Title: Decoding a black hole

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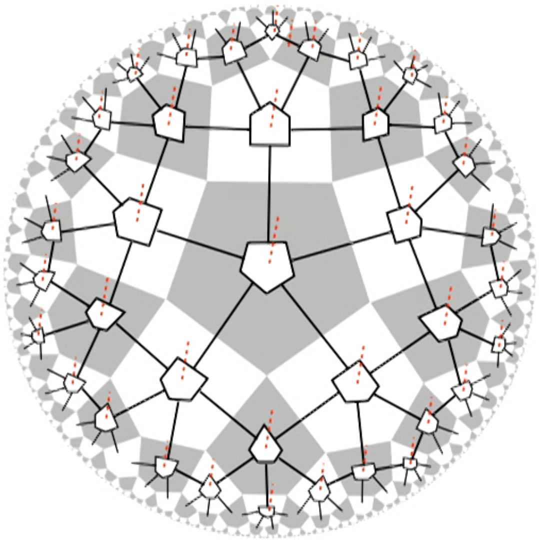
Abstract: <p>It is commonly believed that quantum information is not lost in a black hole. Instead, it is encoded into non-local degrees of freedom in some clever way; like a quantum error-correcting code. In this talk, I will discuss how one may resolve some paradoxes in quantum gravity by using the theory of quantum error-correction. First, I will introduce a simple toy model of the AdS/CFT correspondence based on tensor networks and demonstrate that the correspondence between the AdS gravity and CFT is indeed a realization of quantum codes. I will then show that the butterfly effect/scrambling in black holes can be interpreted as non-local encoding of quantum information and can be quantitatively measured by out-of-time ordered correlations. Finally I will describe a simple decoding protocol for reconstructing a quantum state from the Hawking radiation and suggest a physical interpretation as a traversable wormhole in an AdS black hole. The decoding protocol also provides an attractive platform for laboratory experiments for measuring out-of-time ordered correlation functions as it clearly distinguishes unitary scrambling from non-unitary decoherence. </p>

Decoding a black hole



Beni Yoshida (Perimeter Institute)

@ Perimeter (April 2017)

Collaborators

HEP

QI

QI

Daniel Harlow (Harvard → MIT) | Fernando Pastawski (Caltech → Berlin) | John Preskill (Caltech)

CMT

HEP

???

Xiao-liang Qi (Stanford) | Daniel Roberts (IAS Princeton) | Alexei Kitaev (Caltech)

Thanks to

Patrick Hayden, Pavan Hosur, Michael Walter and many others for discussions …

Quantum gravity meets quantum information ?

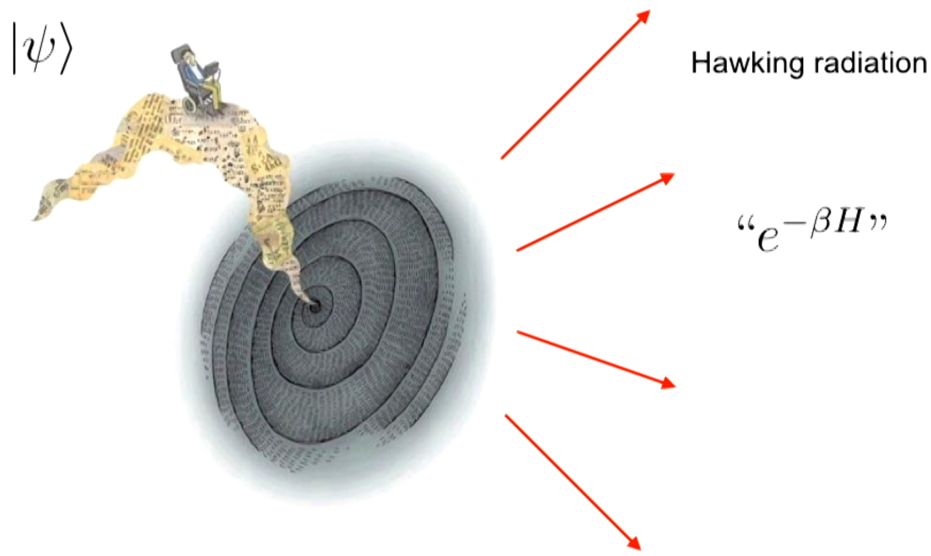
Information loss puzzle

- Quantum mechanics and general relativity are in serious conflicts !

  (a) Quantum mechanics says that information is never lost.

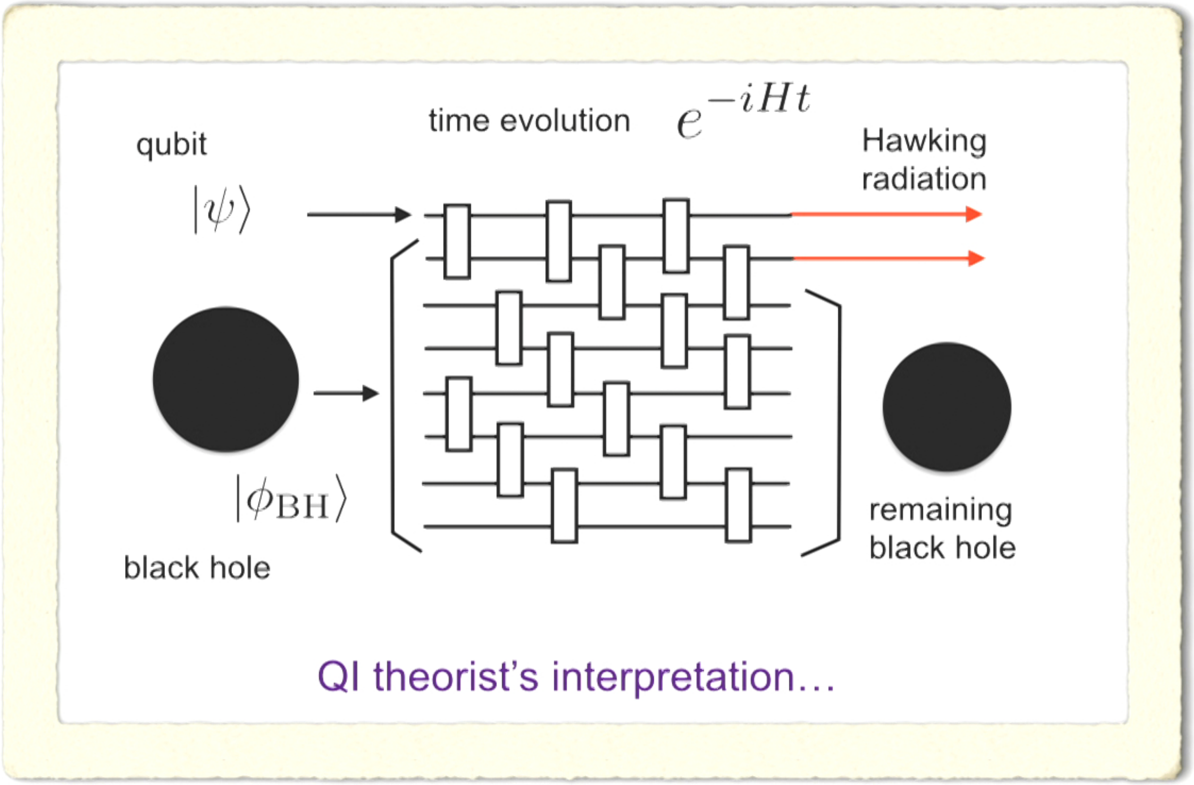
$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

  (b) General relativity says information is lost in black holes.



