}$
$F_{\Delta}(z,\bar{z}) \sim |z|^{-2\Delta_\phi+\Delta} + \cdots$
RAGNAR
CAUTION



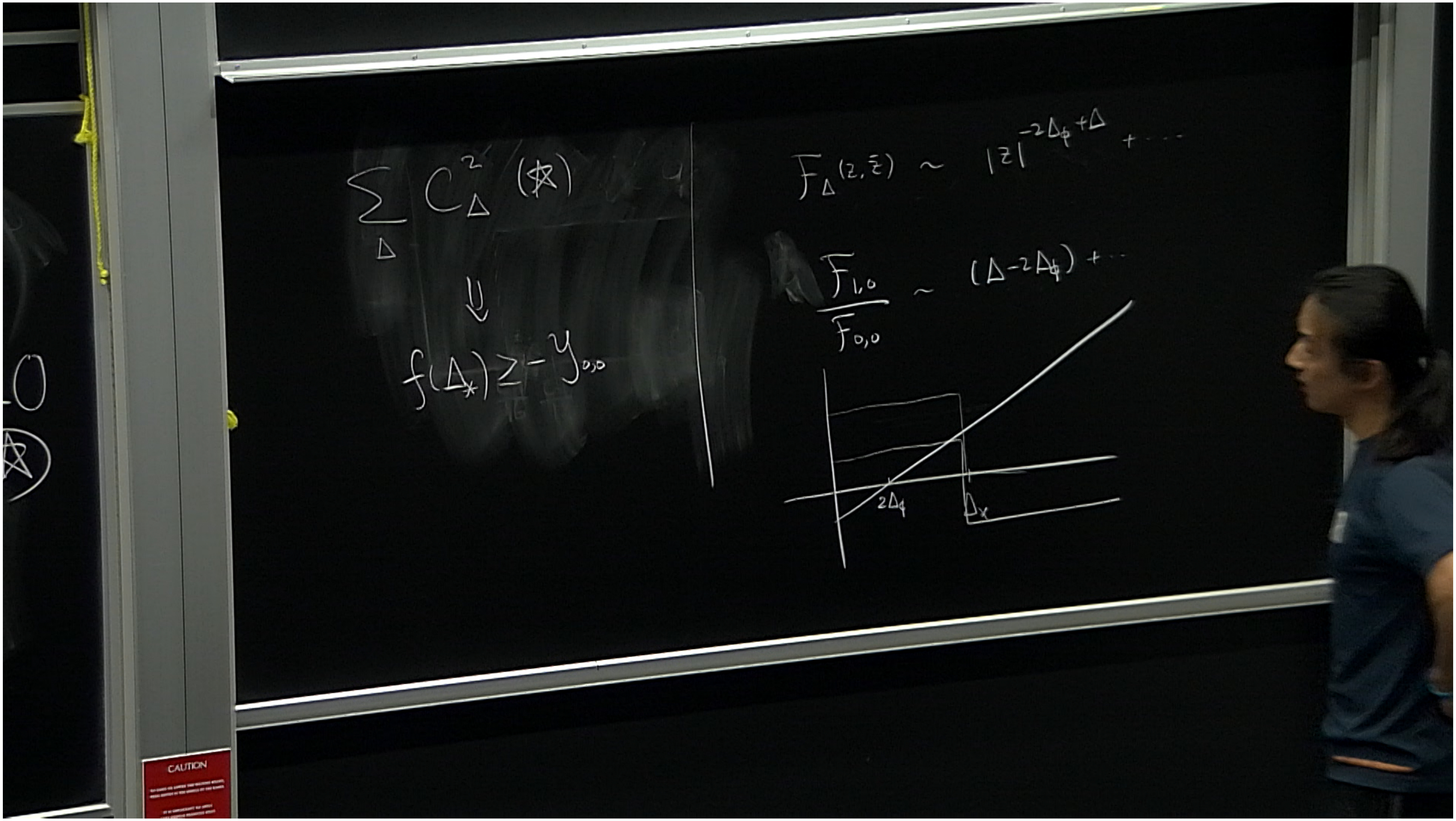
$\sum_{\Delta} C_{\Delta}^{2}$ (★)
$\Downarrow$
$f(\Delta_*) \geq -y_{0,0}$
$F_{\Delta}(z,\bar{z}) \sim |z|^{-2\Delta_\phi+\Delta} + \dots$
$\frac{F_{1,0}}{F_{0,0}} \sim (\Delta - 2\Delta_\phi) + \dots$
CAUTION

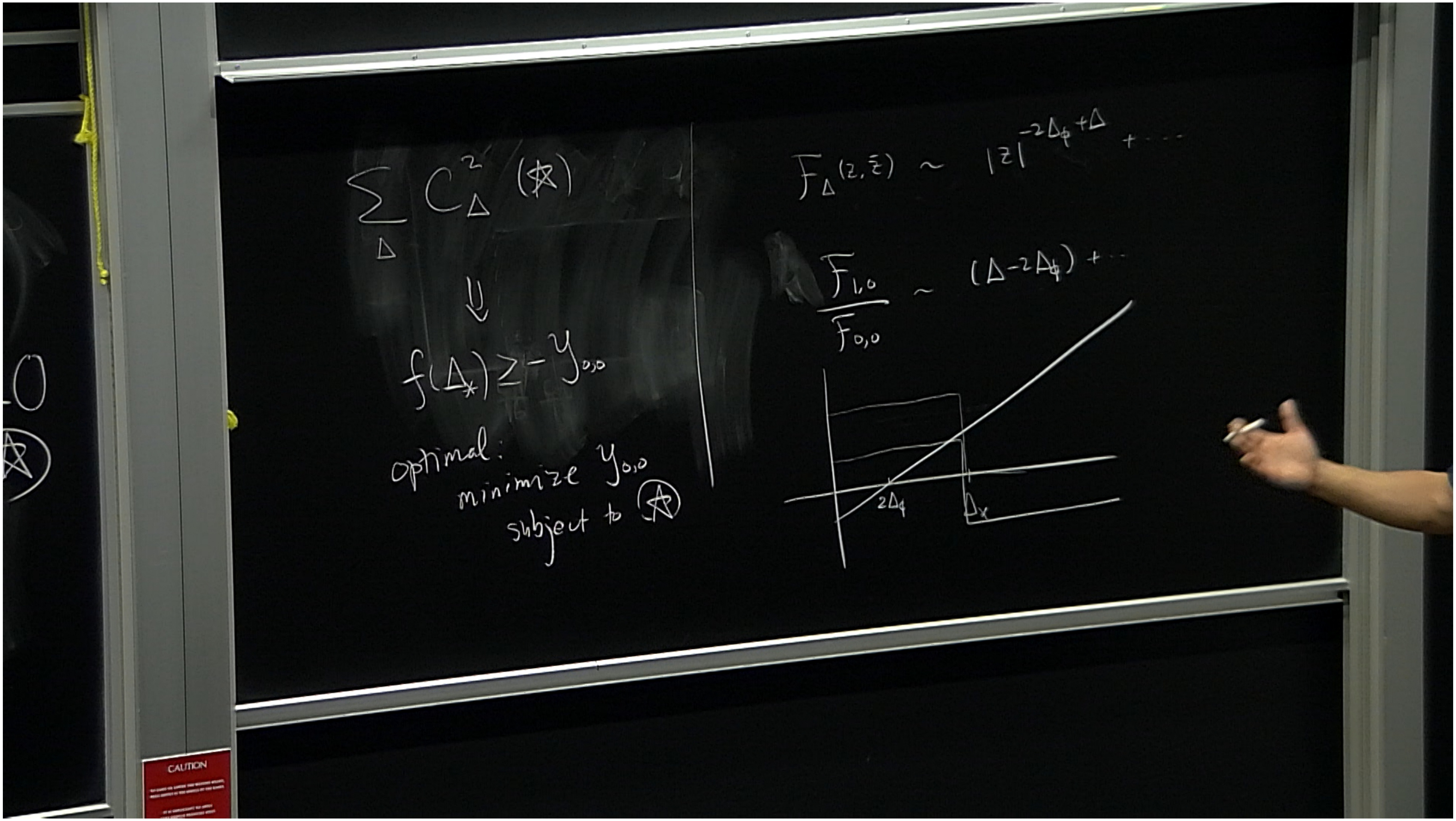


$\sum_{\Delta} C_{\Delta}^{2} (\star)$
$\Downarrow$
$f(\Delta_*) \geq -y_{0,0}$
$F_{\Delta}(z,\bar{z}) \sim |z|^{-2\Delta_\phi+\Delta} + \cdots$
$\frac{F_{1,0}}{F_{0,0}} \sim (\Delta - 2\Delta_\phi) + \cdots$
Δ_*