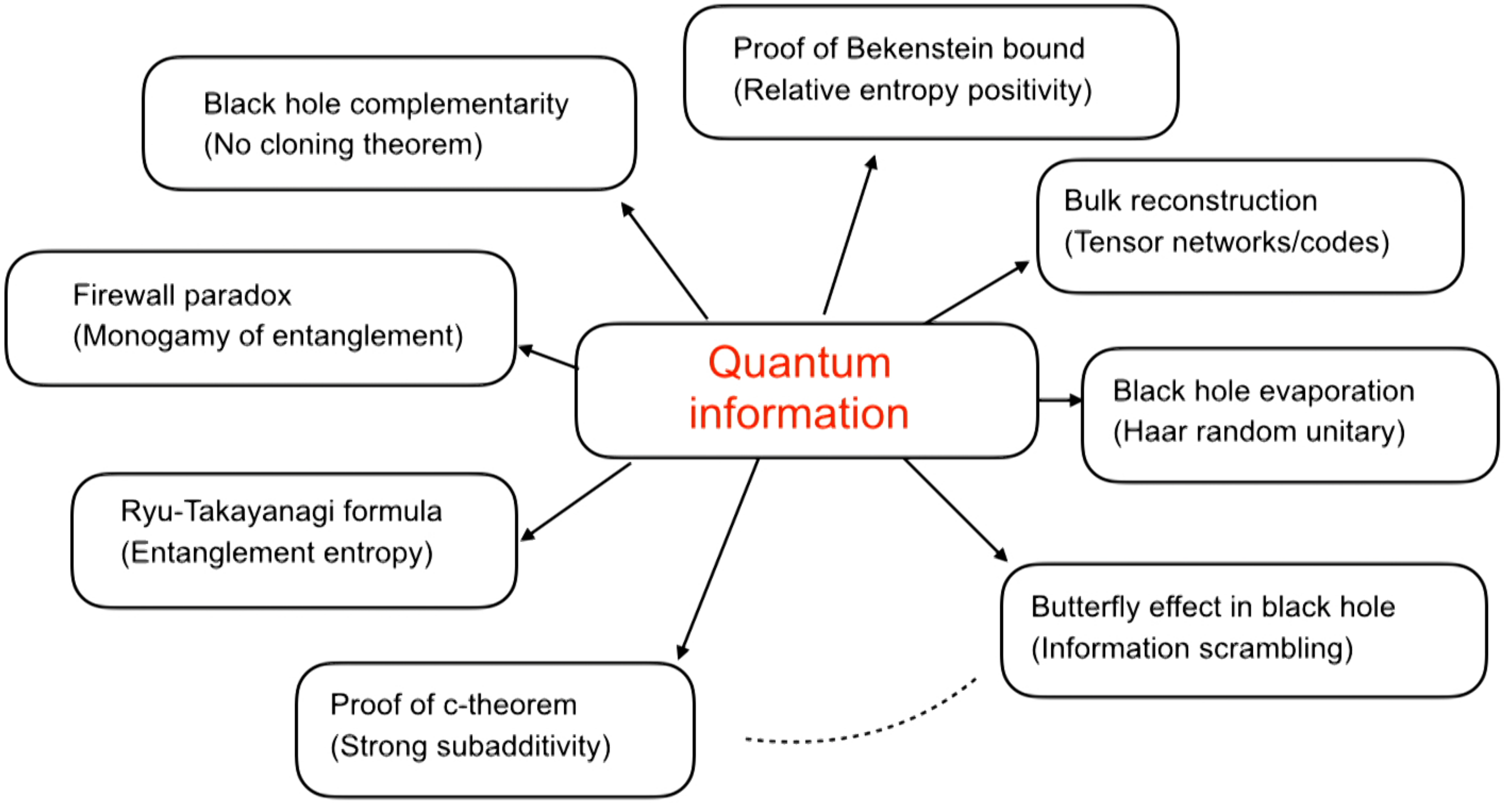
Why quantum information ?

- Information loss puzzle = "Information problem in a quantum system"



Quantum information and HEP

- Many breakthrough ideas in quantum gravity come from quantum information theory



* Definitely there are many other examples.

This talk :

A black hole is a quantum error-correcting code

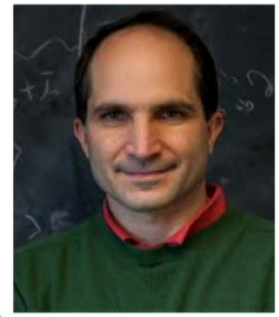
Part 1 :

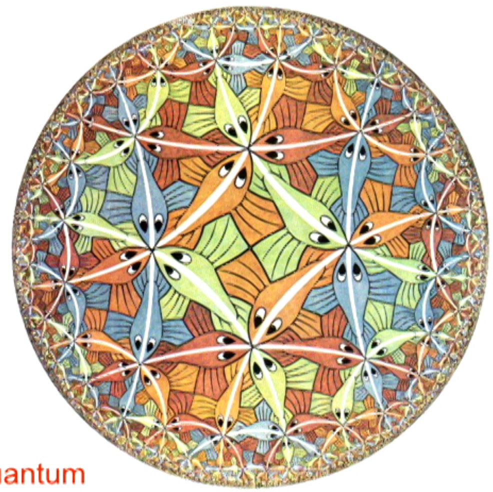
Simple toy model of the AdS/CFT correspondence

(joint with Harlow, Pastawski and Preskill 2015)

Anti-de Sitter/Conformal field theory (AdS/CFT) correspondence

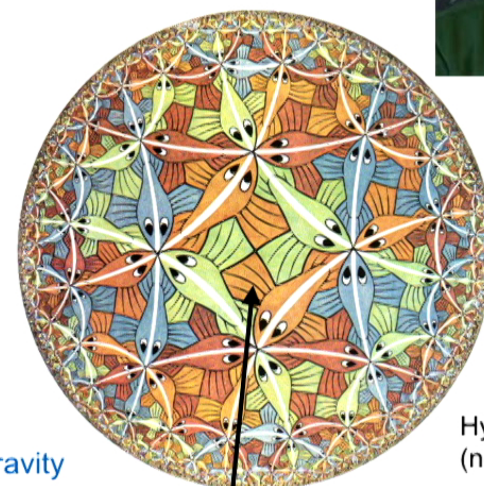
• [Conjecture] Equivalence between string (gravity) theory in bulk and (certain types of) CFT on boundary (Maldacena 1997)





Quantum

Boundary D-dimensional conformal field theory (without gravity)



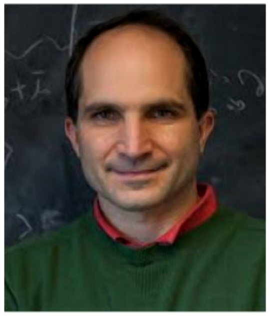
Gravity

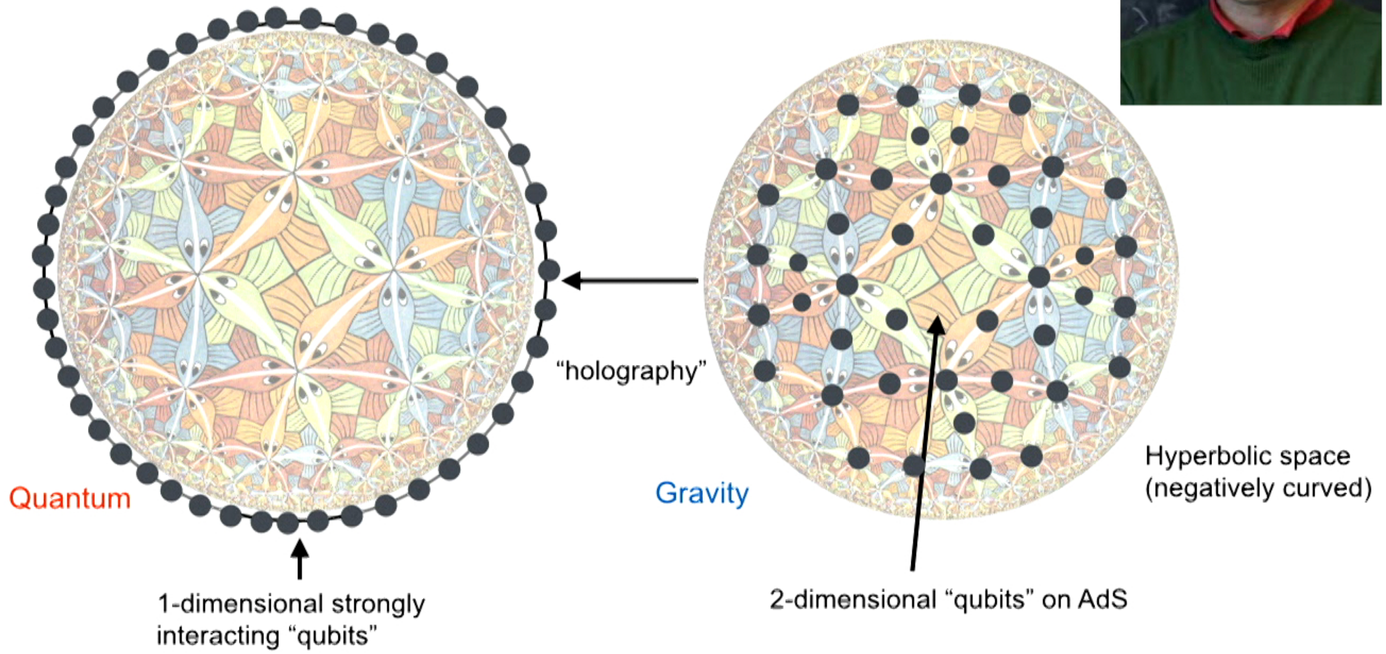
Hyperbolic space (negatively curved)

Bulk (D+1)-dimensional theory with gravity on AdS space

Anti-de Sitter/Conformal field theory (AdS/CFT) correspondence

- [Conjecture] Equivalence between string (gravity) theory in bulk and (certain types of) CFT on boundary (Maldacena 1997)
- [Holography] Bulk degrees of freedom are encoded in boundary, like a hologram.





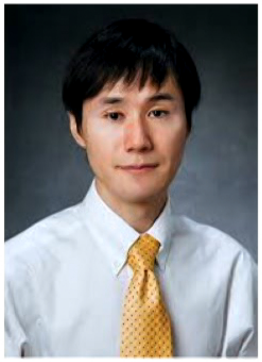
* Finite-dimensional Hilbert space cartoon picture.

Quantum entanglement in AdS/CFT

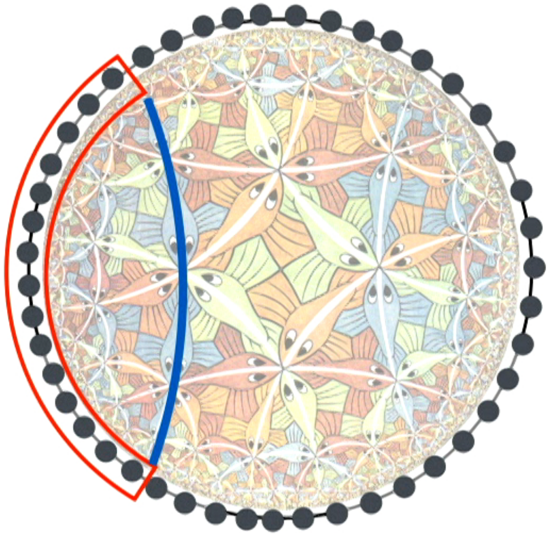
- [Ryu-Takayanagi formula 06]

Quantum → $$S(A) = \frac{1}{4G_N}\min_{\gamma_A}(\mathrm{area}(\gamma_A))$$ ← Gravity

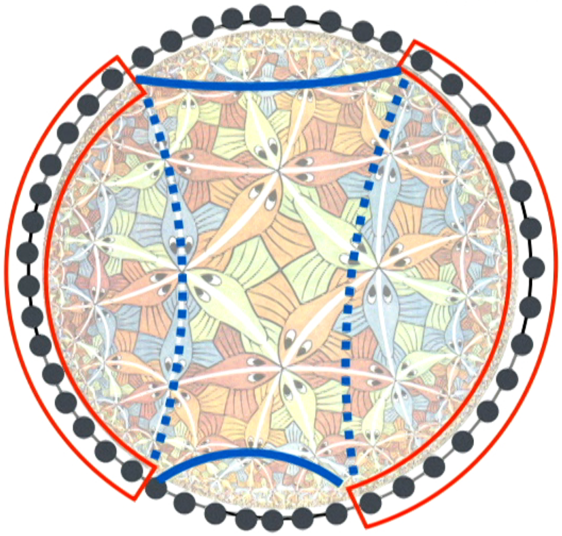
Minimize over spatial bulk surfaces homologous to A



A



A1

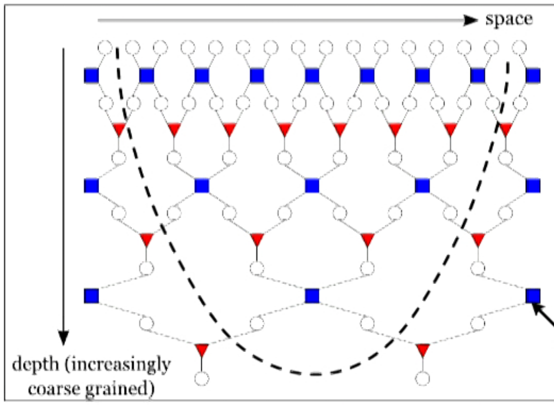


A2

* Leading order in 1/G. * Casini-Huerta-Myers11, Lewkowycz-Maldacena13

MERA (Multiscale entanglement renormalization ansatz)

- Powerful numerical method to study strongly-correlated systems. (Vidal 07)

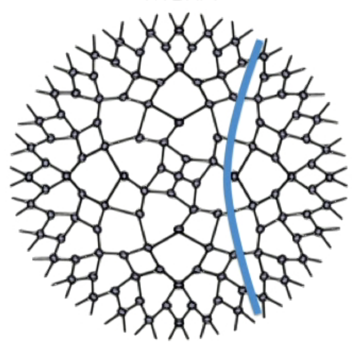


remove short-range entanglement

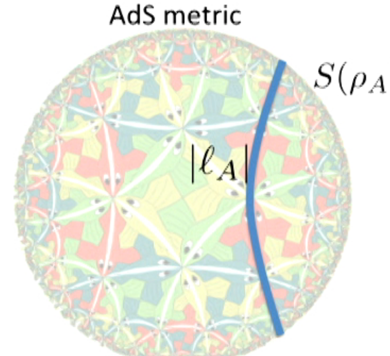
- AdS/CFT correspondence can be explained by a tensor network ?

(Swingle 09)

MERA



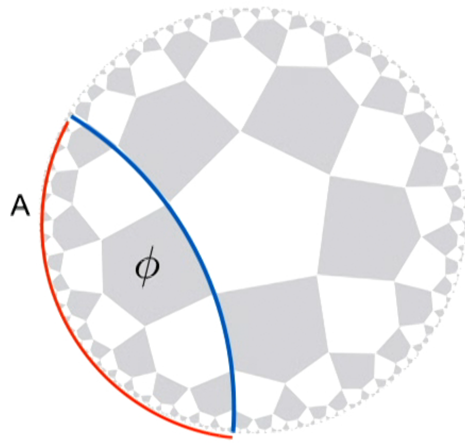
AdS metric



Bulk operator vs boundary operator

- [Entanglement wedge reconstruction]

A bulk operator ϕ can be represented by some integral of local boundary operators supported on A if ϕ is contained inside the entanglement wedge of A.



* HKLL for single intervals * Conjectured for multiple intervals by Aron Wall.

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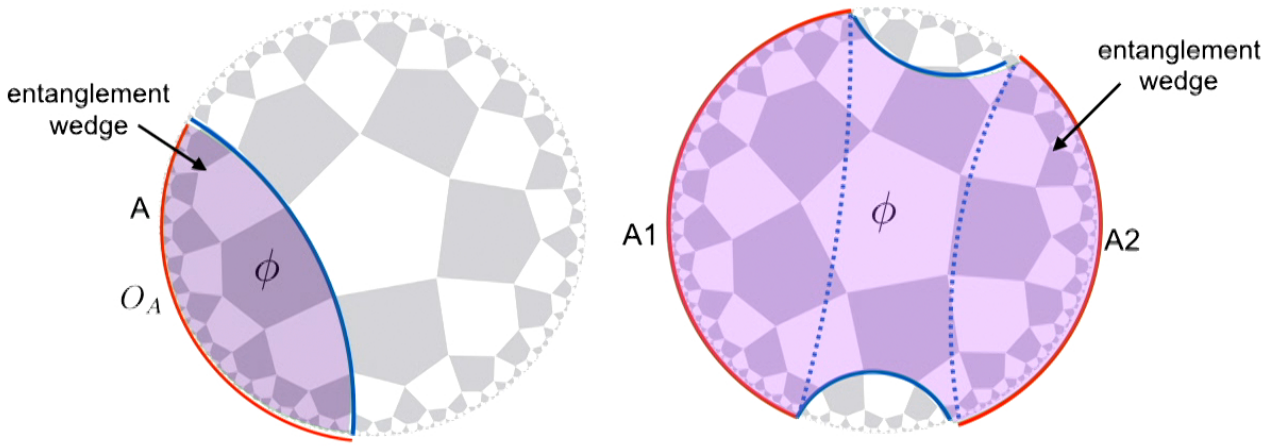