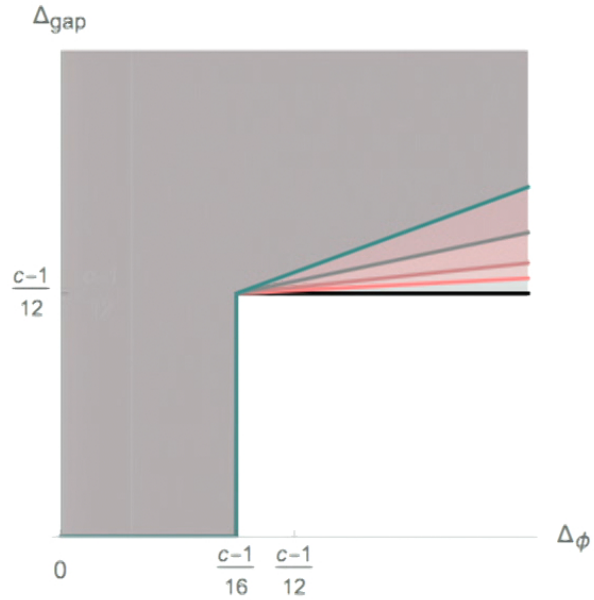
$\sum_{\Delta} C_{\Delta}^{2} (\star)$
$\Downarrow$
$f(\Delta_*) \geq -y_{0,0}$
$F_\Delta(z,\bar{z}) \sim |z|^{-2\Delta_\phi+\Delta} + \dots$
$\frac{F_{1,0}}{F_{0,0}} \sim (\Delta - 2\Delta_\phi) + \dots$
Δ_*


$\sum_{\Delta} C_{\Delta}^{2}$ (✯)
$\Downarrow$
$f(\Delta_*) \geq -y_{0,0}$
$F_{\Delta}(z,\bar{z}) \sim |z|^{-2\Delta_\phi+\Delta} + \cdots$
$\frac{F_{1,0}}{F_{0,0}} \sim (\Delta-2\Delta_\phi) + \cdots$
$2\Delta_\phi$
Δ_*
CAUTION


$\sum_{\Delta} C_{\Delta}^{2}$ (★)
⇓
$f(\Delta_*) \geq -y_{0,0}$
optimal:
minimize $y_{0,0}$
subject to ★
$F_{\Delta}(z,\bar{z}) \sim |z|^{-2\Delta_\phi+\Delta} + \cdots$
$\frac{F_{1,0}}{F_{0,0}} \sim (\Delta - 2\Delta_\phi) + \cdots$
$2\Delta_\phi$
Δ_*
CAUTION

Example of a bound on the OPE gap

Assuming **only scalar** Virasoro primaries in OPE

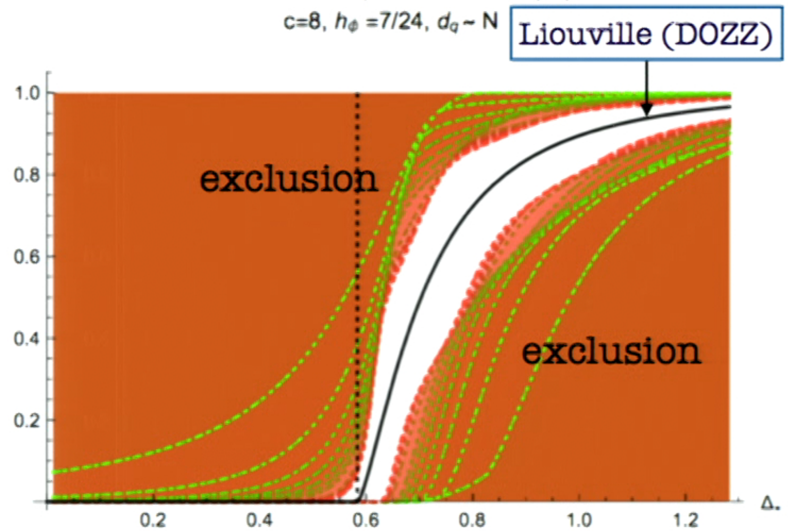


[van Rees, unpublished; Collier, Lin, XY, unpublished]

Claim: assuming unitarity, c>1, **only scalar** primaries, DOZZ structure constants of Liouville theory are the only solution to the crossing equation.

Bounds on the spectral function $f(\Delta_*)$:

$c=8$, $h_\phi = 7/24$, $d_q \sim N$



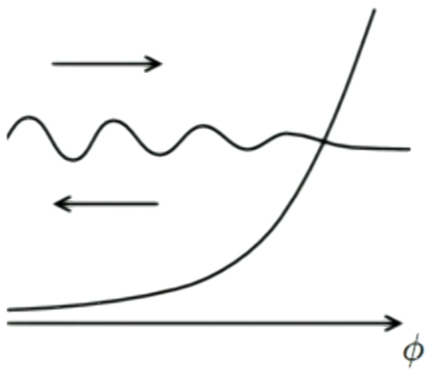
[Collier, Kravchuk, Lin, XY, 1702.00423]

Liouville CFT - a brief recap for millennials

[Seiberg '91, Dorn-Otto '94, Zamolodchikov², '95, Teschner '95, Ponsot-Teschner '99]

$$S_{\text{Liouville}} = \frac{1}{4\pi} \int d^2z \sqrt{g} \left(g^{mn} \partial_m \phi \partial_n \phi + QR\phi + 4\pi\mu e^{2b\phi} \right)$$

$$c = 1 + 6Q^2 \qquad Q = b + b^{-1}$$



Virasoro primary operators take the form

$$\mathcal{V}_\alpha \sim S(\alpha)^{-\frac{1}{2}} e^{2\alpha\phi} + S(\alpha)^{\frac{1}{2}} e^{2(Q-\alpha)\phi}$$

$$\phi \to -\infty$$

$$\alpha = \frac{Q}{2} + iP$$

Liouville CFT - a brief recap for millennials

[Seiberg '91, Dorn-Otto '94, Zamolodchikov^2, '95, Teschner '95, Ponsot-Teschner '99]

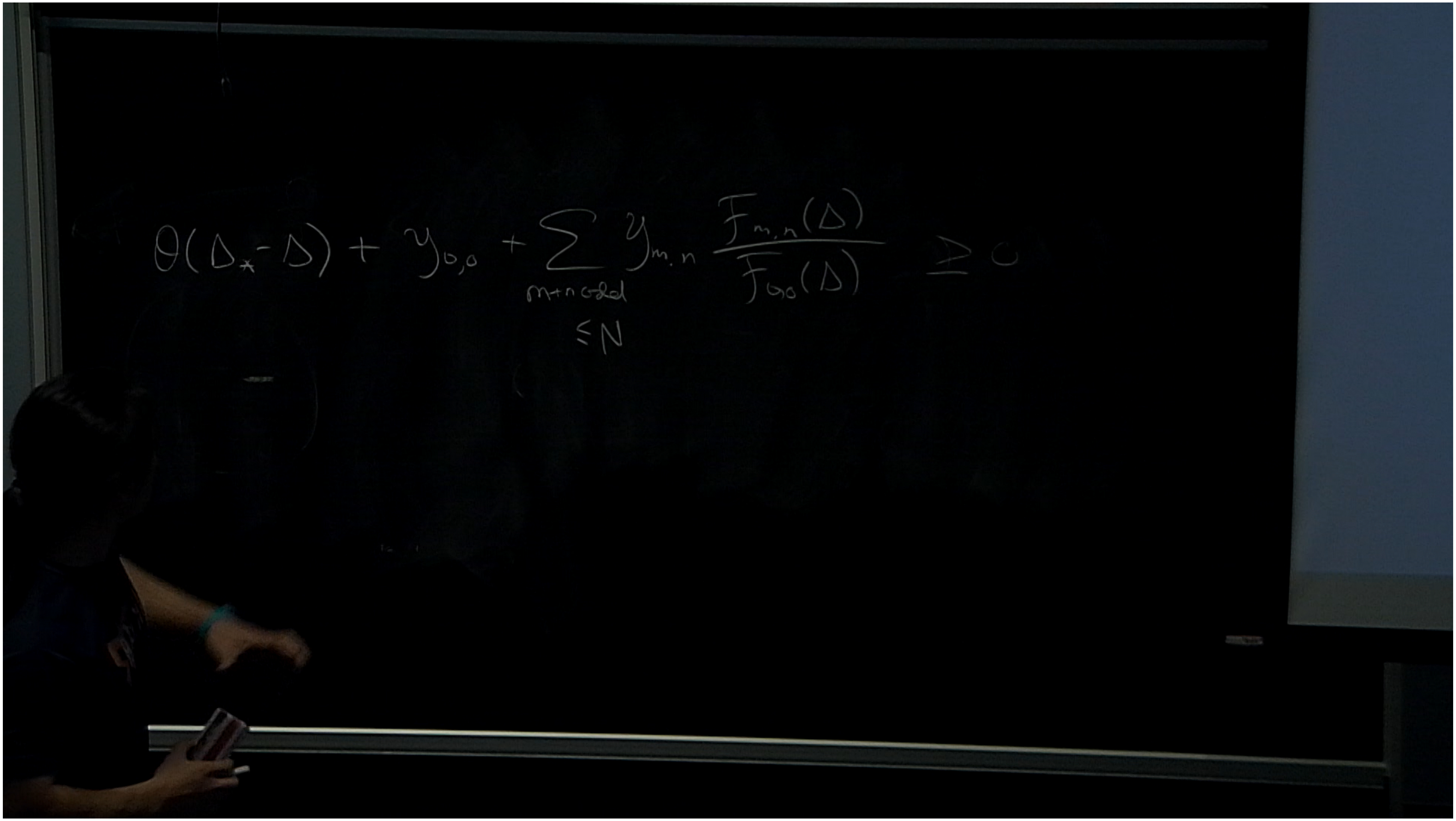
Reflection coefficient:

$$S(\alpha) = -\left(\pi\mu\gamma(b^2)\right)^{(Q-2\alpha)/b}\frac{\Gamma(1-(Q-2\alpha)/b)\Gamma(1-(Q-2\alpha)b)}{\Gamma(1+(Q-2\alpha)/b)\Gamma(1+(Q-2\alpha)b)}$$

DOZZ structure constants:

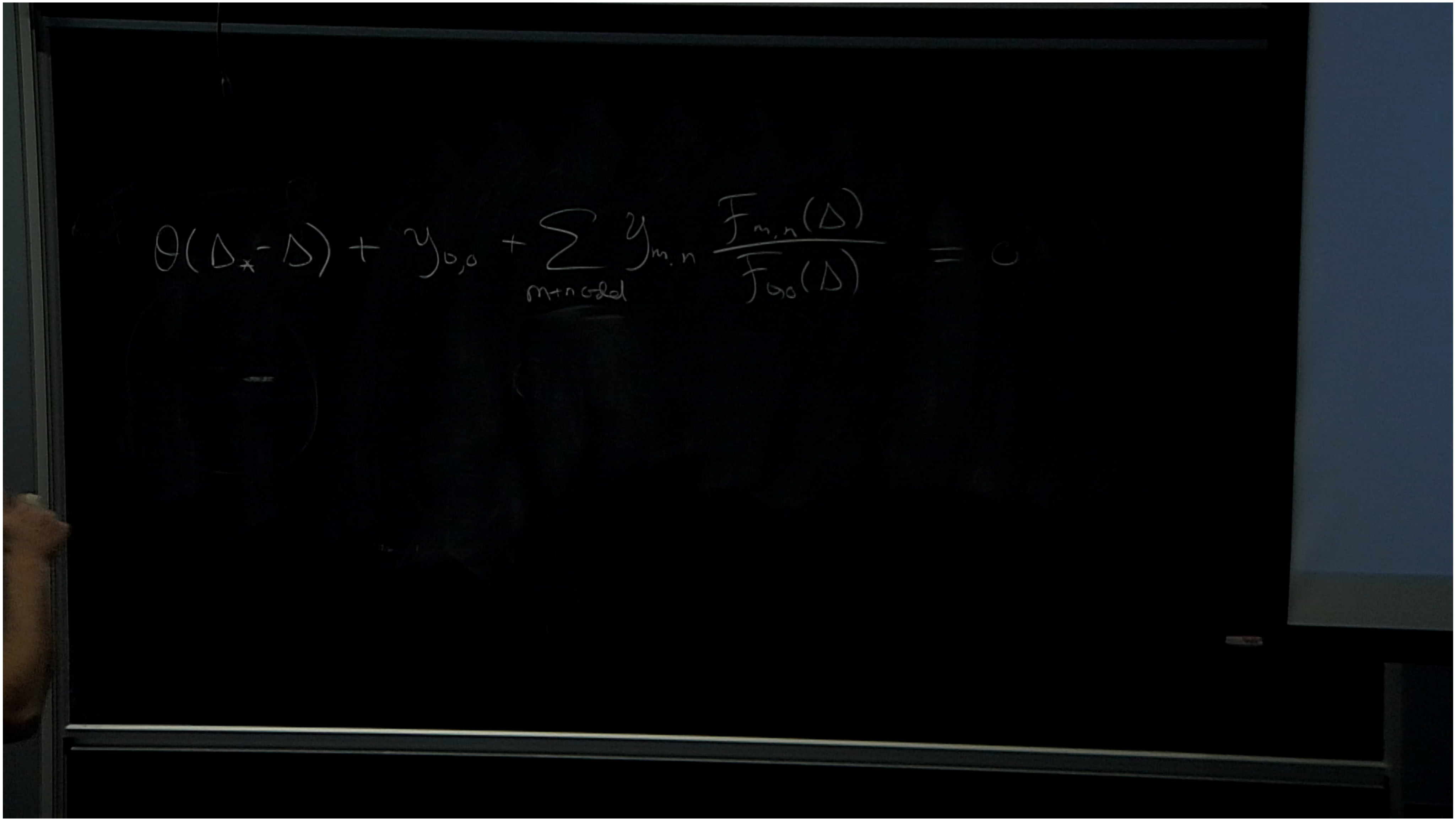
$$\langle \mathcal{V}_{\alpha_1}\mathcal{V}_{\alpha_2}\mathcal{V}_{\alpha_3}\rangle = \prod_{j=1}^{3} S(\alpha_i)^{-\frac{1}{2}}\left[\pi\mu\gamma(b^2)b^{2-2b^2}\right]^{\frac{Q-\sum\alpha_i}{b}}$$
$$\times\frac{\Upsilon_b'(0)\Upsilon_b(2\alpha_1)\Upsilon_b(2\alpha_2)\Upsilon_b(2\alpha_3)}{\Upsilon_b(\sum\alpha_i-Q)\Upsilon_b(\alpha_1+\alpha_2-\alpha_3)\Upsilon_b(\alpha_2+\alpha_3-\alpha_1)\Upsilon_b(\alpha_3+\alpha_1-\alpha_2)}$$

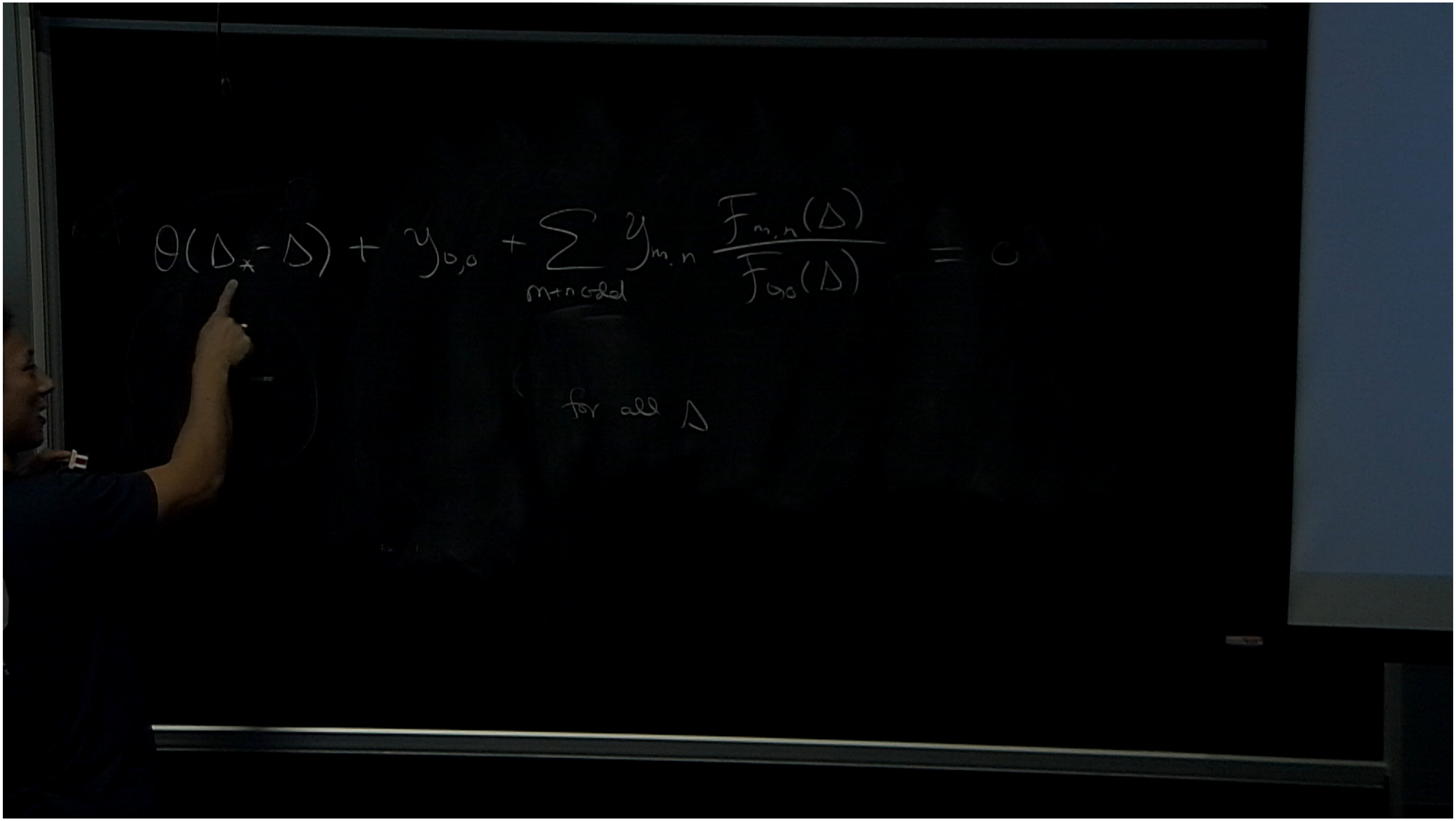
$$\gamma(x) = \frac{\Gamma(x)}{\Gamma(1-x)}$$
$$\log\Upsilon_b(x) = \int_0^\infty dt\, t^{-1}\left[\left(\frac{Q}{2}-x\right)^2 e^{-t} - \frac{\sinh^2\left[\left(\frac{Q}{2}-x\right)\frac{t}{2}\right]}{\sinh\frac{tb}{2}\sinh\frac{t}{2b}}\right],\ 0<\mathrm{Re}(x)<\mathrm{Re}(Q)$$


$\theta(\Delta_* - \Delta) + y_{0,0} + \sum_{\substack{m+n \text{ odd} \\ \leq N}} y_{m,n} \frac{F_{m,n}(\Delta)}{F_{0,0}(\Delta)} \geq 0$

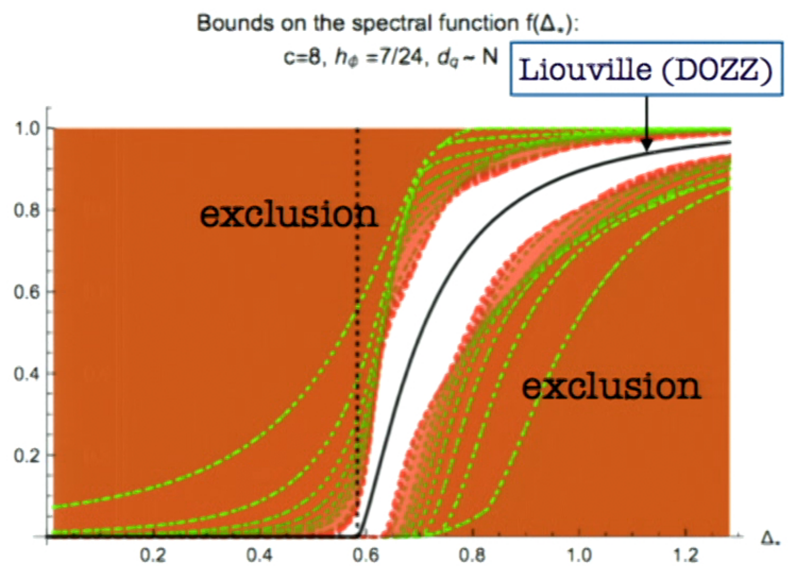
$$\theta(\Delta_* - \Delta) + y_{0,0} + \sum_{\substack{m+n\ \text{odd} \\ \leq N}} y_{m,n} \frac{F_{m,n}(\Delta)}{F_{0,0}(\Delta)} \geq 0$$

$$\leq$$


$\theta(\Delta_* - \Delta) + y_{0,0} + \sum_{m+n\ \text{odd}} y_{m,n} \frac{F_{m,n}(\Delta)}{F_{0,0}(\Delta)} = 0$

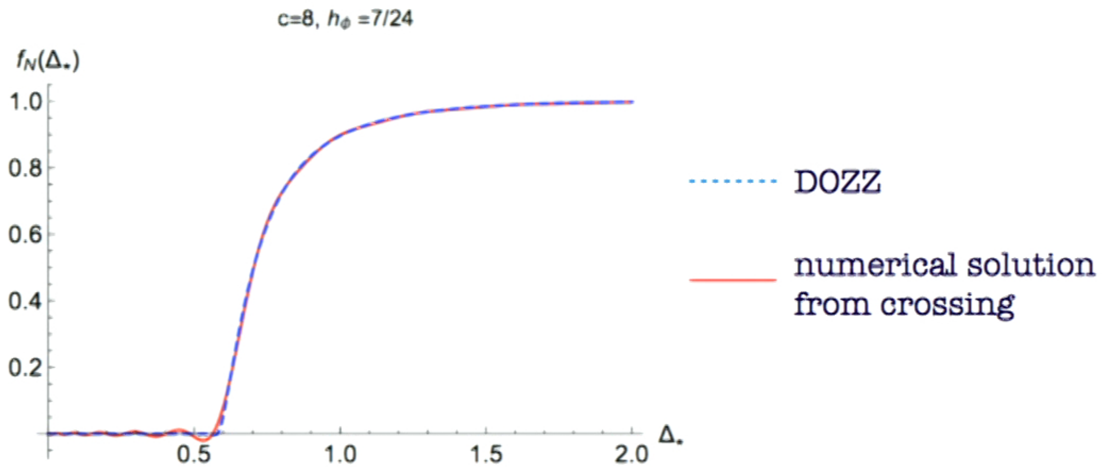

$\theta(\Delta_* - \Delta) + y_{0,0} + \sum_{m+n \text{ odd}} y_{m,n} \frac{F_{m,n}(\Delta)}{F_{0,0}(\Delta)} = 0$
for all Δ

Claim: assuming unitarity, c>1, **only scalar** primaries, DOZZ structure constants of Liouville theory are the only solution to the crossing equation.



[Collier, Kravchuk, Lin, XY, 1702.00423]

Direct numerical solution for the scalar-only spectral function from truncated crossing equation:



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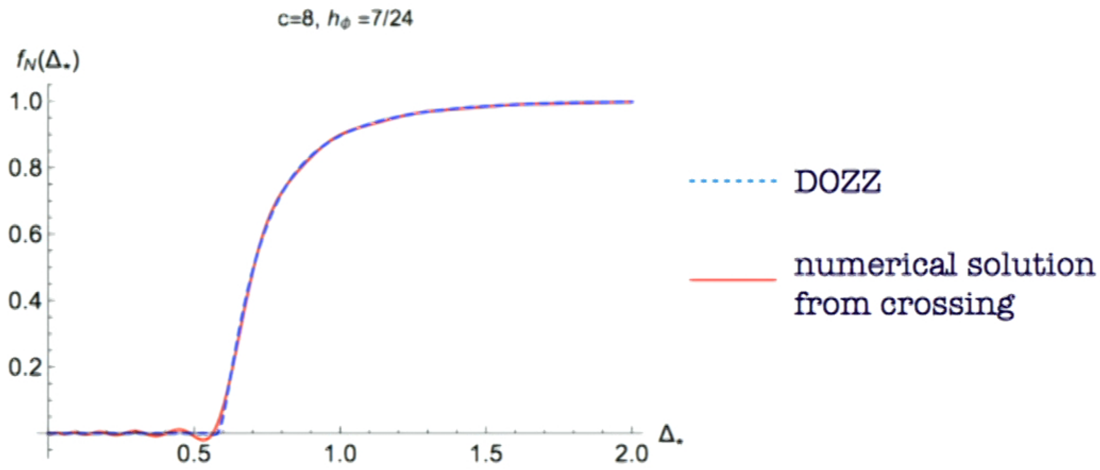
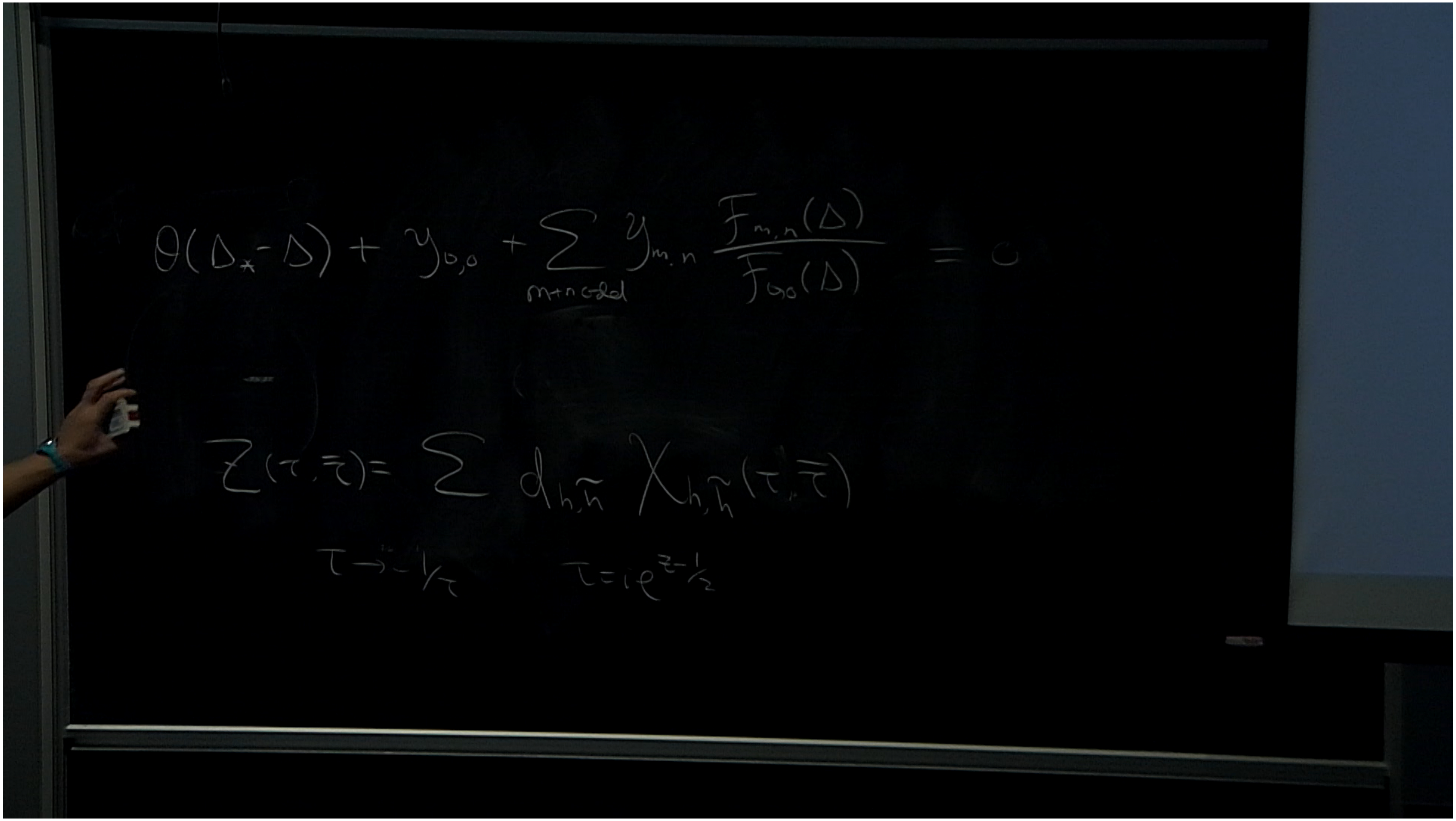
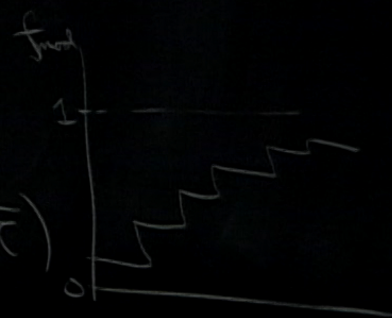


[Collier, Kravchuk, Lin, XY, 1702.00423]


$\theta(\Delta_* - \Delta) + y_{0,0} + \sum_{m+n \text{ odd}} y_{m,n} \frac{F_{m,n}(\Delta)}{F_{0,0}(\Delta)} = 0$
$Z(\tau, \bar{\tau}) = \sum d_{h,\tilde{h}} \chi_{h,\tilde{h}}(\tau, \bar{\tau})$
$\tau \to -1/\tau$
$\tau = i e^{z - 1/2}$

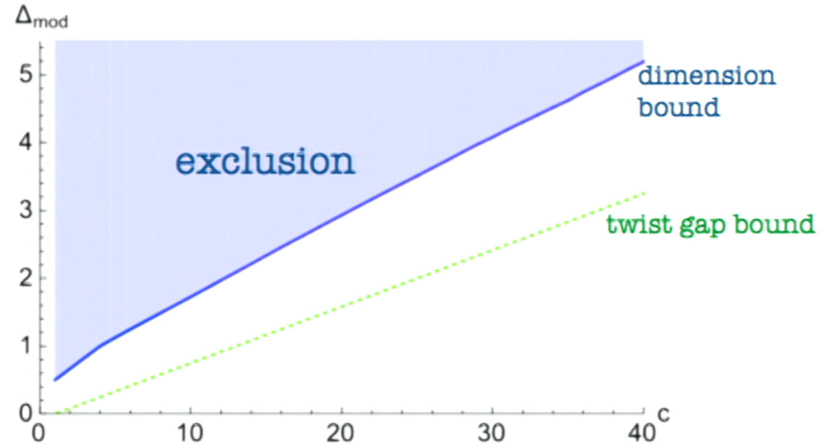
$$\theta(\Delta_* - \Delta) + y_{0,0} + \sum_{m+n\ \text{odd}} y_{m,n} \frac{F_{m,n}(\Delta)}{F_{0,0}(\Delta)} = 0$$

$$Z(\tau,\bar{\tau}) = \sum d_{h,\tilde{h}}\, \chi_{h,\tilde{h}}(\tau,\bar{\tau})$$



$\tau \to -1/\tau \qquad \tau = i e^{z - 1/2}$

Upper bound on gap in operator dimension

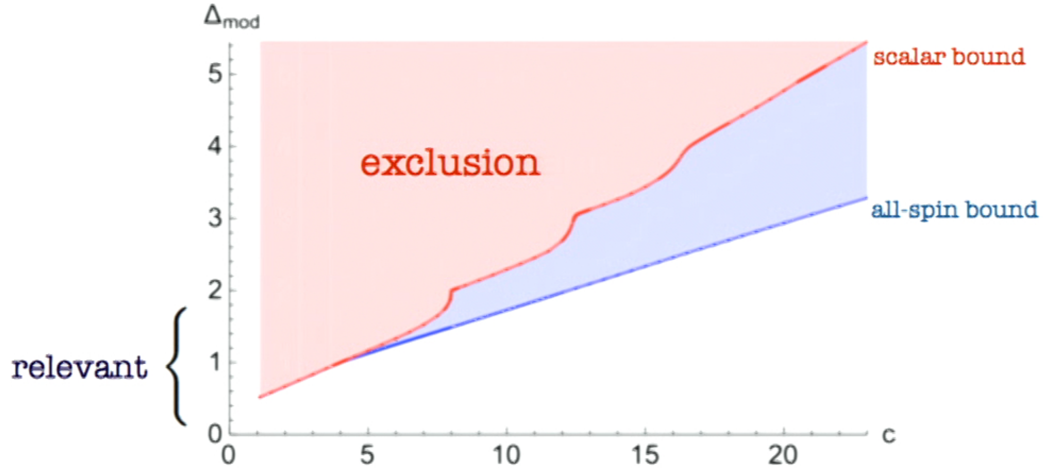


Conjecture: the slope $\frac{d\Delta_{\mathrm{mod}}(c)}{dc}$ is monotonically non-increasing.