$\phi \sim O_A$ $\phi \sim O_{A_1 A_2}$

Remarks for experts

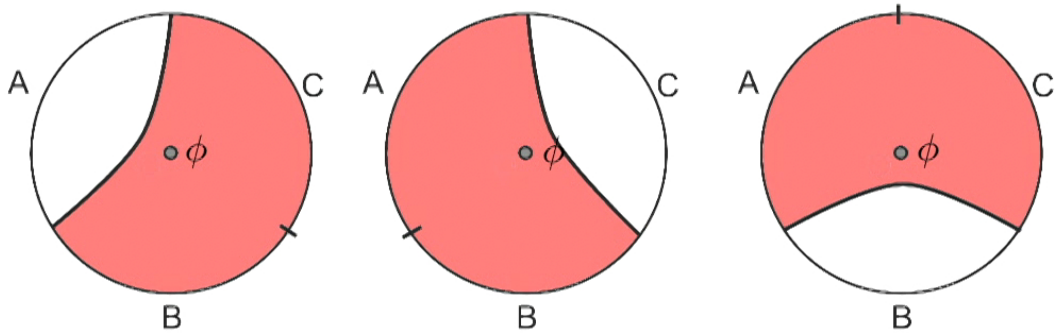
- Entanglement wedge may go beyond black hole horizons (i.e. no firewall ?).
- "Proven" by using a "generalized" RT formula (Jefferis et al, Dong et al, Bao et al 2016)
- No explicit recipe is known for more than one intervals (AdS3) (For higher-dimensions, it's a bit more subtle).

* HKLL for single intervals * Conjectured for multiple intervals by Aron Wall.

Bulk locality paradox

- The reconstruction leads to a <u>paradox</u> !

[Almheiri-Dong-Harlow 14]



* Uses Schur's lemma, assuming finite-dimensional factorizable Hilbert space

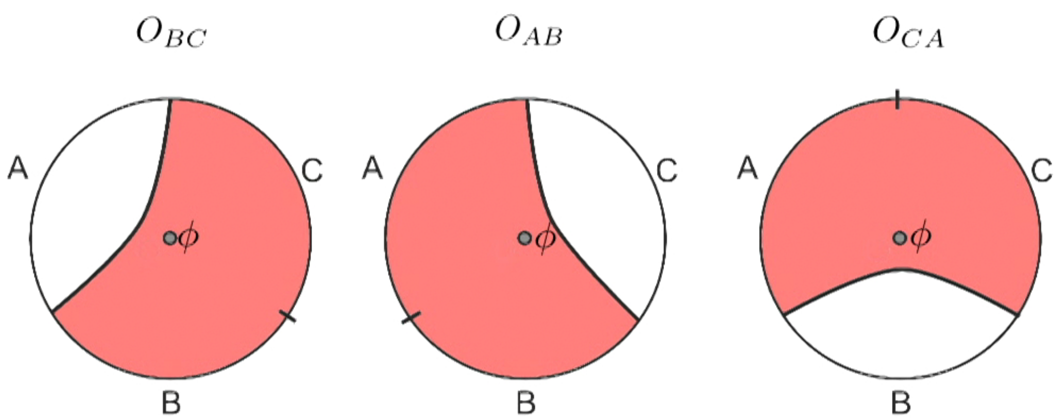
Quantum error-correction in AdS/CFT ?

- The AdS/CFT correspondence can be viewed as a quantum error-correcting code !

  These operators may be different, but act in the same manner in a low energy subspace.

  Recall string operators in lattice gauge theory (or Z_2 spin liquid).

- Quantum secret-sharing code: Alice, Bob and Charlie share a quantum secret.
- Error-correction: Quantum information is protected against erasure of one party.



@ Caltech, 2014 November

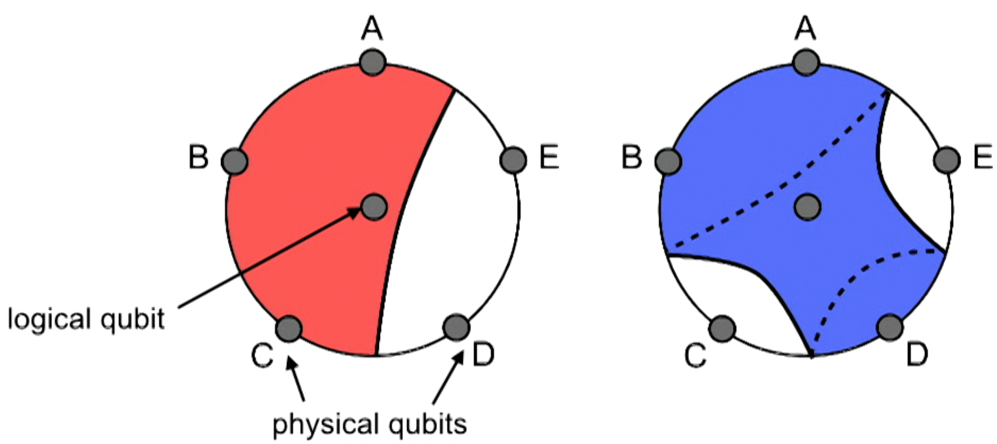
A simple toy model

1 bulk qubit

5 boundary qubits

in total, just 6 qubits

A bulk operator must have corresponding boundary operators on any region with three qubits.



Five-qubit quantum code

- The "simplest" quantum code which is made of qubits.

$$|\xi\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow |\xi\rangle = \alpha|c_0\rangle + \beta|c_1\rangle$$

1-qubit input state — 5-qubit output state

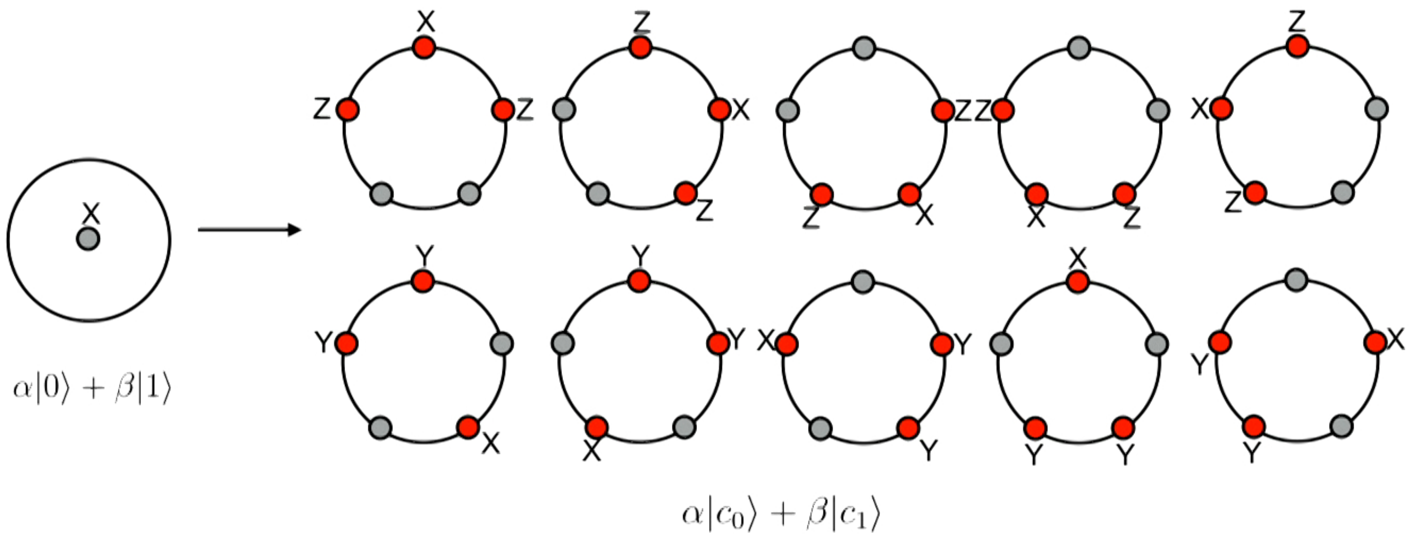
where

$$\begin{aligned}
|c_0\rangle = |00000\rangle \\
&+ |11000\rangle + |01100\rangle + |00110\rangle + |00011\rangle + |10001\rangle \\
&- |10100\rangle - |01010\rangle - |00101\rangle - |10010\rangle - |01001\rangle \\
&- |11110\rangle - |01111\rangle - |10111\rangle - |11011\rangle - |11101\rangle \\
|c_1\rangle = |11111\rangle \\
&+ |00111\rangle + |10011\rangle + |11001\rangle + |11100\rangle + |01110\rangle \\
&- |01011\rangle - |10101\rangle - |11010\rangle - |01101\rangle - |10110\rangle \\
&- |00001\rangle - |10000\rangle - |01000\rangle - |00100\rangle - |00010\rangle
\end{aligned}$$

[DiVincenzo-Shor, Laflamme-Miquel-Paz-Zurek 1996]

Operator correspondence in five-qubit code

- Pauli X operator in the input (bulk) corresponds to following 3-body operators.