Asymptotic slope? $\frac{1}{12} \leq \lim_{c\to\infty} \frac{d\Delta_{\mathrm{mod}}(c)}{dc} < \frac{1}{9}$

[Collier-Lin-XY '16]

Upper bound on dimension of lightest scalar (c<25)



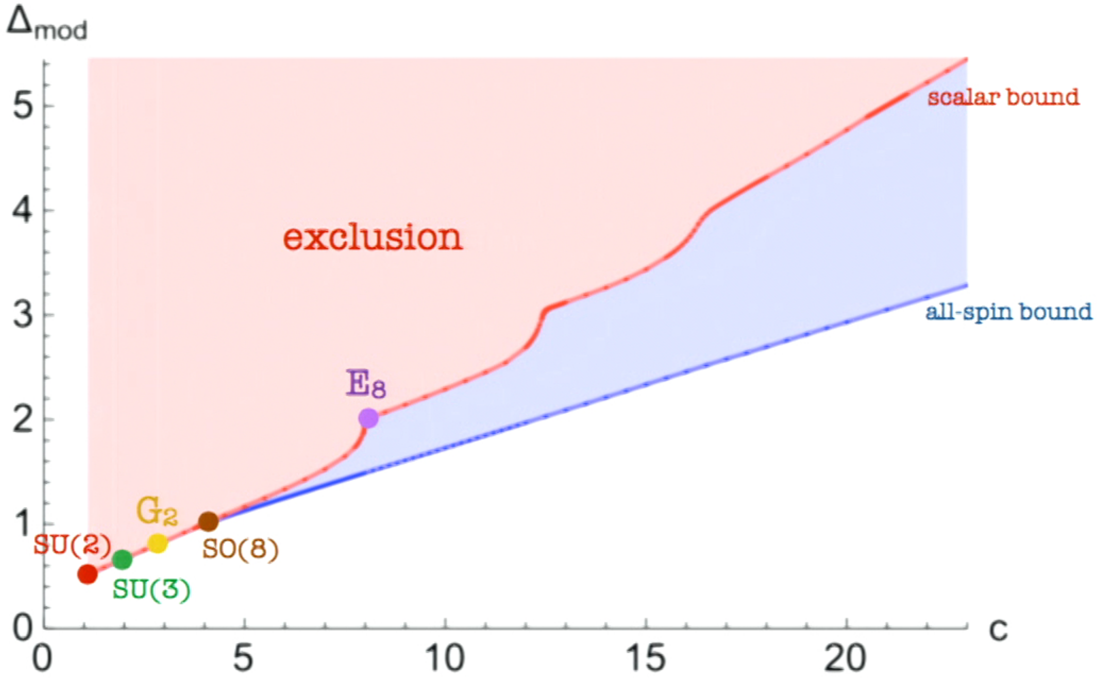
Implies all unitary CFTs with c<8 must admit relevant deformation.

"Perfect metal" does not exist for c≤8.

[Collier-Lin-XY '16]

Upper bound on dimension of lightest scalar (c<25)

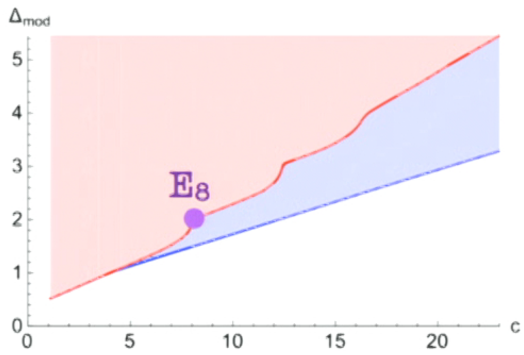
For some values of c, bound saturated by WZW models at level 1.



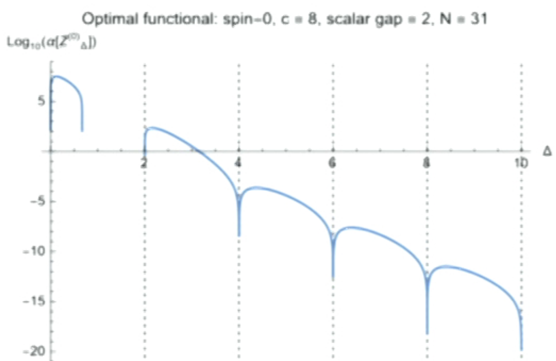
[Collier-Lin-XY '16]

Extremal spectrum

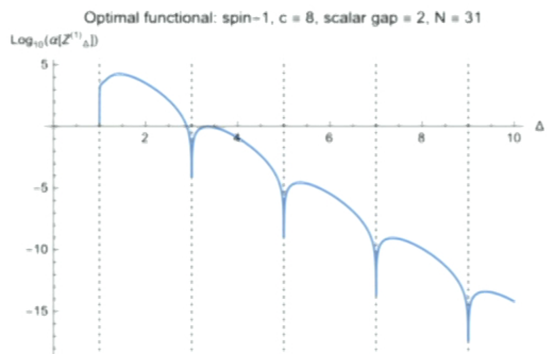
[Collier-Lin-XY '16]



When bound is saturated, zeros of the optimal linear functional acting on Virasoro characters give the spectrum



spin 0 spectrum

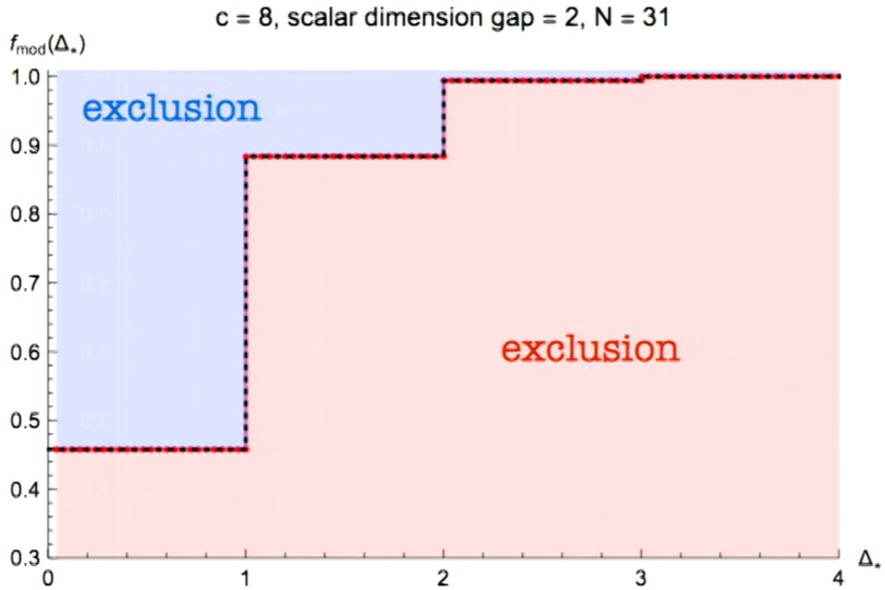


spin 1 spectrum

Modular spectral function

$$f_{\rm mod}(x) = \frac{1}{Z(\tau = -\bar{\tau} = i)} \sum_{s,\Delta \leq x} d_{\Delta,s} \chi_{\Delta,s}(\tau = -\bar{\tau} = i)$$

(assuming maximal gap)

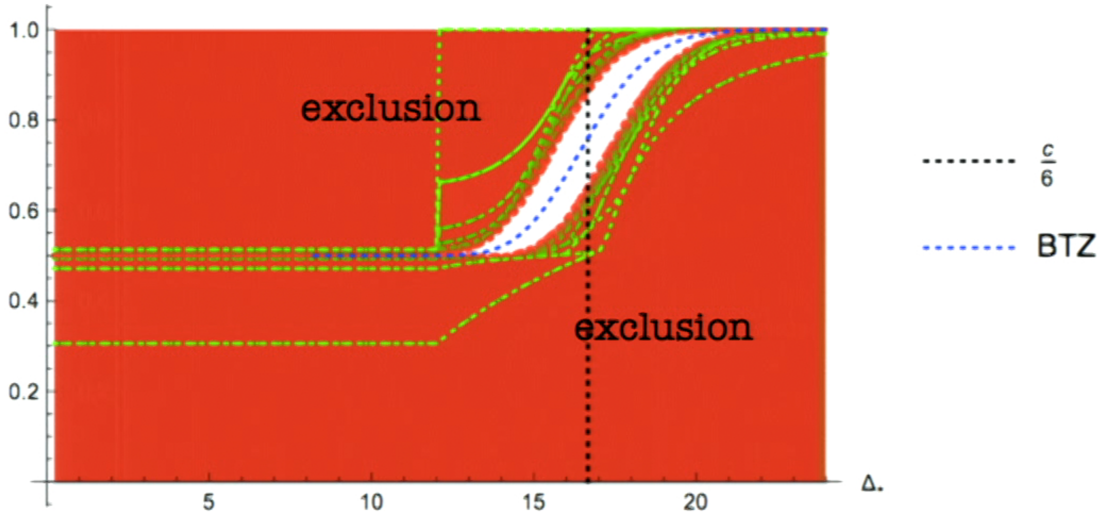


[Collier, Kravchuk, Lin, XY, 1702.00423]

Modular spectral function

Large c, large gap limit?

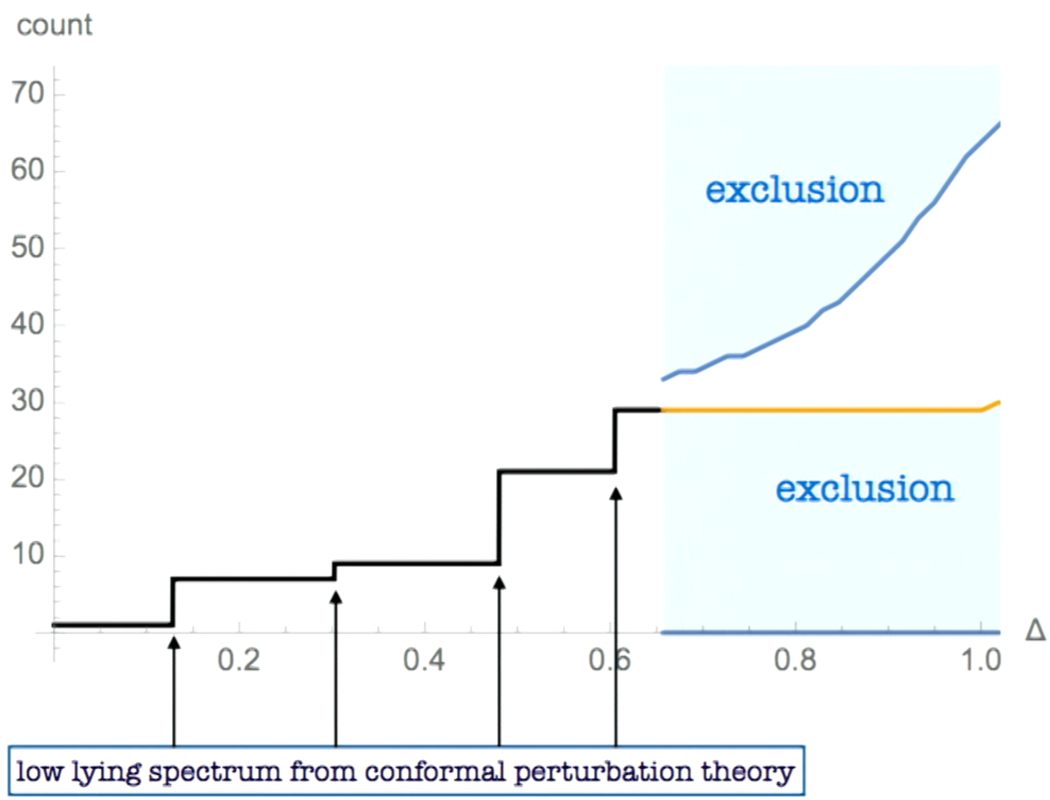
Bounds on the modular spectral function:
c=100, dimension gap = 12 **(close to maximal gap)**



[Collier, Kravchuk, Lin, XY, 1702.00423]

Primary counting function

Coupled Potts model [Dotsenko et al '98] (preliminary)