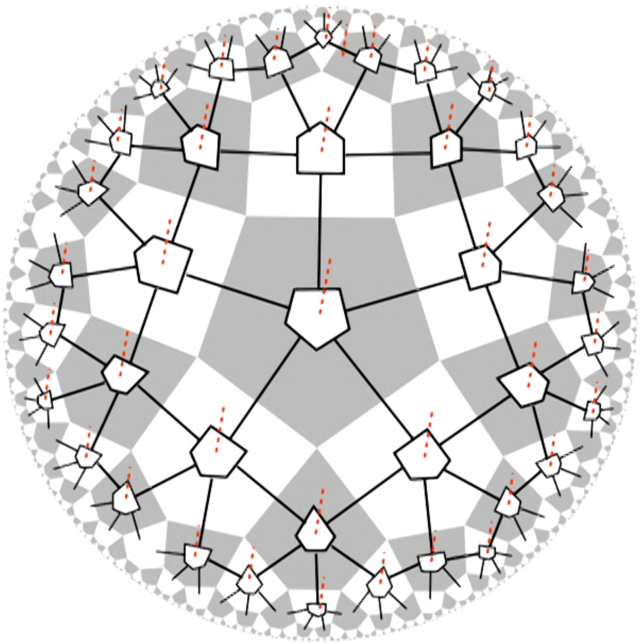


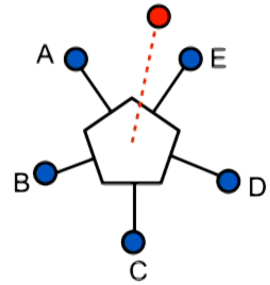
- So, the five-qubit code is a very small toy quantum gravity !

- Error-correction : losing 2 qubits is OK.

A holographic quantum error-correcting code

• A tiling of the five qubit code via tensor network technique



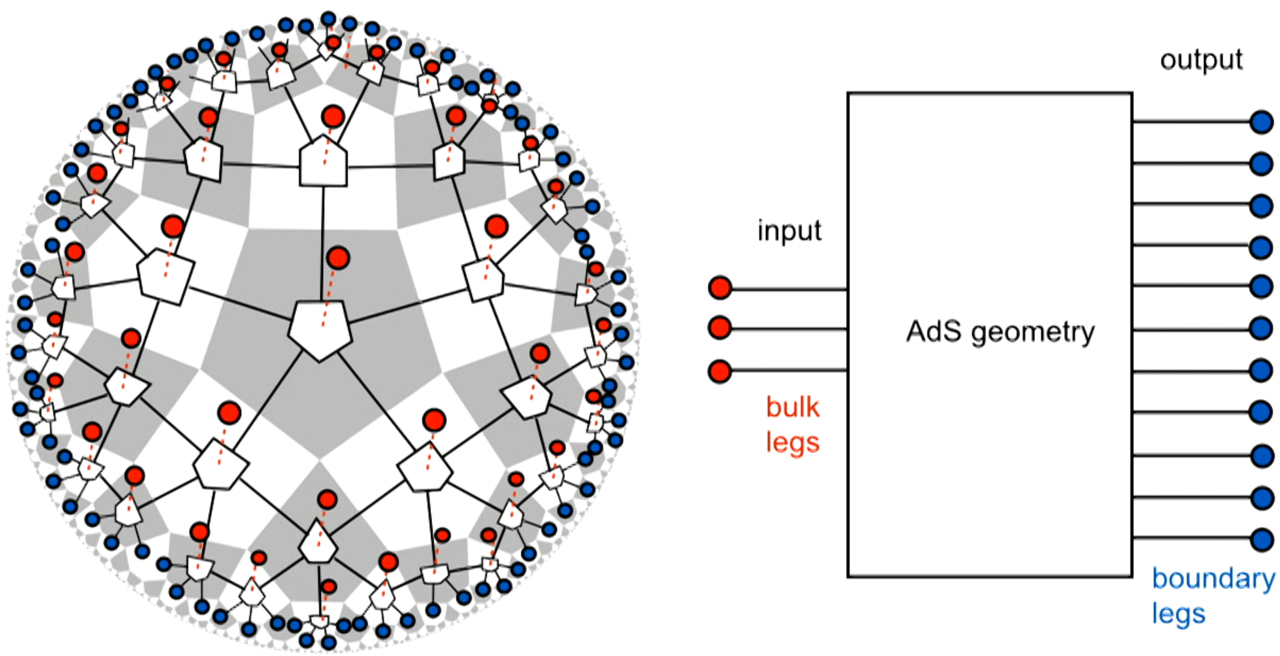


five-qubit code

* Tensor network and AdS/CFT : Vidal07, Swingle12, Qi13, Czech et al15 …

A holographic quantum error-correcting code

- A tiling of the five qubit code via tensor network technique

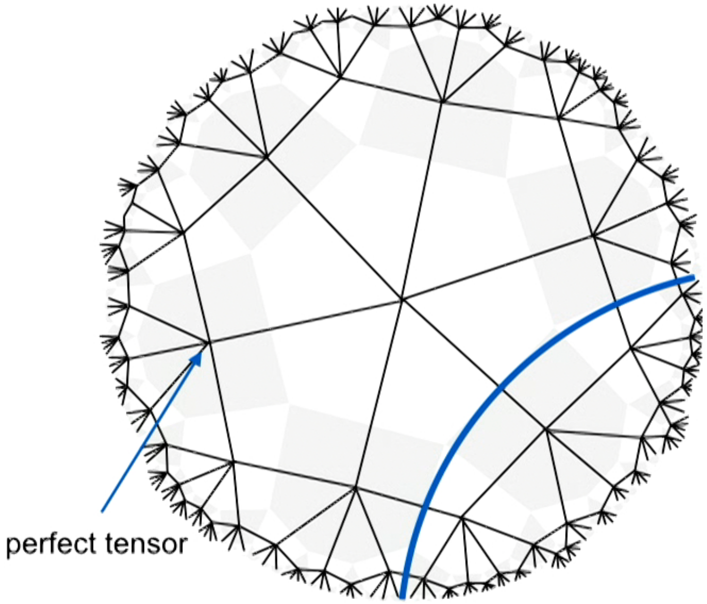


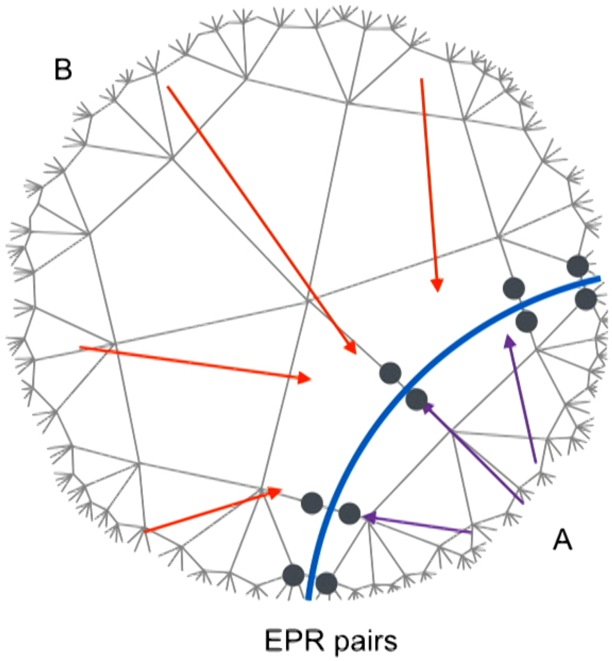
* Tensor network and AdS/CFT : Vidal07, Swingle12, Qi13, Czech et al15 …

Holographic state

- The Ryu-Takayanagi formula holds (tiling without bulk legs)

Coarse-graining (RG transformation) = Distillation of EPR pairs along the geodesic



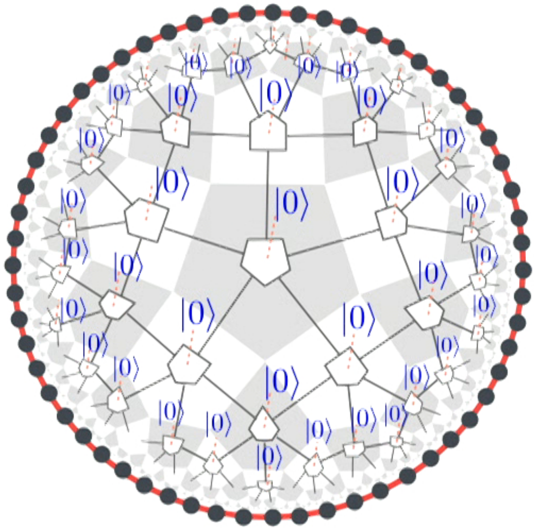


Physical interpretation: Matter and Geometry

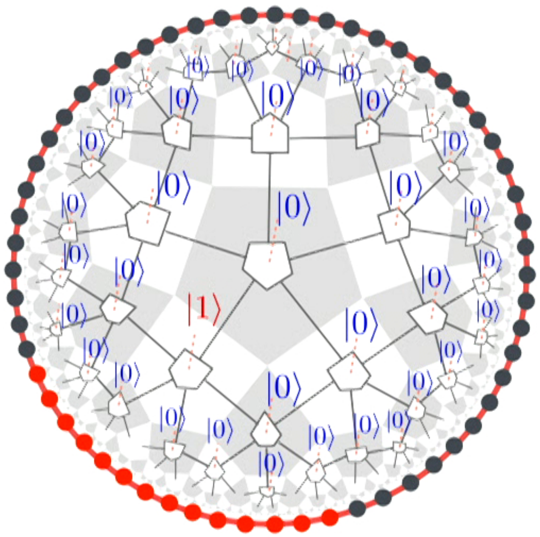
- Qubits on the bulk represent matter fields, coupled to the geometry (via 5-qubit code tensors).
- Bulk Hilbert space $\mathcal{H}_{\text{bulk}}$ is much smaller than boundary Hilbert space $\mathcal{H}_{\text{bdy}}$.

  ⟶ The model captures "perturbations" around a fixed geometry.

- Going outside the codeword space = changing geometries (eg. micro black holes)



Vacuum



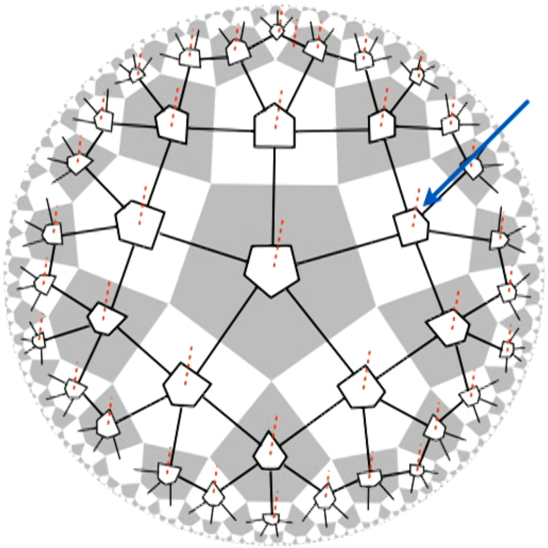
Excited state

Space-time from "something random" ?

- One can use randomly chosen tensors to do the same thing !

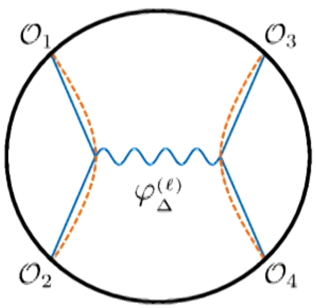
  A random state is nearly maximally entangled along any bipartition (eg: Page's argument, Canonical typicality).

- Analytical solution of random tensor network [Hastings 15, Hayden et al 16]



Pick random tensors at each AdS scale patch

Interesting "1/N" behaviours (2^N: spin dimensions)



Aspects of holographic tensor networks

* Toy models of black holes (smoothness of the horizon? interior of black holes?)

* Complexity = Volume = Action? [Susskind and his friends] (number of tensors = complexity?)

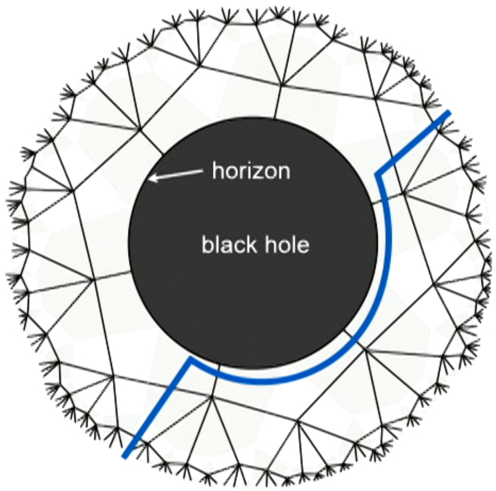
* Generalizations: symmetries, “matrix QM”-like network, sub-AdS locality, kinematic space

* Tensor network in de Sitter space ? (Dark matter problem, as mentioned by Eric Verlinde)

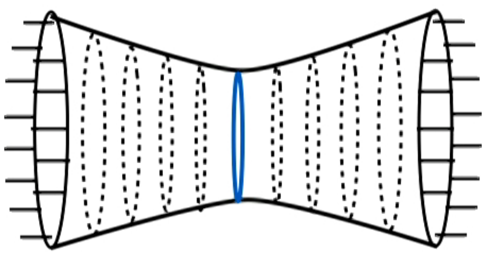
* Difficulty: No Hamiltonian, No dynamics. Need more inputs to be more realistic…

* Experimental realizations ? (eg superconducting qubits)

One-sided black hole



Two-sided black hole



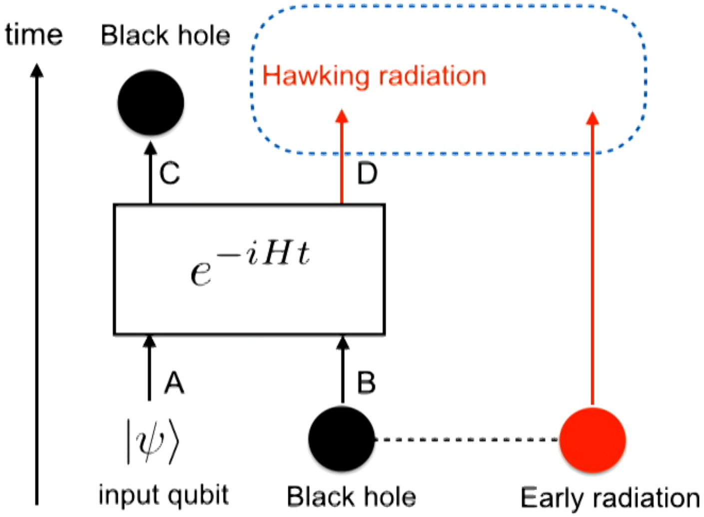
Einstein-Rosen bridge

Part 2 :

Black hole is a quantum error-correcting code

Hayden-Preskill thought experiment

- Suppose that a black hole has already radiated a half of its content
- Bob needs to collect only a few O(1) qubits in order to reconstruct Alice's quantum state.



Black hole reflects a quantum information like a mirror !

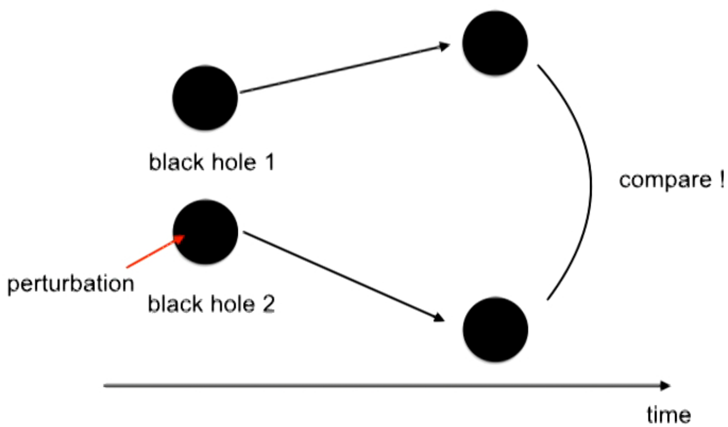
The mirror phenomena occurs only when U is scrambling !