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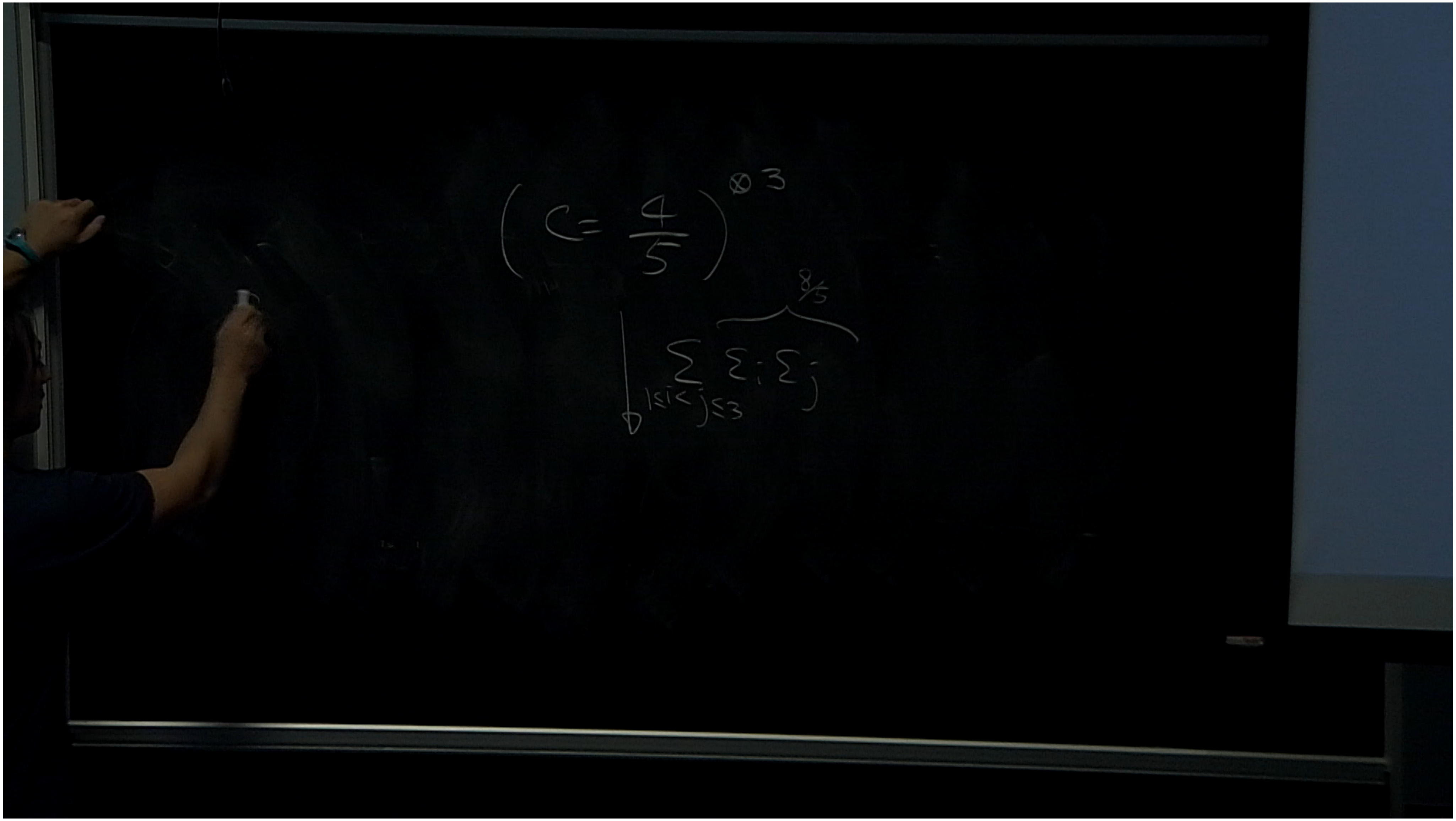
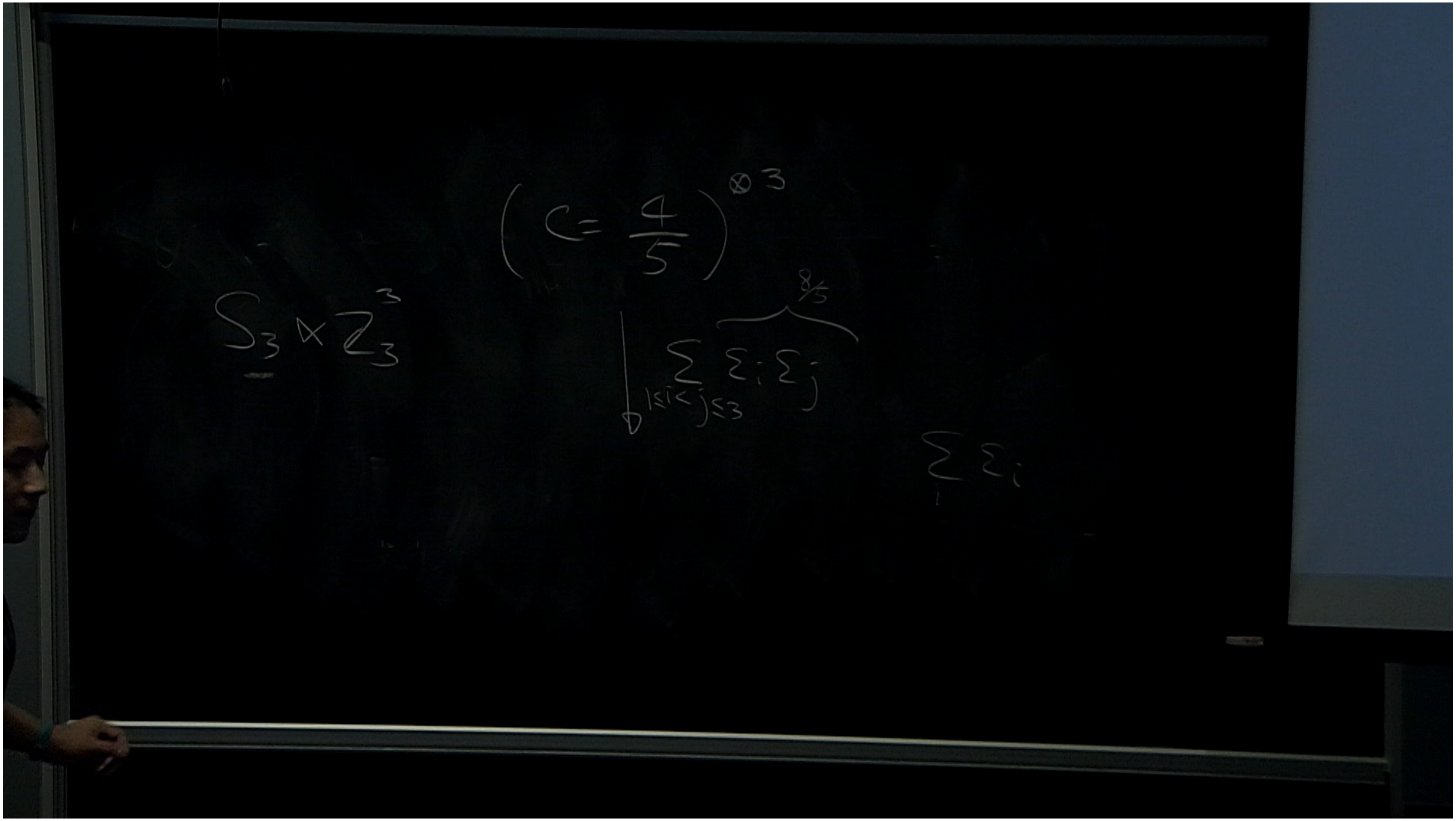
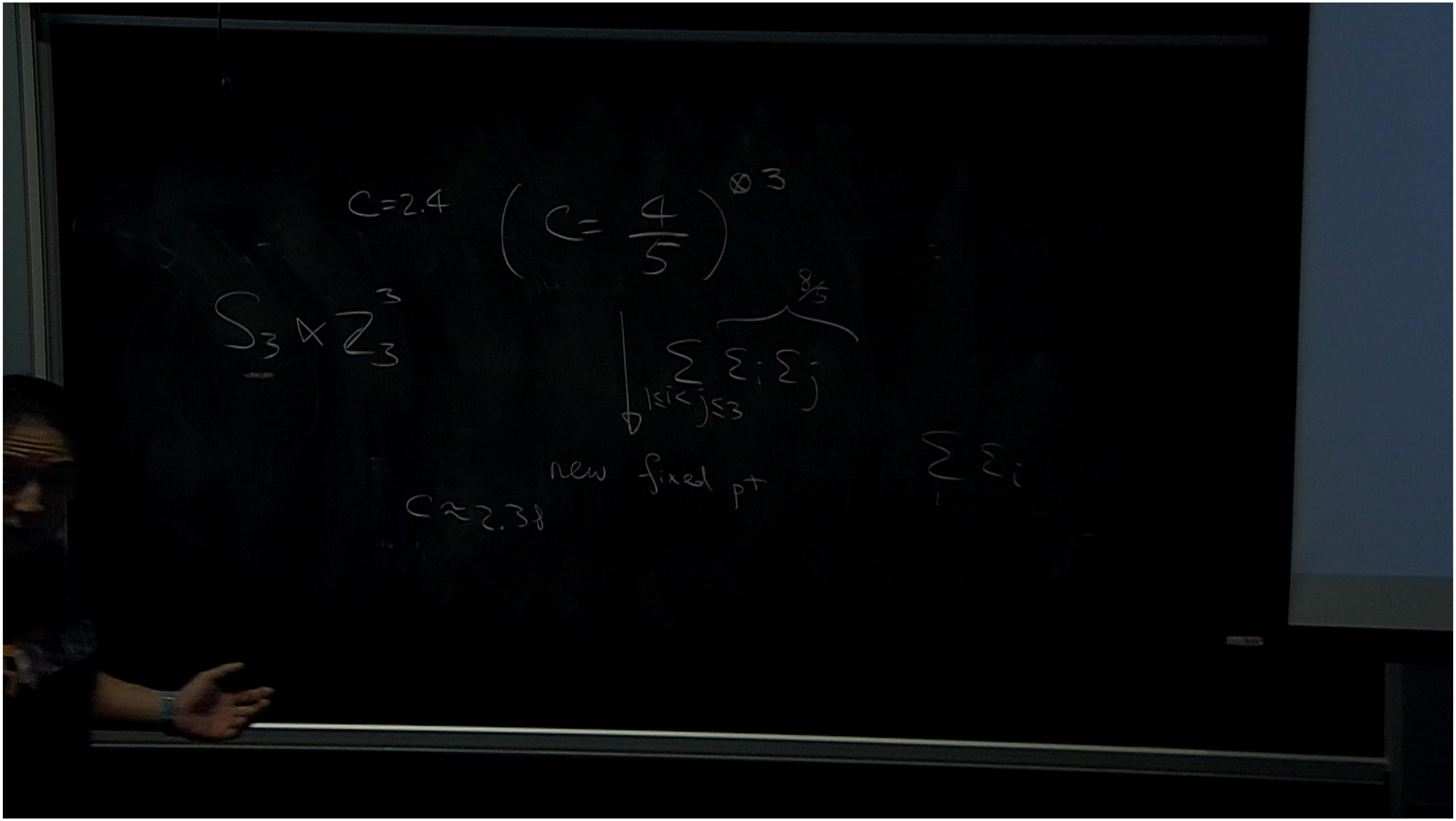
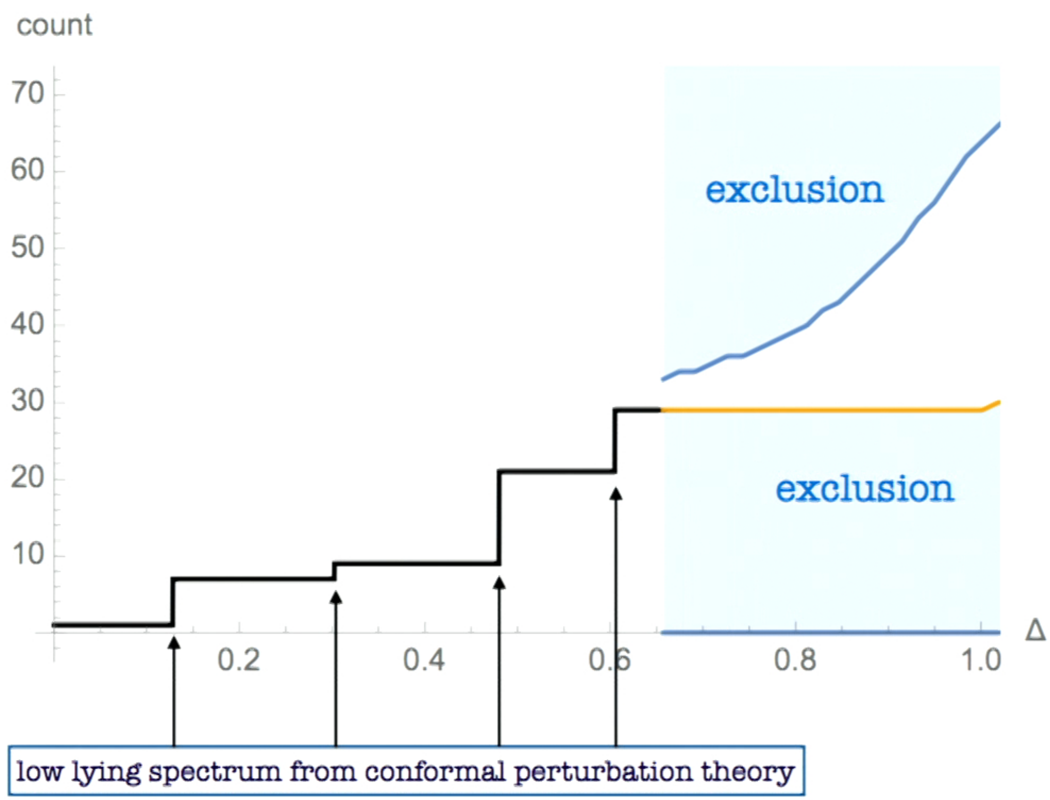

$\left(C = \frac{4}{5}\right)^{\otimes 3}$
$\sum_{1 \le i < j \le 3} \overbrace{\Sigma_i \Sigma_j}^{8/5}$


$S_3 \ltimes \mathbb{Z}_3^3$
$\left(c = \frac{4}{5}\right)^{\otimes 3}$
$\overbrace{\sum_{1 \le i < j \le 3} \varepsilon_i \varepsilon_j}^{8/5}$
$\sum_i \varepsilon_i$


$c=2.4$
$\left(c=\frac{4}{5}\right)^{\otimes 3}$
$S_3 \ltimes Z_3^3$
$8/5$
$\sum_{1\le i<j\le 3} \varepsilon_i \varepsilon_j$
new fixed pt
$c \approx 2.38$
$\sum_i \varepsilon_i$

Primary counting function

Coupled Potts model [Dotsenko et al '98] (preliminary)



Primary counting function

Coupled Potts model [Dotsenko et al '98] (preliminary)