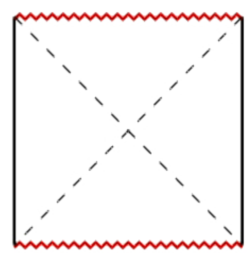
→ Chaos implies easy reconstruction ??

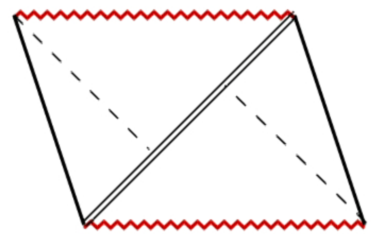
Quantum chaos in black hole

- Shenker-Stanford: Butterfly effect in a black hole
- Perturbation becomes a gravitational shockwave
('t Hooft-Drey)

→ Standard AdS/CFT calculation with shockwave geometry







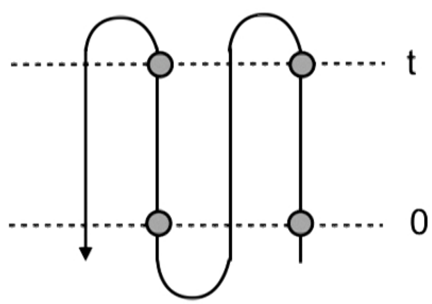
Out-of-time ordered correlation functions

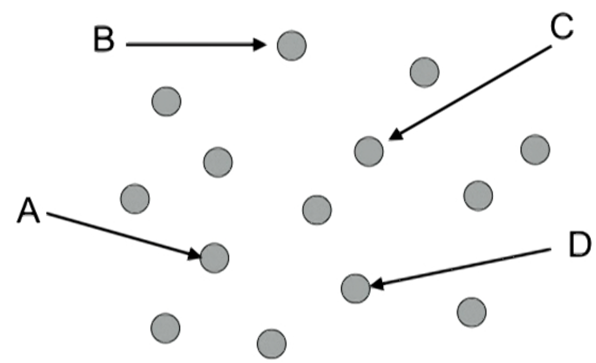
- Black holes have some "hidden" correlations (Kitaev)

$$\mathrm{OTOC} = \langle O_A(0) O_B(t) O_C(0) O_D(t) \rangle = 1 - \frac{1}{n} e^{\lambda t} \qquad \lambda = \frac{2\pi}{\beta}$$

$$O_B(t) = e^{-iHt} O_B(0) e^{iHt}$$
$$O_D(t) = e^{-iHt} O_D(0) e^{iHt}$$

$$t \sim \frac{\beta}{2\pi} \log(n)$$





- SYK model, AdS2 gravity, fast scrambling…

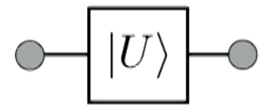
• Previously considered by Larkin and Ovchinikov in 1960s

State-Channel duality

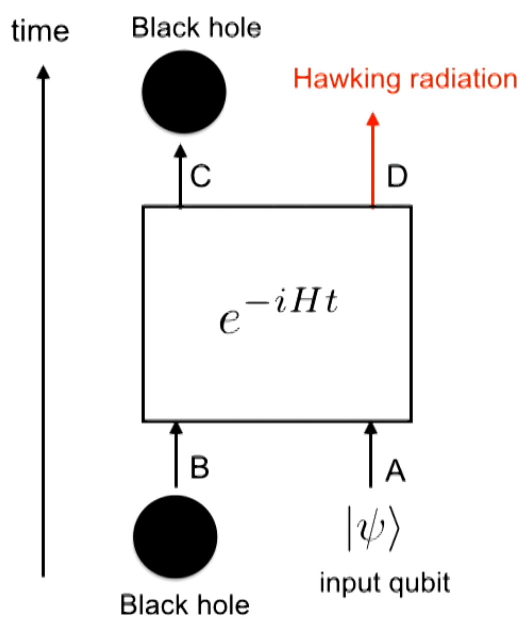
- Unitary operator acting on n qubits can be viewed as a state on 2n qubits.

$$U = \sum_{i,j} U_{ij} |i\rangle\langle j|$$

$$|U\rangle = \sum_{i,j} U_{ij} |i\rangle \otimes |j\rangle$$

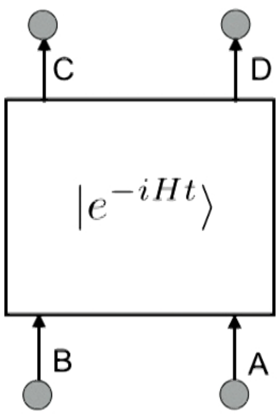


- Viewing the black hole dynamics as a quantum state.



Entanglement between input and output

eg. between A and D = radiation and input



[Choi-Jamilkowski 1960s, Hayden-Preskill,Hartman-Maldacena]

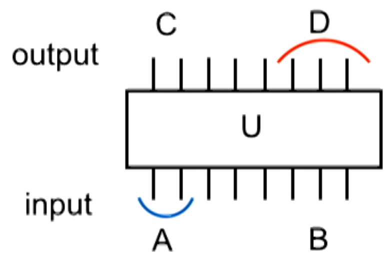
OTOC and entanglement

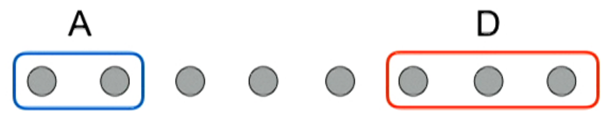
- Average of OTO over local operators A and D at T=infty [Hosur-Qi-Roberts-BY]

$$|\langle O_A(0)O_D(t)O_A(0)O_D(t)\rangle|_{\text{ave}} = 2^{n-a-d-S^{(2)}_{BD}}$$

Renyi-2 entropy

average over O_A, O_D



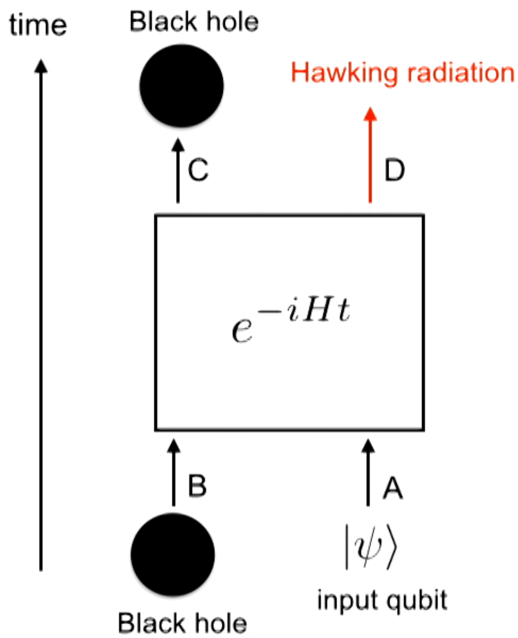


- If $\text{OTO} \simeq 0$ then, $S^{(2)}_{BD}$ is large

This implies the mutual information $I^{(2)}(B,D) = S^{(2)}_B + S^{(2)}_D - S^{(2)}_{BD}$ is small

- If $\text{OTO} \simeq 0$ then, $I^{(2)}(A,C) = S^{(2)}_A + S^{(2)}_C - S^{(2)}_{AC}$ is also small.

Quantum error-correction in black holes



- Hawking-Unruh semiclassical calculation

$$\langle O_A(0) O_A(t) \rangle \to 0$$

$$\Rightarrow \quad I(A, D) \to 0$$

- Kitaev-Shenker-Stanford shockwave calculation

$$\langle O_A(0) O_D(t) O_A(0) O_D(t) \rangle \to 0$$

$$\Rightarrow \quad I(A, C) \to 0$$

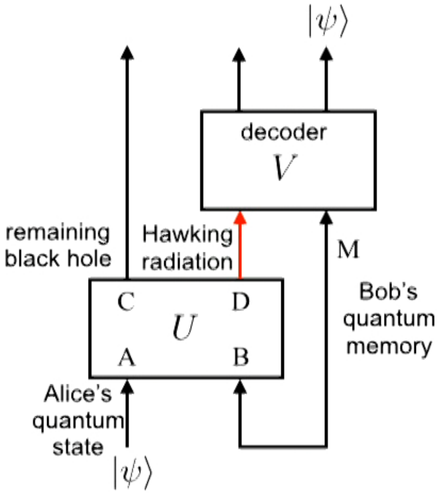
So, a black hole is a quantum error-correcting code.

- OTOCs on A and D can learn about C. We can study "hidden" degrees of freedom.

OTOCs and Hayden-Preskill

- Bob holds a quantum memory M which is maximally entangled with a black hole

- For Alice's m-qubit quantum state, collecting m+e qubits of Hawking radiation is enough to reconstruct the state.



Bob has an access to

B : Black hole initial state

D : Hawking radiation

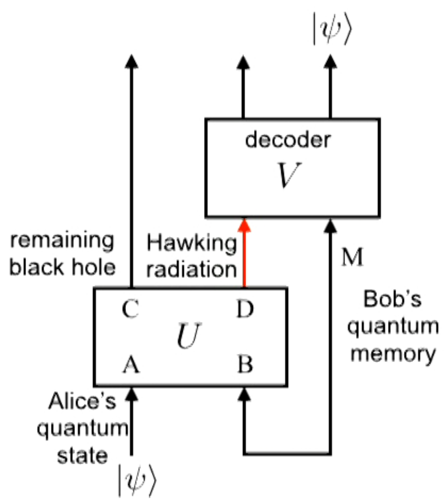
$$-\log_2 |\langle O_A(0)O_D(t)O_A(0)O_D(t)\rangle| = I^{(2)}(A, BD)$$

[Roberts-BY16]

Small OTOC ⟶ Bob's success

Decoding complexity ?

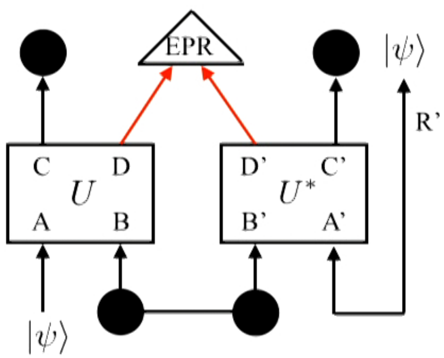
- It is *information theoretically* possible, but…
  - Is V physically implementable ?
  - Is finding V computationally tractable ?



(Note: Hayden-Preskill setup is different from AMPS setup)

Probabilistic decoding protocol

- A very simple decoding protocol, works probabilistically. [Kitaev-BY in prep]
- Bob "teleports" Alice's quantum state to his register qubits via postselection.



Protocol

- Bob prepares an EPR pair, feed one qubit into quantum memory and keep the other as a register
- Bob implements the complex conjugate U*
- Bob collects a pair of the Hawking radiation from two sides of an entangled black hole.
- Bob performs Bell measurements

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \quad \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \quad \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle).$$

- If he measured an EPR pair, he immediately obtains a faithful reconstruction of Alice's state.

(The protocol works as long as U is strongly scrambling)

Decoding and out-of-time ordered correlators

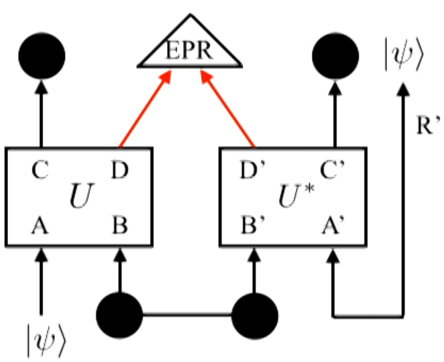
- Success probability can be computed from OTOCs.

$$P_{\mathrm{EPR}} = \langle \mathrm{OTOC} \rangle_{\mathrm{ave}}$$

- Decoding fidelity can be also computed from OTOCs.

$$F_{\mathrm{EPR}} = \frac{1}{|A|^2 \cdot \langle \mathrm{OTOC} \rangle_{\mathrm{ave}}}$$

- Decoding fidelity is sensitive to “unitarity” of U.

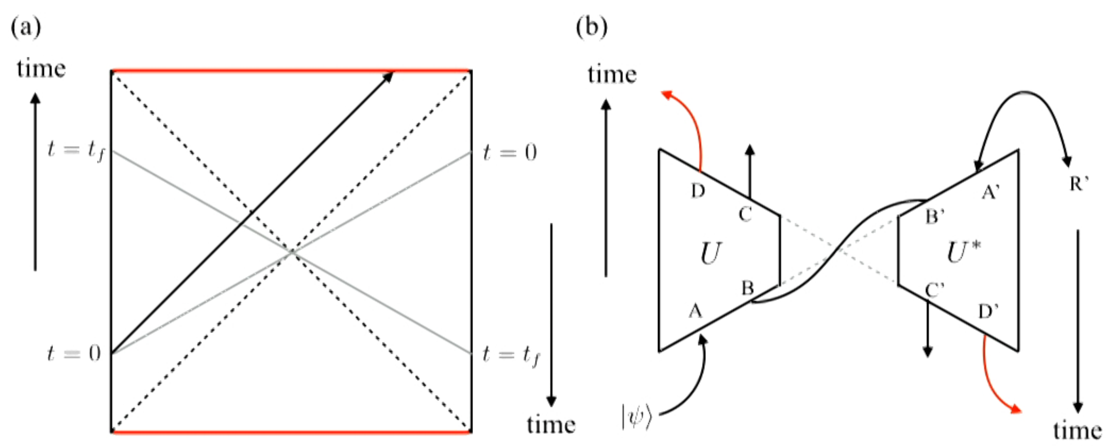


We can distinguish unitary scrambling from non-unitary decoherence.

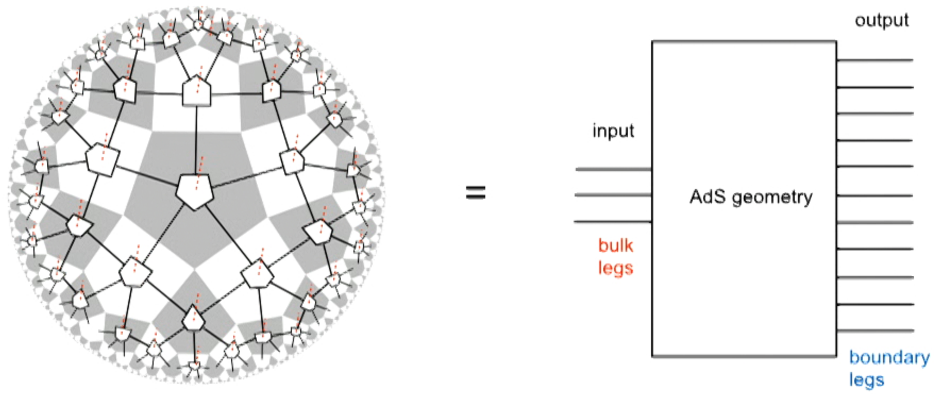
Ongoing discussions with experimentalists !

Traversable wormhole in AdS black hole

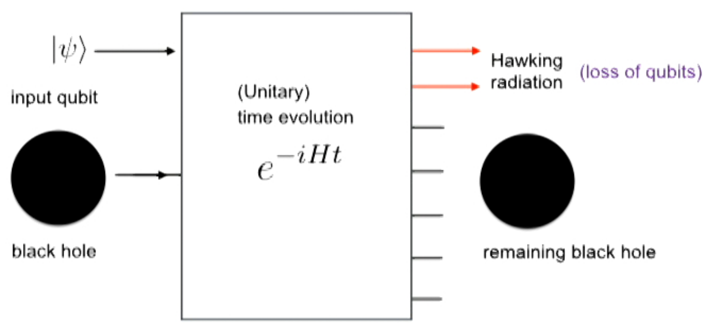
- Decoding protocol can be viewed as a traversable wormhole in AdS black hole.
- Gao-Jafferis-Wall (and followup work by Maldacena-Stanford-Yang) showed that AdS black hole becomes traversable via double trace deformation coupling both sides.
- Update the black hole complementarity.
- Average null energy condition (ANEC), 2nd law of thermodynamics…
- Resolution of firewall paradox ?



Part 1: AdS/CFT correspondence is a quantum error-correcting code.



Part 2: Black hole is a quantum error-correcting code.



$$\langle O_A(0)O_D(t)O_A(0)O_D(t)\rangle$$